The Large Volatility and the Slow Recovery of the Job-Finding Rate

Andrew Yizhou Liu *

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Abstract

I derive the job-finding rate elasticity and convergence in a class of search-and-matching models. The analytic expression of the job-finding rate elasticity shows that besides the fundamental surplus, how agents discount future output flow is also important. On the other hand, the job-finding rate converges fast and co-moves with the aggregate shock, contradicting empirical evidence. The co-movement persists with heterogeneous workers and wage rigidity.

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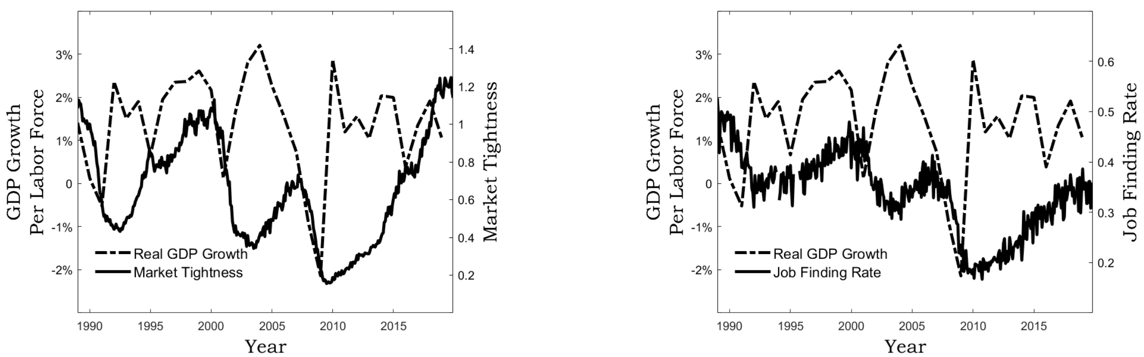
1 Introduction

The three recessions from 1989 to 2019 in the US feature large deterioration and slow recovery of the labor market, in particular the job-finding rate. The large decline in the job-finding rate is extensively studied since Shimer (2005) showed that the Diamond-Mortensen-Pissarides (DMP) search-and-matching model cannot generate empirically consistent job-finding rate volatility, namely the Shimer puzzle. Less is known about the slow recovery aspect in the search-and-matching model. Measuring the aggregate shock as the growth of GDP per labor market participant, Figure 1 shows that both the labor market tightness and the job-finding rate lag the re-bounce in the aggregate shock.

Figure 1: Aggregate Shock, Market Tightness, and Job-Finding Rate



(a) Labor Market Tightness

(b) Job-Finding Rate

Notes. The job-finding rate is constructed as in Shimer (2005) and seasonally adjusted. The vacancy posting data after 2001 is from JOLTS. The vacancy posting data prior to 2001 is from the Help-Wanted Index.

I derive the job-finding rate elasticity and convergence in a class of search-and-matching model which nests the model in Shimer (2005). The model encompasses a range of environments including shock-augmented unemployment benefits and vacancy cost, asset pricing preference, human capital growth, differential labor market exit rates, etc.

I show that the key to generating the large job-finding rate elasticity rests on two things: the output flow difference across employment status, namely the fundamental surplus, and how agents discount the output flow. When the output flow difference is small, or when the discounting of the output flow fluctuates over the cycle, the benefit of opening a vacancy could be sensitive to the aggregate shock and hence the job-finding rate exhibits large elasticity.

Next, I derive the convergence of job-finding rate in the perfect foresight equilibrium. The expression shows that the convergence will be fast and the job-finding rate co-moves with the shock under plausible parameterization. The co-movement carries over to the rational expectation equilibrium, where the half-life of the convergence of job-finding rate is only 3 months.

Adding heterogeneous workers does not break the co-movement, in which the aggregate shock still explains over 80% of the time-series variation in the job-finding rate. As a comparison, empirically the growth of GDP per labor market participant or the growth of average productivity only explains 7% of the fluctuation in the job-finding rate.

The reason for the co-movement lies in the model transmission. The aggregate shock first affects value functions, which impact the job-finding rate. The job-finding rate then leads to changes in other state variables such as the unemployment rate and human capital. However, there is little feedback from the other state variables to the job-finding rate. As a result, every model generated moments co-move with the shock.

Because the job-finding rate is mostly affected by the aggregate shock, neither wage rigidity nor centralized job-search could break the co-movement, which I verify numerically. To break the co-movement, direct impacts on the job-finding rate by factors other the shock are needed. I discuss an extension with firms' hiring standards where unemployment directly affect the job-finding rate. An initial decline in the job-finding rate leads to a decline in the average human capital in the unemployment pool, which in turn decreases the job-finding rate through firms' hiring standards. The feedback loop could result in delayed recovery of the job-finding rate. I leave detail analysis and more extensions to future research.

The paper makes two main contributions. First, the paper derives the analytic expression for the job-finding rate elasticity for a wide range of settings, complementing the work by Shimer (2005), Hagedorn and Manovskii (2008), Ljungqvist and Sargent (2017) and Kehoe et al. (2019). Ljungqvist and Sargent (2017) show how the "fundamental surplus", namely the output flow difference amplifies the job-finding rate elasticity while Kehoe et al. (2019) show that the discounting of the outflow is also important. I combine the two aspects and discuss several ways that the model can generate large job-finding rate elasticity. In particular, I add to Kehoe et al. (2019) by showing that differential labor market exit rates can also generate large job-finding rate elasticity.

The second contribution is the derivation of the convergence of job-finding rate. I show both analytically and numerically that the job-finding rate co-moves with the aggregate shock, contrasting the much weaker empirical correlation. While delayed responses of aggregate statistics such as consumption receive wide attention (see e.g. Brito and Dixon (2013), Auclert et al. (2020)), the convergence of job-finding rate is less studied to the best of my knowledge. Understanding the slow recovery of job-finding rate through the lens of a model is important, as it provides insights on policies that could recover the job-finding rate faster. The extension with firms' hiring standards suggests that one possibility is human capital depreciation during unemployment, which causes the firms to be less willing to hire the endogenous pool of unemployed workers. The possibility has empirical support from Pollmann-Schult (2005) and Quintini (2011) who show that unskilled workers are less likely to be hired during recessions. Related, Barnichon and Figura (2015) show that increasing presence of long-term unemployed workers following the Great Recession explains the decline in the match efficiency.

Outline The organization of the rest of the paper is: Section 2 lays out the model. Section 3 characterizes the equilibrium. Section 4 derives the analytic solution for the job-finding rate elasticity and convergence. It also shows the convergence of job-finding rate numerically in a heterogeneous-agent model. Section 5 concludes.

2 Economy

I present a continuous-time search-and-matching model. The model is closest to Kehoe et al. (2019). I will discuss how the model nests the search-and-matching model in Shimer (2005). The continuous-time setting is mainly for deriving closed form expressions. All results hold in discrete time.[1]

The economy consists of a continuum of firms and workers. Each worker can be employed or unemployed. An employed worker exits the labor market at a rate ϕ_e while an unemployed worker exits at a rate ϕ_u. There is labor market entry at a rate ζ. New entrants are unemployed first.

Firms post vacancies to hire workers. Each worker belongs to one of a large number of identical families that own firms and insure their members against idiosyncratic risks. The assumption makes idiosyncratic risk and hence savings irrelevant. It also implies that workers and firms discount the utility flows using the same stochastic discount factor. There is aggregate risk in the form of productivity shock.

Workers can accumulate human capital. z_{it}^j denotes the human capital of an individual at time t, with $j = \{e, u\}$ denoting labor force status. Human capital growth rates could differ by labor force status:

$$\frac{dz_{it}^j}{z_{it}^j} = g_j dt, \quad j = e, u \tag{1}$$

An unemployed worker finds jobs at a rate $\lambda_{wt}(z)$ which can depend on the worker's human capital. New entrants draw their initial human capital from an exogenous distribution $n_t(z)$. The measure of unemployed workers $u_t(z)$ evolves according to:

$$\frac{\partial u_t(z)}{\partial t} = -\left[\frac{\partial u_t(z)}{\partial z} g_u z + g_u u_t(z)\right] - (\phi_u + \lambda_{wt}(z))u_t(z) + \delta e_t(z) + \zeta n_t(z) \tag{2}$$

The first term is the evolution of the density $u_t(z)$ attributed to human capital growth. The

[1]See the appendix Section A for details and proofs.

second term is the outflow because of labor market exit and employment. The last two terms are the inflow of separated employed workers and new entrants, where $e_t(z)$ is the measure of employed workers. It evolves according to:

$$\frac{\partial e_t(z)}{\partial t} = -\left[\frac{\partial e_t(z)}{\partial z} g_e z + g_e u_t(z)\right] - (\phi_e + \delta) e_t(z) + \lambda_{wt}(z) u_t(z) \tag{3}$$

where δ is the exogenous separation rate.

The aggregate productivity shock follows a Geometric Brownian motion

$$\frac{dA_t}{A_t} = g_a dt + \sigma_a dW_{a,t} \tag{4}$$

The aggregate shock affects the productivity of both employed and unemployed workers with a linear technology. The linear technology implies that the productivity for workers with human capital z is $A_t z$ and the unemployment benefit for workers with human capital z is $bA_t z$. By scaling the unemployment benefit with the aggregate shock, the model does not rely on acyclical opportunity cost of employment, or the fundamental surplus, to generate fluctuations in unemployment and vacancy postings.[2]

Firms post vacancies in submarkets indexed by human capital. Let $v_t(z)$ denote the measure of vacancy for workers with human capital z. The cost of posting vacancies is augmented by the aggregate shock A_t linearly so that firms pay $\kappa A_t z$ for a vacancy directed at a worker with human capital z. The expression implies that the vacancy cost depends on the productivity of employed workers, consistent with the view that recruiting takes time away from production. Another interpretation is that firms' screening costs increase with human capital levels of hired workers.

[2]Namely, the model does not rely on differential effects of shocks to employed and unemployed productivity to generate unemployment fluctuations. See Ljungqvist and Sargent (2017) for a review.

The aggregate resource constraint is

$$C_t = A_t \int z e_t(z) dz + b A_t \int z u_t(z) dz - \kappa A_t \int z v_t(z) dz \tag{5}$$

Labor market flows, along with entry and exit, act as stabilizing forces to aggregate human capitals. In the equilibrium, aggregate human capitals of employed and unemployed workers grow at the same rate as the aggregate shock process A_t.

Each family maximizes the utility of a representative worker. The preference specification has an asset pricing specification that incurs habit X_t:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\beta t} \frac{(C_t - X_t)^{1-\alpha}}{1-\alpha} dt \right] \tag{6}$$

where β is the subjective discount rate. The stochastic discount factor is

$$Q_t = e^{-\beta t} (C_t - X_t)^{-\alpha} \tag{7}$$

It is convenient to express the stochastic discount factor using the surplus consumption ratio $S_t = (C_t - X_t)/C_t$, so that Equation (7) becomes

$$Q_t = e^{-\beta t} (S_t C_t)^{-\alpha} \tag{8}$$

The risk-free rate satisfies $r_{ft} dt = -E_t \frac{dQ_t}{Q_t}$.[3] More generally, for any asset with dividend process $\{D_t\}$, the price of the asset P_t needs to satisfy

$$Q_t P_t = \int_0^\infty Q_{t+s} D_{t+s} ds \tag{9}$$

Equation (9) implies that the instantaneous return of any asset $dR_t = (D_t/P_t dt + dP_t/P_t)$

[3]Families are identical, which implies that the aggregate consumption $\overline{C}_t$ is the same as the family consumption C_t.

needs to satisfy

$$\mathbb{E}(dR_t) = \frac{D_t}{P_t}dt + E_t\left(\frac{dP_t}{P_t}\right) = -E_t\left[\frac{dQ_t}{Q_t} + \frac{dQ_t}{Q_t}\frac{dP_t}{P_t}\right] \tag{10}$$

Equation (10) regulates how firms are valued in the economy. Note that the preference implies that firms transfer all expected profits to workers.

As in Campbell and Cochrane (1999) and Kehoe et al. (2019), I assume that the habit process $\{X_t\}$ is exogenous and specify the process for the log surplus consumption ratio $s_t = \log(S_t)$, which implicitly determines $\{X_t\}$.

$$ds_t = (1-\rho_s)(s - s_t)dt + \sigma_a \lambda_a(s_t)dW_{a,t} \tag{11}$$

where $\lambda_a(s_t)$ is the sensitivity function

$$\lambda_a(s_t) = \frac{1}{S}[1 - 2(s_t - s)]^{1/2} - 1 \tag{12}$$

where S is the long-run surplus consumption ratio and $s = \log(S)$. The log surplus consumption ratio is correlated with the aggregate shock process A_t since their diffusion terms share the same Brownian motion process. Equation (11) is the continuous time analog of the autoregressive process in Campbell and Cochrane (1999) and Kehoe et al. (2019).

Each family maximizes Equation (6) subject to the budget constraint

$$C_t + I_t = W_t + \Pi_t + H_t \tag{13}$$

where I_t is investment in vacancy creation, Π_t is firms' profit, and H_t is unemployment benefit. Since each family is identical, investment is the same as the aggregate cost of vacancy posting, i.e. $I_t = \kappa A_t \int z v_t(z)dz$. Similarly, $W_t + \Pi_t = \int z e_t(z)dz$ and $H_t = \int z u_t(z)dz$.

2.1 A Simpler Model

I show how the model nests the search-and-matching model in Shimer (2005). If the human capital growth is $g_e = g_u = 0$ and the initial human capital distribution is a point mass at 1, the model is a representative agent model.

In Equation (11), if I set $\rho = 1$ and $\lambda_a = 0$, it implies that $X_t = 0$ at all times. The preference is the standard CRRA preference:

$$\mathbb{E}_0\left[\int_0^\infty e^{-\beta t}\frac{C_t^{1-\alpha}}{1-\alpha}dt\right]$$

In addition, if $\alpha = 0$, the agents are risk-neutral, so that there is no need for savings in the absence of families.

If I set $\phi_e = \phi_u = 0$ and $\zeta = 0$, there is no labor market entry or exit. As will be clear in the remaining sections, labor market entry and exit serve two purposes. First, differential labor market exit rates, i.e. $\phi_e \neq \phi_u$, has impacts on the job-finding rate elasticity. Second, with human capital growth, labor market entry and exit guarantee a stationary distribution (Gabaix et al. (2016)).

Further, if I do not augment the unemployment benefit and the vacancy cost with the shock, and set $g_a = 0$ so that the shock follows:

$$dA_t = \sigma_a dW_{a,t}$$

The model is identical to the one in Shimer (2005) modulo the continuous-time setting. The comparison suggests that the difference in the elasticity of job-finding rate between the model and the one in Shimer (2005) must come from the factors above. In the remaining sections, I will discuss how the factors affect the job-finding rate elasticity and convergence.

3 The Competitive Equilibrium

In each submarket, a vacancy specifies a contract of desired human capital level z and wage offer $W_t(z)$, which is the discounted value of wage payment over the course of employment. I focus on the symmetric equilibrium in which firms post the same contract $(z, W_t(z))$ in a submarket. Workers' human capital is public information. The assumption implies that workers with human capital level z will only search in submarket z.

Each submarket has the same matching technology given by a matching function $m_t(u_t(z), v_t(z))$. The job-finding rate is determined by $\lambda_{wt}(z) = m_t(u_t(z), v_t(z))/u_t(z)$. I further assume that the matching function is homogeneous of degree one in $(u_t(z), v_t(z))$. Define the market tightness $\theta_t(z) = v_t(z)/u_t(z)$. The homogeneity assumption implies that the job-finding rate is $\lambda_{wt}(\theta_t(z)) = m_t(1, \theta_t(z))$. The vacancy-filling rate $\lambda_{vt}(\theta_t(z)) = m_t(u_t(z), v_t(z))/v_t(z)$ relates to the job-finding rate by $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{vt}(\theta_t(z))$.

Let $M_t(z)$ denote the post-match value, so that the value of employment is $M_t(z)+W_t(z)$. The post-match value satisfies

$$(\delta + \phi_e)M_t(z) = \delta U_t(z) + \mathbb{E}_t\left[dM_t(z_t) + M_t(z)\frac{dQ_t}{Q_t} + dM_t(z)\frac{dQ_t}{Q_t}\right] \tag{14}$$

The post-match value can be interpreted as worker's outside option, such that the firm cannot contract on the post-match value. Q_t is the stochastic discount factor in Equation (8). $U_t(z)$ is the value of unemployed workers:

$$\begin{aligned}(\lambda_{wt}(z) + \phi_u)U_t(z) =& bA_t z + \lambda_{wt}(z)(M_t(z) + W_t(z)) \\ &+ \mathbb{E}_t\left[dU_t(z_t) + U_t(z)\frac{dQ_t}{Q_t} + dU_t(z)\frac{dQ_t}{Q_t}\right]\end{aligned} \tag{15}$$

Unemployed workers receive unemployment benefit $bA_t z$ and find jobs at a rate $\lambda_{wt}(z)$. The value of unemployment is discounted by the job-finding rate and the labor market exit rate, while the continuation value is affected by the stochastic discount factor.

Firms posting vacancies in submarket z incur a flow cost $\kappa A_t z$. Once a match forms, an employed worker produces output flows $A_t z$ which are linear in the aggregate shock and human capital. Denote the expected revenue of firms in submarket z by $Y_t(z)$ which satisfies

$$(\delta + \phi_e)Y_t(z) = A_t z + \mathbb{E}_t\left[dY_t(z_t) + Y_t(z)\frac{dQ_t}{Q_t} + dY_t(z)\frac{dQ_t}{Q_t}\right] \tag{16}$$

The expected revenue is discounted by exogenous separation and labor market exit. The continuation value is discounted by Q_t, consistent with the assumption that firms are owned by families. In the symmetric equilibrium, the following relation holds in each submarket:

$$\frac{\lambda'_{vt}(\theta_t(z))}{\lambda_{vt}(\theta_t(z))}[Y_t(z) - W_t(z)] = -\frac{\lambda'_{wt}(\theta_t(z))}{\lambda_{wt}(\theta_t(z))}[W_t(z) + M_t(z) - U_t(z)] \tag{17}$$

Equation (17) guarantees efficiency of the competitive equilibrium. Define the elasticity of vacancy-filling rate with respect to the market tightness as

$$\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{vt}(\theta_t(z))/\lambda_{vt}(\theta_t(z)) \tag{18}$$

Equation (17) is equivalent to the Hosios condition under Nash bargaining since $1 - \eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(z)/\lambda_{wt}(\theta_t(z))$.

There is free-entry of firms. The assumption implies that the flow cost of vacancy is equal to the expected profit:

$$\kappa A_t z = \lambda_{vt}(\theta_t(z))[Y_t(z) - W_t(z)] \tag{19}$$

The free-entry condition relates the market tightness to value functions and hence the expected present discounted value of the output flows.

The competitive search equilibrium is defined as a collection of stochastic processes $\{C_t, Q_t, S_t\}_{t\geqslant 0}$ and state-contingent processes $\{W_t(z), M_t(z), U_t(z), Y_t(z), \theta_t(z), e_t(z),$

$u_t(z), v_t(z)\}_{t\geqslant 0}$ such that: i) for each t, taken as given $M_t(z), U_t(z), Y_t(z)$, and Q_t, the wage contract $W_t(z)$ and the market tightness $\theta_t(z)$ satisfy Equation (17), ii) the collection of state-contingent processes $\{W_t(z), M_t(z), U_t(z), Y_t(z)\}_{t\geqslant 0}$ satisfy Equations (14) to (16), iii) the evolution of the human capitals, unemployment, employment, and aggregate shock satisfy Equations (1) to (4), iv) the free-entry condition Equation (19) holds, v) the aggregate resource constraint Equation (5), vi) the stochastic discount factor satisfies Equation (8), vii) the log surplus consumption ratio satisfies Equation (11).

The linear technology implies that value functions are also linear. I write the value of a match $Y_t(z_t) = Y_t z_t$, and $W_t(z_t) = W_t z_t$, $M_t(z_t) = M_t z_t$, $U_t(z_t) = U_t z_t$. Linear value functions simplify the model, as it can aggregate linearly so that the solution does not depend on human capital distributions. Instead, the model's state variables include the aggregate employed human capital $Z_{et} = \int z e_t(z) dz$ and the aggregate unemployed human capital $Z_{ut} = \int z u_t(z) dz$.

Define the total value of a match $\mu_{et} = Y_t + M_t$. Define the joint outside option $\mu_{ut} = U_t$.[4] The economy is characterized by the following five equations plus Equation (11):

$$\left[(\delta + \phi_e - g_e) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_{et}\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1)\right]\mu_{et} = A_t + \delta\mu_{ut} + \mathbb{E}_t(d\mu_{et}) \tag{20}$$

$$\left[(\eta_t\lambda_{wt} + \phi_u - g_u) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_{ut}\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1)\right]\mu_{ut} = bA_t + \eta_t\lambda_{wt}\mu_{et} + \mathbb{E}_t(d\mu_{ut}) \tag{21}$$

$$\kappa A_t = (1-\eta_t)\lambda_{vt}(\mu_{et} - \mu_{ut}) \tag{22}$$

$$\frac{\mathrm{d}Z_{et}}{\mathrm{d}t} = (g_e - \phi_e - \delta)Z_{et} + \lambda_{wt}Z_{ut} \tag{23}$$

$$\frac{\mathrm{d}Z_{ut}}{\mathrm{d}t} = (g_u - \phi_u - \lambda_{wt})Z_{ut} + \delta Z_{et} + \zeta \tag{24}$$

where $\varepsilon_{e,u}$ are the elasticity of the value functions with respect to the log surplus consump-

[4] μ_{ut} is the joint outside option since the firms' outside option is 0.

tion ratio, evaluated at the steady state:

$$\varepsilon_{et} = \frac{\partial \mu_{et}}{\partial s_t}\frac{1}{\mu_{et}}, \quad \varepsilon_{ut} = \frac{\partial \mu_{ut}}{\partial s_t}\frac{1}{\mu_{ut}}$$

The constant ζ in Equation (24) comes from the normalization $\int z n_t(z) dz = 1$.

The first two equations specify the evolution of value functions. I do not need to separately write the value functions for workers and firms because their value functions are proportional to each other, implied by the efficiency condition Equation (17).

The third equation relates the job-filling rate to the value functions. Because the model is efficient, the job-filling rate depends on the match surplus, rather than how the surplus is distributed between workers and firms. The last two equations and Equation (11) specify the evolution of the state variables.

By assuming that the job-finding rate is determined by a Cobb-Douglas matching function $m(u,v) = \overline{m} u^{\eta} v^{1-\eta}$, I can write Equation (22) as

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right) \tag{25}$$

Equation (25) will be the basis to analyze the volatility and convergence of job-finding rate. I refer to $(\mu_{et} - \mu_{ut})/A_t$ as the normalized match surplus. The elasticity of job-finding rate will be large if the normalized march surplus is sensitive to the aggregate shock. Intuitively, the incentive to create vacancies and hence the job-finding rate elasticity is high if the benefit of opening an extra vacancy is large, summarized by the normalized match surplus. It is similar to the user cost of labor in Kudlyak (2014).

Define

$$\begin{aligned}
\Delta_{et} &= \varepsilon_{et} \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1) \\
\Delta_{ut} &= \varepsilon_{ut} \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1)
\end{aligned}$$

Equation (20) and Equation (21) is a dynamic system that depends on the stochastic discount factor process $\{Q_t\}_{t\geqslant 0}$:

$$\mathbb{E}_t\begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} \delta + \phi_e - g_e & -\delta \\ -\eta\lambda_{wt} & \eta\lambda_{wt} + \phi_u - g_u \end{bmatrix}}_{B_1\text{: search model}} + \underbrace{\begin{bmatrix} -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \Delta_{et} & 0 \\ 0 & -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \Delta_{ut} \end{bmatrix}}_{B_2\text{: preference}} \right) \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{26}$$

In particular, since the risk-free rate is $r_{ft} = -\mathbb{E}(dQ_t)/Q_t$, Equation (26) shows that the effective discount rate for the total value of match μ_{et} is $\left(\delta + \phi_e + r_{ft} + \Delta_{et} - g_e\right)$. The effective discount rate for the joint outside option μ_{ut} is $\left(\eta\lambda_{wt} + \phi_u + r_{ft} + \Delta_{ut} - g_u\right)$.

4 Characterization of Volatility and Convergence

In this section, I derive the elasticity and convergence of the job-finding rate. I discuss how different settings affect the elasticity. I then characterize the convergence rate in the perfect-foresight equilibrium and show that the convergence is fast. I show numerically that the fast convergence carries to the rational expectation equilibrium. I also show that the fast convergence holds when the model cannot aggregate and the dynamics depend on human capital distributions. Finally, I show that when the model is inefficient with wage rigidity or centralized job search, the convergence is still fast.

4.1 The Elasticity of Job-Finding Rate

In Equation (26), the matrix that determines the evolution of the dynamic system consists of two parts. The first matrix B_1 depends only on the search side of the model while the second one depends only on the preference. I first characterize the preference part.

Lemma 1. *The risk-free rate to a first order approximation in the log surplus consumption ratio s_t is*

$$r_{ft} = -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) \approx -\left[a_Q + b_Q(s_t - s)\right] \tag{27}$$

where

$$a_Q = -\beta - \alpha g_a + \frac{1}{2}\alpha\sigma_a^2\left(\frac{\alpha}{S^2} + 1\right) \text{ and } b_Q = \alpha(1-\rho_s) - \frac{\alpha\sigma_a^2(\alpha + S - 1)}{S^2} \tag{28}$$

Compare to a model with risk-neutral preference, several differences standard out. First, the stochastic discount factor with risk-neutral preference is $-\beta$ with $g_a = \lambda_a = 0$ and $\rho_s = 1$, namely the subjective discount rate. With the asset pricing preference Equation (6), the stochastic discount factor is time varying. In particular, the shock affects how agents discount future output flows.

Proposition 1 characterizes the elasticity of job-finding rate with respect to the aggregate shock near the steady state.

Proposition 1. *The elasticity of the job-finding rate with respect to the surplus consumption ratio s_t near $s_t = s$ is*

$$\frac{\partial \log(\lambda_{wt})}{\partial s_t} = \underbrace{\frac{b_Q e^{\Theta_1}}{1-\rho_s}}_{\text{Preference}} \underbrace{\left[\frac{c_l}{-(\gamma_l + a_Q + \Delta_e + \Theta_2)} + \frac{c_h}{-(\gamma_h + a_Q + \Delta_u + \Theta_2)}\right]\overline{\mu}^{-1}}_{\text{Search Side of the Model}} \tag{29}$$

where $\gamma_h < \gamma_l < 0$ are two eigenvalue of the matrix B_1 in Equation (26), c_l and c_h are weights of the eigenvalues given in the appendix, Θ_1 and Θ_2 depend on model parameters, Δ_e and Δ_u are evaluated at the steady state, and $\overline{\mu}$ is the long-run average match surplus without shocks.

To understand Proposition 1, I re-write the normalized match surplus formally as

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \int_t^{+\infty} e^{-\int_t^s R_u du} \frac{A_s}{A_t}(1-b)ds \tag{30}$$

Namely, it is the cumulative sum of the output flow difference between employed and unemployed workers, $\{A_s(1-b)/A_t\}_s$, discounted by the discount rate process R_t which is related to the preference and the search side of the model. The job-finding rate elasticity depends on how the output flow difference responds to the aggregate shock and how the discount rate process responds to the shock.

In the simpler model in Section 2, the unemployment benefit and the vacancy cost are not scaled by the shock. The job-finding rate elasticity depends on the match surplus without normalization:

$$\mu_{et} - \mu_{ut} = \int_t^{+\infty} e^{-Rs}(A_s - b)ds \tag{31}$$

Note that with risk neutrality, the discount rate process R_t is constant. The integral is equal to a constant times $(A_t - b)$. Taking the log and then the derivative, the elasticity of $(\mu_{et} - \mu_{ut})$ with respect to the shock is

$$\frac{\partial \log(\mu_{et} - \mu_{ut})}{\partial \log(A_t)} = \frac{A_t}{A_t - b} \tag{32}$$

Equation (32) reiterates the results in Shimer (2005), Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2017) that the elasticity depends on the ratio between the aggregate productivity and the difference between the aggregate productivity and the unemployment benefit. In particular, the fundamental surplus $A_t - b$ is the only factor for generating large job-finding rate elasticity.[5]

The intuition is that, with risk neutrality, the shock only affects the output flow differences, not the discounting of the output flow. Hence, the job-finding rate elasticity is large if the output flow difference is small, making the match surplus sensitive to the aggregate technology.

When the unemployment benefit and the vacancy posting cost are augmented by the aggregate shock, the job-finding rate elasticity will be zero in the simpler model because the output flow difference does not respond to the shock. In particular, the job-finding rate

[5]In discrete time, other factors could influence the elasticity of job-finding rate in the simpler model. However, their effect is bounded by 1. See Ljungqvist and Sargent (2017) for details.

elasticity depends on the normalized match surplus:

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \int_t^{+\infty} e^{-Rs} \frac{A_s}{A_t}(1-b)ds \tag{33}$$

Integrating the expression, the normalized match surplus is $(1-b)$ and does not respond to the shock.

With the asset pricing preference, the model no longer relies on the fundamental surplus to generate a large job-finding rate elasticity. In particular, even when the vacancy posting cost and the unemployment benefit are augmented by the shock, the job-finding rate still responds to the shock because the discount rate process is time varying. R_t depends on the search side of the model and the preference, given by B_1 and B_2 in Equation (26). Integrating Equation (30) leads to the expression in Equation (29).

Potential Sources of Large Job-Finding Rate Elasticity

I first discuss the preference part in Equation (29). The parameters that affect the preference part of the elasticity include the long-run surplus consumption ratio S and the auto-correlation of the log surplus consumption ratio ρ_s. The job-finding rate elasticity is decreasing in the long-run surplus consumption ratio S and increasing in the auto-correlation of the log surplus consumption ratio ρ_s.

Smaller long-run surplus consumption ratio would lead to larger volatility of the stochastic discount factor, amplifying the response of job-finding rate. Intuitively, with a small long-run surplus consumption ratio, small changes in consumption can have large impact on the stochastic discount factor and hence the discounting of the output flow.

Higher auto-correlation of the log surplus consumption ratio means that the effect of the shock on the discount rate process R_t is long lasting, so that the accumulated effect on the normalized match surplus is large.

I turn to the search side of the model. The elasticity depends on the ratio between the weights and the eigenvalues, $-c_l/\gamma_l$ and $-c_h/\gamma_h$. I refer to $-c_l/\gamma_l$ as the long-run multiplier and $-c_h/\gamma_h$ as the short-run multiplier, where γ_l and γ_h are the two eigenvalues of B_1.

The smaller eigenvalue γ_h is the high frequency component of the model. It relates to short-run discounting, which includes job-finding and separation. In particular, the eigenvalue is at least as big as $\eta\lambda_w$ in absolute value. Since the monthly job-finding rate is around 0.46 and the usual parameter choice for η is around 0.5, the eigenvalue γ_h is large in magnitude under plausible parameterization. The intuition is that, the output flow difference is discounted by job finding in the short run. If the workers can find jobs fast, an extra vacancy does not increase the job-finding rate by a large percent. Hence, the short-run benefit of opening an extra vacancy, given by the multiplier $-c_h/\gamma_h$, is small.

The larger eigenvalue γ_l is the low frequency component of the model. It relates to long-run discounting, which includes human capital growth and labor market exit. Since γ_l is close to zero, the long-run multiplier $-c_l/\gamma_l$ can be large if c_l is not zero.

The weight c_l depends on the difference between human capital growth rates and labor exit rates across employment status. With identical human capital accumulation and labor market exit rate, i.e. $g_e - g_u = \phi_e - \phi_u = 0$, the weight on the larger eigenvalue c_l is 0. Intuitively, since the output flow across employment status has the same discount rate process, the difference cancels out and does not interact with the shock. When there are differential human capital growth rates or labor market exit rates, the output flow difference interacts with the shock because creating an extra vacancy and hence employment can lead to higher human capital growth or lower labor market exit rates, resulting in large long-run benefits.

The analysis shows that the key to amplification of the shock lies in the discounting of the output flow by employment status. In particular, it depends on whether creating a vacancy has long-run benefit, in addition to the higher output of employed workers compared to unemployed workers. The long-run benefit can be in the form of higher human capital growth rates or lower labor market exit rates. This points way to other settings that could amplify the shock's effect. For example, in search-and-matching models embedding network (see e.g. Galenianos (2014), Arbex et al. (2019)), employed workers can refer other workers and increase the effective job-finding rate, which could lead to a large job-finding rate elasticity.

4.2 The Convergence of Job-Finding Rate

I derive analytically the convergence of job-finding rate in a perfect foresight equilibrium. The perfect foresight equilibrium concerns only value functions and sheds light on the convergence in the rational expectation equilibrium by certainty equivalence. I then numerically study the convergence in the rational expectation equilibrium and verify that a lot of insights are carried over.

In the simpler model, the transition depends only on the high-frequency component of the search side of the model. Under the parameterization in Table 1, the half-life of the job-finding rate is 3 months. This implies that the job-finding rate co-moves with the aggregate shock.

Table 1: Parameterization

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
$\overline{m}$, efficiency of match technology	0.55	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.5	Mean unemployment rate	5.9	5.9
Assigned Parameters				
β, time preference factor	0.001	η, matching function elasticity	0.5	
b, home production parameter	0.6	$\phi_e = \phi_u$, exit rate	0.0028	
δ, separation rate	0.028			

Notes. Table 1 shows the parameter values for the simpler model in Section 2. %p.a. means percent per annum.

I turn to the full model and define the perfect foresight equilibrium as follows:

1. The trajectory of the shock is deterministic.

2. The deviation from the steady state is small so that human capitals and the job-finding rate can be considered constant.

With the definition, the dynamic system in the perfect foresight equilibrium depends only on the deterministic aggregate shock:

$$\begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = \begin{bmatrix} r_{ft} + \delta + \phi_e - g_e & -\delta \\ -\eta\lambda_w & r_{ft} + \eta\lambda_w + \phi_u - g_u \end{bmatrix} \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{34}$$

where $r_{ft} = -a_Q - b_Q(s_t - s)$. Proposition 2 characterizes the convergence speed of the job-finding rate.

Proposition 2. *Let $\mu_t = (\mu_{et} - \mu_{ut})/A_t$ denote the normalized match surplus. Let $\overline{\mu}$ denote its steady state value. The normalized match surplus satisfies*

$$\mu_t - \overline{\mu} = \Omega_t \left(c_l e^{\gamma_l t} + c_h e^{\gamma_h t}\right)(\mu_0 - \overline{\mu}) \tag{35}$$

The half-life τ is the solution to the equation

$$\Omega_\tau (c_l e^{\gamma_l \tau} + c_h e^{\gamma_h \tau}) = \frac{1}{2} \tag{36}$$

where Ω_τ is given in the Appendix Section A. With $\gamma_h < \gamma_l < 0$, the half-life is increasing in c_l and decreasing in c_h:

$$\frac{\partial \tau}{\partial c_l} > 0, \quad \frac{\partial \tau}{\partial c_h} < 0$$

Proposition 2 suggests that a larger long-run weight reduces the rate of convergence. The intuition is that the long-run dynamics of the model, such human capital accumulation, has a slower rate of convergence compared to the job-finding rate. When the model transition places more weight on the long-run dynamics, its convergence will be slower.

Using the parameter values in Table 2, the half-life is again 3 months. The number is close to the one in the simpler model because the long-run weight, compared to the short-run weight, is negligible. The result means that without the other state variables, namely human capitals, the job-finding rate converges fast to the steady state and co-moves with the aggregate shock.

Table 2: Parameterization

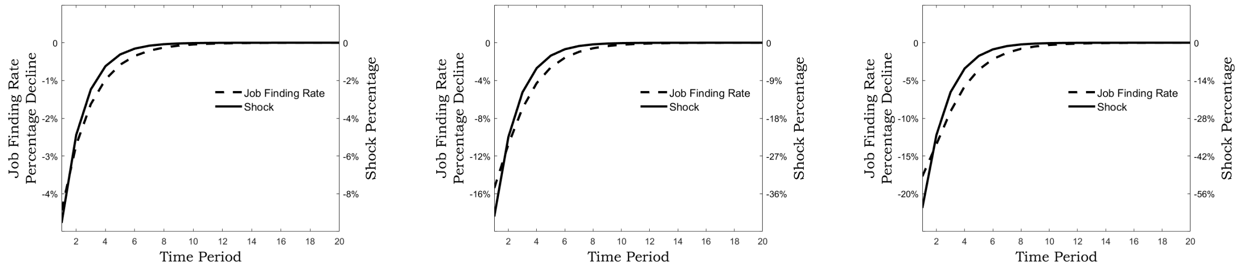
Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
$\overline{m}$, efficiency of match technology	0.55	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.975	Mean unemployment rate	5.9	5.9
β, time preference factor	0.001	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.057	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	5	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.63
δ, separation rate	0.028	Autocorrelation unemployment rate	0.97	0.98
η, matching function elasticity	0.5	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, employed human capital growth (%p.a.)	3.5	Autocorrelation job-finding rate	0.94	0.98
ρ_s, persistence of state	0.9944	S.d. unemployment rate	0.75	0.75
$\phi_e = \phi_u$, exit rate	0.0028			

Notes. Table 2 shows the parameter values for the simpler model in Section 2. %p.a. means percent per annum.

The co-movement would carry over to the rational expectation equilibrium in Section 3 if the model mainly transits through the aggregate shock. I consider the following numerical experiment: I impose shocks such that the job-finding rate declines by 5%, 10%, and 20% respectively. The shocks die down to half of the value in 3 months. I compare the convergence paths of the job-finding rate and the shock. Figure 2 shows that regardless of the size of shocks, the job-finding rate co-moves with the aggregate shock in the rational expectation equilibrium.

The co-movement not only illustrates that the convergence of job-finding rate is fast, but also indicates that there is little transmission through human capitals. The results contrast the slow recovery of the job-finding rate in recent recessions, unless one assumes that the empirical counterpart of the aggregate shock in the model has almost identical movements as the job-finding rate in the data, which takes more than 3 years to recover to

Figure 2: Convergence of the Job-Finding Rate



(a) Small Shock (b) Medium Shock (c) Large Shock

Notes. Figure 2 shows the convergence of job-finding rate when the initial decline is 5%, 10%, and 20%. The model is parameterized as in Table 2. Regardless of the size of shocks, the job-finding rate co-moves with the shock.

its pre-cession level in the last three recessions.

Human Capital Distribution and the Convergence of Job-Finding Rate

One possibility that the transmission depends almost entirely on the aggregate shock is that the movement in aggregate human capitals is small. If the model's states include human capital distributions, the transmission could be more dependent on human capitals.

To that end, I specify a nonlinear production technology $A_t f(z)$. To simplify analysis, the production technology is multiplicative in the aggregate shock. The result does not depend on the specification, as will be clear in later part of the section.

With the nonlinear production technology, the model can no longer aggregate linearly. Denote $\mu_{et}(z) = Y_t(z) + M_t(z)$ the total value of a match. Denote $\mu_{ut}(z) = U_t(z)$ the joint outside option. The model reduces to the following equations together with Equation (11):

$$(\delta + \phi_e)\mu_{et}(z) = A_t f(z) + \delta\mu_{ut}(z) + \mathbb{E}_t\left[d\mu_{et}(z) + \mu_{et}(z)\frac{dQ_t}{Q_t} + d\mu_{et}(z)\frac{dQ_t}{Q_t}\right] \tag{37}$$

$$(\eta_t\lambda_{wt}(z) + \phi_u)\mu_{ut}(z) = bA_t f(z) + \eta_t\lambda_{wt}(z)\mu_{et}(z) + \mathbb{E}_t\left[d\mu_{ut}(z) + \mu_{ut}(z)\frac{dQ_t}{Q_t} + d\mu_{ut}(z)\frac{dQ_t}{Q_t}\right] \tag{38}$$

$$\kappa A_t f(z) = (1 - \eta_t)\lambda_{vt}(z)[\mu_{et}(z) - \mu_{ut}(z)] \tag{39}$$

$$\frac{\partial e_t(z)}{\partial t} = -\frac{\partial e_t(z)}{\partial z}g_e z - (g_e + \delta + \phi_e)e_t(z) + \lambda_{wt}(z)u_t(z) \tag{40}$$

$$\frac{\partial u_t(z)}{\partial t} = -\frac{\partial u_t(z)}{\partial z}g_u z - (g_u + \lambda_{wt}(z) + \phi_u)u_t(z) + \delta e_t(z) + \zeta n_t(z) \tag{41}$$

The first two equations specify the value functions. The free-entry condition Equation (39) implies that the job-finding rate can differ by human capital, whereas in Section 4 it is equalized across submarkets.

The last two equations specify the evolution of human capital distributions. The expressions are similar to Equation (23) and Equation (24). The density loss due to human capital growth involves an extra term $(\partial e_t(z)/\partial z)g_e z$, indicating that the density $e_t(z)$ moves to higher level human capitals. When dealing with the aggregate human capital, the reshuffling of the density does not matter.

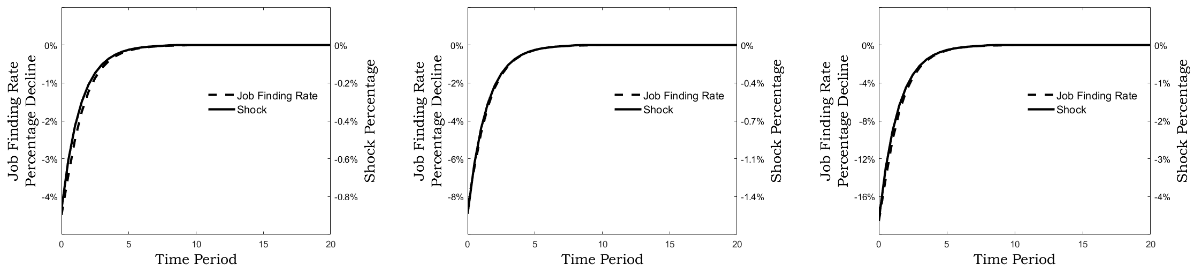
The model does not afford analytic solutions. I solve the model using the technique in Ahn et al. (2017). I use two production functions with $f(z) = z^{1/2}$ and $f(z) = z^2$. The first one implies decreasing return to scale. The second one is less common, but is useful for illustration purpose. The parameter values remain the same as in Table 2.

Figure 3 plot the job-finding rate convergence. The initial declines of job-finding rate are 5%, 10%, and 20% respectively, with the half-life of the shock equal to 3 months. I calculate the aggregate job-finding rate, defined as

$$\lambda_{wt} = \overline{m}\left(\int \theta_t(z)u_t(z)dz\right)^{1-\eta} = \left(\int \lambda_{wt}(z)^{\frac{1}{1-\eta}}u_t(z)dz\right)^{1-\eta} \tag{42}$$

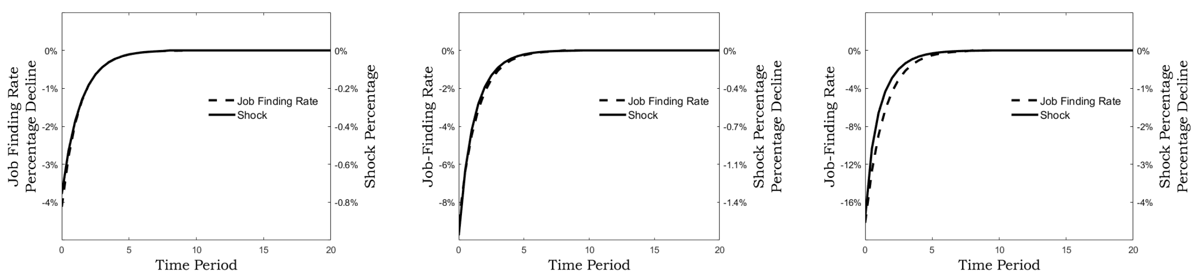
This is the model counterpart of the empirical job-finding rate. Regardless of the size of the shock and the production technology, the job-finding rates co-move with the aggregate shock.

Figure 3: Job-Finding Rate Convergence with Human Capital Distribution



Small Shock | Medium Shock | Large Shock

(a) Decreasing Return-to-Scale Technology



Small Shock | Medium Shock | Large Shock

(b) Increasing Return-to-Scale Technology

Notes. Figure 3 shows the convergence of job-finding rate when the production technology is $f(z) = z^2$ (increasing return-to-scale) and $f(z) = z^{1/2}$ (decreasing return-to-scale). The size of shocks implies that the initial declines in the job-finding rate are 5%, 10%, and 20%. The model is parameterized as in Table 2. The non-linear technology implies that the model dynamics depend on human capital distributions. However, the convergence of the job-finding is still fast, regardless of the size of shocks.

The result is surprising since the distribution of unemployed workers directly enters the aggregate job-finding rate. To see why the model fails to generate the slow convergence of the job-finding rate, I focus on the aggregate market tightness $\boldsymbol{\theta}_t \equiv \int \theta_t(z)u_t(z)dz$. The change in the aggregate market tightness has the following decomposition:

$$d\boldsymbol{\theta}_t = \int d\theta_t(z)u_t(z)dz + \int \theta_t(z)du_t(z)dz \tag{43}$$

The convergence hence also depends on the market tightness in individual submarkets and the measure of unemployment. To generate a slow convergence, at least one of them needs to have delayed recovery after the shock.

To study the convergence of market tightness and the measure of unemployment, I simulate the model using $f(z) = z^{1/2}$ as the production function.[6] Since theoretically there are infinitely many submarkets, plotting the path of each submarket market tightness is forbidden while selecting a few submarkets is arbitrary. Instead, I run the following regression of the submarket market tightness on the aggregate shock.

$$\theta_{it} = \alpha_i + \beta A_t + \epsilon_{it} \tag{44}$$

where i is the submarket index. In the numerical simulation, I discretize the model to have 100 submarkets and simulate the model for 2000 periods. The statistics of interest is the R-squared of the regression. It reflects how much time-series variation in the submarket market tightness is explained by the aggregate shock.

The R-squared is 0.84. The value is high given that the submarket market tightness embeds heterogeneity. As a comparison, empirically the R-squared of the regression of the *aggregate* market tightness on the productivity shock is 0.07 from 1989 to 2019. The comparison suggests that the submarket market tightness depends almost entirely on the aggregate shock in all submarkets, even though the marginal productivity differs.

Similarly, I regress the measure of unemployment in each submarket against the aggregate shock using

$$u_{it} = \alpha_i + \beta A_t + \epsilon_{it} \tag{45}$$

The resulting R-squared is 0.99, suggesting that all time series variations in the measure of unemployment are explained by the aggregate shock. Namely, every submarket synchronizes the time and magnitude of unemployment rate changes. As a result, the

[6]The results with $f(z) = z^2$ are similar and available upon request.

aggregate market tightness and hence the job-finding rate co-moves with the shock.

The results suggest that there is little feedback from human capital to the job-finding rate. More specifically, the model transmission is the following. The aggregate shock affects value functions, which shift job-finding rates in each submarket. Job-finding rates then affect the evolution of the human capital distributions. In particular, job-finding rates decline by similar magnitude in each submarket, so that the impact on the measure of unemployment is uniform across submarkets. After the aggregate shock dissipates, value functions recover fast by Proposition 2, leading to fast convergence of the job-finding rate.[7] Hence, almost all dynamics are driven by the aggregate shock, including aggregate human capitals.

By the transmission, the co-movement with the aggregate shock would carry over to models with wage rigidity or centralized job search. The reason is because there is no feedback from other factors to the job-finding rate. As a result, the job-finding rate is only affected by value functions, which converges fast under plausible parameterization. I verify the co-movement numerically in the Appendix Section C.

Extension for a Slow Convergence of Job-Finding Rate

To generate a slow convergence of the job-finding rate, it is important to have some feedback from human capital to the job-finding rate. Moreover, the feedback needs to differ by human capital levels. I briefly discuss one extension that could generate a slow convergence of the job-finding rate and leave more extensions for future research.

[7]To verify the transmission, I examine the percent change dispersion in the value functions. More specifically, define

$$VD_t(z) \equiv (\mu_{et}(z) - \mu_{ut}(z))/A_t \tag{46}$$

as the normalized value function difference. According to Equation (39), the difference determines the job-finding rate in each submarket. If the percent declines are similar for different submarkets, job-finding rates decline would also be similar. I hence calculate the standard deviation of the percent decline across submarkets as

$$STD_t = std(VD_t(z)/VD_{t-1}(z)) \tag{47}$$

For the simulation, the largest standard deviation in the value functions percent decline is 0.002%. The small standard deviation suggests that the value functions in each submarket decline by similar magnitude, so do job-finding rates. The unemployed human capital distribution remains almost unchanged.

I introduce firms' hiring standards as follows. When a worker and a firm meet, the worker draws an idiosyncratic match quality shock q_t. Firms only accept match quality draws that exceed the hiring standard. In particular, the probability that a match is formed is:

$$\overline{m}_t = \mathbb{P}(q_t z \geqslant p_t), \quad q_t \sim N(\mu_q, \sigma_q) \tag{48}$$

where p_t is firms' hiring standard. With this, the job-finding rate becomes

$$\lambda_{wt}(z) = \overline{m}_t \theta_t(z)^{1-\eta} \tag{49}$$

With the hiring standard, human capital directly affects job-finding rates, differentially for each submarket. The aggregate shock first affects value functions, which lower the job-finding rate. Average human capital falls because of the lower job-finding rate and because unemployed workers have slower human capital growth, which further decreases the job-finding rate and delays the recovery.[8]

The interpretation of the extension is that, the match efficiency is endogenously determined by workers' human capital and firms' hiring standards. During recessions, average human capital decreases in the unemployment pool while firms' hiring standards are more stringent (Pollmann-Schult (2005), Quintini (2011)). The feedback loop could lead to persistent low match efficiency and hence a low job-finding rate, even if the aggregate technology recovers. The view is consistent with Barnichon and Figura (2015) who show that the match efficiency declines during recessions because of increasing presence of low match efficiency workers in the unemployment pool.

[8]For details, see Liu (2021).

5 Conclusion

This paper studies the job-finding rate elasticity and convergence in a class of efficient search-and-matching models. The paper derives analytic solution for the job-finding rate elasticity, and show that the key for a large elasticity lies in the response of the output flow and how the agents discount the output flow.

The analytic expression for the job-finding rate convergence in the perfect foresight equilibrium suggests that the job-finding rate co-moves with the aggregate shock, which carries over to the rational expectation equilibrium. The co-movement persists even when the model dynamics depend on human capital distributions or when the wage is rigid.

The reason for the co-movement is a lack of feedback from other factors such as human capital to the job-finding rate. Without the feedback, all dynamics are driven by the shock, including the movement in human capital distributions. The model is hence unable to generate the slow convergence of the job-finding rate, which has been the case in the three recessions from 1989 to 2019. Being able to break the comovement in the search-and-matching model is important for understanding the slow recovery of the job-finding rate and is left for future research.

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Appendices

A Proofs

A.1 Proof of Lemma 1

Proof. The risk-free rate is

$$r_{ft} = -\mathbb{E}\left(\frac{dQ_t}{Q_t}\right)$$

where Q_t is the stochastic discount factor

$$Q_t = e^{-\beta t}(S_t C_t)^{-\alpha}$$

Using Ito's lemma, the stochastic discount factor satisfies the following stochastic differential equation (SDE):

$$\begin{aligned} dQ_t = &-\beta e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} dt - \alpha e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} \frac{dS_t}{S_t} - \alpha e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} \frac{dC_t}{C_t} \\ &+ \frac{1}{2} e^{-\beta t} \alpha(\alpha+1) S_t^{-\alpha} C_t^{-\alpha} \frac{(dS_t)^2}{S_t^2} + \frac{1}{2} e^{\beta t} \alpha(\alpha+1) S_t^{-\alpha} C_t^{-\alpha} \frac{(dC_t)^2}{C_t^2} \\ &+ e^{-\beta t} \alpha^2 S_t^{-\alpha} C_t^{-\alpha} \frac{dS_t}{S_t} \frac{dC_t}{C_t} \end{aligned} \tag{A.1}$$

where

$$\frac{dS_t}{S_t} = (1-\rho_s)(s - s_t)dt + \sigma_a \lambda(s_t) dW_t$$

and

$$\frac{dC_t}{C_t} \approx \frac{dA_t}{A_t} = g_a dt + \sigma_a dW_t$$

Here the approximation ignores the changes in the human capital. In the steady state, the human capital is constant. Close to the steady state, the approximation is appropriate.

Substituting in the SDE for S_t and C_t to equation (A.1), the stochastic discount factor satisfies

$$
\begin{aligned}
dQ_t = & \left(-\beta - \alpha g_a + \frac{1}{2}\alpha^2\sigma_a^2\frac{1}{S^2}\right)Q_t dt + \left[\alpha(1-\rho_s) + \frac{\alpha^2\sigma_a^2}{S^2}\right]Q_t(s_t - s) \\
& - \underbrace{\alpha\sigma_a(\lambda(s_t)+1)dW_t}_{\text{local martingale}}
\end{aligned}
\tag{A.2}
$$

When taking the expectation, the local martingale vanishes, so that the risk-free rate satisfies

$$
r_{ft} = -\mathbb{E}\left(\frac{dQ_t}{Q_t}\right) = \underbrace{-\beta - \alpha g_a + \frac{1}{2}\alpha^2\sigma_a^2\frac{1}{S^2}}_{a_Q} + \underbrace{\left[\alpha(1-\rho_s) + \frac{\alpha^2\sigma_a^2}{S^2}\right]}_{b_Q}(s_t - s) \tag{A.3}
$$

□

A.2 Proof of Proposition 1

Proof. By equation (25), the elasticity of the job-finding rate with respect to the shock s_t is equal to the elasticity of $\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)$. The latter evolves according to equation (26). We can solve the system forward by making two assumptions.

Assumption 1 The job-finding rate is approximately equal to its steady-state value λ_w.

Assumption 2 The human capitals are constant.

The first assumption makes the job-finding rate in the transition matrix B_1 constant. The dynamic system becomes analytically tractable. The second assumption allows me to drop the expectation operator on the left-hand side of equation (25). Under the assumption, the dynamic system only depends on the log surplus consumption ratio process s_t, which is an Ornstein-Uhlenbeck (O-U) process. The dynamic system is hence an integrated O-U process which has a closed-form solution.

The dynamic system equation (26) can further be written as

$$\mathbb{E}_t \begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = (B_1 + B_2 + B_3) \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{A.4}$$

where

$$B_1 = \begin{bmatrix} \delta + \phi_e - g_e & -\delta \\ -\eta\lambda_w & \eta\lambda_w + \phi_u - g_u \end{bmatrix}$$
$$B_2 = \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_u \end{bmatrix}$$
$$B_3 = \begin{bmatrix} -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) & 0 \\ 0 & -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) \end{bmatrix}$$

B_3 commutes with B_1 and B_2, but B_1 and B_2 do not commute. And hence, the exponential, by the Zassenhaus formula, can be written as

$$\exp(B_1 + B_2) \approx \exp(B_1)\exp(B_2)\exp\left(\frac{1}{2}(B_1B_2 - B_2B_1)\right) \tag{A.5}$$

The matrix $(B_1B_2 - B_2B_1)/2$ is

$$\begin{bmatrix} 0 & -\delta(\Delta_u - \Delta_e) \\ -\eta\lambda_w(\Delta_e - \Delta_u) & 0 \end{bmatrix} \tag{A.6}$$

The matrix is approximately 0 if $\Delta_e \approx \Delta_u$. In other words, the matrix is approximately 0 if the elasticity of the total match revenue and the outside option with respect to the log surplus consumption ratio is close near the steady state. Numerically, this is indeed the case. So I first derive the result by assuming that $\Delta_u = \Delta_e$.

With the simplification, the dynamic system equation (A.4) can be solved with

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = \int_t^{+\infty}\left(c_l e^{(\gamma_l+\Delta_e)(u-t)} + c_h e^{(\gamma_h+\Delta_u)(u-t)}\right)\mathbb{E}_t\left(\frac{Q_u}{Q_t}\right)du \tag{A.7}$$

where

$$\gamma_l = \frac{(g_e+g_u)-(\delta+\phi_e)-(\eta\lambda_w+\phi_u)}{2} + \frac{1}{2}\sqrt{\left[(g_e-g_u)-(\delta+\phi_e)+(\eta\lambda_w+\phi_u)\right]^2+4\delta\eta\lambda_w}$$
$$\gamma_h = \frac{(g_e+g_u)-(\delta+\phi_e)-(\eta\lambda_w+\phi_u)}{2} - \frac{1}{2}\sqrt{\left[(g_e-g_u)-(\delta+\phi_e)+(\eta\lambda_w+\phi_u)\right]^2+4\delta\eta\lambda_w}$$

are the two eigenvalues of the matrix B_1. The weights c_l, c_h are given by

$$c_l = \frac{(\gamma_l+\phi_u-g_u)[(1-b)\eta\lambda_w - b(\gamma_h+\eta\lambda_w-g_u)]}{(\gamma_l-\gamma_h)\eta\lambda}$$
$$c_h = \frac{(\gamma_h+\phi_t-g_u)\left[b(\gamma_l+\phi_u-g_u)-(1-b)\eta\lambda_w\right]}{(\gamma_l-\gamma_h)\eta\lambda}$$

By lemma 1, we can integrate equation (A.2) and write Q_u/Q_t as

$$\begin{aligned}\frac{Q_u}{Q_t} &= \int_t^u \left(a_Q + b_Q(s_\tau - s)\right)d\tau \\ &= a_Q(u-t) + b_Q\int_t^u (s_\tau - s)d\tau\end{aligned}$$

Plug the expression into equation (A.7), we can rewrite the equation as

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = \int_t^{+\infty}\left(c_l e^{(\gamma_l+\Delta_e+a_Q)(u-t)} + c_h e^{(\gamma_h+\Delta_u+a_Q)(u-t)}\right)\mathbb{E}_t\left(e^{b_Q\int_t^u(s_\tau-s)d\tau}\right)du \tag{A.8}$$

The integral has two parts. The first part is an exponential integrand. The second part involves an expectation operator. In particular, since $\{s_t\}$ is an O-U process, the second part is the exponential of an integrated O-U process. Let the integrated O-U process be denoted by

$$G_u = \int_t^u (s_\tau - s)d\tau$$

The integrated O-U process is a Gaussian process (see e.g. Barndorff-Nielsen and Shephard (2003)). For Gaussian processes, the expected value of its exponential is characterized by its mean and variance, which are:

$$\begin{aligned}
\mathbb{E}_t(G_u) &= (1-\rho_s)(s_t - s) - \frac{(s_t - s)e^{-(1-\rho_s)(u-t)}}{1-\rho_s} \\
Var_t(G_u) &= \frac{b_Q \sigma_a^2 \lambda^2(s_t)}{(1-\rho_s)^2}(u-t) - \frac{3}{2}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} + \frac{3}{2}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} e^{-(1-\rho_s)(u-t)}
\end{aligned}$$

I will use M_u and V_u to denote the mean and the variance of the O-U process. The expected value is

$$\mathbb{E}_t e^{G_u} \approx e^{M_u + \frac{1}{2}V_u} = e^{\Theta_1 + \Theta_2(u-t) - \Theta_3 e^{-(1-\rho_s)(u-t)}} \tag{A.9}$$

where the constants (fix time t) Θ_1, Θ_2, Θ_3 are

$$\begin{aligned}
\Theta_1 &= \frac{b_Q(s_t - s)}{1-\rho_s} - \frac{3}{4}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} \\
\Theta_2 &= \frac{b_Q\sigma_a^2\lambda^2(s_t)}{2(1-\rho_s)^2} \\
\Theta_3 &= b_Q\left[\frac{s_t - s}{1-\rho_s} - \frac{3}{4}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3}\right]
\end{aligned}$$

Plug equation (A.9) into and integrate, we can derive that integral is equal to

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \frac{c_l e^{\Theta_1}}{-(\gamma_l + \Delta_e + a_Q + \Theta_2)} + \frac{c_h e^{\Theta_1}}{-(\gamma_h + \Delta_u + a_Q + \Theta_2)} \tag{A.10}$$

The ratio $(\mu_{et} - \mu_{ut})/A_t$ is related to the log surplus consumption ratio since Θ_1 and Θ_2 are functions of the log surplus consumption ratio. Taking the derivative with respect to s_t of equation (A.10), the derivative is

$$
\begin{aligned}
\frac{\partial\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)}{\partial s_t} =& \frac{b_Q}{1-\rho_s}\left[\frac{c_l e^{\Theta_1}}{-(\gamma_l+\Delta_e+a_Q+\Theta_2)}+\frac{c_h e^{\Theta_1}}{-(\gamma_h+\Delta_u+a_Q+\Theta_2)}\right] \\
&-\alpha^2\sigma_a^4\frac{\partial(\lambda(s_t)(\lambda(s_t)+1))}{\partial s_t}\left[\frac{c_l\varepsilon_e e^{\Theta_1}}{(\gamma_l+\Delta_e+a_Q+\Theta_2)^2}+\frac{c_h\varepsilon_u e^{\Theta_1}}{(\gamma_h+\Delta_u+a_Q+\Theta_2)^2}\right]
\end{aligned}
\tag{A.11}
$$

The second term on the right-hand side is approximately 0 because σ_a^4 is small. By dropping the second term and denoting the long-run value of $(\mu_{et}-\mu_{ut})/A_t$ to be $\overline{\mu}$, the elasticity is

$$
\frac{\partial\log(\lambda_{wt})}{\partial s_t}=\underbrace{\frac{b_Q e^{\Theta_1}}{1-\rho_s}}_{\text{Preference}}\underbrace{\left[\frac{c_l}{-(\gamma_l+\Delta_e+a_Q+\Theta_2)}+\frac{c_h}{-(\gamma_h+\Delta_u+a_Q+\Theta_2)}\right]\overline{\mu}^{-1}}_{\text{Search Model}}
\tag{A.12}
$$

The first part of the elasticity is due to the preference. The asset pricing preference implies that the log surplus consumption ratio enters the integral. The second part is due to the search model, because the transition matrix B_1 which determines the eigenvalues and the weights are determined by the search side of the model.

This completes the proof when $\Delta_e=\Delta_u$. While this is numerically accurate, I cannot prove that $\Delta_e=\Delta_u$. In the remaining of the proof, I show the elasticity when the equality is not assumed.

Since both B_1 and the matrix in equation (A.6) are non-diagonal, I need to first diagonalize the two matrices. B_1 is diagonalized in the above proof. The matrix in equation (A.6), after diagonalization, is

$$
B_2=H\begin{bmatrix}\frac{1}{2}i\nu & 0\\ 0 & -\frac{1}{2}i\nu\end{bmatrix}H^{-1}
\tag{A.13}
$$

where

$$
\nu=\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1)
$$

is the eigenvalue for the matrix B_2. The dynamic system reduces to

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = \int_t^{+\infty} \Big[(c_{l,1}+ic_{l,2})e^{(\gamma_l+\frac{1}{2}i\nu)(u-t)} + (c_{l,1}-ic_{l,2})e^{(\gamma_l-\frac{1}{2}i\nu)(u-t)} + (c_{h,1}+ic_{h,2})e^{(\gamma_h+\frac{1}{2}i\nu)(u-t)} + (c_{h,1}-ic_{h,2})e^{(\gamma_h-\frac{1}{2}i\nu)(u-t)}\Big]\mathbb{E}_t\left(\frac{Q_u}{Q_t}\right)dt \tag{A.14}$$

where

$$\begin{aligned}
c_{l,1} =& \frac{(\gamma_l+\phi_u-g_u)[\eta\lambda_w - b(\gamma_h+\phi_u+\eta\lambda_w-g_u)]}{2\eta\lambda_w(\gamma_l-\gamma_h)} \\
c_{l,2} =& \frac{(\gamma_l+\phi_u-g_u)(\gamma_h+\phi_u+\eta\lambda_w-g_u)}{2\eta\lambda_w(\gamma_l-\gamma_h)}\frac{\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}{\delta\varepsilon_u} + \frac{b\delta\varepsilon_u(\gamma_l+\phi_u-g_u)}{2(\gamma_l-\gamma_h)\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|} \\
c_{h,1} =& \frac{(\gamma_h+\phi_u-g_u)[b(\gamma_l+\phi_u+\eta\lambda_w-g_u)-\eta\lambda_w]}{2\eta\lambda_w(\gamma_l-\gamma_h)} \\
c_{h,2} =& -\frac{(\gamma_l+\phi_u+\eta\lambda_w-g_u)(\gamma_h+\phi_u-g_u)}{2\eta\lambda_w(\gamma_l-\gamma_h)}\frac{\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}{\delta\varepsilon_u} - \frac{b\delta\varepsilon_u(\gamma_h+\phi_u-g_u)}{2(\gamma_l-\gamma_h)\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}
\end{aligned}$$

Equation (A.14) shows that the normalized surplus $(\mu_{et}-\mu_{ut})/A_t$ can be solved by a Fourier transform of the exponential of the eigenvalues of B_1. Solving the Fourier transform, the normalized surplus is

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = e^{\Theta_1}\left\{\frac{\left[-c_{l,1}(\gamma_l+\Delta_e+a_Q+\Theta_2)-c_{l,2}\nu/2\right]}{(\gamma_l+\Delta_e+a_Q+\Theta_2)^2+\nu^2/4} + \frac{\left[-c_{h,1}(\gamma_h+\Delta_u+a_Q+\Theta_2)-c_{h,2}\nu/2\right]}{(\gamma_h+\Delta_u+a_Q+\Theta_2)^2+\nu^2/4}\right\} \tag{A.15}$$

Taking the derivative, one gets an expression that is similar to equation (A.11) which is omitted. □

A.3 Proof of Proposition 2

Proof. The normalized match surplus, $\mu_t = (\mu_{et}-\mu_{ut})/A_t$, is given in the proof of proposition 1, namely section A.2. Section A.2 implies that the normalized match surplus satisfies

$$\mu_t - \overline{\mu} = \left[c_l e^{(\gamma_l + a_Q)t} + c_h e^{(\gamma_h + a_Q)t}\right] \mathbb{E}_0\left(e^{b_Q \int_0^t (s_\tau - s) d\tau}\right)(\mu_0 - \overline{\mu}) \tag{A.16}$$

where μ_0 is the initial value of the normalized match surplus. The expectation on the right-hand side is given in the proof of proposition 1. Plugging in the expression for the expectation, the normalized match surplus is

$$\mu_t - \overline{\mu} = \Omega_t \left(c_l e^{\gamma_l t} + c_h e^{\gamma_h t}\right)(\mu_0 - \overline{\mu}) \tag{A.17}$$

where

$$\Omega_t = e^{\Theta_1 - \Theta_3 e^{-(1-\rho_s)t} + (a_Q + \Theta_2)t}$$

Ω_t does not depend on human capital growth or labor market exit rate. By definition, the half-life τ is implicitly given by the equation

$$\Omega_\tau (c_l e^{\gamma_l \tau} + c_h e^{\gamma_h \tau}) = \frac{1}{2} \tag{A.18}$$

Taking the derivative of τ with respect to c_l, noting that Ω_τ does not vary with c_l, we can show that the derivative is positive. Similarly, the derivative of τ with respect to c_h is negative. □

B Human Capital Growth Rates and the Job-Finding Rate Elasticity

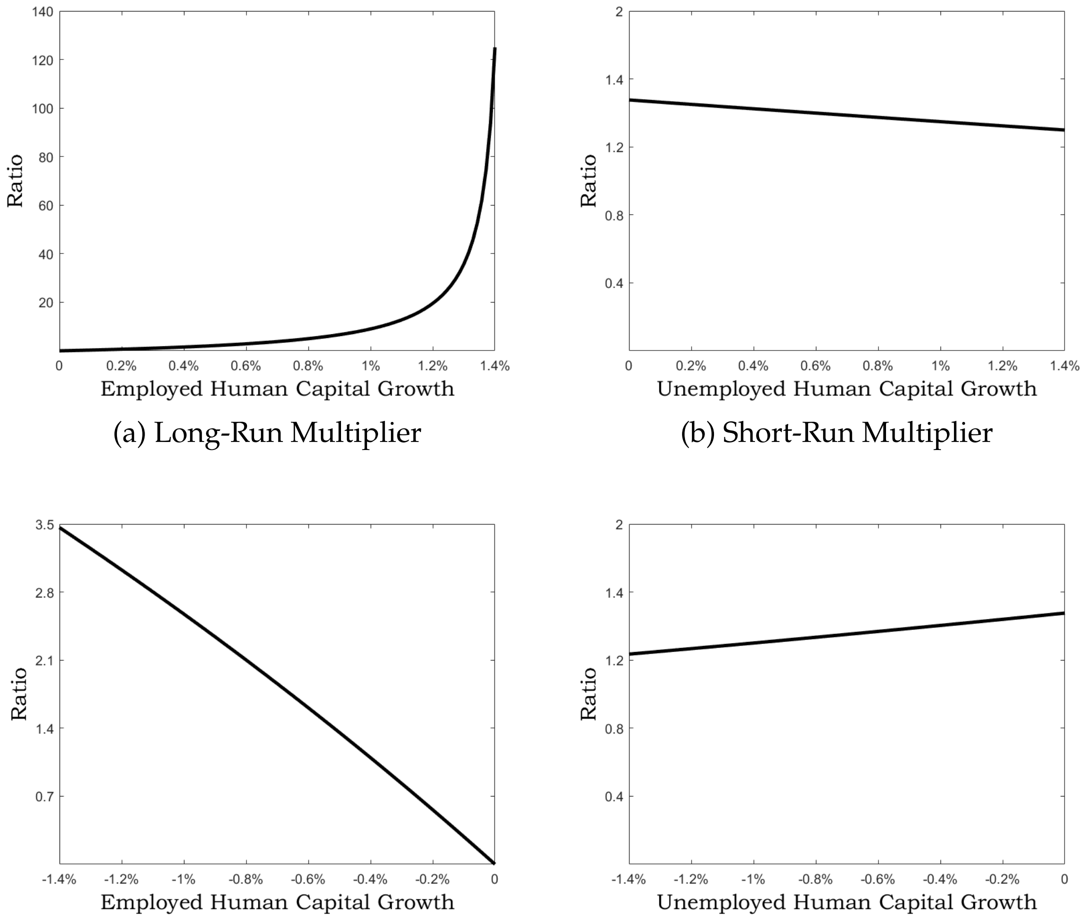
By the discussion in section 4, the elasticity is affected by the parameter values of the human capital growth, the labor market exit rates, etc. In this subsection, I explore in detail how the human capital growth rates affect the elasticity. I fix the other parameters as in table 2 and vary g_e or g_u. Figure 4 plots the long-run and the short-run multipliers as the human capital grow rates vary. In figure 4 (a) and (b), I set $g_u = 0$ and vary g_e. In figure 4 (c) and (d), I set $g_e = 0$ and vary g_u. Note that while the weights c_h and c_l depend only on the difference $g_e - g_u$, the eigenvalues differ by varying g_e or g_u.

In both cases, I vary the human capital growth so that the difference $g_e - g_u$ ranges from 0 to 1.4% per month, which corresponds to 18% human capital growth differential per annum. In figure 4 (a), there is a singularity in the long-run multiplier as the employed human capital growth rate approaches 20% per annum. Near the singularity, the long-run multiplier can be very large, leading to a large job-finding elasticity. In contrast, the short-run effect is bounded between 1.2 and 1.4, so there is little amplification.

In figure 4 (b), while the short-run multiplier is similarly bounded between 1.2 and 1.4, the long-run multiplier is substantially smaller than in figure 4 (a). Even when the human capital depreciates at about 20% a year, the long-run multiplier is less than 3.5. This means that the model might not generate large job-finding rate volatility for workers whose human capital depreciates fast during unemployment.

The multipliers are small even when I vary other parameter values. In particular, I vary the unemployment benefit parameter b between 0.1 and 0.9. The sum of the long-run and short-run multipliers is never greater than 6.

I verify whether the small multipliers translate to small job-finding rate volatility numerically. I solve the model and fix the other parameters as in table 2 but set $g_e = 0$ and change the unemployed human capital growth g_u to be -0.014, corresponding to a 18%



(a) Long-Run Multiplier (b) Short-Run Multiplier

(c) Long-Run Multiplier (d) Short-Run Multiplier

Figure 4: Human Capital Growth and the Multipliers

decline per annum. The model job-finding rate standard deviation is 3.3 percentage points, half of the overall job-finding rate standard deviation in the data. Slower human capital depreciation leads to even smaller job-finding rate volatility.

If I assume that the less-educated workers have relatively higher human capital depreciation, the discrepancy between the model and the data is larger: while the overall job-finding rate standard deviation is around 6.6 percentage points, the standard deviation for the high-school workers is around 8.1 percentage points using CPS from 1992 to 2017.

To understand the model's difficulty in producing the large volatility with fast human capital depreciation, I first state corollary 1.

Corollary 1. *The sum of the weights c_l and c_h satisfies*

$$c_l + c_h = 1 - b \tag{B.1}$$

The long-run weight c_l is increasing in $g_e - g_u$:

$$\frac{\partial c_l}{\partial (g_e - g_u)} > 0$$

The sum of the weight is equal to the gain in productivity when employed, referred to as the fundamental surplus in Ljungqvist and Sargent (2017). The fixed sum means that the long-run weight is bounded by $(1 - b)$. On the other hand, if the difference in human capital growth rates is large across employment status, more weight is assigned to the long-run benefit. In effect, the job-finding rate elasticity from the search side of the model can be decomposed into the following sum:

$$\underbrace{\text{Long-run Weight} \times \text{Long-run Benefit}}_{\text{Long-run Multiplier}} + \underbrace{\text{Short-run Weight} \times \text{Short-run Benefit}}_{\text{Short-run Multiplier}}$$

The short-run multiplier is small for plausible parameter values, and the long-run weight is bounded. A large job-finding rate elasticity then requires the long-run benefit to be large, or the long-run discount rate, given by the low-frequency eigenvalue, to be small.

The long-run benefit is large only when there is large human capital growth for the employed. The intuition is clear: while the weight depends only on the difference $g_e - g_u$, the eigenvalue and hence the long-run benefit depends on how the difference comes about. An analogy is that, suppose $g_e - g_u = 0.1$. Then $(1 + 0.1)^t - 1$ is much larger than $1 - (1 - 0.1)^t$ when t is large. Corollary 2 states the result.

Corollary 2. *The model has difficulty in producing large job-finding rate elasticity when the human capital growth on the job is small but the depreciation is large when unemployed.*

One argument is that the same aggregate shock might have differential effects. The workers whose human capital depreciates fast might receive a larger shock in effect, so that their job-finding rate is more volatile. The argument suggests that the model should change the variance of the aggregate shock for workers with fast human capital depreciation. However, the fact that the same shock leads to much smaller job-finding rate volatility in the model suggests that the model lacks transition mechanism that amplifies the shock for the workers with fast human capital depreciation.

C Job-Finding Rate Convergence When Wage is Rigid

I show numerically that in a variant of the model in Section 4 with human capital distribution and wage rigidity, the job-finding rate co-moves with the aggregate shock.

With wage rigidity, there is no longer perfect risk-sharing between workers and firms. To simplify the analysis and focus on the converge of job-finding rate, I assume that workers and firms are risk-neutral with objective discount rate β. The model hence no longer features asset pricing preference, which is shown to be unrelated to the job-finding rate convergence. I also assume that the aggregate shock $\{A_t\}$ follows:

$$dA_t = \sigma_a dW_{a,t}$$

The unemployment benefit and the vacancy cost are not scaled by the shock. Note that scaling does not affect the convergence.

Let $V_t(z)$ denote the value of employed workers. Let $U_t(z)$ denote the value of unemployed workers. $J_t(z)$ is the value of firms. The system of equations that characterize the economy is:

$$(\delta + \phi_e + \beta - g_e)V_t(z) = w_t(z) + \delta U_t(z) + \mathbb{E}_t[dV_t(z)] \tag{C.1}$$

$$(\lambda_{wt}(z) + \phi_u + \beta - g_u)U_t(z) = bf(z) + \lambda_{wt}(z)V_t(z) + \mathbb{E}_t[dU_t(z)] \tag{C.2}$$

$$(\delta + \phi_e + \beta)J_t(z) = e^{A_t} f(z) - w_t(z) + \mathbb{E}_t[dJ_t(z)] \tag{C.3}$$

$$\kappa f(z) = (1 - \eta)\lambda_{vt}(z)J_t(z) \tag{C.4}$$

$$w_t(z) = \max\left\{\underset{w}{\operatorname{argmax}}\,[V_t(z) - U_t(z)]^{\eta}[J_t(z)]^{1-\eta}, \chi w_{t-1}(z)\right\} \tag{C.5}$$

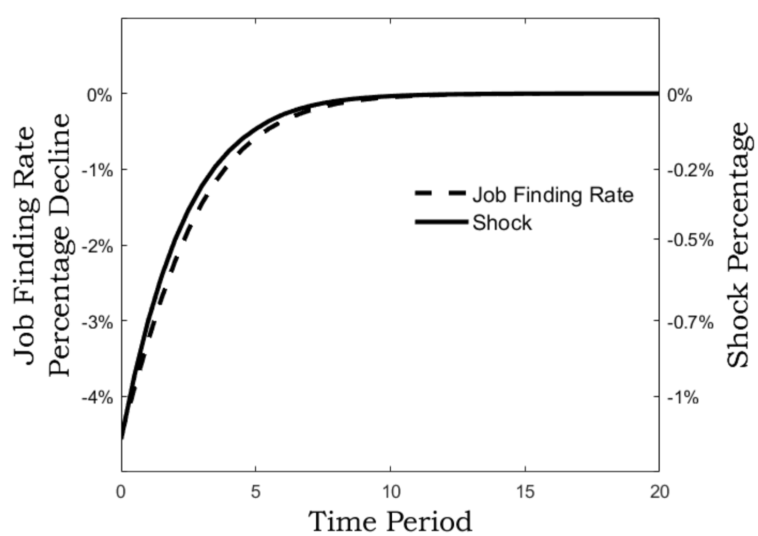
$$\frac{\partial e_t(z)}{\partial t} = -\frac{\partial e_t(z)}{\partial z} g_e z - (g_e + \delta + \phi_e)e_t(z) + \lambda_{wt}(z)u_t(z) \tag{C.6}$$

$$\frac{\partial u_t(z)}{\partial t} = -\frac{\partial u_t(z)}{\partial z} g_u z - (g_u + \lambda_{wt}(z) + \phi_u)u_t(z) + \delta e_t(z) + \zeta n_t(z) \tag{C.7}$$

The first three equations are the value functions of employed workers, unemployed workers, and firms. The fourth equation is the free-entry condition. The fifth equation is the wage determination equation. In particular, the wage is determined by Nash bargaining, together with a wage rigidity condition that the wage cannot be lower than χ fraction of the last period's wage. The last two equations are the evolution of the employed and the unemployed human capital distribution.

I set the production technology to be $f(z) = z^{1/2}$. I impose a one-time negative shock that decreases the job-finding rate by 5%. The half-life of the shock is 3 months. With the example, Figure 5 plots the job-finding rate and the aggregate shock paths. The co-movement is similar to the ones in Figure 2. The intuition is stated in Section 4: the wage rigidity does not change the model transmission. There is still not enough feedback from other factors to the job-finding rate and the elasticity of the job-finding rate to the shock is similar in all submarkets. To see this, I use the metric in Equations (46) and (47). The standard deviation in the value function difference is 0.4% for the 5% decline in the job-finding rate. Note that the deviation is considerably larger than the one without wage rigidity (0.002%), suggesting that the wage rigidity creates more differential responses of job-finding rates across submarkets. However, it is still not enough to shift human capital distributions.

Figure 5: Job-Finding Rate Convergence



Lastly, I run the regression in Equation (44) to see how much time-series variation is explained by the aggregate shock. The R-squared is 0.61. Consistent with the analysis above, less variation in the submarket market tightness is explained by the aggregate shock. However, the explanatory power is still large compared to the empirical R-squared at 0.07. I conclude that adding wage rigidity to the model in Section 2 is not able to generate the slow convergence of job-finding rate.