Assignment 1 - Andrew Chan

Question 1

The total weight of the four mice will be 120 grams, give or take 5 grams or so.
Calculation:

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Weight of the 4 mice = 2 * 30g = 120g
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Standard error for 1 mice is 5g

$$\sqrt{\frac{(5^2) + (5^2) + (5^2) + (5^2)}{4}} = \sqrt{\frac{100}{4}} = 5g$$

Question 2

In another Harris Poll of 1144 adult Americans, 306 people felt that the U.S. Constitution should be amended to have presidential elections decided by popular vote rather than by the electoral college. Find the statistic p and estimate the standard error SEp.

$$p ≈ \frac{306}{1144} ≈ 27% $$
$$ SEp = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.27(1-0.27)}{1144}} = \sqrt{\frac{0.27(0.73)}{1144}} = \sqrt{0.00017229} \approx 0.0131 \approx 1.3%$$

Question 3

The symbol for the parameter is π. The value of the parameter is unknown. The collection of 1010 registered voters is called the sample. The symbol for the statistic is p. The value of the statistic is 45%. The standard error is approximately 1.5%. The 95% margin of error is approximately 3%.

$$SEp = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.45(1-0.45)}{1010}} = \sqrt{\frac{0.45(0.55)}{1010}} = \sqrt{\frac{0.24}{1010}} = \sqrt{0.00024} \approx 0.015 \approx 1.5%$$

Question 4

(d) 352 is a statistic and 29 is a parameter.

Question 5

Recall from calculus the techniques of finding maximum values of functions.
a. Show that the maximum of the function f(x)=x(1−x)f(x)=x(1−x) occurs when x=12x=21.
b. Use this to prove the inequality in Formula 1.1: $$ \sqrt{\frac{\pi(1-\pi)}{n}} \leq \frac{1}{2{\sqrt{n}}}$$

(a)

Function definition

$$ f(x) = x(1 - x) $$

Expand the function

$$ f(x) = x - x^2$$

Find the derivative

$$ f(x)' = 1 - 2x $$

Find critical points by setting f(x)' = 0

$$ 1 - 2x = 0 => x = \frac{1}{2} $$

Check maximum by looking at second derivative

$$ f(x)'' = -2 $$ Since -2 < 0, the function is concave down which means that x = 1/2 is the maximum.

Plug the x back in to find the maximum

$$ f(\frac{1}{2}) = \frac{1}{2}(1 - \frac{1}{2}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

Therefore the maximum is at x = 1/2 and the maximum value is 1/4

(b)

Use the result from part a

We know that the maximum value of $\pi(1 - \pi)$ is 1/4, and occurs when $\pi = 1/2$ That means that
$$ \pi(1-\pi) \leq \frac{1}{4} $$

Substitute the inequality

Since $$ \pi(1-\pi) \leq \frac{1}{4} $$ We can substitute $$ \sqrt{\frac{\pi(1-\pi)}{n}} \leq \sqrt{\frac{\frac{1}{4}}{n}} $$

Simplify

$$ \sqrt{\frac{\frac{1}{4}}{n}} = \sqrt{\frac{1}{4n}} = \frac{1}{2\sqrt{n}}$$

Therefore

$$ \sqrt{\frac{\pi(1-\pi)}{n}} \leq \frac{1}{2{\sqrt{n}}}$$

Therefore the inequality is proven