Advanced Autonomous Systems

Kinematic Process Model for our UGV Platform

The states to be estimated are: $\mathbf{X} = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix}$ E1

We consider the following Process Model, for our platform. It is a discrete time version of the continuous 2D kinematic model of a tricycle or a car.

$$\mathbf{X}(k+1) = f\left(\mathbf{X}(k), \mathbf{u}(k)\right)$$

$$\Downarrow$$

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + T \cdot \begin{bmatrix} v(k) \cdot \cos(\phi(k)) \\ v(k) \cdot \sin(\phi(k)) \\ \omega(k) \end{bmatrix}$$

$$\mathbf{X}(k) = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix}, \quad \mathbf{u}(k) = \begin{bmatrix} v(k) \\ \omega(k) \end{bmatrix}$$

In which ***T*** is the sample time of our discrete version, the index ***k*** means that it corresponds to time $k \cdot T$; $v(k)$ and $\omega(k)$ are, respectively, the measured speed (longitudinal velocity) and angular rate at time ***k.***

The measurements ***v*** and ***w*** are the inputs of this process model. Note that, in this case, we may not actually control the process model inputs, but we are able to know them (e.g. by measuring them). In our case, we know the inputs by measuring them through sensors (gyroscope and speed meter).

This process model is, as usually happens, imperfect. There are many sources of error in our model. We can mention some relevant ones:

1) Although we measure the inputs, those measurements are polluted with noise.
2) The platform may not be operating in a pure 2D contexts, i.e. the terrain is not flat, suspension's effects, etc.
3) The velocity vector at the point (x,y) may not be parallel to the heading vector, e.g. due to skidding and platform's suspension effects, etc.

If we applied the process model considering the measured inputs (that are polluted versions of the actual inputs) we obtain the following approximated result:

$$\mathbf{X}(k+1) = f\left(\mathbf{X}(k), \mathbf{u}(k)\right) = f\left(\mathbf{X}(k), \mathbf{u}_m(k) + \delta\mathbf{u}(k)\right) = f\left(\mathbf{X}(k), \mathbf{u}_m(k)\right) + \delta f$$

$$\delta f = \left(f\left(\mathbf{X}(k), \mathbf{u}(k)\right) - f\left(\mathbf{X}(k), \mathbf{u}_m(k)\right)\right)$$

where $\mathbf{u}(k)$ is the real input to the system and $\mathbf{u}_m(k)$ is its known value (e.g. measured value); $\delta\mathbf{u}(k)$ is the error, the discrepancy between the assumed inputs, and the real inputs. Obviously, we do not have the real values of the inputs to evaluate this error. However, we may have the statistical properties of the error $\delta\mathbf{u}(k)$. Now, what we really need are the statistical properties of δf . How to get them?

If the uncertainty $\delta\mathbf{u}(k)$ is Gaussian (or approximately Gaussian) then we can approximate the uncertainty δf as a Gaussian uncertainty as well. We just apply a linearization of the function ***f***. One way to linearize it, is by a first order Taylor.

$$\delta f = \left.\frac{\partial f}{\partial \mathbf{u}}\right|_{\mathbf{u}=\mathbf{u}_{measured}} \cdot \delta\mathbf{u} = \mathbf{F}_{\mathbf{u}} \cdot \delta\mathbf{u}$$

If we have the covariance of the error $\delta\mathbf{u}$ (we call it $\mathbf{P}_{\mathbf{u}}$) then we can obtain the covariance of the uncertainty δf (exclusively due to the noise $\delta\mathbf{u}$); we call it $\mathbf{Q}_{\mathbf{u}}$; we obtain $\mathbf{Q}_{\mathbf{u}}$ according to $\mathbf{Q}_{\mathbf{u}} = \mathbf{F}_{\mathbf{u}} \cdot \mathbf{P}_{\mathbf{u}} \cdot \mathbf{F}_{\mathbf{u}}{}^T$. Note that in this equation we use the notation $\mathbf{P}_{\mathbf{u}}$ for indicating the covariance of the noise which pollutes the inputs, and we use $\mathbf{Q}_{\mathbf{u}}$ to indicate the covariance of the uncertainty/noise of the process model, exclusively as consequence of the uncertainty that pollutes our knowledge about the inputs.

In our case of study, we know that the speed and angular rate, which we measure, are noisy. We know the standard deviation of those noises (so we also have their joint covariance matrix $\mathbf{P}_{\mathbf{u}}$). Now we apply the concept to our process model:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + T \cdot \begin{bmatrix} v(k)\cdot\cos(\phi(k)) \\ v(k)\cdot\sin(\phi(k)) \\ \omega(k) \end{bmatrix}$$

$$\mathbf{F}_{\mathbf{u}} = \left.\frac{\partial f}{\partial(v,\omega)}\right|_{\substack{\mathbf{x}=\hat{\mathbf{x}},\\ v=v_m,\omega=\omega_m}} = \left.\begin{bmatrix} T\cdot\cos(\phi(k)) & 0 \\ T\cdot\sin(\phi(k)) & 0 \\ 0 & T \end{bmatrix}\right|_{\phi(k)=\hat{\phi}(k)}$$

This Jacobian matrix is evaluated at the current "point of operation", i.e. current expected values of the system state and the currently assumed inputs (i.e. the one we measure). You can see that, in this particular case, it only involves the estimated value of $\phi(k)$, because the process model is linear respect to the rest of the variables.

In this section we have presented the way of considering the errors which are present in the assumed inputs of the process model. This situation is usual in our case, where we use the measured speed and angular rate as inputs in our kinematic model of the platform.

However, there are other errors in our assumed kinematic model; e.g. we assume that the displacement of the platform is in the direction of the heading; however, that assumption is wrong. We can model this discrepancy by assuming the existence of an additional additive noise, which affects the *x* and *y* components of the model. We will discuss about this matter in class.