UCONN
UNIVERSITY OF CONNECTICUT

Department of Computer Science and Engineering

Data Structures and Object-Oriented Design

(CSE – 2050)

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Quick Recap

- Trees
  - Root, Parent, Child, leaf node, Depth/level, etc
- Binary Search Trees
  - Each node can have at most two child
  - Parent is greater than the left and smaller than the right
- Operations –
  - Insert
  - Search – Max, Min
  - Delete – Leaf node, Node having one child, Node having two children
- Traversal – Pre-order, In-order, Post-order

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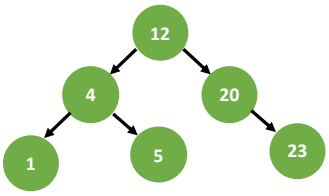
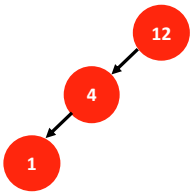
Quick Recap

- Balanced and Imbalanced Trees



BALANCED TREE



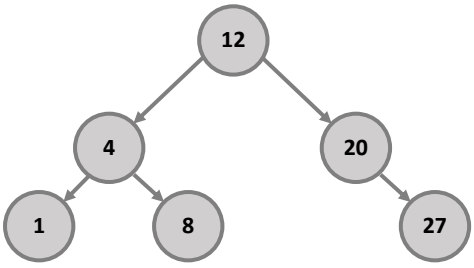
IMBALANCED TREE

Binary Search Trees

- To determine whether the tree is balanced or not, we have to measure its height
- Height of a tree (or a node) is the longest path from the root (or from the node) to a leaf node

height = max (left child's height , right child's height) + 1

The height of a NULL node is -1
➔ leaf nodes have height 0.



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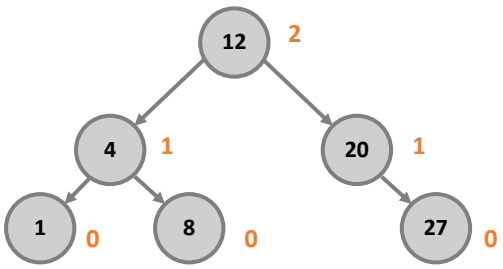
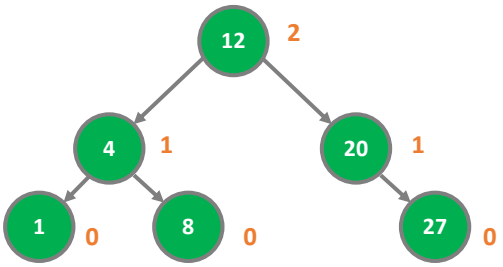
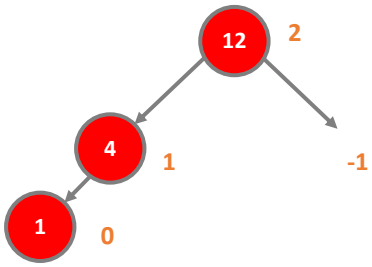


Binary Search Trees

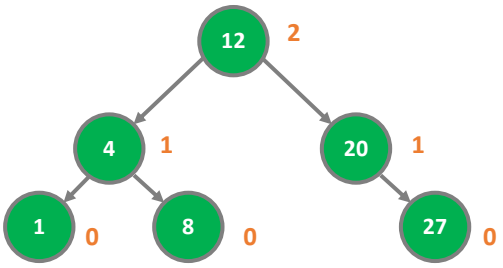


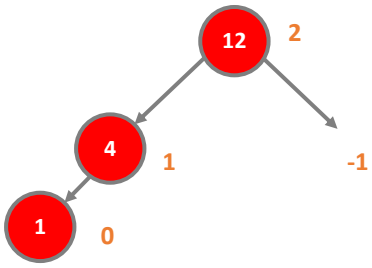


- Height of both trees is 2!

How can we determine programmatically which one of these trees is imbalanced?

Binary Search Trees



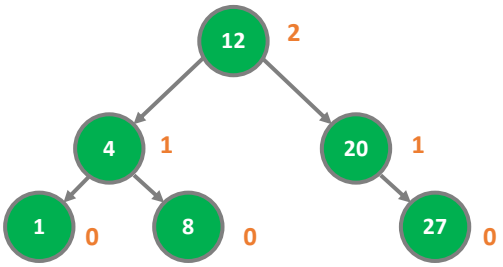


- Compare the heights of left and right subtrees by calculating the balance factor below:

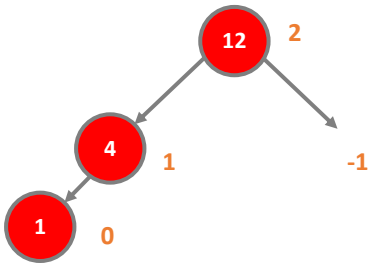
Balance factor: $\mathbf{h_{left} - h_{right}}$

- The tree heights on both sides can differ at most by 1

Binary Search Trees

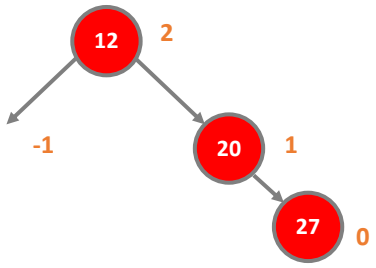


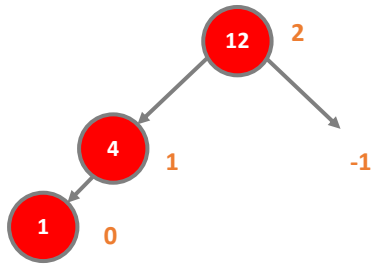
Balance factor: **1 – 1 = 0**



Balance factor: **1 – (–1) = 2**

Binary Search Trees





Balance factor: **-1 – 1 = -2**

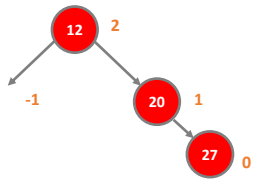
Right-heavy case
→ Rotate **left** to balance

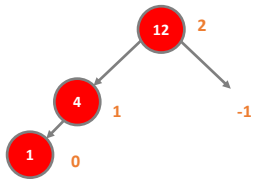
Balance factor: **1 – (–1) = 2**

Left-heavy case
→ Rotate **right** to balance

Binary Search Trees

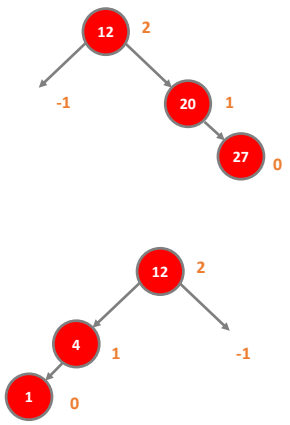
- Keep tracking the height of each node in BST
- Calculate the balance factors for the nodes
- Make rotations if necessary to balance search trees
- -ve balance factor → right-heave tree → rotate left
- +ve balance factor → left-heave tree → rotate right





Binary Search Trees | AVL Trees

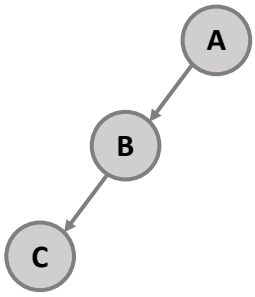
- Keep tracking the height of each node in BST
- Calculate the balance factors for the nodes
- Make rotations if necessary to balance search trees
- -ve balance factor → right-heave tree → rotate left
- +ve balance factor → left-heave tree → rotate right
- Balanced data structure invented by Adelson-Velsky and Landis
- Has guaranteed O(logN) running time for all operations



AVL Trees | Rotations

- Right rotation on root node (with value 5)
- Left rotation on root node (with value 3)
- The BST properties remain same
- The in-order traversal remains same after rotations

AVL Trees — Rotations — Case 1



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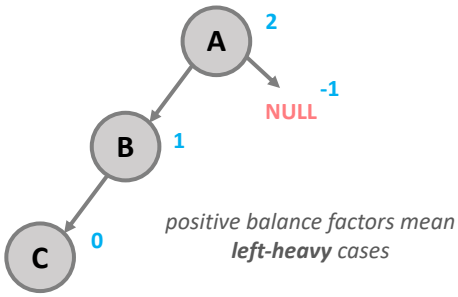
AVL Trees — Rotations — Case 1



positive balance factors mean ***left-heavy*** *cases*

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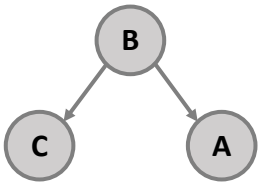
AVL Trees

Rotations

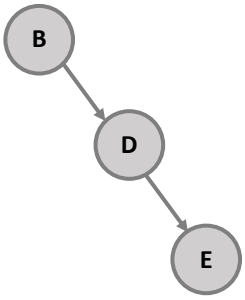
Case 1



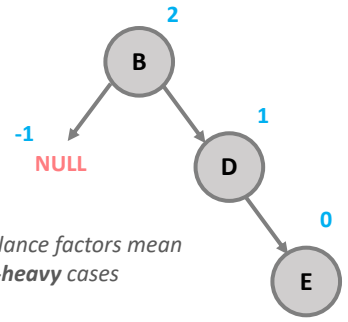
AVL Trees

Rotations

Case 2

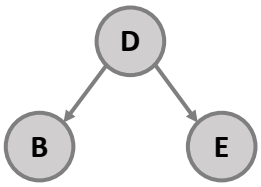


AVL Trees — Rotations — Case 2

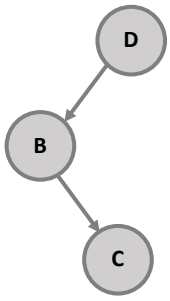


negative balance factors mean ***right-heavy*** *cases*

AVL Trees — Rotations — Case 2

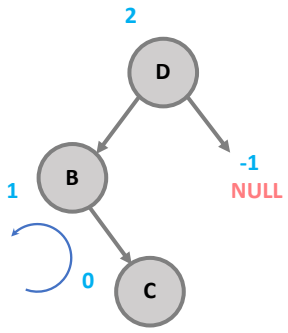


AVL Trees — Rotations — Case 3



AVL Trees — Rotations — Case 3

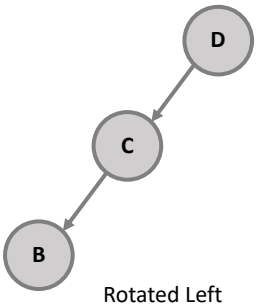
Balance factor of root → 2
→ left-heavy
- Determine BF of left child
- if –ve
  - → left-right heavy
  - → Rotate parent left
  - → Rotate grand-parent right
- if +ve
  - → left-left heavy
  - → Rotate grand-parent right



AVL Trees

Rotations

Case 3

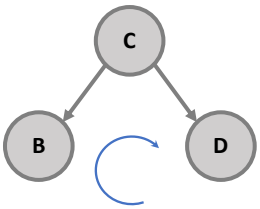


Rotated Left

AVL Trees

Rotations

Case 3

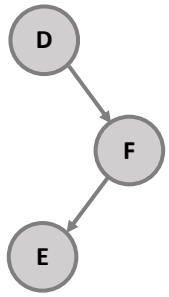


Rotated right

AVL Trees

Rotations

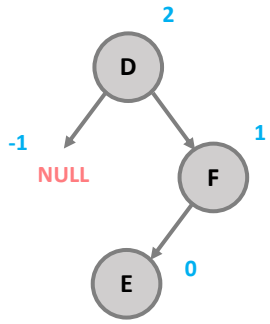
Case 4



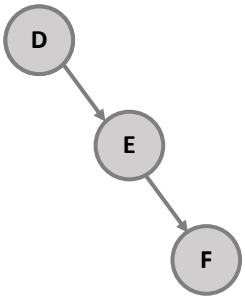
AVL Trees

Rotations

Case 4



AVL Trees | Rotations
Case 4



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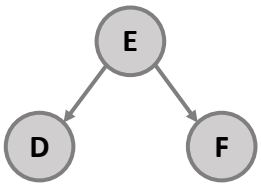
AVL Trees | Rotations
Case 4



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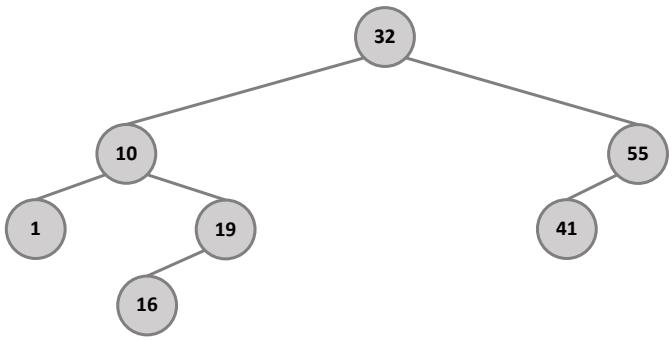
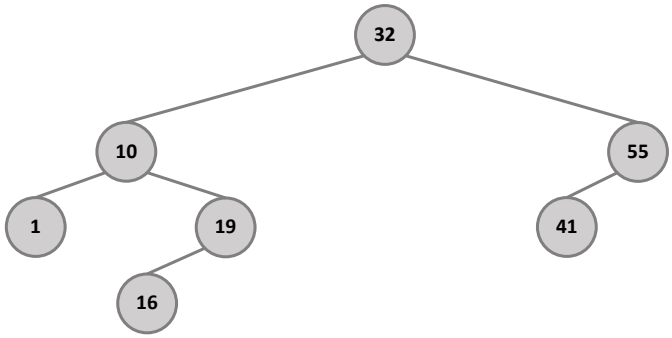
AVL Trees Insertion



AVL Trees Insertion

INSERT(12)



AVL Trees Insertion

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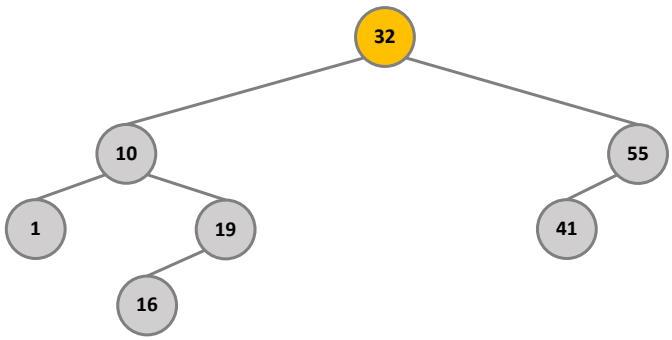

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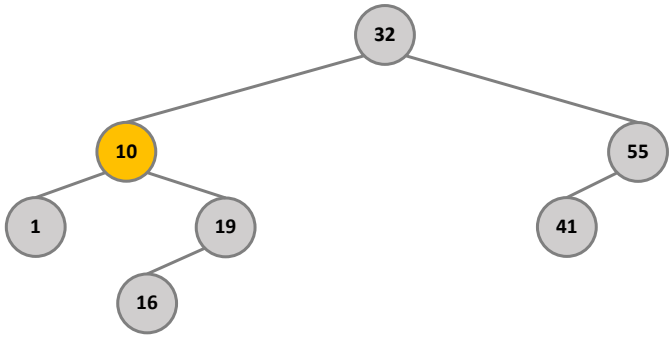

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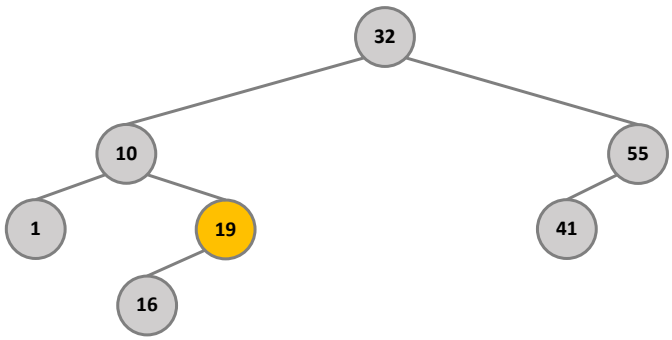

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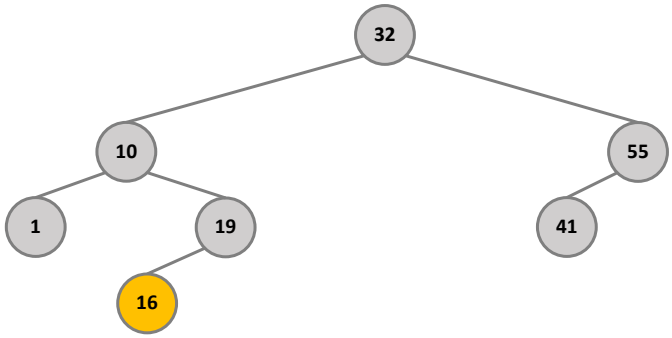

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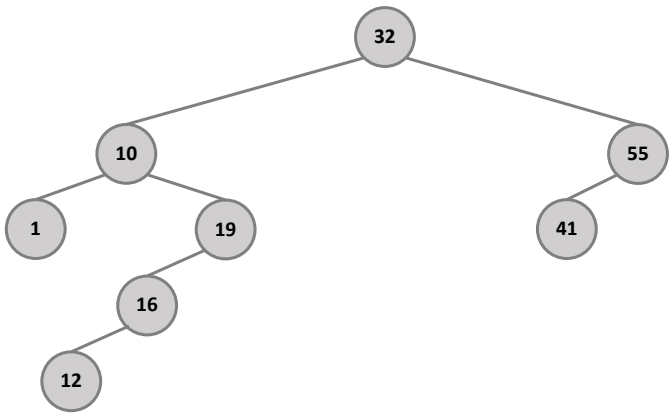

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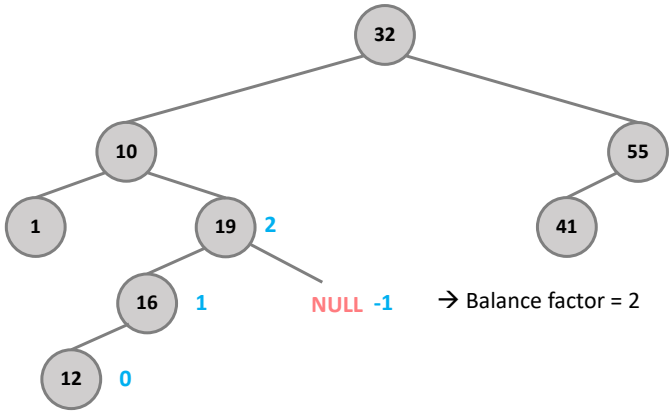

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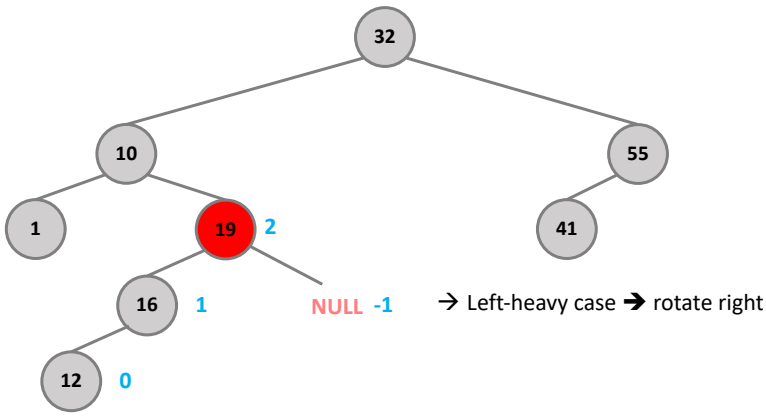
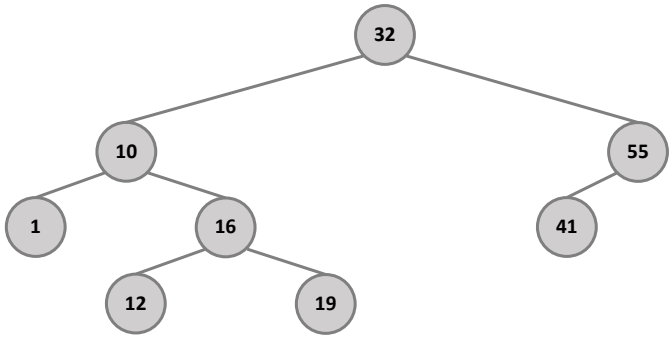


→ Left-heavy case ➔ rotate right

AVL Trees Insertion

INSERT(12)



AVL Trees Insertion

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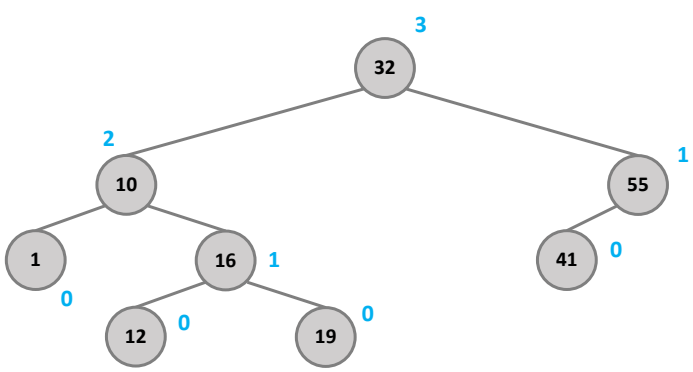

Balance factor for every node differs by 1

AVL Trees

- Implementation will be a part of assignment 3
  - Same as BST implementation except:
    - Height and Balance factor must be calculated for every node