Analyzing Voter Registration in the 2016 Presidential Election

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1 Introduction

Political campaigns analyze historical election data to understand how demographic factors influence voter registration rates. This information can inform optimal advertising strategies which can drum up more votes and win elections, especially during high profile election years when voter turnout can be substantially higher. In this report 2016 election data from North Carolina counties was used to investigate how geography, gender, age and party affiliation influence registration behavior. The structure of the paper is as follows…

First, I provide some brief background on hierarchical modeling and motivate their usage under the voter registration context. Next, I review the data used in the analysis and discuss challenges arising from dated population estimates. Lastly, I specify several inference models and review their results for main questions of interest.

2 Background Information

A hierarchical model is a statistical inference tool that replicates the natural nesting structure of observed data. This structure can violate traditional modeling assumptions, like i.i.d. errors, that can lead to poor uncertainty estimation if ignored. To address these shortcomings, hierarchical models define parameters at different group levels and permits them to vary, which accounts for group heterogeneity and can improve uncertainty estimation. For this analysis, voter registration and population data are generally aggregated by geography (most naturally by county) during collection. Individuals may display different registration tendencies depending on the urban/rural environment of a particular geography, which lends itself well to hierarchical modeling, allowing parameters to vary by grouping structure.

Hierarchical models can be estimated using Bayesian or Frequentist methods. Under Bayesian estimation, parameters are treated as random quantities and derived from posterior distributions using Bayes rule. Posteriors provide more direct uncertainty quantification, via credible regions, that avoid the interpretation caveats of Frequentist based confidence intervals. However, this interpretability comes with an increased computational cost as Bayesian inference relies on expensive sampling methods compared to likelihood based optimization. This analysis predominantly uses Bayesian estimation methods using conjugate priors to avoid excessive computational costs. I'll now review the data used for the analysis.

3 Data Overview

To investigate main questions of interest data from the 2010 U.S. Census was was enhanced with North Carolina voter registration records from the 2016 Presidential election. To combine the data, demographic field values were standardized, as the coding structure varies slightly between State and Federal agencies. In total, there were 6,858 county level observations with unknown gender (~15%) which lack a population estimate from the Census data and these records were dropped. Irrelevant registration fields denoting precinct location were also removed. Metadata information for the combined dataset and a sample observation, can be viewed in Table 1.

Table 1: Metadata Information

Field Name	Description	Sample
Geography	*County in North Carolina*	*BURKE*
Age	*Age Demographic Category*	*26−40*
Gender	*Gender Demographic Category*	*Female*
Hispanic	*Demographic Indicator of Hispanic Origin*	*Hispanic*
Race	*Race Demographic Category*	*WhiteAlone*
VoterFreq	*Number of Registered Voters in Specified Demographics*	*14*
Freq	*Total Population Count for Specified Demographics*	*128*
TotalCountyPopulation	*Total County Population*	*90912*
PartyCd	*Political Party Affiliation*	*DEM*

3.1 Population Migration

The 2010 Census is assumed to represent the voter population during the 2016 election, however, **16.3%** of all observations have more registered voters than the demographic population estimates from the Census. Are we observing potential voter fraud or is historical Census information from six years ago too dated to reflect accurate estimates? To understand the scope of this issue, the difference between total registered voters and Census estimates were computed and aggregated by county. Figure 1 shows summaries for geographies with more than five observations for ease of viewing. Conincidentally, **16%** of counties have median population difference greater than **100 individuals**, however, **70%** of counties have median difference less than **50** and **35%** have medians less than **10**. The majority of moderately small differences assuages some concerns of major population shifts over the six year period. Population estimates for the "invalid" observations are set to the sum of registered voters, but could also be inflated with a correction factor learned from other observations.

To minimize error, 30 counties were randomly selected using sampling weights inversely proportional to the percentage of faulty observations (total registered voters > demographic population). This framework can be extended to bootstrap resample different

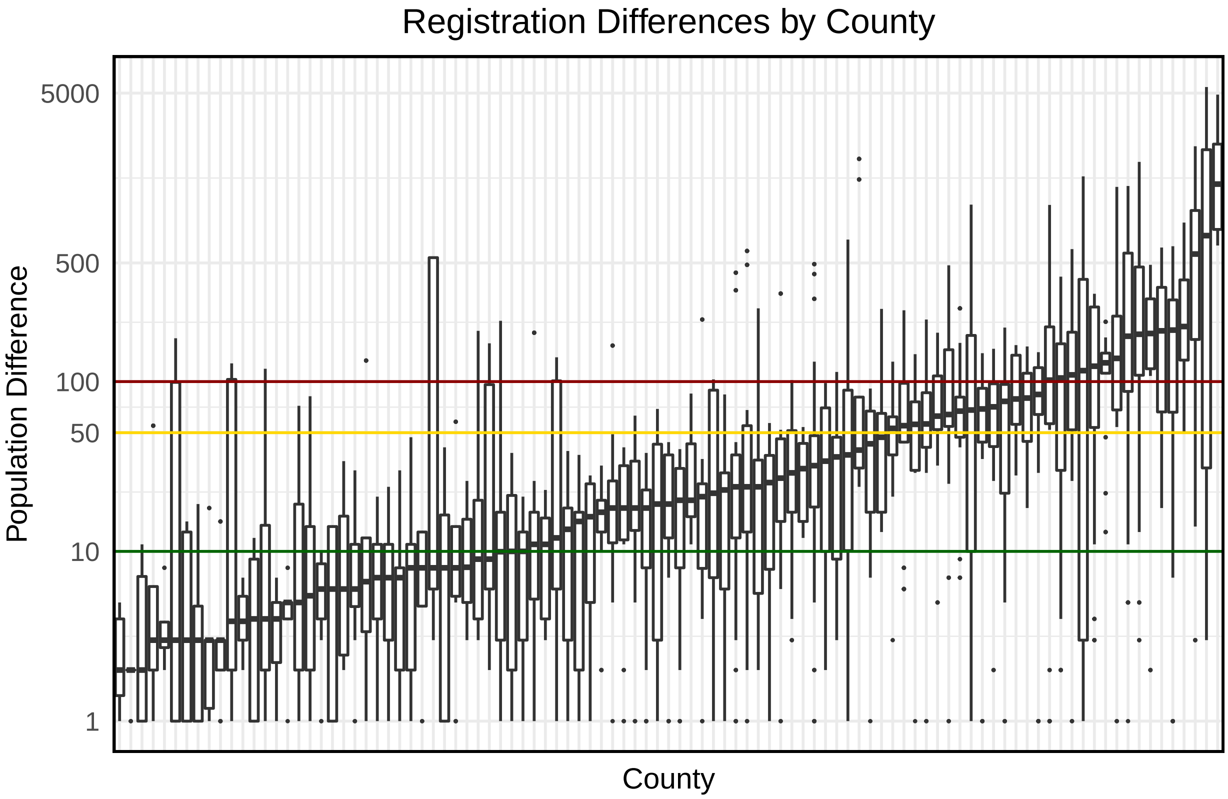


Figure 1: Registration Differences by County

county combinations for parameter estimation which reduces variation by steering our model towards more accurate data. An additional benefit of sub-sampling is a reduction in computation time for MCMC sampling for parameter estimation in Bayesian models.

3.2 Motivating a Multilevel Model

We are interested in understanding how demographic covariates affect the number of registered voters ("successes") relative to respective population estimates ("trials"), suggesting a binomial model. However, we do not know the political party affiliation for unregistered voters, as the Census does not ask about political affiliation and this membership is considered latent. This is information is necessary to draw conclusions on party membership registration influence. As an initial estimate for this missing quantity, I impute it based on the original population Census estimates and the political party distribution for registered voters. This new quantity is used in place of the original estimates for inference. To evaluate modeling decisions, like where to apply random effects, an exploratory data analysis (EDA) was conducted. Figure 2 is a facet plot of different binomial regressions on voter registration probability for two demographic categories and color coded by county. (A) fits regressions by county, age group and has substantially different trend lines across and within facets. (B) fits regressions by county, race and has much lower variability across facets. Plots for additional covariates can by found in the Appendix. Gender effects are consistent across geography, party affiliation displays consistent trends as does hispanic indicator. Continuous fields, like total county population, were also explored in the regression analysis, but were not included in the final model as including it lead to difficulty generating valid sampling distributions in brms (even with scaling and prior adjustments).

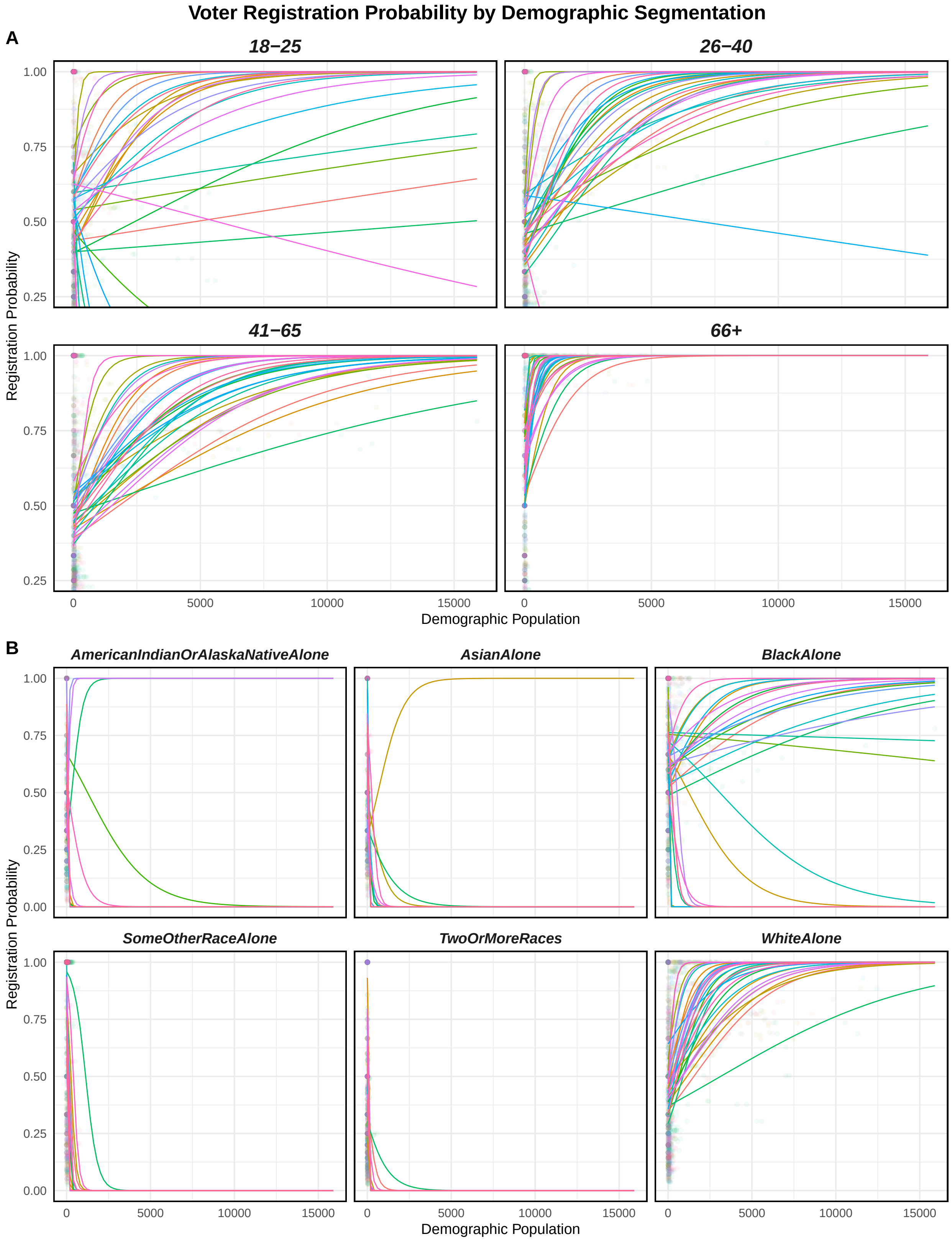


Figure 2: Voter Registration Behavior for (A) Age Category and (B) Race

4 Multilevel Models

4.1 Model Specifications

The modeling framework can now be specified as follows. Let (y_i, n_i, p_i) denote the number of voter registrations, demographic population and registration probability respectively for observation i. y_i is assumed to be distributed as a binomial random variable.

$$y_i \sim Binomial\left(n_i,\ p_i\right)$$

Individuals can only register to vote once, but binomial trials sample independently with replacement. The independent draw assumption seems reasonable, as the probability of any one individual registering to vote is most likely independent of any other individual's decision (more on this later). Additionally, $\sim 70\%$ of demographic categories in our dataset have populations $n_i \geq 30$. However, the demographic samples can be considered population estimates and may warrant a hypergeometric distribution, but this induces a dependence between y_i's which is unwarranted based on our independent registration assumption. Under the binomial model, several multilevel structures with varying complexity were evaluated. Table 3 displays the specifications.

Table 2: Random Effect Structures

	Fixed Effect Fields	**Random Intercept Fields**	**Random Slope Fields**
Model I	Gender, Hispanic, Race, PartyCd, Age	Geography	
Model II	Hispanic, Race, Age	PartyCD	Gender
Model III	Gender, Hispanic, Race	PartyCD	Age

4.1.1 Model I

Let μ denote a global intercept corresponding to a baseline demographic category with the following values: 'Female', 'Hispanic', 'AmericanIndianOrAlaskaNativeAlone', 'DEM' and '18-25'. Let θ_j denote a **county** random effect intercept for observations, i, in county j, X_i a vector of demographic indicator variables corresponding to column 1 in Table 2 and β the corresponding fixed effect estimates.

$$Logit(p_{ij})) = \mu + \theta_j + X_i\beta$$

Prior Specifications

$$\mu \sim N\left(0,1\right),\ \beta \sim N\left(0,1\right),\ \theta_j \sim N\left(0,\sigma\right)$$

$$\sigma \sim HalfCauchy\left(0, \frac{1}{2}\right)$$

4.1.2 Model II

In accordance with the model specification in Table 2, let μ denote a global intercept corresponding to a baseline demographic category with the following values: 'Hispanic', 'AmericanIndianOrAlaskaNativeAlone' and '18-25'. Let Ω_p denote the **party affiliation** random effect for party p, let g_{ipk} denote the **gender** category, k, for observation i in political party p and Γ_{pk} the random effect. Finally, let X_i be a vector of demographic indicator variables corresponding to column 1 in Table 2 and β the corresponding fixed effect estimates.

$$Logit(p_{ipk})) = \mu + \Omega_p + g_{ipk}\Gamma_{pk} + X_i\beta$$

Prior Specifications

$$\Omega_p \sim N\left(0, \sigma\right), \; \Gamma_{pk} \sim N\left(0, \gamma\right), \; \sigma/\gamma \sim HalfCauchy\left(0, \frac{1}{2}\right)$$

$$\mu \sim N\left(0, 1\right), \; \beta \sim N\left(0, 1\right)$$

4.1.3 Model III

In accordance with the model specification in Table 2, let μ denote a global intercept corresponding to a baseline demographic category with the following values: 'Female', 'Hispanic' and 'AmericanIndianOrAlaskaNativeAlone'. Let Ω_p denote the **party affiliation** random effect for party p, let a_{ipk} denote the **age** category, k for observation i in political party p and α_{pk} the random effect slope. Finally, let X_i be a vector of demographic indicator variables corresponding to column 1 in Table 2 and β the corresponding fixed effect estimates.

$$Logit(p_{ipk})) = \mu + \Omega_p + a_{ipk}\alpha_{pk} + X_i\beta$$

Prior Specifications

$$\Omega_p \sim N\left(0, \omega\right), \; \alpha_{pk} \sim N\left(0, \tau\right), \; \omega/\tau \sim HalfCauchy\left(0, \frac{1}{2}\right)$$

$$\mu \sim N\left(0, 1\right), \; \beta \sim N\left(0, 1\right)$$

5 Results

MCMC diagnostics, like trace plots, can be viewed in the Appendix for each model. The main questions of interest concern demographic factors influencing registration probabilities. We are particularly interested in the following italicized questions.

5.1 *How did different demographic subgroups register to vote?*

The fixed effect estimates from each model are plotted in Figure 3 with 95% credible regions (CR). Note, not all models include identical fixed effects, but the overlapping estimates tend to display more shrinkage with increased model complexity. Starting from the top of Figure 3, all Race indicators have significant impact on registration rate. Party affiliation displays no significant effect. Hispanic and Gender estimates are also significant. Older individuals are also more likely to register, but this can be attributed to having more opportunities/elections to do so.

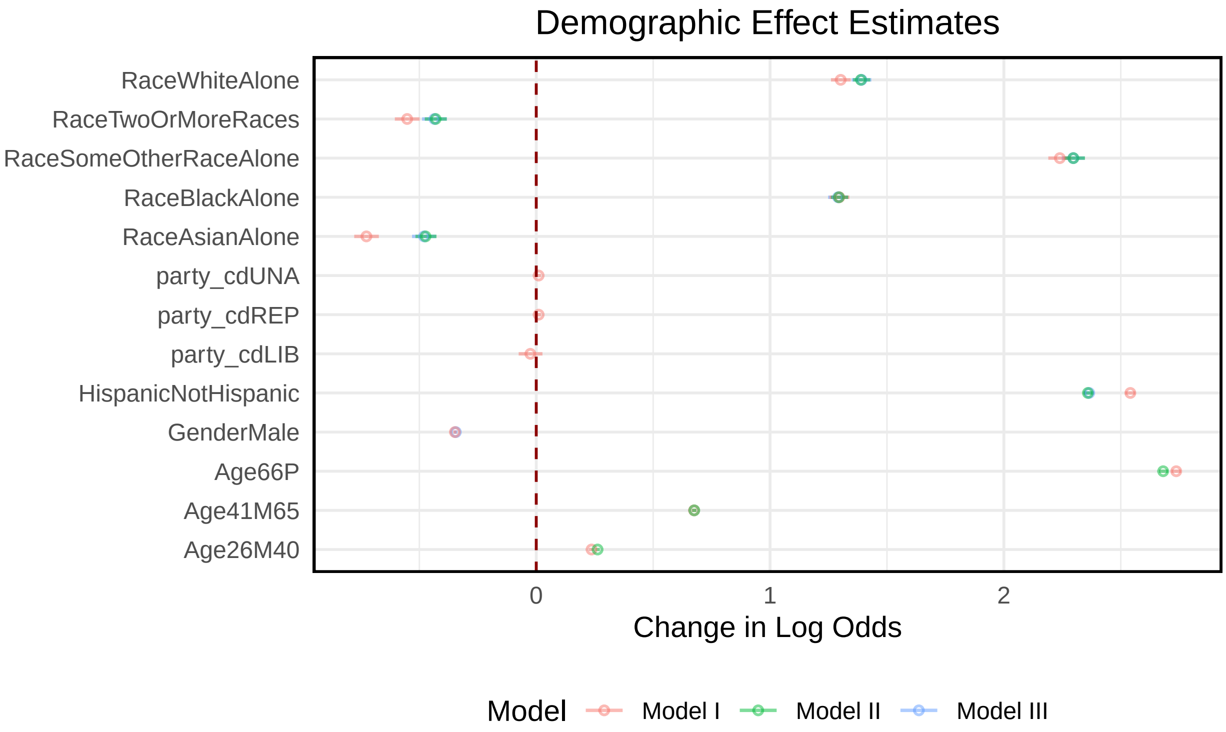


Figure 3: Fixed Effect Estimates From Each Model

5.2 *Did the overall odds of registering differ by county in 2016?*

Only 30 counties are included in this analysis and the list can be viewed in the Appendix. The random intercepts from Model I, with 95% CR are displayed in Figure 4. Tyrrell, the only county in the dataset without any "invalid" observations has the lowest registration effect, while **43%** of counties have no detectable effect.

5.3 *How did the registration rates differ between genders for different party affiliations?*

Using Model II we can look at the gender effect within each political party from random slope estimates and assess the relative change in log odds from the baseline gender (Female). Figure 5 shows 95% CRs and point estimates. Males across all political parties are less likely to register than corresponding females and republican males register at the highest rates.

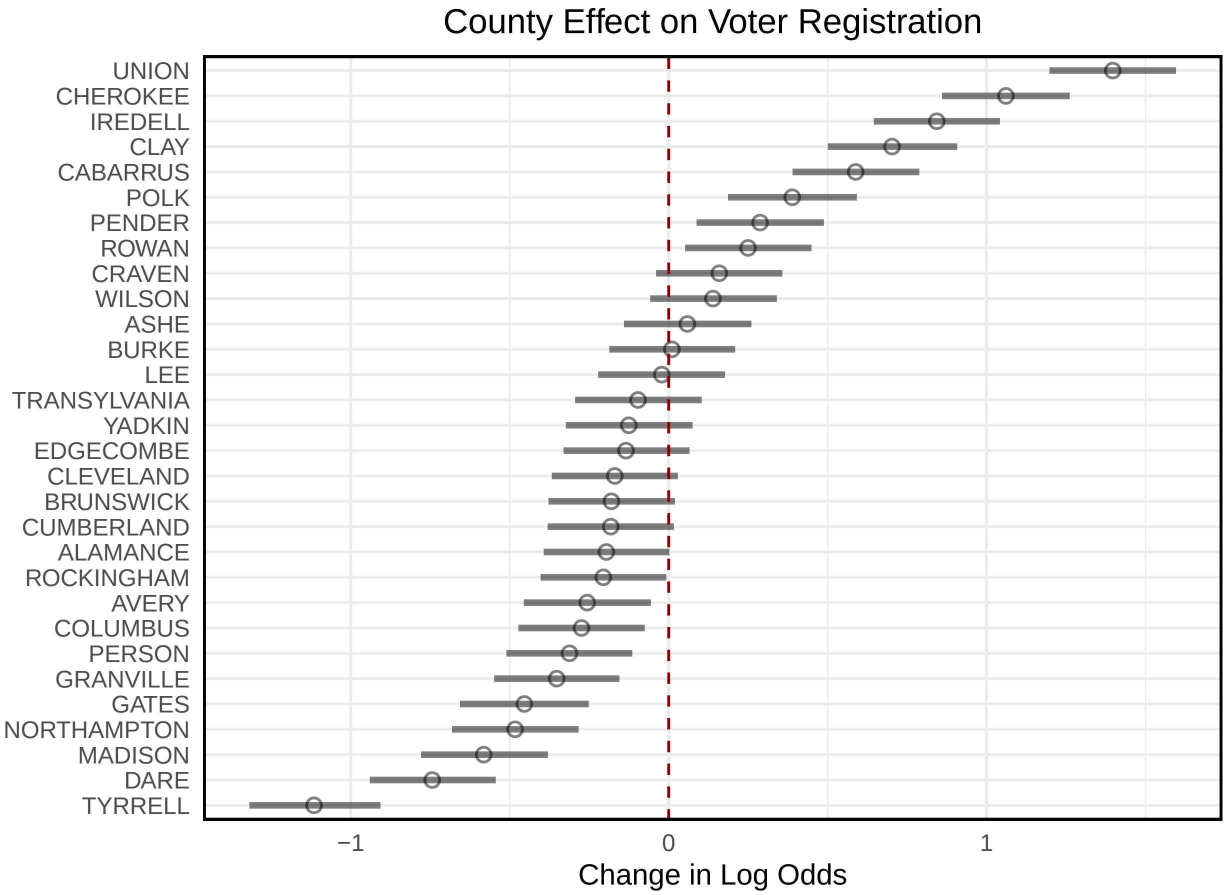


Figure 4: County Level Intercept Estimates from Model II

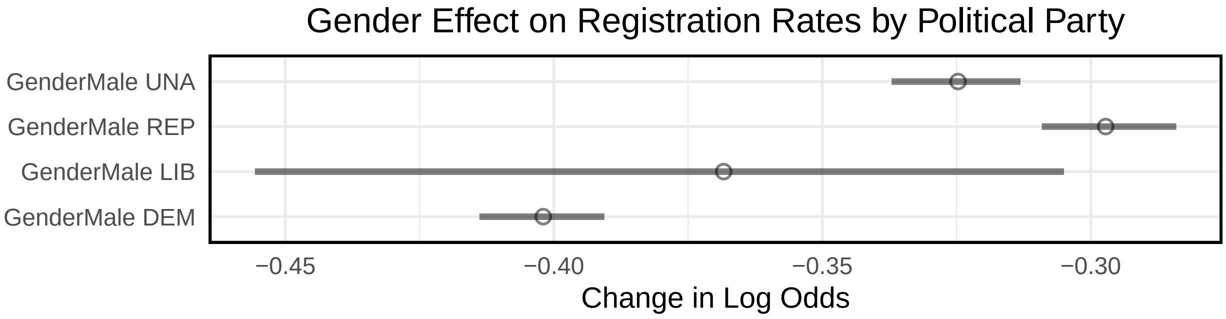


Figure 5: Random Slope Estimates (Model II) for Gender across Political Party

5.4 *Registration differences between age groups by party affiliation?*

Using Model III we can look at the age effect within each political party from random slope estimates assess the relative change in log odds from the baseline age (18-25). Figure 6 shows 95% CRs and point estimates. In almost all age categories republicans are more likely to register than other political parties. We also see registration rate across all parties as age increases.

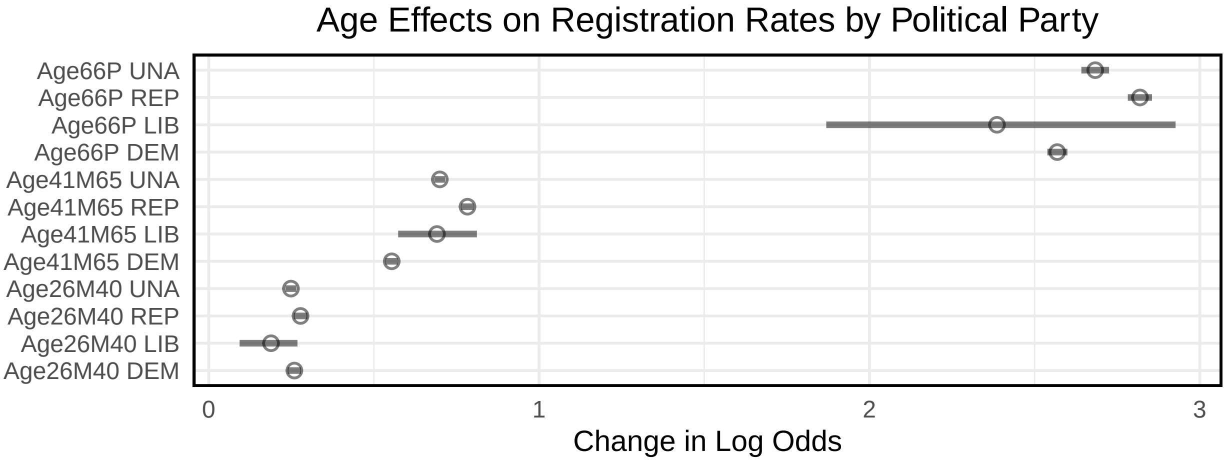


Figure 6: Random Slope Estimates (Model III) for Age across Political Party

5.5 Predictive Accuracy

In addition to inference, we are interested in predicting registered voters for each demographic combination. 70% of our data is unseen by the model and constitutes the test set. For ease of viewing, I've randomly selected one unseen county and display the predictive accuracy for every demographic combination in Figure 7. The error is computed as the difference between the actual number of registered voters and the model prediction. Dashed lines on Figure 7 denote average error. Surprisingly, Model I has the best predictive performance, with average error roughly 10 individuals, but median error of less than one.

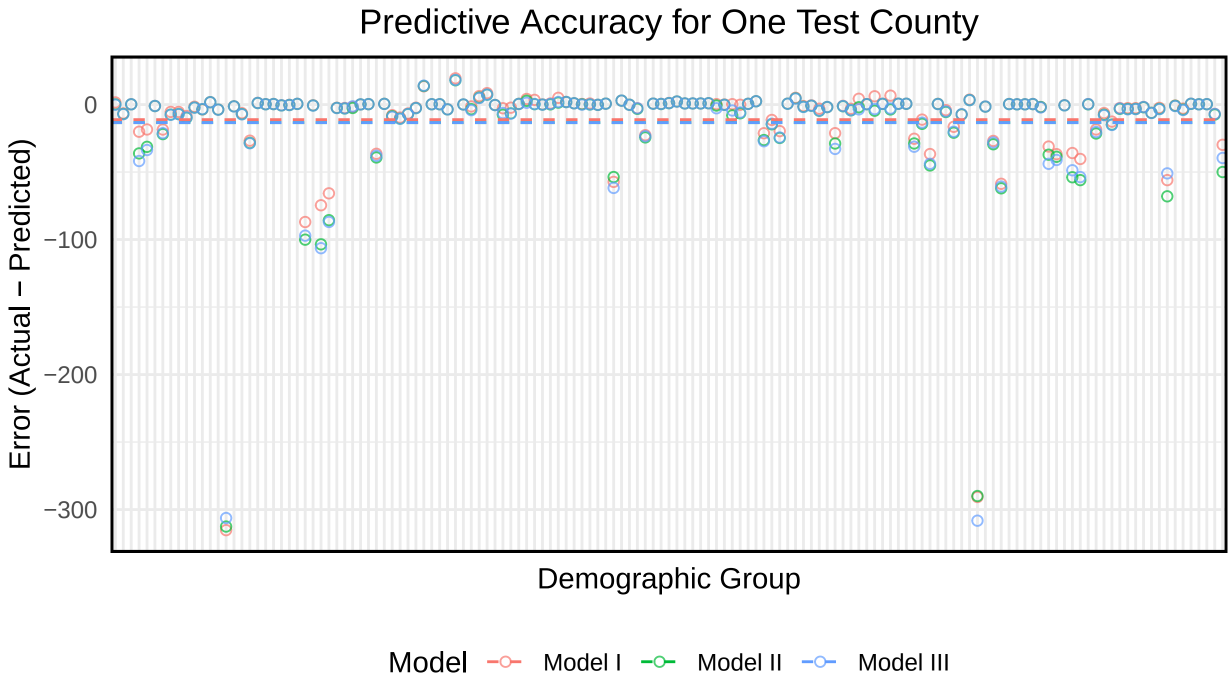


Figure 7: Predictive Accuracy for One Unseen County Across All Demographic Groups

6 Limitations & Conclusion

Several limitations have been mentioned throughout and are summarized. First, we have concerns about the viability of the 2010 Census for population estimates in 2016 as some observations have more voter registrations than population estimates. A weighted sub-sampling strategy was introduced to minimize the statistical impact, but only addresses detectable observations where $registration > population$. A valid concern is that all demographic populations undergo some change over the six year period. A potential solution is to impute or average the demographic populations in 2016 using estimates from both the 2010 and 2020 Censuses.

Another limitation is from the sub-sampling strategy which limits our data usage to only 30% of available counties and introduces additional variability. To alleviate this, one could leverage the entire dataset by either acquiring more computation power, letting models run for longer (most complex model takes $\sim$ 12 hours to run), adopt a bootstrap resampling procedure to run the model multiple times with different county combinations (also requires more computation time), and/or specify simpler models with less random effects that take less time to run.

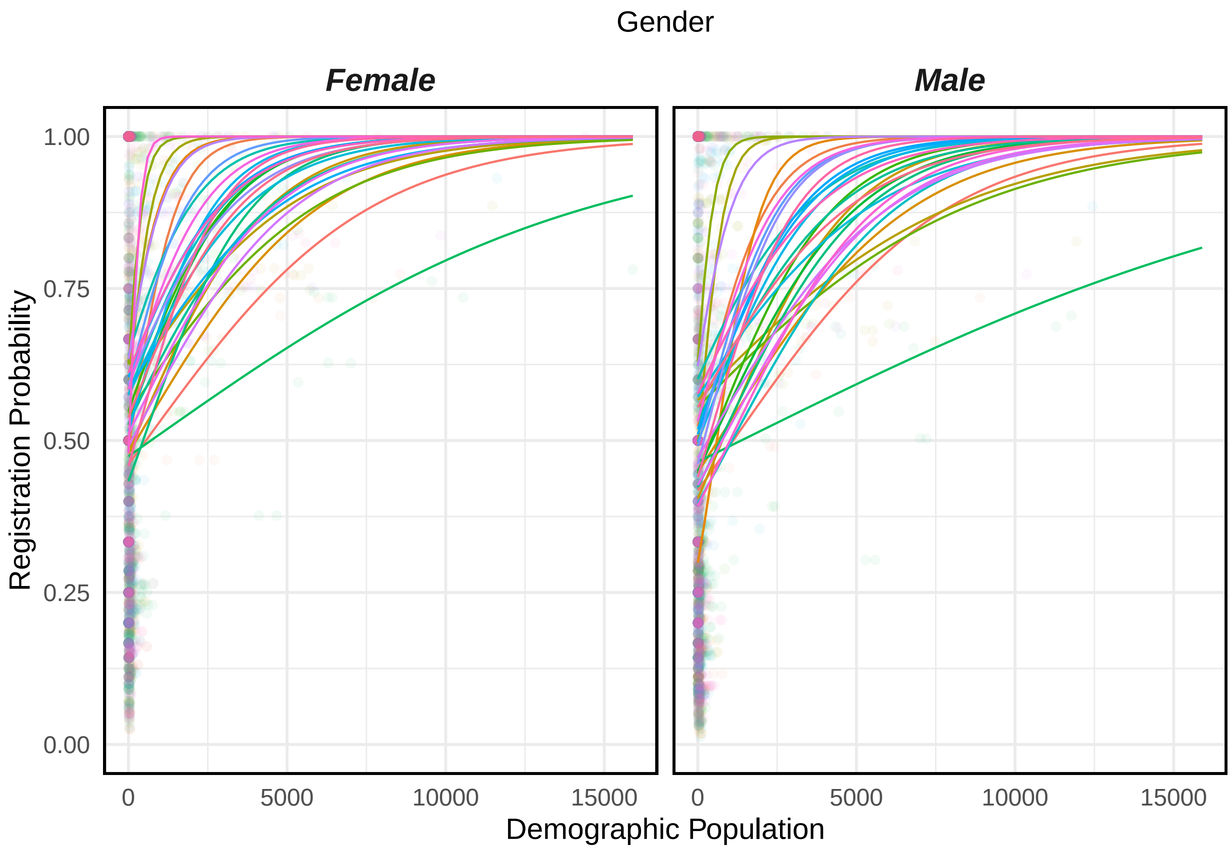
An additional limitation is the unknown population estimates for political party affiliations. The Census data only quantifies innate categories and does not contain information regarding party affiliation. These totals were imputed from available fields based on the party distribution from registered voters within each demographic group. This introduces another source of error, but is necessary if we want to make inferences about party affiliation.

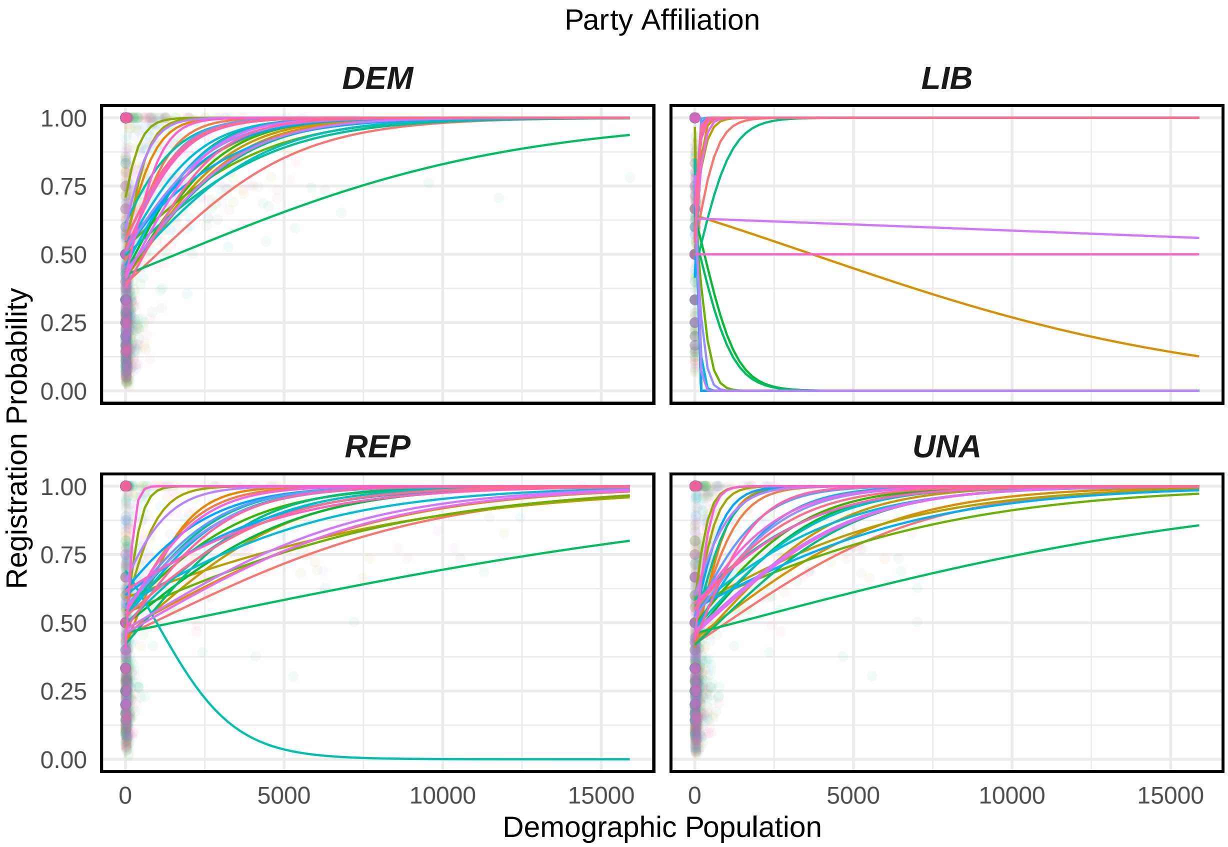
Lastly, the Gelman book indicates overdisperion is almost always a problem in binomial regression models as there is no independent variance term. If overdispersion is present, our estimates will be overconfident. Time constraints prevented testing for overdispersion and applying an uncertainty adjustment to estimates.

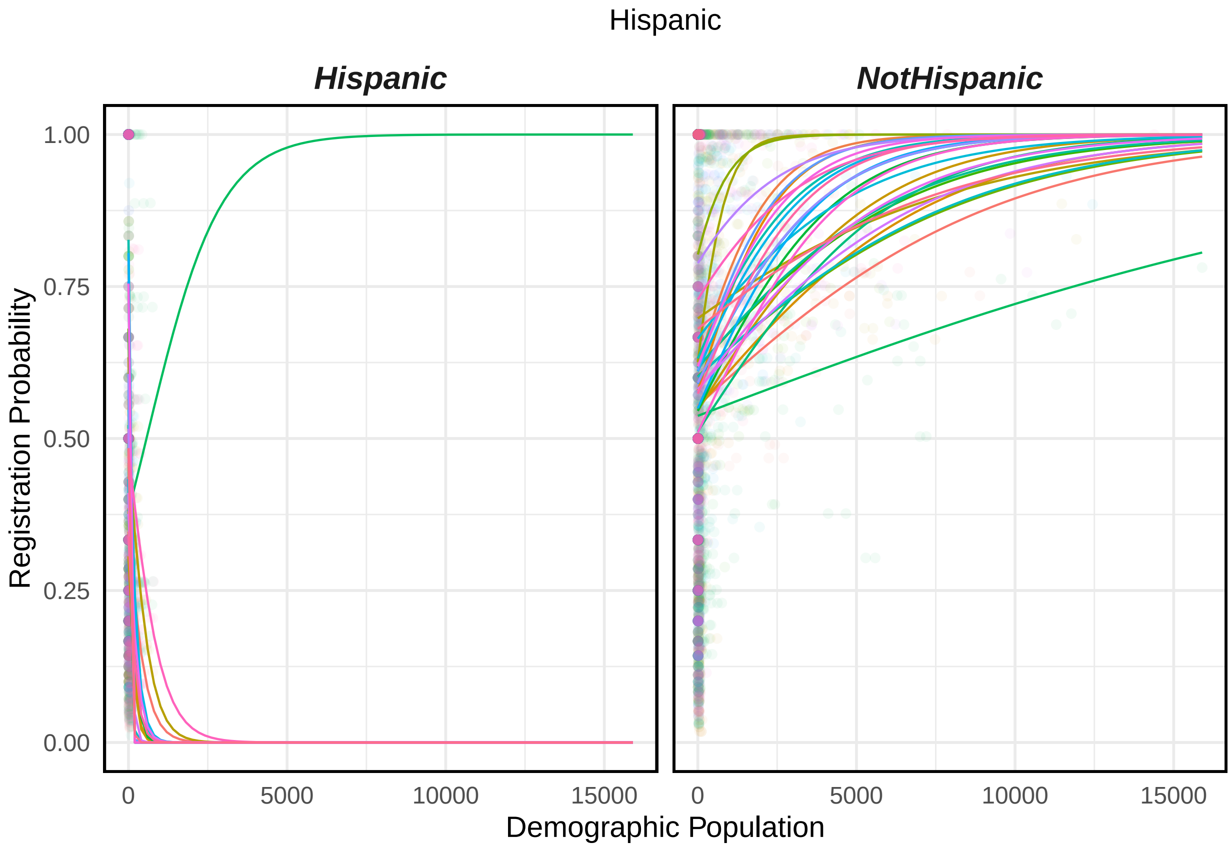
Historical voting records and Census population estimates were used to evaluate demographic differences in registration tendencies. Three Bayesian multilevel models were created to address specific questions of interest using different effect structures. The full dataset was sub-sample to improve computation time, but may lead to higher variance estimates. Preliminary results suggest significant differences in registration tendencies across demographic categories in North Carolina.

7 Appendix

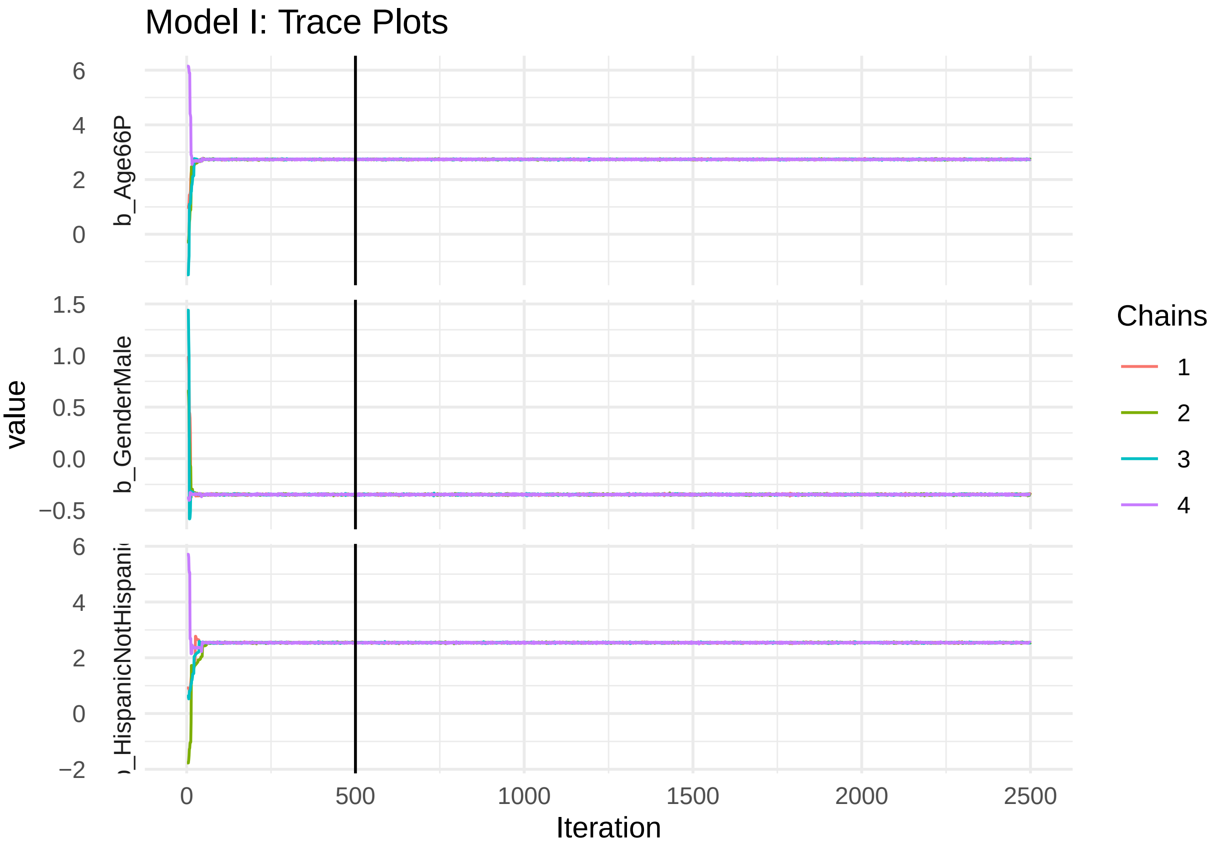
Additional Covariate Plots

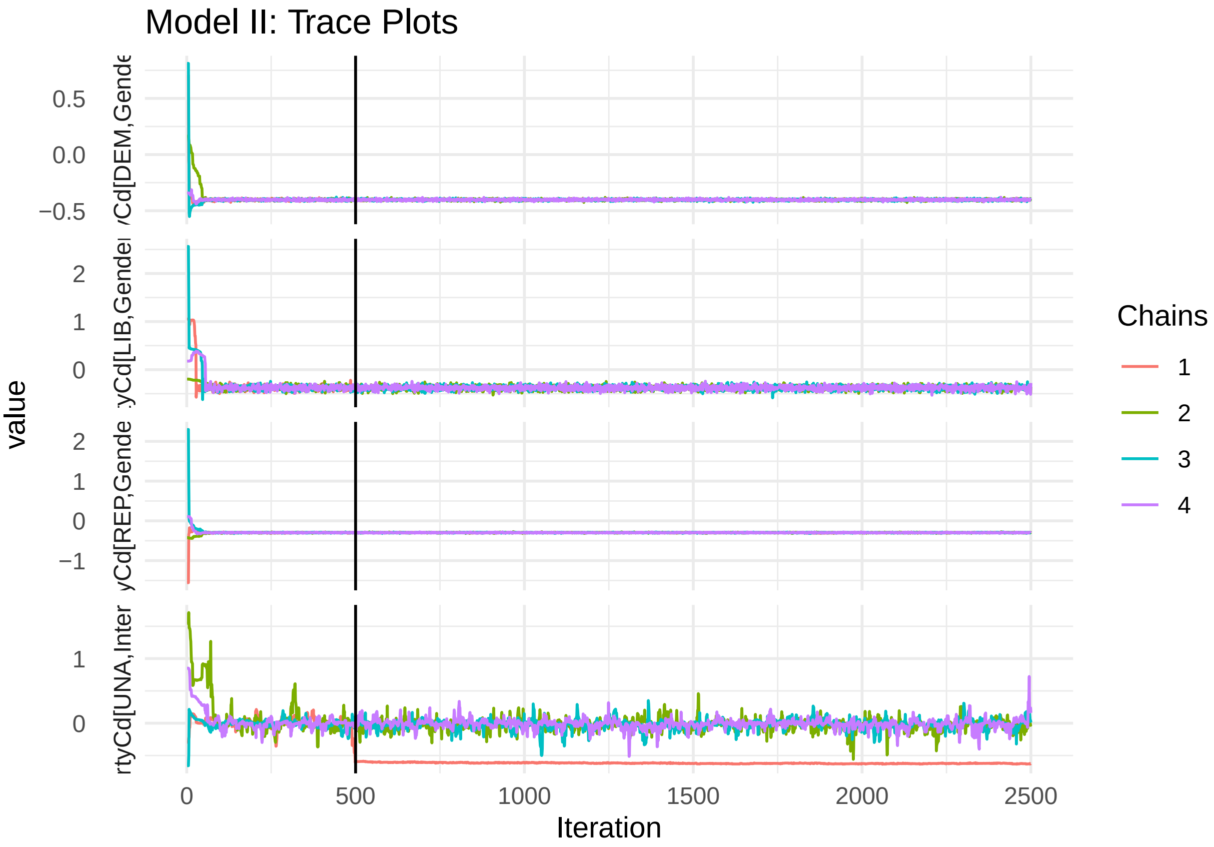


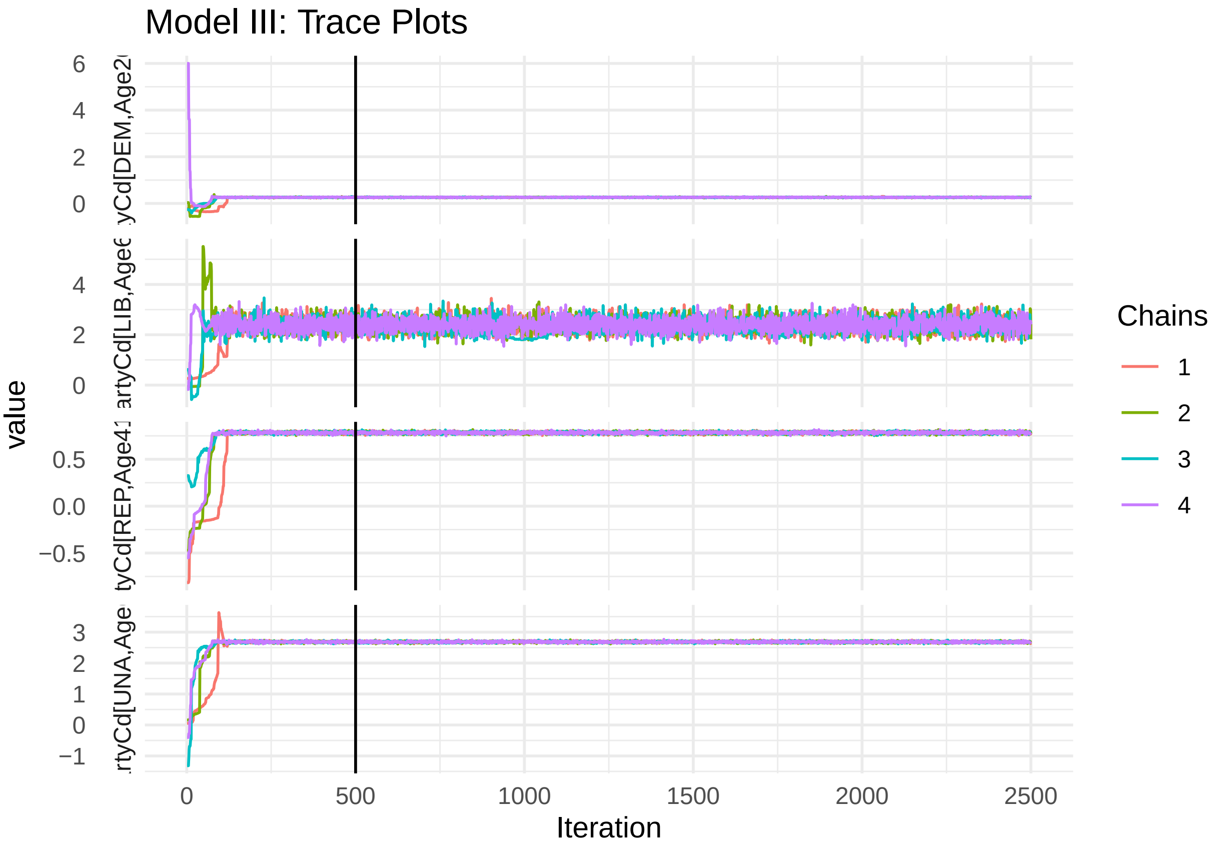

Party Affiliation
DEM
LIB
REP
UNA
Registration Probability
0.00
0.25
0.50
0.75
1.00
0
5000
10000
15000
Demographic Population


Hispanic
Hispanic
NotHispanic
Registration Probability
1.00
0.75
0.50
0.25
0.00
0
5000
10000
15000
0
5000
10000
15000
Demographic Population

Trace Plots


Model I: Trace Plots
value
b_Age66P
6
4
2
0
b_GenderMale
1.5
1.0
0.5
0.0
−0.5
b_HispanicNotHispani
6
4
2
0
−2
0
500
1000
1500
2000
2500
Iteration
Chains
1
2
3
4


Model II: Trace Plots
value
/Cd[DEM,Gende
0.5
0.0
−0.5
yCd[LIB,Gende
2
1
0
yCd[REP,Gende
2
1
0
−1
rtyCd[UNA,Inter
1
0
0
500
1000
1500
2000
2500
Iteration
Chains
1
2
3
4



County Sample List

Geography
ALAMANCE
ASHE
AVERY
BRUNSWICK
BURKE
CABARRUS
CHEROKEE
CLAY
CLEVELAND
COLUMBUS
CRAVEN
CUMBERLAND
DARE
EDGECOMBE
GATES
GRANVILLE
IREDELL

Geography
LEE
MADISON
NORTHAMPTON
PENDER
PERSON
POLK
ROCKINGHAM
ROWAN
TRANSYLVANIA
TYRRELL
UNION
WILSON
YADKIN