EE3980 Algorithm
HW7. Grouping Friends

106061225 楊宇羲

1. Problem Description:

In this assignment, we are going to analyze about a special component that exists in a graph, which is called the “strongly connected component”, which is considered to be able to applied in the field of communication. In a directed graph, we can define a strongly component as a component in a graph that each vertices can mutually reach every other vertices. In other words, is a vertex A is able to reach B with a given path but B can’t get through A with any possible route, then the SCC that contains vertex A definitely will not contain vertex B. In this report, besides discussing how to determine all the SCCs in a graph, I will also focus on the time complexity and space complexity toward best case, average case and worst case of the implementation of this algorithm, then plot the result about relation between process time and graph size. We are provided with 10 lists of names which are consist of different size number of name and connection of each name in it. When scan in all the data in it, we have to be able to print out the number of total subgroup number and each element in a subgroup with no order requirement.

2. Approach:

The further explanation of how to obtain SCC will be discuss in the next part analysis. To approach our expected result, a definition of structure used for storing adjacent list is necessary, which is defined as below:

```
typedef struct CONN CONN;
struct CONN {
        int index;
        struct CONN *next;
};
```

I am going to declare several dynamically allocated global lists and array used for storing the core name list, transpose list and the separation (root of each tree in the forest). The main function pseudo code and used global variable will be as below:

```
CONN **adlist, **adlistT, *roots;        // adjacent list, transposed list, subgroup roots
Int N, M, SUBSUM;                        // size of name, size of connection, total subgroup
Int *f, *s, *visited;                    // store first sort, second sort, check visit
```

```
Algorithm main(void)
{
    ReadData();                    // read the date and the price of the corresponding date
    t := GetTime();                // start counting time
    SCC();
    t := GetTime() - t;            // stop counting time
    curr := roots;                 // curr is a CONN used to travel the list
    while (curr->next != NULL) {
        Write (curr, f[i]);        // f[I] store the name in topological order
        i++;
    }
}
```

There are also some function needed to be written to carry out the DFS algorithm and store the name list in order to print out the SCC.

```
Algorithm DFS_Call (graph)
{
    for I := 1 to N do {
        visited[I] := 0;           // not visited
        f[I] := 0;                 // store list initialize
    }
    for I := 1 to N do {
        if (visited[I] == 0) then {
            top_sort(I, f);
            store root of tree in roots;
            SUBSUM++;
        }
    }
}
```

```
Algorithm top_sort (v, graph)
{
    visited[v] := 1;
    curr = graph->next;
    while (curr) do {
        if (visited[curr->index] == 0) then {
            top_sort(curr->index, graph);
        }
        curr = curr->next;
    }
    visited[I] := 2;
    add v to head of f list;
}
```

After the the DFS functions are set up, we can eventually call the SCC function and perform DFS on the adjacent list and transposed adjacent list to get the SCCs.

```
Algorithm SCC (void)
{
        for I from 1 to N do s[I] = I;
        DFS_Call (adlist);
        for I from 1 to N do f[I] = s[I];
        DFS_Call(adlistT);
}
```

In this homework, we are going to use adjacent list to represent every edges exist in the graph, which is represented as below:

```
A  -> B -> C -> D -> NULL
B  -> C ->A -> NULL
C  -> A -> NULL
D  -> B ->C ->NULL
```

In comparison with adjacent array, lists only takes little space since there aren't many edges to store if it is directional graph. When reading data, we store the edges in the *adlist* and *adlistT* simultaneously in the ReadData function, so that there is no need to build the transpose list in the SCC function.

After all functions used to determine the SCCs are done, we can run the SCC function for **one** time then use the *GetTime* function to calculate the process time and finally being able to analyze the relation about the edge number, name size and the process speed.

3. Analysis:

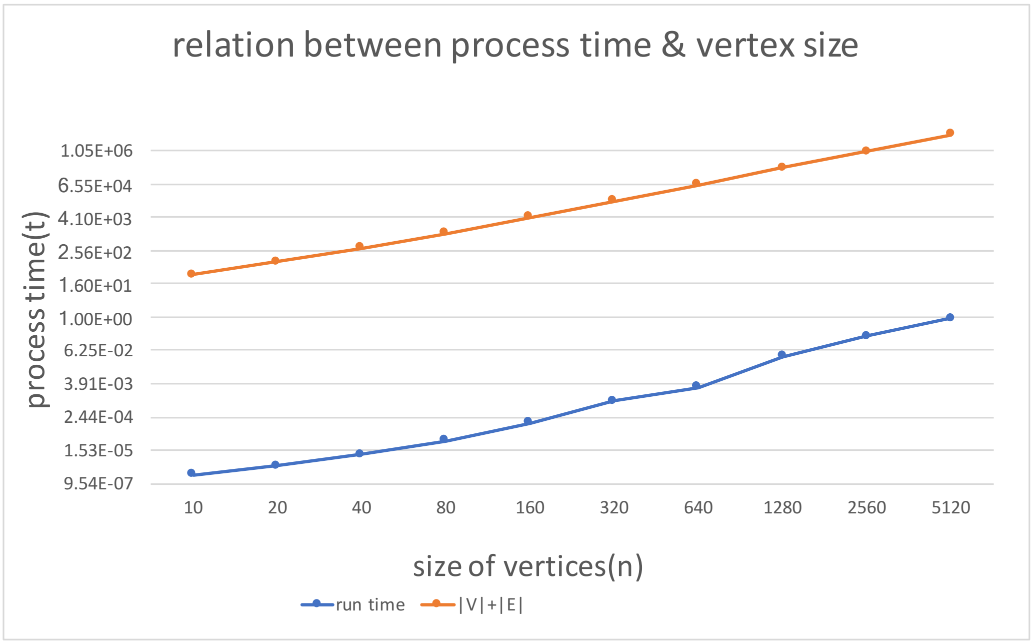
In this part we are going to analyze why performing DFS in adjacent list and transposed list can obtain the SCC. If we start from one random vertex and perform DFS on this directed graph, we might obtain a spanning tree or maybe several spanning trees, which indicates which vertices are reachable. However, SCC need to be "mutually" reachable, which infers that every point the original point can reach must also be able to reach the original point. Applying the property that a graph has the same SCC as the transposed graph (meaning every edges' direction is being opposite). So we can also perform second time DFS toward the transposed graph and double check to obtain the correct number of subgroup of strongly connected components.

Here is an example of what performing only one time of DFS will causes:

Suppose there is a graph consists of 4 vertices A, B, C, D. A is able to reach every vertex but other vertex is not able to reach any vertex. If we perform DFS on A, then we will get only one SCC which is the whole graph. However, the fact Is that there are four SCCs which each vertices make up their own SCC. Once we apply DFS one more time on A, but transposed graph, we can then get the correct answer which is 4 SCC.

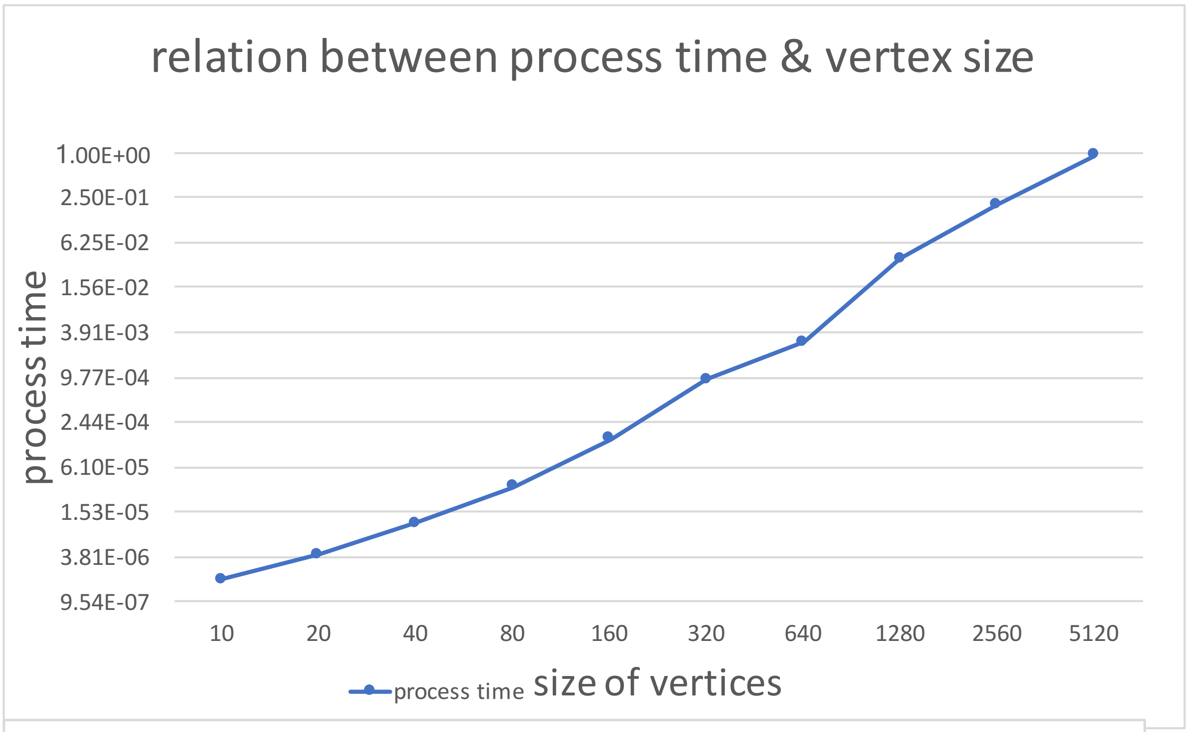
The total time complexity is O(E+V), where E is the number of edges and V is the number of vertices. Since we call the SCC function one time to get the result, and the SCC function calls two time of DFS_Call, which goes through the whole vertices to get the forest consists of spanning trees.

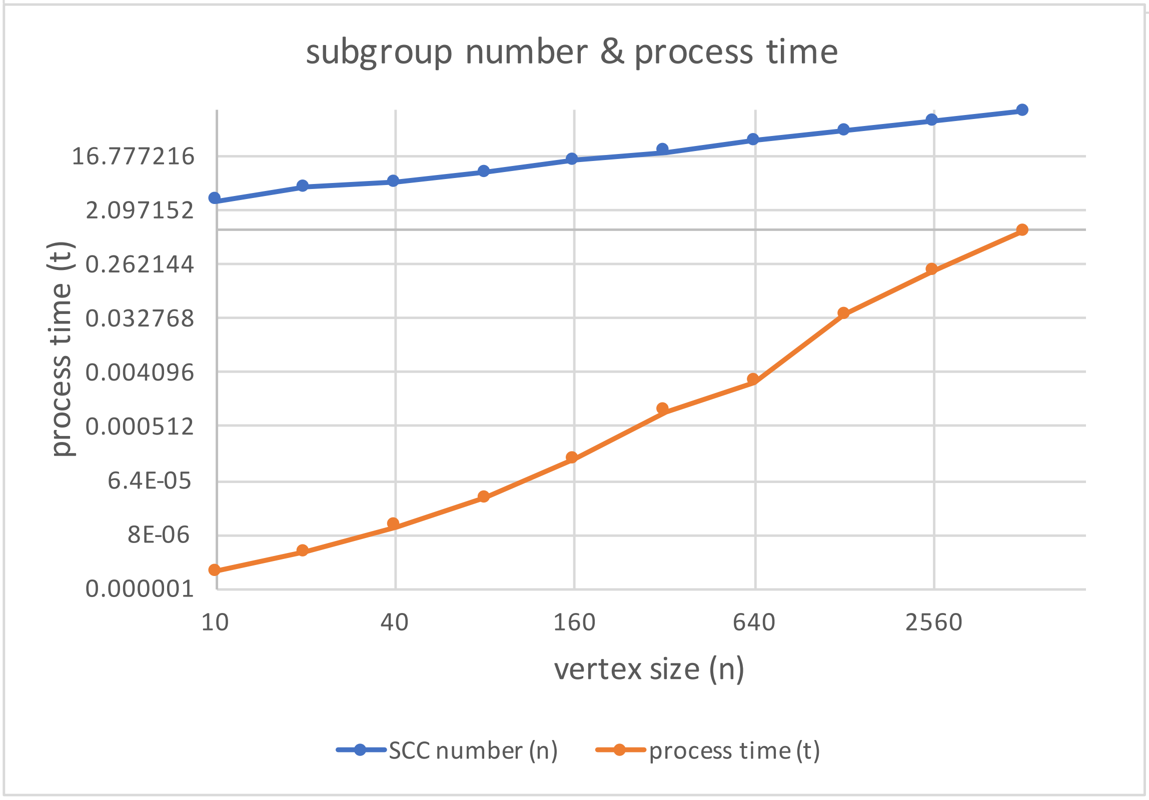
4. Results:



vertex size(n)	edge size(n)	SCC number (n)	process time (t)
10	25	3	1.90E-06
20	79	5	4.05E-06
40	274	6	1.09E-05
80	996	9	3.29E-05
160	3926	14	1.44E-04
320	15547	20	9.21E-04
640	61786	30	2.88E-03

1280	246515	43	3.73E-02
2560	984077	63	2.03E-01
5120	3934372	92	9.13E-01





5. Observation and conclusion:

We can tell that the result mostly matches with the analysis, the table below is the time and space complexity of this algorithm:

	SCC
Time complexity	O(E+V)
Space complexity	O(E+V)