EE3980 Algorithms

Homework 10. Coin Set Design, II

Due: May 22, 2021

This homework is a continuation of the last one. In the last homework, we have assumed that we have 4 types of coins with different values and a greedy method is used to find the minimum number coins for any dollar amount (D) between 1 and 99. Assuming the coin values are C_1, C_2, C_3 and C_4 and the number of each type of coins are x_1, x_2, x_3 and x_4. Then the problem can be formulated as

$$\begin{aligned} \text{minimize} \qquad & Ncoin = \sum_{i=1}^{4} x_i, \\ \text{subject to} \qquad & D = \sum_{i=1}^{4} x_i C_i \\ \text{and} \qquad & x_i \in \mathbb{Z} \text{ and } x_i \geq 0. \end{aligned}$$

Let $g_n(D)$ be the function that returns the minimum number of coins, using n types of coins, $1 \leq n \leq 4$, then one can derive the following recursive equation of our minimum-coin problem.

$$\begin{aligned} g_1(D) &= D, \\ g_n(D) &= \min_{x_n=0}^{\lfloor D/C_n \rfloor} \{x_n + g_{n-1}(D - x_n \cdot C_n)\} \qquad n > 1. \end{aligned}$$

And our our goal is to find $g_4(D)$ since we have 4 types of coins.

Your assignment is to write 3 functions to calculate $g_n(D)$ using dynamic programming approaches:

```
int NCoinDP_R(int D, int Ncoin, int Coins[]); // DP recursive approach
int NCoinDP_TD(int D, int Ncoin, int Coins[]); // DP top-down approach
void NCoinDP_BU(int D, int Ncoin, int Coins[]); // DP bottom approach
```

Note that in the bottom-up approach, the function `NCoinDP_BU` may need to be called only once for all possible value of d, $1 \leq d \leq D$.

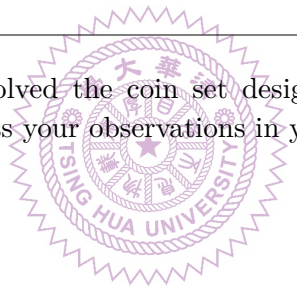
Using these functions, please do the following:

1. Given $\{C_1, C_2, C_3, C_4\} = \{1, 5, 10, 50\}$, find the average number of coins for $D = 1$ to 99.
2. Assuming C_4 is a variable find its value that minimizes the average for $D = 1$ to 99.
3. Assuming C_3 is a variable find its value that minimizes the average for $D = 1$ to 99.
4. Assuming both C_3 and C_4 are variables find their values that minimizes the average for $D = 1$ to 99.

The output of your program should be as following:

```
$ a.out
Original coin set:
  DP recursive: {1, 5, 10, 50} average is 5.05051, CPU time: yyyyyy sec
  DP top-down:  {1, 5, 10, 50} average is 5.05051, CPU time: yyyyyy sec
  DP bottom-up: {1, 5, 10, 50} average is 5.05051, CPU time: yyyyyy sec
Replacing $50:
  DP recursive: {1, 5, 10, C4} average is x.xxxxx, CPU time: yyyyyy sec
  DP top-down:  {1, 5, 10, C4} average is x.xxxxx, CPU time: yyyyyy sec
  DP bottom-up: {1, 5, 10, C4} average is x.xxxxx, CPU time: yyyyyy sec
Replacing $10:
  DP recursive: {1, 5, C3, 50} average is x.xxxxx, CPU time: yyyyyy sec
  DP top-down:  {1, 5, C3, 50} average is x.xxxxx, CPU time: yyyyyy sec
  DP bottom-up: {1, 5, C3, 50} average is x.xxxxx, CPU time: yyyyyy sec
Replacing $10 and $50:
  DP recursive: {1, 5, C3, C4} average is x.xxxxx, CPU time: yyyyyy sec
  DP top-down:  {1, 5, C3, C4} average is x.xxxxx, CPU time: yyyyyy sec
  DP bottom-up: {1, 5, C3, C4} average is x.xxxxx, CPU time: yyyyyy sec
```

In these two homework we have solved the coin set design problem using two different approaches, you are encouraged to discuss your observations in your report.



Notes.

1. One executable and error-free C source file should be turned in. This source file should be named as `hw010.c`.
2. A report file in `pdf` format is also needed. This file should be named as `hw010a.pdf`.
3. Submit your `hw010.c` and `hw010a.pdf` on EE workstations using the following command:

   ~ee3980/bin/submit hw010 hw010.c hw010a.pdf

   where `hw010` indicates homework 10.
4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.