Optimal Rule Enforcement*

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Most recent draft here.

Abstract

While rules are often seen as rigid, their enforcement can be flexible. We develop a model to study the optimal enforcement of rules in an employment relationship, identifying two categories of relational contracts: manager-led enforcement and worker-led accountability. The manager-led approach, involving cycles of rule enforcement, arises when enforcement costs are low. The worker-led approach, used when enforcement costs rise and surplus grows, relies on worker self-regulation, with the threat of termination motivating effort. In a subcategory of the worker-led approach, rules are enforced initially, before letting the worker to self-regulate.

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> "Knowing when to bend the rules is one of the hallmarks of an experienced decision maker."
>
> —James March, 1994. *A Primer on Decision Making*.

1 Introduction

The management literature is littered with criticisms of rules. They lead to rigidity, stifle innovation, and create unnecessary bureaucracy. Reed Hastings, the founder of Netflix, went so far as to famously advocate for a no-rule-rule approach, suggesting that abolishing rules unleashes creativity and autonomy within organizations (Hastings and Meyer 2020).[1] Despite the criticisms, rules persist in most organizations because they fulfill essential functions. Rules provide structure, ensure consistency, and create a framework that aligns actions with organizational goals. Without rules, organizations risk descending into disorder (Turco 2016).

Although rules are often thought of as rigid and hard constraints, in practice they are often more flexible than they appear. The enforcement of rules is rarely automatic or uniform; rather, it is carried out by managers who exercise discretion in deciding when and how to apply them. Importantly, enforcing rules involves both tangible and intangible costs. Managers must deploy resources to monitor and ensure compliance and often face psychological and relational costs when disciplining employees. Any parent who has enforced a rule with their children can attest to the emotional toll involved.

The dilemma that managers (or any rule enforcers) face is that they prefer not to enforce the rule as long as the workers provide good behavior. But if the rule is not enforced, then the workers have incentive to shirk and lower their performance. The key challenge is to exercise discretion (to lower the enforcement cost) while maintaining a functioning organizational environment.

An example of selective rule enforcement is described in the classic work of Gouldner (1954), who studies the behaviors of workers and managers in a mining company. The company had a "no-floating around" rule, specifying that "workers must stay at their workplace, except to go to the washroom or to eat". But managers did not strictly follow this rule. When they felt that everything was going smoothly, they would allow workers to walk around to smoke or talk to a friend. Only when the manager felt that the worker was not performing well did the manager invoke the rule. In Goulder's framework, "formal rules

[1] An earlier famous example of minimal rules is Nordstrom, which states in its employee handbook that "Rule #1: Use good judgment in all situations. There will be no additional rules" (Spector and McCarthy 2012).

gave supervisors something with which they could 'bargain' in order to secure informal cooperation from workers" (Gouldner 1954, p. 173). In other words, the actual enforcement of rules arise out of a relational contract between the manager and the workers.

In this paper, we formally examine the dynamics of rule enforcement through the lens of a repeated interaction between a middle manager (she) and a worker (he). At the beginning of each period, the manager has the option to enforce a rule that requires the worker to exert effort. However, enforcing the rule comes at a cost to the manager[2]. If the manager chooses not to enforce the rule, the worker is free to decide whether to put in effort. The workers effort cost fluctuates over time and is determined by the state of the world, which is only known to him. When the worker's effort cost is high, it is efficient that the worker does not put in effort. Finally, each party can choose to leave the relationship at the end of each period.

We characterize the optimal relational contracts, which fall into two distinct categories. The first category can be described as the "rule-based" approach. In this case, the manager actively manages the relationship by enforcing rules after the worker's behavior has deteriorated. However, this enforcement is not permanent. After a period of strict rule enforcement, the manager eventually relaxes the rules, allowing the worker to return to his previous behavior: exerting effort only when his (privately observed) effort cost is low. Over time, the workers performance will naturally decline again due to the stochastic nature of the effort cost, causing the manager to enforce the rule. Thus, under the rule-based approach, the relationship cycles between periods of strict rule enforcement, which limits flexibility and adaptability, and periods of relaxed enforcement, during which the worker may slack off. This cyclical pattern reflects the inherent trade-off between maintaining control through rules and allowing for adaptation and discretion.

The cyclical pattern of rule enforcement is closely aligned with observations from various organizational settings. For example, in the mining company studied by Gouldner, managers only enforced rules after a noticeable decline in worker performance. This approach to management, where rules are imposed when productivity or behavior slides, mirrors the "rule-based" pattern we describe. Managers allow for a certain degree of flexibility until performance dips below acceptable levels, at which point enforcement becomes necessary to restore order.

This cyclical pattern also resonates with the governance model in China. Scholars have documented a phenomenon known as "campaign-style mobilization" as a mechanism of governance, wherein the state periodically mobilizes resources and intensifies enforcement

[2]A familiar example for many readers might be that a professor displaying a "No Phones Allowed" slide at the start of each class.

(Zhou 2022). These mobilization periods can roughly be understood as periods of strict rule enforcement, followed by phases of relaxation. (cite papers by Xiaodong in footnote.) Much like in organizational settings, this ebb and flow of enforcement in governance reflects an adaptation to fluctuating circumstances.

Although scholars have long criticized this pattern, described in Chinese as *yi-guan-jiu-si, yi-fang-jiu-luan*, meaning "control leads to stagnation, freedom leads to disorder", these patterns are simply unavoidable. Enforcing rules too rigidly stifles flexibility, while relaxing them too much leads to disorder. The adherence to either strict control or relaxed flexibility is difficult to maintain over the long run, but this cyclical approach may, in fact, be the best that managers or governments can achieve under the constraints they face. The cyclical pattern thus reflects an ongoing negotiation between these two extremes, allowing for a dynamic, if imperfect, equilibrium.

The second category of relational contracts is the "exit-based" approach. In this case, managers do not rely on rule enforcement in the long run to maintain order. Instead, the primary mechanism for ensuring effort from the worker is the threat of termination, or *exit*. The possibility of being fired serves as a disciplinary tool, motivating the worker to manage their own behavior without the need for constant oversight or rule enforcement by the manager.

Under this approach, the worker may initially choose not to exert effort when the cost of doing so is high, knowing that there is some tolerance for this in the short term. However, after a period of lower effort - due to high effort costs - the worker enters a phase where they begin to exert effort again, even when the cost of effort remains high. This change in behavior stems from the worker's understanding that prolonged slacking will eventually lead to the termination of the relationship. Hence, the fear of exit leads the worker to self-regulate and proactively put in effort to avoid termination.

In essence, the exit-based approach does not require managers to actively enforce rules in the way they would in a rule-based system. Instead, the threat of exit creates an incentive for the worker to maintain their productivity. The worker internalizes the potential consequences of sustained underperformance, effectively self-managing through a balance between the current cost of effort and the future risk of termination. This dynamic makes the exit-based approach more hands-off for managers, as they do not need to intervene frequently. Instead, the relationship is governed by the implicit understanding that continued employment is contingent on the worker's ability to maintain adequate effort, even in challenging circumstances when the cost is high.

Our analysis allows us to clearly delineate when each of the two approaches–rule-based and exit-based–comes into play. Naturally, when the cost of enforcing rules is relatively low, the rule-based approach is more likely to be used. Specifically, we show that there exists a cutoff enforcement cost: the rule-based approach is employed if and only if the enforcement cost falls below this threshold. In this scenario, the manager finds it worthwhile to enforce rules actively, as the cost of doing so is manageable and the benefits (i.e., improved worker effort) outweigh the enforcement cost.

When the enforcement cost exceeds this threshold, the exit-based approach is used. Our analysis further reveals two distinct subcategories within the exit-based approach, depending on the size of the enforcement cost. First, when the enforcement cost is sufficiently highthat is, above a second, higher thresholdthe manager never enforces the rules throughout the relationship. In this case, the cost of enforcing effort is prohibitively expensive, so the manager relies entirely on the threat of termination to motivate the worker. The relationship is thus governed purely by the exit mechanism, with no rule enforcement involved.

Interestingly, when the enforcement cost lies between the two thresholds, a different dynamic emerges within the exit-based approach. In this middle range, the manager will enforce rules at the beginning of the relationship for a limited period of time, even though this enforcement is not necessarily optimal for the overall long-term value of the relationship. This initial use of enforcement reflects the managers own short-term incentive: she prefers that the worker puts in effort.

This feature of early enforcement is closely related to the idea of deferred rewards in the dynamic game literature. By enforcing the rules at the outset of the relationship, the manager effectively increases the workers future payoff. Once the initial enforcement period is over, the worker gains more flexibility of not putting in effort when the cost of effort is high. In other words, the initial phase of rule enforcement functions as a way to build up the workers "credit" or trust within the relationship, allowing for greater discretion in the future. This early enforcement period, therefore, acts as an upfront cost the worker must bear, where high effort early on is effectively "paid" in exchange for more autonomy and flexibility later in the relationship.

In addition to the effects of enforcement costs, we also show that the exit-based approach is more likely to be adopted when the players are more patient, or equivalently, when there is greater surplus in the relationship. Patience, in this context, means that both the manager and the worker place a higher value on future payoffs relative to immediate rewards. For the worker, this translates into a greater appreciation of the ongoing relationship, which in turn makes the threat of exit more potent. The worker is less likely to risk termination because they recognize the future benefits of maintaining the relationship, even if doing so requires more effort or self-discipline in the present.

As a result, the management of the relationship increasingly relies on proactive self-regulation by the worker rather than direct enforcement of rules by the manager. The worker, motivated by the desire to preserve the long-term relationship, takes on the responsibility of adjusting their behavior to meet performance expectations, even in the absence of strict supervision. This shift in responsibility reflects a deeper level of trust within the relationship, where the worker is entrusted to manage their own effort levels based on the implicit understanding that their future within the organization depends on it.

This finding is consistent with existing management literature, which suggests that rules and strict enforcement mechanisms are used less frequently in high-performing organizationsthose with higher surplus (add cites? possibly Gibbons and Henderson?). In these organizations, workers are often granted more autonomy and flexibility, as the emphasis shifts away from rigid rule enforcement toward self-management and self-motivation. By shifting the responsibility of behavioral regulation to the worker, these organizations enjoy a culture of accountability and self-motivation, where the worker's contribution to the relationship is actively maintained without the need for constant managerial intervention.

This finding is consitent with existing management literature, which suggests that rules and strict enforcement mechanisms are used less frequently in high-performing organizationsthose with greater surplus. In such organizations, workers are typically granted more autonomy and flexibility, as the focus shifts from rigid rule enforcement to self-management. By transferring the responsibility for regulating behavior to the workers, these organizations cultivate a culture of accountability and self-motivation, where employees actively sustain their contributions to the relationship without the need for constant managerial oversight. However, this autonomy comes with a price: workers may still need to exert effort even when the cost is high, as maintaining their role in the organization sometimes requires short-term sacrifices for long-term benefits.

2 Related Literature

It is widely acknowledged that ultimate decisions should be made by those who can adapt their actions to local information without strict control from rules (Hayek 1945). There is a growing literature studying the cost of control. The empirical study by Falk and Kosfeld (2006) shows that workers respond negatively to rule enforcement and perceive that as lack of trust and limitation of autonomy. Following their study, there are some theoretical models explaining the hidden cost of control from the perspective of signaling (Sliwka 2007, Ellingsen and Johannesson 2008) or reciprocity (Von Siemens 2013). Li, Mukherjee and Vasconcelos (2023) study a dynamic setting where the rule cannot be taken away once established and show that the adoption of new rules will incur a dynamic cost and make organizations more rigid over time. Despite the cost of rules, the economics and the management literature still provide the reasons for the wide-spread use of rules in practice. Rule serves as the bargaining chip for management to induce desired outcome (Gouldner 1954). Work routines can help organizations to achieve less misunderstanding and better coordination (Nelson and Winter 1985). Sticking to rules can help transaction parties to preserve trust and maintain reputation in unforeseen circumstances (Kreps et al. 1990). Relatedly, there are some papers studying how to motivate lower managers to acquire local information and truthfully report to top management (Alonso, Dessein and Matouschek 2008, Rantakari 2008, Rantakari 2012, also see Gibbons, Matouschek and Roberts 2013 for a survey).

In contrast to Li, Mukherjee and Vasconcelos (2023), we study a setting where the rule can be enforced or ignored in every period. We contribute to this literature by investigating when should the manager enforce the rule to regulate the worker and how this is affected by enforcement cost. We highlight how enforcement cost can affect the manager's choice between the rule-based approach and the exit-based approach. In particular, when the cost is sufficiently high, it is optimal for the manager not to implement the rule at all and use the threat of exit instead.

The literature of rule is closely related to the incomplete contracting literature on delegation. There are two fundamentally different control systems: ex post performance control and ex ante action planning (Mintzberg 1989). Both rule and delegation can be a format of ex ante action planning. Starting from the seminal work by Aghion and Tirole (1997), most of the literature on delegation assumes principals can commit to various allocations of control rights through contracts and examines the formal allocation of those control rights (see, e.g., Dessein 2002, and, for a survey, Bolton and Dewatripont 2013). However, typically courts do not enforce contracts when transaction parties are in the same organization (see, e.g., Bolton and Dewatripont 2013, and Aghion, Bloom and Van Reenen 2014). Given this fact, there are some papers studying the informal allocation of control rights when top management commits through noncontractual means (see, e.g., Aghion and Tirole 1997, Baker, Gibbons and Murphy 1999 and Alonso and Matouschek 2007). A closely related work is Li, Matouschek and Powell (2017), which models power as a relational contract and explores a setting where the optimal allocation of power is not stationary. We follow this paper in studying a relational contract with dynamic setting. In contrast to it, however, we introduce the rule by modeling it as an instruction for the worker to strictly follow and explore how manager's decision on rule enforcement changes over time.

Another element of our paper that aligns with the delegation literature is that we rule out monetary transfer. In our setting, middle manager will not pay the worker. Instead, the worker is paid a fixed wage by the firm. Usually middle managers do not have the discretion to pay the workers. The literature on mechanism design without transfers explores the optimal design of contracts when parties among the same organization are constrained by managerial or legal issues (see Holmström 1984, Melumad and Shibano 1991, and, in a dynamic context, Guo and Hörner 2015 and Lipnowski and Ramos 2020).

Our paper is also related to the growing literature on the economics of relationships; see Samuelson (2006) and Mailath and Samuelson (2006) for reviews. Many papers have shown that relationships can improve over time (Ghosh and Ray 1996, Kranton 1996, Watson 1999, Mailath and Samuelson 2001, Watson 2002, Chassang 2010, Yang 2013, Halac 2014). However, relationships may also deteriorate because of a worsening production environment (Garrett and Pavan 2012, Halac and Prat 2016) or limited allocation of authority constrained by past promises (Li, Matouschek and Powell 2017). Finally, relationships can cycle between reward and punishment phases in the long run when parties have private information (Padró i Miquel and Yared 2012, Li and Matouschek 2013, Zhu 2013, Fong and Li 2017).[3] In our model, the relationship also oscillates between reward and punishment phases when players are sufficiently patient. Our focus is that during the punishment phase, the manager can choose between the rule-based action and the exit-based action.

Finally, our paper is also broadly related to the literature on the interaction between formal and informal incentives (see, e.g., Baker, Gibbons and Murphy 1994, Schmidt and Schnitzer 1995, Che and Yoo 2001). This literature assumes that an agent's private action is reflected by both verifiable and non-verifiable signals.[4] Hence, the optimal incentive scheme should combine both formal and informal incentives, and the two forms of incentives can be substitutes as well as complements. We consider a setting where formal contracts are not available. When the rule is not enforced, the worker is driven by informal incentives through repeated interactions. When the rule is enforced, the manager can force the worker to exert effort instead of using continuation payoff to motivate him.

[3]There is a growing literature on dynamic games with one-sided private information; see Mailath and Samuelson (2006) for a general review and Malcomson (1999) and Malcomson (2013) for surveys of applications in labor and organizational economics. In some models, long-run dynamics do not cycle between reward and punishment phases. Instead, they involve termination of the relationship or convergence to an efficient steady (see, e.g., Clementi and Hopenhayn 2006, Biais et al. 2007, DeMarzo and Fishman 2007).

[4]Note that an exception is Kvaløy and Olsen (2009), who study a setting with endogenous verifiability.

3 The Model

Consider a long-term relationship between a manager and a worker. Time is discrete and denoted as $t \in \{1, 2, \ldots, \infty\}$. In each period, the manager and the worker play a stage game characterized by four components: *technology*, *contract*, *payoffs*, and *randomization*.

Technology: The worker chooses an effort level $e_t \in \{0, 1\}$. If $e_t = 1$, he generates an output of $Y_t = y$. Otherwise, if $e_t = 0$, the output is zero. Both effort and output are publicly observable. However, the cost of effort is private. It depends on an underlying state $\theta_t \in \{G, B\}$, known only to the worker, and is given by $c(e_t) = c\mathbf{1}_{\theta_t=G} + C\mathbf{1}_{\theta_t=B}$. Assume $c < C$ so that the effort cost is lower in the good state (G) than in the bad state (B). The state is independently drawn in each period with $\mathbf{P}(\theta_t = G) = p \in (0, 1)$.

Contract: At the beginning of each period, the manager decides whether to offer a contract and, if she offers one, whether to enforce a rule that requires the worker to exert effort. Denote the offer of the contract as $d_t^m \in \{0, 1\}$ with one meaning "yes" and zero "no." Denote the decision of rule enforcement as $\gamma_t \in \{0, 1\}$. If $\gamma_t = 1$, the worker is forced to make $e_t = 1$ regardless of the state θ_t, because otherwise he would be severely punished according to the prescribed rule. In contrast, if $\gamma_t = 0$, the worker is free to make effort conditional the state. To enforce the rule, the manager must incur a cost $D \geq 0$. This cost can be interpreted as the physical expense of close monitoring or the manager's mental cost that results from the tension between her and the worker.

After receiving the contract offer and knowing the manager's decision on rule enforcement, the worker decides whether to accept them. Denote his decision as $d_t^w \in \{0, 1\}$, where $d_t^w = 0$ means rejection and $d_t^w = 1$ acceptance.

Payoffs: In each period, if the manager does not offer a contract or the agent declines it, the period ends with both parties receiving their outside option payoffs, normalized to zero. Instead, if a contract is offered and accepted, production occurs according to the previously specified technology. The worker receives a fixed wage w, while the manager obtains the output. To focus on the dynamics of rule enforcement, we abstract from monetary incentives by assuming that the wage is exogenous and not paid by the manager. This setup reflects scenarios where middle managers have limited control over wage-setting. Accounting for production, the worker's effort cost, and the manager's enforcement cost, we express their stage payoffs as follows:

$$\hat{u}_t = d_t^m d_t^w [w - c(e_t)], \text{ and } \hat{\pi}_t = d_t^m d_t^w (Y_t - \gamma_t D).$$

Randomization: At the end of the period, the manager and the worker observe the realization $x_t \in [0,1]$ of a public randomization device. We assume that they can also observe a realization of the randomization device at the beginning of period 1. We denote this realization by x_0. The randomization device guarantees that the set of equilibrium payoffs is convex, which is commonly used in the literature. The timing of the stage game is summarized in Figure 1.

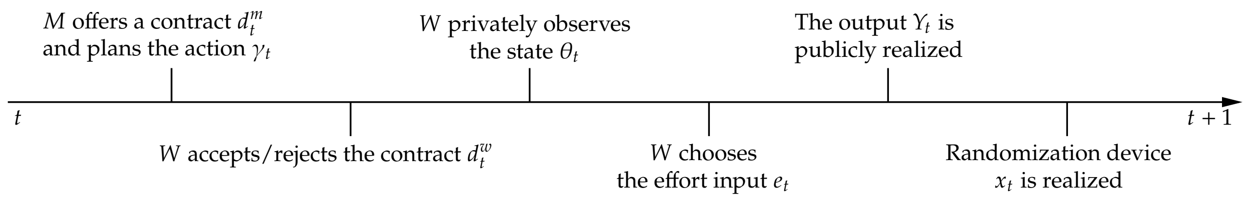

Figure 1: Timeline of the Stage Game

The stage game repeats in each period, with the manager and the worker sharing a common discount factor $\delta \in (0,1)$. Thus, at the beginning of each period, the expected payoffs for the worker (u_t) and the manager π_t are given by

$$u_t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\hat{u}_t, \text{ and } \pi_t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\hat{\pi}_t.$$

Note that these payoffs are normalized to per-period average values through multiplication by $1-\delta$.

Following Levin (2003), we define a relational contract as a pure strategy Perfect Public Equilibrium (henceforth PPE). This restriction is without loss of generality because our game has only one-sided private information, and is therefore a game with product monitoring structure. In this case, every sequential equilibrium outcome is also a PPE outcome (see Mailath and Samuelson 2006, p. 330).

Formally, denote the public history at the beginning of period $t+1$ as h_{t+1}, where $h_{t+1} = \{d_\tau^m, \gamma_\tau, d_\tau^w, e_t, x_t\}_{\tau=1}^t$. Let $\mathcal{H}_{t+1} = \{h_\tau\}_{\tau=1}^{t+1}$ be the set of public histories of period $t+1$. Note that $\mathcal{H}_1 = \phi$. A public strategy of the manager is a sequence of functions $\{D_t^m, \Gamma_t\}_{t=1}^\infty$, where $D_t^m : \mathcal{H}_t \to \{0,1\}$ and $\Gamma_t : \mathcal{H}_t \to \{0,1\}$. Similarly, a public strategy of the worker is a sequence of functions $\{D_t^w, E_t\}_{t=1}^\infty$, where $D_t^w : \mathcal{H}_t \cup \{d_t^m, \gamma_t\} \to \{0,1\}$, and $E_t : \mathcal{H}_t \cup \{d_t^m, d_t^w, \gamma_t, \theta_t\} \to \{0,1\}$. Their strategies compose a PPE if they compose a Nash equilibrium given any public history $h_t \in \mathcal{H}_t$.

A relational contract is called *optimal* if it is a PPE that maximizes the manager's payoff in the first period. Our goal is to characterize the set of optimal relational contracts.

4 Preliminaries

This section characterizes the set of perfect public equilibrium (PPE) payoffs, denoted as $\mathcal{E}$. Following Abreu, Pearce and Stacchetti (1990), we observe that any equilibrium payoff pair $(u, \pi) \in \mathcal{E}$ can be supported by pure actions or by randomization among equilibrium payoff pairs that are generated by pure actions. When a pure action is used, the players receive a pair of flow payoffs generated in the stage game and expect to receive a pair of associated continuation payoffs in the future. Similarly, when an equilibrium payoff pair is supported by randomization, the players select a pure action after observing the realization of the randomization device at the end of the previous period. We proceed by first introducing the available pure actions, then specifying the constraints for a pure action to support an equilibrium payoff. Finally, we formulate the maximization problem that characterizes $\mathcal{E}$ and its frontier.

Actions. In each period, if the manager offers a contract and the worker accepts it (i.e., $d_t^m d_t^w = 1$), they can choose from four actions:

1. Forced Effort (F): The manager enforces the rule, compelling the worker to exert effort regardless of the state.
2. Proactive Effort (P): The manager does not enforce the rule, but the worker still exerts effort in all states.
3. Adaptive Effort (A): The manager does not enforce the rule, and the worker exerts effort only in the good state.
4. Shirking (S): The manager does not enforce rule, and the worker makes no effort.

Alternatively, either party can opt out of the contract ($d^m = 0$ or $d^w = 0$), denoted as action O. We characterize $\mathcal{E}$ using these five actions: O, F, P, A, and S. Note that we exclude the case where the manager does not enforce the rule and the worker exerts effort only in the bad state, as this is not incentive-compatible for the worker.

Constraints. Any action $j \in \{O, F, P, A, S\}$ supporting an equilibrium payoff must satisfy three constraints:

1. Promise-keeping constraint: The equilibrium payoff decomposes into a flow payoff from the action j and a continuation payoff (u_j, π_j). The continuation payoff represents the discounted value of future actions.
2. No-deviation constraint: Players have no incentive to deviate from the action j. This ensures that the worker is willing to exert effort in line with the action.

3. Self-enforcing constraint: The continuation payoff must belong to the PPE payoff set, i.e., $(u_j, \pi_j) \in \mathcal{E}$. This constraint, combined with the no-deviation constraint, ensures that the supported payoff is indeed an equilibrium payoff.

The details of these constraints are omitted here and can be found in Appendix A.

The Maximization Problem. Now we formulate our maximization problem that characterizes $\mathcal{E}$ and, in particular, its frontier. Define the PPE payoff frontier as

$$\pi(u) := \max\{\pi' : (u, \pi') \in \mathcal{E}\}.$$

This frontier is well defined because Abreu, Pearce and Stacchetti (1990) says that $\mathcal{E}$ is compact. Furthermore, because the players can randomize their behavior with the randomization device x_t, $\mathcal{E}$ is convex. This implies that $\pi(u)$ is a concave function. To characterize $\pi(u)$, we need to pin down how the continuation payoffs evolve on the frontier.

Lemma 1. *For any $(u, \pi(u))$ that is supported by action $j \in \{O, F, P, A, S\}$, the continuation payoffs $(u_j(u), \pi_j(u))$ are also on the frontier, i.e., $\pi_j(u) = \pi(u_j(u))$. In particular, the worker's continuation payoff has the following properties:*

(*i*) *If $(u, \pi(u)$ is supported by forced effort, then*

$$u_F(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C] > u.$$

(*ii*) *If $(u, \pi(u))$ is supported by proactive effort, then*

$$u_P(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C] > u.$$

Moreover, $u_P(u) \geq \frac{1-\delta}{\delta}C$ and $u \geq (1-\delta)[w + p(C-c)]$.

(*iii*) *If $(u, \pi(u))$ is supported by adaptive effort, then*

$$u_{A,l}(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}w < u$$

and

$$u_{A,h}(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}(w-c),$$

where $u_{A,h}(u) > u$ if and only if $u > w - c$.

(iv) *If* $(u, \pi(u))$ *is supported by shirking, then*

$$u_S(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}w < u.$$

(v) *If* $(u, \pi(u))$ *is supported by outside option, then*

$$u_O(u) = \frac{1}{\delta}u > u.$$

Lemma 1 characterizes the continuation payoffs for each equilibrium payoff and its supporting action. First, it shows that for any payoffs on the PPE payoff frontier, the associated continuation payoffs also stay on the frontier. This is because the manager's actions are publicly observable, and there is no need to punish her by moving below the frontier (see, e.g., Spear and Srivastava 1987, Levin 2003). We can therefore trace out the entire equilibrium action sequences on the frontier without worrying about payoffs below the frontier.

Second, the lemma gives detailed characterization of the worker's continuation payoff. For actions F, P, S, and O, the results follow directly from their respective promise-keeping constraints. For action A, the result is derived as follows. On one hand, to incentivize the worker to exert effort in the good state, the continuation payoff conditional on effort must be sufficiently higher than that of no effort. On the other hand, however, the concave function $\pi(u)$ is maximized by minimizing the gap between the two possible continuation payoffs. Consequently, the worker's incentive constraint binds at the optimal gap. The continuation payoffs are then determined by solving the promise-keeping constraints.

For any u, define the manager's highest payoff as $f_j(u)$ if it is supported by action $j \in \{F, P, A, S, O\}$. With Lemma 1, we can express $f_j(u)$ as:

$$f_F(u) = (1-\delta)(y - D) + \delta\pi(u_F(u)),$$

$$f_P(u) = (1-\delta)y + \delta\pi(u_P(u)),$$

$$f_A(u) = (1-\delta)py + \delta[p\pi(u_{A,h}(u)) + (1-p)\pi(u_{A,l}(u))],$$

$$f_S(u) = (1-\delta)w + \delta\pi(u_S(u)), \text{ and}$$

$$f_O(u) = \delta\pi(u_O(u)).$$

Since the PPE payoff frontier gives the highest payoff for the manager by using some pure action or randomizing between different pure actions, it is characterized by the following constrained maximization problem:

$$\pi(u) = \max_{\alpha_j \geq 0, u_j \in [0,w]} \sum_{j \in \{F,P,A,S,O\}} \alpha_j f_j(u_j)$$

$$s.t. \quad \sum_{j \in \{F,P,A,S,O\}} \alpha_j = 1$$

$$\sum_{j \in \{F,P,A,S,O\}} \alpha_j u_j = u$$

If any of the weight α_j equals one, $(u, \pi(u))$ is supported by a pure action from $\{F, P, A, S, O\}$. Otherwise, it is generated by randomization. We will characterize the PPE payoff frontier by choosing these weights. For our analysis, we make two assumptions.

Assumption 1. (i) $c < w < pc + (1-p)C$; (ii) $w > pc + \frac{p^2}{1-p}C$.

Assumption 2. (i) $\delta \geq \frac{C}{C+w}$; (ii) $\delta w \geq \frac{c}{\delta} + (1-\delta)p(C-c)$; (iii) $\delta w \geq c + (1-\delta)(C - \frac{c}{p})$.

Part (i) of Assumption 1 indicates that in the stage game, the worker prefers the outside option to proactive effort, but prefers adaptive effort to the outside option. Part (ii) of Assumption 1 and Part (i) of Assumption 2 together ensure that proactive effort (action P) may support the PPE payoff frontier for certain parameter values. Additionally, Part (ii) of Assumption 2 guarantees that the optimal relational contract permits a choice between forced effort (action F) and proactive effort (action P) when the worker fails to make adaptive effort. Finally, Part (iii) of Assumption 2 ensures that the manager prefers starting the equilibrium from proactive effort than from adaptive effort. Alternatively, if adaptive effort were preferred, the range of possible dynamics would be reduced, as we will discuss after presenting our main results.

5 Optimal Relational Contract

How can a manager induce effort from an under-performing worker? Two main approaches are commonly considered: the *rule-based* approach and the *exit-based* approach. The rule-based approach involves the manager enforcing rules to force the worker to exert effort (action F). In contrast, the exit-based approach relies on the worker's proactive effort (action P), which is motivated by the threat of termination.

This section examines the optimal choice between the rule-based approach and the exit-based approach. We begin with a proposition that provides an overview of the preferred approach, followed by a detailed analysis of the dynamics under each strategy.

Proposition 1. *There exists a cutoff $\underline{D}(\delta) \geq 0$, such that:*

(*i*) *When $D < \underline{D}(\delta)$, the manager adopts the rule-based approach* (*i.e., any payoff on the PPE payoff frontier is not supported by proactive effort*).[5]

(*ii*) *When $D \geq \underline{D}(\delta)$, the manager adopts the exit-based approach* (*i.e., some payoffs on the PPE payoff frontier are supported by proactive effort*).

The choice between the two approaches is essentially the choice between forced effort and proactive effort. Both actions will deliver the same output as the worker will exert effort regardless of the state. The downside of the forced effort is that the manager has to pay a fixed cost. Hence, ceteris paribus, proactive effort is preferred by the manager. However, proactive effort is not always feasible as the worker will not be motivated when his payoff is too low. To provide enough incentive for the worker to proactively exert effort, sufficient rent must be allowed for the worker. In contrast, the manager can simple pay a cost and force the worker to put in effort even if the worker's payoff is very low. Therefore, the manager can use the forced effort when the worker's utility is very low and the proactive effort is not feasible. This requires lower continuation payoff for the worker and deters future inefficient actions for the manager like adaptive effort or shirking.

When considering rule enforcement, the manager has to balance the trade-off between the enforcement cost and the benefit from deterring inefficient actions. As is shown in Proposition 1, when the enforcement cost is sufficiently low, the benefit of forced effort dominates. Therefore, the manager will use the rule-based approach to regulate the worker. In contrast, when the enforcement cost is high, the manager will take the exit-based approach and wait for the worker to proactively behave well.

In the following part, we investigate these two approaches by characterizing the optimal relational contract, that is, the PPE that maximizes the manager's expected payoff. For this purpose, we first characterize the PPE payoff frontier by solving the constrained maximization problem aforementioned, and then describe the optimal relational contract.

5.1 Rule-based Approach

The following lemma characterizes the PPE payoff frontier when the enforcement cost is sufficiently low.

Lemma 2. *When $D < \underline{D}(\delta)$, there exist cutoffs $0 < \underline{u}^A < \overline{u}^A < w$, such that:*

[5] When there is a tie between the two approaches, we break the tie by favoring the exit-based approach.

(*i*) *For* $u \in [0, \underline{u}^A)$*, the payoff frontier is linear and supported by randomization between* $(0, \pi(0))$ *and* $(\underline{u}^A, \pi(\underline{u}^A))$*. We have* $\pi(0) > 0$ *and the payoff* $(0, \pi(0))$ *is supported by force effort.*

(*ii*) *For* $u \in [\underline{u}^A, \overline{u}^A]$*,* $\pi(u) = f_A(u)$ (*i.e., the payoff frontier is supported by adaptive effort*).

(*iii*) *For* $u \in (\overline{u}^A, w]$*, the frontier is linear and supported by randomization between* $(\overline{u}^A, \pi(\overline{u}^A))$ *and* $(w, 0)$*. We have* $\pi(w) = 0$ *and the payoff* $(w, 0)$ *is supported by* S (*i.e., the worker always shirks*).

Figure 2 illustrates the three regions described in Lemma 2. In the middle region, the payoffs at the frontier are supported by playing pure action adaptive effort. Also, the $(0, \pi(0))$ payoff pair is the only point on the frontier that is supported by playing forced effort, and the $(w, 0)$ payoff pair is the only point on the frontier that is supported by playing shirking. In the other two regions, the payoffs are sustained through randomization. Without loss of generality, we assume that in the regions where randomization is used, the players randomize only between the endpoints of the two adjacent regions that are sustained by pure actions.

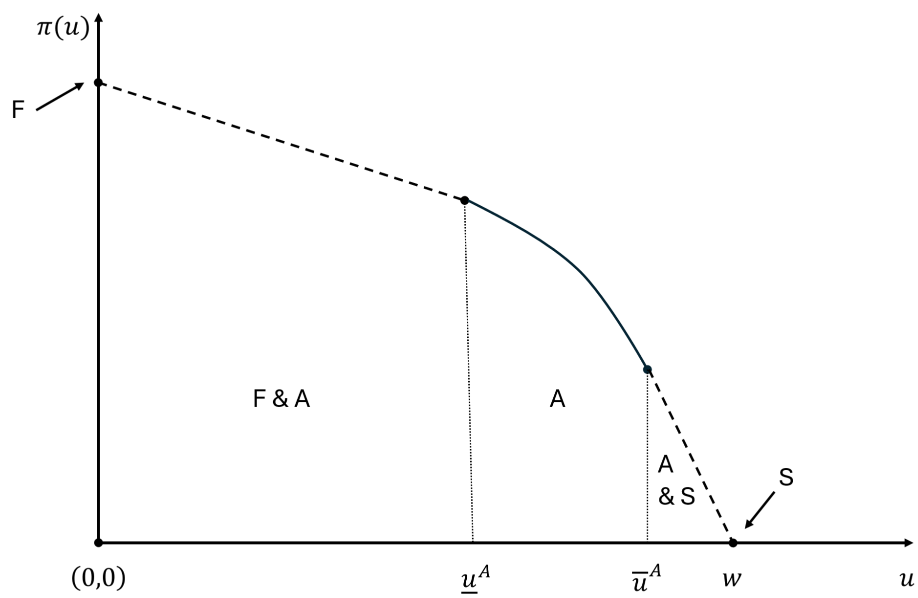

Figure 2: PPE payoff frontier: $D < \underline{D}(\delta)$

Proposition 2. *When* $D < \underline{D}(\delta)$*, the optimal relational contract satisfies the following.*

First period: The worker's and manager's payoffs are given by $(0, \pi(0))$*. The parties engage in forced effort. The worker's continuation payoff is given by* $u_F(0)$*.*

Subsequent periods: The agent's and the principal's expected payoffs are given by $u \in \{0\} \cup [\underline{u}^A, \overline{u}^A] \cup \{w\}$ *and* $\pi(u)$*. Their actions and continuation payoffs depend on what region* u *is in.*

(*i*) *If* $u = 0$*, the parties choose forced effort. The worker's continuation payoff is given by* $u_F(0) > 0$*.*

(*ii*) *If* $u \in [\underline{u}^A, \overline{u}^A]$*, the parties choose adaptive effort. If the worker puts in effort, his continuation payoff is given by* $u_{A,h} \geq u$*. If, instead, he shirks, his continuation payoff is given by* $u_{A,l} < u$*.*

(*iii*) *If* $u = w$*, the parties engage in shirking action. The worker's continuation payoff is given by* $u_S(w) = w$*.*

The proposition shows that players start out by engaging in forced effort. The worker is forced to exert effort by rule. When the adaptive effort is used later on, to motivate the worker to deliver her desired output, the manager increases his continuation payoff whenever he exerts effort, and she decreases his continuation payoff whenever he shirks.

To see how the manager optimally increases the worker's continuation payoff, suppose the state is good for a number of consecutive periods so that the worker keeps exerting effort. The players continue to engage in adaptive effort, and the worker's continuation payoff keeps increasing, until the parties reach a period when the worker's continuation payoff passes the threshold $\overline{u}^A$. At the end of that period, the parties engage in randomization to determine their actions in the following period. Depending on the outcome of this randomization, the parties either continue to engage in adaptive effort, or they move to shirking action. Finally, once the play has moved to shirking action, it remains there in all subsequent periods.

To see how the manager optimally decreases the worker's continuation payoff, suppose instead that the state is bad for a number of consecutive periods so that the worker shirks. The players continue to engage in adaptive effort, and the worker's continuation payoff keeps decreasing, until the parties read a period when the worker's continuation payoff passes the threshold $\underline{u}^A$. At the end of that period, the parties engage in randomization to determine their actions in the following period. Depending on the outcome of this randomization, the parties either continue to engage in adaptive effort, or they move to forced effort.

Corollary 1. *Suppose that* $D < \underline{D}(\delta)$*. Then, in the long run, if* $\frac{1-\delta}{\delta}[pc + (1-p)C - w] \leq w - c$*, the relationship will alternate between using forced effort and adaptive effort; if* $\frac{1-\delta}{\delta}[pc + (1-p)C - w] > w - c$*, the relationship will stay at shirking.*

This corollary shows that when players are sufficiently patient, the manager will frequently use rule to regulate the worker in the long run. It is easier for the manager to enforce the rule than provide enough future incentive to motivate the worker. Hence, we refer to this as "rule-based" approach.

The worker will be forced to exert effort if he was not performing well, while he can choose effort adaptive to the sate when he did a great job. This pattern is consistent with what Gouldner (1954) found in his case study. There was a "No-floating" rule in the company he investigated. Workers were supposed to stay at their workplace unless they need go to washroom or eat. But managers selectively enforced the rule in practice. Workers were allowed to walk around if the manager thought everything was smooth. But the manager would invoke the rule when workers were not performing well.

5.2 Exit-based Approach

Lemma 3. *Let* $u^* = \operatorname{argmax}_{u\in[0,w]} \pi(u)$*. There exists a cutoff* $\overline{D}(\delta) > \underline{D}(\delta)$*, such that when* $D > \overline{D}(\delta)$*,* $u^* = \underline{u}^P$*.*

This lemma shows that when the enforcement cost is sufficiently high, the optimal relational contract starts with using proactive effort. The following lemma characterizes the PPE payoff frontier in this scenario.

Lemma 4. *When* $D > \overline{D}(\delta)$*, there exist cutoffs* $0 < \underline{u}^P \leq \overline{u}^P \leq \underline{u}^A < \overline{u}^A < w$*, such that:*

(*i*) *For* $u \in [0, \underline{u}^P)$*, the payoff frontier is linear and supported by randomization between* $(0, \pi(0))$ *and* $(\underline{u}^P, \pi(\underline{u}^P))$*. We have* $\pi(0) \geq 0$ *and the payoff* $(0, \pi(0))$ *is supported by* F *or* O*.*[6]

(*ii*) *For* $u \in [\underline{u}^P, \overline{u}^P]$*,* $\pi(u) = f_P(u)$ (*i.e., the payoff frontier is supported by proactive effort*).

(*iii*) *For* $u \in (\overline{u}^P, \underline{u}^A)$*, the frontier is linear and supported by randomization between* $(\overline{u}^P, \pi(\overline{u}^P))$ *and* $(\underline{u}^A, \pi(\underline{u}^A))$*.*

(*iv*) *For* $u \in [\underline{u}^A, \overline{u}^A]$*,* $\pi(u) = f_A(u)$ (*i.e., the payoff frontier is supported by adaptive effort*).

(*v*) *For* $u \in (\overline{u}^A, w]$*, the frontier is linear and supported by randomization between* $(\overline{u}^A, \pi(\overline{u}^A))$ *and* $(w, 0)$*. We have* $\pi(w) = 0$ *and the payoff* $(w, 0)$ *is supported by* S (*i.e., the worker always shirks*).

Figure 3 illustrates the five regions described in Lemma 4.[7] In the second and fourth regions, the payoffs at the frontier are supported by playing pure actions, proactive effort and adaptive effort respectively. Also, the $(0, \pi(0))$ payoff pair is the only point on the frontier that is supported by playing forced effort or outside option, and the $(w, 0)$ payoff

[6]When D is large enough, $\pi(0) = 0$ and $(0, 0)$ is supported by players taking outside option. Otherwise, $\pi(0) > 0$ and $(0, \pi(0))$ is supported by using forced effort.

[7]This figure only shows the case when $(0, \pi(0))$ is supported by using forced effort. The figure when $(0, 0)$ is supported by O is the same except that the frontier is a straight line between $(0, 0)$ and $(\underline{u}^P . \pi(\underline{u}^P))$

pair is the only point on the frontier that is supported by playing shirking. In the other three regions, the payoffs are sustained through randomization between endpoints of the two adjacent regions.

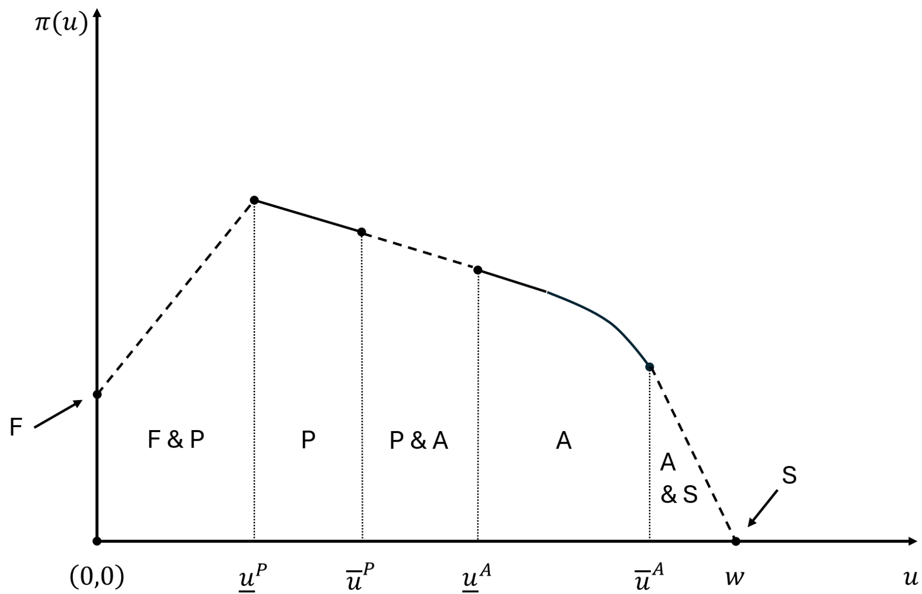

Figure 3: PPE payoff frontier: $D > \overline{D}(\delta)$

Proposition 3. *When $D > \overline{D}(\delta)$, the optimal relational contract satisfies the following.*

First period: The worker's and the manager's payoffs are given by $(\underline{u}^P, \pi(\underline{u}^P))$. The parties engage in proactive effort. The worker's continuation payoff is given by $u_P(0)$.

Subsequent periods: The agent's and the principal's expected payoffs are given by $u \in [\underline{u}^P, \overline{u}^P] \cup [\underline{u}^A, \overline{u}^A] \cup \{w\}$ and $\pi(u)$. Their actions and continuation payoffs depend on what regions u is in.

(*i*) *If $u \in [\underline{u}^P, \overline{u}^P]$, the parties choose proactive effort. The worker's continuation payoff is given by $u_P(u) > u$.*

(*ii*) *If $u \in [\underline{u}^A, \overline{u}^A]$, the parties choose adaptive effort. If the worker puts in effort, his continuation payoff is given by $u_{A,h} \geq u$. If, instead, he shirks, his continuation payoff is given by $u_{A,l} < u$.*

(*iii*) *If $u = w$, the parties engage in shirking action. The worker's continuation payoff is given by $u_S(w) = w$.*

This proposition shows that players start out by engaging in proactive effort. The worker voluntarily exerts effort regardless of the state. To incentivize him to do that, the worker is rewarded with higher continuation payoff in the future. When the adaptive effort is used later on, the dynamics are quite similar to those in the previous scenario. The only difference is that when the worker's continuation payoff passes the threshold $\underline{u}^A$, the parties randomize between proactive effort and adaptive effort. Since the enforcement cost is sufficiently high, the manager will wait for the worker to proactively work hard. Knowing that he is not doing well, the worker worries that he might be fired by the manager. Hence, the worker will proactively exert effort even if the state is still bad.

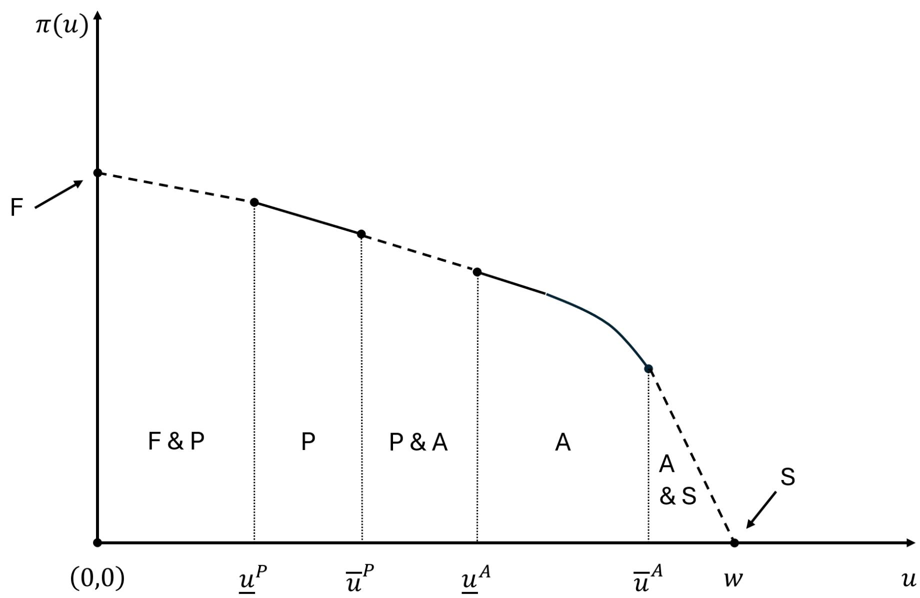

Figure 4: PPE payoff frontier: $\underline{D}(\delta) \leq D \leq \overline{D}(\delta)$

Figure 4 illustrates the scenario when the enforcement cost is moderate. The frontier is quite similar to the scenario when the enforcement cost is sufficiently high. The only difference is the optimal relational contract starts from using forced effort.

Proposition 4. *When $\underline{D}(\delta) \leq D \leq \overline{D}(\delta)$, the optimal relational contract satisfies the following.*

First period: The worker's and the manager's payoffs are given by $(0, \pi(0))$. The parties engage in forced effort. The worker's continuation payoff is given by $u_F(0)$.

Subsequent periods: The agent's and the principal's expected payoffs are given by $u \in \{0\} \cup [\underline{u}^P, \overline{u}^P] \cup [\underline{u}^A, \overline{u}^A] \cup \{w\}$ and $\pi(u)$. Their actions and continuation payoffs depend on what regions u is in.

- (i) *If $u = 0$, the parties engage in forced effort. The worker's continuation payoff is given by $u_F(0) > u$.*

- (ii) *If $u \in [\underline{u}^P, \overline{u}^P]$, the parties choose proactive effort. The worker's continuation payoff is given by $u_P(u) > u$.*

(*iii*) *If $u \in [\underline{u}^A, \overline{u}^A]$, the parties choose adaptive effort. If the worker puts in effort, his continuation payoff is given by $u_{A,h} \geq u$. If, instead, he shirks, his continuation payoff is given by $u_{A,l} < u$.*

(*iv*) *If $u = w$, the parties engage in shirking action. The worker's continuation payoff is given by $u_S(w) = w$.*

Since the enforcement cost is not too high, it is optimal for the manager to implement the rule at the very beginning of the relationship to extract more rent. Without forced effort, the relationship would start from using proactive effort, leading to the expected payoffs $(\underline{u}^P, \pi(\underline{u}^P))$ for the parties. With forced effort, the worker's payoff drops to zero while the manager's payoff increases to $\pi(0)$. In the long run, the worker is still motivated by the threat of exit when performance is not good. Hence, this is a special case in "exit-based" approach, and forced effort is only used at the beginning of the relationship to extract rent.

Corollary 2. *Suppose that $D \geq \underline{D}(\delta)$. Then, in the long run, if $\frac{1-\delta}{\delta}C \leq w - c$, the relationship will alternate between using proactive effort and adaptive effort; if $\frac{1-\delta}{\delta}C > w - c$, the relationship will stay at shirking.*

This corollary shows that when players are sufficiently patient, the manager will just wait for the worker to proactively exert effort when his performance is not good. In the long run, after a number of consecutive periods of poor performance, the worker is motivated by the threat of exit. Hence, we refer to this as "exit-based" approach.

5.3 Effect of Patience

Proposition 5. *When δ increases, both $\underline{D}(\delta)$ and $\overline{D}(\delta)$ will decrease.*

This proposition shows that rule is less likely to be used when players are more patient. From our previous discussion, we know that $\underline{D}(\delta)$ determines whether rule will be used in the long run while $\overline{D}(\delta)$ determines whether rule will be used in the short run.

When players get more patient, they put more weight on future payoff. Hence, the worker will cherish the relationship more, making the threat of exit more powerful. Therefore, with the same enforcement cost, it is better for the manager to motive the worker by using termination threat instead of enforcing the rule with a cost. As a result, rule is less likely to be used to regulate the worker in the long run.

At the beginning of the the relationship, the worker can be motivated to use proactive effort for longer periods because of more patience. Proactive effort deliver the same output to the manager as forced effort, while forced effort comes with a cost. Therefore, using forced effort will extract less rent when the worker gets more patient. As a result, rule is less likely to be used for rent-extraction in the short run.

6 Discussion

Interim rule enforcement. An inherent feature of our main model is that the manager announces rule enforcement before the worker decides to accept the contract. The manager's discretion is therefore constrained by the possibility that the worker may decline the contract upon learning of rule enforcement. However, in practice, managers may enforce rules after workers have accepted contracts. We model this alternative setup by repositioning the manager's rule enforcement decision (γ_t) to occur after the worker's participation decision (d_t^w) but before the realization of the private state (θ_t). We call this setup *interim rule enforcement*, contrasting it with the original setup of *ex-ante rule enforcement*.

In this scenario, unguarded intuition may suggest that the manager have more incentives to enforce rules, as the manager enjoys larger discretion in forcing the worker to exert effort. The following proposition examines the scenario and the intuition.

Proposition 6. *Suppose $\delta \geq \frac{(1-p)w}{w-pc}$. Then, interim rule enforcement enhances the manager's incentive to enforce rules: both $\underline{D}(\delta)$ and $\overline{D}(\delta)$ are weakly greater for any δ. It also reduces the manager's payoff from the optimal relational contract: for D that weakly exceeds $\underline{D}(\delta)$ under interim rule enforcement, the manager's optimal payoff is weakly decreased compared with ex-ante rule enforcement.*

Proposition 6 shows that the manager is indeed more willing to enforce rules under interim rule enforcement than under ex-ante rule enforcement. However, it also shows that the intuition mentioned above the proposition is wrong. As the manager's payoff from the optimal relational contract is actually decreased, the tendency for rule enforcement cannot be simply attributed to profit maximization.

In contrast, the proposition reveals a credibility problem for the manager: interim rule enforcement makes her less credible in back-loading incentives for the worker. By announcing rule enforcement after the worker accepts the contract, the manager can secure a minimum flow payoff of $(1-\delta)(y-D)$, conditional on worker participation. This narrows the range of feasible continuation payoffs for the worker. For instance, shirking (action S) is no longer self-enforcing, as it yields zero payoff to the manager. Consequently, the worker's maximum continuation payoff now falls strictly below w.

Recall that the efficacy of the exit-based approach in inducing effort relies on the threat of termination. However, the manager's inability to credibly back-load incentives weakens this threat because it is less appealing to the worker to continue the relationship. As a result, the manager must enforce rules more frequently to elicit effort from the worker. Given that the manager would prefer the exit-based approach under ex-ante rule enforcement (when

$D \geq \underline{D}'(\delta))$, we conclude that her payoff is diminished due to the credibility problem under interim rule enforcement.

7 Conclusion

This paper explores the optimal enforcement of rules in employment relationships. Optimal relational contracts fall into two categories: manager-led enforcement and worker-led accountability. When enforcement costs are low, the manager-led approach is used, characterized by cycles of rule enforcement and relaxation. In this setting, the manager plays an active role in maintaining the worker's performance by enforcing the rule when necessary to ensure effort. However, as enforcement costs increase and the surplus of the relationship grows, the worker-led approach will be chosen. Here, the threat of termination serves as the motivator, causing the worker to self-regulate and maintain effort without managerial supervision in the long run.

Furthermore, we identify a subcategory within the worker-led approach where rules are enforced at the beginning of the relationship. This temporary enforcement helps establish future flexibility. Our findings align with empirical studies that suggest that high-performing organizations, which typically have greater surplus, rely less on rigid rules and more on worker autonomy.

For future research, it would be valuable to explore how managerial characteristics, which influence enforcement costs, interact with the production environment and the surplus of the relationship. This could provide further insight into when and how organizations transition between different enforcement strategies, potentially linking management styles to specific industry or market conditions.

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A Constraints for Pure Actions

This appendix contains the constraints that any PPE payoff pair $(u, \pi) \in \mathcal{E}$ should satisfy.

Forced Effort F. —A payoff pair (π, u) can be supported by Forced Effort (F) if the following constraints are satisfied.

(i) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Hence, we have the following self-enforcing constraint:

$$(SE_F) \qquad (u_F, \pi_F) \in \mathcal{E},$$

where u_F and π_F are the continuation payoffs when forced effort (F) is used in the stage game.

(ii) No Deviation: There are two kinds of deviations: off-schedule and on-schedule. Off-schedule deviations can be observed by both players, while on-schedule deviations can only be observed by the one with private information.

We assume the worst punishment for off-schedule deviation, which is outside option forever for both players. To prevent any off-schedule deviation, we have the following IR constraints:

$$(IR_F) \qquad u \geq 0, \pi \geq 0.$$

Given the contract, the worker, who has private information, cannot deviate on-schedule when the rule is enforced.

(iii) Promise Keeping: The PPE payoff should be decomposed consistently. That is, the payoff should equal the weighted sum of the current stage payoff and the associated future payoffs. Hence, the promise-keeping constraints are given by

$$(PK_F^M) \qquad \pi = (1 - \delta)(y - D) + \delta\pi_F$$

for the manager and

$$(PK_F^W) \qquad u = (1 - \delta)[w - pc - (1 - p)C] + \delta u_F$$

for the worker.

Proactive Effort P. —A payoff pair (π, u) can be supported by Proactive Effort (P) if the following constraints are satisfied.

(i) Feasibility: Similarly, we have the following self-enforcing constraint for the continuation payoffs:

$$(SE_P) \qquad (u_P, \pi_P) \in \mathcal{E},$$

where u_P and π_P are the continuation payoffs when proactive effort (P) is used in the stage game.

(ii) No Deviation: Given the contract, any deviation is off-schedule. Hence, we have the following IR constraints:

$$(IR_P) \qquad u \geq 0, \pi \geq 0.$$

To ensure that the worker always exerts effort, we have the following IC constraints:

$$(IC^W_{P,G}) \qquad -(1-\delta)c + \delta u_P \geq 0,$$

$$(IC^W_{P,B}) \qquad -(1-\delta)C + \delta u_P \geq 0.$$

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK^M_P) \qquad \pi = (1-\delta)y + \delta \pi_P$$

for the manager and

$$(PK^W_P) \qquad u = (1-\delta)[w - pc - (1-p)C] + \delta u_P$$

for the worker.

Adaptive Effort A. —A payoff pair (π, u) can be supported by Adaptive Effort (A) if the following constraints are satisfied.

(i) Feasibility: There might be two different outcomes. Hence, we have the following self-enforcing constraint for the continuation payoffs:

$$(SE_A) \qquad (u_{A,h}, \pi_{A,h}), (u_{A,l}, \pi_{A,l}) \in \mathcal{E},$$

where $u_{A,h}$ and $\pi_{A,h}$ are the continuation payoffs when adaptive effort (A) is used and there is output, and $u_{A,l}$ and $\pi_{A,l}$ are the continuation payoffs when A is used and there is no output.

(ii) No Deviation: To avoid off-schedule deviation, we have the following IR constraints:

$$(IR_A) \qquad u \geq 0, \pi \geq 0.$$

With private information on states, the worker might deviate on-schedule by misreporting the state. Therefore, we have the following IC constraints:

$$(IC^W_{A,G}) \qquad -(1-\delta)c + \delta u_{A,h} \geq \delta u_{A,l},$$

$$(IC^W_{A,B}) \qquad \delta u_{A,l} \geq -(1-\delta)C + \delta u_{A,h}.$$

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK^M_A) \qquad \pi = (1-\delta)py + \delta[p\pi_{A,h} + (1-p)\pi_{A,l}]$$

for the manager and

$$(PK^W_A) \qquad u = (1-\delta)(w - pc) + \delta[pu_{A,h} + (1-p)u_{A,l}]$$

for the worker.

Shirking S. —A payoff pair (π, u) can be supported by Shirking (S) if the following constraints are satisfied.

(i) Feasibility: Similarly, we have the following self-enforcing constraint for the continuation payoffs:

$$(SE_S) \qquad (u_S, \pi_S) \in \mathcal{E},$$

where u_S and π_S are the continuation payoffs when shirking (S) is used in the stage game.

(ii) No Deviation: Given the contract, any deviation is off-schedule. Hence, we have the following IR constraints:

$$(IR_S) \qquad u \geq 0, \pi \geq 0.$$

The worker has no incentive to exert effort as it incurs extra cost and destroys future payoff.

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK_S^M) \qquad \pi = \delta \pi_S$$

for the manager and

$$(PK_S^W) \qquad u = (1-\delta)w + \delta u_S$$

for the worker.

Outside Option O. —A payoff pair (π, u) can be supported by Outside Option (O) if the following constraints are satisfied.

(i) Feasibility: Similarly, we have the following self-enforcing constraint for the continuation payoffs:

$$(SE_O) \qquad (u_O, \pi_O) \in \mathcal{E},$$

where u_O and π_O are the continuation payoffs when outside option (O) is used in the stage game.

(ii) No Deviation: One-sided deviation will not change the result. Therefore, we do not need worry about deviations here.

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK_O^M) \qquad \pi = \delta \pi_O$$

for the manager and

$$(PK_O^W) \qquad u = \delta u_O$$

for the worker.

B Proofs

This appendix contains the proofs of main model omitted in the text.

Before characterize the PPE frontier, we need following two lemmas that describe the properties of PPE payoff set.

Lemma B.1. *The PPE payoff set $\mathcal{E}$ has the following properties:* (*i*) *it is compact;* (*ii*) *the frontier $\pi(u)$ is concave;* (*iii*) $\inf\{u : (u, \pi) \in \mathcal{E}\} = 0$ *and* $\sup\{u : (u, \pi) \in \mathcal{E}\} = w$*;* (*iv*) *payoff pair* (u, π) *belongs to $\mathcal{E}$ if and only if $u \in [0, w]$ and $\pi \in [0, \pi(u)]$.*

Proof of Lemma B.1. Part (i): There are finite actions that the players can choose, so the payoff set $\mathcal{E}$ is compact by standard arguments.

Part (ii): The public randomization device makes sure that the frontier $\pi(u)$ is concave.

Part (iii): It is easy to verify that $(0, 0)$ is self-enforcing and satisfies all the constraints for outside option. Hence, $(0, 0)$ belongs to PPE payoff set and can be supported by outside option. Similarly, $(w, 0)$ is self-enforcing and can be supported by shirking. Notice that the worker's minimum payoff is 0, which is obtained by taking outside option forever. Hence, $\inf\{u : (u, \pi) \in \mathcal{E}\} = 0$. The worker's maximum payoff is w, which is achieved by shirking forever. Hence, $\sup\{u : (u, \pi) \in \mathcal{E}\} = w$.

Part (iv): Since $(0, 0)$ and $(w, 0)$ are self-enforcing, any payoff pair on the line segment between them belongs to PPE payoff set by randomization. If (u, π) belongs to $\mathcal{E}$, then it must be that $u \in [0, w]$. By the definition of $\pi(u)$, π cannot be larger than $\pi(u)$. By the IR constraints, π cannot be negative. If $u \in [0, w]$ and $\pi \in [0, \pi(u)]$, (u, π) belongs to $\mathcal{E}$ and can be supported by randomizing between $(u, 0)$ and $u, \pi(u)$. ■

Lemma B.2. *For any payoff pair $(u, \pi(u))$ on the frontier, the associated continuation payoffs* (*along the equilibrium path*) *remain on the frontier.*

Proof of Lemma B.2. The frontier is supported by either pure actions or randomizing between pure actions. Hence, we just need to show that this is true if $(u, \pi(u))$ is supported by a pure action. Assume that $(u, \pi(u))$ is supported by pure action P. We prove that by contradiction. Suppose that the continuation payoff (u_P, π_P) does not remain on the frontier. By definition, it must be that $\pi_P < \pi(u_P)$. Now consider another strategy profile which is also supported by P in this stage, but the corresponding continuation payoff is $(u_P, \hat{\pi}_P)$. $\hat{\pi}_P = \pi_P + \varepsilon$, where ε is a small positive value such that $\pi_P + \varepsilon \leq \pi(u_P)$. Following the promise-keeping constraints PK_P^M and PK_P^W, the payoffs under this alternative strategy profile are given by $\hat{u} = u$ and $\hat{\pi} = \pi(u) + \delta\varepsilon > \pi(u)$. It can be checked that this alternative strategy profile satisfies all the constraints and hence constitutes a PPE. $\hat{\pi} > \pi(u)$ contradicts the definition of $\pi(u)$. Therefore, it must be that $\pi_P = \pi(u_P)$. This argument also applies to other pure actions. ■

Proof of Lemma 1. Part (i): When forced effort is used, neither of the players will deviate as long as their payoff is not negative. Directly from PK_F^W, we have the worker's continuation payoff as

$$u_F(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C].$$

Notice that the worker's payoff cannot be larger than w. By SE_F, we have $u_F(u) \leq w$. This leads to the range of u when F is used: $u \in [0, w-(1-\delta)[pc+(1-p)C]]$. Since $w < pc + (1-p)C$ by Assumption 1 (i), for any u when F is used, $u_F(u) > u$.

Part (ii): When proactive effort is used, we get the worker's continuation payoff

$$u_P(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C]$$

directly from PK_P^W. Based on $IC_{P,G}^W$ and $IC_{P,B}^W$, we have $u_P \geq \frac{1-\delta}{\delta}C$. Notice that $u_P \leq w$. Hence, we have $u \in [(1-\delta)[w + p(C-c)], w-(1-\delta)[pc+(1-p)C]]$ when P is used. Since $w < pc + (1-p)C$ by assumption, for any u when P is used, $u_F(u) > u$.

Part (iii): When adaptive effort is used, the worker's continuation depends on the output. We first show that $IC_{A,G}^W$ must be binding. $IC_{A,G}^W$ shows that $u_{A,h} - u_{A,l} \geq \frac{1-\delta}{\delta}c > 0$. Suppose that $IC_{A,G}^W$ is slack, which means that $u_{A,h} - u_{A,l} > \frac{1-\delta}{\delta}c$. Consider an alternative strategy where $\hat{u}_{A,h} = u_{A,h} - (1-p)\varepsilon$ and $\hat{u}_{A,l} = u_{A,l} + p\varepsilon$. ε is positive and small enough such that $\hat{u}_{A,h} \geq \hat{u}_{A,l}$. This new strategy yields a payoff $(u, \hat{\pi})$. By concavity of the frontier, we have $p\pi_{A,h} + (1-p)\pi_{A,l} \leq p\hat{\pi}_{A,h} + (1-p)\hat{\pi}_{A,l}$. Therefore, by PK_A^M, we have $\hat{\pi} \geq \pi$. Therefore, the new strategy satisfies all the constraints that a PPE payoff must abide by when it is supported by A and yields a weakly higher payoff for the manager. This means that, when $IC_{A,G}^W$ is slack, either (u, π) is not on the frontier of $\mathcal{E}$, which is a contradiction, or we can construct a new PPE where $IC_{A,G}^W$ binds and both players get the same payoff as before. Since $IC_{A,G}^W$ binds, together with PK_A^W, we can get

$$u_{A,l}(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}$$

and

$$u_{A,h}(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}(w-c).$$

By SE_A, the worker's continuation payoff must satisfy $0 \leq u_{A,l}(u) < u_{A,h}(u) \leq w$. This means that $u \in [(1-\delta)w, w-(1-\delta)c]$. Therefore, $u_{A,l}(u) < u$ since $u < w$. When $u > w - c$, it can be verified that $u_{A,h}(u) > u$. When $u_{A,h}(u) > u$, it must be $u > w - c$.

Part (iv): When shirking is used, the worker gets w and the manager gets nothing.

Neither of them will deviate. Directly from PK_S^W, we have

$$u_S(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}w.$$

By SE_S, we have $u_S(u) \in [0, w]$. Hence, the worker's payoff must satisfy $u \in [(1-\delta)w, w]$. It is easy to verify that $u_S(u) < u$.

Part (v): When outside option is used, the players' payoff is zero. Hence, neither of them has any incentive to deviate. Directly from PK_O^W, we have

$$u_O(u) = \frac{1}{\delta}u.$$

By SE_O, we have $u_O(u) \in [0, w]$, which leads to $u \in [0, \delta w]$. It is easy to verify that $u_O(u) > u$. ■

Lemma B.3. *$\pi(w) = 0$ and $(w, 0)$ is sustained by shirking. Furthermore, if for some $\tilde{u} < w$, $(\tilde{u}, \pi(\tilde{u}))$ is sustained by S, then for all $u \geq \tilde{u}$, $\pi(u) = f_S(u)$. Hence, there exists a cutoff $\overline{u}^A$ such that $\pi(u)$ is a straight line between $(\overline{u}^A, \pi(\overline{u}^A))$ and $(w, 0)$, and $\pi(u) = f_A(u)$ if and only if $u \geq (1-\delta)w + \delta\overline{u}^A$.*

Proof of Lemma B.3. **Step 1.** As $(w, 0)$ is an extreme point, it must be supported by a pure action. In the proof of Lemma 1, we discussed the range of u when each pure action is used. Hence, when $u = w$, the frontier can only be sustained by L. This is because w is the highest payoff for the worker, and it can only be obtained by using S forever. Hence, we have $\pi(w) = 0$. So the unique PPE that supports $(w, \pi(w))$ is one where the worker never exerts effort. Hence, $(w, \pi(w)) = (w, 0)$.

Step 2. From Lemma 1 and (PK_S^M) we have

$$f_S(u) = (1-\delta)w + \delta\pi(u_S(u)) = (1-\delta)w + \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w)$$

for all $u \in [(1-\delta)w, w]$ (i.e., for all u where $f_S(u)$ is well-defined). Hence,

$$f'_{S+}(u) = \pi'_+(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w) \geq \pi'_+(u)$$

for all $u \in ((1-\delta)w, w)$, where the inequality follows from the concavity of $\pi(u)$. But as $\pi(\tilde{u}) = f_S(\tilde{u})$, this implies that $f_S(u) \geq \pi(u)$ for all $u \geq \tilde{u}$. But as $\pi(u) \geq f_S(u)$, we have $\pi(u) = f_S(u)$ for all $u \geq \tilde{u}$.

Step 3. As $\pi(u) = f_S(u)$ for all $u \geq \tilde{u}$, we have $\pi'_+(u) = f'_{S+}(u)$. So, from step 2 above, $f'_{S+}(u) = \pi'_+(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w) = \pi'_+(u)$, and since $\pi(u)$ is concave, this implies that $\pi(u)$ is a straight line passing through $(w, 0)$ and extends at least up to the point $(\tilde{u}, \pi(\tilde{u}))$. Denote the left-most end-point of this line as $(\overline{u}^A, \pi(\overline{u}^A))$.

Step 4. Take any $(u, \pi(u))$ such that $u \geq (1-\delta)w + \delta\overline{u}^A$. We claim that such a payoff is sustainable by S. Note that the associated continuation payoffs $(u_S, \pi_S) = (\frac{1}{\delta}u - \frac{1-\delta}{\delta}w, \pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w))$, and hence, (SE_S) is satisfied. Finally, (PK_S^M) holds as $\pi(u) = \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w)$ since $\pi(u)$ is linear.

Step 5. But if $u < (1-\delta)w + \delta\overline{u}^A$, then the payoff $u, \pi(u)$ cannot be sustained by S. The argument is as follows. If $\overline{u}^A > 0$, we have

$$\pi(u) > (1-\delta)\pi(w) + \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w) = \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}w).$$

The inequality follows from the fact that $\pi(u')$ is concave and the segment starting from $(w, 0)$ is linear if and only if $u' > \overline{u}^A$ whereas $u < (1-\delta)w + \delta\overline{u}^A$. Also the equality follows from $\pi(w) = 0$. But this implies that PK_S^M is violated, and hence, $(u, \pi(u))$ cannot be supported by S. And if $\overline{u}^S = 0$, the proof is immediate as by PK_S^W any point sustained by S requires $u_S(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}w \geq 0 = \overline{u}^A$. ■

Lemma B.4. *If $f_A(u') \geq f_P(u')$ for some u', then $f_A(u) \geq f_P(u)$ for all $u \geq u'$.*

Proof of Lemma B.4. For any u, where $f_P(u)$ and $f_A(u)$ are well-supported, we have

$$f_A(u) - f_P(u) = -(1-\delta)(1-p)y + \delta[p\pi(u_{A,h}(u)) + (1-p)\pi(u_{A,l}(u)) - \pi(u_P(u))].$$

From Lemma 1, we have

$$\begin{aligned}
u_{A,h}(u) &= \frac{1}{\delta}u - \frac{1-\delta}{\delta}(w-c), \\
u_{A,l}(u) &= \frac{1}{\delta}u - \frac{1-\delta}{\delta}w, \\
u_P(u) &= \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C].
\end{aligned}$$

This implies that

$$f'_{A+} - f'_{P+} = p\pi'(u_{A,h}(u)) + (1-p)\pi'(u_{A,l}(u)) - \pi'(u_P(u)) \geq 0$$

since $u_{A,l} < u_{A,h} < u_P$ and $\pi(u)$ is concave. In other words, if $f_A(u') \geq f_P(u')$ for some u', then $f_A(u) \geq f_P(u)$ for all $u \geq u'$. ■

Lemma B.5. *Adaptive effort is used on the payoff frontier. In particular,* $\overline{u}^A = w - (1-\delta)c$*, and* $(\overline{u}^A, \pi(\overline{u}^A))$ *is sustained by* A*.*

Proof of Lemma B.5. We have shown that $f_A(u)$ is well-defined for $u \in [(1-\delta)w, w-(1-\delta)c]$. We first characterize part of the PPE frontier when A is not used. Then we show that the manager's payoff can be improved by using A at $u = w-(1-\delta)c$. Finally, we show that $\overline{u}^A = w-(1-\delta)c$.

Step 1. Suppose adaptive effort is not used, the upper boundary of the feasible set is the line segment connecting $(w - pc - (1-p)C, y)$ and $(w, 0)$. We claim that the frontier is on the boundary when $u \in [(1-\delta)[w + p(C-c)], w]$. Let $\hat{u} = (1-\delta)[w+p(C-c)]$ and $\hat{\pi} = \frac{\delta w - (1-\delta)p(C-c)}{pc+(1-p)C} y$. It can be verified that $(\hat{u}, \hat{\pi})$ is on the upper boundary of the feasible set. Since $(w, 0)$ is self-enforcing, we just need to show that $(\hat{u}, \hat{\pi})$ belongs to PPE set. By PK_P^W, $u_P(\hat{u}) = \frac{1}{\delta}\hat{u} - \frac{1-\delta}{\delta}[w - pc - (1-p)C]$. Since $\delta \geq \frac{C}{C+w}$ by Assumption 2 (i), $u_P(\hat{u}) \leq w$. By PK_P^M, $\pi_P(\hat{\pi}) = \frac{1}{\delta}\hat{\pi} - \frac{1-\delta}{\delta} y$. It can be verified that $(u_P(\hat{u}), \pi_P(\hat{\pi}))$ lies on the boundary of the feasible set. Hence, SE_P is satisfied. It is easy to verify that other constraints are also satisfied. Therefore, $(\hat{u}, \hat{\pi})$ belongs to PPE set. As a result, the frontier is on the boundary when $u \in [(1-\delta)[w + p(C-c)], w]$.

Step 2. Let $\tilde{u} = w - (1-\delta)c$. If adaptive effort is not used, the manager's payoff is $\frac{(1-\delta)c}{pc+(1-p)C} y$ when $u = \tilde{u}$. We now show that the manager's payoff can be improved by using A. By Lemma 1, $u_{A,h}(\tilde{u}) = w$ and $u_{A,l}(\tilde{u}) = w - \frac{1-\delta}{\delta}c$. It can be shown that $u_{A,l}(\tilde{u}) > w - c > \hat{u}$, where the first inequality is from Assumption 2 (i), and the second inequality is from Assumption 2 (ii). Hence, $\pi(u_{A,h}(\tilde{u})) \geq 0$ while $\pi(u_{A,l}(\tilde{u})) \geq \frac{(1-\delta)c}{\delta[pc+(1-p)C]} y$ based on the frontier in step 1 above. By PK_A^M, we have $\pi(\tilde{u}) \geq (1-\delta)y + (1-p)\frac{(1-\delta)c}{pc+(1-p)C} y > \frac{(1-\delta)c}{pc+(1-p)C} y$. Therefore, the manager's payoff can be improved by using A at $\tilde{u} = w - (1-\delta)c$.

Step 3. We claim that A is used when $u = \overline{u}^A$. If not, $(\overline{u}^A, \pi(\overline{u}^A))$ must be sustained by P as F and O will yield a lower payoff for the manager. Then the frontier must be a straight line connecting $(w - pc - (1-p)C, y)$ and $(w, 0)$, which can be improved by using A as is shown in step 2 above. Then we show that $\overline{u}^A = w - (1-\delta)c$.

Step 4. Suppose $\overline{u}^A > \tilde{u} = w - (1-\delta)c$. As $(\overline{u}^A, \pi(\overline{u}^A))$ is an extreme point, it must be sustained by a pure action. However, we have shown in the proof of Lemma 1 that the frontier can only be sustained by S when $u > \tilde{u}$. Meanwhile, we have shown that $(\overline{u}^A, \pi(\overline{u}^A))$ cannot be sustained by S. Hence, it must be $\overline{u}^A \leq \tilde{u} = w - (1-\delta)c$.

Step 5. Suppose $\overline{u}^A < w - (1-\delta)c$. Then $u_{A,h}(\overline{u}^A) < w$. Let s be the slope of the line segment between $(\overline{u}^A, \pi(\overline{u}^A))$ and $(w, 0)$. By the definition of $\overline{u}^A$ and the concavity of $\pi(u)$, we have $\pi'_-(\overline{u}^A) > \pi'_+(\overline{u}^A) = s$. Since $u_{A,l}(\overline{u}^A) < \overline{u}^A$, we have $\pi'_+(u_{A,l}(\overline{u}^A)) \geq \pi'_-(\overline{u}^A) > s$ by the concavity of $\pi(u)$. We also have $\pi'_{A,h}(\overline{u}^A) \geq s$ by concavity of $\pi(u)$. Hence, we have

$$f'_{A+}(\overline{u}^A) = p\pi'_+(u_{A,h}(\overline{u}^A)) + (1-p)\pi'_+(u_{A,l}(\overline{u}^A)) > ps + (1-p)s = s = \pi'_+(\overline{u}^A)$$

Since $\pi(\overline{u}^A) = f_S(\overline{u}^A)$ by step 3 above, we must have $f_S(u) > \pi(u)$ for some $u > \overline{u}^A$. This contradicts the definition of $\pi(u)$. Hence, $\overline{u}^A = w - (1-\delta)c$. ■

To characterize the frontier, we first check the case where forced effort is not available.

Lemma B.6. *$\pi(0) = 0$ and $(0,0)$ is sustained by outside option. Furthermore, if for some $\tilde{u} > 0$, $(\tilde{u}, \pi(\tilde{u}))$ is sustained by O, then for all $u \leq \tilde{u}$, $\pi(u) = f_O(u)$. Hence, there exists a cutoff $\underline{u}^P$ such that $\pi(u)$ is a straight line between $(0,0)$ and $(\underline{u}^P, \pi(\underline{u}^P))$, and $\pi(u) = f_O(u)$ if and only if $u \leq \delta\underline{u}^P$.*

Proof of Lemma B.6. **Step 1.** As $(0, \pi(0))$ is an extreme point, it must be sustained by a pure action. From the proof of lemma 1, we have shown that P, A or S cannot be used when $u = 0$. Since F is not used in this scenario, we know that $(0, \pi(0))$ must be sustained by O. From PK_O^W we have $u_O(0) = \frac{1}{\delta}0 = 0$. So, the unique PPE that supports $(0, \pi(0))$ is one where both players take their outside options in all periods. Hence, $(0, \pi(0)) = (0, 0)$.

Step 2. From Lemma 1 and PK_O^M we have

$$f_O(u) = \delta\pi(u_O(u)) = \delta\pi(u/\delta)$$

for all $u \in [0, \delta w]$ (i.e., for all u where $f_O(u)$ is well-defined). Hence,

$$f'_{O-}(u) = \pi'_-(u/\delta) \leq \pi'_-(u)$$

for all $u \in (0, \delta w)$, where the inequality follows from the concavity of $\pi(u)$. But as $\pi(\tilde{u}) = f_O(\tilde{u})$, this implies that $f_O(u) \geq \pi(u)$ for all $u \leq \tilde{u}$. But as $\pi(u) \geq f_O(u)$, we have $\pi(u) = f_O(u)$ for all $u \leq \tilde{u}$.

Step 3. As $\pi(u) = f_O(u)$ for all $u \leq \tilde{u}$, we have $\pi'_-(u) = f'_{O-}(u)$. So, from step 2 above, $f'_{O-}(u) = \pi'_-(u/\delta) = \pi'_-(u)$, and since $\pi(u)$ is concave, this implies that $\pi(u)$ is a straight line passing through $(0,0)$ and extends at least up to the point $(\tilde{u}, \pi(\tilde{u}))$. Denote the right-most end-point of this line as $(\underline{u}^P, \pi(\underline{u}^P))$.

Step 4. Take any $(u, \pi(u))$ such that $u/\delta \leq \underline{u}^P$. We claim that such a payoff is sustainable by O. Note that the associated continuation payoffs $(u_O, \pi_O) = (u/\delta, \pi(u\delta))$, and hence, SE_O is satisfied. Finally, PK_O^M holds as $\pi(u) = \delta\pi(u/\delta)$ since $\pi(u)$ is linear.

Step 5. But if $u/\delta > \underline{u}^P$, then the payoff $(u, \pi(u))$ cannot be sustained by O. The argument is as follows. If $\underline{u}^P < w$, we have

$$\pi(u) > (1-\delta)\pi(0) + \delta\pi(u/\delta) = \delta\pi(u/\delta).$$

The inequality follows from the fact that $\pi(u')$ is concave and the segment starting from $(0,0)$ is linear if and only if $u' \le \underline{u}^P$ whereas $u/\delta > \underline{u}^P$. Also the equality follows from $\pi(0) = 0$. But this implies that PK_O^M is violated, and hence, $(u, \pi(u))$ cannot be supported by O. And if $\underline{u}^P = w$, the proof is immediate as by PK_O^W any point sustained by O requires $u_O(u) = u/\delta \le w = \underline{u}^P$. ∎

Lemma B.7. *Proactive effort is used on the payoff frontier. In particular,* $\underline{u}^P = (1-\delta)[w + p(C-c)]$ *and* $(\underline{u}^P, \pi(\underline{u}^P))$ *is sustained by* P. *Furthermore, the frontier is not supported by* A *for* $u \le \underline{u}^P$.

Proof of Lemma B.7. Suppose to the contrary that proactive effort is not used. We first show that the frontier can be improved by using P at $u = (1-\delta)[w + p(C-c)]$. Then we show that $\underline{u}^P = (1-\delta)[w + p(C-c)]$.

Step 1. Since P is not used, the upper boundary of the feasible set when $0 \le u \le w - pc$ is the straight line connecting $(0,0)$ and $(w - pc, py)$. Let $\tilde{u} = (1-\delta)[w + p(C-c)]$ and $\tilde{\pi} = \frac{p[w+p(C-c)]}{w-pc}(1-\delta)y$. It can be verified that $(\tilde{u}, \tilde{\pi})$ is on the upper boundary of the feasible set. This implies that the manager's payoff is at most $\tilde{\pi}$ is P is not used. In the proof of Lemma 1, we have shown that $f_P(u)$ is well-defined when $u = \tilde{u}$. By PK_P^M, we have

$$f_P(\tilde{u}) = (1-\delta)y + \delta\pi(u_P(\tilde{u})) \ge (1-\delta)y > \tilde{\pi}.$$

The first inequality is from the fact that $\pi(u_P(\tilde{u})) \ge 0$ and the second inequality is from Assumption 1 (ii). Therefore, the frontier can be improved by using P when $u = \tilde{u}$.

Step 2. Recall that $(\underline{u}^P, \pi(\underline{u}^P))$ is denoted in Lemma B.6 as the right-most end-point of the straight line starting from $(0,0)$. Suppose that $\underline{u}^P < (1-\delta)[w + p(C-c)]$, then $(\underline{u}^P, \pi(\underline{u}^P))$ must be sustained by A. Let s be the slope of the line segment between $(0,0)$ and $(\underline{u}^P, \pi(\underline{u}^P))$. From Assumption 2 (ii), we know that $(1-\delta)[w + p(C-c)] < w - c$. From Lemma 1 and PK_A^W, we have $u_{A,l}(\underline{u}^P) < u_{A,h}(\underline{u}^P) < \underline{u}^P$. By Lemma 1 and PK_A^W, we have

$$f'_{A+}(\underline{u}^P) = p\pi'_+(u_{A,h}(\underline{u}^P)) + (1-p)\pi'_+(u_{A,l}(\underline{u}^P)) = ps + (1-p)s = s > \pi'_+(\underline{u}^P).$$

The inequality is from the concavity of $\pi(u)$ and the fact that $(\underline{u}^P, \pi(\underline{u}^P))$ is the right-most point of the straight line. This is a contradiction as $f'_{A+}(\underline{u}^P)$ cannot be larger than $\pi'_+(\underline{u}^P)$ given $f_A(\underline{u}^P) = \pi(\underline{u}^P)$. ∎

Lemma B.8. *The PPE payoff frontier $\pi(u)$ can be divided into five regions. There exist cutoffs $0 < \underline{u}^P \leq \overline{u}^P \leq \underline{u}^A \leq \overline{u}^A < w$, with $\underline{u}^P < \underline{u}^A$, such that:*

(*i*) *For $u \in [0, \underline{u}^P)$, the payoff frontier is linear and supported by randomization between $(0,0)$ and $(\underline{u}^P, \pi(\underline{u}^P))$. We have $\pi(0) = 0$ and the payoff $(0,0)$ is supported by O (i.e., players taking the outside option).*

(*ii*) *For $u \in [\underline{u}^P, \overline{u}^P]$, $\pi(u) = f_P(u)$ (i.e., the payoff frontier is supported by proactive effort).*

(*iii*) *For $u \in (\overline{u}^P, \underline{u}^A)$, the frontier is linear and supported by randomization between $(\overline{u}^P, \pi(\overline{u}^P))$ and $(\underline{u}^A, \pi(\underline{u}^A))$.*

(*iv*) *For $u \in [\underline{u}^A, \overline{u}^A]$, $\pi(u) = f_A(u)$ (i.e., the payoff frontier is supported by adaptive effort).*

(*v*) *For $u \in (\overline{u}^A, w]$, the frontier is linear and supported by randomization between $(\overline{u}^A, \pi(\overline{u}^A))$ and $(w, 0)$. We have $\pi(w) = 0$ and the payoff $(w, 0)$ is supported by S (i.e., the worker always shirks).*

Proof of Lemma B.8. For expositional clarity, we will prove parts (i) and (v) first, then prove part (ii) and (iv), and finally prove part (iii).

Part (i): That $\pi(0) = 0$, the existence of $\underline{u}^P$, and the linearity of $\pi(u)$ between $(0,0)$ and $(\underline{u}^P, \pi(\underline{u}^P))$ are proved in Lemma B.6. By virtue of linearity, any payoff in this line segment can be supported by randomization between the two end points. Hence, without loss of generality, we can also assume that O is played on the frontier only to support $(0,0)$ payoff.

Part (v): That $\pi(0) = 0$, the existence of $\overline{u}^A$, and the linearity of $\pi(u)$ between $(\overline{u}^A, \pi(\overline{u}^A))$ and $(w, 0)$ are proved in Lemma B.3. Similarly, we can assume that S is played on the frontier only to support $(w, 0)$ payoff.

Part (ii) and Part (iv): Lemma B.5 shows that $\overline{u}^A = w - (1-\delta)c$ and that $(\overline{u}^A, \pi(\overline{u}^A))$ is sustained by A. Lemma B.7 shows that $\underline{u}^P = (1-\delta)[w + p(C-c)]$ and that $(\underline{u}^P, \pi(\underline{u}^P))$ is sustained by P. Lemma B.7 also implies that A will not be used for $u \leq \underline{u}^P$, meaning that $\underline{u}^P < \underline{u}^A$. Lemma B.4 implies (along with the fact that $f_P(u)$ and $f_A(u)$ are concave functions) that the set of u values such that $(u, \pi(u))$ is supported by each of these two actions (P and A) are intervals (potentially containing a single point only) on $[(1-\delta)[w + p(C-c)], w-(1-\delta)c]$. Moreover, if $(u, \pi(u))$ is supported by P and $(u', \pi(u'))$ is supported by A, then it must be that $u' > u$. Note that there might exist $\tilde{u} \in ((1-\delta)[w+p(C-c)], w-(1-\delta)c)$ such that $(\tilde{u}, \pi(\tilde{u}))$ can be sustained by both P and A. In that case, let $\overline{u}^P = \underline{u}^A$ be the left most value of u such that $(u, \pi(u))$ can be sustained by A.

Part (iii): If $\overline{u}^P < \underline{u}^A$, it implies that any payoff $(u, \pi(u))$ where $u \in (\overline{u}^P, \underline{u}^A)$ cannot be supported by any of the pure actions. But such $(u, \pi(u)) \in \mathcal{E}$ as $\mathcal{E}$ is convex. So, $(u, \pi(u))$

must be supported by randomization between a payoff that is supported by P and one that is supported by A. Consequently, $\pi(u)$ is linear on this interval. Also, it is without loss of generality to assume that we randomize between the end points $(\overline{u}^P, \pi(\overline{u}^P))$ and $(\underline{u}^A, \pi(\underline{u}^A))$. ■

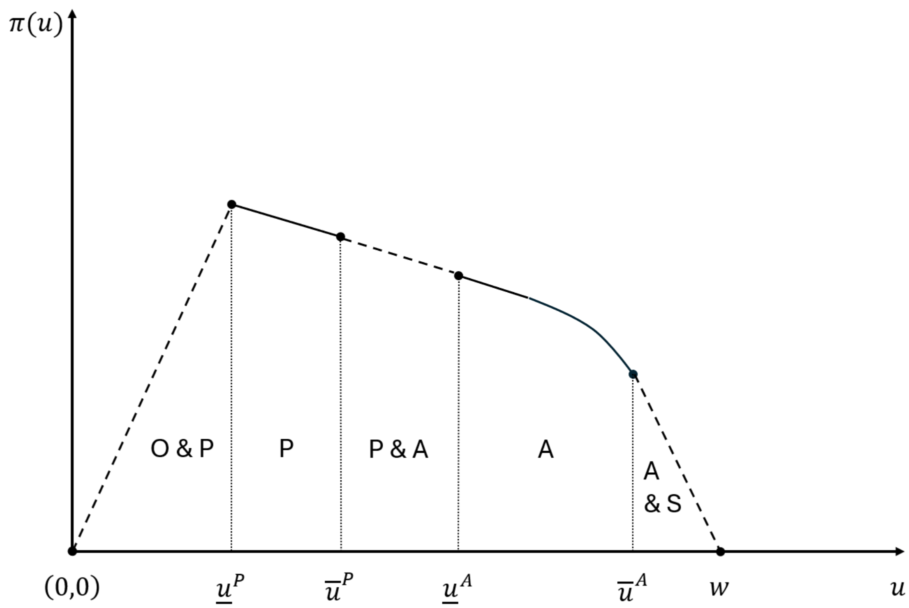

Figure 5: Benchmark Frontier

Figure 5 illustrates the five regions described in Lemma B.8. In two regions in the middle, the payoffs at the frontier are supported by pure actions, by playing P and A, respectively. Also, the $(0, 0)$ payoff pair is the only point on the frontier that is supported by playing O, and the $(w, 0)$ payoff pair is the only point on the frontier that is supported by playing S. In the other three regions, the payoffs are sustained through randomization.

Lemma B.9. *For $u \in [(1-\delta)[w + p(C - c)], w - c]$, the frontier is linear. Furthermore, the payoff $(w - c, \pi(w - c))$ is supported by A.*

Proof of Lemma B.9. We first show that $\pi(u)$ is linear for $u \in [\underline{u}^P, \underline{u}^A]$ (recall that $\underline{u}^P = (1-\delta)[w + p(C - c)]$). Then we show that $(w - c, \pi(w - c))$ is supported by A. Finally, we show that the frontier is linear for $u \in [(1-\delta)[w + p(C - c)], w - c]$.

Step 1. We consider two scenarios: $\underline{u}^P = \overline{u}^P$ and $\underline{u}^P < \overline{u}^P$.

Step 1.1. Suppose that that $\underline{u}^P = \overline{u}^P$. From Lemma B.8, we know that the frontier is supported by randomization for $u \in [\overline{u}^P, \underline{u}^A]$. Since $\underline{u}^P = \overline{u}^P$, the frontier is linear for $u \in [\underline{u}^P, \underline{u}^A]$.

Step 1.2. Now we consider the other scenario: $\underline{u}^P < \overline{u}^P$. From Lemma 1 and PK_A^M we have

$$f_P(u) = (1-\delta)y + \delta\pi(u_P(u)) = (1-\delta)y + \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C])$$

for all $u \in [\underline{u}^P, \overline{u}^P]$. Hence,

$$f'_{P-}(u) = \pi'_-(u_P(u)) \leq \pi'_-(u)$$

for all $u \in (\underline{u}^P, \overline{u}^P]$. The inequality follows from the concavity of $\pi(u)$ and the fact that $u_P(u) > u$. As $\pi(\overline{u}) = f_P(\overline{u})$, this implies that $\pi(u) \leq f_P(u)$ for all $u \in [\underline{u}^P, \overline{u}^P]$. But as $\pi(u) \geq f_P(u)$, we have $\pi(u) = f_P(u)$ for all $u \in [\underline{u}^P, \overline{u}^P]$. Hence, we have $f'_{P-}(u) = \pi'_-(u_P(u)) = \pi'_-(u)$. Since $\pi(u)$ is concave, $\pi(u)$ must be a straight line for $u \in [\underline{u}^P, u_P(\overline{u}^P)]$. But we know that $u_P(\overline{u}^P) > \overline{u}^P$ and $\pi(u)$ is a straight line for $u \in [\overline{u}^P, \underline{u}^A]$ by randomization. Hence, the frontier must be linear for $u \in [\underline{u}^P, \underline{u}^A]$.

Step 2. Suppose that $(w-c, \pi(w-c))$ is not supported by A. Then we must have $\underline{u}^A > w - c$. For any $u \in [w-c, \underline{u}^A]$, from Lemma 1 and PK_A^M we have

$$f_A(u) = (1-\delta)py + \delta[p\pi(u_{A,h}(u)) + (1-p)\pi(u_{A,l}(u))].$$

Notice that $\underline{u}^P < u_{A,l}(u) < u \leq u_{A,h}(u)$. The first inequality follows from Assumption 2 (ii) and the fact that $w - c \leq u$. The last two inequalities directly follows from Lemma 1. Hence,

$$f'_{A-}(u) = p\pi'_-(u_{A,h}(u)) + (1-p)\pi'_-(u_{A,l}(u)) \leq \pi'_-(u_{A,h}(u)) = \pi'_-(u).$$

The inequality follows from the concavity of $\pi(u)$, and the last equality follows from that $\pi(u)$ is linear since it is supported by randomization for $u \in (\underline{u}^P, \underline{u}^A)$. Since $\pi(\underline{u}^A) = f_A(\underline{u}^A)$, this implies that $f_A(u) \geq \pi(u)$ for $u \in [w-c, \underline{u}^A]$. But as $\pi(u) \geq f_A(u)$, we must have $\pi(u) = f_A(u)$ for $u \in [w-c, \underline{u}^A]$. Namely, $(w-c, \pi(w-c))$ is supported by A. This implies that $\underline{u}^A \leq w - c$.

Step 3. Suppose that the frontier is not linear for $u \in [(1-\delta)[w + p(C-c)], w-c]$. Let $(\hat{u}, \pi(\hat{u}))$ be the left-most point where $\pi'_-(\hat{u}) > \pi'_+(\hat{u})$. Notice that $\underline{u}^A \leq \hat{u} < w - c$. Similarly, from Lemma 1, we have

$$\pi'_+(\hat{u}) \geq f'_{A+}(\hat{u}) = p\pi'_+(u_{A,h}(\hat{u})) + (1-p)\pi'_+(u_{A,l}(\hat{u})) \geq \pi'_-(\hat{u}).$$

The first inequality follows from the concavity of $\pi(u)$ and the second inequality follows from the fact that $u_{A,l}(\hat{u}) < u_{A,h}(\hat{u}) < \hat{u}$ as $\hat{u} < w - c$. Hence, the frontier is linear for

$u \in [(1-\delta)[w + p(C-c)], w-c]$. ■

Lemma B.10. *Let (u^*, π^*) denote the payoff pair where the manager gets the highest payoff. When forced effort is not available, we must have $u^* = \underline{u}^P$.*

Proof of Lemma B.10. **Step 1.** Lemma B.8 has shown that O is played on the frontier only to support $(0,0)$ and that S is played on the frontier only to support $(w,0)$. Hence, we must have $u^* \in [\underline{u}^P, \overline{u}^A]$. Lemma B.9 has shown that the frontier is a straight line between $(\underline{u}^P, \pi(\underline{u}^P))$ and $(w-c, \pi(w-c))$. Now we show that $u^* = \underline{u}^P$ by contradiction.

Step 2. Suppose that u^* is not $\underline{u}^P$, then we must have $u^* \geq w - c$ by concavity of $\pi(u)$. Notice that if $\pi(\underline{u}^P) = \pi(w-c)$, we just let $u^* = w - c$. Since $w - c \leq u^* < w$, we must have

$$\pi(\underline{u}^P) > (1-\delta)y + \delta \frac{w-(1-\delta)C/\delta}{w-u^*}\pi(u^*),$$

where the inequality follows from PK_P^M, the concavity of $\pi(u)$, and the fact that $\pi(w) = 0$.

Step 3. To show that $\pi(\underline{u}^P) > \pi(u^*)$, we just need to show that

$$(1-\delta)y \geq [1 - \delta \frac{w-(1-\delta)C/\delta}{w-u^*}]\pi(u^*).$$

Since $(u^*, \pi(u))$ is supported by A, we must have $\pi(u^*) \leq py$. Hence, we just need to show that

$$1 - \delta \geq [1 - \delta \frac{w-(1-\delta)C/\delta}{w-u^*}]p.$$

Since $u^* \geq w - c$, we just need to show that

$$c(1-\delta) \geq [c - \delta w + (1-\delta)C]p,$$

which is guaranteed by Assumption 2 (iii). ■

Now we can characterize the frontier when forced effort is available to support the frontier. Before proving Lemma 1, we need the following lemmas.

Lemma B.11. *If the PPE payoff pair $((1-\delta)[w+p(C-c)], \pi((1-\delta)[w+p(C-c)]))$ is supported by proactive effort, then any PPE payoff pair $(u, \pi(u))$ satisfying $u \geq (1-\delta)[w+p(C-c)]$ can be sustained as an equilibrium which does not involve forced effort.*

Proof of Lemma B.11. **Step 1.** We first show that $(u, \pi(u))$ is not supported by F if $u \geq (1-\delta)[w+p(C-c)]$. Notice that $f_F(u)$ and $f_P(u)$ is well-defined on $u \in [(1-\delta)[w+p(C-c)], w-(1-\delta)[pc+(1-p)C]]$. Suppose that $(\tilde{u}, \pi(\tilde{u}))$ is supported by F. From PK_F^M and

PK_P^M we must have $f_P(\tilde{u}) \geq f_F(\tilde{u}) = \pi(\tilde{u})$. Other constraints are the same. Hence, we can always use P to support $(\tilde{u}, \pi(\tilde{u}))$ if it can be supported by F.

Step 2. We then show that the worker's continuation payoff satisfies $u_j(u) \geq (1-\delta)[w + p(C-c)]$, where $j \in \{P, A, S\}$. Notice that if $(u, \pi(u))$ is supported by randomization, it must be randomizing between P and A or A and S. Hence, we just need to focus on the pure actions. From Lemma 1 we know that $u_P(u) > u \geq (1-\delta)[w + p(C-c)]$. From Lemma B.3 we know that $u_S(u) \geq \overline{u}^A > (1-\delta)[w + p(C-c)]$. Hence, we just need to show that $u_{A,l}(u) \geq (1-\delta)[w + p(C-c)]$ as $u_{A,h} > u_{A,l}$.

Step 3. From Lemma 1, $u_{A,l}(u)$ increases in u. Hence, we just need to show that $u_{A,l}(\underline{u}^A) \geq (1-\delta)[w + p(C-c)]$. Suppose on the contrary that $u_{A,l}(\underline{u}^A) < (1-\delta)[w + p(C-c)]$. Since $((1-\delta)[w + p(C-c)], \pi((1-\delta)[w + p(C-c)]))$ is supported by P, we have $(1-\delta)[w + p(C-c)] = \underline{u}^P$ by Lemma B.7. From Lemma 1, we have

$$f'_{A+}(\underline{u}^A) = p\pi'_+(u_{A,h}(\underline{u}^A)) + (1-p)\pi'_+(u_{A,l}(\underline{u}^A)) > \pi'_+(\underline{u}^A),$$

where the inequality follows from the concavity of $\pi(u)$ and the fact that $u_{A,l}(\underline{u}^A) < u_{A,h}(\underline{u}^A) < \underline{u}^A < w - c$. Since $\pi(\underline{u}^A) = f_A(\underline{u}^A)$, this implies $f_A(u) > \pi(u)$ for $u > \underline{u}^A$. This is a contradiction. Hence, we must have $u_{A,l}(u) \geq (1-\delta)[w + p(C-c)]$. ■

Lemma B.12. *Suppose proactive effort is used to support the frontier. For $u \in (w-c, \overline{u}^A)$, $\pi'_+(u) < \pi'_+(w-c)$. Furthermore, if $\underline{u}^P < \overline{u}^P$, then $u_P(\overline{u}^P) = w - c$ and the frontier for $u \in [\underline{u}^P, w-c]$ falls on the light segment connecting $(w - pc - (1-p)C, y)$ and $(w - pc, py)$.*

Proof of Lemma B.12. **Step 1.** From Lemma B.9, we know that the frontier is linear for $u \in [\underline{u}^P, w-c]$. Suppose that there exists $\tilde{u} > w - c$ such that $\pi'_-(\tilde{u}) = \pi'_-(w-c)$. By concavity, the frontier must be a straight line for $u \in [\underline{u}^P, \tilde{u}]$. From Lemma 1 and PK_A^M, we have

$$\pi'_-(\tilde{u}) = p\pi'_-(u_{A,h}(\tilde{u})) + (1-p)\pi'_-(u_{A,l}(\tilde{u})),$$

for $u \in (\underline{u}^A, \overline{u}^A]$. Since $\tilde{u} > w - c$, we have $\underline{u}^P < u_{A,l}(\tilde{u}) < \tilde{u} < u_{A,h}(\tilde{A})$. This implies that

$$\pi'_-(u_{A,h}(\tilde{u})) = \pi'_-(w-c),$$

meaning that the frontier is a straight line for $u \in [\underline{u}^P, u_{A,h}(\tilde{u})]$. Since $u_{A,h}(\tilde{u}) > \tilde{u}$, the same logic applies to any $u \in [\underline{u}^A, \overline{u}^A]$. However, we know that $(\overline{u}^A, \pi(\overline{u}^A))$ is the left-most end-point a straight line, which means $\pi'_-(\overline{u}^A) > \pi'_-(u_{A,h}(\overline{u}^A))$. Hence, for $u \in (w-c, \overline{u}^A]$, we must have $\pi'_+(u) < \pi'_+(w-c)$.

Step 2. Suppose that $u_P(\overline{u}^P) > w - c$. We know that from Step 1 above, $\pi'_-(\overline{u}^P) > \pi'_-(u_P(\overline{u}^P))$, which contradicts the result from Step 1.2 of Lemma B.9. Now we assume that $u_P(\overline{u}^P) < w-c$. Then let $\hat{u} = \overline{u}^P + \delta\varepsilon$, where $0 < \varepsilon < w-c-u_P(\overline{u}^P)$. We claim that $(\hat{u}, \pi(\hat{u}))$ can be supported by P. From PK_P^W, we have $u_P(\hat{u}) = u_P(\overline{u}^P) + \varepsilon < w - c$. Hence, SE_P is satisfied. It can be verified that other constraints are also satisfied. Therefore, $(\hat{u}, \pi(\hat{u}))$ is also a PPE supported by P, which contradicts the definition of $\overline{u}^P$. Hence, we must have $u_P(\overline{u}^P) = w - c$.

Step 3. Since the frontier is a straight line for $u \in [\underline{u}^P, w-c]$, we know that $(\overline{u}^P, \pi(\overline{u}^P))$, $(w-c, \pi(w-c))$, $(u_{A,l}(w-c), \pi(u_{A,l}(w-c)))$, and $(u_{A,h}(w-c), \pi(u_{A,h}(w-c)))$ are on the same line. Together with PK_A^W and PK_A^M, it is easy to verify that $(\overline{u}^P, \pi(\overline{u}^P))$, $(w-c, \pi(w-c))$, and $(w-pc, py)$ are on the same line. Similarly, we must have $(w-pc-(1-p)C, y)$, $(\overline{u}^P, \pi(\overline{u}^P))$, and $(w-c, \pi(w-c))$ are on the same line. Therefore, the frontier for $u \in [\underline{u}^P, w-c]$ falls on the light segment connecting $(w-pc-(1-p)C, y)$ and $(w-pc, py)$. ■

Lemma B.13. *If $\pi(0) > 0$, then $(0, \pi(0))$ is sustained by F. Furthermore, if for some $\tilde{u} > 0$, $(\tilde{u}, \pi(\tilde{u}))$ is sustained by F, then for all $u \le \tilde{u}$, $\pi(u) = f_F(u)$. Hence, if P is used to support the frontier, there exists a cutoff $\underline{u}^P$ such that $\pi(u)$ is a straight line between $(0, \pi(0))$ and $(\underline{u}^P, \pi(\underline{u}^P))$, and $\pi(u) = f_F(u)$ if and only if $0 \le u \le \max\{(1-\delta)[w-pc-(1-p)C] + \delta\underline{u}^P, 0\}$. If P is not used to support the frontier, there exists a cutoff $\underline{u}^A$ such that $\pi(u)$ is a straight line between $(0, \pi(0))$ and $(\underline{u}^A, \pi(\underline{u}^A))$, and $\pi(u) = f_F(u)$ if and only if $0 \le u \le \max\{(1-\delta)[w-pc-(1-p)C] + \delta\underline{u}^A, 0\}$.*

***Proof of Lemma B.13*.** **Step 1.** As $(0, \pi(0))$ is an extreme point, it must be supported by a pure action. In the proof of Lemma 1, we have shown that when $u = 0$, it cannot be sustained by P, A, or S. Suppose $(0, \pi(0))$ is sustained by O, $\pi(0)$ must be 0 by PK_O^M. Since $\pi(0) > 0$, it must be supported by F. Next, we will focus on the case when P is used to support the frontier and discuss two scenarios: (i) there exists some $\tilde{u} > 0$, such that $(\tilde{u}, \pi(\tilde{u}))$ is sustained by F; (ii) for all $\tilde{u} > 0$, $(\tilde{u}, \pi(\tilde{u}))$ is not sustained by F.

Step 2. Suppose that there exists some $\tilde{u} > 0$, such that $(\tilde{u}, \pi(\tilde{u}))$ is sustained by F.

Step 2.1. From Lemma 1 and (PK_F^M) we have

$$f_F(u) = (1-\delta)(y-D) + \delta\pi(u_F(u)) = (1-\delta)(y-D) + \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[w-pc-(1-p)C])$$

for all $u \in [0, w-(1-\delta)[pc+(1-p)C]]$ (i.e., for all u where $f_F(u)$ is well-defined). Hence,

$$f'_{F-}(u) = \pi'_-(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[w-pc-(1-p)C]) \le \pi'_-(u)$$

for all $u \in (0, w-(1-\delta)[pc+(1-p)C])$, where the inequality follows from the concavity of

$\pi(u)$. But as $f_F(\tilde{u}) = \pi(\tilde{u})$, this implies that $f_F(u) \geq \pi(u)$ for all $u \leq \tilde{u}$. But as $\pi(u) \geq f_F(u)$, we have $\pi(u) \geq f_F(u)$ for all $u \leq \tilde{u}$.

Step 2.2. As $\pi(u) = f_F(u)$ for all $u \leq \tilde{u}$, we have $f'_{F-}(u) = \pi'_-(u)$. So from step 2.1 above, $f'_{F-}(u) = \pi'_-(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C]) = \pi'_-(u)$, and since $\pi(u)$ is concave, this implies that $\pi(u)$ is a straight line passing through $(0, \pi(0))$ and extends at least up to the point $(\tilde{u}, \pi(\tilde{u}))$. Denote the right-most end-point of this line as $(\underline{u}^P, \pi(\underline{u}^P))$.

Step 2.3. We claim that $(1-\delta)[w-pc-(1-p)C]+\delta\underline{u}^P > 0$. Since $\tilde{u} > 0$, consider a payoff pair $(\varepsilon, \pi(\varepsilon))$ where $0 < \varepsilon < \tilde{u}$. It is on the straight line connecting $(0, \pi(0))$ and $(\tilde{u}, \pi(\tilde{u}))$. Denote the slope of the straight line as s. Hence, it is sustained by F and $\pi'_-(\varepsilon) = \pi{+}'(\varepsilon) = s$. From Lemma 1 and (PK_F^M), we can also get $\pi'_-(u_F(\varepsilon)) = \pi{+}'(u_F(\varepsilon)) = s$. This implies that $\pi'(\varepsilon) = \pi'(u_F(\varepsilon)) = s$. Hence, it must be $\underline{u}^P \geq u_F(\varepsilon)$ by concavity. Since $\varepsilon > 0$, by PK_F^W, we have $\underline{u}^P \geq u_F(\varepsilon) > \frac{1-\delta}{\delta}[pc + (1-p)C - w]$. That is, $(1-\delta)[w - pc - (1-p)C] + \delta\underline{u}^P > 0$.

Step 2.4. Take any $(u, \pi(u))$ such that $u \leq (1-\delta)[w-pc-(1-p)C]+\delta\underline{u}^P$. We claim that such a payoff is sustainable by F. Note that the associated continuation payoffs $(u_F, \pi_F) = (\frac{1}{\delta}u - \frac{1-\delta}{\delta}[pc-(1-p)C-w], \pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[pc-(1-p)C-w]))$, and hence, SE_F is satisfied. Finally, (PK_F^M) for the manager holds as $\pi(u) = (1-\delta)(y-D) + \delta\pi(\frac{1}{\delta}u - \frac{1-\delta}{\delta}[pc - (1-p)C - w])$ since $\pi(u)$ is linear.

Step 2.5. We claim that $(w - pc - (1-p)C, y - D)$, $(0, \pi(0))$ and $(\underline{u}^P, \pi(\underline{u}^P))$ are on the same line. As is in step 2.3 above, we denote the slope between $(0, \pi(0))$ and $(\underline{u}^P, \pi(\underline{u}^P))$ as s. Now we just need to show that the slope between $(w - pc - (1-p)C, y - D)$ and $(0, \pi(0))$ is also s. since $(1-\delta)[w-pc-(1-p)C]+\delta\underline{u}^P > 0$, we have $u_F(0) = \frac{1-\delta}{\delta}(pc+(1-p)C-w) < \underline{u}^P$. This means that the slope between $(0, \pi(0))$ and the associated continuation payoffs $(u_F(0), \pi(u_F(0)))$ is also s. By PK_F^M and PK_F^W, we have

$$
\begin{aligned}
s &= (\pi(u_F(0)) - \pi(0))/u_F(0) \\
&= \left[\frac{1}{\delta}\pi(0) - \frac{1-\delta}{\delta}(y-D) - \pi(0)\right] / \left\{\frac{1-\delta}{\delta}[pc - (1-p)C - w]\right\} \\
&= [(y-D) - \pi(0)]/[w - pc - (1-p)C].
\end{aligned}
$$

This shows that the slope between $(w-pc-(1-p)C, y-D)$ and $(0, \pi(0))$ is also s, implying that $(w - pc - (1-p)C, y - D)$, $(0, \pi(0))$ and $(\underline{u}^P, \pi(\underline{u}^P))$ are on the same line.

Step 2.6. But if $u > (1-\delta)[w - pc - (1-p)C] + \delta\underline{u}^P$, then the payoff $(u, \pi(u))$ cannot be sustained by F. The argument is as follows. If $\underline{u}^P < w$, we have

$$\pi(u) > (1-\beta_1)\pi(0) + \beta_1\pi(u_F(u)),$$

where $\beta_1 = \delta u / \{u - (1-\delta)[w - pc - (1-p)C]\}$, and $u_F(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C]$

by PK_F^W. The inequality is from the fact that $\pi(u')$ is concave and the segment starting from $(0, \pi(0))$ is linear if and only if $u' \leq \underline{u}^P$ whereas $u_F(u) > \underline{u}^P$. From step 2.5, we know that $(w - pc - (1-p)C, y - D)$, $(0, \pi(0))$ and $(u, \pi(u))$ are on the same line. Hence, we have

$$\pi(0) = (1 - \beta_2)[w - pc - (1-p)C] + \beta_2 u,$$

where $\beta_2 = [pc + (1-p)C - w]/[u - w + pc + (1-p)C]$. This means that

$$\pi(u) > (1 - \delta)(y - D) + \delta\pi(u_F(u)).$$

But this implies that PK_F^M is violated, and hence, $(u, \pi(u))$ cannot be supported by F. And if $\underline{u}^P = w$, the proof is immediate as by (PK_F^W) any point sustained by F requires $u_F(u) = \frac{1}{\delta}u - \frac{1-\delta}{\delta}[w - pc - (1-p)C] \leq w = \underline{u}^P$.

Step 3. Now we suppose that for any $\tilde{u} > 0$, $(\tilde{u}, \pi(\tilde{u}))$ is not sustained by F.

Step 3.1. We claim that the frontier $\pi(u)$ is supported by randomization when $0 < u < (1-\delta)[w + p(C - c)]$. In the proof of Lemma 1, we have shown that the frontier cannot be sustained by pure actions P or S when $0 < u < (1-\delta)[w + p(C - c)]$. By Lemma B.7, we know that the frontier cannot be sustained by Since it is not sustained by F, we just need to show that it cannot be sustained by O. Suppose there exists $0 < \tilde{u} < (1-\delta)[w + p(C - c)]$, such that $\tilde{u}$ is sustained by O. By concavity of $\pi(u)$, we have

$$\pi_+^{'}(\tilde{u}) \geq (\pi(u_O(\tilde{u})) - \pi(\tilde{u}))/(u_O(\tilde{u}) - \tilde{u}) = \pi(\tilde{u})/\tilde{u}.$$

Since $(0, \pi(0))$ is sustained by F and $\pi(0) > 0$, we have

$$\pi_-^{'}(\tilde{u}) \leq (\pi(\tilde{u}) - \pi(0))/(\tilde{u} - 0) < \pi(\tilde{u})/\tilde{u}.$$

This implies that $\pi_-^{'}(\tilde{u}) < \pi_+^{'}(\tilde{u})$, which contradicts the concavity of $\pi(u)$. Hence, $\pi(u)$ is supported by randomization when $0 < u < (1-\delta)[w + p(C - c)]$. This implies that $\pi(u)$ is a straight line passing through $(0, \pi(0))$ and extends at least up to the point $((1-\delta)[w + p(C - c)], \pi((1-\delta)[w + p(C - c)]))$. Denote the right-most end-point of this line as $(\underline{u}^P, \pi(\underline{u}^P))$.

Step 3.2. In this scenario, $\pi(u) = f_F(u)$ if and only if $u = 0$. Hence, we just need to show that $(1-\delta)[w - pc - (1-p)C] + \delta\underline{u}^P \leq 0$. The argument is as follows. Suppose that $(1-\delta)[w - pc - (1-p)C] + \delta\underline{u}^P > 0$, Consider the following payoff pair $(\varepsilon, \pi(\varepsilon))$, where $0 < \varepsilon < (1-\delta)[w - pc - (1-p)C] + \delta\underline{u}^P$. From PK_F^W, the associated continuation payoff for the worker is $u_F(\varepsilon) = \frac{1}{\delta}\varepsilon - \frac{1-\delta}{\delta}[w - pc - (1-p)C] < \underline{u}^P$, and hence, SE_F is satisfied. Finally, PK_F^M holds as $\pi(\varepsilon) = (1-\delta)(y - D) + \delta\pi(u_F(\varepsilon))$ since $\pi(u)$ is linear when $u \leq \underline{u}^P$.

This implies that $(\varepsilon, \pi(\varepsilon))$ is also supported by F, which is a contradiction. Hence, we have $(1-\delta)[w - pc - (1-p)C] + \delta \underline{u}^P \leq 0$.

Step 4. The same logic also applies to the case when P is not used to support the frontier. In that case, adaptive action will be used when $u \geq \underline{u}^A$. ■

Proposition 1. *There exists a cutoff $\underline{D}(\delta) \geq 0$, such that:*

- (i) *When $D < \underline{D}(\delta)$, the manager adopts the rule-based approach (i.e., any payoff on the PPE payoff frontier is not supported by proactive effort).*[8]

- (ii) *When $D \geq \underline{D}(\delta)$, the manager adopts the exit-based approach (i.e., some payoffs on the PPE payoff frontier are supported by proactive effort).*

Proof of Proposition 1. To prove this, we start from the frontier in the benchmark case, where forced effort is not used. We try to see how action F affects the frontier.

Step 1. If $\underline{u}^P < \overline{u}^P$, it is trivial that $\underline{D}(\delta) = 0$. This follows from Lemma B.12 and the fact that the light segment connecting $(w - pc - (1-p)C, y)$ and $(w - pc, py)$ is the upper boundary of feasible set.

Step 2. If $\underline{u}^P = \overline{u}^P$, we know that $u_P(\underline{u}^P) > w - c$. From Lemma B.9 we know that the frontier is a straight line between $(\underline{u}^P, \pi(\underline{u}^P))$ and $(w-c, \pi(w-c))$. Let s denote the slope of this line. From Lemma B.10, we know that $s < 0$. Let $u_c = u_P(\underline{u}^P) = u_F(\underline{u}^P)$. From PK_P^W and Lemma B.2, we know that $(u_c, \pi(u_c))$ is on the PPE frontier. Let s' denote the slope of the line between $(\underline{u}^P, \pi(\underline{u}^P))$ and $(u_c, \pi(u_c))$. Since $u_P(\underline{u}^P) > w - c$, we have $s' < s$ by concavity. We try to characterize $\underline{D}(\delta)$ in two different scenarios.

Step 3.1. Suppose δ is sufficiently large so that $u_F(0) = \frac{1-\delta}{\delta}[pc + (1-p)C - w] < w - c$. We claim that $\underline{D}(\delta) = (s - s')[\underline{u}^P - w + pc + (1-p)C]$, where $\underline{u}^P = (1-\delta)[w + p(C-c)]$.

Step 3.2. We first show that when $D < \underline{D}(\delta)$, the frontier is not sustained by using P. For any $0 < \varepsilon \leq \frac{(w-c)(s-s')}{(1-p)(C-c)}[\underline{u}^P - w + pc + (1-p)C]$, let $\tilde{D} = \underline{D}(\delta) - \frac{(1-p)(C-c)}{w-c}\varepsilon$. Hence, $0 \leq \tilde{D} < \underline{D}(\delta)$. When $D = \tilde{D}$, suppose that P is used to support the frontier, then $(\underline{u}^P, \pi(\underline{u}^P))$ must be supported by P. From Lemma B.11, we know that the frontier for $u \geq \underline{u}^P$ will not be sustained by using F. Let $\pi_0 = \pi(\underline{u}^P) - \underline{u}^P s + \varepsilon$. It is obvious that the slope of the line segment between $(0, \pi_0)$ and $(w-c, \pi(w-c))$ is smaller than s. This implies that if $(0, \pi_0)$ belongs to PPE payoff set, then the manager's payoff can be improved at $u = \underline{u}^P$ by randomizing between $(0, \pi_0)$ and $(w-c, \pi(w-c))$. Since $u_F(0) = \frac{1-\delta}{\delta}[pc + (1-p)C - w] < w - c$ from Step 3.1, it can be verified that the continuation payoff of $(0, \pi_0)$ falls on the line segment between $(0, \pi_0)$ and $(w-c, \pi(w-c))$, as $(w - pc - (1-p)C, y - \tilde{D})$, $(0, \pi_0)$ and $(w-c, \pi(w-c))$

[8] When there is a tie between the two approaches, we break the tie by favoring the exit-based approach.

are on the same line. Hence, $(0, \pi_0)$ belongs to the PPE payoff set and is supported by F. Therefore, when $D < \underline{D}(\delta)$, the frontier is not sustained by using P.

Step 3.3. Then we show that when $D > \underline{D}(\delta)$, the frontier must be sustained by using P. Suppose, on the contrary, there exists $\tilde{D} < \underline{D}(\delta)$, such that the frontier can be supported without P. In that case, part of the upper boundary of the feasible set must be the line connecting $(w - pc - (1-p)C, y - \tilde{D})$ and $(w - pc, py)$. However, if P is used, $(\underline{u}^P), \pi(\underline{u}^P)$ is on the line connecting $(w - pc - (1-p)C, y - \underline{D}(\delta))$ and $(w - pc, py)$. Hence, the frontier can be improved by using P. This implies that when $D > \underline{D}(\delta)$, the frontier must be sustained by using P.

Step 3.4. It is easy to verify that when $D = \underline{D}(\delta)$, using P or not will not change the frontier.

■