Optimal Talent Hoarding*

Jin Li Yunchou Zhang

Abstract

This paper develops a relational contracting model to study how the managers can best motivate and keep their workers when the worker's promotion opportunity is the manager's private information. Managers would like to keep the capable workers as long as possible. But doing so unduly will de-motivate the worker. The optimal relational contract has three phases: Hoarding, Promotion, and Coasting. In the first phase, talent hoarding occurs so that the worker will not get promoted even if the promotion opportunity is available. Effort is efficient while job allocation is not. In the second phase, the worker exerts effort and gets promoted when there is an opportunity. Both effort and job allocation are efficient. In the third phase, the worker gets promoted when there is an opportunity, but he will not put effort. Job allocation is efficient while effort is not. While the total working duration remains the same, more capable workers suffer from more severe talent hoarding. A higher frequency of opportunity empowers the manager to make better promise, leading to more talent hoarding.

Keywords: talent hoarding, relational contract, internal labor market, promotion

JEL Codes: D86, J33, J41, M12, M51

*Contact information: Jin Li (jli1@hku.hk), Yunchou Zhang (zyc616@connect.hku.hk), Faculty of Business and Economics, The University of Hong Kong, Pokfulam Road, Hong Kong SAR. We are grateful for the helpful conversations with Cheng Chen, Nuno Barros De Oliveira, Florian Englmaier, Matthias Fahn, Luis Garicano, Ingrid Haegele, John Bodian Klopfer, Alan Kwan, Wing Tung Lam, Virginia Minni, Arijit Mukherjee, Juan Pantano, Uta Schönberg, Wing Suen, Michael B. Wong, Yanhui Wu, Junjie Zhou, and seminar participants at HKU. All remaining errors are ours.

> "None of us get to 'own' an employee. If someone's hiring an employee from your team, that's not poaching. That's two managers collaborating for the win of the company."
>
> —Chuck Edward, CVP of HR at Microsoft, 2020. *Global Talent Trends.*

1 Introduction

Internal advancement opportunity serves as the keystone of talent retention. As noted by Cappelli (2008), frustration with limited advancement prospects is a significant driver of employees seeking opportunities elsewhere. It is important to recognize that sometimes employees cannot move upward not because organizations are constrained by limited opportunities, but rather because managers actively impede their movement to other internal positions. Such behavior of managers is referred to as talent hoarding.

Talent hoarding is a prevalent issue in organizations, and it arises from managers leveraging their private information on promotion opportunities. As is highlighted by LinkedIn (2020), 70% of talent professionals agree that"Managers don't want to let go of good talent." Lighthouse and Cornerstone (2023) report that when employers perceive their managers as talent hoarders, they are "267% more likely to say employees do not have visibility into career opportunities." Schneider Electric serves as an illustrative example of such situation. Andrew Saidy, the vice president of talent digitization at Schneider, explains that managers can hoard talent by requiring employees to obtain their approval for role changes, participation in other projects, or finding a mentor (Anderson 2020).

However, it is important to recognize that talent hoarding can also lead to negative consequences for managers themselves. Existing research predominantly focuses on the role of managers in talent hoarding, while neglecting the responses of workers (see, for instance, Haegele 2022, Friebel and Raith 2022). When talented individuals anticipate no prospects for promotion, they may seek employment opportunities in other organizations. Even if they choose to stay, their motivation to exert effort may diminish, subsequently reducing team production. As a result, managers face an inter-temporal trade-off between the immediate benefits of talent hoarding and the future costs associated with decreased worker motivation. To gain a deeper understanding of how managers carry out talent hoarding, we have developed a relational contracting model that aims to identify the optimal approach for managing this trade-off.

In this paper, we examine a business unit consisting of a risk-neutral manager (she) and a risk-neutral worker (he). At the beginning of each period, the manager promises a promotion decision conditional on the promotion opportunity. The opportunity of promotion arrives with a publicly known probability. Then the worker decides on whether to exert effort or not. Effort and output are publicly observable. After that, the manager privately observes the promotion opportunity and makes the final decision on promotion. Due to the manager's possession of private information, she may falsely claim the absence of a promotion opportunity, even if it does exist. As a consequence, the worker may experience a gradual loss of motivation due to the lack of information. The relationship between the manager and the current worker concludes with the occurrence of a promotion, after which the game continues. The manager will be randomly assigned a replacement worker, initiating a new relationship with the newly appointed individual.

The optimal relational contract in our setting reveals a back-loaded promotion pattern with three possible phases: Hoarding, Promotion, and Coasting. Initially, talent hoarding occurs as the manager refrains from promoting the worker, even in the presence of opportunities. Despite knowing the lack of promotion, the worker continues to exert effort, driven by the hope of future advancement. Therefore, worker's effort is efficient while job allocation is not. During this phase, the manager's continuation payoff decreases while the worker's continuation payoff increases. This means that the manager has to compensate the worker in the future in exchange for the current benefits. At some point, the relationship transitions to Promotion Phase, where promotion promises are included in the contract. The worker maintains effort, believing that promotion will occur whenever an opportunity arises. Both effort and allocation are efficient. If the opportunity fails to materialize, the manager's continuation payoff further diminishes. Continuously claiming the absence of opportunities undermines the manager's credibility, leading to a deterioration in the manager-worker relationship over time. After consecutive periods of no opportunity, the relationship proceeds to Coasting Phase, where the worker punishes the manager through shirking. During this phase, job allocation is efficient while the worker's effort is not. This inefficiency arises due to information asymmetry, as the manager alone observes the arrival of opportunities. Consequently, the worker may lose trust in the manager's repeated claims of no opportunity.

Our model also sheds light on several factors that influence the severity of talent hoarding. First, more capable workers will suffer more from talent hoarding. Although the total duration of working remains the same, the manager tends to extend the duration of Hoarding Phase and reduce the duration of Promotion Phase. With a higher output, the manager will try to lock in the benefit from the worker by hoarding him longer. Second, talent hoarding is positively associated with the frequency of opportunities. Increasing opportunity frequency may seem like a deterrent for managers to lie about opportunities. However, the worker's expectation of earlier promotion in Promotion Phase increases tolerance towards talent hoarding in the initial phase. Third, the threat of firing the worker will not help the manager to hoard the worker longer. Rather, it will make the worker worry about getting fired when he stops putting effort as promised by the manager, leading to efficiency loss in Promotion Phase. Last but not least, by endogenizing the manager's future benefit, our model predicts that the manager will refrain from talent hoarding if they can readily find a replacement talent.

2 Related Literature

This paper contributes to the literature of internal labor market (see Gibbons 1996, Gibbons and Waldman 1999, Prendergast 1999, Lazear and Oyer 2013, Waldman 2013, Lazear 2018, for reviews). Particularly, our paper is related to the tournament literature beginning with the seminal work by Lazear and Rosen (1981). Lots of research has emphasized the importance of promotion as incentives to elicit effort (Malcomson 1984, Rosen 1986, MacLeod and Malcomson 1988). Additionally, some studies have argued for the superiority of internal promotions over external recruitment (Chan 1996, Waldman 2003, Kräkel and Schöttner 2012, Auriol, Friebel and Lammers 2012). However, recent research has shown that the provision of promotion incentives may not always be effective due to limited opportunities within an organization (Ke, Li and Powell 2018, Bertrand et al. 2020, Bianchi et al. 2022). Building upon this line of inquiry, our model focuses on the mismatch between promotion opportunities and workers, providing another reason why internal labor markets may fail. We argue that even if there are abundant opportunities, managers cannot commit to promotions as rewards for effort due to information asymmetry, leading to a loss of worker motivation.

Our paper also contributes to the literature on misallocation of human capital. Previous research has highlighted the importance of matching workers with positions that suit them best (Rosen 1982, Geanakoplos and Milgrom 1991, Hsieh et al. 2019). A growing body of literature has examined the role of managers in talent selection and its impact on workers' careers (Bandiera, Barankay and Rasul 2007, Lazear, Shaw and Stanton 2015, Frederiksen, Kahn and Lange 2020, Fenizia 2022, Adhvaryu, Kala and Nyshadham 2022, Minni 2022). Recent studies in management pay attention to the phenomenon of talent hoarding (Dineen, Ling and Soltis 2011, Gardner et al. 2018, Kraichy and Walsh 2021). Haegele (2022) provides the first empirical evidence on talent hoarding and shows that workers' applications for promotions increased by 123% after managers' rotations. Waldman (1984) and Garicano and Rayo (2017) provide different reasons for not promoting workers. A recent work by by Friebel and Raith (2022) studies the optimal incentive contract for managers to advocate cross-divisional mobility without considering workers as strategic players. Our paper extends this literature by examining workers' reactions to talent hoarding, shedding light on the optimal strategies employed by managers to balance the trade-off between hoarding talents and motivating workers.

Additionally, our paper is related to the extensive literature on relational contracts (see Malcomson 1999, Mailath and Samuelson 2006, Malcomson 2013, for reviews). Some research studies dynamic games with one-sided private information (Clementi and Hopenhayn 2006, Fuchs 2007, Padró i Miquel and Yared 2012, Fong and Li 2017). Li and Matouschek (2013) explore a similar framework, where the principal possesses private information on shocks. There is also some literature on dynamic games without monetary transfers (Deb, Pai and Said 2018, Lipnowski and Ramos 2020, Ely and Szydlowski 2020, Guo and Hörner 2021, Solan and Zhao 2021). A closely related paper is Li, Matouschek and Powell (2017), in which the agent is motivated by the benefit from the promised increase in future power. We study a relational contracting model where the principal has the private information on promotion opportunity, and the agent is motivated by future benefit from promotion. We try to shed light on the conflict between managers and workers raised by the asymmetric information on promotion opportunities. The relationship between the manager and the current worker ends when promotion happens, but the game continues with a new worker as a replacement. This setting helps us to understand under what circumstances managers tend to hoard talents more.

3 The Model

A risk-neutral manager and a risk-neutral worker interact repeatedly until the worker gets promoted. Time is discrete and denoted as $t = \{1, 2, \dots, \infty\}$. We will first describe the stage game and then talk about the repeated game.

3.1 The Stage Game

We will introduce the stage game by describing the following three key components: *technology*, *contract*, and *payoffs*.

Technology: The worker decides on his effort level $e_t \in \{0, 1\}$. If the worker chooses to put effort ($e_t = 1$), his output is $Y_t(e_t) = ae_t$, and the corresponding cost is $c_t(e_t) = ce_t$. One can think of a as the worker's ability, implying that a more capable worker produces more output. After the worker decides on the effort level, the output $Y_t(e_t)$ is realized and can be observed by both players. Here we assume that the manager does not participate in production. Therefore, all of the team production comes from the worker's output.

Contract: In each period t, the manager decides whether to offer a contract to the worker. Let $d_t^m \in \{0,1\}$ denote the manager's decision, where $d_t^m = 1$ if the manager makes such an offer, and $d_t^m = 0$ otherwise.

Since the arrival of promotion opportunity is manager's private information, explicit contracts on promotion are not feasible. Instead, we consider a relational contract which promises a promotion decision $d_t^o \in \{0,1\}$ conditional on the worker putting effort. $d_t^o = 1$ means the manager will promote the worker when there is an opportunity. $d_t^o = 0$ means the worker will not get promoted regardless of the opportunity. Let $\rho_t \in \{o, n\}$ be the realization of the promotion opportunity in period t, where $\rho_t = o$ if there is an opportunity, and $\rho_t = n$ if there is none. We assume that ρ_t is identically and independently distributed across periods and $Pr(\rho_t = o) = \theta \in (0,1)$. The worker cannot be promoted if there is no opportunity.

Upon receiving the contract offer, the worker decides whether he will accept the offer or not. Let $d_t^w \in \{0,1\}$ denote the worker's decision, where $d_t^w = 1$ if the worker accepts the offer, and $d_t^w = 0$ otherwise. Once accepting the contract, the worker decides on his effort level, and then the output is realized.

After that, the manager privately observes the realization of the promotion opportunity and makes the final decision on promotion. The final promotion decision is denoted as d_t^p, where $d_t^p = 1$ if promotion happens, and $d_t^p = 0$ otherwise. Since the promotion opportunity is the manager's private information, the manager might tell the worker that there is no opportunity when there is actually one. That is, d_t^p can be different from d_t^o.

Finally, as a common assumption in the literature, a public randomization device will generate a realization $x_t \in [0,1]$ at the end of each period. We assume that there is also a randomization device before the first period. The realization results of the randomization device can be observed by both players so that they can publicly randomize at the beginning of each period. This will make sure that the set of equilibrium payoffs is convex.

The timing of the stage game is summarized in Figure 1.

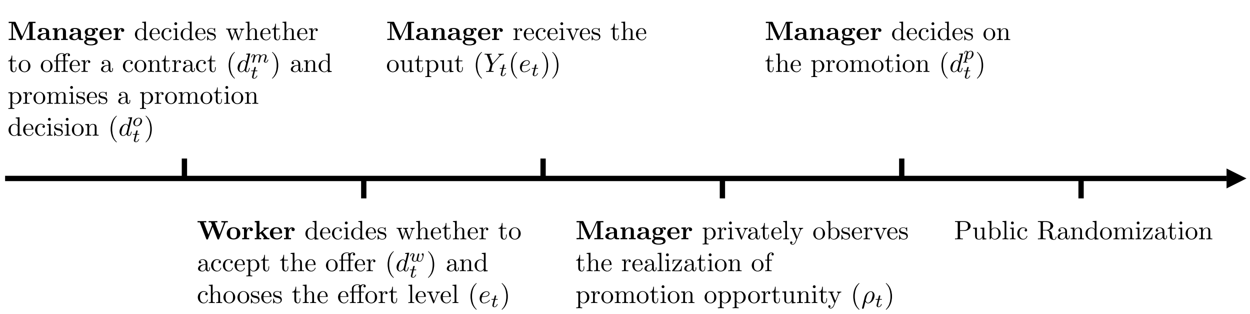

Figure 1: Timeline of the Stage Game

Payoffs: Both players are risk neutral. If $d_t^m d_t^w = 0$, both of them will get their outside options in that period, which are normalized to be 0 for simplicity. Then the game will move to next period. Otherwise, they will engage in the stage game. We assume that the manager's payoff is the team production, which is simply the worker's output Y_t. The worker bears the cost of production c_t.

When promotion happens, the worker yields a promotion benefit b in the next period. This can be considered as his future wages from the new position. Similarly, the manager gets a payoff of B in the next period, which can be seen as the future production from her business unit. We will endogenize the value of B and explain the logic behind in our extension part later.

Given the worker's effort e_t and the manager's final decision on promotion d_t^p, their stage payoffs are:

$$\hat{u}_t = E[1_{\{\rho_t = o\}} d_t^p b | e_t] - c_t(e_t)$$

and

$$\hat{\pi}_t = E[1_{\{\rho_t = o\}} d_t^p B | e_t] + Y_t(e_t).$$

3.2 The Repeated Game

The manager and the worker share the same discount factor $\delta \in (0, 1)$. Assume that promotion happens at the end of period T. At the beginning of any period t, where $t \leq T$, the expected payoffs for the manager and the worker are given by:

$$u_t = (1 - \delta) E \left[\sum_{\tau = t}^{T} \delta^{\tau - t} [-c_\tau(e_\tau) d_\tau^m d_\tau^w] + \delta^{T - t + 1} b \right]$$

and

$$\pi_t = (1 - \delta) E \left[\sum_{\tau = t}^{T} \delta^{\tau - t} [Y_\tau(e_\tau) d_\tau^m d_\tau^w] + \delta^{T - t + 1} B \right].$$

As a common practice in the literature, the right-hand side is multiplied by $(1 - \delta)$ to normalize the payoffs to per-period average values.

Following the convention, we define a relational contract as a pure-strategy Perfect Public Equilibrium (henceforth PPE). The manager and the worker use public strategies only. That means the manager will not take her previous private information into consideration. Both players make decisions only based on public information. The strategies form a Nash Equilibrium for every subgame following any public history. Our restriction to pure strategies is without loss of generality because our game has only one-sided private information, and is therefore a game with product monitoring structure. In this case, there is no need to consider private strategies because every sequential equilibrium outcome is also a PPE outcome (see, for instance, Mailath and Samuelson 2006, p. 330).

Formally, we denote the public history at the beginning of period $t+1$ as h_{t+1}, where $h_{t+1} = \{d^m_\tau, d^o_\tau, d^w_\tau, e_\tau, d^p_\tau, x_\tau\}_{\tau=1}^{t}$. Let $\mathcal{H}_{t+1} = \{h_\tau\}_{\tau=1}^{t+1}$ be the set of public histories of period $t+1$. Note that $\mathcal{H}_1 = \phi$. A public strategy for the manager is a sequence of functions $\{D^m_t, D^o_t, D^p_t\}_{t=1}^{\infty}$, where $D^m_t : \mathcal{H}_t \to \{0,1\}$, $D^o_t : \mathcal{H}_t \to \{0,1\}$, and $D^p_t : \mathcal{H}_t \cup \{d^m_t, d^o_t, d^w_t, e_t, \rho_t\} \to \{0,1\}$. A public strategy for the worker is a sequence of functions $\{D^w_t, E_t\}_{t=1}^{\infty}$, where $D^w_t : \mathcal{H} \cup \{d^m_t, d^o_t\} \to \{0,1\}$, and $E_t : \mathcal{H} \cup \{d^m_t, d^o_t, d^w_t\} \to \{0,1\}$.

We define an optimal relational contract as a PPE of this game that maximizes the manager's first-period equilibrium payoff. Our goal is to characterize the set of optimal relational contracts.

Assumption 1. (i) $a > c > 0$; (ii) $\delta b > c > 0$; (iii) $a > B > 0$; (iv) $\frac{b}{c} \geq \frac{(1-\delta)(1-\theta)}{\theta}\frac{B}{a} + \frac{(1-\delta)(1+\delta)(1-\delta+\delta\theta)}{\delta^2\theta}$.

Part (i) guarantees that to exert effort is efficient for the business unit as a whole. Part (ii) means that the worker should be willing to put effort when he knows for sure that promotion will happen in the current period. Part (iii) indicates that the manager has the incentive to keep the worker when he works hard and promote him when he shirks. The manager expects a smaller future benefit than what she can get from the current worker's output. This can result from the cost of training or the cost of getting familiar with new team members. We will discuss the effect of replacement later. Part (iv) ensures that the worker's promotion benefit is large enough so that he is willing to work without promotion during certain periods along the equilibrium path. Meanwhile, the manager's future benefit after promotion cannot be too large. It guarantees the existence of talent hoarding along the equilibrium path. Finally, note that as long as $\delta(1+\delta\theta) \geq 1$, part (ii)-(iv) can hold simultaneously.

4 Benchmark

In this section, we consider two benchmark cases: First Best and Public Information. In the First Best scenario, we show that talented workers should be promoted to the new position. In the Public Information scenario, we show that the worker can be motivated to exert effort when the promotion opportunity is public information.

4.1 First Best

Assume that the output of the new position is $y_n ew(e_t) = (1+k)a$, where $k > 0$ and a is the worker's ability. The cost of effort in the new position is $c_t(e_t) = ce_t$. Let $\tilde{a}$ denote the expected ability of workers from external labor market. The following lemma describes the efficient allocation in first best case. Detailed proofs can be found in the appendix.

Lemma 1. *In the first best case, promotion is efficient when $a > \tilde{a}$.*

This lemma shows that talents should be promoted to the new position, which fits them better. Since talented workers produce more in the new position with the same cost, internal promotion can generate more surplus for the firm than external hiring. Hence, from the perspective of the whole organization, internal promotion delivers the first best efficient result.

4.2 Public Information

Now we focus on the manager-worker relationship. Assume that the realization of promotion opportunity is public information. We will impose the most severe punishment for deviation. In the event of any deviation, both players will receive their outside options indefinitely. The following lemma describes a stationary equilibrium under this scenario.[1]

Lemma 2. *If ρ is public information, there exists a stationary equilibrium such that: (i) the worker will exert effort; and (ii) the manager will promote the worker with probability $p = c/(\theta\delta b)$ if there is an opportunity.*

This lemma demonstrates that the worker can be motivated to always exert effort under public information. Since the realization of the opportunity is publicly known, the worker has the power to penalize the manager when she lies about the opportunity. If the manager deviates, she would forfeit all future payoffs, thereby ensuring the manager's telling the truth. Meanwhile, the worker has no incentive to shirk as he will lose the opportunity of promotion. As a result, the manager can safely reduce the promotion probability as long as the expected benefit for the worker ($\theta\delta bp$) is sufficient to compensate the cost of effort (c).

5 Preliminaries

In this section, we formulate a maximization problem to determine the properties of the PPE payoff set. We follow the recursive method by Abreu, Pearce and Stacchetti (1990) to

[1]There are a set of equilibria under this scenario that can motivate the worker to exert effort. We just use this stationary equilibrium as an example for illustration.

characterize the PPE payoff set. Let $\mathcal{E}$ denote the PPE payoff set. Any equilibrium payoff pair $(\pi, u) \in \mathcal{E}$ can be supported by pure actions or by randomization among several equilibrium payoff pairs that are generated by pure actions. When a pure action is used, the players receive a pair of flow payoffs generated in this stage game and a pair of continuation payoffs in the future accordingly. On the other hand, if an equilibrium payoff pair is supported by randomization, each player selects a pure action after observing the realization of the randomization device at the end of the previous period.

	$e = 1$	$e = 0$
$d^o = 1$	Effort, Promotion (E_2)	No effort, Promotion (E_3)
$d^o = 0$	Effort, No promotion (E_1)	No effort, No promotion (E_0)

Table 1: Pure Actions

In each period, the players can opt out of the game by not entering the contract ($d^m = 0$ or $d^w = 0$). This is called "Outside Option" (O) and gives both players a payoff of zero. Apart from "Outside Option", there are four possible action profiles shown in Table 1. The first action profile is "Effort, Promotion" and we denote it as E_2. In this case, the manager promotes the worker once there is an opportunity ($d^o = 1$), and the worker always exerts effort ($e = 1$). The second one is "No effort, Promotion", which is denoted by E_3. In this case, the manager promotes the worker if possible ($d^o = 1$), but the worker does not have the motivation to work ($e = 0$). The third one is "Effort, No promotion", which is denoted by E_1. Under this action profile, the manager does not promote the worker ($d^o = 0$), regardless of the realization of the opportunity. However, the worker still exerts effort ($e = 1$). The last one is "No effort, No promotion" and is denoted by E_0. In this case, the manager does not promote the worker ($d^p = 0$) and the worker refuses to put effort ($e = 0$).

Notice that the PPE frontier will not be supported by E_0. This is because when the worker punishes the manager by shirking, the manager will be better off by getting rid of the worker as soon as possible. The detailed proof is in our appendix. In the rest of this article, we will only discuss the following four actions: E_1, E_2, E_3 and O.

There are three conditions that an action should satisfy if it can support the equilibrium payoff. First, let (π_c^j, u_c^j) denote the continuation payoffs of the players, where $j \in \{E_1, E_2, E_3, O\}$. This is the discounted value of all future payoffs. The equilibrium payoff should equal the sum of the flow payoff from the action and the associated continuation payoff. This is often referred to as the promise-keeping constraint. It reflects that current and future actions deliver the promised payoff.

Second, neither player wants to deviate from his or her action. This is often referred to as the no-deviation constraint. It guarantees the worker's incentive to exert effort and the manager's incentive to tell the truth.

Last but not least, the continuation payoff for each player should also belong to the equilibrium payoff set. This is often referred to as the self-enforcing constraint. It means that we can repeat the procedure and look for the actions in the next period. In our appendix, we describe the detailed constraints that help us to characterize the PPE frontier.

Now we characterize the frontier of the PPE payoff set following the method in Abreu, Pearce and Stacchetti (1990). We define the payoff frontier as

$$u(\pi) \equiv sup\{u' : (\pi, u') \in \mathcal{E}\}.$$

In our appendix, we show that there are two important properties of the PPE payoff set. First, it is compact and has a well-defined concave frontier. Hence, the PPE payoff set $\mathcal{E}$ can be fully characterized by the frontier $u(\pi)$. Second, for any payoff pair on the frontier, the associated continuation payoffs also fall on the frontier. This allows us to trace out the entire equilibrium action sequences on the frontier.

The next lemma describes the properties of the continuation payoffs for the manager when each action is used in the stage game. Particularly, it shows how the continuation payoffs evolve for the manager.

Lemma 3. *For any payoff pair (π, u) on the PPE payoff frontier $u(\pi)$, the manager's continuation payoff has the following properties.*

(i) If (π, u) is supported by E_1, then

$$\pi_c^{E_1}(\pi) = \frac{1}{\delta}[\pi - (1-\delta)a] < \pi.$$

(ii) If (π, u) is supported by E_2, then

$$\pi_c^{E_2}(\pi) = \frac{1}{\delta(1-\theta)}[\pi - (1-\delta)a - \delta\theta B] < \pi.$$

(iii) If (π, u) is supported by E_3, then

$$\pi_c^{E_3}(\pi) = \frac{1}{\delta(1-\theta)}(\pi - \delta\theta B),$$

where $\pi_c^{E_3} \geq \pi$ if and only if $\pi \geq \frac{\delta\theta B}{1-\delta(1-\theta)}$, and equation happens when $\pi = \frac{\delta\theta B}{1-\delta(1-\theta)}$.

(iv) If (π, u) is supported by outside option (O), then

$$\pi_c^O(\pi) = \frac{1}{\delta}\pi > \pi.$$

The continuation payoffs for the manager are derived from the promise-keeping constraints. Part (i) shows that the manager's continuation payoff decreases when E_1 is used. This is derived from the fact that $\pi < a$. Part (ii) shows that the manager's continuation payoff decreases when E_2 is used. This is derived from the truth-telling constraint that $B \leq \pi_c^{E_2}$. Part (iii) shows that the manager's continuation payoff increases when $\pi \geq \pi_{E_3}$, where $\pi_{E_3} = \frac{\delta\theta B}{1-\delta(1-\theta)}$.[2] When E_1 and E_2 are used, the manager gets the output a, leading to the decrease in her continuation payoff. In contrast, when E_3 and O are used, the manager gets no stage payoff, which increases her continuation payoff.

Now we describe the constrained optimization problem that helps us to solve for the PPE payoff frontier. For a given manager's payoff π, we define the worker's highest payoff as $f_j(\pi)$ if it is supported by a pure action j, where $j \in \{E_1, E_2, E_3, O\}$. Then for each action, we have the following expressions:

$$f_S(\pi) = (1-\delta)(-c) + \delta u(\pi_c^S(\pi)),$$

$$f_T(\pi) = (1-\delta)(-c) + \delta[\theta b + (1-\theta)u(\pi_c^T(\pi))],$$

$$f_R(\pi) = \delta[\theta b + (1-\theta)u(\pi_c^R(\pi))],$$

and

$$f_O(\pi) = \delta u(\pi_c^O(\pi)).$$

For any $\pi \in [0, \overline{\pi}]$, we have the following constrained maximization problem that characterizes the PPE payoff frontier.

$$u(\pi) = \max_{\alpha_j \geq 0, \pi_j \in [0,\overline{\pi}]} \sum_{j \in \{E_1,E_2,E_3,O\}} \alpha_j f_j(\pi_j)$$

$$s.t. \quad \sum_{j \in \{E_1,E_2,E_3,O\}} \alpha_j = 1$$

$$\sum_{j \in \{E_1,E_2,E_3,O\}} \alpha_j \pi_j = \pi$$

[2] We only focus on the case that $\pi \geq \pi_{E_3}$ because we can randomize between E_3 and O when $\pi < \pi_{E_3}$.

If any of the weight α_j equals one, the payoff pair $(\pi, u(\pi))$ is generated by a pure action $j \in \{E_1, E_2, E_3, O\}$. Otherwise, the payoff pair $(\pi, u(\pi))$ is generated by randomization. The PPE frontier is obtained by optimally choosing the corresponding weights.

6 The Optimal Relational Contract

In this section, we characterize the optimal relational contract by solving the constrained maximization problem aforementioned. To this end, we first characterize the PPE payoff frontier, then we describe the optimal relational contract as the PPE that maximizes the manager's payoff. After that, we compare the efficiency with benchmark cases, and discuss the impacts of exogenous variables on the severity of talent hoarding.

6.1 PPE Payoff Frontier

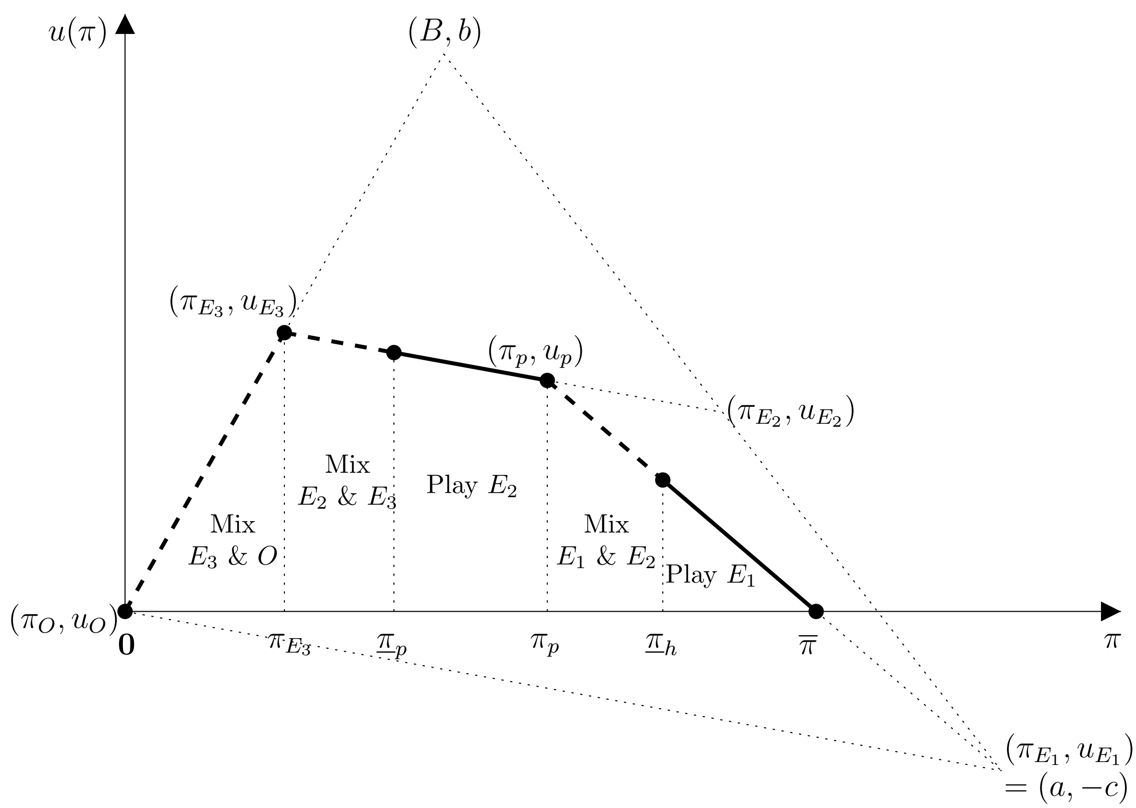

Figure 2: PPE Payoff Frontier

Proposition 1. *The payoff frontier $u(\pi)$ can be divided into five regions. There exist cutoffs $0 < \pi_{E_3} < \underline{\pi}_p < \pi_p < \underline{\pi}_h < \overline{\pi}$, such that:*

(i) For $\pi \in [0, \pi_{E_3})$, the payoff frontier is supported by randomization between E_3 outside option (O). The payoff $(0, 0)$ is supported by outside option (O).

(ii) For $\pi \in [\pi_{E_3}, \underline{\pi}_p)$, the payoff frontier is supported by randomization between E_2 and E_3.[3] *The payoff (π_{E_3}, u_{E_3}) is supported by E_3.*

(iii) For $\pi \in [\underline{\pi}_p, \pi_p]$, the payoff frontier is supported by E_2.

(iv) For $\pi \in (\pi_p, \underline{\pi}_h)$, the payoff frontier is supported by randomization between E_1 and E_2.

(v) For $\pi \in [\underline{\pi}_h, \overline{\pi}]$, the payoff frontier is supported by E_1.

Figure 2 illustrates the PPE payoff frontier described in the above proposition. This proposition shows that the frontier can be divided into five regions. In two of these regions, the frontier payoffs are supported by pure actions. For the remaining three regions, the frontier payoffs are supported by randomization, and the payoffs will end up at one of the endpoints of the region. For simplicity, we assume that the players will randomize between the endpoints after observing the realization of the randomization device.

6.2 Optimal Relational Contract

Now we can describe the optimal relational contract. Specifically, we will talk about how the relationship between the two players evolves over time.

Proposition 2. *The optimal relational contract contains the following three phases:*

(i) The relationship starts from Hoarding Phase where: (a) The manager's payoff π starts at $\overline{\pi}$. (b) The worker exerts effort, but the manager does not promote him even if there is an opportunity. (c) The manager's continuation payoff decreases. The relationship stays in this phase as long as $\pi \geq \underline{\pi}_h$. If $\pi < \underline{\pi}_h$, the relationship will move to Promotion Phase with positive probability.

(ii) In Promotion Phase: (a) The worker exerts effort, and the manager promotes him when there is an opportunity. (b) If there is no promotion opportunity, the manager's continuation payoff decreases. The relationship stays in this phase as long as $\pi \geq \underline{\pi}_p$. If $\pi < \underline{\pi}_p$, the relationship will move to Coasting Phase with positive probability.

[3]There can be other possible ways to support the frontier in this line segment. Part (iii) of Lemma 3 indicates that when π is close to π_{E_3}, the payoff pair can be supported by E_3 and the continuation payoff of the manager will increase. Here we use randomization for simplicity.

(iii) *In Coasting Phase: (a) The worker shirks, but the manager still promotes him when there is an opportunity. (b) If there is a promotion opportunity, the worker gets promoted. Otherwise, the relationship remains in this phase.*[4]

This proposition illustrates that the optimal relational contract shows a pattern of back-loaded promotion. The reason is that the manager will maximize her expected payoff by locking in the output and deferring promotion as much as possible.

Under the optimal relational contract, E_1 is used at the very outset, indicating that talent hoarding occurs during the first phase. The manager can obtain the output without promoting the worker. The worker is aware that he will not get promoted even if there is an opportunity. However, he is willing to exert effort as long as the expected benefit from future promotion can compensate the cost. To see how it works, the manager can incentivize the worker by promising a promotion in the future. As a result, the worker's continuation payoff increases over time while the manager's continuation payoff keeps decreasing. At this stage, the manager has no incentive to promote the worker immediately because her promotion benefit is smaller than the worker's output. This will continue until the manager's continuation payoff reaches the cutoff $\underline{\pi}_h$. At that time, the players will decide on which action to play, based on the realization of the randomization device. If they choose E_1 eventually, the relationship stays in Hoarding Phase. Otherwise, E_2 is chosen, and the relationship moves to Promotion Phase.

In Promotion Phase, E_2 is used. That means talent hoarding ceases when the manager's expected payoff drops to "π_p". To motivate the worker, promising a future promotion is no longer sufficient, and the manager must promote the worker whenever an opportunity arises. The manager has no incentive to lie about the promotion opportunity because of the gap between the continuation payoff and her promotion benefit. If there is indeed no opportunity, the manager's continuation payoff further decreases, and the worker's continuation payoff keeps increasing. This will continue until either a promotion opportunity arrives, or the manager's continuation payoff reaches the cutoff $\underline{\pi}_p$. At that time, the players again will decide on which action to play, based on the realization of the randomization device. If they continue choosing E_2, the relationship stays in Promotion Phase. Otherwise, E_3 is chosen, and the relationship moves to Coasting Phase.

Finally, when the relationship moves to Coasting Phase, it will stay in this phase until promotion happens because E_3 is self-enforcing. After a series of periods with no opportunity in Promotion Phase, the worker can no longer be motivated by promotion. The manager

[4] As is mentioned earlier, the pure action E_3 might be used when $\pi > \pi_{E_3}$. In that case, the relationship might move back to Promotion Phase. Then along the equilibrium path, the worker's action might alternate between shirking and working. For simplicity, we will not talk about this implementation.

should promote the worker once there is an opportunity, even though the worker does not exert effort. The manager's only way to gain a positive expected payoff is to promote the worker and realize the promotion benefit. That is why the manager will not consider keeping the shirking worker.

Along the equilibrium path, the manager's continuation payoff drops until the relationship moves into the third phase. Since it measures the future value of the relationship, we can use it as a proxy for the worker's trust. This is common in literature on relational contracting with private information (see Li and Matouschek 2013, Li, Matouschek and Powell 2017, for instance). In the first two phases, the worker will lose some trust in the manager every time the manager claims that there is no opportunity. Such trust is reducing in an increasing rate, as is shown in the trust curve plotted in the appendix. This indicates that the manager's promise on promotion is becoming less credible over time. Since resting is enough to punish the manager, the worker will not totally lose trust and turn to outside option. Therefore, the worker's trust will stay at a low level (π_{E_3}) until promotion.

While inefficient in terms of total surplus, E_3 is used along the equilibrium path in Coasting Phase. This aligns with related literature of repeated games with imperfect monitoring (see, for instance, Levin 2003, Fuchs, Green and Levine 2022). Note that outside option (O) is also self-enforcing, but it is not used along the equilibrium path as E_3 can credibly punish the manager.

In contrast to some literature on relational contracts (see, for instance, Padró i Miquel and Yared 2012, Fong and Li 2017), the optimal relational contract in our setting does not alternate between punishment and reward. While punishment is indeed present in our optimal contract, causing the manager's continuation payoff to decrease over time, reward only occurs when the worker is promoted. At this point, the manager realizes the promotion benefit, which exceeds the continuation payoff when there is no opportunity. Therefore, in our setting, reward only happens once at the end of the relationship.

6.3 Efficiency

	First Best	Public Information	Private Information		
			Hoarding	Promotion	Coasting
Allocation	Efficient	Inefficient	Inefficient	Efficient	Efficient
Effort	Efficient	Efficient	Efficient	Efficient	Inefficient

Table 2: Efficiency

From the perspective of the organization, talent hoarding is not efficient. In this part, we want to dig into the dynamics of efficiency along the equilibrium path. Table 2 shows the comparison of efficiency between the optimal relational contract and the benchmark cases. We discuss two dimensions of efficiency: allocation efficiency and effort efficiency.

Efficient allocation, as is shown in the first best case, means that talented workers should be promoted to the new position. In the benchmark case with public information, allocation is not efficient. The manager wants to maximize her payoff as long as the worker is motivated to exert effort. Therefore, with positive probability the worker cannot get promoted when there is an opportunity. Under the optimal relational contract described in Proposition 2, allocation is not efficient in Hoarding Phase. Talent hoarding happens in this phase because the worker can be motivated by future promotion. However, when the relationship enters Promotion Phase, the manager has to promote the worker if there is an opportunity. When the worker shirks in Coasting Phase, the manager wants to get rid of him as soon as possible. Therefore, allocation is efficient in the last two phases.

Efficient effort means that the worker should exert effort, since the output is higher than the cost of effort. In the benchmark case with public information, effort is efficient because the worker can be motivated when the benefit from promised promotion can cover the cost. In the setting of private information, the worker can be motivated by the promised promotion in the first two phases. However, effort gets inefficient in Coasting Phase, during which the worker loses trust in the manager. As a result, the worker shirks as a punishment and effort is inefficient.

6.4 Comparative Statics

Now we can explore how various factors influence the manager's choice of talent hoarding. We first justify why we use the ratio u_p/c as a measure of talent hoarding. Then we examine how exogenous parameters affect this ratio.

Recall that (π_p, u_p) is the rightmost payoff pair on the frontier that is supported by E_2. Under the optimal relational contract, this pair represents the expected payoffs for the players when it is the first time for the worker to be eligible for immediate promotion. Therefore, u_p is the worker's expected payoff when the manager decides not to hoard the worker. On the other hand, when E_1 is used, the worker incurs a cost c for exerting effort and receives a negative payoff in each period. Under the optimal relational contract, which maximizes the manager's expected payoff, the relationship starts from the payoff pair $(\overline{\pi}, 0)$. This indicates that the worker's expected payoff at the beginning is zero. Note that the worker always puts effort in Hoarding Phase. Therefore, a larger u_p/c implies a longer duration of talent hoarding.

Corollary 1. *Talent hoarding will be more severe when the worker is more productive.*

This corollary reveals that workers with higher productivity are more likely to be retained within the business unit for a longer duration. This prediction is consistent with the finding in Haegele (2022). Note that a change in worker's productivity will only affect the manager's payoffs without changing the worker's incentive. Since the manager reaps all the output from the worker, an increase in a provides more incentive for the manager to defer promotion. As a result, the worker will shirk as a punishment after exerting effort for the same periods. As is shown in the trust curve in the appendix, the trust of different workers with different ability will eventually decrease to the same level with the same duration. Hence, facing a worker with higher ability, the manager will extend the duration of Hoarding Phase by promising the worker a higher continuation payoff in Promotion Phase. The worker can get this higher continuation payoff because he is allowed to stop putting effort earlier if there is no promotion opportunity. Therefore, a more capable worker will experience a longer duration of Hoarding Phase, while the duration of Promotion Phase will decrease accordingly.

Corollary 2. *Talent hoarding will be more severe when the promotion opportunity arrives more frequently.*

This corollary shows that more frequent promotion opportunities result in more severe talent hoarding. When the relationship progresses to Promotion Phase, the worker is promoted at the first available opportunity. From the worker's perspective, a higher θ allows him to realize the promotion benefit b sooner. This means that the manager can promise a higher continuation payoff in Hoarding Phase, relaxing the worker's incentive constraint. Meanwhile, the manager also receives her promotion benefit B earlier in Promotion Phase. Since this benefit is smaller than the output from the worker, the manager has a greater incentive to delay promotion by extending the duration of Hoarding Phase, resulting in more talent hoarding. This, however, will hurt the manager's credibility more in Promotion Phase. Knowing that there will be more opportunities, the worker loses trust faster when the manager claims that there is no opportunity. As is shown in the trust curve, the worker's trust drops in a higher rate during Promotion Phase.

Ke, Li and Powell (2018) posits that a key reason for workers' frustration with advancement opportunities is that organizations are constrained by limited opportunities. We propose an additional reason: the mere increase in the frequency of opportunities may not be effective if workers are unable to observe them. Rather, it could potentially exacerbate the situation, leaving workers worse off. In such scenarios, it becomes crucial to establish an internal labor market that connects workers with opportunities.

7 Discussion

In this section, we extend our main model to consider the effects of layoff and replacement. After that, we also briefly talk about the effects of different firm policies.

7.1 Layoff

Assume that the manager can lay off the worker at no cost. Then the manager can randomly hire a worker from external labor market and get a future value B. The worker, however, will take the outside option and get no benefit. Figure 3 shows the PPE frontier when we allow the manager to fire the worker at no cost.

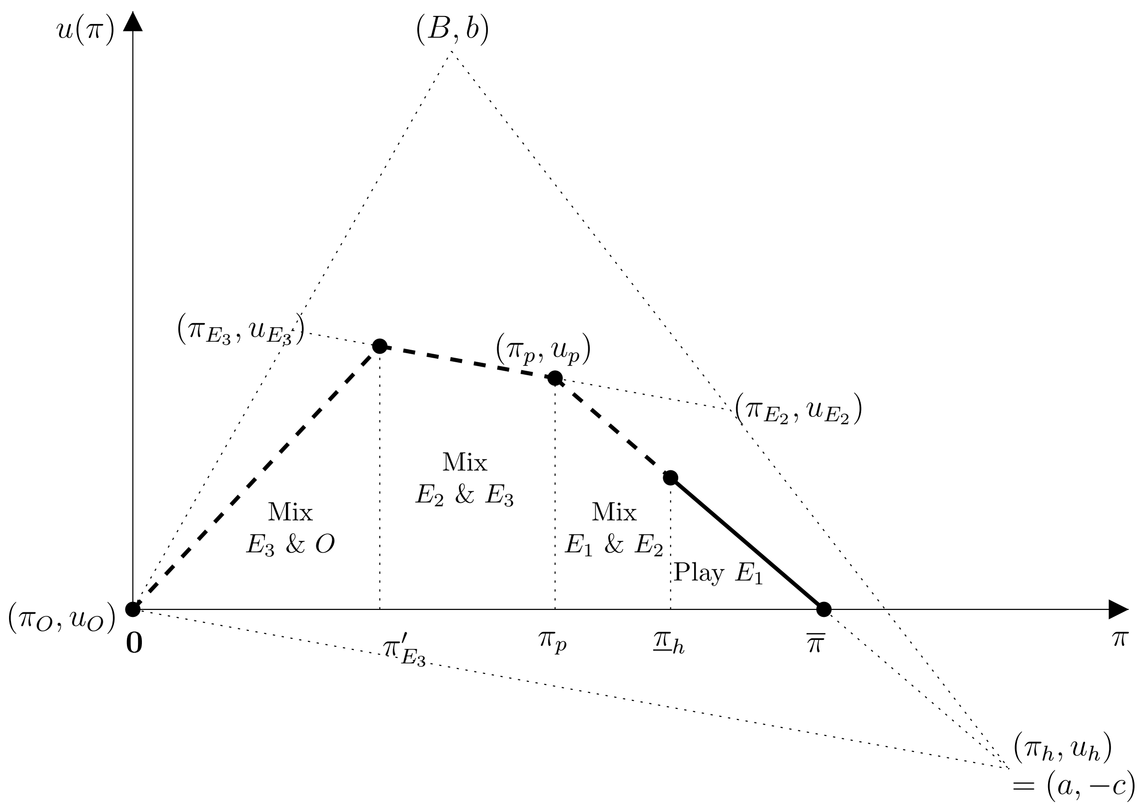

Figure 3: Layoff

Proposition 3. *When the manager can layoff the worker at no cost, the optimal relational contract contains the following three phases:*

(i) The relationship starts from Phase 1 (Hoarding Phase) where: (a) The manager's payoff π starts at $\overline{\pi}$. (b) The worker exerts effort, but the manager does not promote him even if there is an opportunity. (c) The manager's continuation payoff decreases. The

relationship stays in this phase as long as $\pi \geq \underline{\pi}_S$. *If* $\pi < \underline{\pi}_S$, *the relationship will move to Phase 2 with positive probability.*

(ii) In Phase 2: (a) The worker exerts effort, and the manager promotes him when there is an opportunity. (b) If there is no promotion opportunity, the relationship moves to Phase 3.

(iii) In Phase 3: (a) The worker exerts effort with positive probability and rests otherwise. (b) If there is a promotion opportunity, the worker gets promoted. Otherwise, the relationship remains in this phase.

This proposition shows that the worker will not be fired under the optimal relational contract. The reason is that by randomizing between E_2 and E_3, the manager is indifferent while the worker gains a positive expected payoff. Therefore, firing the worker will not be used to support the frontier. The detailed proof can be found in the appendix.

The first phase is the same as Hoarding Phase in our main model. E_1 is used at the very beginning, showing that the worker is motivated by future promotion. The manager has no incentive to fire the worker at this stage, given that the continuation payoff is higher than the promotion benefit B.

The second phase only lasts for one period. That is, after exerting effort once, the worker cannot be well motivated if there is no promotion opportunity. If keeping use E_2, the manager's continuation payoff will drop below B, which provides the incentive for the manager to fire the worker. Knowing that, the worker will lose the motivation to further exert effort. As a result, the relationship will move to the third phase if there is no promotion opportunity. This is also shown in the trust curve in the appendix. When the worker's trust reduces to π_p, it will immediately drop to π'_{E_3} and stay there until promotion.

In the third phase, E_2 and E_3 are randomly used until there is an opportunity. This means that the worker will not always exert effort or rest. Same as before, the worker cannot be motivated to continuously exert effort. However, the worker cannot keep shirking either. To avoid being fired, the worker should exert effort in the future to compensate the manager if E_3 is used. On the other hand, when the manager's continuation payoff increases above B, the manager has the incentive to hide the promotion opportunity. Meanwhile, when the manager's continuation payoff drops below B, the manager has incentive to fire the worker. Therefore, randomizing between E_2 and E_3 can make the manager indifferent while help the worker to obtain the promotion benefit when there is an opportunity.

The threat of firing has been used as a powerful instrument to elicit effort (see, for instance, Shapiro and Stiglitz 1984). However, it does not help the manager to hoard the worker longer. Rather, it makes the manager's promise on future rest less credible, leading to a much shorter duration of the second phase, when both allocation and effort are efficient. Knowing that the manager has the option to fire him, the worker will worry about getting fired when using E_3. As a result, the worker will not keep working after the first phase, causing more efficiency loss to the firm.

7.2 Replacement

Assume that the manager has the option to hire a new worker after promoting an existing one. To see how replacement affects talent hoarding, we further assume that there are two types of workers: High or Low. High-type workers have a productivity level of $\overline{a}$ when they exert effort, and the cost of effort is c. In contrast, Low-type workers are unable to exert effort. If a worker does not exert effort, both the output and the cost are zero. The probability of hiring a high-type worker is q, and the probability of hiring a low-type worker is $1-q$. For simplicity, this model assumes a binary worker type. However, the same logic could be applied to a scenario with a continuous range of worker types.

Based on the results in Section 6, the manager's expected payoff from hiring a high-type worker is given by

$$\overline{\pi}(B) = \frac{\theta b\overline{a}^2 + \theta Bc\overline{a}}{\theta b\overline{a} + (1-\delta+\delta\theta)c\overline{a} - (1-\delta)(1-\theta)Bc}.$$

If the manager hires a low-type worker, she cannot get any benefit until the worker is promoted. In that case, the manager's expected payoff is given by

$$\pi_R(B) = \frac{\delta\theta}{1-\delta+\delta\theta}B.$$

Therefore, the manager's expected payoff from promotion is given by

$$B = q\overline{\pi}(B) + (1-q)\pi_R(B).$$

After rearranging it, we can get $\overline{\pi}(B) = \kappa B$, where $\kappa = \frac{1-\delta+\delta\theta q}{(1-\delta+\delta\theta)q} > 1$. This means that the manager's expected benefit from promotion is smaller than what she can get from a high-type worker. Then we can solve the above equation for the expressions of B. Substituting B in u_p, we can get the expected payoff for the high-type worker at the beginning of Promotion Phase.

Corollary 3. *There will be less talent hoarding if it is easier for the manager to hire a talented worker.*

In line with the previous part, we use u_p/c to measure the duration of talent hoarding and find that u_p/c decreases in p. This suggests that a manager is less inclined to retain high-performing employees when the likelihood of hiring another high-performing worker increases. As is shown in the trust curve, the duration of Hoarding Phase is shorter while the duration of Promotion Phase is longer. In an ideal scenario where a perfect replacement can be found at no cost (i.e., $q = 1$), the manager would be indifferent and promote the worker at every opportunity. Given that the manager has less intention to hoard talents, the manager's claim of no opportunity in Promotion Phase sounds more credible to the worker. Conversely, if the manager anticipates significant difficulty in hiring another high-performing worker, they may compel the current worker to exert effort until the worker is no longer motivated.

The effect of opportunity frequency on talent hoarding remains consistent when we take replacement into consideration. The expressions of B and u_p are complicated, so we did not calculate the comparative statics for θ. Instead, we plotted the diagram of u_p/c against θ and found similar results as Corollary 2. The diagram is omitted here and can be found in the appendix.

7.3 Firm Policies

In this part, we discuss some firm policies that might help mitigate talent hoarding or motivate workers.

First, the manager can provide monetary compensation to the worker when there is no promotion opportunity. Such monetary transfer can provide additional motivation for the worker to exert effort. Meanwhile, it can also reduce the manager's intention to hoard talents because hoarding them is more costly. However, it cannot eliminate talent hoarding completely. As long as the worker can get high enough benefit from promotion, the manager can always motivate the worker to exert effort without immediate compensation. Therefore, we do not consider monetary compensation in our main model.

Second, the firm can reward the manager for promotion. As is described in Friebel and Raith (2022), Haidilao, a Chinese chain restaurant, motivates the managers by relating their income to the profitability of restaurants ran by their subordinates. This provides a strong incentive for managers to help their subordinates accumulate human capital and become managers of new restaurants. However, this cannot eliminate talent hoarding if the reward is not large enough. On the opposite, too much reward might lead to over-promotion.

Third, the firm can require the managers to rotate randomly. As is suggested in Haegele (2022), the manager's rotation can largely reduce talent hoarding. When the relationship between the manager and the worker is broken by rotation, the manager can no longer hoard the worker. However, this cannot stop the manager from hoarding talents before rotating to a new team. What's worse, the manager might push the worker harder since she does not need to worry about future punishment from the worker.

Finally, the firm can create an open internal labor market such that workers can apply for new positions directly without getting approval from their managers. Consider the case of Schneider, as mentioned in the introduction. To combat talent hoarding, Schneider launched their Open Talent Market system, creating an internal labor market for talents. "It [Open Talent Market] has created a de-biased process because the decisions are not in one manager's hands. [...] We want employees to feel empowered that they're the ones in the driver's seat of their career," said Dean Summlar, vice president of human resource at Schneider (Gloat 2023). By making promotion opportunities public, the firm can refrain managers from talent hoarding.

8 Conclusion

Talent hoarding is widespread in organizations. The primary conflict arises from the private information held by managers. Managers are privately informed of promotion opportunities but may act to prevent workers from accessing this information. To address this conflict, we consider a scenario where the manager privately observes the arrival of promotion opportunities. We also take the potential response of the worker into account. By examining how managers optimally balance the trade-off between retaining talents and motivating workers, our model provides insights into the emergence and dissolution of talent hoarding. This helps us to better understand under which situations talent hoarding can be more severe. For instance, being more productive may lead to more talent hoarding. Moreover, a higher frequency of opportunities may not necessarily be beneficial for the worker. We also discuss the effects of different firm policies. With the option of firing the worker, the manager's promise on future rest becomes less credible, leading to more efficiency loss. When the manager can easily get a replacement from external labor market, she has less intention to hoard the talented worker.

This paper tries to shed light on the dynamics of the relationship between the manager and the worker when talent hoarding happens. We suggest some directions for future research on this topic. First, we have derived some theoretical predictions on how exogenous factors affect talent hoarding. These predictions could be tested empirically with suitable data. Second, Keller and Dlugos (2023) show that managers who promote their workers more often attract more and better workers. Together with our analysis on replacement, this implies that there might be multiple equilibria in the talent market. Managers who have a reputation for promoting workers can easily find a replacement, so they do not need to hoard talents. Conversely, managers who hoard talents will face difficulties in hiring new ones, making them less willing to promote their workers. Third, the manager might try to internalize the externality of promotion by creating more career ladders within her business unit. In this way, the worker can still contribute to the same unit after promotion and maintain his effort level. More research is needed to explore whether such policies can mitigate the conflict of interest between managers and workers.

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Appendix

Appendix A

This appendix contains the constraints that any PPE payoff pair $(\pi, u) \in \mathcal{E}$ should satisfy.

Effort, No promotion (E_1). —A payoff pair (π, u) can be supported by E_1 if the following constraints are satisfied.

(i) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Hence, we have the following self-enforcing constraint:

$$(SE^{E_1}) \qquad (\pi_c^{E_1}, u_c^{E_1}) \in \mathcal{E},$$

where $\pi_c^{E_1}$ and $u_c^{E_1}$ are the continuation payoffs when E_1 is used in the stage game.

(ii) No Deviation: There are two kinds of deviations: off-schedule and on-schedule. Off-schedule deviations can be observed by both players, while on-schedule deviations can only be observed by the one with private information.

When off-schedule deviation happens, we assume the worst punishment, which is outside option (O) in the future. The manager might deviate by promoting the worker if the benefit B is high enough. Therefore, the non-reneging constraint for the manager is

$$(NR^{E_1}) \qquad \delta\pi_c^{E_1} \geq \delta B.$$

The worker might deviate by shirking as it is less costly. The worker's incentive constraint is given by

$$(IC^{E_1}) \qquad u \geq 0.$$

Since the worker has no private information, he will not deviate on schedule. The manager cannot deviate on schedule either because the promised promotion decision is $d^o = 0$. No matter what the realization of promotion opportunity is, the manager will not promote the worker. Otherwise, it is an off-schedule deviation. Hence, the manager will always tell the truth if E_1 is used.

(iii) Promise Keeping: The PPE payoff should be decomposed consistently. That means the payoffs should equal the weighted sum of the current stage payoffs and the corresponding future payoffs. Therefore, the promise-keeping constraints are given by

$$(PK_m^{E_1}) \qquad \pi = (1-\delta)a + \delta\pi_c^{E_1}$$

for the manager and

$$(PK_w^{E_1}) \qquad u = (1-\delta)(-c) + \delta u_c^{E_1}$$

for the worker.

Effort, Promotion (E_2). —A payoff pair (π, u) can be supported by E_2 if the following constraints are satisfied.

(i) Feasibility: Similarly, we have the following self-enforcing constraint for the continuation payoffs:

$$(SE^{E_2}) \qquad (\pi_c^{E_2}, u_c^{E_2}) \in \mathcal{E},$$

where $\pi_c^{E_2}$ and $u_c^{E_2}$ are the continuation payoffs when E_2 is used in the stage game.

(ii) No Deviation: The manager will not deviate off schedule as she will lose future benefits from promotion. Since effort is costly, the incentive constraint for the worker is given by

$$(IC^{E_2}) \qquad u \geq 0.$$

The worker, who has no private information, will not deviate on schedule. However, the manager might deviate on schedule when there is a promotion opportunity. If promotion happens, the manager will get the future benefit of B, which is assumed to be smaller than the worker's output a. To avoid on-schedule deviation, the truth-telling constraint for the manager is given by

$$(TT^{E_2}) \qquad \delta B \geq \delta \pi_c^{E_2}.$$

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK_m^{E_2}) \qquad \pi = (1-\delta)a + \delta[\theta B + (1-\theta)\pi_c^{E_2}]$$

for the manager and

$$(PK_w^{E_2}) \qquad u = (1-\delta)(-c) + \delta[\theta b + (1-\theta)u_c^{E_2}]$$

for the worker.

No effort, Promotion (E_3). —A payoff pair (π, u) can be supported by E_3 if the following constraints are satisfied.

(i) Feasibility: The self-enforcing constraints are given by

$$(SE^{E_3}) \qquad (\pi_c^{E_3}, u_c^{E_3}) \in \mathcal{E},$$

where $\pi_c^{E_3}$ and $u_c^{E_3}$ are the continuation payoffs when E_3 is used in the stage game.

(ii) No Deviation: The worker has no incentive to deviate off schedule as it will incur a cost and forfeit the future payoffs. The manager will not deviate off schedule either. The worker cannot deviate on schedule, but the manager could. So the truth-telling constraint for the manager is given by

$$(TT^{E_3}) \qquad \delta B \geq \delta \pi_c^{E_3}.$$

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK_m^{E_3}) \qquad \pi = \delta[\theta B + (1-\theta)\pi_c^{E_3}]$$

for the manager and

$$(PK_w^{E_3}) \qquad u = \delta[\theta b + (1-\theta)u_c^{E_3}]$$

for the worker.

Outside Option O. —Finally, a payoff pair (π, u) can be supported by outside option (O) if the following constraints are satisfied.

(i) Feasibility: The self-enforcing constraints are given by

$$(SE^{O}) \qquad (\pi_c^{O}, u_c^{O}) \in \mathcal{E},$$

where π_c^O and u_c^O are the continuation payoffs when outside option (O) is used in the stage game.

(ii) No Deviation: One-sided deviation will not change the result. Therefore, we don't worry about deviations when players use outside option (O).

(iii) Promise Keeping: The promise-keeping constraints are given by

$$(PK_m^{O}) \qquad \pi = \delta \pi_c^{O}$$

for the manager and

$$(PK_w^O) \qquad\qquad u = \delta u_c^O$$

for the worker.

Appendix B

This appendix contains the proofs of main model omitted in the text.

Proof of Lemma 1. In the first best case, exerting effort is always efficient given $c < a$. Now we compare the total surplus of internal promotion and external hiring.

If the new position is filled by internal promotion, the total surplus is

$$\tilde{a} + (1+k)a - 2c.$$

If the new position is filled by external hiring, the total surplus is

$$a + (1+k)\tilde{a} - 2c.$$

It is obvious that in the first best case, internal promotion generates more surplus when $a > \tilde{a}$. ■

Proof of Lemma 2. In this setting, because there is no private information, any deviation will yield the most severe punishment, namely outside option forever. Hence, we first show that the worker will always exert effort and then show that the manager will not deviate.

For the worker, the IC constraint at any period is

$$-c + \theta\delta p b \geq 0$$

This shows that the worker will not deviate as long as $p \geq \frac{c}{\theta\delta b}$.

For the manager, the IC constraint at any period is

$$a\theta\delta(1-p) \geq 0$$

This shows that the manager will not deviate as long as $p \leq 1$.

To maximize the manager's payoff, the optimal stationary equilibrium requires that $p = \frac{c}{\theta\delta b}$. ■

Lemma 4. *E_0 will not be used to support the frontier.*

Proof of Lemma 4. Assume that a payoff pair (π, u) on the frontier is supported by E_0 with the continuation payoff pair (π_c, u_c). The non-reneging condition for the manager is given by:

$$\pi_c \geq B$$

We also have the following promise-keeping conditions:

$$\pi = 0 + \delta \pi_c$$

$$u = 0 + \delta u_c$$

Given $\delta \leq 1$, we have $u_c \geq u$. Since u_{E_3} is the highest value of the worker's payoff and the frontier is concave, it must be $\pi_c \leq \pi_{E_3}$. Notice that $\pi_{E_3} = \frac{\delta\theta}{1-\delta+\delta\theta} B < B$. This contradicts to the non-reneging condition. Hence, E_0 will not be used to support the frontier. ■

Hence, the feasible set is the convex hull of the payoff set of the four pure actions: E_1, E_2, E_3 and O. That is, for any payoff pair (π, u) in the feasible set,

$$(\pi, u) \in Conv(\{(\pi_{E_1}, u_{E_1}), (\pi_{E_2}, u_{E_2}), (\pi_{E_3}, u_{E_3}), (\pi_O, u_O)\}).$$

Before proving Lemma 3, we need following two lemmas that describe the properties of PPE payoff set.

Lemma 5. *The PPE payoff set $\mathcal{E}$ has the following properties: (i) it is compact; (ii) the frontier $u(\pi)$ is concave; (iii) the extremal point $(\overline{\pi}, u(\overline{\pi}))$ satisfies $u(\overline{\pi}) = 0$; and (iv) payoff pair (π, u) belongs to $\mathcal{E}$ if and only if $\pi \in [0, \overline{\pi}]$ and $u \in [0, u(\pi)]$.*

Proof of Lemma 5. Part (i): There are finite actions that the players can choose, so the payoff set $\mathcal{E}$ is compact based on standard arguments.

Part (ii): The public randomization device makes sure that the frontier $u(\pi)$ is concave.

Part (iii): Since $(\overline{\pi}, u(\overline{\pi}))$ is the extremal point, it must be supported by using the pure action E_1. Suppose that $u(\overline{\pi}) > 0$, we have the corresponding continuation payoffs:

$$\pi_c(\overline{\pi}) = \frac{1}{\delta}[\overline{\pi} - (1-\delta)a]$$
$$u_c(u(\overline{\pi})) = \frac{1}{\delta}[u(\overline{\pi}) - (1-\delta)(-c)]$$

Now consider another payoff pair $(\tilde{\pi}, \tilde{u}) = (\overline{\pi}+\varepsilon, u(\overline{\pi}) - \frac{u(\overline{\pi})+c}{a-\overline{\pi}}\varepsilon)$, where $0 < \varepsilon < \min\{\frac{a-\overline{\pi}}{u(\overline{\pi})+c} u(\overline{\pi}), (1-\delta)(a-\overline{\pi})\}$. It can be verified that $0 < \tilde{u} < u(\overline{\pi})$. Using the same pure action E_1, the corresponding continuation payoffs are:

$$\pi_c(\tilde{\pi}) = \frac{1}{\delta}[\overline{\pi} - (1-\delta)a + \varepsilon]$$
$$u_c(\tilde{u}) = \frac{1}{\delta}\left[\frac{a-\overline{\pi}-\varepsilon}{a-\overline{\pi}}(u(\overline{\pi})+c) - \delta c\right]$$

It can be verified that $(\pi_c(\tilde{\pi}), u_c(\tilde{u}))$ falls on the line segment between $(\overline{\pi}, u(\overline{\pi}))$ and $(\pi_c(\overline{\pi}), u_c(u(\overline{\pi})))$. Hence, $(\pi_c(\tilde{\pi}), u_c(\tilde{u}))$ belongs to the PPE payoff set. This, in turn, shows that $(\tilde{\pi}, \tilde{u})$ should also belong to the PPE payoff set. However, $\tilde{\pi} > \overline{\pi}$ contradicts the definition of the extremal point.

Part (iv): First of all, $(0,0)$ is self-enforcing and it can be supported by the outside option. Because of randomization, any payoff pair on the line segment between $(0,0)$ and $(\overline{\pi}, 0)$ can be supported as a PPE payoff. Similarly, any payoff pair on the line segment between $(\pi, 0)$ and $(\pi, u(\pi))$ can be supported as a PPE payoff. ■

The first part of this lemma is derived from the fact that there are only a finite number of actions. The second part follows from the existence of the randomization device. Part (iii) and (iv) imply that the PPE payoff set $\mathcal{E}$ can be fully characterized by the frontier $u(\pi)$.

Hence, to determine the PPE payoff set, we only need to characterize the PPE frontier $u(\pi)$. To this end, we should figure out for each $(\pi, u(\pi)) \in \mathcal{E}$, whether it is supported by a pure action $j \in \{S, T, R, O\}$ or by randomization. Particularly, if it is supported by a pure action, we need determine the corresponding continuation payoff. The next lemma shows the property of the continuation payoffs for any payoff pair on the frontier, regardless of the pure action used.

Lemma 6. *For any payoff pair $(\pi, u(\pi))$ on the frontier, the associated continuation payoffs (along the equilibrium path) remain on the frontier.*

Proof of Lemma 6. Since the frontier is supported by pure actions or randomizing among pure actions, we just need to show that this is true if $(\pi, u(\pi))$ is supported by a pure action. Assume that $(\pi, u(\pi))$ is supported by the pure action E_1. We prove that by contraction. Suppose that the continuation payoffs (π_c, u_c) does not remain on the frontier. Then it must be $u_c < u(\pi_c)$. Now consider another strategy profile also supported by E_1, but the corresponding continuation payoff is $(\pi_c, \hat{u}_c)$, where $\hat{u}_c = u_c + \varepsilon$. ε is a small positive value such that $u_c + \varepsilon \leq u(\pi_c)$. Following the promise-keeping constraints PK_m^S and PK_w^S, the payoffs under this alternative strategy profile are given by $\hat{\pi} = \pi$ and $\hat{u} = u(\pi) + \delta\varepsilon > u(\pi)$. It can be checked that this alternative strategy profile satisfies all the constraints and therefore constitutes a PPE. $\hat{u} > u(\pi)$ contradicts the definition of $u(\pi)$. Hence, it must be that $u_c = u(\pi_c)$. The similar argument also applies to other pure actions. ■

This lemma shows that the continuation payoffs should always fall on the frontier under any optimal relational contract. That means payoff pairs on the frontier are sequentially optimal. This is because we only have one-sided private information. Since the worker's action and output are publicly observed, there will not be joint punishment along the equilibrium path. This property is similar to the results in Spear and Srivastava (1987) and the first part of Levin (2003). In contrast, it is necessary to use joint punishment when more than one party have private information as, for instance, in Green and Porter (1984), Athey and Bagwell (2001), and the second part of Levin (2003).

Now we can talk about Lemma 3, which describes how the manager's continuation payoff evolves over time.

Proof of Lemma 3. Part (i): As we discussed in the paper, the manager will tell the truth when E_1 is used. The non-reneging constraint (NR^{E_1}) requires that $\pi_c^{E_1} \geq B$. Following the promise-keeping constraint of manager ($PK_m^{E_1}$), the continuation payoff of the manager is given by $\pi_c^{E_1}(\pi) = \frac{1}{\delta}[\pi - (1-\delta)a]$. Hence, we have $\pi \geq (1-\delta)a + \delta B$. Recall that a is the manager's payoff when the worker exerts effort. Then we must have $\pi < a$, which leads to $\pi_c^{E_1}(\pi) < \pi$.

Part (ii): When E_2 is used, the truth-telling constraint (TT^{E_2}) requires that $\pi_c^{E_2} \leq B$. Following the promise-keeping constraint of manager ($PK_m^{E_2}$), the continuation payoff of the manager is given by $\pi_c^{E_2}(\pi) = \frac{1}{\delta(1-\theta)}[\pi - (1-\delta)a - \delta\theta B]$. Since $\pi_c^{E_2} \leq B$, we have $\pi \leq (1-\delta)a + \delta B$. By the assumption that B is smaller than a, it can be checked that $\pi_c^{E_2} < \pi$ when $\pi \leq (1-\delta)a + \delta B$.

Part (iii): When E_3 action is used, the truth-telling constraint (TT^{E_3}) also requires that $\pi_c^{E_3} \leq B$. Following the promise-keeping constraint of manager ($PK_m^{E_3}$), the continuation payoff of the manager is given by $\pi_c^{E_3}(\pi) = \frac{1}{\delta(1-\theta)}[\pi - \delta\theta B]$. Since $\pi_c^{E_3} \leq B$, we have $\pi \leq \delta B$. It can be checked that $\pi_c^{E_3} \geq \pi$ if and only if $\pi \geq \frac{\delta\theta B}{1-\delta(1-\theta)}$, and equation happens when $\pi = \frac{\delta\theta B}{1-\delta(1-\theta)}$. Notice that $\frac{\delta\theta B}{1-\delta(1-\theta)} < \delta B$.

Part (iv): When "Outside Option" is used, both players get zero for the current period. Directly following the promise-keeping constraint of manager (PK_m^O), the continuation payoff of the manager is given by $\pi_c^O(\pi) = \frac{1}{\delta}\pi$. Since $\delta < 1$, we have $\pi_c^O > \pi$. ■

Now we calculate the expected payoffs for the following four actions:E_1, E_2, E_3 and O. It is straightforward that $\pi_O = u_O = 0$, $\pi_{E_1} = a$ and $u_{E_1} = -c$. If E_2 is used, the manager gets a for this period and expects to promote the worker with probability θ. So we have

$$\pi_{E_2} = (1-\delta)a + \delta[\theta B + (1-\theta)\pi_{E_2}]$$

$$\Rightarrow \pi_{E_2} = \frac{1-\delta}{1-\delta+\delta\theta}a + \frac{\delta\theta}{1-\delta+\delta\theta}B$$

Similarly, we have

$$u_{E_2} = \frac{1-\delta}{1-\delta+\delta\theta}(-c) + \frac{\delta\theta}{1-\delta+\delta\theta}b$$

$$\pi_{E_3} = \frac{\delta\theta}{1-\delta+\delta\theta}B$$

$$u_{E_3} = \frac{\delta\theta}{1-\delta+\delta\theta}b$$

Hence, it can be checked that (π_{E_3}, u_{E_3}) is the highest point in the feasible set.

Before the proof of the first proposition, we first prove the following lemmas.

Lemma 7. *The payoff pairs (π_O, u_O) and (π_{E_3}, u_{E_3}) are self-enforcing, and they are supported by outside Option (O) and E_3 respectively.*

Proof of Lemma 7. It can be checked that (π_O, u_O) can be supported by outside option and the continuation payoff is itself. If satisfies all the constraints and constitutes a PPE. Therefore, (π_O, u_O) is self-enforcing and supported by outside option. The same logic also applies to (π_{E_3}, u_{E_3}). ■

Lemma 8. *There exists a cutoff $\underline{\pi}_p$ such that $u(\pi)$ is a straight line on $[\pi_{E_3}, \pi_p]$ and $u(\pi) = f_{E_2}(\pi)$ for all $\pi \in [\underline{\pi}_p, \pi_p]$.*

Proof of Lemma 8. Notice that the line segment between (π_{E_3}, u_{E_3}) and (π_p, u_p) is on the upper boundary of the feasible set. Besides, (π_{E_3}, u_{E_3}) is self-enforcing and supported by E_3. Hence, we just need to show that (π_p, u_p) can be supported by E_2.

Since we have calculated the values of $\pi_{E_3}, u_{E_3}, \pi_{E_2}, u_{E_2}$, we know the slope of the line segment between (π_{E_3}, u_{E_3}) and (π_p, u_p) is $-\frac{c}{a}$. with $\pi_p = (1-\delta)a + \delta B$, we have

$$u_p = \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{\delta(1-\delta)(1-\theta)}{1-\delta+\delta\theta}\frac{Bc}{a} - (1-\delta)c$$

If (π_p, u_p) is supported by E_2, the corresponding continuation payoff pair can be calculated by the promise-keeping constraints $PK_m^{E_2}$ and $PK_w^{E_2}$:

$$\pi_c^{E_2} = B$$
$$u_c^{E_2} = \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a}$$

It can be verified that the continuation payoff pair also falls on the line segment. Hence, all the constraints are satisfied and (π_p, u_p) is supported by E_2.

The same analysis also applies for points on the left until $\underline{\pi}_p$, where $\pi_c^{E_2}(\underline{\pi}_p) = \pi_{E_3}$. The payoff pairs between $(\underline{\pi}_p, u(\underline{\pi}_p))$ and (π_p, u_p) can be supported by E_2, so $u(\pi) = f_{E_2}(\pi)$ for all $\pi \in [\underline{\pi}_p, \pi_p]$. The rest part can be supported by randomizing between (π_{E_3}, u_{E_3}) and $(\underline{\pi}_p, u(\underline{\pi}_p))$. ■

Lemma 9. *There exist cutoffs π_p and $\underline{\pi}_h$ such that $u(\pi)$ is a straight line on $[\pi_p, \overline{\pi}]$ and $u(\pi) = f_{E_1}(\pi)$ for all $\pi \in [\underline{\pi}_h, \overline{\pi}]$.*

Proof of Lemma 9. First, the manager's truth-telling constraints TT^{E_2} and TT^{E_3} and non-reneging constraint NR^{E_1} indicate that when $\pi > (1-\delta)a + \delta B$, the frontier can only be supported by E_1 if it is supported by a pure action. Let $\pi_p = (1-\delta)\alpha + \delta B$.

Next, we show that there exists a cutoff π^* such that $u(\pi) = f_{E_1}(\pi)$ on $[\pi^*, \overline{\pi}]$. Suppose, to the contrary, $f_{E_1}(\pi) < u(\pi)$ for $\pi \in [\pi^*, \overline{\pi})$. Then the frontier must be supported by randomization. Choose a small enough $\varepsilon > 0$ such that $(\overline{\pi} - \varepsilon, u(\overline{\pi} - \varepsilon))$ is supported by randomization. Let the slope be denoted as s. Notice that $(\overline{\pi}, 0)$ is the right most point, so it must be supported by the pure action E_1. Then we have $u(\pi) = c + s(\pi - a)$. Recall that $f_{E_1} = (1-\delta)(-c) + \delta u(\pi_c^{E_1}(\pi))$. For all $\pi \in [(1-\delta)(-c) + \delta(\overline{\pi} - \varepsilon), \overline{\pi})$, we have $u_{E_1} \geq \overline{\pi} - \varepsilon$ and

$$
\begin{aligned}
f_{E_1} &= (1-\delta)(-c) + \delta u(\pi_c^{E_1}(\pi)) \\
&= (1-\delta)(-c) + \delta u\left(\frac{1}{\delta}[\pi - (1-\delta)a]\right) \\
&= c + s(\pi - a) \\
&= u(\pi)
\end{aligned}
$$

This contradicts to our assumption that $f_{E_1}(\pi) < u(\pi)$. Therefore, $u(\pi) = f_{E_1}(\pi)$ on $[\pi^*, \overline{\pi}]$. Notice that for any $\pi \in (\pi^*, \overline{\pi}]$, the left derivative shows that

$$
\begin{aligned}
u^-(\pi) &= f_{E_1}^- \\
&= \delta u^-(\pi_c^{E_1}(\pi))\pi_c^{E_1 -}(\pi) \\
&= u^-(\pi_c^{E_1}(\pi))
\end{aligned}
$$

Let $\underline{\pi}_h = \pi^*$. Specifically, we require that $\pi_c^{E_1}(\underline{\pi}_h) = \pi_p$. Hence, $u(\pi)$ is a straight line on $[\underline{\pi}_h, \overline{\pi}]$.

Finally, we argue that $u(\pi)$ is a straight line on $[\pi_p, \overline{\pi}]$. $u(\pi)$ is supported by randomizing $(\pi_p, u(\pi_p))$ and $(\pi^*, u(\pi^*))$ for $\pi \in (\pi_p, \pi^*)$. Then this should be a straight line. Let this slope be s'. If $s' > s$, for any $\pi \in (\pi^*, \pi^* + \varepsilon)$, where $0 < \varepsilon < (1-\delta)(a - \pi^*)$, we must have

$u(\pi_c^{E_1}) < u_c^{E_1}$. This contradicts to the definition of $u(\pi)$. Besides, $s' < s$ contradicts to the concavity of the frontier. Hence, $u(\pi)$ is a straight line on $[\pi_p, \overline{\pi}]$. ■

Proof of Proposition 1. The previous lemmas (lemma 7-9) have characterized the payoff frontier and shown the properties described in this proposition.

Part (i): Lemma 7 has shown that payoff pairs (π_O, u_O) and (π_{E_3}, u_{E_3}) are self-enforcing. Hence, we can simply randomize between (π_O, u_O) and (π_{E_3}, u_{E_3}) when $\pi \in [\pi_O, \pi_{E_3})$.

Part (ii): Lemma 8 has shown that the frontier can be supported by randomizing between (π_{E_3}, u_{E_3}) and $(\underline{\pi}_p, u(\underline{\pi}_p))$ when $\pi \in [\pi_{E_3}, \underline{\pi}_p)$.

Part (iii): Lemma 8 has shown that the frontier can be supported by pure action E_2 when $\pi \in [\underline{\pi}_p, \pi_p]$.

Part (iv): Lemma 9 has shown that the frontier can be supported by randomizing between $(\pi_p, u(\pi_p))$ and $(\underline{\pi}_h, u(\underline{\pi}_h))$ when $\pi \in (\pi_p, \underline{\pi}_h)$.

Part (v): Lemma 9 has shown that the frontier can be supported by pure action E_1 when $\pi \in [\underline{\pi}_h, \overline{\pi}]$. ■

Proof of Proposition 2. Part (i): The manager maximizes her expected payoff, so the relationship starts with using E_1, and the manager's expected payoff is $\overline{\pi}$. Lemma 3 shows that the value of manager's continuation payoff decreases every time E_1 is used. This continues until $\pi < \underline{\pi}_h$, where the frontier is supported by randomizing between (π_p, u_p) and $(\underline{\pi}_h, u(\underline{\pi}_h))$. With positive possibility (π_p, u_p) is used, and the relationship moves to next phase.

Part (ii): Once the relationship moves to Promotion Phase, the manager can no longer use future promotion to motivate the worker. Hence, E_2 must be used, and promotion happens whenever there is an opportunity. Lemma 3 shows that the manager's continuation payoff keeps decreasing when there is no opportunity. This continues until $\pi < \underline{\pi}_p$, where the frontier is supported by randomizing between (π_{E_3}, u_{E_3}) and $(\underline{\pi}_p, u(\underline{\pi}_p))$. With positive possibility (π_{E_3}, u_{E_3}) is used, and the relationship moves to next phase.

Part (iii): Since (π_{E_3}, u_{E_3}) is self-enforcing, the relationship will stay here until promotion happens. ■

As is mentioned earlier, we use u_p/c as a proxy for talent hoarding.

$$u_p/c = \frac{\delta\theta}{1-\delta+\delta\theta}\frac{b}{c} - \frac{\delta(1-\delta)(1-\theta)}{1-\delta+\delta\theta}\frac{B}{a} - (1-\delta)$$

Proof of Corollary 1.

$$\frac{\partial(u_p/c)}{\partial a} = \frac{\delta(1-\delta)(1-\theta)}{1-\delta+\delta\theta}\frac{B}{a^2} > 0$$

■

Proof of Corollary 2.

$$\frac{\partial(u_p/c)}{\partial\theta} = \frac{\delta(1-\delta)}{(1-\delta+\delta\theta)^2}\left(\frac{b}{c}+\frac{B}{a}\right) > 0.$$

■

Appendix C

This appendix contains the proofs of the discussion on layoff.

To prove Proposition 3, we first characterize the new frontier when the manager is allowed to fire the worker at no cost.

Lemma 10. *When the manager is allowed to fire the worker, there exist cutoffs π_p and $\underline{\pi}_h$ such that $u(\pi)$ is a straight line on $[\pi_p, \overline{\pi}]$ and $u(\pi) = f_S(\pi)$ for all $\pi \in [\underline{\pi}_S, \overline{\pi}]$.*

Proof of Lemma 10. Based on Lemma 9, we just need to show that for any $\tilde{\pi} \in (\pi_p, \overline{\pi})$, firing the worker will not be used to supported the frontier. Notice that $\pi_p = (1-\delta)a + \delta B$. By firing the worker, the manager's payoff is at most $(1-\delta)a + \delta B$. Therefore, firing the worker will not be used to supported the frontier when the manager's payoff is higher than π_p. ■

Lemma 11. *When the manager is allowed to fire the worker, there exits a cutoff π'_{E_3} such that $u(\pi)$ is a straight line on $[\pi'_{E_3}, \pi_p]$.*

Proof of Lemma 11. Let (π_p, u_p) be supported by E_2. We can calculate the associated continuation payoff $(B, \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a})$. Let (π'_{E_3}, u'_{E_3}) be the payoff pair that is supported by E_3 and the associated continuation payoff is also $(B, \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a})$. When the manager's continuation payoff is B, she is indifferent and will not fire the worker. Hence, (π_p, u_p) can be supported by E_2 and (π'_{E_3}, u'_{E_3}) can be supported by E_3 if $(B, \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a})$ belongs to PPE.

It can be verified that (π'_{E_3}, u'_{E_3}) and (π_p, u_p) fall on the upper boundary of the feasible set. Moreover, $(B, \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a})$ falls on the line segment between (π'_{E_3}, u'_{E_3}) and (π_p, u_p). Therefore, $(B, \frac{\delta\theta}{1-\delta+\delta\theta}b - \frac{1-\delta}{1-\delta+\delta\theta}\frac{cB}{a})$ can be supported by randomization between (π'_{E_3}, u'_{E_3}) and (π_p, u_p). (π'_R, u'_R) satisfies all the constraints and is supported by E_3. (π_p, u_p) satisfies all the constraints and is supported by E_2.

Since the line segment between (π'_{E_3}, u'_{E_3}) and (π_p, u_p) is already on the upper boundary of feasible set, firing the worker cannot improve the frontier since it is already on the upper boundary. Hence, $u(\pi)$ is a straight line between (π'_{E_3}, u'_{E_3}) and (π_p, u_p). ■

Lemma 12. *When the manager is allowed to fire the worker, $u(\pi)$ is a straight line on $[0, \pi'_{E_3}]$.*

Proof of Lemma 12. Notice that $(0,0)$ is self-enforcing and supported by outside option. Therefore, the line segment between $(0,0)$ and $(\pi'_{E_3}, uE_3{}')$ belongs to PPE set. Let the slope of the line segment be s.

For any $\tilde{\pi} \in (0, \pi'_{E_3})$, since $\tilde{\pi} < \pi'_{E_3}$, the manager's continuation payoff π^j_C will be lower than B for $j \in \{E_1, E_2, E_3\}$. This provides the manager with the incentive to fire the worker. Hence, The frontier cannot be supported by E_1, E_2 or E_3. If $(\tilde{\pi}, u(\tilde{\pi}))$ is supported by outside option, we must have $\tilde{\pi} \geq \delta\pi'_{E_3}$ by the same logic. Suppose $u(\tilde{\pi}) > s\tilde{\pi}$, it is clear that the associated continuation payoff falls beyond the feasible set. Therefore, the frontier cannot be supported by outside option either.

It is easy to calculate that $\pi_{E_3} = \delta B$. If firing is used, the continuation payoff for the manager is B. So for any $\tilde{\pi} \in (0, \pi'_{E_3})$, the frontier cannot be supported by firing the worker.

Therefore, the frontier must be supported by randomizing between $(0,0)$ and $(\pi'_{E_3}, uE_3{}')$. That means $u(\pi)$ is a straight line on $[0, \pi'_{E_3}]$. ■

Lemma 13. *The payoff frontier $u(\pi)$ can be divided into four regions. There exist cutoffs $0 < \pi'_{E_3} < \pi_p < \underline{\pi}_h < \overline{\pi}$, such that:*

(i) For $\pi \in [0, \pi'_{E_3})$, the payoff frontier is supported by randomization between outside option (O) and E_3. The payoff $(0,0)$ is supported by outside option (O).

(ii) For $\pi \in [\pi'_{E_3}, \pi_p]$, the payoff frontier is supported by randomization between E_2 and E_3. The payoff $(\pi'_{E_3}, u(\pi'_{E_3}))$ is supported by E_3, and the payoff (π_p, u_p) is supported by E_2.

(iii) For $\pi \in (\pi_p, \underline{\pi}_h)$, the payoff frontier is supported by randomization between E_1 and E_2.

(iv) For $\pi \in [\underline{\pi}_h, \overline{\pi}]$, the payoff frontier is supported by E_1.

Proof of Lemma 13. The previous lemmas (lemma 10-12) have characterized the payoff frontier and shown the properties described in this lemma.

Part (i): Lemma 12 has shown that the frontier can be supported by randomizing between $(0,0)$ and (π'_{E_3}, u'_{E_3}) when $\pi \in [0, \pi'_{E_3})$.

Part (ii): Lemma 11 has shown that the frontier can be supported by randomizing between (π'_{E_3}, u'_{E_3}) and (π_p, u_p) when $\pi \in [\pi'_{E_3}, \pi_p]$.

Part (iii): Referring to lemma 9, Lemma 10 has shown that the frontier can be supported by randomizing between (π_p, u_p) and $\underline{\pi}_h, u(\underline{\pi}_h)$ when $\pi \in (\pi_p, \underline{\pi}_h)$.

Part (iv): Referring to lemma 9, Lemma 10 has shown that the frontier can be supported by pure action E_1 when $\pi \in [\underline{\pi}_h, \overline{\pi}]$. ■

Proof of Proposition 3. Part (i): The first phase (Hoarding Phase) is the same as that in Proposition 2.

Part (ii): When $\pi = \pi_p$, E_2 is used and the manager's continuation payoff drops to B. If E_2 is used again, the continuation payoff will further drop below B, leading the manager to fire the worker. Therefore, the relationship will immediately move to next phase if there is no opportunity.

Part (iii): The action alternates between E_2 and E_3 until promotion opportunity arrives. When $\pi = \pi_p$, E_2 is used and the manager's continuation payoff drops to B. When $\pi = \pi'_{E_3}$, E_3 is used and the manager's continuation payoff increases to B. Therefore, the manager's payoff will not drop below π'_{E_3}. ■

Appendix D

This appendix contains the proofs of the discussion on replacement.

Proof of Corollary 3. We first calculate B as a function of $\theta, \delta, \overline{a}, b, c, q$. For simplicity, let $k = \frac{1}{\kappa} = \frac{(1-\delta+\delta\theta)q}{1-\delta+\delta\theta q}$. Hence, by solving $B = k\overline{\pi}$, we have

$$\begin{aligned}
B = & \frac{\theta b\overline{a} + (1-\delta+\delta\theta)c\overline{a} - k\theta c\overline{a}}{2(1-\delta)(1-\theta)c} \\
& - \frac{\sqrt{[k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a}]^2 - 4k\theta b\overline{a}^2(1-\delta)(1-\theta)c}}{2(1-\delta)(1-\theta)c}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial B}{\partial k} = & -\frac{\theta c\overline{a}}{2(1-\delta)(1-\theta)c} \\
& - \frac{[k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a}]\theta c\overline{a} - 2\theta b\overline{a}^2(1-\delta)(1-\theta)c}{\sqrt{[k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a}]^2 - 4k\theta b\overline{a}^2(1-\delta)(1-\theta)c}}
\end{aligned}$$

Next, we want to show that $\frac{\partial B}{\partial k}$ is positive.

$$\begin{aligned}
& \frac{\partial B}{\partial k} > 0 \\
\Longleftrightarrow & \sqrt{[k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a}]^2 - 4k\theta b\overline{a}^2(1-\delta)(1-\theta)c} \\
& < 2b\overline{a}(1-\delta)(1-\theta) - k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a} \\
\Longleftrightarrow & \sqrt{[k\theta c\overline{a} - \theta b\overline{a} - (1-\delta+\delta\theta)c\overline{a}]^2 - 4k\theta b\overline{a}^2(1-\delta)(1-\theta)c} \\
& < (1-\delta+\delta\theta)(2b\overline{a} + c\overline{a}) - k\theta c\overline{a} - \theta b\overline{a}
\end{aligned}$$

Now we show that $(1-\delta+\delta\theta)(2b\overline{a} + c\overline{a}) - k\theta c\overline{a} - \theta b\overline{a}$ is positive so that we can square both sides. Notice that $k < 1$, it then follows

$$\begin{aligned}
&(1-\delta+\delta\theta)(2b\overline{a}+c\overline{a}) \\
=&(1-\delta)(1-\theta)(2b\overline{a}+c\overline{a})+2\theta b\overline{a}+\theta c\overline{a} \\
>&\theta b\overline{a}+k\theta c\overline{a}
\end{aligned}$$

Hence, by squaring both sides, we get

$$\begin{aligned}
&\frac{\partial B}{\partial k} > 0 \\
\iff & -4k\overline{a}^2 bc\theta(1-\theta)(1-\delta) \\
& < -2k\overline{a}^2 bc\theta(1-\theta)(1-\delta)+4b\overline{a}(1-\delta)^2(1-\theta)^2 \\
& + 2b\overline{a}(1-\theta)(1-\delta)[\theta b\overline{a}+(1-\delta+\delta\theta)c\overline{a}]
\end{aligned}$$

It is apparent that $\frac{\partial B}{\partial k} > 0$. It is easy to check that $\frac{\partial(u_p/c)}{\partial B} < 0$ and $\frac{\partial k}{\partial q} > 0$. By chain rule, we have $\frac{\partial(u_p/c)}{\partial q} < 0$. ■

Figure 4 shows the effect of θ when replacement is introduced. We simulate the result using the following values for the parameters:

$$\{\overline{a}, b, c, \delta, q\} = \{10, 8, 2, 0.8, 0.5\}.$$

Then we plot the diagram of $\frac{u_p}{c}$ when θ changes.

This diagram shows that u_p/c increases in θ. The effect of the frequency of opportunities is consistent after taking replacement into consideration.

Appendix E

This appendix contains the figures of the discussion on trust. Time is discrete in our model, but we plot continuous trust curves for better illustration.

First of all, we check how the trust decreases in our main model. The manager's payoff starts from $\overline{\pi} = \frac{\theta ba^2+\theta Bca}{\theta ba+(1-\delta+\delta\theta)ca-(1-\delta)(1-\theta)Bc}$, which represents the initial trust level. After the first phase, the trust level goes to $\pi_p = (1-\delta)a+\delta B$. Finally, the trust level will stay at $\pi_{E_3} = \frac{\delta\theta}{1-\delta+\delta\theta}B$. Let $\Delta\pi^j$ denote per period change in trust when action j is used. Hence, we have

$$\begin{aligned}
\Delta\pi^{E_1} &= \pi_c^{E_1} - \pi = \frac{1-\delta}{\delta}\pi - \frac{1-\delta}{\delta}a \\
\Delta\pi^{E_2} &= \pi_c^{E_2} - \pi = \frac{1-\delta(1-\theta)}{\delta(1-\theta)}\pi - \frac{1}{\delta(1-\theta)}[(1-\delta)a+\delta\theta B]
\end{aligned}$$

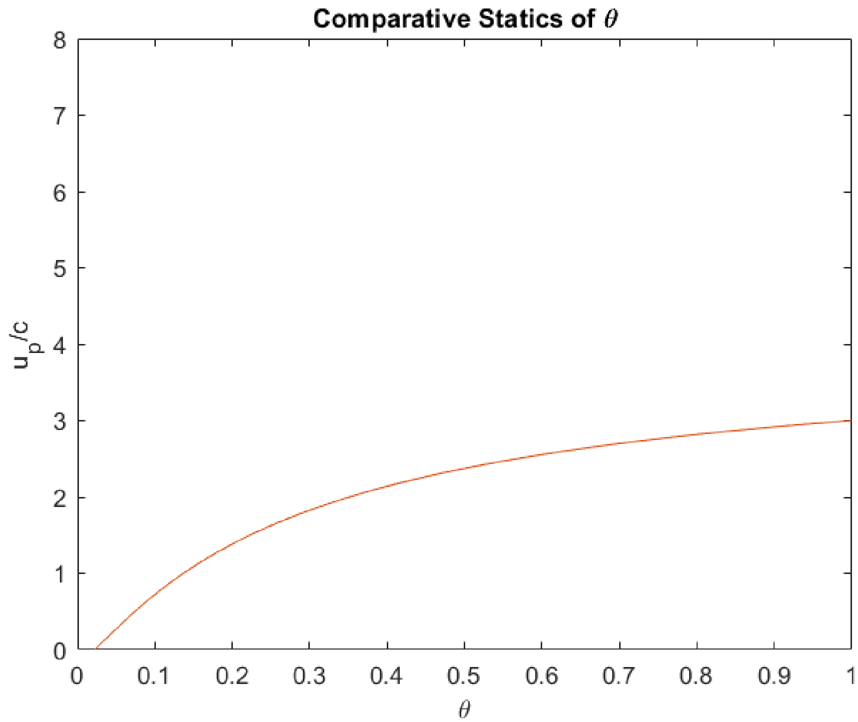

Figure 4: Comparative statics of $\frac{u_p}{c}$ on θ

Notice that $\Delta\pi^{E_1}$ and $\Delta\pi^{E_2}$ are negative and increase in π. When $\pi = \pi_p$, $\Delta\pi^{E_1}$ equals $\Delta\pi^{E_2}$. Therefore, we can plot the following trust curve in Figure 5 for our main model.

Then, we check how worker's ability affects the change of trust. It can be shown that $\overline{\pi}$ and π_p increase in a, while π_{E_3} is independent of a. According to Corollary 1, the duration of the first phase increases in a. Since u_{E_3}/c is also independent of a, the duration of the first two phases is the same. Given the same π, $\Delta\pi^{E_1}$ and $\Delta\pi^{E_2}$ are smaller with a larger a. Hence, we can plot the following trust curve in Figure 6 to show how it changes when worker's ability increases.

Next, we check how the frequency of opportunity affects the change of trust. It can be shown that $\overline{\pi}$ and π_{E_3} increase in θ, while π_p is independent of θ. According to Corollary 2, the duration of the first phase increases in θ. It is also easy to verify that the total duration of the first two phases increases while the duration of the second phase decreases. Given the same π, $\Delta\pi^{E_1}$ stays the same while $\Delta\pi^{E_2}$ is smaller with a larger θ. Hence, we can plot the following trust curve in Figure 7 to show how it changes when worker's ability increases.

Next, we check how layoff affects the change of trust. When the manager is allowed to fire the worker, the first phase of the dynamics is the same. However, when the relationship enters the second phase, it immediately jumps to the third phase in the next period. At the end, the worker's trust stays in a higher level compared to our main model. Hence, we can

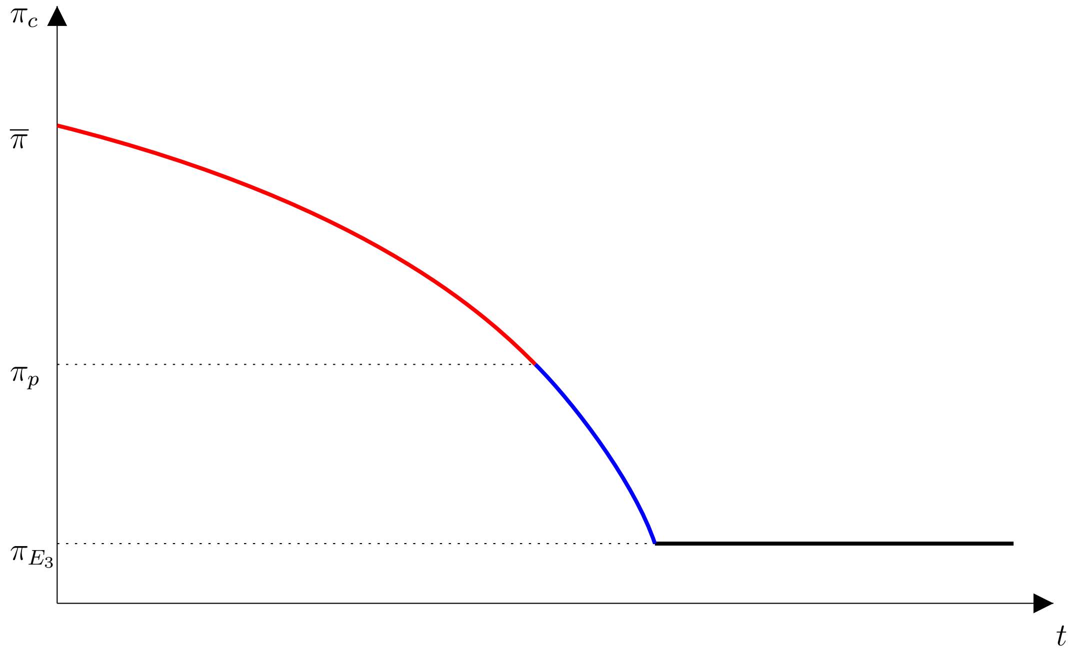

Figure 5: Trust curve: main model (The red curve means the first phase, and the blue curve means the second phase.)

plot the following trust curve in Figure 8 to show how it changes when the manager can fire the worker without any cost.

Finally, we check how the external labor market affects the change of trust. When the manager can easily find a good replacement from external labor market (q is higher), the promotion benefit B becomes larger. Therefore, $\overline{\pi}$, π_p and π_{E_3} will increase. It can be verified that u_p/c decreases in B while u_{E_3}/c is independent of B. Therefore, the duration of the first phase is shorter while the total duration of the first two phases remain the same. Given the same π, $\Delta\pi^{E_1}$ stays the same while $\Delta\pi^{E_2}$ is smaller. Hence, we can plot the following trust curve in Figure 9 to show how it changes when the proportion of talents is higher in external labor market.

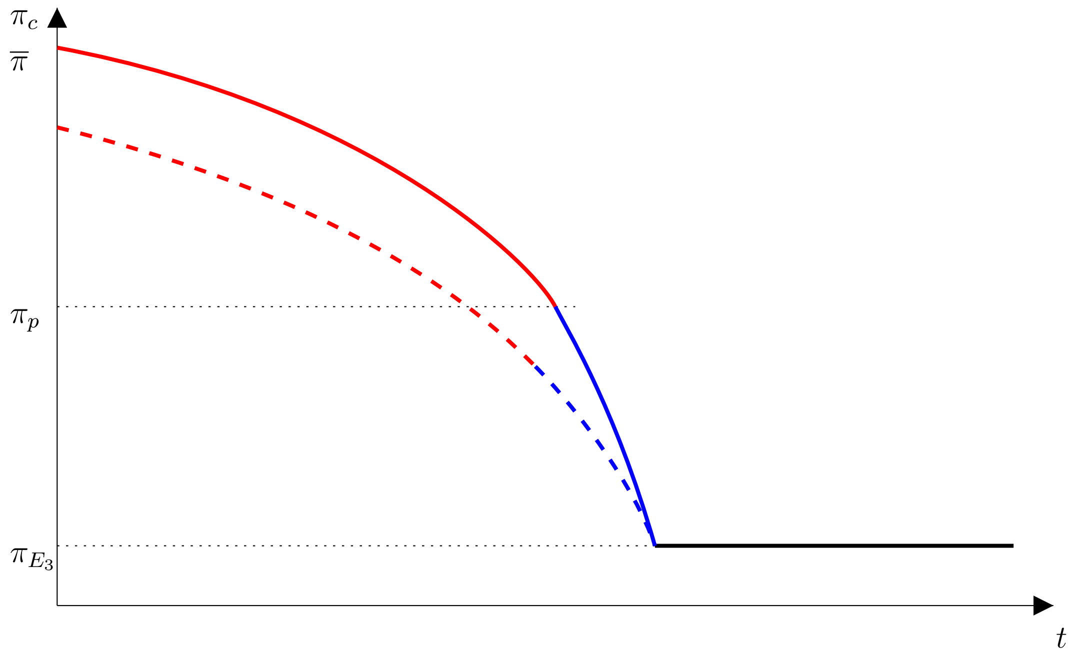

Figure 6: Trust curve: ability (The red curve means the first phase, and the blue curve means the second phase. The dashed curves show the main model for comparison.)

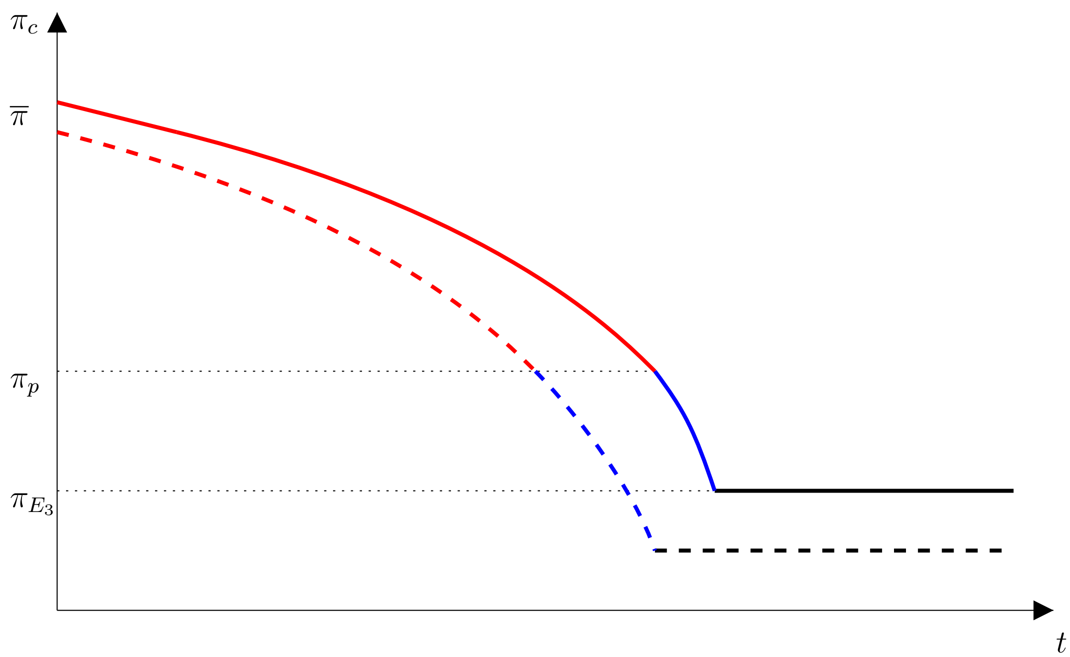

Figure 7: Trust curve: frequency (The red curve means the first phase, and the blue curve means the second phase. The dashed curves show the main model for comparison.)

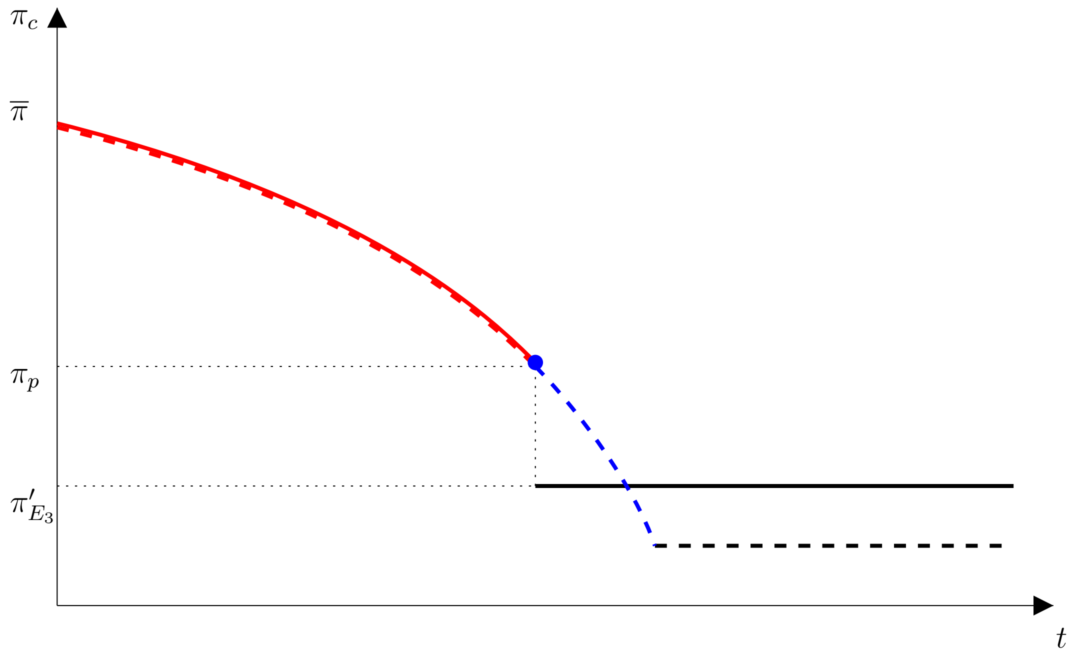

Figure 8: Trust curve: layoff (The red curve means the first phase, and the blue curve means the second phase. The dashed curves show the main model for comparison.)

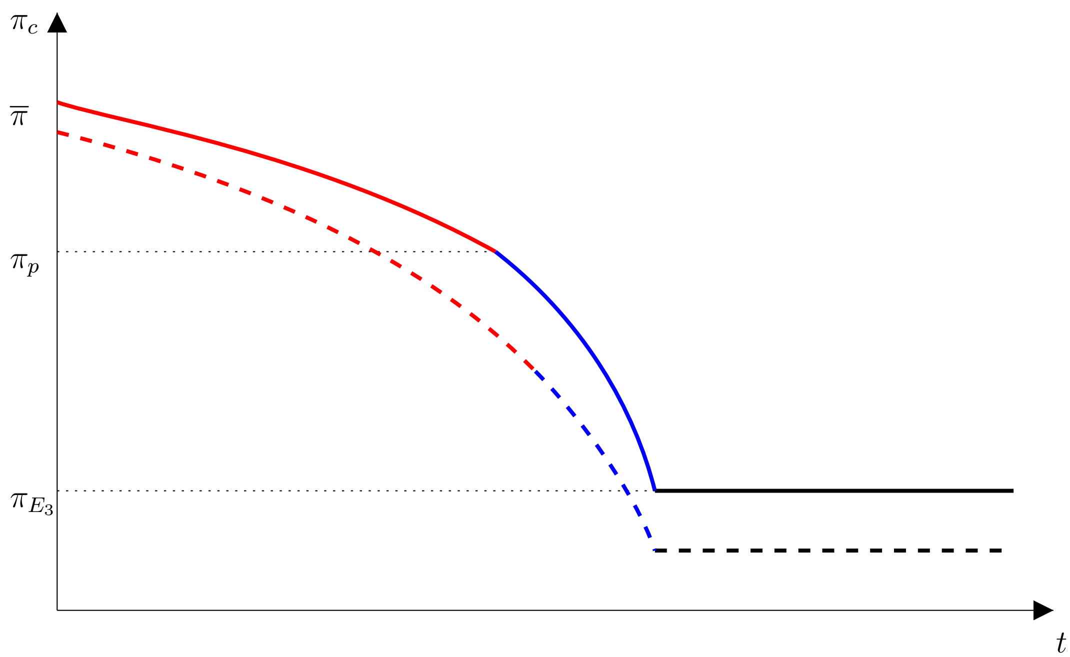

Figure 9: Trust curve: replacement (The red curve means the first phase, and the blue curve means the second phase. The dashed curves show the main model for comparison.)