Notes for MIT 18.03: Differential Equations

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1 Recitation 1 & Lecture 1: Direction Fields, Existence & Uniqueness of Solutions

1.1 Readings

Edwards & Penney, Chapters 1.1, 1.2, 1.3, and 1.4.

1.2 Lecture & Reading Notes

1.2.1 Differential Equations & Separability

In the the first part of the course, we will only study ODEs involving the first derivative:

$$y' = F(x, y) \tag{1}$$

Example:

$$y' = 2x, \tag{2}$$

which has solution $x^2 + C$. Such a solution is called the **general solution**. Another example *(memorize this!)*:

$$y' = kx \tag{3}$$

With general solution $y = Ce^{kx}$. This will be a central example in the course; in fact, a reasonable defintion of the function e^x is that it is the solution to the differential equation $y' = y$ such that $y(1) = 0$.

Separation of variables will also be important. The protocol is:

1. Pull all variables to either side.
2. Integrate.
3. Amalgamate all constants and solve (if possible) for y in terms of x.

Most real-life equations aren't solvable, though. So, we often resort to graphical/numerical methods.

1.2.2 Graphical Methods

The differential equation $y' = F(x, y)$ defines a slope at every point in the plane. This is a **direction field** or **slope field**.

A **solution** of a differential equation is a function whose graph has the given slope at every point it goes through.

To specify a particular solution of an ODE you must give an **initial condition**: when x takes on a certain value, y takes on a specified value.

Theorem 1.2.1: Existence and Uniqueness Theorem for ODEs

Suppose that both the function $f(x, y)$ and $D_y f(x, y)$ are continuous on some rectangle R in the xy-plane containing the point (a, b) in its interior. Then for some open interval I containing the point a, the IVP

$$\frac{dy}{dx} = f(x, y), \quad y(a) = b \tag{4}$$

has one and only one solution defined on the interval I.

Direction fields are used to visualize the qualitative behavior of solutions to differential equations, and this is what we often want to know.

2 Lecture 2: Numerical Methods

2.1 Readings

Edwards & Penney 6.1, 6.2.

2.2 Lecture & Reading Notes

The study of differential equations has three parts.

1. Analytic, exact, symbolic methods.
2. Quantitative methods (direction fields, isoclines, etc).
3. Numerical methods.

We focus on numerical methods today.

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