Advanced Autonomous Systems
Estimating the "speed"

We call "speed" (which is a scalar) the longitudinal component of the velocity vector.
We consider the case in which no speed measurements are available (e.g. no such sensor is installed on the platform). This concept is also useful in cases in which the speed measurements are available but are of low quality or are sampled at an inadequate sampling rate. In such cases, the speed can be estimated, as well as the rest of the states of interest.

We propose the following augmented process model.

The states to be estimated are four: $\mathbf{X} = \begin{bmatrix} x & y & \phi & v \end{bmatrix}^T$ E1

The following proposed Process Model, is called "constant velocity" kinematic model,

$$\frac{d}{dt}\mathbf{X}(t) = \frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \\ \phi(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} v(t)\cdot\cos(\phi(t)) \\ v(t)\cdot\sin(\phi(t)) \\ \omega(t) \\ 0 \end{bmatrix}, \quad \mathbf{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ \phi(t) \\ v(t) \end{bmatrix} \in \mathbb{R}^4, \quad \mathbf{u}(t) = \omega(t) \in \mathbb{R}^1 \qquad \text{E2}$$

An approximated discrete time model (based on Euler's approximation), for a sample time **T**, is the following one,

$$\mathbf{X}(k+1) = f\left(\mathbf{X}(k), \mathbf{u}(k)\right)$$
$$\Downarrow \qquad \text{E3}$$
$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \\ v(k) \end{bmatrix} + T\cdot\begin{bmatrix} v(k)\cdot\cos(\phi(k)) \\ v(k)\cdot\sin(\phi(k)) \\ \omega(k) \\ 0 \end{bmatrix}$$

In which the state $v(k)$ is the longitudinal velocity (the longitudinal component of the velocity vector, we also call it "speed") of the platform (which, in previous projects, was provided by a sensor, i.e. it was one of the process model's inputs).

We considered this assumption: **CONSTANT SPEED**. We could, in theory, use an accelerometer for measuring accelerations, and avoid this simplification. However, we do not try that approach in this case.
In addition, this is an approximated discrete time model, which we obtained by applying Euler approximation.

Uncertainty in the Process Model

1) The platform may experiment translation in the transversal direction (perpendicular to the heading direction).

2) Uncertainty that pollutes the inputs of the process model (i.e. measured angular rates, $\omega(k)$), as we had considered in the previous projects.

3) Real longitudinal velocity **is not** constant, i.e. there exist acceleration. However, the acceleration is bounded. For our UGV, the acceleration is bounded to be $|a(t)| < a_{\max} = 1.2 \text{ m/s}^2$. Consequently, we can assume that

$$\Delta v(k) = v(k) - v(k-1)$$
$$-T \cdot a_{\max} < \Delta v(k) < T \cdot a_{\max}$$

Although this error, due to our assumption that the speed is constant (i.e. $\Delta v(k) = v(k) - v(k-1) = 0$), is not Gaussian white noise; we assume it is (here, we introduce an additional approximation), because it does usually fluctuate around 0 and it is bounded. We assume then the following process model, for the longitudinal velocity component,

$$v(k+1) = v(k) + T \cdot 0 + \xi_{dv}(k)$$
$$\sigma_{dv} = T \cdot a_{\max}$$

In which the component $\xi_{dv}(k)$ represents the uncertainty, which pollutes the process model of the speed state. The proposed value for σ_{dv} approximates the standard deviation of this uncertainty.

The **Q** matrix, as result of the assumptions that we have mentioned, is then

$$\mathbf{Q} = \begin{bmatrix} .. & .. & .. & 0 \\ .. & .. & .. & 0 \\ .. & .. & .. & 0 \\ 0 & 0 & 0 & (T \cdot a_{\max})^2 \end{bmatrix} \qquad \text{E4}$$

The upper 3x3 submatrix of **Q**, which is not explicitly shown in E4, corresponds to the covariance of the process model of the states (x, y, ϕ). We obtain it by the same way we applied in previous cases.

Observation Model

Provided that we have available observations of range or/and bearing, the system would be observable, i.e. the full state vector, including the longitudinal velocity, would be estimated.
The observation model does not change, respect to the previous cases (in which the longitudinal velocity is not estimated). The observations related to each observed landmark are range and bearing (as before):

$$\mathbf{Y} = \begin{bmatrix} r \\ \alpha \end{bmatrix} \qquad \text{E5}$$

Their observation models are:

$$h(\mathbf{X}) = \begin{bmatrix} h_1(x,y,\phi,v) \\ h_2(x,y,\phi,v) \end{bmatrix} = \begin{bmatrix} \sqrt{(x_a - x)^2 + (y_a - y)^2} \\ \tan^{-1}(y_a - y, x_a - x) - \phi + \pi/2 \end{bmatrix}$$

$$\Downarrow \qquad \text{E6}$$

$$= \begin{bmatrix} \sqrt{(x_a - x)^2 + (y_a - y)^2} \\ \tan^{-1}(y_a - y, x_a - x) - \phi + \pi/2 \end{bmatrix} = \begin{bmatrix} r \\ \alpha \end{bmatrix}$$

Note: The function $\tan^{-1}(y,x)$ is actually implemented by the function $\text{atan2}(y,x)$ in Matlab and other programming languages. It is actually the argument (angle) of the 2D vector (x,y).

By linearization of the observation function, $h(x,y,\phi,v)$ (expressed in E6), at the PRIOR expected value of the estimates of (x,y,ϕ,v), we obtain the associated **H** matrix.

Note that now we have four states, consequently the **H** matrix has a size 2x4 (in place of 2x3, considered in the previous projects, in which the state vector had dimensionality =3).

Consequently,

$$\mathbf{H} = \left.\frac{\partial h(\mathbf{X})}{\partial \mathbf{X}}\right|_{\mathbf{X}=\hat{\mathbf{X}}} = \left.\left[\begin{array}{c|c|c|c} \dfrac{\partial h_1}{\partial x} & \dfrac{\partial h_1}{\partial y} & \dfrac{\partial h_1}{\partial \phi} & \dfrac{\partial h_1}{\partial v} \\ \hline \dfrac{\partial h_2}{\partial x} & \dfrac{\partial h_2}{\partial y} & \dfrac{\partial h_2}{\partial \phi} & \dfrac{\partial h_1}{\partial v} \end{array}\right]\right|_{\mathbf{X}=\hat{\mathbf{X}}} \qquad \text{E7}$$

$$\mathbf{H} = \left.\left[\begin{array}{c|c|c|c} -\dfrac{(x_a - x)}{\sqrt{(x_a - x)^2 + (y_a - y)^2}} & -\dfrac{(y_a - y)}{\sqrt{(x_a - x)^2 + (y_a - y)^2}} & 0 & 0 \\ \hline \dfrac{(y_a - y)}{(x_a - x)^2 + (y_a - y)^2} & \dfrac{-(x_a - x)}{(x_a - x)^2 + (y_a - y)^2} & -1 & 0 \end{array}\right]\right|_{\mathbf{X}=\hat{\mathbf{X}}} \qquad \text{E8}$$

Note for students: You should verify that the partial derivatives in Equation E8 are correct.
Question for students: The 4th column is populated by zeroes, why? Does it mean that we could not observe its associated state, ***v***?

Additional details about this estimation approach will be discussed in class. For instance, we will describe how to initialize this EKF process.

Note: Even in cases in which we do actually measure the speed, we may have a slow sensor (whose sample rate is not the one we need); additionally, the measurements from that sensor may be noisy; which means it is convenient to use a “filtered” version of it, i.e. we estimate it, in place of using it directly. In such a case, we can use the speed measurements for implementing observations (updates).