First, for the null hypothesis, we assume complete independence between the three variables.

If we refer to rows, columns and planes as r, c, p, (with dimensions R, C, P) with each cell containing the observed value o_{rcp} then we can define the total, t, for a particular row, r, as:

$$t_{r++} = \sum_{c=1}^{C} \sum_{p=1}^{P} o_{rcp}$$

(where a subscript of + indicates summation over the appropriate index)

Similarly for columns and planes:

$$t_{+c+} = \sum_{r=1}^{R} \sum_{p=1}^{P} o_{rcp}$$

$$t_{++p} = \sum_{r=1}^{R} \sum_{c=1}^{C} o_{rcp}$$

The expected value for a given cell, e_{rcp} is then:

$$e_{rcp} = \frac{t_{r++} \times t_{+c+} \times t_{++p}}{N^2}$$

(where N is the total number of observations).

We then calculate the chi-squared value as normal:

$$\chi^2 = \sum_{r=1}^{R} \sum_{c=1}^{C} \sum_{p=1}^{P} \frac{(o_{rcp} - e_{rcp})^2}{e_{rcp}}$$

The number of degrees of freedom, D, is simply:

$$D = (R-1)(C-1)(P-1)$$

The calculation of the expected values is based on information at:

- `web.ntpu.edu.tw/~cflin/Teach/Cate/06CateUEN05ThreeWayPPT.pdf`
- `onlinecourses.science.psu.edu/stat504/book/export/html/102`
- Lienert & Wolfrun (1980) *Biometrical Journal* **22**:159–167 (`onlinelibrary.wiley.com/doi/10.1002/bimj.4710220209/pdf`).