

# Review: number systems



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CSE3666: Introduction to Computer Architecture

# Outline

- Binary numbers
  - Binary to decimal
  - Decimal to binary
- Hexadecimal numbers
  - Hexadecimal to decimal
  - Decimal to hexadecimal

`0b1101110` =

`0xC0FFEE` =

<i>n</i>	<i>2<sup>n</sup></i>
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

# Decimal number

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- We are familiar with decimal numbers
  - The radix is 10
- There are ten digits: 0, 1, 2, ..., 8, 9

Example:      3 6 6 6

$$\begin{aligned} 3666 &= 3 \times 10^3 + 6 \times 10^2 + 6 \times 10^1 + 6 \times 10^0 \\ &= 3000 + 600 + 60 + 6 \end{aligned}$$

In this example, each 6 carries different values.

Each time a digit moves to left by one place, the value is increased by 10.

Does the radix have to be 10?

# Binary numbers

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- If the radix is 2, we have binary numbers
  - We only need two digits, 0 and 1
  - Each digit is also called a **bit** (a **binary digit**)

Given a sequence of bits:

$$b_{n-1} b_{n-2} \dots b_2 b_1 b_0$$

The value is

$$b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \dots + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

$b_{n-1}$  is the most significant bit (MSB).  $b_0$  is the least significant bit (LSB).

# Example

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- Convert a binary number to a decimal number

11010101

128	64	32	16	8	4	2	1
b7	b6	b5	b4	b3	b2	b1	b0
1	1	0	1	0	1	0	1

← Weights

The answer is:

$$\begin{aligned} &1 \times 2^7 + 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^0 \\ &= 128 + 64 + 16 + 4 + 1 \\ &= 213 \end{aligned}$$

# Decimal to binary

- Convert decimal number 78 to a binary number.
- Human method

78

128	64	32	16	8	4	2	1
b7	b6	b5	b4	b3	b2	b1	b0

← Weights

Bits we need to figure out

We start from b0.  
What is the value of b0?

# Decimal to binary - 2

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78

128	64	32	16	8	4	2	1
b7	b6	b5	b4	b3	b2	b1	b0
							0

The right most bit is 0 because 78 is even, or when 78 is divided by 2, the remainder is 0

$$78 = 39 * 2 + 0$$

How about the rest of the bits?

# Decimal to binary: Complete steps

We repeat on the quotient we get, until it is 0.

Number	Quotient // 2	Remainder % 2	Expression
78	39	0	$78 = 39 * 2 + 0$
39	19	1	$39 = 19 * 2 + 1$
19	9	1	$19 = 9 * 2 + 1$
9	4	1	...
4	2	0	
2	1	0	
1	0	1	

The answer is 0b 100 1110.

We will use 0b or a subscript of 2 for bits



# Hexadecimal

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- The radix is 16
  - There are 16 digits: 0-9, and a - f

<b>Hex digits</b>	0 – 9	a	b	c	d	e	f
<b>Decimal value</b>	0 – 9	10	11	12	13	14	15

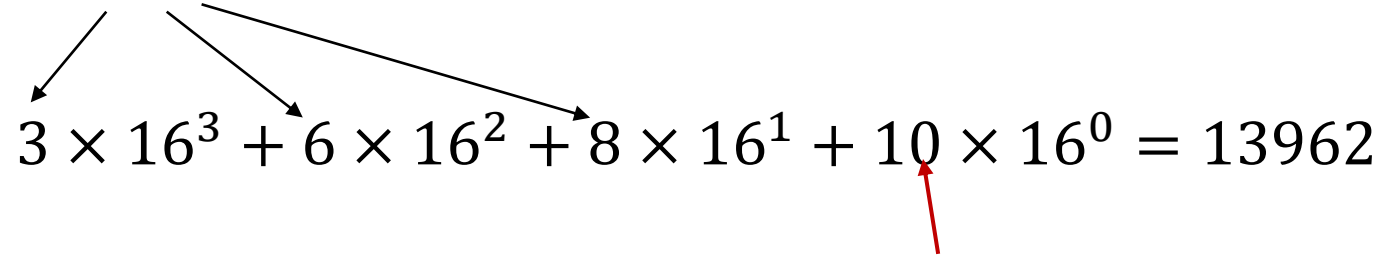
- Use 0x or a subscript of 16 to indicate hexadecimal numbers  
0xABCD or ABCD<sub>16</sub>

## Conversion between hexadecimal and decimal

The method is similar to the conversion between binary and decimal  
The difference: the radix is 16

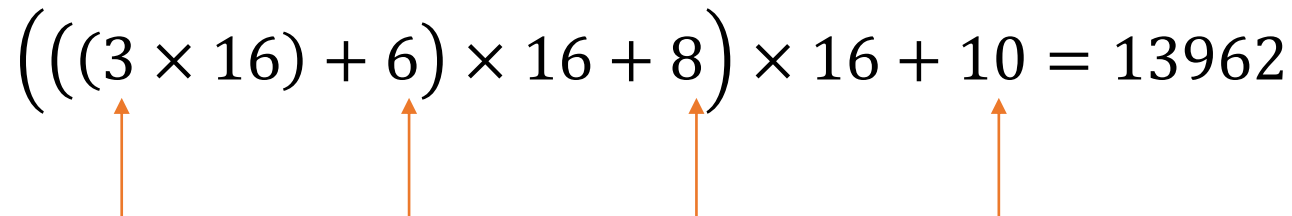
# Example: hex to decimal

Convert 0x368A to a decimal number


$$3 \times 16^3 + 6 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 = 13962$$

The right most digit A is  
10 in decimal

Another method, which is commonly used in computer programs


$$\left( \left( (3 \times 16) + 6 \right) \times 16 + 8 \right) \times 16 + 10 = 13962$$

# Example: decimal to hex

## Convert 846 to a hexadecimal number

We divide the number by 16, repeatedly

Number	Quotient // 16	Remainder % 16	Expression
846	52	14	$846 = 52 * 16 + 14$
52	3	4	$52 = 3 * 16 + 4$
3	0	3	$3 = 0 * 16$

So  $846 = 0x34E$ .

The lowest digit is 0xE (14) and the highest is 3.

# Number in other radices

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- The methods are similar to what we have learned for binary and hexadecimal numbers.

Octal numbers (radix is 8)

$$72_8 = 7 \times 8 + 2 = 58$$

Ternary numbers (radix is 3)

$$121_3 = 1 \times 3^2 + 2 \times 3 + 1 = 16$$

## Exercises:

Convert 58 back to an octal number and 16 back to a ternary number.

# Exercise

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- Convert a hexadecimal number to a decimal number
- Convert a hexadecimal number to a decimal number

Randomly pick a number.

Exercises:

[Binary Numbers \(zhijieshi.github.io\)](https://zhijieshi.github.io)

Config: select the number of bits and uncheck “bits are signed”.