EE3980 Algorithm

HW4. Stock Short Selling

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1. Problem Description:

When it comes to buying and selling stocks, everyone would wish to make money by selling at a high price while buying it at a low price. Being provided with a chart with everyday's price on, we have to be able to figure out the max value array to maximize the profit we get. So what is maximum sub array? Given an array with size n, we have to choose two integer *i, j* in the range of n such that the sum from A[i] to A[j] gets the biggest value. However, to solve the max sub-array problem directly involves tedious loop calculation that requires three layers of for loop since setting the start point and end point already brings us $C_2^n = \frac{n(n-1)}{2}$ combinations, where n is the dataset size. After adding the implementation of calculating sum ,the total process step can be unbelievably cumbersome, the detailed steps will be discussed in analysis. As the dataset size are growing bigger, the process time will grow to an extremely large size as well (in accurate, at the time complexity of $\Theta(n^3)$). Hence there is a necessity to do some improvement toward the method of calculating subarray problem like this, thus the divide & conquer approach are introduced here with a significant decrease in processing time.

2. Approach:

First of all, instead of merely returning the max value, I choose to create a type of structure that contains the max value ,start point and end point, so that it will be easier to do the recursive part and there will be no need to initialize global variable besides the data array and its size. For doing analysis, we will use the GetTime to record the brute-force method for one time (since performing hundreds time can take more than an hours to run), and set repetition time to 1000 to run the divide & conquer approach, then there are 9 datasets with each dataset the size growing exponentially, but note that we are going to compare with data's change so that we make sure to get max profit, which is *data[i] - data[i-1]*, so after we obtain the result we have to shift the selling day one day earlier. We are then required to carry out the two methods to find the correct max sub array to create the maximized profit and record the process time simultaneously. After the two methods are all completed, we can compare the run time of the former one and the average time of the latter one ,then combine it to a line chart not only to compare the speed of them but also observe the growing tendency of the two methods while the dataset size are growing exponentially in the power of 2. The main function's pseudo code and some notices are provided below:

a) We are going to find the value that has the largest absolute value with negative sign since we will sell first then buy it, so we are going to seek for the minimum value to create max profit.

b) We are comparing each data's change value, so as we found the start point and end point we have to minus 1 to start point so that it provides the correct data.

```
Algorithm main()
{
        ReadData();                         // read the date and the price of the corresponding date
        t := GetTime();                     // start counting time
        result := SubArrayBT();             // implement brute force approach
        t := GetTime() - t;                 // stop counting time
        Write (t, result);                  // print outcome
        t := GetTime();                     // start counting time
        for i := 1 to Repeat do{            // repeat R times
                result := SubArray();       // implement divide & conquer method
        }
        t := GetTime() - t;                 // stop counting time
        Write (t/Repeat, result);           // print outcome
}
```

3. Analysis:

As mentioned in the problem description, the pseudo code of brute force approach is:

```
Algorithm MaxSubArrayBT (data, n)
{
        result.max := 0;
        result.sell := 1;
        result.buy := n;
        for i := 1 to n do {
                for j := i to n do {
                        sum := 0;
                        for k := i to j do {
                                sum := sum + data[k];
                        }
                        if sum < result.max then {
                                result.max := sum;
                                result.sell := i;
                                result.buy := j;
                        }
                }
```

```
	}
	return result;
}
```

Where result structure are initialized as:

```
Struct result{
	int sell, buy;
	double max;
};
```

The outer two loop determines the position of start index and end index. With all the combination possibilities, we calculate sum of each of them, then return the start point and end point that creates the max value. This is the most direct thought to carry out the result, however, the total execute steps are going to be $\frac{n(n-1)}{2}$ times the average steps to calculate sum, which is *n* time. The difference between best case and worst case is not obvious since the algorithm **always** runs through all the combinations and calculate the sum, so we can obtain the total time complexity to be $\Theta(n^3)$. So as we are seeking s1.dat's result it may seem to be not that slow, but when we are carrying out s9.dat you can just leave your computer and take a rest due to long time of waiting. The total space complexity will be O(n), which is the size of array and variables for loop, sum, sell & buy.

On the other hand, the coding of divide & conquer method could be more difficult than the former one since it requires recursive solution and cross boundary condition, the pseudo code will look like this:

```
Struct result{
	int sell, buy;
	double max;
};

Algorithm MaxSubArray (data, begin, end)
{
	if begin = end then{				// terminal condition
		result.sell := begin;
		result.buy := end;
		result.max := data[begin];
		return result;
	}
	mid := (begin + end) / 2;
	lsum := MaxSubarray(data, begin, mid);		// left half sum
	rsum := MaxSubarray(data, mid +1, end);		// right half sum
	xsum := MaxSubarrayXB(data, begin, mid, end);	// sum that contains mid
```

```
        if lsum <=rsum and rsum <= xsum then return lsum;          // left is max
        else if rsum <=lsum and rsum <= xsum then return rsum;     // right is max
        return xsum;                                               // cross boundary is max
}
```

And the cross boundary function will be defined like this:

```
Algorithm MaxSubArrayXB (data, begin, mid, end)
{
        leftsum := 0;                               // initialize the lower half
        result.sell := mid;                         // initialize the sell date
        sum := 0;                                   // initialize sum
        for i := mid to begin step -1 do {          // start from mid
                sum := sum + data[i];
                if  sum > leftsum then {
                        leftsum := sum;             // obtain left half max
                        result.sell := i;           // obtain sell date
                }
        }
        rightsum := 0;                              // initialize upper half
        result.buy := mid + 1;                      // initialize buy date
        sum := 0;                                   // initialize sum
        for i := mid + 1 to end do {                // start from mid
                sum := sum + data[i];
                if  sum > rightsum then {
                        rightsum := sum;            // obtain right half max
                        result.buy := i;            // obtain buy date
                }
        }
        result.max := rightsum + leftsum;           // put it together
        return result;                              // return
}
```

The whole algorithm works like this: Every time it calls the recursive function, we will compare the left part sum, right part sum, and the sum that may cross the boundary. As it calls until the size are divided to 1, it returns its own change value, then calls back until the first layer of recursive function has the result. After the terminal condition is triggered, we can then compare the *lsum, rsum, xsum* of each layer, so that the dominating calculation is reduced, replaced by more comparisons. Each layer's function's comparisons contains 2 side function and 1 cross boundary function, which implies that the total comparisons of an entire recursive function can be obtained as follow with some assumed premises:

(a let the total dataset size n represented as a power of 2: 2^k

(b the total comparison per one cross boundary function will be n time since it only calculates the sum with a given range.

$$2 \cdot T(\frac{n}{2}) + T_{xb}(n)$$

$$= 2 \cdot (2 \cdot T(\frac{n}{4}) + T_{xb}(\frac{n}{2})) + n$$

$$= 2 \cdot (2 \cdot T(\frac{n}{4}) + \frac{n}{2}) + n$$

$$=4 \cdot (2 \cdot T(\frac{n}{8}) + \frac{n}{4}) + 2n$$

$$= \cdots$$

$$= 2^k \cdot T(1) + k \cdot n$$

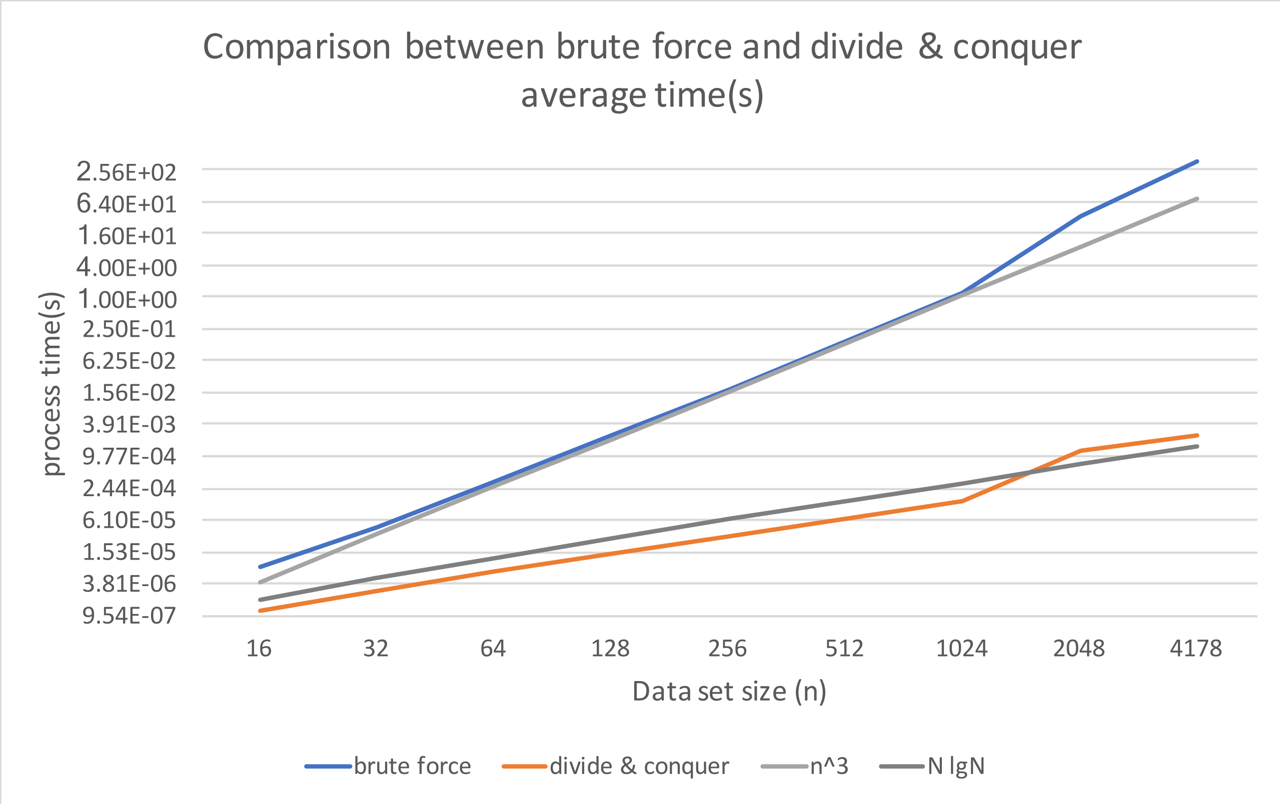
$$= n + n \cdot log(n)$$

Since the above derivation is based on the average case, so we can get the final result of the time complexity of divide conquer approach to be $\Theta(n \cdot \lg n)$. The total space complexity will be O(n), which also varies linearly with the increase of size of array.

Need to be more complete.

4. Results:

dataset size(n)	best sell date	sell price (USD)	best buy date	buy price (USD)	maximum earning per share (USD)	brute force process time (s)	divide & conquer average time(s)
16	2004/8/23	54.8694	2004/9/3	50.1598	4.7096	8.10E-06	1.12E-06
32	2004/8/23	54.8694	2004/9/3	50.1598	4.7096	4.31E-05	2.83E-06
64	2004/11/1	98.3185	2004/11/10	84.1899	14.1286	3.15E-04	6.29E-06
128	2004/11/1	98.3185	2004/11/22	82.8056	15.5129	2.34E-03	1.44E-05
256	2005/7/21	157.4561	2005/8/22	137.4292	20.0269	1.86E-02	3.10E-05
512	2006/1/10	236.5452	2006/3/13	169.0518	67.4934	1.47E-01	6.63E-05
1024	2007/11/6	372.0435	2008/3/10	207.4504	164.5931	1.15E+00	1.44E-04
2048	2007/11/6	372.0435	2007/11/24	129.1186	242.9249	3.29E+01	1.20E-03
4178	2020/2/19	1524.8700	2020/3/23	1054.1300	470.7400	3.64E+02	2.40E-03



5. Observation and conclusion:

As seen in the result, the actual speed and the tendency line are mostly matched with the theorem and my analysis (n^3 and $n \cdot lg(n)$). Hence it is proved that divide & conquer method can improve the performance when solving max sub array problem.

	space complexity	time complexity
brute force	O(n)	$\Theta(n^3)$
divide & conquer	O(n)	$\Theta(n \cdot lg(n))$

```
$ gcc hw04.c
$ a.out < s7.dat
N = 1024
Brute-force approach: CPU time 0.415807 s
  Sell: 2007/11/6 at 372.0435
  Buy: 2008/3/10 at 207.4504
  Earning: 164.5931 per share.
Divide and Conquer: CPU time 4.87969e-05 s
  Sell: 2007/11/6 at 372.0435
  Buy: 2008/3/10 at 207.4504
  Earning: 164.5931 per share.
```

Good, solution is correct.

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score: 77.0

- Overall report writing
  - English writing needs more practice.
- Approach
  - C implementation is different from the pseudo codes – the arguments for the functions should match each other.
- Time/Space complexity
  - Space complexity analysis for divide-and-conquer approach should be more complete.
- Program format can be improved

hw04.c

```c
// EE3980 HW04 Stock Short Selling
// 106061225, 楊宇羲
// 2021/3/29

#include <stdio.h>
#include <stdlib.h>
#include <sys/time.h>

typedef struct STKprice {                 // Stock's price & date
    int year, month, day;
    double price, change;
} STKprice;

typedef struct RSLT {                     // Contains start, end date and maximum
    int sell, buy;
    double max;
} RSLT;

void ReadData(void);                      // read everyday's price
double GetTime(void);                     // Get local time
RSLT MaxSubArrayBF(void);                 // brute-force sub array solution
RSLT MaxSubArray(int begin, int end);     // divide & conquer approach
RSLT MaxSubArrayXB(int begin, int mid, int end);    // cross boundary condition

STKprice *data;                           // array to store price
int N;                                    // size of array

int main(void)
{
    int i = 0;                            // for loop
    int R = 1000;                         // Repetition time
    double t;                             // time variable
    RSLT result;                          //  start & end date and maximum

    ReadData();                           // read in data

    t = GetTime();                        // start counting CPU time
    result = MaxSubArrayBF();             // brute-force solution for 1 time
    t = GetTime() - t;                    // end count time
```

```
    printf("Brute-force approach: CPU time %g s\n", t); // print outcome

    // selling day should shift to the day before
    printf("  Sell: %d/%d/%d at %.4lf\n", data[result.sell - 1].year
                                    , data[result.sell - 1].month
            X,X data[result.sell - 1].month
  ',' should not lead a line
                                    , data[result.sell - 1].day
            X,X data[result.sell - 1].day
  ',' should not lead a line
                                    , data[result.sell - 1].price);
            X,X data[result.sell - 1].price);
  ',' should not lead a line
    printf("  Buy: %d/%d/%d at %.4lf\n", data[result.buy].year
                                   , data[result.buy].month
            X,X data[result.buy].month
  ',' should not lead a line
                                   , data[result.buy].day
            X,X data[result.buy].day
  ',' should not lead a line
                                   , data[result.buy].price);
            X,X data[result.buy].price);
  ',' should not lead a line
    printf("  Earning: %.4lf per share.\n", -1 * result.max);

    t = GetTime();                      // start counting time
    for (i = 0; i < R; i++)              // repeat 1000 times
    {
        result = MaxSubArray(0, N-1);   // divide and counquer approach
        result = MaxSubArray(0, N - 1);   // divide and counquer approach
    }
    t = (GetTime() - t) / R;            // end counting time

    printf("Divide and Conquer: CPU time %g s\n", t);   // print outcome

    // selling day should shift to the day before
    printf("  Sell: %d/%d/%d at %.4lf\n", data[result.sell - 1].year
                                    , data[result.sell - 1].month
            X,X data[result.sell - 1].month
  ',' should not lead a line
```

```
                                            , data[result.sell - 1].day
            X,X data[result.sell - 1].day
  ',' should not lead a line
                                            , data[result.sell - 1].price);
            X,X data[result.sell - 1].price);
  ',' should not lead a line
    printf("  Buy: %d/%d/%d at %.4lf\n", data[result.buy].year
                                           , data[result.buy].month
            X,X data[result.buy].month
  ',' should not lead a line
                                           , data[result.buy].day
            X,X data[result.buy].day
  ',' should not lead a line
                                           , data[result.buy].price);
            X,X data[result.buy].price);
  ',' should not lead a line
    printf("  Earning: %.4lf per share.\n", -1 * result.max);
    return 0;
}

void ReadData(void)                         // read in data
{
    int i = 0;                              // for loop

    scanf("%d", &N);                        // read in size of array
    printf("N = %d\n", N);

    data = (STKprice*)malloc(N * sizeof(STKprice)); // dynamic array allocation

    for (i = 0; i < N; i++)                 // scan in data
    {
        scanf("%d %d %d", &data[i].year, &data[i].month,&data[i].day);
        scanf("%d %d %d", &data[i].year, &data[i].month, &data[i].day);
        scanf("%lf", &data[i].price);

        // derive the change of today and yesterday except the first day
        if (i != 0) data[i].change = data[i].price - data[i-1].price;
        if (i != 0) data[i].change = data[i].price - data[i - 1].price;
        else data[i].change = 0;
    }
}
```

```

double GetTime(void)                          // get local time
{
    struct timeval tv;

    gettimeofday(&tv, NULL);
    return tv.tv_sec + 1e-6 * tv.tv_usec;     // sec + micro sec
}

RSLT MaxSubArrayBF()                          // brute-force solution
RSLT MaxSubArrayBF(void)                          // brute-force solution
{
    int i, j, k;                              // for loops
    RSLT result;                              // for storing low, high, change

    result.max = 0.0;                         // initializations of result
    result.sell = 0;
    result.buy = N - 1;

    for (i = 0; i < N; i++)                   // determine start index
    {
        for (j = i; j < N; j++)               // determine end index
        {
            double sum = 0;                   // start counting sum
Do not mix declarations with statements
            for (k = i; k <= j; k++)          // from start to end
            {
                sum += data[k].change;        // derive sum
            }
            if (sum < result.max)
            {
                result.max = sum;             // found max, update informations
                result.sell = i;
                result.buy = j;
            }
        }
    }

    return result;                            // return result
}

```

```
RSLT MaxSubArray(int begin, int end)        // divide and conquer approach
{
    RSLT result;                            // store information
    if (begin == end)                       // terminal condition
    {
        result.sell = begin;
        result.buy = end;
        result.max = data[begin].change;
        return result;
    }
    int mid = (begin + end) / 2;            // determine middle index
```

Do not mix declarations with statements

```

    RSLT lsum = MaxSubArray(begin, mid);    // check left sum
    RSLT rsum = MaxSubArray(mid + 1, end);  // check right sum
    RSLT xsum = MaxSubArrayXB(begin, mid, end); // check cross boundary sum

    // comparisons of the above three sums
    if ((lsum.max <= rsum.max) && (lsum.max <= xsum.max))
    {
        return lsum;
    }
    else if ((rsum.max <= lsum.max) && (rsum.max <= xsum.max))
    {
        return rsum;
    }
    return xsum;
}

RSLT MaxSubArrayXB(int begin, int mid, int end) // cross boundary condition
{
    RSLT result;                            // store information
    int i;                                  // for loop
    double sum;                             // for checking sum

    double lsum = 0.0;                      // left hand side's sum
    result.sell = 0;                        // initialize sell's date
    sum = 0.0;

    for (i = mid; i > begin; i--)
    {
```

```
        sum += data[i].change;              // counting sum
        if (sum < lsum)                     // determine left max
        {
            lsum = sum;                     // update left sum
            result.sell = i;                // update sell's date
        }
    }

    double rsum = 0.0;                      // right hand side's sum
Do not mix declarations with statements
    result.buy = mid + 1;                   // initialize buy's date
    sum = 0.0;

    for (i = mid + 1; i < end; i++)
    {
        sum += data[i].change;              // counting sum
        if (sum < rsum)                     // determine right sum
        {
            rsum = sum;                     // update right sum
            result.buy = i;                 // update buy date
        }
    }

    result.max = rsum + lsum;               // update total sum
    return result;
}
```