Dot product of vectors | Coursera

Dot product of vectors

Practice Quiz • 15 min

Congratulations! You passed!

TO PASS 80% or higher

GRADE

100%

Dot product of vectors

TOTAL POINTS 6

1.

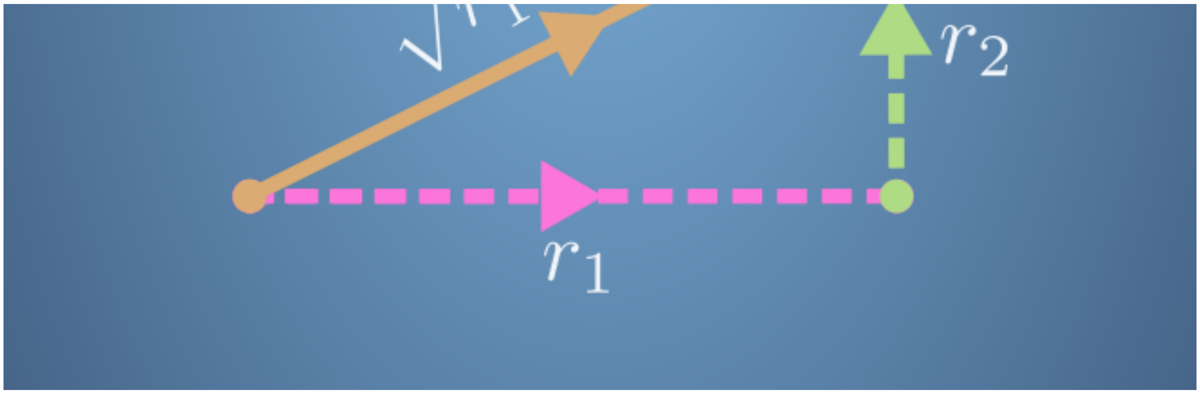
Question 1

As we have seen in the lecture videos, the dot product of vectors has a lot of applications. Here, you will complete some exercises involving the dot product.

We have seen that the size of a vector with two components is calculated using Pythagoras' theorem, for example the following diagram shows how we calculate the size of the orange vector

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

r = [r_1 r_2]:



In fact, this definition can be extended to any number of dimensions; the size of a vector is the square root of the sum of the squares of its components. Using this information, what is the size of the vector

$$\begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \end{bmatrix}$$

s = $\begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \end{bmatrix}$?

1 / 1 point

- ( ) $|\mathbf{s}| = \sqrt{10}$
- ( ) $|\mathbf{s}| = 30$
- (•) $|\mathbf{s}| = \sqrt{30}$
- ( ) $|\mathbf{s}| = 10$

✓ **Correct**

The size of the vector is the square root of the sum of the squares of the components.

2.

Question 2

Remember the definition of the dot product from the videos. For two n component vectors, $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$.

What is the dot product of the vectors

$$\begin{bmatrix} -5 \\ 3 \\ 2 \\ 8 \end{bmatrix}$$

$\mathbf{r} = \begin{bmatrix} -5 \\ 3 \\ 2 \\ 8 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$?

1 / 1 point

- $\mathbf{r} \cdot \mathbf{s} = 1$

- $\begin{bmatrix} -5 \\ 6 \\ -2 \\ 0 \end{bmatrix}$

  $\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} \mathbf{-5} \\ \mathbf{6} \\ \mathbf{-2} \\ \mathbf{0} \end{bmatrix}$

$$\begin{bmatrix} -4 \\ 5 \\ 1 \\ 9 \end{bmatrix}$$

$\mathbf{r} \cdot \mathbf{s} = \begin{bmatrix} -4 \\ 5 \\ 1 \\ 9 \end{bmatrix}$

$\mathbf{r} \cdot \mathbf{s} = -1$

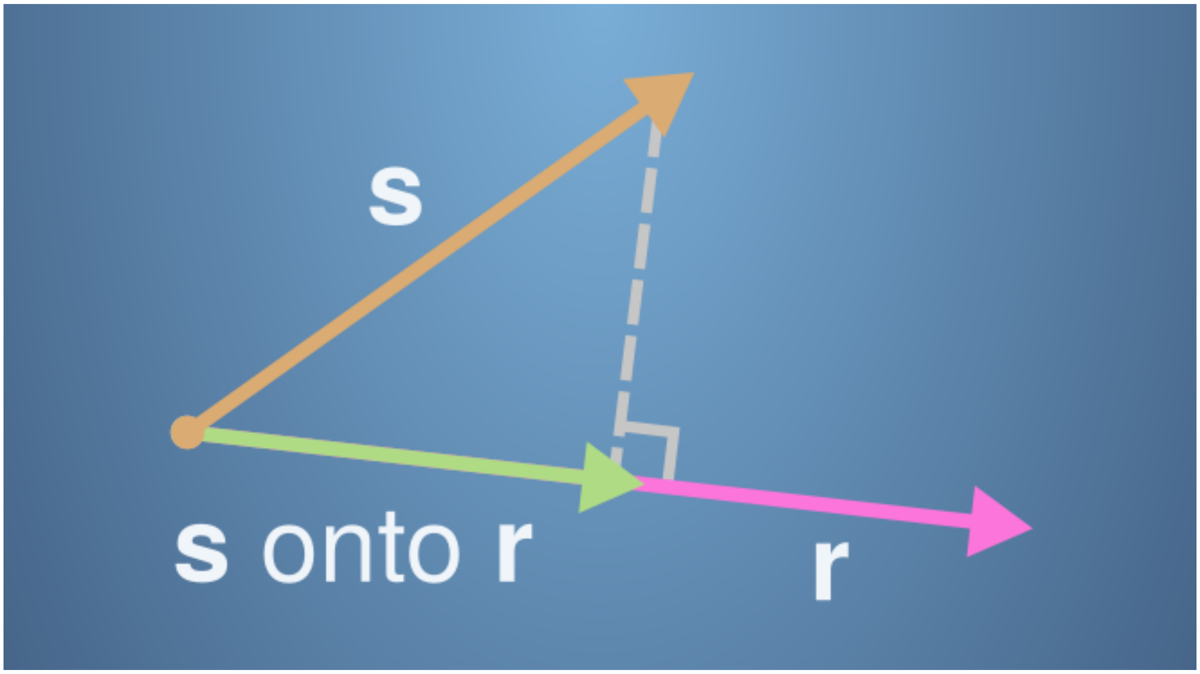
Correct

The dot product of two vectors is the total of the component-wise products.

3.

Question 3

The lectures introduced the idea of projecting one vector onto another. The following diagram shows the projection of **s** onto **r** when the vectors are in two dimensions:



Remember that the scalar projection is the *size* of the green vector. If the angle between **s** and **r** is greater than $\pi/2$, the projection will also have a minus sign.

We can do projection in any number of dimensions. Consider two vectors with three components,

$$\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

r = $\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the scalar projection of **s** onto **r**?

1 / 1 point

- [ ] $-\frac{1}{2}$

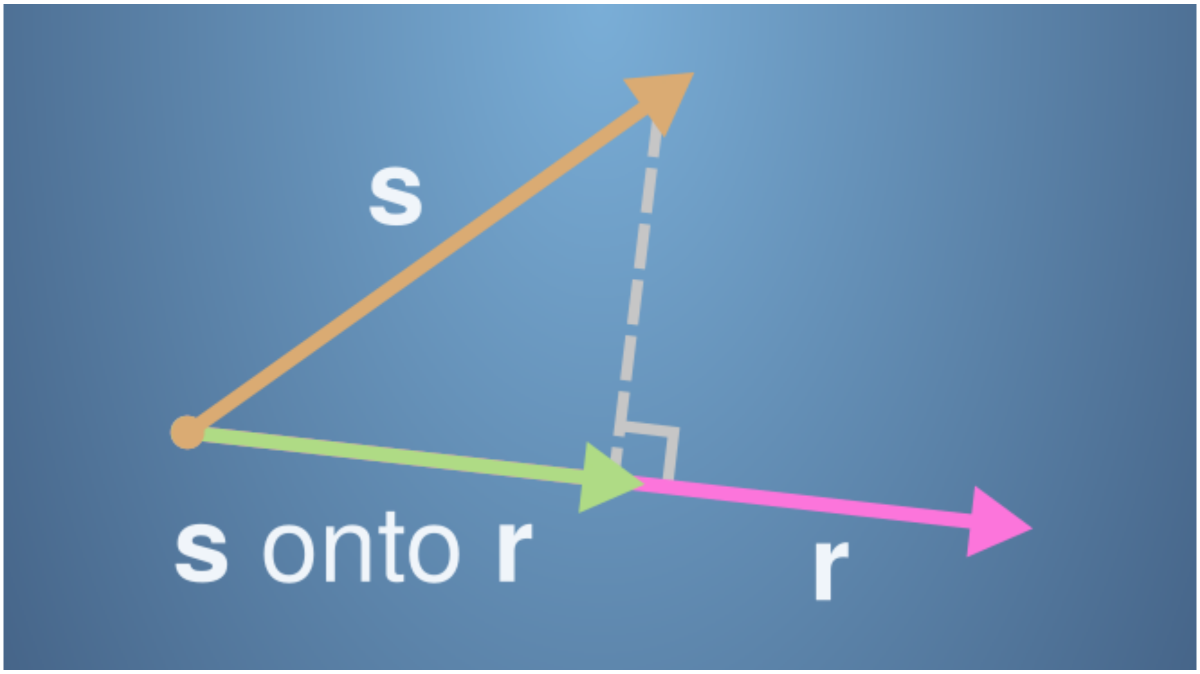
- [x] 2
- [ ] -2
- [ ] $\frac{1}{2}$

Correct

The scalar projection of of $\mathbf{s}$ onto $\mathbf{r}$ can be calculated with the formula $\frac{\mathbf{s} \cdot \mathbf{r}}{|\mathbf{r}|}$

4.
Question 4

Remember that in the projection diagram, the vector projection *is* the green vector:



Let

$$\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

$\mathbf{r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and let $\mathbf{s} = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$.

What is the vector projection of **s** onto **r**?

1 / 1 point

$$\begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 30 \\ -20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 30 \\ -20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ -8 \\ 0 \end{bmatrix}$$

○ $\begin{bmatrix} \mathbf{6} \\ \mathbf{-8} \\ \mathbf{0} \end{bmatrix}$

Correct

The vector projection of **s** onto **r** can be calculated with the formula $\frac{\mathbf{s} \cdot \mathbf{r}}{\mathbf{r} \cdot \mathbf{r}}\mathbf{r}$.

5.

Question 5

Let

$$\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$$

a = $\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 12 \end{bmatrix}$.

Which is larger, $|\mathbf{a}+\mathbf{b}|$ or $|\mathbf{a}|+|\mathbf{b}|$?

1 / 1 point

- ◉ $|\mathbf{a}+\mathbf{b}| < |\mathbf{a}|+|\mathbf{b}|$
- ○ $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}|+|\mathbf{b}|$
- ○ $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}|+|\mathbf{b}|$

Correct

In fact, it has been shown that $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$ for every pair of vectors $\mathbf{a}$ and $\mathbf{b}$. This is called the triangle inequality; try to think about it in the 2d case and see if you can understand why.

6.

Question 6

Which of the following statements about dot products are correct?

1 / 1 point

- [ ] **The scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ is always the same as the scalar projection of $\mathbf{r}$ onto $\mathbf{s}$.**

- [x] **The size of a vector is equal to the square root of the dot product of the vector with itself.**

Correct

We saw in the video lectures that $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$.

- [x] **We can find the angle between two vectors using the dot product.**

Correct

We saw in the lectures that $\mathbf{r} \cdot \mathbf{s} = |\mathbf{r}||\mathbf{s}| \cos \theta$, where θ is the angle between the vectors. This can then be used to find θ.

- [x] **The vector projection of $\mathbf{s}$ onto $\mathbf{r}$ is equal to the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ multiplied by a vector of unit length that points in the same direction as $\mathbf{r}$.**

Correct

The vector projection is equal to the scalar projection multiplied by $\frac{\mathbf{r}}{|\mathbf{r}|}$.

- [ ] **The order of vectors in the dot product is important, so that $\mathbf{s} \cdot \mathbf{r} \neq \mathbf{r} \cdot \mathbf{s}$.**