Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
ABCDEF		2410605

Summary Sheet

Contents

1 Introduction

1.1 Problem Background

Modern people's understanding of the ocean, especially the deep sea, is far less than that of the land. Deep-sea exploration is to comprehensively study the mysteries of the ocean and the earth, exploring the natural conditions of the deep ocean, such as the appearance of the seabed, ocean currents, as well as the biological and economic resources contained in the seabed. The deep-sea space has complex and special environmental characteristics, its sea surface Marine meteorology and sea water movement are changeable, and the sea bottom has no light, high pressure, low temperature and no oxygen. The severe Marine environment, equipment failure, human factors and other factors make the deep sea major sudden safety accidents hover at a high level for a long time. In order to reduce the loss of deep-sea accident and find out the cause of the accident, it is necessary to carry out rescue and search and salvage the accident equipment at the first time.

1.2 Restatement of the Problem

According to the requirements of MCMS, we are supposed to support their submersible safety system in the following aspects.

- Develop a model to predict the position of the submersible over time. Through the analysis of uncertain factors, consider the auxiliary positioning information and the corresponding acquisition equipment.
- Under the premise of considering economy and practicality, adding additional search equipment to the main vessel and the rescue vessel.
- By using the information in the positioning model, recommend the initial deployment point and search mode of the equipment in order to minimize the search time, and determine the probability of finding the submersible based on the time and cumulative search results.
- Extend the model to different marine environment and the environment with identified disturbances.

1.3 Our work

2 Assumptions and Justification

3 Notations

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4 Model I: Submersible Location Prediction Model

4.1 Submersible configuration

In order to simplify the model, through data search and comparison, we set the submersible as a capsule-like shape, and the specific structure and size are shown in the figure below (in meters).

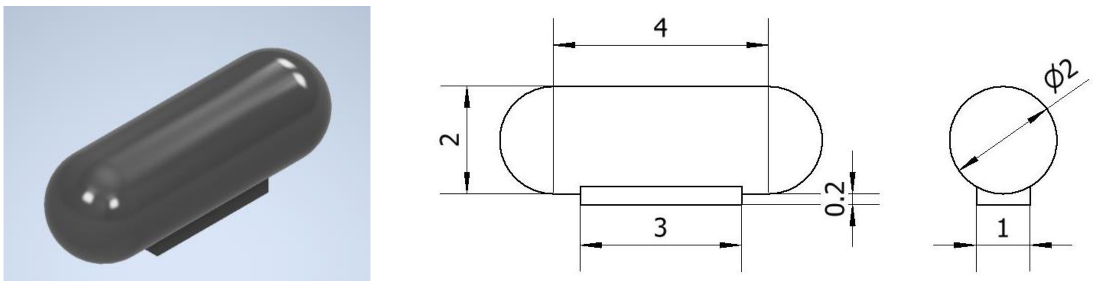


Figure 2 Schematic Diagram of Submarine Model

We can see that the structure of the submersible consists of two parts, the main body of the capsule, and a piece of ballast iron suspended below the body. 压载铁的作用 On this basis, we make further assumptions as follows

Table 2 Detailed Parameters of Submarine

Length	Width	Height	Full load displacement	Empty Weight	Water Storage Place
6m	2m	2.2m	16.7552m^3	$1.0\times10^4 kg$	$6m^3$

As a result, the total weight of the submersible m can be expressed as follows：

$$m = m_{empty} + m_{water} + m_{iron} + m_{others}$$

Where m_{empty} is the weight of the empty ship, m_{iron} is the weight of the ballast iron, m_{others} is the weight of other weights such as personnel and equipment, and m_{water} is the weight of water contained in the water storage tank.

4.2 State of the Ionian Sea

The data we use to describe the state of Ionian Sea include historical ocean currents data, density of sea water and the depth of Ionian Sea. The data sources are summarized in Table 3.

Table 3 Data Source Collocation

Database Names	Database Websites	Data Type
HYCOM	https://www.hycom.org/dataserver/gofs-3pt1/analysis	Currents
HURRICAN	https://hurricanescience.org/science/basic/water/index.html	Density
GBECO	https://www.gebco.net/data_and_products	Depth

4.2.1 Currents

Ocean current is a force that cannot be ignored in the ocean, which refers to the regular horizontal flow of sea water in a certain direction at a relatively stable speed, and is the main form of sea water movement. There are three main influencing factors, namely wind, density and compensation. Given the operating area of the submersible, wind and compensation effects have less effect on ocean currents, and density differences in layers of similar or the same depth are not enough to have large effects. Therefore, we believe that the current data at a certain point tend to be stable as a whole and do not affect the change of seasons over time. On the basis of the above cognition, we obtained the ocean current data at the depth of 4000 meters in the Ionian Sea, and plotted the flow field and velocity characteristic pattern of the ocean current at this depth.

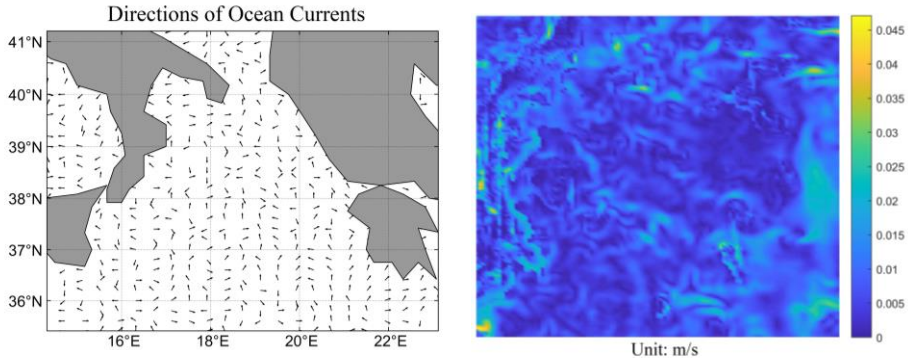


Figure 3 Direction and Value of Ocean Currents in Ionian Sea at $H_0 = 4000m$

It can be analyzed from Figure 3 that the ocean current field in the Ionian Sea is basically disordered in direction, and the remote region still maintains a certain degree of overall direction, while the direction of the ocean current around the land plate is relatively chaotic and does not have overall directivity. And also, it can be seen that the ocean current velocity is basically constant at 0.008m/s at a depth of 4000m.

Considering that the search location is in the deep sea, since the ocean current velocity varies with depth and the degree of change is large, the surface current velocity may reach several meters per second, and the deep-sea current velocity is only a few centimeters per second, we use an exponential function to describe the change of ocean current velocity with depth:

$$v_{sea}(x,y,h(t)) = v(x,y,H_0)e^{\frac{k(H_0-h(t))}{H_0}}$$

where $H_0 = 4000m$, $h(t)$ is the current depth, $v(x,y,H_0)$ is the ocean current velocity data of 3000m depth in the Ionian Sea obtained above, and the constant k can be determined by the sea level current velocity (that is, when $h(t) = 0$).

Since ocean currents are not completely invariable, for the sake of accurate modeling, we determine a velocity magnitude deviation σ in the range of $(-0.3, 0.3)$，so the equation can be further refined as

$$v_{sea}(x,y,h(t)) = (1+\sigma)v_{sea}(x,y,H_0)e^{\frac{k(H_0-h(t))}{H_0}}$$

Assuming that the Angle between the ocean current direction and the Y-axis is α, and considering the directional deviation θ of magnitude （$-20°$，$20°$）, the value range of the ocean current direction can be expressed as

$$(\sin(\alpha+\theta), \cos(\alpha+\theta))$$

4.2.2 Density of Sea Water

Through the International one-atmosphere equation of state of seawater (Frank J. Millero, Alain Poisson, 1981, Oceanographic Research Papers, 625-629), we can determine the density of seawater at the target location, and the equation form is

$$\rho = \rho_0 + As + Bs^{\frac{3}{2}} + Cs^2$$

where s is the salinity of seawater ρ_0 is the density of water, and A, B and C are all functions of temperature T.

- Temperature

Temperature is also an important consideration in the state of the deep-sea environment, and by finding and fitting the data, it is possible to plot the temperature with the depth of the water, which is also known as thermocline. It can be clearly seen from Figure 4 that the initial temperature decreases significantly with the increase of depth, and then turns to a steady and slow decrease and continues after it drops to 5℃.

- Salinity

Similar to temperature, we perform a similar analysis on seawater salinity to find data for a dataset where a graph of changes can be plotted, which is also known as Halocline. It can be analyzed from the figure that the salinity initially decreases significantly with increasing depth, but after reaching a critical value of about 34.2%, it continues to rise at a lower rate of change than before.

Since salinity and temperature are both known, the corresponding density values can be obtained by calculation, and the data can be fitted to plot the density of seawater with depth.

According to the analysis of the curve trend in the above figure, except for a small decrease near the sea surface, the density increases as a whole with the increase of depth, and the increase rate is large at the shallow layer, and the rate slows down at about 2000 meters.

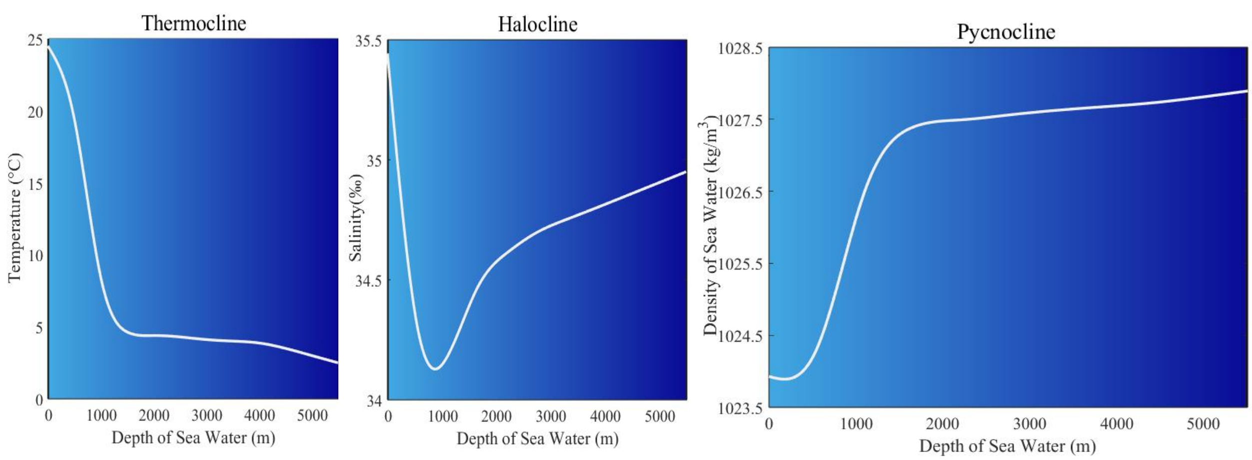


Figure 4 Changes in Seawater Temperature, Salinity, and Density with Depth

4.2.3 Geography of the Sea Floor

The shape of the sea floor is also an important consideration, with fluctuations in the shape of the sea floor determining that the deepest depth can vary from region to region, which in turn can produce different Marine environments. Based on the search data fitting, we have mapped the sea floor characteristic pattern of the Ionian Sea in Figure 5.

Through the analysis of the above figure, it can be seen that the overall depth of the Ionian Sea is basically more than 3000m, and the deepest depth can reach about 5000m. On the whole, compared with other regions, the eastern south region of the Ionian Sea has a deeper seabed.

海水深度可以表示为

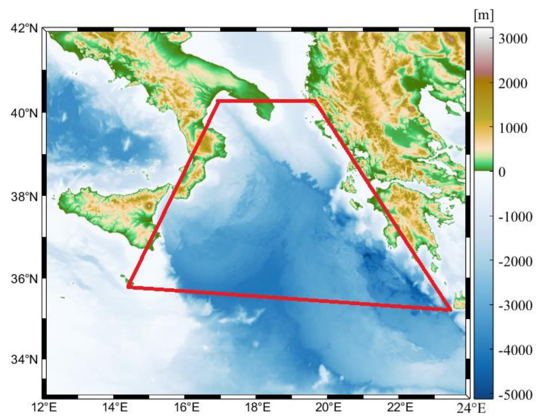


Figure 5 Topographic Map of Ionian Sea

4.3 Dynamic analysis of submersibles

动力学分析
第一步说明失事时位置（x,y,h），说明失事时速度 0，0，0，船头朝北
受力分析 三维+2 个 2 维 一个竖直方向、一个水平方向（俯视）
竖直方向上：一个重力、一个浮力、阻力和运动方向相反，开始的时候重力大于浮力
水平方向上：阻力

最后说明：重量、洋流方向和大小的随机性都会对潜水器的位置造成影响
Weight

$$G = mg$$

Floatage

$$F = \rho(h) g V_{sub}$$

FrictionC=0.03 类比鱼

$$f = \frac{1}{2} C \rho S v^2$$

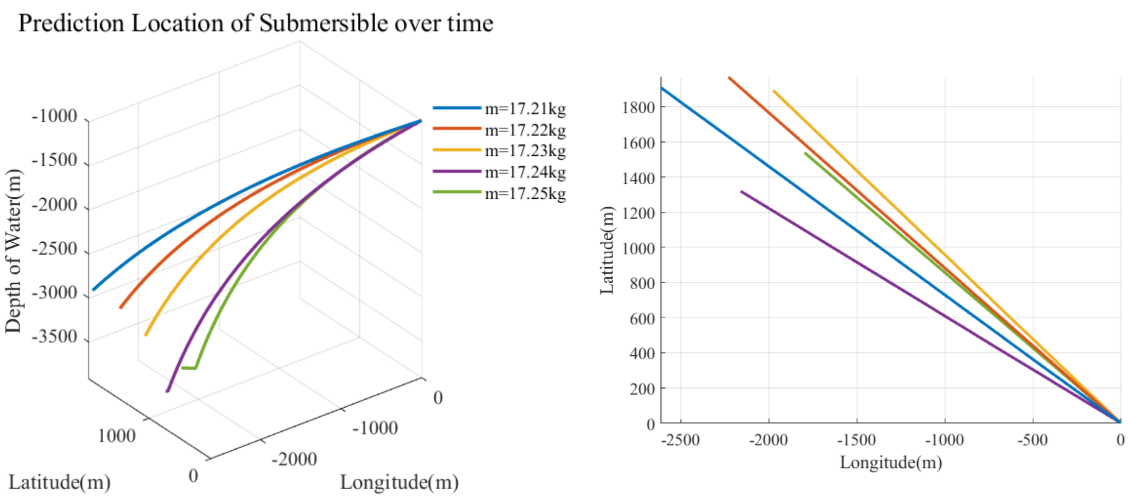
$$mg - \rho(h) g V_{sub} - \frac{1}{2}\rho(h) C_{sub} S \left(\frac{\mathrm{d}h}{\mathrm{d}t}\right)^2 = \frac{\mathrm{d}^2 h}{\mathrm{d}t^2}$$

$$\frac{1}{2} C \rho(h) S_x \left[v_{sea}(x,y,h(t)) \sin(\alpha_{x,y} + \theta_{x,y}) - \frac{\mathrm{d}x}{\mathrm{d}t}\right]^2 = m \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

$$\frac{1}{2} C \rho(h) S_y \left[v_{sea}(x,y,h(t)) \cos(\alpha_{x,y} + \theta_{x,y}) - \frac{\mathrm{d}y}{\mathrm{d}t}\right]^2 = m \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

$$h(t) \leqslant H_{sea}(x,y)$$

$$
\begin{cases}
m\dfrac{\mathrm{d}^2 h}{\mathrm{d}t^2} = mg - \rho(h)gV_{sub} - \dfrac{1}{2}\rho(h)C_{sub}S_z\left(\dfrac{\mathrm{d}h}{\mathrm{d}t}\right)^2 \\
m\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \dfrac{1}{2}\rho(h)C_{sub}S_x\left[v_{sea}(x,y,h(t))\sin(\alpha_{x,y}+\theta_{x,y}) - \dfrac{\mathrm{d}x}{\mathrm{d}t}\right]^2 \\
m\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \dfrac{1}{2}\rho(h)C_{sub}S_y\left[v_{sea}(x,y,h(t))\cos(\alpha_{x,y}+\theta_{x,y}) - \dfrac{\mathrm{d}y}{\mathrm{d}t}\right]^2 \\
h(t) \leqslant H_{sea}(x,y) \\
\sigma \in (-0.3, 0.3), \theta_{x,y} \in \left(-\dfrac{\pi}{9}, \dfrac{\pi}{9}\right)
\end{cases}
$$

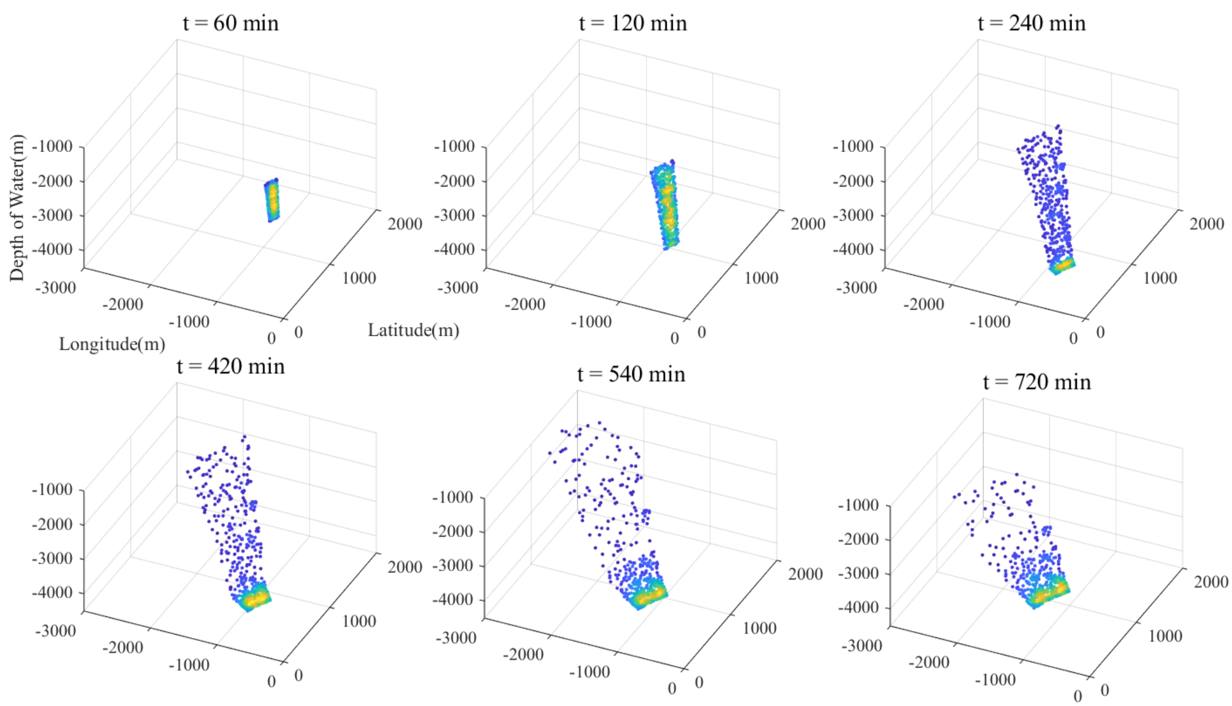


4.4 Model Evaluation of Uncertainty

蒙特卡洛方法

Uncertainties:

- Weight
- Current 大小&方向



4.5
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AUV 自主式：6000m
- 1 名 AUV 操作员：负责计划、执行和监控 AUV 任务。
- 1 名科学家或研究员：负责分析和解释从 AUV 传感器中获得的科学数据。
- 1 名技术支持人员：负责解决 AUV 系统问题，确保设备正常运行

占用体积：3m^3, 质量 2000kg
价格：800,000 美元

ROV 遥控式：4500m
- 2 名 ROV 操作员：负责 ROV 的操控和任务执行。
- 1 名 ROV 技术支持：负责 ROV 设备的维护和问题解决。
- 1 名船上技术支持：包括电气工程师和机械工程师，确保 ROV 设备和系统正常运行。
- 1 名科学家或研究员：负责分析 ROV 收集的数据。

体积：2m^3,质量 1300kg
价格：400,000 美元

声呐

实时操控：ROV>AUV 说明越到后面，越需要实时操纵，主船<救援船，重量权重：主船<救援船

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Reference

密度[1] Frank J. Millero, Alain Poisson, International one-atmosphere equation of state of seawater, Deep Sea Research Part A. Oceanographic Research Papers, Volume 28, Issue 6, 1981, Pages 625-629
[2]