Assignment 2 - Andrew Chan

Question 1

Write out all 70 possibilities:
1: OOOOXXXX 2: OOOXOXXX 3: OOOXXOXX 4: OOOXXXOX 5: OOOXXXXO 6: OOXOOXXX 7: OOXOXOXX 8: OOXOXXOX 9: OOXOXXXO 10: OOXXOOXX
11: OOXXOXOX 12: OOXXOXXO 13: OOXXXOOX 14: OOXXXOXO 15: OOXXXXOO 16: OXOOOXXX 17: OXOOXOXX 18: OXOOXXOX 19: OXOOXXXO 20: OXOXOOXX
21: OXOXOXOX 22: OXOXOXXO 23: OXOXXOOX 24: OXOXXOXO 25: OXOXXXOO 26: OXXOOOXX 27: OXXOOXOX 28: OXXOOXXO 29: OXXOXOOX 30: OXXOXOXO
31: OXXOXXOO 32: OXXXOOOX 33: OXXXOOXO 34: OXXXOXOO 35: OXXXXOOO 36: XOOOOXXX 37: XOOOXOXX 38: XOOOXXOX 39: XOOOXXXO 40: XOOXOOXX
41: XOOXOXOX 42: XOOXOXXO 43: XOOXXOOX 44: XOOXXOXO 45: XOOXXXOO 46: XOXOOOXX 47: XOXOOXOX 48: XOXOOXXO 49: XOXOXOOX 50: XOXOXOXO
51: XOXOXXOO 52: XOXXOOOX 53: XOXXOOXO 54: XOXXOXOO 55: XOXXXOOO 56: XXOOOOXX 57: XXOOOXOX 58: XXOOOXXO 59: XXOOXOOX 60: XXOOXOXO
61: XXOOXXOO 62: XXOXOOOX 63: XXOXOOXO 64: XXOXOXOO 65: XXOXXOOO 66: XXXOOOOX 67: XXXOOOXO 68: XXXOOXOO 69: XXXOXOOO 70: XXXXOOOO

Question 2

Consider the raw data of men's heights of Example 2.5 on page 27.

a. The mean is **69.148** and the standard deviation is **2.587**

b. Compare the raw data to the three common rules. The data seems to follow a bell shape distribution. 68% of the data falls within 1 standard deviation. $69.148 \pm 2.587 => [66.561, 71.735]$ 95% of the data falls within 2 stand deviations. $69.148 \pm 2(2.587) => [63.974, 74.322]$ And 99.7% of the data falls within 3 standard deviations. $69.148 \pm 3(2.587) => [61.387, 76.909]$

c. If one of the men were to be randomly selected and you had to estimate his height, your best guess would be **69.148** inches; your estimate would differ from his actual height by approximately **2.587** inches or so.

d. Compute the estimated standard error.

$$SE = \frac{s}{\sqrt{n}} = \frac{2.587}{\sqrt{33}} \approx \frac{2.587}{\sqrt{5.744}} \approx 0.4504$$

Where s is the sample standard deviation and n is the sample size e. The population mean μ is approximately **69.148** give or take **0.4504** or so.

Question 3

If a fair die is rolled three times, find the probabilities of the following events:

a. All of the rolls show an even number of dots. There are 3 even numbers so the chance of rolling an even number once is 3/6 = 1/2. Since we are looking at all three rolls and they are independent of each other, we can multiply the chances all together. $P(\text{all rolls even}) = (\frac{1}{2})^3 = \frac{1}{8}$

b. The last two rolls show an even number of dots. Using the previous logic the probability for the last 2 rolls are just multiplied together to get $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ And since the first roll doesn't matter, it stays as 1/4.

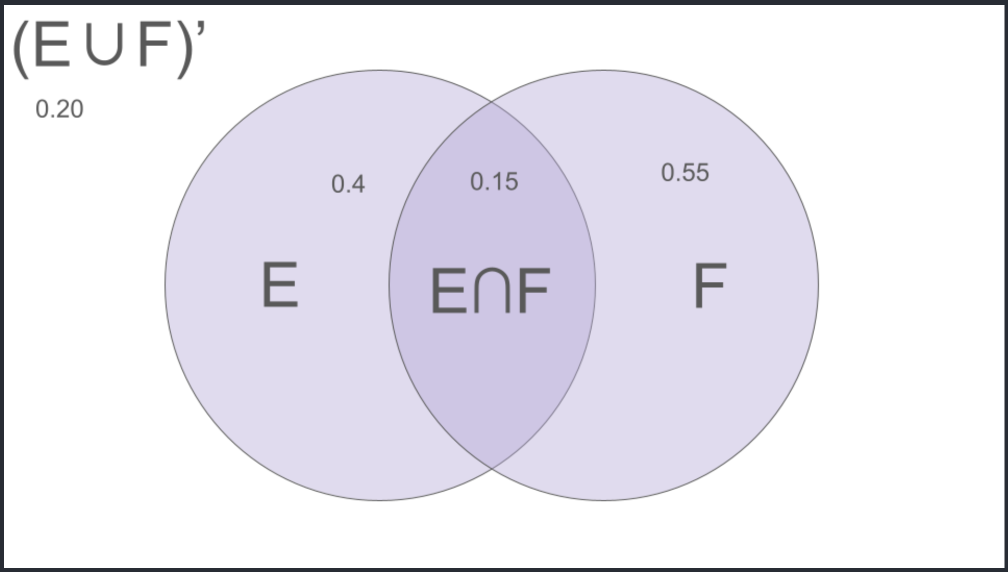
c. The third roll shows an even number of dots. Only the last roll matters $P(\text{last roll even}) = \frac{1}{2}$

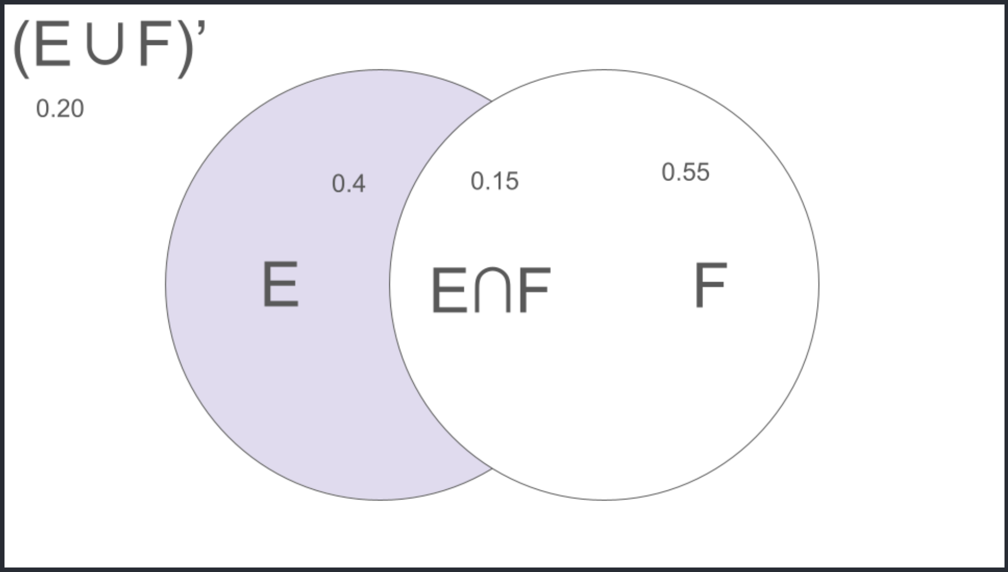
d. Every roll shows a single dot. The probability of rolling a single dot, aka a 1, is 1/6. And since they are all independent we can mutiply all 3 rolls together. $P(\text{all single dot}) = (\frac{1}{6})^3 = \frac{1}{216}$

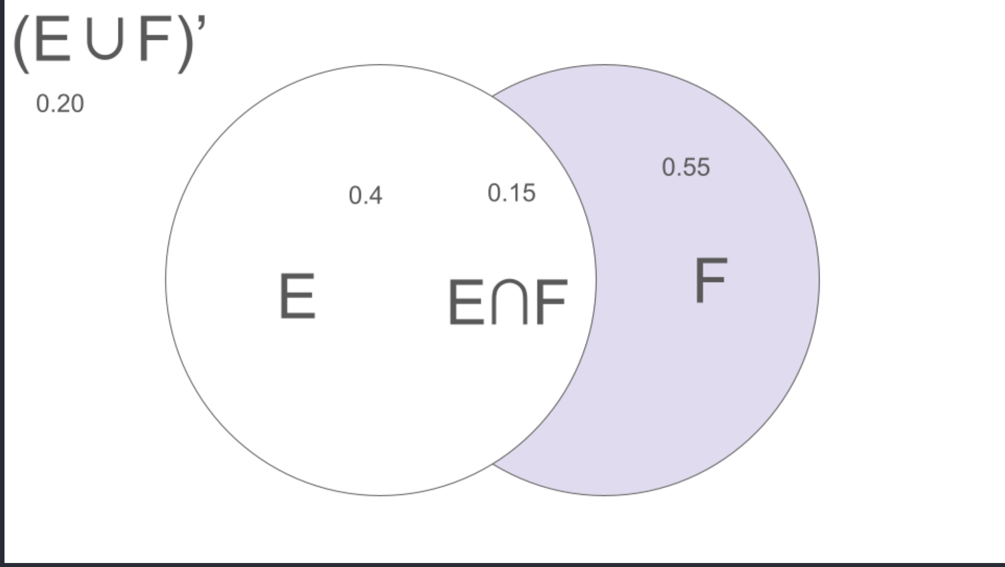
e. Every roll shows the same number of dots. Since they all have to match and are being rolled one at a time, the means that the first die can be anything. Since it can be anything, it won't effect the probability. So, we can just say that the last two rolls will have a 1/6 chance of matching the first roll, whatever it may be. We can notate it like this $P(\text{all rolls match}) = (\frac{1}{6})^2 = \frac{1}{36}$

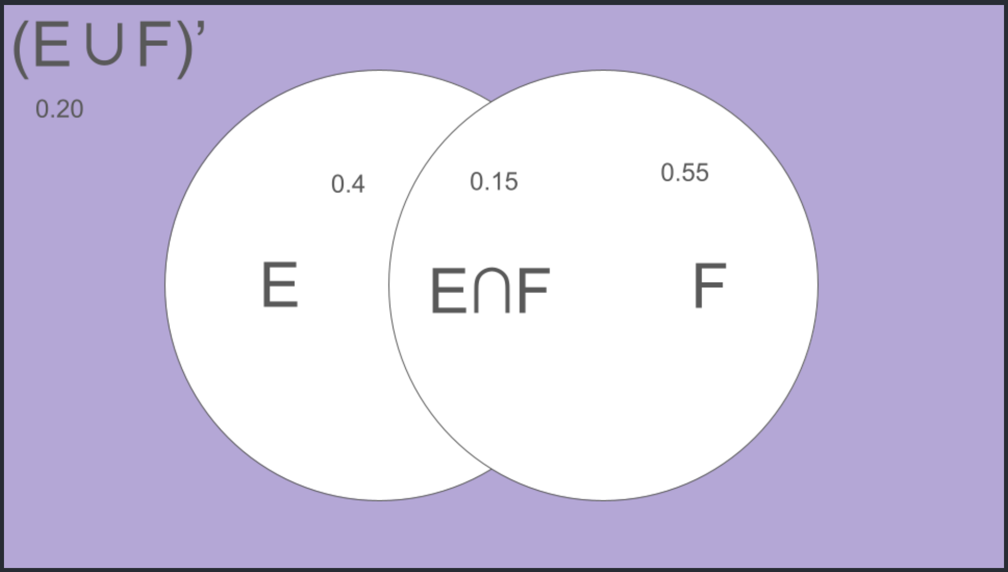
Question 4

Sketch a Venn diagram and label the probabilities of the regions $E \cup F$, $E \cap F'$, $F \cap E'$, and $(E \cup F)'$, if $Pr(E) = 0.40$, $Pr(F) = 0.55$, and $Pr(E \cap F) = 0.15$.

$Pr(E \cup F) = 0.80$ 

$Pr(E \cap F') = 0.25$ 

$Pr(F \cap E') = 0.40$ 

$Pr((E \cup F)') = 0.20$ 

Question 5

Suppose that the outcome set of an experiment is the set of positive integers and the events are all possible subsets of the integers. For each positive integer i, let E_i be the event that i is the outcome. Prove from the axioms of probability that it is impossible for to have $Pr(E_i) = p$ for all i, where p is a fixed constant value.

Step 1 $S = \{1, 2, 3, ...\}$ Each outcome i is representated by the event E_i where E_i = $\{i\}$ Assume $P(E_i) = p$, a constant probability for all i

From the **axioms of probability**, we know:

1. Axiom 2: $P(S) = 1$
2. Axiom 3: $P(S) = \sum_{i=1}^{\infty} P(E_i)$, since $E_1, E_2, ...$ are disjoint events.

Step 2

1. Substituting $P(E_i) = p$ into the formula for $P(S)$, we get: $P(S) = \sum_{i=1}^{\infty} P(E_i) = \sum_{i=1}^{\infty} p$.
2. Since p is constant, the summation simplifies to: $P(S) = p + p + p + ... = \sum_{i=1}^{\infty} p = p \cdot \infty$.
3. This result means that $P(S)$ diverges to infinity unless $p = 0$.

Step 3 Case 1: $p > 0$ If $p > 0$, the sum $p \cdot \infty$ becomes infinite This violates the normalization axiom:

$P(S) = 1$ Therefore $p > 0$ is not possible

Case 2: $p = 0$ If $p = 0$, the sum becomes $P(S) = \sum_{i=1}^{\infty} 0 = 0$ This violates the normalization axiom: $P(S) = 1$ Therefore $p = 0$ is also not possible

Step 4 Since neither $p > 0$ nor $p = 0$ satisfies the axioms of probability, it is impossible for $P(E_i) = p$ for all i, where p is a fixed constant value.

Question 6

An experiment has six possible outcomes. An outcome set for this experiment is $S = \{o1, o2, o3, o4, o5, o6\}$. Suppose the probabilities assigned to the single possible outcome are $Pr(\{o1\}) = 0.04$, $Pr(\{o2\}) = 0.20$, $Pr(\{o3\}) = 0.10$, $Pr(\{o4\}) = 0.40$, $Pr(\{o5\}) = 0.20$, and $Pr(\{o6\}) = 0.06$. Let $A = \{o1, o4, o6\}$, $B = \{o3, o4, o5\}$, and $C = \{o2, o5\}$ denote three events.

a. Which of these events are mutually exclusive?
By definition, two events are mutually exclusive if the intersection of both of them results in an empty set. A and C are mutually exclusive since $A \cap B$ is empty. Also, they have no elements in common.

b. Which are independent? By definition, two events are independent if $Pr(X \cap Y) = Pr(X) \cdot Pr(Y)$

Calculate Pr $Pr(A) = Pr(\{o_1\}) + Pr(\{o_4\}) + Pr(\{o_6\}) = 0.04 + 0.40 + 0.06 = 0.50$ $Pr(B) = Pr(\{o_3\}) + Pr(\{o_4\}) + Pr(\{o_5\}) = 0.10 + 0.40 + 0.20 = 0.70.$ $Pr(C) = Pr(\{o_2\}) + Pr(\{o_5\}) = 0.20 + 0.20 = 0.40.$

Check independence
$Pr(A \cap B) = Pr(\{o_4\}) = 0.40, Pr(A) \cdot Pr(B) = 0.50 \cdot 0.70 = 0.35$

Since $Pr(A \cap B) \neq Pr(A) \cdot Pr(B)$, A and B are **not independent**.

$Pr(A \cap C) = 0 \,(\text{mutually exclusive}), Pr(A) \cdot Pr(C) = 0.50 \cdot 0.40 = 0.20$

Since $Pr(A \cap C) \neq Pr(A) \cdot Pr(C)$, A and C are **not independent**.

$Pr(B \cap C) = Pr(\{o_5\}) = 0.20, Pr(B) \cdot Pr(C) = 0.70 \cdot 0.40 = 0.28.$

Since $Pr(B \cap C) \neq Pr(B) \cdot Pr(C)$, B and C are **not independent**.

Therefore, none of the events are independent.

c. Compute $Pr(B \cup C)$.
Formula: $Pr(B \cup C) = Pr(B) + Pr(C) - Pr(B \cap C)$ $Pr(B) = 0.70$ $Pr(C) = 0.40$ $Pr(B \cap C) = Pr(\{o5\}) = 0.20$

$Pr(B \cup C) = 0.70 + 0.40 - 0.20 = 0.90$

$Pr(B \cup C) = 0.90$