Characteristic polynomials, eigenvalues and eigenvectors | Coursera

Characteristic polynomials, eigenvalues and eigenvectors

Practice Quiz • 30 min

✓

Congratulations! You passed!

TO PASS 80% or higher

GRADE

80%

Characteristic polynomials, eigenvalues and eigenvectors

TOTAL POINTS 10

1.

Question 1

Given a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, recall that one can calculate its eigenvalues by solving the characteristic polynomial $\lambda^2 - (a+d)\lambda + (ad - bc) = 0$. In this quiz, you will practice calculating and solving the characteristic polynomial to find the eigenvalues of simple matrices.

For the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

○ $\lambda^2 - 3\lambda - 2 = 0$
$\lambda_1 = 1, \lambda_2 = -2$

○ $\lambda^2 + 3\lambda - 2 = 0$
$\lambda_1 = -1, \lambda_2 = 2$

● $\lambda^2 - 3\lambda + 2 = 0$
$\lambda_1 = 1, \lambda_2 = 2$

○ $\lambda^2 + 3\lambda + 2 = 0$
$\lambda_1 = -1, \lambda_2 = -2$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

2.

Question 2

Recall that for a matrix A, the eigenvectors of the matrix are vectors for which applying the matrix transformation is the same as scaling by some constant.

For

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

0 / 1 point

☑ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

☐ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

1
−1
[ ]

☐ $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

0
3
[ ]

☑ $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

0
2
[ ]

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

You didn’t select all the correct answers

3.
Question 3

For the matrix

$$\begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

$A = [\begin{smallmatrix} 3 & 4 \\ 0 & 5 \end{smallmatrix}]$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

○ $\lambda^2 + 8\lambda + 15 = 0$

$\lambda_1 = -3, \lambda_2 = -5$

○ $\lambda^2 + 8\lambda - 15 = 0$

$\lambda_1 = 3, \lambda_2 = -5$

○ $\lambda^2 - 8\lambda - 15 = 0$

$\lambda_1 = -3, \lambda_2 = 5$

● $\lambda^2 - 8\lambda + 15 = 0$

$\lambda_1 = 3, \lambda_2 = 5$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

4.

Question 4

For the matrix

$$\begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☐ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

☑ $\begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

☑ $\begin{bmatrix}3\\0\end{bmatrix}$

3
0
[]

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

☑ $\begin{bmatrix}2\\1\end{bmatrix}$

2
1
[]

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

5.

Question 5

For the matrix

$$\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

● $\lambda^2 - 5\lambda + 4 = 0$
$\lambda_1 = 1, \lambda_2 = 4$

○ $\lambda^2 + 5\lambda + 4 = 0$
$\lambda_1 = -1, \lambda_2 = -4$

○ $\lambda^2 - 5\lambda - 4 = 0$
$\lambda_1 = -1, \lambda_2 = 4$

○ $\lambda^2 + 5\lambda - 4 = 0$
$\lambda_1 = 1, \lambda_2 = -4$

✓ **Correct**

Well done! This matrix has two distinct eigenvalues.

6.

Question 6

For the matrix

$$\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☐ $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

☑ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

☐ $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

3
[2]

☑ $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

3
[1]

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

7.

Question 7

For the matrix

$$\begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$$

$A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

0 / 1 point

$\lambda^2 + 25 = 0$

○ $\lambda_1 = -5, \lambda_2 = 5$

$\lambda^2 - 25 = 0$

● $\lambda_1 = \lambda_2 = 5$

$\lambda^2 - 25 = 0$

○ $\lambda_1 = -5, \lambda_2 = 5$

$\lambda^2 + 25 = 0$

○ $\lambda_1 = \lambda_2 = -5$

! **Incorrect**

Be careful when calculating the characteristic polynomial and finding its roots.

8.

Question 8

For the matrix

$$\begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$$

$A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$ as immediately above, select all eigenvectors of this matrix.

1 / 1 point

☑ $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$

4
[−1]

✓ **Correct**

Well done! One way to check that a vector is an eigenvector is to simply apply the matrix transformation and see if this is the same as multiplying by a scalar. Another way is to calculate the eigenvector by hand, as in the lecture videos.

☑ $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

−1
[−1]

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

☑ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1
[1]

✓ **Correct**

Well done! Recall that if a vector is an eigenvector of a matrix, then so is any (non-zero) multiple of that vector.

☐ $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

[0 2]

9.

Question 9

For the matrix

$$\begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$$

$A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

● $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = 1$

○ $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = -1$

○ $\lambda^2 - 2\lambda + 1 = 0$

$\lambda_1 = -1, \lambda_2 = 1$

$\lambda^2 - 2\lambda + 1 = 0$

○ No real solutions.

✓ **Correct**

Well done! This matrix has one repeated eigenvalue - which means it may have one or two distinct eigenvectors (which are not scalar multiples of each other).

10.

Question 10

For the matrix

$$\begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$$

$A = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

1 / 1 point

$\lambda^2 + \lambda + 1 = 0$

● No real solutions.

$\lambda^2 + \lambda - 1 = 0$

○ $\lambda_1 = \frac{-\sqrt{5}-1}{2}, \lambda_2 = \frac{\sqrt{5}-1}{2}$

$\lambda^2 - \lambda - 1 = 0$

○ $\lambda_1 = \frac{1-\sqrt{5}}{2}, \lambda_2 = \frac{1+\sqrt{5}}{2}$

$\lambda^2 - \lambda + 1 = 0$

○ No real solutions.

✓ **Correct**

Well done! This matrix has no real eigenvalues, so any eigenvalues are complex in nature. This is beyond the scope of this course, so we won't delve too deeply on this.