Optimal Student Loan Design with Labor Market Entry Shock

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Abstract

While returns to educational investment in the form of lifetime income crucially depend on labor market entry shocks, current student loan design does not consider the aspect. I study the optimal Mirrleesian taxation with observable labor market entry shocks. The optimal policy is entry-shock dependent and can be implemented by an income- and entry-shock-dependent student loan repayment schedule. Calibrating the model to the US economy, the optimal policy gains 0.2% of lifetime consumption compared to the entry-shock independent policy. The optimal policy can be well approximated by fixed repayment rates and a lump-sum payment which depend on the entry shocks. The approximating policy can be implemented by an automatic loan consolidation program upon labor market entry.

Keywords: labor market entry shock, optimal taxation, optimal student loan design.

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1 Introduction

Student debt has grown to be the largest non-mortgage consumer debt in the United States, reaching \$1.5 trillion in the fourth quarter of 2019.[1] The outstanding debt is 7.5% of the US GDP, affecting 45 million borrowers. Among the borrowers, 11 percent are 90 days or more delinquent on their student debts (Maggio et al. (2019)). These numbers raise concerns about a student loan crisis. The research has focused on debt relief that aims to lighten the debt burden. This paper identifies a missing component in the line of research, the labor market entry shocks, which is shown to have large persistent effect on life-time incomes (Oreopoulos et al. (2012); Altonji et al. (2016); Oyer (2006); Kahn (2010)). While the entry shocks have potential implication on the student loan design, the current policies do not take them into consideration.

To demonstrate the lack of relation between labor market entry shocks and student loan design, I regress past student loan interest rates against current and future unemployment rates. The results suggest no significant correlation between the student loan interest rates and the unemployment rates. The income-contingent student loan repayment rate does not depend on the entry shock either. The lack of correlation implies a lack of cross-cohort insurance in the student loan design. In particular, I show in the paper that the current entry-shock independent income-contingent repayment cannot provide the optimal amount of cross-cohort insurance. The intuition is that the probability of earning the same level of income is different when one enters the labor market at different times. Two people who earn the same but enter the labor market at different times might face different distortions in their labor supply decisions and hence different optimal income-contingent student loan repayment rate.[2]

I study the optimal Mirrleesian taxation in an environment with observable labor market entry shocks. There are two periods, the college and the working period. Agents are

[1] See https://www.newyorkfed.org/microeconomics/topics/student-debt.
[2] The details are in the appendix section A.

ex ante identical in the college period. Upon labor market entry, they observe the entry shocks which affect the distribution of earnings ability. After labor market entry, each agent draws an earnings ability which is private information.

The constrained optimal policy is entry-shock dependent. The optimal allocation can be implemented using a student loan, a history independent tax schedule, and an income- and entry-shock-contingent student loan repayment schedule. By making the student loan design entry-shock dependent, the policy provides the optimal amount of cross-cohort insurance. A first-order approximation of the optimal policy reduces to a simple student loan repayment design with fixed repayment rates and lump-sum payments depending on the entry shocks.

I calibrate the model to the U.S. economy. The quantitative analysis provides several insights. First, within each cohort, the total student loan repayment is increasing with income. There is partial loan forgiveness for a non-trivial fraction of the borrowers, while high income earners pay back more than they owe. 41% of the agents with the bad entry shock and 28% of the agents with the good entry shock receive partial loan forgiveness. The result suggests that the income-contingent student loan is providing within cohort insurance. Second, fixing the income level, the student loan repayment rates are higher for the good shock cohort than those for the bad shock cohort. The difference in the student loan repayment rates provides the optimal amount of cross-cohort insurance. Third, if I restrict the policy to be entry-shock independent, there are still both within cohort insurance and cross-cohort insurance, because the budget constraint pools the resources. However, the cross-cohort insurance is not optimal because for the cohort with the bad entry shock, the entry-shock independent labor income tax is higher than the optimal one. Compared to the entry-shock independent policies, the entry-shock dependent policies achieve welfare gains equal to 0.2% of lifetime consumption.

I consider two real world implementations. First, the total repayment is restricted to be less than the net present value of the student loan. The policy can achieve 83% of the welfare gain. The result is because the low income earners, who share most of the welfare gain, face similar allocation in the restricted policy as in the optimal policy. Second, besides the cap on repayment, I use a second-order approximation of the optimal repayment policy. The approximation implies fixed student loan repayment rates and a lump-sum entry-shock dependent payment. The simple policy can obtain 55% of welfare gain. The second policy is attractive because it differs from the current US student loan design by only adding entry-shock dependent repayment rates and lump-sum fees. The simple policy can be implemented by an automatic student loan consolidation program at the labor market entry.

The paper extends the study on optimal student loan design by considering labor market entry shocks. The literature shows robust evidence that the labor market entry shock affects life-time income (Altonji et al. (2016); Oyer (2006); Oreopoulos et al. (2012); Liu et al. (2016); Kahn (2010); Bedard and Herman (2008)). However, the current student loan design does not consider the aspect. The literature on the student loan design has focused on limited commitment, costly state verification, moral hazard, etc(Lochner and Monge-Naranjo (2016)). To the best of my knowledge, the correlation between labor market entry shocks and income, and hence repayment ability is largely missing in the discussion.

A growing literature shows the importance of income-contingent student loan repayments (Gary-Bobo and Trannoy (2015), Findeisen and Sachs (2016)). The paper contributes to the literature by showing that different cohorts should have different income-contingent repayment schedules, rather than a uniform one.

The paper employs an applied mechanism applied approach and derives the optimal Mirrleesean taxation. The literature dates back to Mirrlees (1971), and Piketty and Saez (2013) gives a recent survey. In the paper, the earnings ability can be viewed as idiosyncratic human capital risk. There is an extensive literature on the design of optimal taxation under human capital risks (see e.g. Best and Kleven (2013); Stantcheva (2017); Kapicka and Neira (2019)). The paper adds to the literature by combining human capital risks with labor market entry shocks and study the policy implication on the student loan design.

Outline The outline of the paper is the following. Section 2 lays out the model. Section 3 derives the optimal labor wedge and savings wedge. Section 4 discusses the implementation of the optimal policies. Section 5 quantitatively studies the welfare gain of the optimal policies compared to the current US student loan repayment design. Section 6 concludes.

2 Model

I study the optimal Mirrleesian tax in an environment with observable labor market entry shocks. Individuals are ex ante identical.There are two periods. The first period is the college. The agents observes labor market entry shock in this period. The second period is the working period.

In the college period, the utility function is u^e with discount rate β^e. Agents choose consumption c^e. When the agents enter the labor market, they draw an unobservable earnings ability θ that determines life-time income $y(\theta) = \theta \times l$, in which l is labor supply. The distribution of the earning ability $f(\theta|\sigma)$ depends on the labor market entry shock σ. It has a compact support in $[\overline{\theta}, \underline{\theta}]$.

In the working period, the agents consume and supply labor, with utility function given by $u^w(c^w, y(\theta)/\theta)$ and discount rate β^w. The agents' lifetime discounted utility is

$$\beta^e u^e(c^e) + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} u^w\left(c^w, \frac{y(\theta)}{\theta}\right) f(\theta|\sigma) d\theta \tag{1}$$

subject to a resource constraint:

$$-\beta^e c^e + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} (y(\theta) - c^w) f(\theta|\sigma) d\theta \geqslant 0 \tag{2}$$

Because the earnings ability is private information, the agents might not truthfully report the earnings ability θ. Denote the agents' reporting strategy by r. The agents will truthfully report their earnings ability if the following incentive-compatible constraint holds:

$$u^w\left(c^w(\theta), \frac{y(\theta)}{\theta}\right) \geqslant u^w\left(c^w(r), \frac{y(r)}{\theta}\right)$$

I relax the incentive-compatible constraint by using the first order approach. Define the promised utility to be $v^w(\theta)$. To simplify notation, I suppress the dependency of the promised utility on the labor market entry shock. The envelop theorem gives the necessary condition for incentive compatibility

$$\frac{\partial v^w(\theta)}{\partial \theta} = -u_l^w\left(c^w(\theta), \frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2} \tag{3}$$

If the lifetime income function $y(\theta)$ is increasing in θ, equation (3) is also the sufficient condition for the incentive-compatible constraint.[3]

For simplicity, I assume there are only two states of the labor market entry shocks. σ_g represents the good shock and σ_b the bad shock. The entry shocks affect the distribution of the earnings ability. Specifically, the earnings ability CDF with the good shock $F(\theta|\sigma_g) \equiv \int_{\underline{\theta}}^{\theta} f(x|\sigma_g)dx$ first-order stochastically dominates the one with the bad shock $F(\theta|\sigma_b)$: $F(\theta|\sigma_g) \leqslant F(\theta|\sigma_b)$. The assumption implies that the probability of drawing higher earnings ability with the good entry shock is at least as good as that of the bad entry shock.

The simple framework abstracts away from heterogeneity that could be important for student loan design, such as ability, major, parental income, etc. All agents differ by only one dimension, namely the labor market entry shocks. While other aspects are important, I focus on the labor market entry shock in a clean and transparent way to show its implication on the student loan design.

[3] I verify the monotonicity of $y(\theta)$ numerically.

3 Constrained Optimal Allocation

The section characterizes the constrained Pareto-efficient allocation. The planning problem gives the maximization problem the social planner faces. The optimal wedges are derived to study the distortions in the economy.

3.1 The Planning Problem

The social planner needs to maximize individual utilities under incentive-compatible constraint. The individuals differ ex ante only in the labor market entry shocks. The probability of the good entry shock σ_g is m_g, and the probability of the bad entry shock σ_b is m_b. By the law of large numbers, m_g and m_b are also the measure of agents with the good entry shock and the bad entry shock. The planner places the weight on the individual's utility based on the distribution of the labor market entry shocks. Assuming the social planner has access to a risk-free asset with the interest rate implying the agents' discount rate, the social planner's problem is

$$\begin{aligned} \mathcal{L} = \min_{\mu,\mu'} \max_{v^w,y,c^e} \sum_{i=g,b} m_i \left\{ \beta^e u^e(c_i^e) + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} v_i^w(\theta) f(\theta|\sigma_i) d\theta \right\} \\ + \lambda \sum_{i=g,b} m_i \left\{ -\beta^e c_i^e + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} \left(y_i - \zeta^w(v_i^w, \frac{y_i}{\theta}) \right) f(\theta|\sigma_i) d\theta \right\} \\ - \sum_{i=g,b} \int_{\underline{\theta}}^{\overline{\theta}} \left[\mu_i'(\theta) v_i^w(\theta) - \mu_i(\theta) u_l^w \left(\zeta^w(v_i^w, \frac{y_i}{\theta}), \frac{y_i}{\theta} \right) \frac{y_i}{\theta^2} \right] d\theta \end{aligned} \tag{4}$$

ζ^w is the consumption in the working period, which is implicit determined by $v^w = u^w(\zeta^w, y/\theta)$. Equation (4) rewrite the incentive-compatible constraint by integration by parts and using the boundary conditions $\mu(\underline{\theta}) = \mu(\overline{\theta}) = 0$.

Equation (4) shows that the social planner maximizes the total discounted utility of the agents in the economy, subject to the budget constraint that the total consumption cannot exceed the total output.

There are two source of risk sharing. First, let the social planner pool the resources but restrict the policy such that y_i cannot condition on the entry shocks. Under the restriction, the consumption and labor supply allocation depends only on the realization of the earnings ability but not the entry shocks, and hence the allocation is identical across cohorts, similar to the current US system. The risk-sharing under the restriction comes from resource pooling, i.e. the social planner pools the income of the good shock cohort and the bad shock cohort. Second, the social planner conditions labor supply and hence income y_i on the entry shocks. The conditioning implies additional transfers across cohorts.

3.2 Labor Wedge

Define the labor wedge as

$$\tau_l = 1 - \frac{u_l^w(c_i^w(\theta), y_i(\theta)/\theta)}{u_c^w(c_i^w(\theta), y_i(\theta)/\theta)\theta}$$

The labor wedge measures the distortion in the agents' labor supply decision. To understand the distortions and to shed light on implementation of the optimal allocations, lemma 1 derives the labor wedge in the constrained optimal solution.

Lemma 1. *The optimal labor wedge is*

$$\frac{\tau_l}{1-\tau_l} = \frac{\mu_i(\theta)u_c^w}{\lambda\beta^w m_i f(\theta|\sigma_i)\theta}\frac{1+\epsilon^u}{\epsilon^c}, \quad i = g, b \tag{5}$$

ϵ^u *and* ϵ^c *are the uncompensated and compensated labor supply elasticity:*

$$\epsilon^u = \frac{-u_l^w/l + (u_l^w)^2/(u_c^w)^2}{-u_{ll}^w - (u_l^w)^2 u_{cc}^w/(u_c^w)^2}$$
$$\epsilon^c = \frac{-u_l^w/l}{-u_{ll}^w - (u_l^w)^2 u_{cc}^w/(u_c^w)^2}$$

The Lagrangian multiplier for the incentive-compatible constraint has the representation

$$\frac{\mu_i(\theta)}{\beta^w\lambda m_i} = \int_\theta^{\overline{\theta}} \exp\left(-\int_\theta^x \frac{u_{cl}^w(s)}{u_c^w(s)}\frac{y_i(s)}{s}ds\right)\left(\frac{1}{u_c^w(x)} - \frac{1}{\lambda}\right) f(x|\sigma_i)dx, \quad i = g, b \tag{6}$$

Equation (5) shows that the labor wedge, and hence the distortion in the labor supply decision is increasing in the Lagrangian multiplier of the incentive compatibility constraint μ_i. A large μ_i implies that the agent has an incentive to report downward the earnings ability to supply less labor. For truthful reporting, the social planner allocates less labor supply to the agent than would be the case when the earnings ability is observable, resulting in a large distortion and hence larger labor wedge. For the same earnings ability, the incentive compatibility multiplier could differ by the entry shocks.

The labor wedge also differs across cohorts by the probability of the shock m_i and the probability that a particular earnings draw occurs $f(\theta|\sigma_i)$. If the probability of the bad shock m_b is large, the labor wedge for the cohort with the bad shock decreases. Similarly, a large density of the income draw $f(\theta|\sigma_i)$ damps the labor wedge. The former captures across-cohort insurance. When more agents face the bad entry shock, it puts more pressure on the overall budget constraint. The optimal labor income tax should be lower to encourage labor supply. The latter captures within-cohort insurance. If a particular income draw has a high probability, each agent with the income draw should face less labor income tax because each burdens a smaller share of the incentives for truthful reporting.

Another link between the cohorts is the Lagrangian multiplier λ on the budget constraint. After plugging equation (6) into equation (5), the result shows that the labor wedge is increasing in the Lagrangian multiplier. When resource is scarce, the Lagrangian multiplier, which is the shadow price of resource, is large. Because of limited resources, it is more difficult to promise a higher utility in the working period, which in turn distorts labor supply by the incentive compatible constraint. Because the social planner pools the resource together, when one cohort experiences a decrease in the resource, it decreases the ability to promise a higher utility for the other cohort, leading to more distortion in the labor supply decision

The labor wedge is decreasing in the compensated labor elasticity but increasing in the uncompensated labor elasticity. These results imply that the labor distortion is larger for more risk-averse agents. When the utility function in the working period $u^w(c^w, y/\theta)$ is separable in consumption and labor, the cross derivative u^w_{cl} vanishes. Equation (6) becomes

$$\frac{\mu_i(\theta)}{\beta^w \lambda m_i} = \int_{\theta}^{\overline{\theta}} \left(\frac{1}{u^w_c(x)} - \frac{1}{\lambda} \right) f(x|\sigma_i) dx$$

By setting $\theta = \underline{\theta}$, the equation reduces to the inverse Euler equation. There is a simple relation in the incentive compatible constraint multiplier and the probability of the shock across cohorts:

$$\frac{\mu_g(\theta)}{\mu_b(\theta)} = \frac{m_g \int_{\theta}^{\overline{\theta}} \left(\frac{\lambda}{u^w_c(x)} - 1 \right) f(x|\sigma_g) dx}{m_b \int_{\theta}^{\overline{\theta}} \left(\frac{\lambda}{u^w_c(x)} - 1 \right) f(x|\sigma_b) dx} \tag{7}$$

Equation (7) suggests that the incentive-compatible constraint of one cohort affects the other by the Lagrangian multiplier λ on the budget constraint.

The labor wedge captures the main feature of the model and sheds light on the optimal policy. The agents can also save in the college period. I leave the results on the savings wedge in the appendix.

4 Implementation

With the simple framework, I show two implementations. In the first one, the optimal allocation is implemented by a grant schedule $G(\sigma)$, a history-dependent labor tax $T_l(\sigma)$, a savings tax T_s. In the second one, the optimal allocation is implemented by a student loan schedule $L(\sigma)$, a history-independent labor income tax schedule $T(y)$, a savings tax T_s, and a loan repayment schedule $R(y, \sigma)$ which is both income-contingent and history dependent.

4.1 History Dependent Labor Income Tax

I implement the optimal allocation with zero savings as the following: the social planner first offer grant $G(\sigma) = c^e(\sigma)$ to all agents. From the first order conditions, the consumption in the college period does not vary with the labor market entry shock. The social planner provides the same grant amount regardless of the potential entry shocks agents face. The result arises because of two reasons. First, the social planner assigns Pareto weight according to the probability of entry shocks, which effectively leads to utility equalization across cohort prior to the labor market entry shock. Second, there are enough instruments to achieve the optimal allocation in the working period.

The savings tax is set such that the agents have no incentive to save in the college period. Specifically, the savings tax is the upper envelop of the savings wedge for all θ and σ: $T_s = \sup_{\theta,\sigma} \tau_s(\theta, \sigma)$. In the working period, the planner imposes a history-dependent labor tax $T_l(y, \sigma)$ given by equation (5). The consumption in the working period is then $c^w(\sigma) = y - T_l(y, \sigma)$.

The short-coming of the implementation is that the labor income tax schedule depends on both the labor market entry shock σ and the income level y, whereas the current US tax system is history-independent. Next I show the alternative implementation with a history-independent labor income tax but a history-dependent student loan repayment schedule.

4.2 History Independent Labor Income Tax

I separate the labor income tax in section 4.1 into two parts, with one part being a history-independent labor tax and the other part being a history-dependent loan repayment schedule. Let $T(y)$ denote the new labor income tax schedule and $R(y, \sigma)$ denote the loan repayment schedule, then $T_l(y, \sigma) = T(y) + R(y, \sigma)$. The grant is replaced by a student loan schedule $L(\sigma) = c^e(\sigma)$ in the college period. Similar to section 4.1, the social planner provides the same amount of student loans regardless of the entry shocks.

The implementation is based on the idea that income-contingent student loan repayment is effectively labor income tax. Using a first-order Taylor approximation on $R(y, \sigma) \approx \alpha y + \beta\sigma$, it has the following interpretation: αy is the income-contingent student loan repayment. In the US, α ranges from 10% to 20% in the income-contingent repayment plans. $\beta\sigma$ shifts the intercept of the loan repayment schedule based on the entry shocks. It can be viewed as a lump-sum repayment in addition to the the income-contingent repayment. Rewriting the Taylor approximation as $R(y, \sigma) \approx (\alpha + \beta\sigma/y)y$ and define $(\alpha + \beta\sigma/y)$ as $r(y)$, the first-order approximation can be viewed as a single income-contingent student loan repayment, with the repayment rate $r(y)$ declining as the income increases.

Because I abstract away from higher order terms in the Taylor expansion, the repayment rate α does not depend the entry shocks. However, if I include an interaction term $\gamma y\sigma$, the repayment schedule becomes $R(y, \sigma) \approx (\alpha + \gamma\sigma)y + \beta\sigma$, implying that the repayment rates can depend on the labor market entry shocks.

To sum up, the optimal policy can be implemented using a student loan L, a labor income tax schedule $T(y)$, a loan repayment $R(y, \sigma)$ which specifies repayment that depends on both the income level and the labor market entry shocks. In the first-order approximation, the loan repayment schedule is an income-contingent repayment which has the same repayment rate across cohorts plus a lump-sum fee depending on the entry shocks.

5 Quantitative Analysis

I quantitatively evaluate the welfare gain of the entry-shock dependent policy compared to an entry-shock independent one. I also study whether approximations to the optimal implementation can achieve most of the gain.

5.1 Parameterization

Agents are ex ante homogeneous. The only heterogeneity is the labor market entry shock. The entry shocks have two levels, namely the good entry shock σ_g and the bad entry shock σ_b. I base the differences in expected lifetime earnings on Oreopoulos et al. (2012) and Kahn (2010), which suggest that the average lifetime earnings difference because of the labor market entry shocks is 3.5%.[4]

The parametric form of the lifetime earnings distribution is Pareto-Lognormal PlN(μ, γ^2, λ). μ and γ^2 governs the Lognormal distribution while λ is the Pareto tail parameter. Pareto-Lognormal distribution arises as the stationary distribution of a Geometric Brownian Motion (GBM) killed at an exponentially distributed stopping time. It is suitable for modeling income and earnings distributions because the density matches the empirical income distribution well. I vary $\mu(\sigma)$ so that the average lifetime earnings is 5% higher with the good shock. γ is calibrated to match the variance of wages in the Current Population Survey (CPS) from 2008 to 2017. λ is chosen so that the tails in the model income distribution and the empirical wage distribution match.

Figure 1 plots the calibrated CDFs and PDFs, with lifetime income on the x-axis. The differences in the income distribution of different entry shocks are mostly on the probabilities of low earnings. With the bad entry shock, the CDF puts more weight on the lower end of the income distribution. For high income levels the distinction between the CDFs disappears. The smaller gap at high income levels is consistent with the empirical evidence in Oreopoulos et al. (2012), Kahn (2010), and Liu et al. (2016) who show that the entry shocks mainly affect the workers with low earnings ability. The optimal labor wedge depends on the PDFs of the income distribution. In figure 1 (b), the PDFs comparison reiterates that the differences are concentrated at low income levels.

I choose the annual discount rate $\beta = 0.98$, which implies a 2% annual interest rate. An agent lives for 47 years (age 18 to 65) in the model. The college period takes 4 years. Her discount factor for the college period is $\beta^e = \sum_{t=1}^{4} \beta^{t-1}$. The discount factor for the working period β^w is $\sum_{t=5}^{47} \beta^{t-1}$. The average length of the recession is 12 months between 1970 to 2010 and there were 6 recessions, which implies that the probability of the bad shock is $m_b = 0.15$.

[4]Here the bad shock is a 3.5 percentage points increase in the unemployment rate, which is the average unemployment rate increase in the recessions from 1970 to 2010.

The utility function for the working period is

$$u^w(c, y/\theta) = \frac{\left(c - \frac{(y/\theta)^\delta}{\delta}\right)^{1-\rho}}{1-\rho} \tag{8}$$

The utility function has no income effects. I follow Findeisen and Sachs (2016) to set $\delta = 3$ and $\rho = 2$. For the college period, the utility function is

$$u^c(c) = \frac{c^{1-\rho}}{1-\rho} \tag{9}$$

in which $\rho = 2$.

5.2 Optimal Policies: Features and Implementation

The optimal labor income tax depend on earnings ability θ and the labor market entry shocks σ.

Figure 2 shows that the optimal labor tax rate decreases as the income rises. The negative relation between the labor income tax and income is similar to Findeisen and Sachs (2016). The figure also shows that the labor income tax is higher with the good entry shock. The gap shrinks as income increases, suggesting that the distortion by the labor market entry shock declines with income. The result is consistent with high earnings ability being less affected by the entry shocks.

Section 4.2 suggests that the labor income tax can be decomposed into a history-independent labor tax and a history-dependent student loan repayment schedule. The history-independent labor income tax schedule is indeterminate. To better compare with the US system, I set a fixed repayment rate at 10% with the bad entry shock. The implied interest rate on the student loan is 4%, which is smaller than the average student loan interest rate from 1992 to 2018 at 6%.[5] The resulting history independent tax schedule is $T(y) = T(y, \sigma_b) - 0.1y$

[5]The implied interest rate is calculated as $i(\sigma) = \int R(y, \sigma) f(\theta|\sigma) d\theta / L - 1$.

and the student loan repayment schedule is $R(y, \sigma_i)$. In particular, $R(y, \sigma_b) = 0.1y$ and $R(y, \sigma_g) = T(y, \sigma_g) - T(y)$.

Figure 3 (a) plots the history-independent labor income tax schedule, in comparison with the optimal labor income tax schedule. The gap between the optimal labor income tax and the history independent tax schedule is the student loan repayment rate. In figure 3 (b), for each income level, the loan repayment rate is higher with the good entry shock. The average loan repayment rate is 11.5% with the good entry shock. The repayment rate decreases as the income rises, whereas the total repayment increases.

The implied student loan interest rate is 6% with the good entry shock, which is the same as the current average student loan interest rate and 50% higher than the implied interest rate with the bad entry shock. Two factors contribute to the large increase. The first one is that for each income level, the loan repayment rate is higher with the good entry shock. This is the level effect. The level effect accounts for only 13% of the increase in the interest rate. The second factor is the difference in the income density. The agents are more likely to have good income draws with the good shock. For a fixed repayment rate, a higher income leads to more repayment, which increases the implied student loan interest rate. This is the composition effect, which accounts for 87% of the increase in the student loan interest rate.

A drawback of the implementation is that high income earners could repay more than the present value of the student loan debt, in contrast with the current US system. I discuss a realistic implementation with a cap on repayment in section 5.4. The increasing total repayment provides within cohort insurance. The low income earners pay back less than the amount they owe, suggesting that the implementation provides partial loan forgiveness.

5.3 The Welfare Gains from Entry Shock Dependent Policies

The current US student loan repayment policy does not depend on the labor market entry shocks. Section 5.3 studies the welfare gains from basing the policy on the entry shocks.

Lemma 2 characterizes the optimal labor wedge with the additional restriction that the labor wedge cannot differ across entry shocks. The restriction essentially means that the policy cannot be history dependent.

Lemma 2. *Fix the student loan interest rate across entry shocks, the optimal labor wedge is*

$$\frac{\tau_l}{1-\tau_l} = \frac{\sum_{i=g,b} \mu_i u_c^w}{\lambda \beta^w \theta \sum_{i=g,b} m_i f(\theta|\sigma_i)} \left(\frac{1+\epsilon^u}{\epsilon^c} \right) \tag{10}$$

Equation (10) shows that when the labor income tax cannot differ across entry shocks, it is a weighted average of the optimal labor wedge in equation (5).

With the extra restriction, the policy can be implemented by a history independent labor income tax schedule $T(y)$ and an income-contingent student loan repayment schedule with a fixed repayment rate. For calculating the welfare gain compared to history dependent policies, the division into labor income tax and student loan repayment is irrelevant.

There is still insurance across cohort when the policy does not condition on entry shocks because the social planner pools the income of both cohorts. The policy is very similar to the current US system, where the student loan repayment is income contingent but does not depend on entry shocks. Essentially, the high income earners are insuring the low income earners in the current design. However, in the history-independent policy the high income earners with the bad shock are providing too much insurance compared to the case with the history-dependent policy. The higher effective labor income tax leads to sub-optimal labor supply allocation.

Compare to the entry-shock independent policy, the optimal policies in section 5.2 lead to a 0.2% welfare gain. The magnitude is smaller than the gain of the income-contingent student loan in Findeisen and Sachs (2016). The reasons are two fold. First, the entry-shock independent labor wedge still provides some amount of insurance, because the social planner pulls the income together. This is similar to the current policy design. Second, the only source of heterogeneity in the model is the entry shocks, whereas in Findeisen and Sachs (2016) agents also differ in ability. The 0.2% welfare improvement is close to the gain of introducing the income-contingent loan when the agents have a high elasticity of substitution in Findeisen and Sachs (2016).

5.4 Real World Policies: Cap on Repayment

The student loan repayment in section 5.2 has the drawback that the high income earners pay back more than they owe. I add an extra constraint in this section that the repayment cannot exceed the net present value of the student loan.

Figure 5 shows that there are partial loan forgiveness for low income earners. The total repayment is higher with the good entry shock at low income level. In other words, the loan forgiveness is more generous with the bad entry shock in the sense that the repayment rate is lower and full repayment happens at higher income levels. The calculation shows that 41% of the agents receive loan forgiveness with the bad entry shock while 28% have loan forgiveness with the good shock.

The real world policy captures 83% of the welfare gain compared to the entry-shock independent policy in section 5.3. Because the low income earners with the bad shock face the same policy in the two implementation, the result suggests that the low income earners capture most of the welfare gain from the entry-shock dependent student loan repayment, consistent with them having larger labor supply distortion.

The entry-shock dependent loan repayment rate decreases with income for the good shock cohorts. While the total repayment still increases with income, the design has few real world examples. One alternative is the approximation in **??**, which suggests that the entry-shock dependent policies can be approximated by a history-independent labor income tax $T(y)$, a fixed student loan repayment rate α which depends on the entry shock, and a one-time lump sum payment β for the good shock cohorts. The approximation can achieve 55% of the welfare gain compared to the entry-shock independent policy, which amounts to 0.11% gain in lifetime consumption.[6]

The approximation differs from the current US student loan design by only fixed repayment rates that depend on entry shocks and a lump-sum payment at labor market entry. The policy can be implemented by an automatic loan consolidation program at the labor market entry, which determines the fixed repayment rates and the lump-sum consolidation fee. Although the current student loan design allows for loan consolidation, the repayment rate and the interest rate cannot decrease. The loan consolidation program for private loans is not designed for cross-cohort insurance either. Given the relative ease of implementing the approximating policies, the 0.1% gain in welfare is large.

6 Conclusion

I study the optimal Mirrleesian taxation in the environment in which the agents differ only in observable labor market entry shocks. The constrained optimal allocations are entry-shock dependent. The optimal allocation can be implemented using a student loan, an entry-shock independent tax schedule, and an income- and entry-shock contingent student loan repayment schedule.

I calibrate the model to the U.S. economy. The quantitative analysis shows that, compared to the entry-shock independent policies, the optimal entry-shock dependent policies achieve a 0.2% increase in lifetime consumption. I consider two real world implementations. First, the total repayment is restricted to be less than the present value of the total student loan owed. The policy can achieve 83% of the welfare gain. The low income earners face the same allocations as in the optimal policy, suggesting that the distortion and hence the welfare gain concentrate on the low income earners. In the second real world policy, I fixed the repayment rates to be independent of income but depends on the entry

[6] The comparison is as follows: I use the second-order approximation policy to calculate the allocations for both the good and the bad entry shock cohort, then I verify the allocations satisfy the resource constraint and the incentive-compatible constraint of the good entry cohort. The allocations are compared with those in the entry-shock independent implementation.

shocks, in addition to the cap on repayments. There is also a lump-sum payment at the labor market entry. The simple policy can achieve a 0.11% increase in lifetime consumption, which amounts to 55% of total welfare gain. The second policy is attractive because it differs from the current US system by only adding fixed repayment rates and a lump-sum fee that depend on the labor market entry shocks, which can be implemented by an automatic loan consolidation program at the labor market entry.

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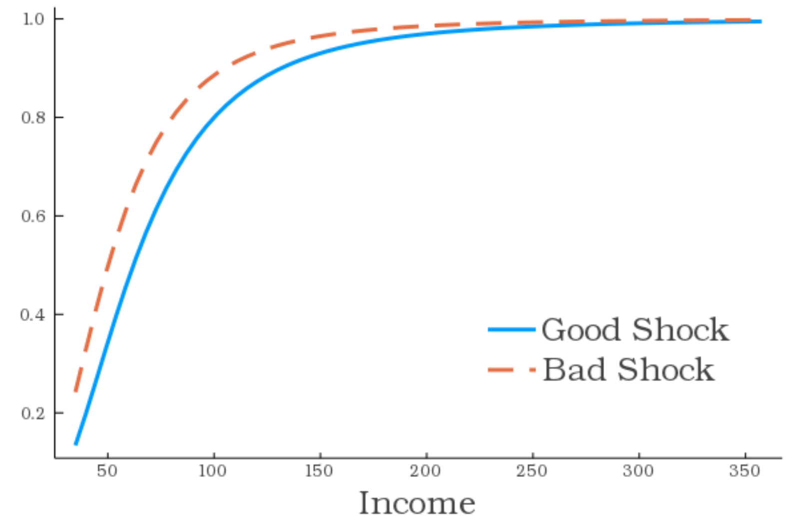
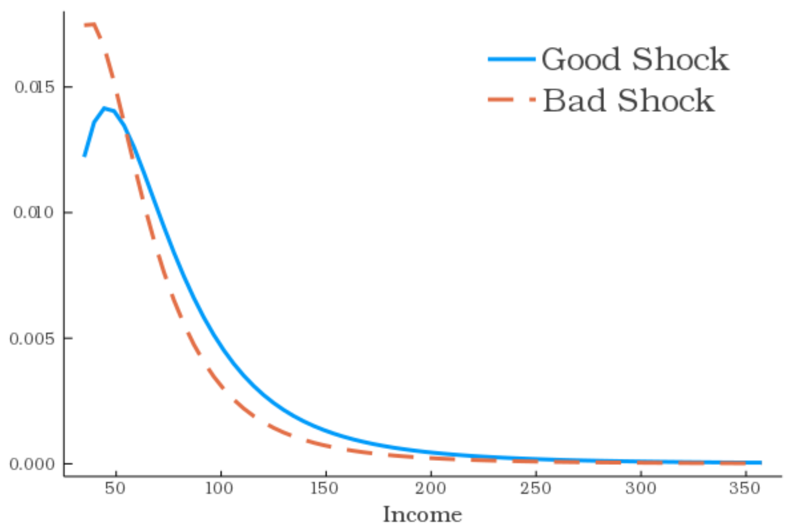
Tables and Figures



(a) CDF Comparison



(b) PDF Comparison

Figure 1: Probability Density Functions

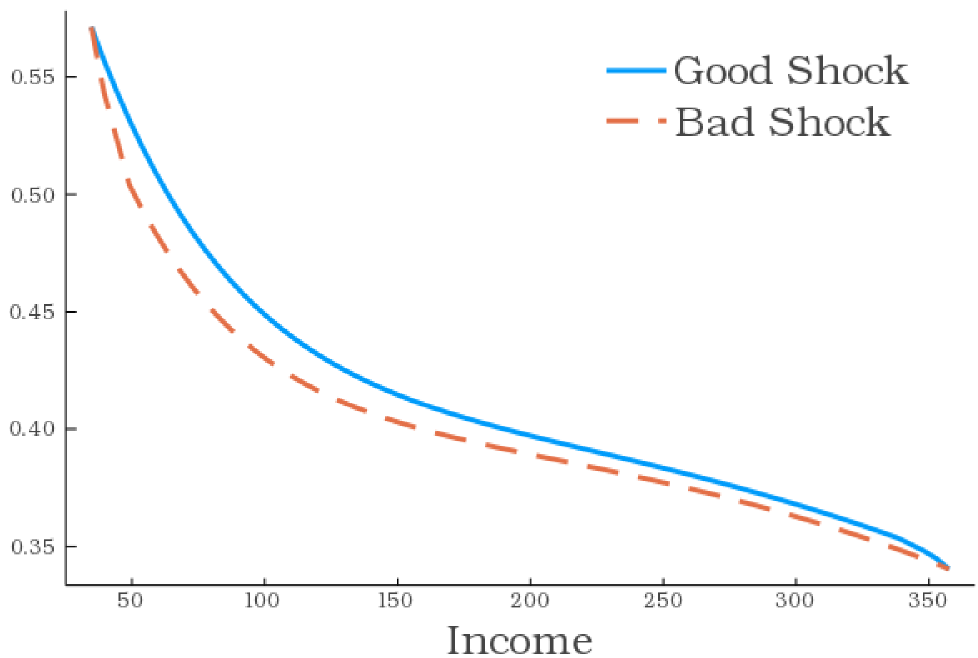
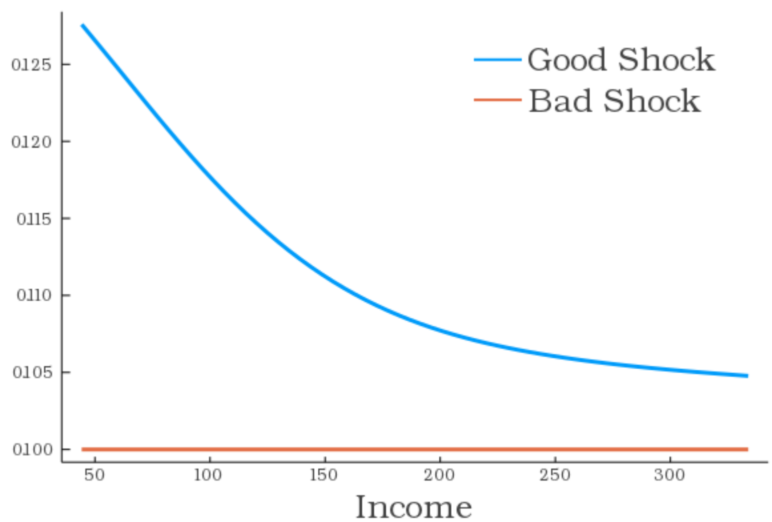


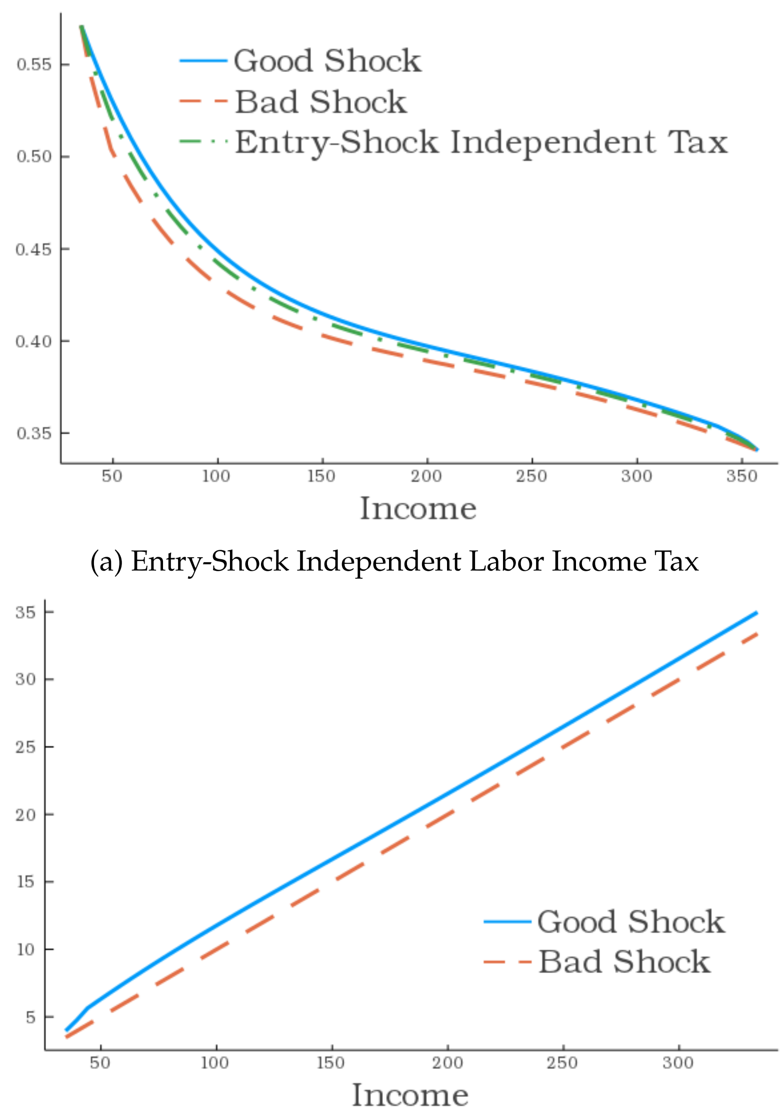
Figure 2: Labor Tax Across Cohorts

(a) History Independent Labor Income Tax



(b) Repayment Rates

Figure 3: Student Loan Repayment



(a) Entry-Shock Independent Labor Income Tax

(b) Entry-Shock Independent Repayment Rates

Figure 4: Entry-Shock Independent Policies

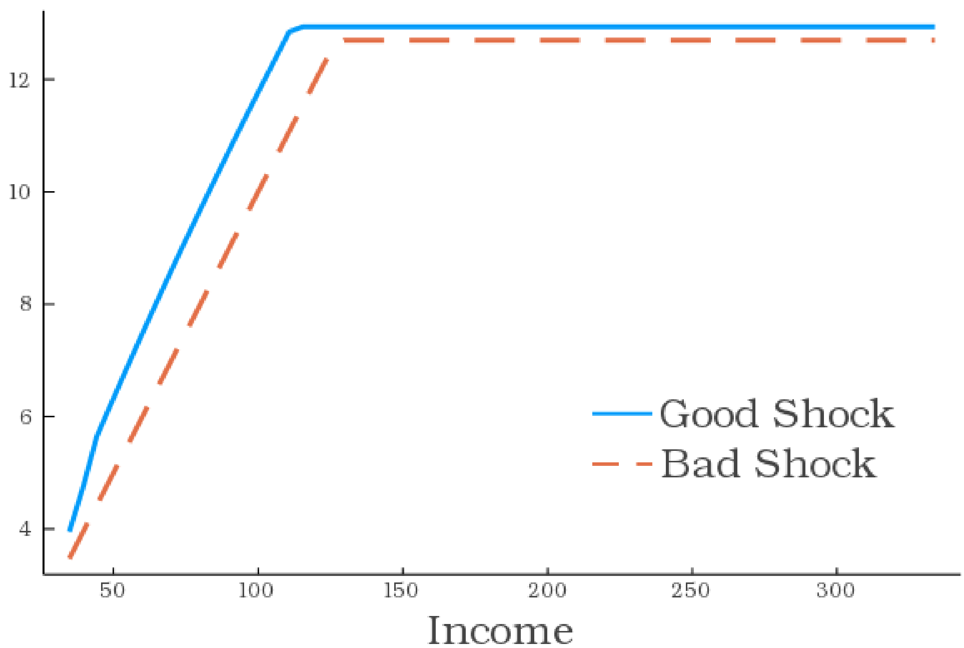


Figure 5: Real World Total Loan Repayment

Appendices

A Correlation Between Student Loan Interest Rates and Labor Market Entry Shocks

The current student loan design lacks consideration of the labor market entry shocks. For example, the current US student loan interest rates are determined at the time of loan application. They are reset annually and pegged to the 91-day T-Bill, 12-month T-Bill or Constant Maturity Treasury. Once they are determined, the interest rates are fixed for the repayment periods. While the interest rates take expectations about the future entry shocks into account, they do not resolve the problem that borrowers who take out loans at the same time belong to different cohorts. While the borrowers can consolidate their federal student loans, they cannot get lower interest rates. There is private student loan consolidate which could lead to lower interest rates, but private student loans account for less than 8% of the total outstanding. Further, private loan consolidation is not meant for cross-cohort insurance. The qualification for consolidation is idiosyncratic, and high income earners usually receive lower rates.

To further illustrate the point, I regress the student loan interest rates against the annual average unemployment rate. Table A.1 shows that there is no correlation between the interest rates and the current and future unemployment rates. Table A.1 also suggests that there is no correlation between the interest rates and the current and future GDP growth rates.

Table A.1: Correlation Between Student Loan Interest Rates and Unemployment Rates

	(1) Current	(2) 1 Year Ahead	(3) 2 Years Ahead	(4) 3 Years Ahead	(5) 4 Years Ahead
			Unemployment Rates		
	0.019	0.118	0.202	0.182	0.041
	(0.140)	(0.130)	(0.143)	(0.150)	(0.156)
			GDP Growth Rates		
	0.191	0.011	0.067	0.214	0.387
	(0.202)	(0.171)	(0.170)	(0.188)	(0.160)
Observations	26	25	24	23	22

Notes. The sample period is 1992 to 2018. The standard error is heteroskadasticity robust.

B Proofs

Proof of lemma 1.

Proof. The social planner's problem equation (4) is

$$\begin{aligned}\mathcal{L} = \min_{\mu,\mu'} \max_{v^w,y,c^e} \sum_{i=g,b} m_i \Big\{ \beta^e u^e(c_i^e) + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} v_i^w(\theta) f(\theta|\sigma_i) d\theta \Big\} \\ + \lambda \sum_{i=g,b} m_i \Big\{ -\beta^e c_i^e + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} \Big(y_i - \zeta^w(v_i^w, \frac{y_i}{\theta}) \Big) f(\theta|\sigma_i) d\theta \Big\} \\ - \sum_{i=g,b} \int_{\underline{\theta}}^{\overline{\theta}} \Big[\mu_i'(\theta) v_i^w(\theta) - \mu_i(\theta) u_l^w \Big(\zeta^w(v_i^w, \frac{y_i}{\theta}), \frac{y_i}{\theta} \Big) \frac{y_i}{\theta^2} \Big] d\theta\end{aligned} \tag{B.1}$$

Note that I use integration by parts to rewrite the incentive compatible constraint. Taking the first order conditions with respect to c_i^e, y_i, and v_i^w, I derive the following equations:

$$\begin{aligned}\left[\frac{\partial \mathcal{L}}{\partial c_i^e}\right] &: u'(c_i^e) = \lambda \\ \left[\frac{\partial \mathcal{L}}{\partial y_i}\right] &: \lambda m_i \beta^w f(\theta|\sigma_i) \left[1 + \frac{u_l}{u_c \theta}\right] + \mu_i(\theta) \left[-u_{cl}^w \frac{u_l^w y_i(\theta)}{u_c^w \theta^3} + u_{ll}^w \frac{y_i(\theta)}{\theta^3} + \frac{u_l^w}{\theta} \right] = 0 \\ \left[\frac{\partial \mathcal{L}}{\partial v_i^w}\right] &: \beta^w m_i f(\theta|\sigma_i) - \lambda m_i \beta^w \frac{f(\theta|\sigma_i)}{u_c^w} - \mu_i'(\theta) \frac{u_{cl}^w y_i(\theta)}{u_c^w \theta^2} = 0\end{aligned} \tag{B.2}$$

The labor wedge is $1 - \tau_L = -u_l^w / (u_c^w \theta)$. Substitute the expression of the labor wedge into equation (B.2), we arrive at equation (5).

From the first order condition with respect to v_i^w, we have a differential equation for $\mu_i(\theta)$ with the boundary conditions $\mu_i(\underline{\theta}) = \mu_i(\overline{\theta}) = 0$. Solving for $\mu_i(\theta)$ gives equation (6).

□

Proof of lemma 2.

Proof. For the policy to be entry-shock independent, it is equivalent to letting the income to be entry-shock independent: $y_g(\theta) = y_b(\theta)$. The social planner's problem under the restriction is

$$\begin{aligned}\mathcal{L} = \min_{\mu,\mu'} \max_{v^w,y,c^e} \sum_{i=g,b} m_i \Bigg\{ \beta^e u^e(c_i^e) + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} v_i^w(\theta) f(\theta|\sigma_i) d\theta \Bigg\} \\ + \lambda \sum_{i=g,b} m_i \Bigg\{ -\beta^e c_i^e + \beta^w \int_{\underline{\theta}}^{\overline{\theta}} \Big(y - \zeta^w(v_i^w, \frac{y}{\theta}) \Big) f(\theta|\sigma_i) d\theta \Bigg\} \\ - \sum_{i=g,b} \int_{\underline{\theta}}^{\overline{\theta}} \Big[\mu_i'(\theta) v_i^w(\theta) - \mu_i(\theta) u_l^w \Big(\zeta^w(v_i^w, \frac{y}{\theta}), \frac{y}{\theta} \Big) \frac{y}{\theta^2} \Big] d\theta\end{aligned} \tag{B.3}$$

Taking the first order condition with respect to y, we have

$$\lambda \sum_{i=g,b} m_i \beta^w f(\theta|\sigma_i) \Big[1 + \frac{u_l^w}{u_c^w \theta} \Big] + \sum_{i=g,b} \mu_i \Big[-\frac{u_{cl}^w u_l^w y}{u_c^w \theta^3} + \frac{u_{ll}^w y}{\theta^3} + \frac{u_l^w}{\theta^2} \Big] = 0 \tag{B.4}$$

Rearranging the terms, we arrive at equation (10). □

Savings wedge.

While I implement the optimal policy with 0 savings, I derive the savings wedge for completeness. The savings wedge is $\tau_S = 1 - u_c^e / \int_{\underline{\theta}}^{\overline{\theta}} u_c^w(c_i^w, y_i(\theta)/\theta) f(\theta|\sigma_i) d\theta$. From the first order conditions equation (B.2), notice that

$$\begin{aligned}\lambda =& \frac{\mu_i(\theta) u_l^w}{\theta^2} \Big(\frac{1+\epsilon^u}{\epsilon^c} \Big) \frac{1}{f(\theta|\sigma_i)\tau_L m^i} \\ \lambda =& \int_{\underline{\theta}}^{\overline{\theta}} u_c^w f(\theta|\sigma_i) d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\mu_i(\theta)}{\beta^w m_i} \Big(u_{cc}^w c_i' + u_{cl}^w \frac{y_i}{\theta^2} \Big) d\theta\end{aligned}$$

Substituting the expression for τ_S, we have

$$\frac{\tau_S}{1-\tau_S} = \frac{\lambda - \int_{\underline{\theta}}^{\overline{\theta}} \frac{\mu_i}{m_i \beta^w} \Big(u_{cc}^w c_i' + \frac{u_{cl}^w y_i}{\theta^2} \Big) d\theta}{u_c^e(\theta)}$$