Mathematical Analysis of a Simple Pendulum's Motion

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I. INTRODUCTION

Discovered by Galileo, the pendulum has served as the backbone to many scientific discoveries. Notably, Jean Foucault, a 19th century scientist, used the pendulum to prove that the earth rotates on its axis. In addition, Christain Huygens, a Dutch scientist, added pendulums to clocks back in the year 1660.

The purpose of this lab report was to determine if the motion of a pendulum followed the damped harmonic oscillator model. Various tools, such as video analysis software, were used to quantitatively analyze and derive data pertaining to the motion of a pendulum. The pendulum's symmetrical motion, the period's dependence on the amplitude, the Q factor, the period versus pendulum length, and the Q factor's dependence on length were analyzed. In addition, an appropriate initial release angle for the period Versus pendulum length and Q factor versus pendulum length experiments had to be determined.

I.A. Pendulum's Symmetrical Motion

The pendulum's period versus angle relationship can modeled via the following relationship:

$$T = T_o(1 + B\theta + C\theta^2) \tag{1}$$

in which, from a theoretical perspective, the B value should be consistent with zero, which would indicate that the pendulum is exhibiting symmetrical motion. This exhibition of symmetrical motion would follow the damped harmonic oscillator model. The obtained B value satisfied this symmetrical motion as B was 0.001200±0.000006, which was consistent with zero.

From a theoretical perspective, the pendulum's period can be derived using the following equation:

$$T \approx 2\sqrt{L} \tag{2}$$

This equation can be used to assess the accuracy of the previous equation. In this case, the theoretical and experimental values for the period were similar as the theoretical value and experimental value for T_o were $T_o = 0.72366$ seconds and $T_o = 1.079 \pm 0.005$ seconds, respectively.

I.B. Period's Dependence on the Amplitude

The pendulum's period versus angle relationship can modeled via the following equation:

$$T = T_o(1 + B\theta + C\theta^2) \tag{3}$$

in which, from a theoretical perspective, the C value should be consistent with zero, which would indicate that the period is independent of the amplitude. This independence between the period and the amplitude would follow the damped harmonic oscillator model. The obtained C value indicated that the period is independent of the amplitude as C was 0.1397±0.0006, which was consistent with zero.

I.C. Q Factor

The pendulum's Q factor can be found using the following equation:

$$Q = \pi \times \frac{\tau}{T}[4] \tag{4}$$

in which the Q factor's tau and period values were derived using Python. In this case, the obtained Q value was $Q_1 = 47 \pm 2$.

In addition, the pendulum's Q factor can also be found by counting the number of oscillations until the amplitude reached its 35 percent equivalent, then multiplying that number by 3. In this case, the obtained Q value was $Q_2 = 150 \pm 1$.

The difference between the two aforementioned Q factors can be determined using the following equation:

$$\frac{|Q_1 - Q_2|}{\frac{Q_1+Q_2}{2}} \times 100 \tag{5}$$

In this case, the percentage difference between the two Q values was 103±4%.

I.D. Initial Release Angle for Period Versus Pendulum Length and Q Factor Versus Pendulum Length Experiments

A range of initial angles in which the variation in period is indistinguishable from experimental error can be determined using the following equation:

$$T(\theta) - T_o < u_T \tag{6}$$

In this case, using the data obtained from the Angle vs. Period experiment, it was found that a range of initial release angles from 0 to 0.1 ±0.009 radians satisfied this requirement.

I.E. Period versus Pendulum Length

For small amplitudes of oscillation, the pendulum's period versus length relationship can modeled via the following relationship:

$$T = kL^n \tag{7}$$

The obtained k and n values were k = 1.8±0.2 $\frac{seconds}{m^n}$ and n = 0.44±0.05.

From a theoretical perspective, the obtained experimental values for k and n should resemble the theoretical values in the following equation:

$$T \approx 2\sqrt{L} \tag{8}$$

in which k = 2 $\frac{seconds}{m^n}$ and n = 0.5. If the experimental and theoretical values are similar to each other, then it can be claimed that the period of the pendulum does increase when the length of the pendulum is increased. This simultaneous increase follows the damped harmonic oscillator model. This was the case as the percentage difference between the theoretical values for k and n were 13±11% and 11±11%, respectively, which were consistent with zero.

The difference between the theoretical and experimental values was quantified using the following equation:

$$\frac{|Value_1 - Value_2|}{\frac{Value_1+Value_2}{2}} \times 100 \tag{9}$$

The pendulum's period versus length relationship can also modeled via the following relationship:

$$log(T) = n(log(L)) + log(k) \tag{10}$$

I.F. Q Factor's Dependence on Length

For small amplitudes of oscillation, the pendulum's Q factor versus length relationship can modeled via the following relationship:

$$Q = Q_o(1 + BL + CL^2) \tag{11}$$

in which, from a theoretical perspective, the C value should not be consistent with zero, which would indicate that the Q factor increases when the length of the pendulum increases. This simultaneous increase would follow the damped harmonic oscillator model. This was the case as the C value was 3700±90 m^{-2}, which was not consistent with zero.

II. METHODS AND PROCEDURES

My pendulum setup consisted of a cotton twine string with an estimated mass of 0.1 ±0.01 grams attached to a bob with a mass of 22.50±0.01 grams. The aforementioned string was also tied to a structure consisting of 1.91±0.05 centimetres x 25.40±0.05 centimetres polyvinyl chloride pressure pipes connected together using two 1.27±0.05 centimetre wide chlorinated polyvinyl chloride tee pipe fittings, five 1.27±0.05 centimetre wide chlorinated polyvinyl chloride 90-degree elbow pipe fittings, and one 1.91±0.05 centimetre wide chlorinated polyvinyl chloride 90-degree street elbow pipe fitting. The last mentioned pipe fitting required *Gorilla Duct Tape*[1] to connect it to the topmost and rightmost pipes.

In addition, the polyvinyl chloride pressure pipes that were in contact with the table were duct taped to the table to stabilize the base of the pendulum, which prevented the pendulum from oscillating in an elliptical motion. The aforementioned adjustment inherently reduced the occurrence of the collection of faulty data.

Furthermore, cotton twine was used as the material for my string as cotton twine was an affordable string that didn't deform when tied to the bob. The string was tied to the top pipe using a Taut-line hitch knot as the aforementioned knot rendered the string's length adjustable. Figure 1 depicts the entire setup. The bob was created using a 5-centimetre-long 1.91±0.05 centimetres x 25.40±0.05 centimetres polyvinyl chloride pressure pipe and nine 10-cent Canadian coins, which were bound to the polyvinyl chloride pressure pipe using *Gorilla Super Glue*[2].

From an intuitive perspective, increasing the weight of the bob by adhering the aforesaid coins to the bob increased the stress on the wire, which decreased the influence of air resistance on the bob's movement patterns. This adjustment prevented the pendulum from oscillating in an elliptical motion. Figure 2 depicts the bob and its respective measurements.

The data obtained from the Q factor experiment was processed using Python whilst the data obtained for the Period vs. Angle, the Q factor versus Pendulum Length, and Pendulum Length vs Period experiments were parsed using Excel. Finally, it is safe to assume that all of my measurements in centimetres were collected using a ruler with a ±0.05 centimetre uncertainty and all of my measurements in radians were collected using a protractor with a ±0.009 radian uncertainty.

Figure 1. A bob was attached to the topmost polyvinyl chloride pipe using cotton twine. The overall structure of the pendulum was made of polyvinyl chloride pressure pipes as they are very durable, tough, resistant to cracking, and affordable. Duct tape was used to stabilize the base of the pendulum and attach the 1.91±0.05 centimetre wide chlorinated polyvinyl chloride 90-degree street elbow pipe fitting to the topmost and rightmost pipes.

Figure 2. A bob was made using a 5-centimetre-long 1.91±0.05 centimetre x 25.40±0.05 centimetre polyvinyl chloride pressure pipe and nine 10-cent Canadian coins, which were bound to the polyvinyl chloride pressure pipe using *Gorilla Super Glue*[2]. It is worth noting that cotton twine was strung around the polyvinyl chloride pressure pipe before gluing the nine 10-cent Canadian coins to the aforementioned pipe.

II.A. Period vs. Angle

For the Period vs. Angle experiment, the bob was released at various angles ranging from $+1.571 \pm 0.009$ radians to -1.571 ± 0.009 radians in increments of 0.175±0.009 radians where the string was held at its maximum length, which was 8.50±0.05 centimetres, with an estimated center of mass 13.00 ± 0.05 centimetres away from the string. A protractor was used for reference in the process of measuring each angle. Each angle's respective period was calculated by manually timing how long the pendulum took to complete a full oscillation starting and ending from when it was at the top of its swing. The time of the aforementioned oscillations was quantified using a stopwatch in which my reaction time for using the stopwatch was ±0.05 seconds. This data collection method was used as it was intuitive, simple, and efficient. In addition, this method required a negligible amount of planning and configuration.

II.B. Q Factor

For the Q factor experiment, the bob was released at an angle of -1.571 ± 0.009 radians where the string was held at its maximum length, which was 8.50 ± 0.05 centimetres, with an estimated center of mass 13.00 ± 0.05 centimetres away from the string. After its release, the bob's trajectory was uninterrupted until the bob stopped moving. An iPhone 12's front camera set to "slow-mo" mode was used to capture a slow-motion video of the bob's movement at 240 frames per second. During the recording process, the camera was set 30.00 ± 0.05 centimetres away from the bob whilst being held in a nearly perfect horizontal manner to capture the pendulum's entire motion. The camera's position was nearly unchanged throughout the recording process.

Furthermore, the angle of the bob was obtained using *Tracker*'s *Autotracker* [3] feature, which is a free video analysis and modelling tool built on the Open Source Physics Java framework [3]. This data collection method was used as it offered the most accurate, precise, consistent, and comprehensive set of data regarding the bob's angle over time.

In addition, *Tracker's Autotracker* [3] was time efficient in relation to other data collection processes as the program automatically tracked and plotted the bob's angle without requiring excessive human intervention [3]. Furthermore, the iPhone 12 was chosen as my video capturing tool as it was the most accessible and reliable video capturing tool at my disposal due to its 240 frames per second video capturing ability and not necessitating any financial expense. Finally, the string's length was set to $8.50 \pm 0.05cm$ to reduce the chances of the occurrence of destabilization from the pendulum's base, which would have resulted in elliptical motion instead of planar motion.

II.C. Pendulum Length versus Period

The pendulum was released at an angle of 0.1 ±0.009 radians with the string being held at the following lengths from the center of mass: 0.05, 0.10, 0.15, 0.20, 0.25, 0.30 ±0.005 meters. The reasoning behind the initial release angle shall be discussed later in this report. A protractor with an uncertainty of ±0.009 radians was used for reference in the process of measuring the initial release angle.

Each length's respective period was calculated by manually timing how long the pendulum took to complete a full oscillation starting and ending from when it was at the top of its swing. The time of the aforementioned oscillations was quantified using a stopwatch in which my reaction time for using stopwatch was ±0.05 seconds. This data collection method was used as it was intuitive, simple, and efficient. In addition, this method required a negligible amount of planning and configuration.

II.D. Q Factor versus Pendulum Length

For the Period versus Pendulum Length experiment, the following lengths from the center of mass: 0.05, 0.10, 0.15, 0.20, 0.25, 0.30 ±0.05 meters were used. An iPhone 12's front camera set to "slow-mo" mode was used to capture a slow-motion video of the bob's movement at 240 frames per second. During the recording process, the camera was set 30.00 ± 0.05 centimetres away from the bob whilst being held in a nearly perfect horizontal manner to capture the pendulum's entire motion. The camera's position was nearly unchanged throughout the recording process.

Furthermore, the pendulum's length was modified according to the aforementioned list of lengths from the center of mass and was released at an initial angle of 0.1 ±0.009 radians. The plot of the angle of the bob over time was obtained using *Tracker′s Autotracker* [3] feature. Then, the respective angle versus time graphs for each length were used to calculate the Q factor for each length. This method was used as the combination of the 240fps slow motion camera and *Autotracker* offered the most consistent, efficient, precise, and comprehensive way to collect data.

III. RESULTS AND ANALYSIS

III.A. Pendulum's Symmetrical Motion (Period vs. Angle)

Plotting the data from the Angle vs. Period experiment, the following graph in figure 3 was obtained:

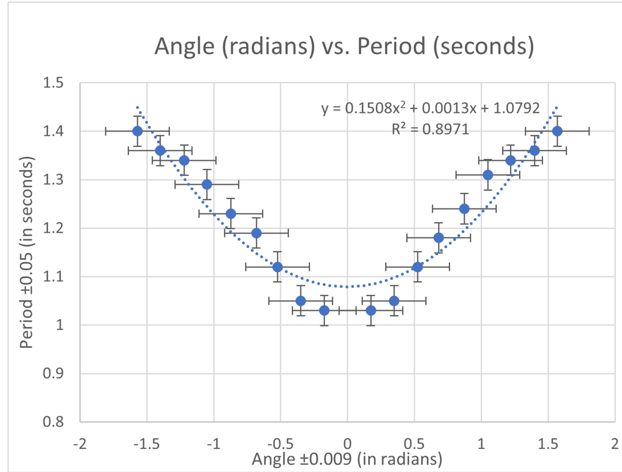

Figure 3. The angle vs. period graph heavily resembles a quadratic function. In addition, the angles range from positive values to negative values to test for asymmetry.

The trendline for the graph in figure 3 seems to suggest the following function,

$$T = T_o(1 + B\theta + C\theta^2) \tag{12}$$

Using Python, the following values were found:

$$T_o = 1.079 \pm 0.005 \text{ seconds}$$
$$\text{B} = 0.001200 \pm 0.000006$$
$$\text{C} = 0.1397 \pm 0.0006$$

In this case, the aforementioned function served as a predictive model for the pendulum's behaviour. It is worth noting that the points obtained in the graph in figure 3 were derived by releasing the pendulum from arbitrarily chosen positive and negative initial angles. This process allowed for testing for asymmetry within the pendulum. In addition, the aforementioned experimental values were close relatively to the theoretical values for T_o, B, C. It is worth noting that the aforementioned estimated center of mass was used as the length in the process of calculating the theoretical values for T_o, B, and C. These theoretical values were:

$$T_o = 0.72366 \text{ seconds}$$
$$\text{B} = 0$$
$$\text{C} = 0.045229$$

The previous statement implies that our values are accurate, thus the experimental values can be relied upon. The theoretical values do not have uncertainty values as they were obtained in a theoretical manner. The discrepancy between the theoretical and experimental values may have originated from a faulty experimental setup. More specifically, the delay between when the bob reached the top of its swing and when the stopwatch was stopped may have negatively affected the accuracy of the experimental data. The theoretical values were derived using the following equation:

$$T \approx 2\sqrt{L} \tag{13}$$

As the value for B is two times larger than its uncertainty, it can be claimed that its value is consistent with zero. This revelation means that our pendulum is exhibiting extremely symmetrical motion. Thus, it can be claimed that the pendulum was symmetrical, which means that it didn't require any additional modifications to remove asymmetry.

III.B. Period's Dependence on the Amplitude

Using the data from the Angle vs. Period experiment, the Python program determined that my fit parameter C was:

$$C = 0.1397 \pm 0.0006$$

for the following function:

$$T = T_o(1 + B\theta + C\theta^2) \tag{14}$$

In this case, the fit parameter C is representative of the period square dependence of the angle. Assuming the value B is zero, if the value C is a nonzero value, then the function for the period is:

$$T = T_o + C\theta^2 \tag{15}$$

Therefore indicating that the period is dependent upon the angle. If the value C is a zero value, then the function for the period:

$$T = T_o \tag{16}$$

Our value C is a nonzero value so there exists a dependence between the period and the angle as it is greater than twice its uncertainty. The criterion used to determine the period's dependence with relation to the amplitude was if the fit parameter C was a nonzero value or not. This means that the period is dependent on the amplitude as the fit parameter C is a nonzero value.

III.C. Q Factor

In this case, the first 4558 points of data were used to plot the graph in figure 4 as it would have been highly inefficient to plot all 16 000 points of data.

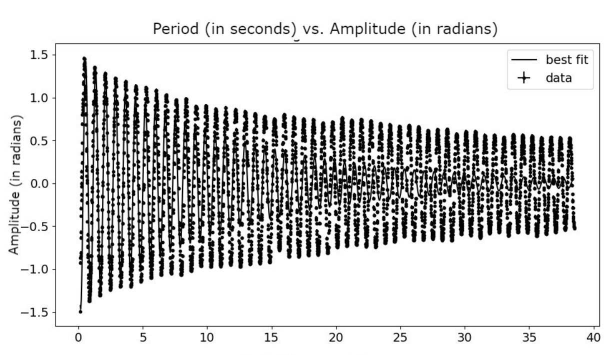

Figure 4. This graph is representative of the first 4558 points of data collected using the experimental method described in the Q Factor section. This graph's peaks resemble the behaviour of a decaying exponential function. This graph does not have error bars as the Python code that was used to plot it didn't have that feature.

In addition, these first points of data were sufficient as they were representative of a 35 percent decrease in amplitude, which will be used in our future calculations for the Q factor.

The Q factor is a parameter that describes how fast an oscillator's amplitude decays over time. A higher Q factor indicates a lower rate of energy loss as the oscillations die out more slowly. A lower Q factor indicates a higher rate of energy loss as the oscillations die out in a quicker manner. The direction of the Q factor was in the direction of the plane of the photograph.

In this experiment, Q was calculated using two different methods. The first method involved using the following formula:

$$Q = \pi \times \frac{\tau}{T}[4] \tag{17}$$

The values for this formula were derived using Python. These values were:

$\tau = 11.6 \pm 0.4$ seconds

T = 0.7734 ± 0.0003 seconds

Using the aforementioned equation and values for τ and T, the value of Q was found to be 47$\pm$2. The uncertainty value for Q was calculated in the following manner, where max()'s output is the highest value between the two inputted values:

$$uncertainty_Q = Q \times max\left(\frac{uncertainty_{Tau}}{\tau}, \frac{uncertainty_T}{T}\right) \tag{18}$$

The second method revolved around counting the number of oscillations until the amplitude reached its 35 percent equivalent, then multiplying that number by 3. A Q value of 150 was obtained using the second method. The graph reaches a 35 percent decrease in its amplitude at the end. This Q value does have uncertainty as it is uncertain when we have reached the peak of the cycle due to the uncertainty related to the measurement of the angle, which was $\pm$0.009 radians. The uncertainty for this method was determined to be $\pm$1 as there were two peaks that were within a range of $\pm$0.009 radians of the 35 percent equivalent of the amplitude. The difference between the two Q factors was significant. Using the following formula and my two Q values, where $Q_1 = 47 \pm 2$ and $Q_2 = 150 \pm 1$:

$$\frac{|Q_1 - Q_2|}{\frac{Q_1 + Q_2}{2}} \times 100 \tag{19}$$

The percentage difference between the two Q factors was 103±4%. This large percentage difference was caused by the Python program not being able to properly find the best fit curve for my data, which decreased the accuracy of the values for tau, and T. These values negatively affected the accuracy of Q as they laid the foundation for computing the Q value in the first approach. My reasoning is reflected in the best fit curve not being able to overlap the collected data. In addition, the uncertainties for both of my Q factors do not agree with each other, which was expected as one method revolved around a faulty fitting of the function and the other revolved around counting. Additionally, a faulty experimental design may have also increased the percentage difference between these two Q values.

In the future, I will have to optimize the Python program to find a better best-fit curve for the collected data and try to identify faults in my experimental design. Finally, the Q factor did have an impact on how I collected my data as my relatively low Q factor meant that the amplitude decayed faster, which means that there fewer points were plotted using the *Tracker* software [3].

III.D. Initial Release Angle for Period Versus Pendulum Length and Q Factor Versus Pendulum Length Experiments

For the rest of the lab report, a range of initial angles in which the variation in period is indistinguishable from experimental error had to be determined. The variation in the period had to be indistinguishable from experimental error, which means that the time variation had to be bigger than the variation in period:

$$T(\theta) - T_o < u_T \tag{20}$$

In this case, using the data obtained from the Angle vs. Period experiment, it was found that a range of initial release angles from 0 to 0.1 ±0.009 radians satisfied this requirement as:

$$u_T = |\ \pm 0.05\ | = 0.05 \text{ seconds}$$
$$\text{T}(\theta) = 1.08 \pm 0.05 \text{ seconds}$$
$$\text{where } \theta = 0.1 \pm 0.009 \text{ radians}$$
$$T_o = 1.079 \pm 0.005 \text{ seconds}$$
$$\text{T}(\theta) \text{ - } T_o < u_T$$
$$0.00 \pm 0.05 < 0.05$$
$$0.00 < 0.05$$

0.00 ±0.05 is consistent with zero as it is less than twice its uncertainty value. Therefore, an initial release angle of 0.1 ±0.009 radians for the Period Versus Pendulum Length and Q Factor Versus Pendulum Length experiments was used. In addition, it can be assumed that parameter C is a zero value as within the aforementioned range, the period is constant. Therefore, C was indirectly important in the process of determining the initial release angle for the aforementioned experiments.

III.E. Period Versus Pendulum Length

Plotting the data from the Period Versus Pendulum Length experiment, the following graph in figure 5 was obtained:

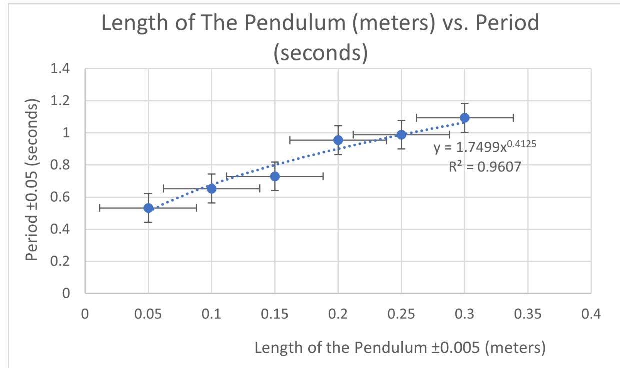

Figure 5. The period versus pendulum length graph resembles a power law function.

The trendline for the graph in figure 5 seems to suggest the following function:

$$T = kL^n \tag{21}$$

Using Python, the following values were found:

$$\text{k} = 1.8 \pm 0.2\ \frac{seconds}{m^n}$$
$$\text{n} = 0.44 \pm 0.05$$

The aforementioned function served as a predictive model for the pendulum's behaviour.

In addition, by plotting the data from the Period Versus Pendulum Length experiment using a log-log plot, the following graph in figure 6 was obtained:

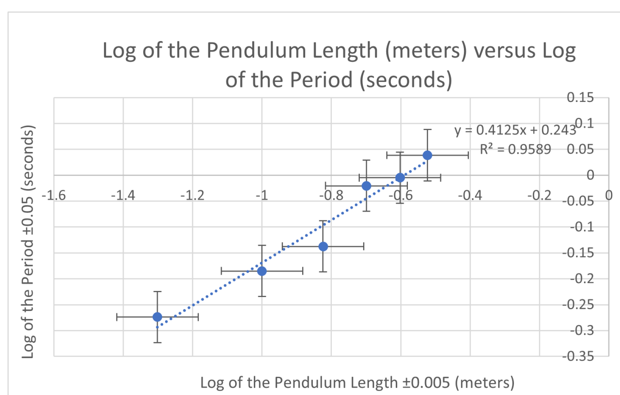

Figure 6. The period versus pendulum length log-log graph strongly resembles a linear function. The trendline for the graph in figure 5 seems to suggest the following function:

$$log(T) = n(log(L)) + log(k) \tag{22}$$

Using Python, the following values were found:

$$k = 1.8 \pm 0.2 \ \frac{seconds}{m^n}$$
$$n = 0.44 \pm 0.05$$

The aforementioned function also served as a predictive model for the pendulum's behaviour. In addition, the log-log plot offers an additional way to visualize the power law function T = kL^n.

From a theoretical perspective, the period of the pendulum as a function of the pendulum length should resemble the following:

$$T \approx 2\sqrt{L} \tag{23}$$

where k = 2
n = 0.5

It is worth noting that the experimental values were very close to the theoretical values for k and n. In terms of percentages, using the following function:

$$\frac{|Value_1 - Value_2|}{\frac{Value_1 + Value_2}{2}} \times 100 \tag{24}$$

The percentage difference between the two n values was 11±11% and the percentage difference between the two k values was 13±11%. The previous statement implies that our values our accurate, thus the experimental values can be relied upon. The theoretical values do not have uncertainty values as they were obtained in a theoretical manner.

The discrepancy between the theoretical and experimental values may have originated from a faulty experimental setup. More specifically, the delay between when the bob reached the top of its swing and when the stopwatch was stopped ±0.05 seconds and the uncertainty of the length of the pendulum ±0.05cm may have negatively affected the accuracy of the experimental data.

As the experimental values for k and n were very close to the theoretical values for k and n as evidenced in the percentage difference calculations, it can be claimed that the theoretical equation for the period as a function of length is consistent with our experimental equation. Therefore, the period of the pendulum does increase when the length of the pendulum is increased in a manner similar to the behaviour of a power law function.

In the future, I will have to use *Tracker's Autotracker* [3] feature to measure the period of the pendulum in relation to its length to decrease the ±0.05 seconds reaction time uncertainty. In addition, purchasing a more precise ruler might help decrease my ±0.05 cm measurement uncertainty.

III.F. Q Factor Versus Pendulum Length

In this experiment, the Q factor was calculated using the second method described in the Q factor section of the lab report. This method was selected as the first method was not accurate due the Python program repeatedly not finding the appropriate fit for my data points.

Plotting the data from the Q Factor versus Pendulum Length experiment, the following graph in figure 6 was obtained:

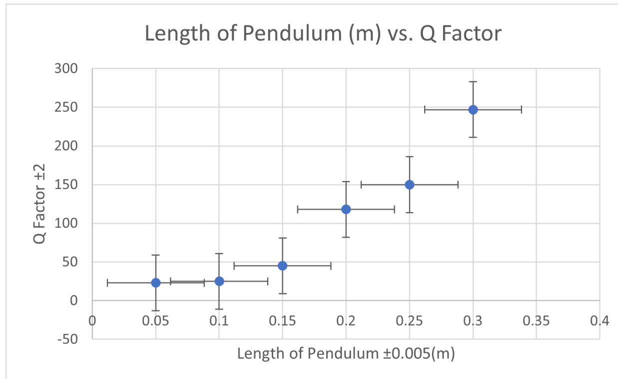

Figure 6. The Q Factor versus Period graph strongly resembles a quadratic function. It is worth noting that the Q Factor increases as the length of the pendulum increases. In addition, the uncertainty for the Q Factor was derived by taking the largest obtained uncertainty of all Q factors.

To determine the relationship between the Q Factor and the Pendulum Length, an appropriate fit had to be found using common functions, such as linear, quadratic, power law, and exponential functions. The following graphs contained in figures 7, 8, 9, and 10 are the fits for the aforementioned data.

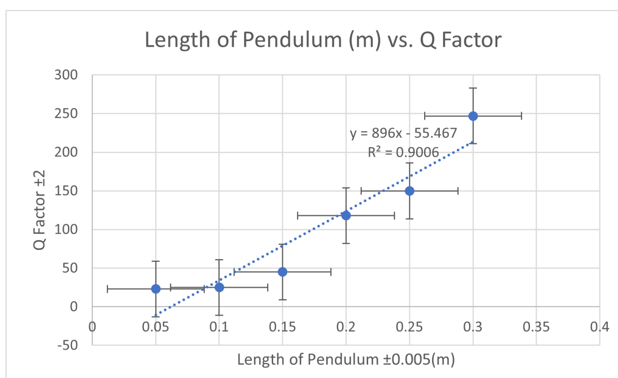

Figure 7. Linear fit for the Q Factor versus Pendulum Length data. It is worth noting that R^2 is 0.9007.

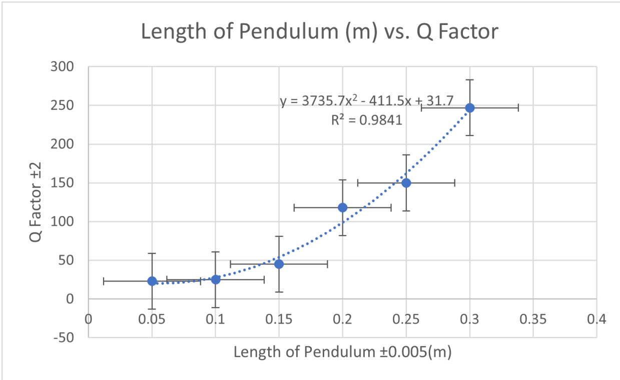

Figure 8. Quadratic fit for the Q Factor versus Pendulum Length data. It is worth noting that R^2 is 0.9841.

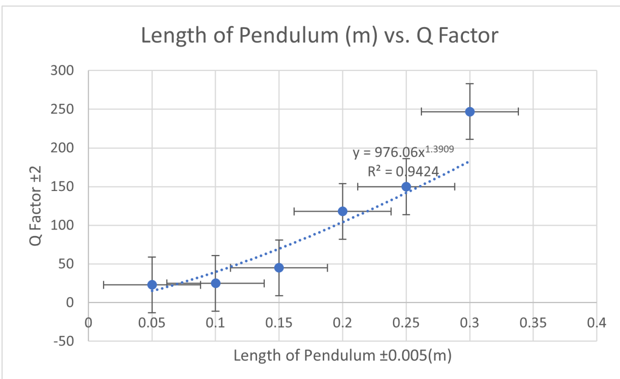

Figure 9. Power Law fit for the Q Factor versus Pendulum Length data. It is worth noting that R^2 is 0.9424.

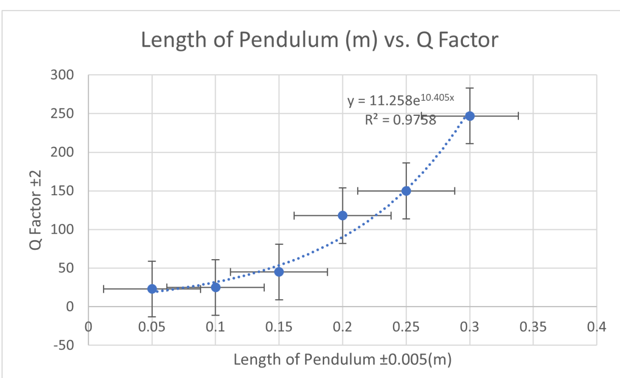

Figure 10. Exponential fit for the Q Factor versus Pendulum Length data. It is worth noting that R^2 is 0.9758.

The quadratic function offers the best fit as R^2, which is representative of how well a function fits the data, is the highest. Therefore, the data seems to follow the following function:

$$Q = Q_o(1 + BL + CL^2) \tag{25}$$

Using Python, the following values were found:

$$Q_o = 32 \pm 26$$
$$\text{B} = \text{-400} \;\; m^{-1} \pm 300$$
$$\text{C} = 3700 \;\; m^{-2} \pm 900$$

In this case, the aforementioned function served as a predictive model for the pendulum's behaviour.

III.G. Q Factor's Dependence on Length

Using the data from the Q Factor versus Pendulum Length experiment, the Python program determined that my fit parameters B and C were:

$$\text{B} = \text{-400} \pm 300 \;\; m^{-1}$$
$$\text{C} = 3700 \pm 900 \;\; m^{-2}$$

for the following function:

$$Q = Q_o(1 + BL + CL^2) \tag{26}$$

In this case, the fit parameter B is consistent with zero as it is less than two times its uncertainty value. In contrast, the fit parameter C, which is representative of the Q factor square the dependence of the length, is not consistent with zero as it is not less than two times its uncertainty value. If the value B is consistent with zero whilst C is not, then the function for the Q factor is:

$$Q = Q_o + CL^2 \tag{27}$$

This function indicates that there exists a dependence between the Q factor and the length of the pendulum. More specifically, my Q factor increases by $Q_o + CL^2$ whenever my length increases. The criterion used to determine the Q factor's dependence with relation to the length was if the fit parameters B and C were nonzero values or not. In this case, the uncertainty stems from my protractor's ± 0.009 radian uncertainty.

In the future, I will have to purchase a more accurate protractor to reduce my ± 0.009 radian uncertainty.

III.H. Uncertainties

In this experiment, the obtained uncertainty values had very little impact on my observations and analysis due to their minuscule magnitude in relation to their respective values. In addition, the experiment was conducted in a rigorous manner, which helped reduce the uncertainty of most values. More specifically, the *Tracker* [3] software drastically increased the accuracy of the data collected as it nearly doubled the frame rate of the recorded video and was able to accurately track the object's position over time. Furthermore, the experimental setup was designed to reduce the chances of the bob experiencing elliptical motion, which helped *Tracker* [3] determine the object's position over time with even more accuracy. This configuration helped reduce the amplitude's uncertainty value, which was ± 0.04 amongst other values. The amplitude was calculated using Python. Unfortunately, the length and the reaction time for the stopwatch, which were ± 0.05 centimetres and ± 0.05 seconds respectively, were the largest experimental source of uncertainty.

III.I. Results' Impact on Future Data Collection Methods

The data collection methods will remain the same as the results were relatively accurate when compared to the theoretical predictions. Although, the Python program will be modified and optimized to find a better best-fit curve for the collected data, which would consequently reduce the uncertainties for τ and the period. In addition, purchasing a more accurate ruler would help decrease the ± 0.05 centimetres uncertainty and using *Autotracker* [3] for my Period versus Pendulum Length and Period versus Angle experiments would help reduce my ± 0.05 seconds reaction time uncertainty.

IV. CONCLUSION

In conclusion, our results and analysis seem to suggest that a pendulum's motion does not completely follow a simple damped harmonic oscillator model.

More specifically, the period is dependent on the amplitude, which does not follow the damped harmonic oscillator model. Although, there exists a dependence between the Q factor and the length of the pendulum, which does follow the damped harmonic oscillator model. The period of the pendulum does increase when the length of the pendulum is increased in a manner similar to the behaviour of a power law function, which does follow the damped harmonic oscillator model. In addition, the pendulum's motion was symmetrical, which does follow the damped harmonic oscillator model.

For future experiments, I will have to use $Tracker's$ *Autotracker* [3] feature to measure the period of the pendulum in relation to its length to decrease the ± 0.05 seconds reaction time uncertainty. In addition, purchasing a more precise ruler might help decrease my ± 0.05 cm measurement uncertainty.
[1] [2] [3] [4]

[1] Gorilla. Gorilla Duct Tape. URL https://www.gorillatough.com/product/black-gorilla-tape/.

[2] Gorilla. Gorilla Super Glue. URL https://www.gorillatough.com/product/gorilla-super-glue-gel/.

[3] Douglas Brown. Tracker 5.0 help. URL https://physlets.org/tracker/tracker_help.pdf.

[4] Michael Richmond. The Q factor of an oscillating system URL http://spiff.rit.edu/classes/phys283/lectures/forced_ii/forced_ii.html.