The Large Volatility and the Slow Convergence of the Job-Finding Rate

Andrew Yizhou Liu *

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Abstract

I derive the elasticity and convergence of the job-finding rate in a class of search-and-matching models. I show that besides the fundamental surplus, how agents discount future output flow is also important for a large job-finding rate elasticity. For convergence, I show that the key to generating empirically consistent job-finding rate time series is time-varying matching efficiency. Specifically, empirically consistent matching efficiency can account for the slow job-finding rate convergence during the Great Recession.

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1 Introduction

The three recessions from 1989 to 2019 in the US feature large deterioration and slow recovery of the labor market, manifested by the job-finding rate. The large decline in the job-finding rate is extensively studied since Shimer (2005) showed that the Diamond-Mortensen-Pissarides (DMP) search-and-matching model cannot generate empirically consistent job-finding rate elasticity, namely the Shimer puzzle. Since then, a large literature proposed prominent solutions.[1] The slow recovery is the subject of several recent studies, all of which emphasizes time-varying matching efficiency.[2]

Of the various ways that generate a large job-finding rate elasticity, what are the keys that are common to all of them? Why do the models with a slow job-finding rate convergence feature time-varying matching efficiency? Can empirically consistent matching efficiency lead to a job-finding rate time series that matches the data? I aim to answer these questions in the current study.

Using a DMP model that nests the one in Shimer (2005), I extend the results in Ljungqvist and Sargent (2017); Kehoe et al. (2019) and show that the keys to a large job-finding rate elasticity are two-fold: the fundamental surplus and time-varying risks. In particular, I derive the analytic expression of the job-finding rate elasticity and show that when augmented with time-varying risks, any difference in the long-run returns to vacancy posting across employment status can generate a large job-finding rate elasticity. Kehoe et al. (2019) emphasizes higher human capital growth rate of the employed workers than that of the unemployed workers. Similarly, if the employed workers are less likely to exit the labor market, the job-finding rate elasticity can be made large. This points to other mechanisms that allow for a large job-finding rate elasticity. For example, if employed workers can refer other workers while unemployed workers cannot as in e.g. Galenianos (2014); Arbex et al. (2019), the job-finding rate elasticity can be large.

[1] See e.g. Hall (2005); Hall and Milgrom (2008); Hagedorn and Manovskii (2008); Pissarides (2009); Hall (2017); Ljungqvist and Sargent (2017); Kilic and Wachter (2018); Kehoe et al. (2019).

[2] See e.g. Sterk (2015); Gavazza et al. (2018); Acharya and Wee (2020); Leduc and Liu (2020).

Turning to the convergence of the job-finding rate, I show that models with a constant matching efficiency always transmit through the aggregate shock. In particular, I derive the analytic expression of the half-life of the job-finding rate and show that the convergence is fast. Combined with the fact that the only time-varying component of the job-finding rate is the aggregate shock near the steady state, the job-finding rate always co-moves with the aggregate shock, implying that a slow convergence is not possible.

The key to a slow convergence of the job-finding rate is to add other time-varying components. Through the lens of the search-and-matching model, this means time-varying matching efficiency or bargaining power. It is difficult to separate the two, but the former is attractive as there is empirical evidence (e.g. Davis et al. (2013)). I derive the half-life under time-varying matching efficiency and show that if the matching efficiency experiences a large decline or a slow recovery following a negative aggregate shock, the job-finding rate features a slow convergence. Using the numbers from the Great Recession, the implied half-life of the job-finding rate is one year, much more consistent with the empirical evidence.[3]

This paper provides the guideline for empirically consistent search-and-matching models. The goal is not to include all possible ways of generating a large job-finding rate elasticity and a slow convergence. Rather, within the framework of a DMP model, I show what components are important for the elasticity and the convergence, respectively. Hence, when thinking about factors that could be important for a large job-finding rate elasticity, one can think of how the factor fits into the model, and use the analytic expression to evaluate whether the extended model could match the job-finding rate elasticity.

Knowledge about whether this factor could account for the slow convergence can be deduced at the same time. For example, a model with wage rigidity can generate a large job-finding rate elasticity, but can the slow adjustment of wages also lead to a

[3]The aggregate matching efficiency inevitably captures variation in the aggregate bargaining power when one uses aggregate data. I aim to show ways through which the convergence of the job-finding rate is slow, not the relative importance of matching efficiency versus bargaining power.

slow convergence of the job-finding rate? Or if the model has heterogeneous workers, some of whom have a lower job-finding rate, can the model feature a slow job-finding rate convergence? The current paper suggests that the answer is no. Instead, either a time-varying matching efficiency or bargaining power is needed.

Outline The organization of the rest of the paper is: Section 2 lays out the model. Section 3 characterizes the equilibrium. Section 4 derives the analytic solution for the job-finding rate elasticity and convergence. Section 5 concludes.

2 Economy

I base the discussion on the continuous-time version of the model in Kehoe et al. (2019), which facilitates showing the keys for generating a large elasticity and a slow convergence. I will discuss how the model nests the search-and-matching model in Shimer (2005), referred to as the simple model. The continuous-time setting is for deriving closed form expressions. All results hold in discrete time. Because there is no novelty, I briefly describe the continuous-time setting and relegate the derivation of the equilibrium to the Appendix Section A.

The economy consists of a continuum of firms and workers. Each worker can be employed or unemployed. An employed worker exits the labor market at the rate ϕ_e while an unemployed worker exits at the rate ϕ_u. There is labor market entry at the rate ζ. New entrants are unemployed first.

Firms post vacancies to hire workers. Each worker belongs to one of a large number of identical families that own firms and insure their members against idiosyncratic risks. The assumption makes idiosyncratic risk and hence savings irrelevant. It also implies that workers and firms discount the utility flows using the same stochastic discount factor. There is aggregate risk in the form of productivity shock.

Workers can accumulate human capital. z_{it}^{j} denotes the human capital of an individual at time t, with $j = \{e, u\}$ denoting labor force status. Human capital growth rates could differ by labor force status:

$$\frac{dz_{it}^{j}}{z_{it}^{j}} = g_j dt, \quad j = e, u \tag{1}$$

An unemployed worker finds jobs at the rate $\lambda_{wt}(z)$ which can depend on the worker's human capital. New entrants draw their initial human capital from an exogenous distribution $n_t(z)$. The measure of unemployed workers $u_t(z)$ evolves according to:

$$\frac{\partial u_t(z)}{\partial t} = -\left[\frac{\partial u_t(z)}{\partial z} g_u z + g_u u_t(z)\right] - (\phi_u + \lambda_{wt}(z))u_t(z) + \delta e_t(z) + \zeta n_t(z) \tag{2}$$

The first term is the evolution of the density $u_t(z)$ attributed to human capital growth. The second term is the outflow because of labor market exit and employment. The last two terms are the inflow of separated employed workers and new entrants, where $e_t(z)$ is the measure of employed workers. It evolves according to:

$$\frac{\partial e_t(z)}{\partial t} = -\left[\frac{\partial e_t(z)}{\partial z} g_e z + g_e u_t(z)\right] - (\phi_e + \delta)e_t(z) + \lambda_{wt}(z)u_t(z) \tag{3}$$

where δ is the exogenous separation rate.

The aggregate productivity shock follows a Geometric Brownian motion

$$\frac{dA_t}{A_t} = g_a dt + \sigma_a dW_{a,t} \tag{4}$$

The aggregate shock affects the productivity of both employed and unemployed workers with a linear technology. The linear technology implies that the productivity for workers with human capital z is $A_t z$ and the unemployment benefit for workers with human capital z is $bA_t z$. By scaling the unemployment benefit with the aggregate shock, the model does not rely on acyclical opportunity cost of employment, or the fundamental surplus, to generate fluctuations in unemployment and vacancy postings.[4]

Firms post vacancies in submarkets indexed by human capital. Let $v_t(z)$ denote the measure of vacancy for workers with human capital z. The cost of posting vacancies is augmented by the aggregate shock A_t linearly so that firms pay $\kappa A_t z$ for a vacancy directed at a worker with human capital z. The expression implies that the vacancy cost depends on the productivity of employed workers, consistent with the view that recruiting takes time away from production. Another interpretation is that firms' screening costs increase with human capital levels of hired workers.

The aggregate resource constraint is

$$C_t = A_t \int z e_t(z) dz + b A_t \int z u_t(z) dz - \kappa A_t \int z v_t(z) dz \tag{5}$$

Labor market flows, along with entry and exit, act as stabilizing forces to aggregate human capitals. In the equilibrium, aggregate human capitals of employed and unemployed workers grow at the same rate as the aggregate shock process A_t.

Each family maximizes the utility of a representative worker. The preference specification has an asset pricing specification that incurs habit X_t:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\beta t} \frac{(C_t - X_t)^{1-\alpha}}{1-\alpha} dt \right] \tag{6}$$

where β is the subjective discount rate. The stochastic discount factor is

$$Q_t = e^{-\beta t} (C_t - X_t)^{-\alpha} \tag{7}$$

It is convenient to express the stochastic discount factor using the surplus consumption

[4]Namely, the model does not rely on differential effects of shocks to employed and unemployed productivity to generate unemployment fluctuations. See Ljungqvist and Sargent (2017) for a review.

ratio $S_t = (C_t - X_t)/C_t$, so that Equation (7) becomes

$$Q_t = e^{-\beta t}(S_t C_t)^{-\alpha} \tag{8}$$

The risk-free rate satisfies $r_{ft}dt = -E_t \frac{dQ_t}{Q_t}$.[5] More generally, for any asset with dividend process $\{D_t\}$, the price of the asset P_t needs to satisfy

$$Q_t P_t = \int_0^\infty Q_{t+s} D_{t+s} ds \tag{9}$$

Equation (9) implies that the instantaneous return of any asset $dR_t = (D_t/P_t dt + dP_t/P_t)$ needs to satisfy

$$\mathbb{E}(dR_t) = \frac{D_t}{P_t}dt + E_t\left(\frac{dP_t}{P_t}\right) = -E_t\left[\frac{dQ_t}{Q_t} + \frac{dQ_t}{Q_t}\frac{dP_t}{P_t}\right] \tag{10}$$

Equation (10) regulates how firms are valued in the economy. Note that the preference implies that firms transfer all expected profits to workers.

As in Campbell and Cochrane (1999) and Kehoe et al. (2019), I assume that the habit process $\{X_t\}$ is exogenous and specify the process for the log surplus consumption ratio $s_t = \log(S_t)$, which implicitly determines $\{X_t\}$.

$$ds_t = (1 - \rho_s)(s - s_t)dt + \sigma_a \lambda_a(s_t) dW_{a,t} \tag{11}$$

where $\lambda_a(s_t)$ is the sensitivity function

$$\lambda_a(s_t) = \frac{1}{S}[1 - 2(s_t - s)]^{1/2} - 1 \tag{12}$$

where S is the long-run surplus consumption ratio and $s = \log(S)$. The log surplus

[5] Families are identical, which implies that the aggregate consumption $\overline{C}_t$ is the same as the family consumption C_t.

consumption ratio is correlated with the aggregate shock process A_t since their diffusion terms share the same Brownian motion process. Equation (11) is the continuous time analog of the autoregressive process in Campbell and Cochrane (1999) and Kehoe et al. (2019).

Each family maximizes Equation (6) subject to the budget constraint

$$C_t + I_t = W_t + \Pi_t + H_t \tag{13}$$

where I_t is investment in vacancy creation, Π_t is firms' profit, and H_t is unemployment benefit. Since each family is identical, investment is the same as the aggregate cost of vacancy posting, i.e. $I_t = \kappa A_t \int z v_t(z) dz$. Similarly, $W_t + \Pi_t = \int z e_t(z) dz$ and $H_t = \int z u_t(z) dz$.

2.1 The Simple Model

I show how the model nests the search-and-matching model in Shimer (2005). If the human capital growth is $g_e = g_u = 0$ and the initial human capital distribution is a point mass at 1, the model is a representative agent model.

In Equation (11), if I set $\rho = 1$ and $\lambda_a = 0$, it implies that $X_t = 0$ at all times. The preference is the standard CRRA preference:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\beta t} \frac{C_t^{1-\alpha}}{1-\alpha} dt \right]$$

In addition, if $\alpha = 0$, the agents are risk-neutral, so that there is no need for savings in the absence of families.

If I set $\phi_e = \phi_u = 0$ and $\zeta = 0$, there is no labor market entry or exit. As will be clear in the remaining sections, labor market entry and exit serve two purposes. First, differential labor market exit rates, i.e. $\phi_e \neq \phi_u$, have impacts on the job-finding rate elasticity. Second, with human capital growth, labor market entry and exit guarantee a stationary distribution

(Gabaix et al. (2016)).

Further, if I do not augment the unemployment benefit and the vacancy cost with the shock, and set $g_a = 0$ so that the shock follows:

$$dA_t = \sigma_a dW_{a,t}$$

The model is identical to the one in Shimer (2005) modulo the continuous-time setting. The comparison suggests that the difference in the elasticity of job-finding rate between the model and the one in Shimer (2005) must come from the factors above. In the remaining sections, I will discuss how the factors affect the job-finding rate elasticity and convergence.

3 The Competitive Equilibrium

Assume the matching technology is given by a standard constant return to scale matching function $Bu^{\eta}v^{1-\eta}$. Appendix Section A derives the competitive equilibrium, which reduces to six equations:

$$\left[(\delta + \phi_e - g_e) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_e \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1)\right]\mu_{et} = A_t + \delta\mu_{ut} + \mathbb{E}_t(d\mu_{et}) \tag{14}$$

$$\left[(\eta\lambda_{wt} + \phi_u - g_u) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_u \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1)\right]\mu_{ut} = bA_t + \eta\lambda_{wt}\mu_{et} + \mathbb{E}_t(d\mu_{ut}) \tag{15}$$

$$\kappa = (1-\eta)\lambda_{vt}\frac{(\mu_{et} - \mu_{ut})}{A_t} \tag{16}$$

$$\frac{\mathrm{d}Z_{et}}{\mathrm{d}t} = (g_e - \phi_e - \delta)Z_{et} + \lambda_{wt}Z_{ut} \tag{17}$$

$$\frac{\mathrm{d}Z_{ut}}{\mathrm{d}t} = (g_u - \phi_u - \lambda_{wt})Z_{ut} + \delta Z_{et} + \zeta \tag{18}$$

$$ds_t = (1-\rho_s)(s - s_t)dt + \sigma_a \lambda_a(s_t) dW_{a,t} \tag{19}$$

μ_{et} is the total value of a match, which is the joint "revenue" of a worker and a firm. μ_{ut} is the total outside option. On the right-hand side of Equation (14), the flow value of μ_{et} is equal to the flow output plus the value of separation and the continuation value. On the left-hand side, the discount factor depends on the separation rate, the labor market exit rate, the human capital growth rate, the risk-free rate, and the covariance between μ_{et} and the log surplus consumption ratio, given by $\varepsilon_e \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t)+1)$. ε_e is the elasticity of μ_{et} with respect to the log surplus consumption ratio, evaluated at the steady state:

$$\varepsilon_e = \frac{\partial \mu_{et}}{\partial s_t} \frac{1}{\mu_{et}} |_{s_t = s}$$

The discount factor suggests that a higher labor market exit rate induces more discounting of future output flows, whereas a higher human capital growth implies less discounting. The intuition is simple: if an employed worker is very likely to exit the labor market, the expected time of a match will be short, and hence the total revenue is small. On the other hand, if an employed worker has a high human capital growth rate, it implies more revenue.

Equation (15) specifies the flow value of the joint outside option μ_{ut}. On the right-hand side, the flow value is equal to the home production plus the value of forming a match and the continuation value, where λ_{wt} is the job-finding rate. On the left-hand side, the discount factor is determined by job-finding, labor market exit, human capital growth, the risk-free rate, and the covariance between the joint outside option and the log surplus consumption ratio.

Equation (16) is the standard free-entry condition, which determines the "price" in the economy, namely the job-finding rate. Specifically, the vacancy-filling rate λ_{vt} relates to the job-finding rate by $\lambda_{vt} = B^{1/(1-\eta)} \lambda_{wt}^{\eta/(\eta-1)}$. $(\mu_{et} - \mu_{ut})/A_t$ is the normalized match surplus. Taking the log on both sides, we arrive at the key equation for characterizing the elasticity and the convergence of the job-finding rate:

$$\log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right) \tag{20}$$

where χ is a constant.[6] In particular, Equation (20) allows us to write the job-finding rate as a function of the normalized match surplus, which can be solved from the dynamic system of Equations (14) and (15):

$$\mathbb{E}_t\begin{bmatrix} d\mu_{et} \\ d\mu_{ut}\end{bmatrix} = \left(\underbrace{\begin{bmatrix} \delta+\phi_e-g_e & -\delta \\ -\eta\lambda_{wt} & \eta\lambda_{wt}+\phi_u-g_u\end{bmatrix}}_{B_1\text{: search model}} + \underbrace{\begin{bmatrix} -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right)+\Delta_{et} & 0 \\ 0 & -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right)+\Delta_{ut}\end{bmatrix}}_{B_2\text{: preference}}\right)\begin{bmatrix}\mu_{et} \\ \mu_{ut}\end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t\end{bmatrix} \tag{21}$$

where

$$\begin{aligned}\Delta_{et} &= \varepsilon_e\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1) \\ \Delta_{ut} &= \varepsilon_u\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1)\end{aligned}$$

Equation (20) also has important economic meaning: the elasticity of the job-finding rate depends on how the normalized match surplus responds to the aggregate shock. Intuitively, the incentive to create vacancies and hence the job-finding rate elasticity is large if the benefit of opening an extra vacancy is large, summarized by the normalized match surplus. On the other hand, the convergence depends on the dynamic system (Equation (21)) once

[6]Specifically,

$$\chi = \frac{1-\eta}{\eta}\log\left(\frac{1-\eta}{\kappa}B^{1/(1-\eta)}\right)$$

the economy is away from the steady state and the shock has disappeared.

The last three equations summarize how the state variables evolve. Z_{et} and Z_{ut} are the aggregate human capital of employed and unemployed workers, respectively, given by $Z_{et} = \int ze_t(z)dz$ and $Z_{ut} = \int zu_t(z)dz$. The change in the aggregate human capital of employed workers depends on human capital growth, labor market exit, separation, and job-finding, which is similar for unemployed workers.[7]

4 Characterization of Volatility and Convergence

4.1 The Elasticity of Job-Finding Rate

In Equation (21), the matrix that determines the evolution of the dynamic system consists of two parts. The first matrix B_1 depends only on the search side of the model while the second one depends only on the preference. I first characterize the preference part.

Lemma 1. *The risk-free rate to a first order approximation in the log surplus consumption ratio s_t is*

$$r_{ft} = -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) \approx -\left[a_Q + b_Q(s_t - s)\right] \tag{22}$$

where

$$a_Q = -\beta - \alpha g_a + \frac{1}{2}\alpha\sigma_a^2\left(\frac{\alpha}{S^2} + 1\right) \text{ and } b_Q = \alpha(1-\rho_s) - \frac{\alpha\sigma_a^2(\alpha + S - 1)}{S^2} \tag{23}$$

Note that the risk-free rate in the simple model is $-\beta$ with $g_a = \lambda_a = 0$ and $\rho_s = 1$, namely the subjective discount rate. With the asset pricing preference Equation (6), the risk-free rate is time varying. In particular, the shock affects how agents discount future output flows.

Proposition 1 characterizes the elasticity of job-finding rate with respect to the aggregate shock near the steady state.

[7] I use the normalization $\int zn_t(z)dz = 1$.

Proposition 1. *The elasticity of the job-finding rate with respect to the surplus consumption ratio s_t near $s_t = s$ is*

$$\frac{\partial \log(\lambda_{wt})}{\partial s_t} = \underbrace{\frac{b_Q e^{\Theta_1}}{1-\rho_s}}_{\text{Preference}} \underbrace{\left[\frac{c_l}{-(\gamma_l + a_Q + \Delta_e + \Theta_2)} + \frac{c_h}{-(\gamma_h + a_Q + \Delta_u + \Theta_2)}\right]\overline{\mu}^{-1}}_{\text{Search Side of the Model}} \tag{24}$$

where $\gamma_h < \gamma_l < 0$ are two eigenvalue of the matrix B_1 in Equation (21) (Equation (B.8)), c_l and c_h are weights of the eigenvalues (Equation (B.9)), Θ_1 and Θ_2 depend on model parameters (Equation (B.12)), Δ_e and Δ_u are evaluated at the steady state, and $\overline{\mu}$ is the long-run average match surplus without shocks.

To understand Proposition 1, I re-write the normalized match surplus formally as

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \int_t^{+\infty} e^{-\int_t^s R_u du} \frac{A_s}{A_t}(1-b)ds \tag{25}$$

Namely, it is the cumulative sum of the output flow difference between employed and unemployed workers, $\{A_s(1-b)/A_t\}_s$, discounted by the discount rate process R_t which is related to the preference and the search side of the model. The job-finding rate elasticity depends on how the output flow difference responds to the aggregate shock and how the discount rate process responds to the shock.

In the simple model, the unemployment benefit and the vacancy cost are not scaled by the shock. The job-finding rate elasticity depends on the match surplus without normalization:

$$\mu_{et} - \mu_{ut} = \int_t^{+\infty} e^{-Rs}(A_s - b)ds \tag{26}$$

Note that with risk neutrality, the discount rate process R_t is constant. The integral is equal to a constant times $(A_t - b)$. Taking the log and then the derivative, the elasticity of

$(\mu_{et} - \mu_{ut})$ with respect to the shock is

$$\frac{\partial \log(\mu_{et} - \mu_{ut})}{\partial \log(A_t)} = \frac{A_t}{A_t - b} \tag{27}$$

Equation (27) reiterates the results in e.g. Shimer (2005); Hagedorn and Manovskii (2008); Ljungqvist and Sargent (2017) that the elasticity depends on the fundamental surplus $A_t - b$, which is the only factor for generating large job-finding rate elasticity.[8]

The intuition is that, with risk neutrality, the shock only affects the output flow differences, not the discounting of the output flow. Hence, the job-finding rate elasticity is large if the fundamental surplus is small, making the match surplus sensitive to the aggregate shock.

When the unemployment benefit and the vacancy posting cost are augmented by the aggregate shock, the job-finding rate elasticity will be zero in the simple model because the fundamental surplus is constant when it is normalized by the aggregate shock. In particular, the job-finding rate elasticity depends on the normalized match surplus:

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \int_t^{+\infty} e^{-Rs} \frac{A_s}{A_t}(1 - b) ds \tag{28}$$

Integrating the expression, the normalized match surplus is $(1 - b)$ and does not respond to the shock.

With the asset pricing preference, the model no longer relies on the fundamental surplus to generate a large job-finding rate elasticity. In particular, even when the vacancy posting cost and the unemployment benefit are augmented by the shock, the job-finding rate still responds to the shock because the discount rate process is time varying. R_t depends on the search side of the model and the preference, given by B_1 and B_2 in Equation (21). Integrating Equation (25) leads to the expression in Equation (24).

[8] In discrete time, other factors could influence the elasticity of job-finding rate in the simple model. However, their effect is bounded by 1. See Ljungqvist and Sargent (2017) for details.

Next, I discuss what factors are important for a large job-finding rate elasticity. To facilitate the discussion, I solve the model numerically using the following parameter values:

Table 1: Parameterization

Panel A: The Simple Model

Parameters		Moments		
Endogenously Chosen		*Targeted*	Data	Model
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
B, matching efficiency	0.55	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.5	Mean unemployment rate	5.9	5.9
Assigned Parameters				
β, time preference factor	0.001	η, matching function elasticity	0.5	
b, home production parameter	0.6	$\phi_e = \phi_u$, exit rate	0.0028	
δ, separation rate	0.028			

Panel B: The Full Model

Parameters		Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
B, matching efficiency	0.55	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.975	Mean unemployment rate	5.9	5.9
β, time preference factor	0.001	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.057	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	5	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned Parameters				
g_e, employed HK growth (%p.a.)	3.5	η, matching function elasticity	0.5	
b, home production parameter	0.6	$\phi_e = \phi_u$, exit rate	0.0028	
δ, separation rate	0.028	ρ_s, persistence of state	0.9944	

Notes. Table 1 shows the parameter values for the simple model and the full model. %p.a. means percent per annum.

4.1.1 Potential Sources of Large Job-Finding Rate Elasticity

Preference

Equation (24) suggests that the preference or the search model parameters both determine the elasticity. The parameters that affect the preference part of the elasticity include the long-run surplus consumption ratio S and the auto-correlation of the log surplus consumption ratio ρ_s. The job-finding rate elasticity is decreasing in the long-run surplus consumption ratio S and increasing in the auto-correlation of the log surplus consumption ratio ρ_s.

Smaller long-run surplus consumption ratio would lead to a larger volatility of the stochastic discount factor, amplifying the response of job-finding rate. Intuitively, with a small long-run surplus consumption ratio, small changes in consumption can have large impacts on the stochastic discount factor and hence the discounting of the output flow.

Higher auto-correlation of the log surplus consumption ratio means that the effect of the shock on the discount rate process is long lasting, so that the accumulated effect on the normalized match surplus is large.

To summarize, if the stochastic discount factor, or more generally how firms price future output flows vary with the aggregate shock, the resulting job-finding rate elasticity could be large. The intuition is straight-forward: even if future output flows are constant, if a negative shock causes the firms to be pessimistic about the future, the expected profits would decline and firms would post fewer vacancies.[9] This suggests other settings that could imply a large job-finding rate elasticity. For example, if the firm stocks are held by risk-averse agents whose consumption is volatile, e.g. wealthy hand-to-mouth agents in Kaplan et al. (2018), the job-finding rate elasticity could be large.

Search Side

I turn to the search side of the model. The elasticity depends on the ratio between the weights and the eigenvalues, $-c_l/\gamma_l$ and $-c_h/\gamma_h$. I refer to c_l as the long-run weight, γ_l as the long-run eigenvalue, and $-c_l/\gamma_l$ as the long-run returns. The use of the term "long-run" will be clear below. Similarly, c_h, γ_h, $-c_h/\gamma_h$ are the short-run weight, eigenvalue, returns, respectively.

The smaller eigenvalue γ_h is the high frequency component of the model. It relates to

[9] This requires departure from rational expectation. See e.g. Preston (2005); García-Schmidt and Woodford (2019); Farhi and Werning (2019).

short-run discounting, namely job-finding and separation. In particular, the eigenvalue is at least as big as $\eta\lambda_w$ in absolute value. Since the monthly job-finding rate is around 0.46 and the usual parameter choice for η is around 0.5, the short-run eigenvalue is large in magnitude under plausible parameterization. The intuition is that, the output flow difference is discounted by job finding in the short run. If the workers can find jobs fast, an extra vacancy does not increase the job-finding rate by much. Hence, the short-run returns of opening an extra vacancy, given by $-c_h/\gamma_h$, is small.

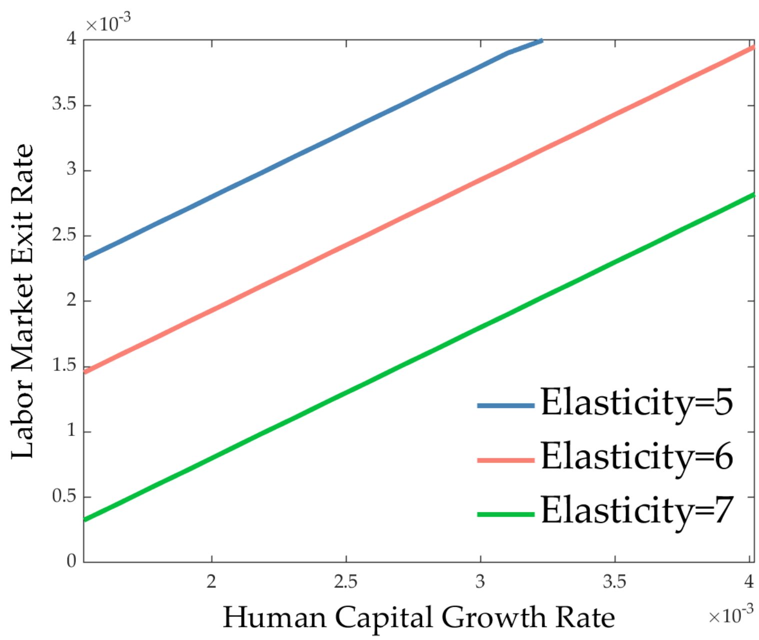
The larger eigenvalue γ_l is the low frequency component of the model. It relates to long-run discounting, which includes human capital growth and labor market exit. Since γ_l is close to zero, the long-run returns $-c_l/\gamma_l$ can be large if c_l is not zero.

The long-run weight c_l depends on the difference between human capital growth rates and labor market exit rates across employment status. Examining the expression for the weights and the eigenvalues (Equations (B.8) and (B.9)), if human capital accumulation and labor market exit rate do not differ by employment status, i.e. $g_e - g_u = \phi_e - \phi_u = 0$, the long-run weight is 0. Intuitively, if the output flow across employment status has the same discount rate process, the difference cancels out and does not interact with the shock. When there are differential human capital growth rates or labor market exit rates, the output flow difference interacts with the shock because creating an extra vacancy can lead to higher human capital growth or lower labor market exit rates, resulting in large long-run returns.

The analysis shows that the key to amplification of the shock lies in the returns by employment status. In particular, it depends on whether creating a vacancy has long-run benefit, in addition to the higher output of employed workers compared to unemployed workers. In the current model, this means higher human capital growth rates or lower labor market exit rates for employed workers. Note that there are other settings that could amplify the effect of the shock. For example, in search-and-matching models embedding network (see e.g. Galenianos (2014), Arbex et al. (2019)), employed workers can refer other workers and increase the effective job-finding rate, which could lead to a large job-finding rate elasticity.

Figure 1 plots the combinations of g_e and ϕ_e that imply the same job-finding rate elasticity, fixing $g_u = 0$ and $\phi_u = 0.004$.[10] These same elasticity curves reveal two results. First, a higher human capital growth rate or a lower labor market exit rate implies a larger elasticity. This is consistent with the discussion above: they both point to larger long-run benefits of opening a vacancy. Second, human capital growth rates and labor market exit rates complement each other in the sense that, a high human capital growth rate and a high labor market exit rate could imply the same elasticity as do a low human capital growth rate and a low labor market exit rate.

Figure 1: Same Elasticity Curves



Notes. Figure 1 plots the combination of human capital growth rates and labor market exit rates that produce the same job-finding rate elasticity. The other model parameters are in Table 1 Panel B.

While human capital growth is emphasized in several studies, the role of labor market

[10]Here ϕ_u should be interpreted as the probability that an unemployed worker exit the labor market, instead of the probability of retirement.

exit rate is less explored in the search-and-matching model.[11] When viewed as the probability of non-participation, the labor market exit rate should differ by employment status, as unemployed workers are more likely to exit the labor force. The model can potentially be used for understanding the effect of policies that affect labor force participation on the job-finding rate, unemployment rate, etc.

4.2 The Convergence of Job-Finding Rate

A large job-finding rate elasticity does not imply a slow rate of convergence. Take the simple model for example, which can generate a large job-finding rate elasticity by the fundamental surplus. However, since the only state variable is the aggregate labor productivity, the job-finding rate comoves with the aggregate shock.

I derive analytically the convergence of job-finding rate in a perfect foresight equilibrium, which sheds light on the convergence in the rational expectation equilibrium by certainty equivalence. I then numerically study the convergence in the rational expectation equilibrium and verify that a lot of insights are carried over.

Define the perfect foresight equilibrium as follows:

1. The trajectory of the shock is deterministic.
2. The deviation from the steady state is small so that human capitals and the job-finding rate can be considered constant.

With the definition, the dynamic system in the perfect foresight equilibrium depends only on the deterministic aggregate shock:

$$\begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = \begin{bmatrix} r_{ft} + \delta + \phi_e - g_e & -\delta \\ -\eta\lambda_w & r_{ft} + \eta\lambda_w + \phi_u - g_u \end{bmatrix} \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{29}$$

[11] Several studies including e.g. Pissarides (1992); Sterk (2015); Kehoe et al. (2019) emphasize human capital growth. For labor market exit rate, there is empirical evidence that it differs by employment status (e.g. Rogerson and Shimer (2011)), and that it is as important as the job-finding rate for explaining the variation in the unemployment rate in most OECD countries (Elsby et al. (2013)).

where $r_{ft} = -a_Q - b_Q(s_t - s)$. Proposition 2 characterizes the convergence speed of the job-finding rate.

Proposition 2. *Let $\mu_t = (\mu_{et} - \mu_{ut})/A_t$ denote the normalized match surplus. Let $\overline{\mu}$ denote its steady state value. The normalized match surplus satisfies*

$$\mu_t - \overline{\mu} = \Omega_t \left(c_l e^{\gamma_l t} + c_h e^{\gamma_h t} \right) (\mu_0 - \overline{\mu}) \tag{30}$$

The half-life τ is the solution to the equation

$$\Omega_\tau (c_l e^{\gamma_l \tau} + c_h e^{\gamma_h \tau}) = \frac{1}{2} \tag{31}$$

where Ω_τ is given in the Equation (B.21). The half-life is increasing in c_l and decreasing in c_h:

$$\frac{\partial \tau}{\partial c_l} > 0, \quad \frac{\partial \tau}{\partial c_h} < 0$$

Proposition 2 suggests that a larger long-run weight reduces the rate of convergence. The intuition is that the long-run dynamics of the model, such as human capital accumulation, has a slower rate of convergence compared to the short-run dynamics. When the model transition places more weight on the long-run dynamics, its convergence will be slower.

To illustrate this point more clearly, consider diagonalizing the transition matrix in Equation (29), ignoring the risk-free rate:

$$\begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = \Gamma \begin{bmatrix} \gamma_l & 0 \\ 0 & \gamma_h \end{bmatrix} \Gamma^{-1} \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{32}$$

where Γ contains the eigenvectors. The solution to the dynamic system could be written formally as

$$\begin{bmatrix} \mu_{et} - \mu_{e0} \\ \mu_{ut} - \mu_{u0} \end{bmatrix} = \int_t^{+\infty} \Gamma \begin{bmatrix} e^{\gamma_l s} & 0 \\ 0 & e^{\gamma_h s} \end{bmatrix} \begin{bmatrix} A_s \\ bA_s \end{bmatrix} ds \tag{33}$$

If Γ is the identity matrix and $A_s = A$, Equation (33) amounts to two separate integrals which can be solved as

$$\begin{bmatrix} \mu_{et} - \mu_{e0} \\ \mu_{ut} - \mu_{u0} \end{bmatrix} = \begin{bmatrix} -\frac{A}{\gamma_l} e^{\gamma_l t} \\ -\frac{bA}{\gamma_h} e^{\gamma_h t} \end{bmatrix}$$

This shows that the rate of convergence depends on the eigenvalues. In particular, if the eigenvalue is close to zero in the case of γ_l, the rate of convergence will be slow. The low-frequency eigenvalue always implies slower rate of convergence than the high-frequency eigenvalue.

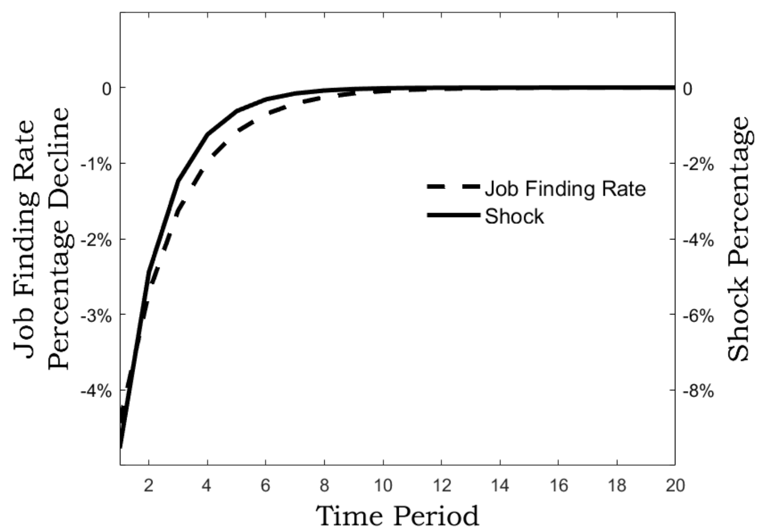
With Γ being the eigenvectors, Equation (33) reduces to Equation (30). The weights c_l and c_h determine whether the low-frequency or the high-frequency part of the model is driving the dynamics of the normalized surplus. When the former is more important, the rate of convergence is slower. The convergence in the simple model is fast because the long-run weight is zero, meaning that the model dynamics depend entirely on the high-frequency part.

To what extend can a larger long-run weight generate a slow rate of convergence? Under the parameterization in Table 1 Panel B, the half-life of the perfect foresight economy is 3 months. The convergence is fast even with the positive long-run weight because it is negligible compared to the short-run weight.

Can a different parameterization generate a slow convergence? I show in the Appendix that $\partial c_l / \partial \gamma_l < 0$. This means that the long-run weight depends on the low-frequency eigenvalue. In particular, if the low-frequency eigenvalue is such that it implies slow convergence and is hence close to zero, the implied long-run weight is small compared to the short-run weight. As a result, the perfect foresight economy always has a fast converging job-finding rate.

This means that without the other state variables, e.g. human capitals, the job-finding rate co-moves with the aggregate shock. The co-movement would carry over to the rational expectation equilibrium if the model mainly transits through the aggregate shock. To that end, I impose a 1% negative shock on the labor productivity, which dies down to half of the value in 3 months. Figure 2 shows that the job-finding rate co-moves with the aggregate shock in the rational expectation equilibrium, suggesting that the aggregate shock drives most of the dynamics in the rational expectation equilibrium.

Figure 2: Convergence of the Job-Finding Rate



Notes. Figure 2 shows the convergence of job-finding rate. The model is parameterized as in Table 1 Panel B.

The rational expectation equilibrium resembles the perfect foresight equilibrium because the dynamics of the job-finding rate depends only on the aggregate shock near the steady state. This can be seen from Equation (20) and Equation (B.13), where the log job-finding rate is linear in the log normalized match surplus and the only time-varying component in the normalized match surplus is the aggregate shock. The deviation from the steady state for other state variables is small, so that the model dynamics are not much different in the rational expectation equilibrium and the perfect foresight equilibrium.

4.2.1 Extension for a Slow Convergence of Job-Finding Rate

The discussion suggests that the key for generating a slow convergence of the job-finding rate is to break the linearity between the log job-finding rate and the log normalized match surplus. Viewing through the lens of Equation (20), either the matching efficiency or the bargaining power needs to be time varying.

Here I focus on time-varying matching efficiency since several recent papers develop theories along this line (e.g. Sterk (2015); Gavazza et al. (2018); Acharya and Wee (2020); Leduc and Liu (2020)). I derive the general formulation and then discuss the extent to which empirically consistent endogenous matching efficiency can generate the slow convergence. Empirically, it is difficult to separate the matching efficiency and the bargaining power. One can hence view the change in the matching efficiency as capturing time-varying bargaining power as well.

As shown above, the elasticity of the job-finding rate is not important for the rate of convergence. I hence resort to the simple model and extend it by assuming that the matching efficiency depends the aggregate shock

$$\lambda_{wt} = B_t \theta_t^{1-\eta} \tag{34}$$

where the dependence is summarized by the time subscript.

Equation (21) simplifies to:

$$\begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = \begin{bmatrix} \delta + \beta & -\delta \\ -\eta B_t \overline{\theta}^{1-\eta} & \eta B_t \overline{\theta}^{1-\eta} + \beta \end{bmatrix} \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{35}$$

where I assume the market tightness is at the steady state value to isolate the effect of the matching efficiency.

Lemma 2 derives the half-life of the extended simple model:

Lemma 2. *With the endogenous matching efficiency in Equation (34), the normalized match surplus satisfies*

$$\mu_t - \overline{\mu} = e^{-(\beta+\delta)t - \eta\overline{\lambda}_w t + (\epsilon_m/100p)(1-e^{-pt})}(\mu_0 - \overline{\mu}) \tag{36}$$

where ϵ_m is the elasticity of the matching efficiency with respect to the aggregate shock, p is the *convergence rate of the matching efficiency, and $\overline{\lambda}_w$ is the steady state job-finding rate. The half-life τ is the solution to the equation*

$$e^{-(\beta+\delta)\tau-\eta\overline{\lambda}_w\tau+(\epsilon_m/100p)(1-e^{-p\tau})} = \frac{1}{2} \tag{37}$$

The half-life is increasing in the elasticity ϵ_m and decreasing in the convergence rate p.

Lemma 2 shows that if the matching efficiency declines by a large magnitude following a negative labor productivity shock, or if it takes longer to return to its steady state, the recovery in the job-finding rate would be slower.

To what extent can empirically consistent matching efficiency response generate the slow convergence of the job-finding rate? I use $\epsilon_m = 10$ and $p = 0.12$. During the Great Recession, labor productivity declined by about 2% while the matching efficiency declined by 50% from 2008 to 2009, so that $\epsilon_m = 10$ might underestimate the elasticity. $p = 0.12$ implies a half-life of 6 months, which also likely underestimate the persistence of the decline in the matching efficiency.[12] However, even with the parameters, the half-life implied by Equation (37) increases to 7 months, which more than doubles what is implied by a constant matching efficiency.[13] With $\epsilon_m = 25$ and $p = 0.1$, the half-life becomes one year, much more consistent with the job-finding rate convergence during the Great Recession.

The stylized example abstracts from the fact that the persistence of the matching efficiency could also be endogenous. In particular, when the matching efficiency affects the evolution of the state variables, the model could feature multiple equilibria that further slow down recovery or even stuck the economy in a corner equilibrium. For example, in Sterk (2015), the matching efficiency depends on the fraction of unemployed workers

[12] Gavazza et al. (2018) shows that the matching efficiency declines by 50% from 2008 to 2009 and does not recover until 2014 (Figure 1).

[13] If the matching efficiency is constant, i.e. $\epsilon_m = p = 0$, we are back to the simple model. Equation (37) becomes

$$e^{-(\beta+\delta)\tau-\eta\overline{\lambda}_w\tau} = \frac{1}{2}$$

Using $\beta = 0.001$, $\delta = 0.028$, $\eta = 0.5$, $\overline{\lambda}_w = 0.46$, the half-life is 3 months. This verifies the validity of Lemma 2.

who cannot find jobs after one period. Because the matching efficiency decline further lowers the job-finding rate, unemployed workers become less likely to find jobs. This complementarity leads to a corner equilibrium where the job-finding rate is low and the unemployment rate is high.

Time-varying matching efficiency thus presents a promising way to account for the slow convergence of the job-finding rate. It also receives empirical support using micro-level data (e.g. Davis et al. (2013)). An empirically consistent search-and-matching model would hence incorporate time-varying matching efficiency, as well as considering time-varying risks.[14]

5 Conclusion

This paper studies the job-finding rate elasticity and convergence in a class of search-and-matching models. The paper derives analytic solution for the job-finding rate elasticity, and show that the key for a large elasticity lies in the response of the output flow and how the agents discount the output flow.

The analytic expression for the job-finding rate convergence in the perfect foresight equilibrium suggests that the job-finding rate co-moves with the aggregate shock, which carries over to the rational expectation equilibrium. The reason for the co-movement is a lack of time-varying components besides the aggregate shock. To generate a slow convergence, either a time-varying matching efficiency or bargaining power is needed.

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[14]By the discussion, wage rigidity cannot generate a slow convergence, either. This is verified numerically in Section C. The intuition is that wage rigidity only affects the extent to which the job-finding rate responds to the shock, but the only time-varying component near the steady state is still the aggregate shock.

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Appendices

A Derivation of the Equilibrium

In each submarket, a vacancy specifies a contract of desired human capital level z and wage offer $W_t(z)$, which is the discounted value of wage payment over the course of employment. I focus on the symmetric equilibrium in which firms post the same contract $(z, W_t(z))$ in a submarket. Workers' human capital is public information. The assumption implies that workers with human capital level z will only search in submarket z.

Each submarket has the same matching technology given by a matching function $m_t(u_t(z), v_t(z)) = Bu_t^{\eta} v_t^{1-\eta}$. The job-finding rate is determined by $\lambda_{wt}(z) = m_t(u_t(z), v_t(z))/u_t(z)$. Define the market tightness $\theta_t(z) = v_t(z)/u_t(z)$. We have $\lambda_{wt}(\theta_t(z)) = m_t(1, \theta_t(z))$. The vacancy-filling rate $\lambda_{vt}(\theta_t(z)) = m_t(u_t(z), v_t(z))/v_t(z)$ relates to the job-finding rate by $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{vt}(\theta_t(z))$.

Let $M_t(z)$ denote the post-match value, so that the value of employment is $M_t(z) + W_t(z)$. The post-match value satisfies

$$(\delta + \phi_e)M_t(z) = \delta U_t(z) + \mathbb{E}_t\left[dM_t(z_t) + M_t(z)\frac{dQ_t}{Q_t} + dM_t(z)\frac{dQ_t}{Q_t}\right] \tag{A.1}$$

The post-match value can be interpreted as worker's outside option, such that the firm cannot contract on the post-match value. Q_t is the stochastic discount factor in Equation (8). $U_t(z)$ is the value of unemployed workers:

$$\begin{aligned}(\lambda_{wt}(z) + \phi_u)U_t(z) =& bA_t z + \lambda_{wt}(z)(M_t(z) + W_t(z)) \\ &+ \mathbb{E}_t\left[dU_t(z_t) + U_t(z)\frac{dQ_t}{Q_t} + dU_t(z)\frac{dQ_t}{Q_t}\right]\end{aligned} \tag{A.2}$$

Unemployed workers receive unemployment benefit $bA_t z$ and find jobs at a rate $\lambda_{wt}(z)$. The value of unemployment is discounted by the job-finding rate and the labor market exit rate, while the continuation value is affected by the stochastic discount factor.

Firms posting vacancies in submarket z incur a flow cost $\kappa A_t z$. Once a match forms, an employed worker produces output flows $A_t z$ which are linear in the aggregate shock and human capital. Denote the expected revenue of firms in submarket z by $Y_t(z)$ which satisfies

$$(\delta + \phi_e)Y_t(z) = A_t z + \mathbb{E}_t\left[dY_t(z_t) + Y_t(z)\frac{dQ_t}{Q_t} + dY_t(z)\frac{dQ_t}{Q_t}\right] \tag{A.3}$$

The expected revenue is discounted by exogenous separation and labor market exit. The continuation value is discounted by Q_t, consistent with the assumption that firms are owned by families. In the symmetric equilibrium, the following relation holds in each submarket:

$$\eta[Y_t(z) - W_t(z)] = (1 - \eta)[W_t(z) + M_t(z) - U_t(z)] \tag{A.4}$$

Equation (A.4) guarantees efficiency of the competitive equilibrium. It is equivalent to the Hosios condition under Nash bargaining.

There is free-entry of firms. The assumption implies that the flow cost of vacancy is equal to the expected profit:

$$\kappa A_t z = \lambda_{vt}(\theta_t(z))[Y_t(z) - W_t(z)] \tag{A.5}$$

The free-entry condition relates the market tightness to value functions and hence the expected present discounted value of the output flows.

The competitive search equilibrium is defined as a collection of stochastic processes $\{C_t, Q_t, S_t\}_{t\geqslant 0}$ and state-contingent processes $\{W_t(z), M_t(z), U_t(z), Y_t(z), \theta_t(z), e_t(z), u_t(z), v_t(z)\}_{t\geqslant 0}$ such that: i) for each t, taken as given $M_t(z), U_t(z), Y_t(z)$, and Q_t, the wage contract $W_t(z)$ and the market tightness $\theta_t(z)$ satisfy Equation (A.4), ii) the collection of state-contingent processes $\{W_t(z), M_t(z), U_t(z), Y_t(z)\}_{t\geqslant 0}$ satisfy Equations (A.1) to (A.3), iii) the evolution of the human capitals, unemployment, employment, and aggregate shock satisfy Equations (1) to (4), iv) the free-entry condition Equation (A.5) holds, v)

the aggregate resource constraint Equation (5), vi) the stochastic discount factor satisfies Equation (8), vii) the log surplus consumption ratio satisfies Equation (11).

The linear technology implies that value functions are also linear. I write the value of a match $Y_t(z_t) = Y_t z_t$, and $W_t(z_t) = W_t z_t$, $M_t(z_t) = M_t z_t$, $U_t(z_t) = U_t z_t$. Linear value functions simplify the model, as it can aggregate linearly so that the solution does not depend on human capital distributions. Instead, the model's state variables include the aggregate employed human capital $Z_{et} = \int z e_t(z) dz$ and the aggregate unemployed human capital $Z_{ut} = \int z u_t(z) dz$.

Define the total value of a match $\mu_{et} = Y_t + M_t$. Define the joint outside option $\mu_{ut} = U_t$.[15] The economy is characterized by the following five equations plus Equation (11):

$$\left[(\delta + \phi_e - g_e) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_e \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1)\right]\mu_{et} = A_t + \delta\mu_{ut} + \mathbb{E}_t(d\mu_{et}) \tag{A.6}$$

$$\left[(\eta_t \lambda_{wt} + \phi_u - g_u) - \mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) + \varepsilon_u \alpha \sigma_a^2 \lambda(s_t)(\lambda(s_t) + 1)\right]\mu_{ut} = bA_t + \eta_t \lambda_{wt} \mu_{et} + \mathbb{E}_t(d\mu_{ut}) \tag{A.7}$$

$$\kappa A_t = (1 - \eta_t)\lambda_{vt}(\mu_{et} - \mu_{ut}) \tag{A.8}$$

$$\frac{\mathrm{d}Z_{et}}{\mathrm{d}t} = (g_e - \phi_e - \delta)Z_{et} + \lambda_{wt} Z_{ut} \tag{A.9}$$

$$\frac{\mathrm{d}Z_{ut}}{\mathrm{d}t} = (g_u - \phi_u - \lambda_{wt})Z_{ut} + \delta Z_{et} + \zeta \tag{A.10}$$

where $\varepsilon_{e,u}$ are the elasticity of the value functions with respect to the log surplus consumption ratio, evaluated at the steady state:

$$\varepsilon_e = \frac{\partial \mu_{et}}{\partial s_t}\frac{1}{\mu_{et}}|_{s_t = s}, \quad \varepsilon_u = \frac{\partial \mu_{ut}}{\partial s_t}\frac{1}{\mu_u}|_{s_t = s}$$

The constant ζ in Equation (A.10) comes from the normalization $\int z n_t(z) dz = 1$.

[15] μ_{ut} is the joint outside option since the firms' outside option is 0.

B Proofs

B.1 Proof of Lemma 1

Proof. The risk-free rate is

$$r_{ft} = -\mathbb{E}\left(\frac{dQ_t}{Q_t}\right)$$

where Q_t is the stochastic discount factor

$$Q_t = e^{-\beta t}(S_t C_t)^{-\alpha}$$

Using Ito's lemma, the stochastic discount factor satisfies the following stochastic differential equation (SDE):

$$\begin{aligned} dQ_t = &-\beta e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} dt - \alpha e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} \frac{dS_t}{S_t} - \alpha e^{-\beta t} S_t^{-\alpha} C_t^{-\alpha} \frac{dC_t}{C_t} \\ &+ \frac{1}{2} e^{-\beta t} \alpha(\alpha+1) S_t^{-\alpha} C_t^{-\alpha} \frac{(dS_t)^2}{S_t^2} + \frac{1}{2} e^{\beta t} \alpha(\alpha+1) S_t^{-\alpha} C_t^{-\alpha} \frac{(dC_t)^2}{C_t^2} \\ &+ e^{-\beta t} \alpha^2 S_t^{-\alpha} C_t^{-\alpha} \frac{dS_t}{S_t} \frac{dC_t}{C_t} \end{aligned} \tag{B.1}$$

where

$$\frac{dS_t}{S_t} = (1-\rho_s)(s - s_t)dt + \sigma_a \lambda(s_t) dW_t$$

and

$$\frac{dC_t}{C_t} \approx \frac{dA_t}{A_t} = g_a dt + \sigma_a dW_t$$

Here the approximation ignores the changes in the human capital. In the steady state, the human capital is constant. Close to the steady state, the approximation is appropriate.

Substituting in the SDE for S_t and C_t to equation (B.1), the stochastic discount factor satisfies

$$
\begin{aligned}
dQ_t = & \left(-\beta - \alpha g_a + \frac{1}{2}\alpha^2\sigma_a^2\frac{1}{S^2}\right)Q_t dt + \left[\alpha(1-\rho_s) + \frac{\alpha^2\sigma_a^2}{S^2}\right]Q_t(s_t - s) \\
& - \underbrace{\alpha\sigma_a(\lambda(s_t)+1)dW_t}_{\text{local martingale}}
\end{aligned}
\tag{B.2}
$$

When taking the expectation, the local martingale vanishes, so that the risk-free rate satisfies

$$
r_{ft} = -\mathbb{E}\left(\frac{dQ_t}{Q_t}\right) = \underbrace{-\beta - \alpha g_a + \frac{1}{2}\alpha^2\sigma_a^2\frac{1}{S^2}}_{a_Q} + \underbrace{\left[\alpha(1-\rho_s) + \frac{\alpha^2\sigma_a^2}{S^2}\right]}_{b_Q}(s_t - s) \tag{B.3}
$$

□

B.2 Proof of Proposition 1

Proof. By equation (20), the elasticity of the job-finding rate with respect to the shock s_t is equal to the elasticity of $\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)$. The latter evolves according to equation (21). We can solve the system forward by making two assumptions.

Assumption 1 The job-finding rate is approximately equal to its steady-state value λ_w.

Assumption 2 The human capitals are constant.

The first assumption makes the job-finding rate in the transition matrix B_1 constant. The dynamic system becomes analytically tractable. The second assumption allows me to drop the expectation operator on the left-hand side of equation (20). Under the assumption, the dynamic system only depends on the log surplus consumption ratio process s_t, which is an Ornstein-Uhlenbeck (O-U) process. The dynamic system is hence an integrated O-U process which has a closed-form solution.

The dynamic system equation (21) can further be written as

$$\mathbb{E}_t \begin{bmatrix} d\mu_{et} \\ d\mu_{ut} \end{bmatrix} = (B_1 + B_2 + B_3) \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} + \begin{bmatrix} -A_t \\ -bA_t \end{bmatrix} \tag{B.4}$$

where

$$B_1 = \begin{bmatrix} \delta + \phi_e - g_e & -\delta \\ -\eta\lambda_w & \eta\lambda_w + \phi_u - g_u \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \Delta_e & 0 \\ 0 & \Delta_u \end{bmatrix}$$

$$B_3 = \begin{bmatrix} -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) & 0 \\ 0 & -\mathbb{E}_t\left(\frac{dQ_t}{Q_t}\right) \end{bmatrix}$$

B_3 commutes with B_1 and B_2, but B_1 and B_2 do not commute. And hence, the exponential, by the Zassenhaus formula, can be written as

$$\exp(B_1 + B_2) \approx \exp(B_1)\exp(B_2)\exp\left(\frac{1}{2}(B_1B_2 - B_2B_1)\right) \tag{B.5}$$

The matrix $(B_1B_2 - B_2B_1)/2$ is

$$\begin{bmatrix} 0 & -\delta(\Delta_u - \Delta_e) \\ -\eta\lambda_w(\Delta_e - \Delta_u) & 0 \end{bmatrix} \tag{B.6}$$

The matrix is approximately 0 if $\Delta_e \approx \Delta_u$. In other words, the matrix is approximately 0 if the elasticity of the total match revenue and the outside option with respect to the log surplus consumption ratio is close near the steady state. Numerically, this is indeed the case. So I first derive the result by assuming that $\Delta_u = \Delta_e$.

With the simplification, the dynamic system equation (B.4) can be solved with

$$\frac{\mu_{et}-\mu_{ut}}{A_t}=\int_t^{+\infty}\left(c_l e^{(\gamma_l+\Delta_e)(u-t)}+c_h e^{(\gamma_h+\Delta_u)(u-t)}\right)\mathbb{E}_t\left(\frac{Q_u}{Q_t}\right)du \tag{B.7}$$

where

$$\begin{aligned}\gamma_l=&\frac{(g_e+g_u)-(\delta+\phi_e)-(\eta\lambda_w+\phi_u)}{2}+\frac{1}{2}\sqrt{\left[(g_e-g_u)-(\delta+\phi_e)+(\eta\lambda_w+\phi_u)\right]^2+4\delta\eta\lambda_w}\\ \gamma_h=&\frac{(g_e+g_u)-(\delta+\phi_e)-(\eta\lambda_w+\phi_u)}{2}-\frac{1}{2}\sqrt{\left[(g_e-g_u)-(\delta+\phi_e)+(\eta\lambda_w+\phi_u)\right]^2+4\delta\eta\lambda_w}\end{aligned} \tag{B.8}$$

are the two eigenvalues of the matrix B_1. The weights c_l, c_h are given by

$$\begin{aligned}c_l&=\frac{\left(\gamma_l+\phi_u-g_u\right)\left[(1-b)\eta\lambda_w-b\left(\gamma_h+\phi_u-g_u\right)\right]}{(\gamma_l-\gamma_h)\eta\lambda}\\ c_h&=\frac{\left(\gamma_h+\phi_u-g_u\right)\left[b\left(\gamma_l+\phi_u-g_u\right)-(1-b)\eta\lambda_w\right]}{(\gamma_l-\gamma_h)\eta\lambda}\end{aligned} \tag{B.9}$$

By lemma 1, we can integrate equation (B.2) and write Q_u/Q_t as

$$\begin{aligned}\frac{Q_u}{Q_t}&=\int_t^u\left(a_Q+b_Q(s_\tau-s)\right)d\tau\\ &=a_Q(u-t)+b_Q\int_t^u(s_\tau-s)d\tau\end{aligned}$$

Plug the expression into equation (B.7), we can rewrite the equation as

$$\frac{\mu_{et}-\mu_{ut}}{A_t}=\int_t^{+\infty}\left(c_l e^{(\gamma_l+\Delta_e+a_Q)(u-t)}+c_h e^{(\gamma_h+\Delta_u+a_Q)(u-t)}\right)\mathbb{E}_t\left(e^{b_Q\int_t^u(s_\tau-s)d\tau}\right)du \tag{B.10}$$

The integral has two parts. The first part is an exponential integrand. The second part involves an expectation operator. In particular, since $\{s_t\}$ is an O-U process, the second part is the exponential of an integrated O-U process. Let the integrated O-U process be denoted by

$$G_u = \int_t^u (s_\tau - s) d\tau$$

The integrated O-U process is a Gaussian process (see e.g. Barndorff-Nielsen and Shephard (2003)). For Gaussian processes, the expected value of its exponential is characterized by its mean and variance, which are:

$$\begin{aligned}
\mathbb{E}_t(G_u) &= (1-\rho_s)(s_t - s) - \frac{(s_t - s)e^{-(1-\rho_s)(u-t)}}{1-\rho_s} \\
Var_t(G_u) &= \frac{b_Q \sigma_a^2 \lambda^2(s_t)}{(1-\rho_s)^2}(u-t) - \frac{3}{2}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} + \frac{3}{2}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} e^{-(1-\rho_s)(u-t)}
\end{aligned}$$

I will use M_u and V_u to denote the mean and the variance of the O-U process. The expected value is

$$\mathbb{E}_t e^{G_u} \approx e^{M_u + \frac{1}{2}V_u} = e^{\Theta_1 + \Theta_2(u-t) - \Theta_3 e^{-(1-\rho_s)(u-t)}} \tag{B.11}$$

where the constants (fix time t) Θ_1, Θ_2, Θ_3 are

$$\begin{aligned}
\Theta_1 &= \frac{b_Q(s_t - s)}{1-\rho_s} - \frac{3}{4}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3} \\
\Theta_2 &= \frac{b_Q\sigma_a^2\lambda^2(s_t)}{2(1-\rho_s)^2} \\
\Theta_3 &= b_Q\left[\frac{s_t - s}{1-\rho_s} - \frac{3}{4}\frac{\sigma_a^2\lambda^2(s_t)}{(1-\rho_s)^3}\right]
\end{aligned} \tag{B.12}$$

Plug equation (B.11) into and integrate, we can derive that integral is equal to

$$\frac{\mu_{et} - \mu_{ut}}{A_t} = \frac{c_l e^{\Theta_1}}{-(\gamma_l + \Delta_e + a_Q + \Theta_2)} + \frac{c_h e^{\Theta_1}}{-(\gamma_h + \Delta_u + a_Q + \Theta_2)} \tag{B.13}$$

The ratio $(\mu_{et} - \mu_{ut})/A_t$ is related to the log surplus consumption ratio since Θ_1 and Θ_2 are functions of the log surplus consumption ratio. Taking the derivative with respect to s_t of equation (B.13), the derivative is

$$
\begin{aligned}
\frac{\partial\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right)}{\partial s_t} =& \frac{b_Q}{1-\rho_s}\left[\frac{c_l e^{\Theta_1}}{-(\gamma_l+\Delta_e+a_Q+\Theta_2)}+\frac{c_h e^{\Theta_1}}{-(\gamma_h+\Delta_u+a_Q+\Theta_2)}\right] \\
& - \alpha^2\sigma_a^4 \frac{\partial(\lambda(s_t)(\lambda(s_t)+1))}{\partial s_t}\left[\frac{c_l \varepsilon_e e^{\Theta_1}}{(\gamma_l+\Delta_e+a_Q+\Theta_2)^2}+\frac{c_h \varepsilon_u e^{\Theta_1}}{(\gamma_h+\Delta_u+a_Q+\Theta_2)^2}\right]
\end{aligned}
\tag{B.14}
$$

The second term on the right-hand side is approximately 0 because σ_a^4 is small. By dropping the second term and denoting the long-run value of $(\mu_{et}-\mu_{ut})/A_t$ to be $\overline{\mu}$, the elasticity is

$$
\frac{\partial \log(\lambda_{wt})}{\partial s_t} = \underbrace{\frac{b_Q e^{\Theta_1}}{1-\rho_s}}_{\text{Preference}} \underbrace{\left[\frac{c_l}{-(\gamma_l+\Delta_e+a_Q+\Theta_2)}+\frac{c_h}{-\left(\gamma_h+\Delta_u+a_Q+\Theta_2\right)}\right]\overline{\mu}^{-1}}_{\text{Search Model}} \tag{B.15}
$$

The first part of the elasticity is due to the preference. The asset pricing preference implies that the log surplus consumption ratio enters the integral. The second part is due to the search model, because the transition matrix B_1 which determines the eigenvalues and the weights are determined by the search side of the model.

This completes the proof when $\Delta_e = \Delta_u$. While this is numerically accurate, I cannot prove that $\Delta_e = \Delta_u$. In the remaining of the proof, I show the elasticity when the equality is not assumed.

Since both B_1 and the matrix in equation (B.6) are non-diagonal, I need to first diagonalize the two matrices. B_1 is diagonalized in the above proof. The matrix in equation (B.6), after diagonalization, is

$$
B_2 = H \begin{bmatrix} \frac{1}{2}i\nu & 0 \\ 0 & -\frac{1}{2}i\nu \end{bmatrix} H^{-1} \tag{B.16}
$$

where

$$
\nu = \sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|\alpha\sigma_a^2\lambda(s_t)(\lambda(s_t)+1)
$$

is the eigenvalue for the matrix B_2. The dynamic system reduces to

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = \int_t^{+\infty} \Big[(c_{l,1}+ic_{l,2})e^{(\gamma_l+\frac{1}{2}i\nu)(u-t)} + (c_{l,1}-ic_{l,2})e^{(\gamma_l-\frac{1}{2}i\nu)(u-t)} + (c_{h,1}+ic_{h,2})e^{(\gamma_h+\frac{1}{2}i\nu)(u-t)} + (c_{h,1}-ic_{h,2})e^{(\gamma_h-\frac{1}{2}i\nu)(u-t)}\Big]\mathbb{E}_t\left(\frac{Q_u}{Q_t}\right)dt \tag{B.17}$$

where

$$\begin{aligned}
c_{l,1} =& \frac{(\gamma_l+\phi_u-g_u)[\eta\lambda_w - b(\gamma_h+\phi_u+\eta\lambda_w-g_u)]}{2\eta\lambda_w(\gamma_l-\gamma_h)} \\
c_{l,2} =& \frac{(\gamma_l+\phi_u-g_u)(\gamma_h+\phi_u+\eta\lambda_w-g_u)}{2\eta\lambda_w(\gamma_l-\gamma_h)}\frac{\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}{\delta\varepsilon_u} + \frac{b\delta\varepsilon_u(\gamma_l+\phi_u-g_u)}{2(\gamma_l-\gamma_h)\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|} \\
c_{h,1} =& \frac{(\gamma_h+\phi_u-g_u)[b(\gamma_l+\phi_u+\eta\lambda_w-g_u)-\eta\lambda_w]}{2\eta\lambda_w(\gamma_l-\gamma_h)} \\
c_{h,2} =& -\frac{(\gamma_l+\phi_u+\eta\lambda_w-g_u)(\gamma_h+\phi_u-g_u)}{2\eta\lambda_w(\gamma_l-\gamma_h)}\frac{\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}{\delta\varepsilon_u} - \frac{b\delta\varepsilon_u(\gamma_h+\phi_u-g_u)}{2(\gamma_l-\gamma_h)\sqrt{\delta\eta\lambda_w}|\varepsilon_e-\varepsilon_u|}
\end{aligned}$$

Equation (B.17) shows that the normalized surplus $(\mu_{et}-\mu_{ut})/A_t$ can be solved by a Fourier transform of the exponential of the eigenvalues of B_1. Solving the Fourier transform, the normalized surplus is

$$\frac{\mu_{et}-\mu_{ut}}{A_t} = e^{\Theta_1}\left\{\frac{\left[-c_{l,1}(\gamma_l+\Delta_e+a_Q+\Theta_2)-c_{l,2}\nu/2\right]}{(\gamma_l+\Delta_e+a_Q+\Theta_2)^2+\nu^2/4} + \frac{\left[-c_{h,1}(\gamma_h+\Delta_u+a_Q+\Theta_2)-c_{h,2}\nu/2\right]}{(\gamma_h+\Delta_u+a_Q+\Theta_2)^2+\nu^2/4}\right\} \tag{B.18}$$

Taking the derivative, one gets an expression that is similar to equation (B.14) which is omitted. □

B.3 Proof of Proposition 2

Proof. The normalized match surplus, $\mu_t = (\mu_{et}-\mu_{ut})/A_t$, is given in the proof of proposition 1, namely section B.2. Section B.2 implies that the normalized match surplus satisfies

$$\mu_t - \overline{\mu} = \left[c_l e^{(\gamma_l + a_Q)t} + c_h e^{(\gamma_h + a_Q)t}\right] \mathbb{E}_0 \left(e^{b_Q \int_0^t (s_\tau - s) d\tau}\right) (\mu_0 - \overline{\mu}) \tag{B.19}$$

where μ_0 is the initial value of the normalized match surplus. The expectation on the right-hand side is given in the proof of proposition 1. Plugging in the expression for the expectation, the normalized match surplus is

$$\mu_t - \overline{\mu} = \Omega_t \left(c_l e^{\gamma_l t} + c_h e^{\gamma_h t}\right)(\mu_0 - \overline{\mu}) \tag{B.20}$$

where

$$\Omega_t = e^{\Theta_1 - \Theta_3 e^{-(1-\rho_s)t} + (a_Q + \Theta_2)t} \tag{B.21}$$

Ω_t does not depend on human capital growth or labor market exit rate. By definition, the half-life τ is implicitly given by the equation

$$\Omega_\tau (c_l e^{\gamma_l \tau} + c_h e^{\gamma_h \tau}) = \frac{1}{2} \tag{B.22}$$

Taking the derivative of τ with respect to c_l, noting that Ω_τ does not vary with c_l, we can show that the derivative is positive. Similarly, the derivative of τ with respect to c_h is negative.

Note that the partial derivative $\partial c_l / \partial \gamma_l < 0$ from Equation (B.8) and Equation (B.9). This means that c_l cannot vary independently from γ_l. When γ_l increases and approaches 0, c_l decreases, so that the long-run weight becomes less important. □

C Job-Finding Rate Convergence under Wage Rigidity

I show numerically that in a variant of the model in Section 4 with human capital distribution and wage rigidity, the job-finding rate co-moves with the aggregate shock.

With wage rigidity, there is no longer perfect risk-sharing between workers and firms. To simplify the analysis and focus on the converge of job-finding rate, I assume that workers and firms are risk-neutral with objective discount rate β. The model hence no longer features asset pricing preference, which is shown to be unrelated to the job-finding rate convergence. I also assume that the aggregate shock $\{A_t\}$ follows:

$$dA_t = \sigma_a dW_{a,t}$$

The unemployment benefit and the vacancy cost are not scaled by the shock. Note that scaling does not affect the convergence.

Let $V_t(z)$ denote the value of employed workers. Let $U_t(z)$ denote the value of unemployed workers. $J_t(z)$ is the value of firms. The system of equations that characterize the economy is:

$$(\delta + \phi_e + \beta - g_e)V_t(z) = w_t(z) + \delta U_t(z) + \mathbb{E}_t[dV_t(z)] \tag{C.1}$$

$$(\lambda_{wt}(z) + \phi_u + \beta - g_u)U_t(z) = bf(z) + \lambda_{wt}(z)V_t(z) + \mathbb{E}_t[dU_t(z)] \tag{C.2}$$

$$(\delta + \phi_e + \beta)J_t(z) = e^{A_t} f(z) - w_t(z) + \mathbb{E}_t[dJ_t(z)] \tag{C.3}$$

$$\kappa f(z) = (1-\eta)\lambda_{vt}(z)J_t(z) \tag{C.4}$$

$$w_t(z) = \max\left\{\underset{w}{\operatorname{argmax}}\, [V_t(z) - U_t(z)]^{\eta}[J_t(z)]^{1-\eta}, \chi w_{t-1}(z)\right\} \tag{C.5}$$

$$\frac{\partial e_t(z)}{\partial t} = -\frac{\partial e_t(z)}{\partial z} g_e z - (g_e + \delta + \phi_e)e_t(z) + \lambda_{wt}(z)u_t(z) \tag{C.6}$$

$$\frac{\partial u_t(z)}{\partial t} = -\frac{\partial u_t(z)}{\partial z} g_u z - (g_u + \lambda_{wt}(z) + \phi_u)u_t(z) + \delta e_t(z) + \zeta n_t(z) \tag{C.7}$$

The first three equations are the value functions of employed workers, unemployed workers, and firms. The fourth equation is the free-entry condition. The fifth equation is the wage determination equation. In particular, the wage is determined by Nash bargaining, together with a wage rigidity condition that the wage cannot be lower than χ fraction of the last period's wage. The last two equations are the evolution of the employed and the unemployed human capital distribution.

I set the production technology to be $f(z) = z$. I impose a one-time negative shock that decreases the job-finding rate by 5%. The half-life of the shock is 3 months. With the example, Figure 3 plots the job-finding rate and the aggregate shock paths. The co-movement is similar to the ones in Figure 2. The intuition is stated in Section 4: the wage rigidity does not change the model transmission. The job-finding rate converges fast understand parameterization, and the only time-varying component is the aggregate shock near the steady state.

Figure 3: Job-Finding Rate Convergence