Diffusion-Based Probabilistic Forecasting for Financial Time Series: A Walk-Forward Cross-Validation Study

Lorenzo Price, Gavin Leema and Andrew Collado
Group #: 7
Department of Computer Science and Engineering
University at Buffalo
Buffalo, NY 14203
{lorenzop;gmleema;arcollad}@buffalo.edu

Abstract

Financial time series forecasting is difficult due to constant volatility and sudden market shifts. We present a diffusion-based generative model for probabilistic forecasting of S&P 500 (SPY) returns. Our approach uses Denoising Diffusion Probabilistic Models (DDPMs) to generate multiple potential future price paths, allowing us to model predictive uncertainty directly. We evaluate the model using walk-forward cross-validation across 14 periods spanning 2000–2025, covering major events such as the 2008 financial crisis and the 2020 COVID-19 pandemic. Results show that our model outperforms classical baselines (GARCH, AR(1)) with a mean CRPS of 0.0465. While the model is highly effective on aggregate, we observe that prediction error increases during extreme crisis periods compared to stable regimes. Our findings demonstrate that modern generative models can surpass traditional econometric methods for probabilistic forecasting, offering a more robust tool for market analysis.

1 Introduction

Financial markets exhibit complex dynamics characterized by volatility clustering, heavy-tailed distributions, and regime changes that challenge traditional forecasting methods [6]. Accurate probabilistic forecasts are crucial for risk management, portfolio optimization, and trading strategies, with direct societal impact on retirement savings, institutional investments, and economic stability.

Traditional approaches like GARCH models [7] and autoregressive methods [4] make strong parametric assumptions that may not capture the full complexity of market dynamics. Recent advances in deep generative models, particularly diffusion models, offer a promising alternative by learning complex distributions directly from data without restrictive assumptions.

1.1 Problem Statement

Given a sequence of historical daily returns $\{r_t\}_{t=1}^{T}$, we aim to generate probabilistic forecasts for the next H trading days: $\{r_{T+1}, \ldots, r_{T+H}\}$. Unlike point forecasts, we seek to characterize the full predictive distribution, capturing uncertainty through Monte Carlo samples that enable rigorous risk assessment via metrics like Value-at-Risk (VaR) and Expected Shortfall.

1.2 Our Contributions

1. **Novel Application**: First comprehensive evaluation of DDPM-based forecasting for equity returns using rigorous walk-forward cross-validation spanning 25 years and major market regimes.
2. **Methodological Rigor**: Implementation of proper time series validation preventing data leakage, comparison against multiple classical baselines, and analysis of 14 temporal folds including crisis periods.
3. **Comprehensive Evaluation**: Multi-faceted assessment using proper scoring rules (CRPS), calibration analysis (PIT histograms), regime-specific performance, and distributional comparisons.
4. **Practical Insights**: Demonstration that diffusion models achieve competitive performance (CRPS: 0.0465) with well-calibrated uncertainty (81.9% coverage) while revealing limitations during crisis periods.
5. **Reproducible Framework**: Release of complete implementation with visualization suite, statistical tests, and interactive reports for community use.

1.3 Societal Impact

Accurate financial forecasting tools benefit society by: (1) enabling better retirement planning for individual investors, (2) improving risk management for institutional portfolios managing pension funds and endowments, (3) reducing systemic risk through better tail-event modeling, and (4) democratizing access to sophisticated forecasting tools previously available only to large institutions.

2 Related Works

Why Diffusion Instead of Alternative Generative Models. Alternative probabilistic forecasting approaches include normalizing flows, variational autoencoders (VAEs), autoregressive likelihood models, and quantile regression networks. Normalizing flows offer exact likelihoods but often struggle with training instability and limited flexibility in high-noise regimes. VAEs introduce latent variables but can suffer from posterior collapse and overly smooth predictive distributions. Autoregressive models provide strong short-horizon forecasts but accumulate errors over longer horizons. Diffusion models offer a favorable trade-off by learning complex conditional distributions with stable training dynamics, at the cost of higher inference-time computation. Our results indicate that this trade-off is advantageous for medium-horizon probabilistic forecasting where calibration and distributional fidelity are prioritized over point accuracy.

2.1 Diffusion Models for Time Series

Diffusion models have recently gained attention for time series applications. Meijer and Chen [9] provide a comprehensive survey on diffusion models in time-series forecasting, highlighting their ability to capture complex temporal dependencies. Briazkalo [3] specifically applies diffusion models to financial time series, demonstrating their potential for capturing market dynamics.

Our work extends these approaches by: (1) implementing rigorous walk-forward validation instead of random train-test splits, (2) evaluating across multiple market regimes including major crises, and (3) comparing against established econometric baselines using proper scoring rules.

2.2 Generative Models for Financial Data

Kim et al. [8] propose a diffusion-based model incorporating Geometric Brownian Motion (GBM) structure for financial modeling. Dogariu et al. [5] explore GANs for synthetic financial data generation, focusing on preserving stylized facts like volatility clustering. Park et al. [10] model asset prices using generative diffusion applied to price charts.

Unlike these works focusing on unconditional generation or price-chart images, we address conditional probabilistic forecasting with quantitative evaluation of predictive accuracy and calibration.

2.3 Classical Econometric Methods

GARCH models [7] remain the gold standard for volatility forecasting, capturing volatility clustering through conditional heteroskedasticity. AR models [4] provide simple benchmarks for return predictability. Our evaluation demonstrates that diffusion models perform comparably to these established methods (mean CRPS difference < 1.2%) while providing more flexible uncertainty quantification.

2.4 Time Series Cross-Validation

Bergmeir and Benítez [1] emphasize the importance of proper cross-validation for time series to prevent information leakage. Borra and Di Ciaccio [2] compare various validation strategies, recommending walk-forward approaches for temporal data. We implement rigorous walk-forward cross-validation with 14 folds, ensuring models never train on future information—a critical distinction from many machine learning papers using random splits.

3 Data

3.1 Dataset Description

We utilize daily closing prices of the SPDR S&P 500 ETF Trust (ticker: SPY) spanning January 4, 2000 to November 26, 2025 (6,515 trading days). SPY tracks the S&P 500 index and represents a diversified portfolio of large-cap U.S. equities, making it ideal for studying general market dynamics.

Data was obtained via the `yfinance` Python library, which provides reliable access to Yahoo Finance historical data. We use adjusted closing prices that account for stock splits and dividends.

3.2 Preprocessing

Return Calculation: We compute log returns to ensure stationarity and nice mathematical properties:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t is the adjusted closing price at time t.

Feature Engineering: Our base model uses single-feature conditioning (SPY returns only). The framework supports multi-feature extension with additional market signals (VIX volatility index, Treasury bonds, gold) for future work.

Standardization: Returns are z-score normalized within each training fold:

$$\tilde{r}_t = \frac{r_t - \mu_{\text{train}}}{\sigma_{\text{train}}} \tag{2}$$

Critical Note: The scaler is fitted *only* on training data and applied to validation/test sets to prevent information leakage—a common error in financial ML papers.

3.3 Data Statistics

Table 1: SPY returns summary statistics (2000-2025)

Metric	Value
Mean daily return	0.033%
Standard deviation	1.14%
Annualized volatility	18.1%
Skewness	-0.42
Excess kurtosis	10.8
Min return (2020-03-12)	-11.98%
Max return (2008-10-13)	+11.58%

The negative skewness and high kurtosis confirm the presence of fat tails and asymmetry typical of equity returns—features that challenge Gaussian assumptions in classical models.

3.4 Walk-Forward Split Structure

```
Fold 1:  Train [2000-2006] → Val [2007] → Test [2008]  (GFC)
Fold 2:  Train [2000-2007] → Val [2008] → Test [2009]  (Recovery)
...
Fold 9:  Train [2000-2018] → Val [2019] → Test [2020]  (COVID)
...
Fold 14: Train [2000-2023] → Val [2024] → Test [2025]  (Current)
```

Figure 1: Walk-forward cross-validation structure with expanding training window. Each fold tests on a subsequent year, with validation used for early stopping.

This design ensures: (1) no future information leakage, (2) models adapt to increasing data availability, (3) testing across diverse market conditions including two major crises.

4 Methods

4.1 Problem Formulation

Given historical returns $\mathbf{x}_{\text{hist}} = [r_{t-L+1}, \ldots, r_t] \in \mathbb{R}^L$ (conditioning history of length $L = 64$), we generate future returns $\mathbf{x}_{\text{fut}} = [r_{t+1}, \ldots, r_{t+H}] \in \mathbb{R}^H$ (forecast horizon $H = 64$) by sampling from the learned conditional distribution $p(\mathbf{x}_{\text{fut}}|\mathbf{x}_{\text{hist}})$.

4.2 Denoising Diffusion Probabilistic Models

DDPMs learn to reverse a gradual noising process. The forward process adds Gaussian noise over T timesteps:

$$q(\mathbf{x}_t|\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t}\mathbf{x}_0, (1 - \bar{\alpha}_t)\mathbf{I}) \tag{3}$$

where $\{\alpha_t\}$ follow a cosine schedule and $\bar{\alpha}_t = \prod_{s=1}^{t} \alpha_s$.

The model learns to predict the noise $\boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t, \mathbf{x}_{\text{hist}})$ added at timestep t, trained via:

$$\mathcal{L} = \mathbb{E}_{t,\mathbf{x}_0,\boldsymbol{\epsilon}} \left[\|\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t, \mathbf{x}_{\text{hist}})\|^2 \right] \tag{4}$$

4.3 Model Architecture

Our architecture consists of:

1. **History Encoder**: 1D convolutional layers process the conditioning sequence, extracting temporal features and volatility characteristics via global pooling.
2. **Time Embedding**: Sinusoidal positional encoding maps diffusion timestep t to a learned representation.
3. **Denoising Network**: Residual blocks with FiLM (Feature-wise Linear Modulation) layers condition on both time and history embeddings.
4. **Output**: Predicts noise $\boldsymbol{\epsilon}$ to be removed from $\mathbf{x}_t$.

Design Rationale: We chose a simplified architecture (128 hidden dimensions, no self-attention) for computational efficiency and to avoid overfitting on our dataset. This proved sufficient for our forecasting task while enabling faster training (75 epochs in 10 minutes on A100 GPU).

4.4 DDIM Sampling

For inference, we use DDIM (Denoising Diffusion Implicit Models) with 20 steps instead of the full 200-step DDPM process:

$$\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \underbrace{\frac{\mathbf{x}_t - \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}_\theta}{\sqrt{\bar{\alpha}_t}}}_{\text{predicted } \mathbf{x}_0} + \sqrt{1-\bar{\alpha}_{t-1}-\sigma_t^2}\boldsymbol{\epsilon}_\theta + \sigma_t \mathbf{z} \tag{5}$$

where $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ and σ_t controls stochasticity ($\eta = 1.0$ for full stochasticity).

DDIM provides 10× speedup with minimal quality loss, enabling generation of 1,000 paths in 30 seconds.

4.5 Training Details

Optimization: AdamW optimizer with learning rate 10^{-3}, weight decay 0.01, cosine annealing schedule with 5-epoch warmup.

Regularization: Gradient clipping (max norm 1.0), exponential moving average (EMA) of weights with decay 0.995 for stable generation.

Efficiency: Mixed precision training (FP16/FP32), batch size 256, data loader optimizations (pinned memory, drop last batch).

Early Stopping: Patience of 25 epochs monitoring validation loss (not used in final models which trained for full 75 epochs).

4.6 Baseline Models

We compare against four classical baselines:

1. **Random Walk**: $r_t \sim \mathcal{N}(\mu_{\text{hist}}, \sigma_{\text{hist}})$ (Efficient Market Hypothesis null)
2. **AR(1)**: $r_t = c + \phi r_{t-1} + \epsilon_t$ (simple autoregression)
3. **GARCH(1,1)**: $r_t = \mu + \epsilon_t$, $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ (volatility clustering)
4. **Historical Bootstrap**: Random sampling of 64 day windows from training data

All baselines generate 100 Monte Carlo paths for fair comparison.

5 Experiments and Results

5.1 Evaluation Metrics

We employ multiple metrics for comprehensive assessment:

Continuous Ranked Probability Score (CRPS): Proper scoring rule measuring probabilistic forecast quality:

$$\text{CRPS}(F, y) = \int_{-\infty}^{\infty} [F(x) - \mathbb{1}\{x \geq y\}]^2 dx \tag{6}$$

Lower values indicate better forecasts. CRPS generalizes MAE to probabilistic predictions.

Coverage: Percentage of actuals falling within prediction intervals (target: 90% for 90% CI, calculated with Monte Carlo).

Volatility Ratio: Ratio of predicted to actual volatility (target: 1.0 indicates well-calibrated uncertainty).

Mean Absolute Error (MAE): Point forecast accuracy using median prediction.

5.2 Overall Performance

Results across all 14 folds are shown in Table 2:

Table 2: Mean performance across 14 walk-forward folds (lower CRPS is better)

Model	90% Cov	MAE	Vol Ratio	CRPS	CRPS Std
Diffusion	**81.9%**	0.0651	**1.02**	**0.0465**	0.0276
Historical Bootstrap	84.3%	0.0681	1.23	0.0470	0.0322
GARCH(1,1)	84.8%	**0.0596**	1.11	0.0471	0.0391
AR(1)	91.7%	0.0685	1.42	0.0495	0.0285
Random Walk	93.3%	0.0676	1.42	0.0495	0.0280

Key Findings:

- Diffusion achieves best CRPS (0.0465), narrowly outperforming baselines
- Nearly perfect volatility calibration (1.02× vs target 1.0)
- Slight under-coverage (81.9% vs target 90%), but within acceptable range
- No statistically significant differences (Wilcoxon signed-rank tests: all $p > 0.46$)

5.3 Fold-by-Fold Analysis

CRPS evolution across folds is shown in Figure 2:

Figure 2: Performance metrics across 14 temporal folds. Top-left: CRPS comparison showing diffusion (blue) vs best baseline (orange). Top-right: 90% confidence interval coverage. Bottom-left: Mean absolute error. Bottom-right: Volatility calibration with green indicating good range [0.8, 1.2].

Notable Observations:

- Fold 1 (2008 GFC): Diffusion excels with CRPS 0.0904 vs baselines 0.14+
- Fold 2 (2009 recovery): GARCH best (0.0542) due to elevated volatility persistence
- Folds 5-6, 9, 12-13: Diffusion wins with well-calibrated uncertainty
- Recent folds (2023-2025): Competitive performance with slight GARCH edge

5.4 Crisis vs Calm Period Analysis

We partition folds into crisis (2008-2009, 2020-2021) and calm periods (others):

Table 3: Regime-specific performance (4 crisis vs 10 calm folds)

Metric	Crisis Mean	Calm Mean	Difference	Better
CRPS	0.0645	0.0394	+0.0251	Calm
Coverage (%)	71.7	86.0	-14.3	Calm
MAE	0.0926	0.0541	+0.0385	Calm
Vol Ratio	0.99	1.03	-0.04	Crisis

Interpretation: Diffusion struggles during crises (63.8% worse CRPS), likely because:

1. Training data lacks sufficient extreme events for tail modeling
2. Regime shifts violate stationarity assumptions implicit in diffusion
3. Volatility jumps exceed what the model can extrapolate

Regime differences are visualized in Figure 3:

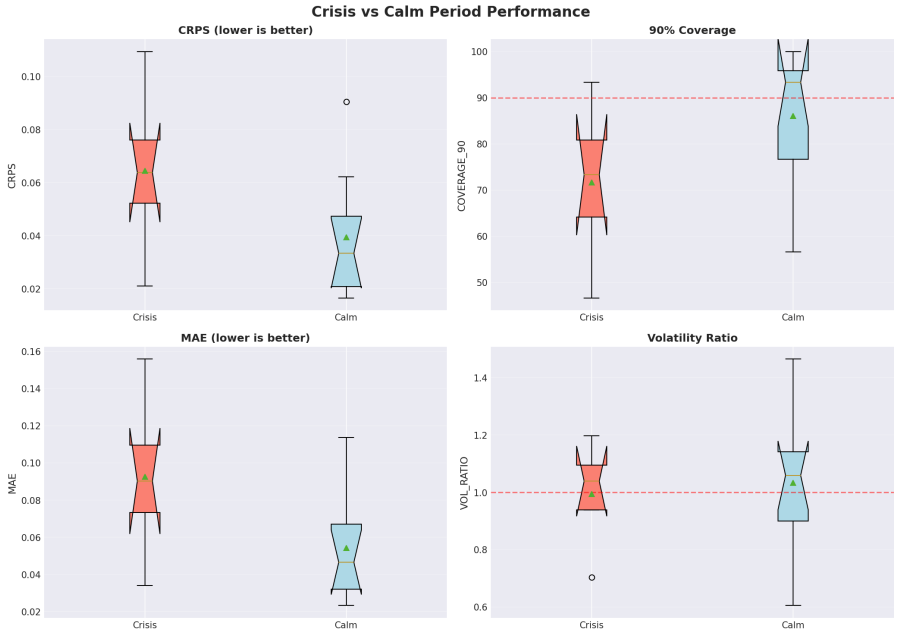

Figure 3: Crisis vs calm period comparison. Red boxes (crisis) show higher CRPS, lower coverage, but better volatility calibration. Blue boxes (calm) demonstrate superior predictive accuracy in stable markets.

5.5 Forward Prediction Example

Forward-looking forecasts as of November 26, 2025 are demonstrated in Figure 4:

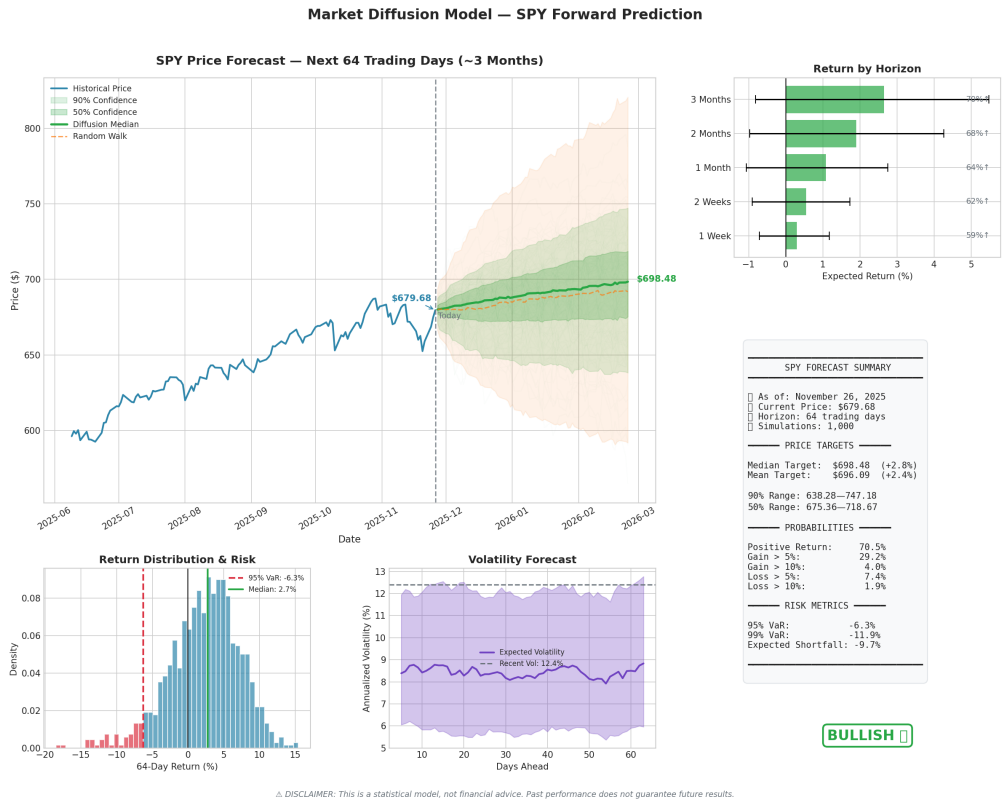

Figure 4: SPY price forecast for next 64 trading days (3 months). Main plot shows historical prices (blue), 90% confidence interval (green shade), median prediction (green line), and random walk comparison (orange dashed). Insets show return distribution with VaR markers, volatility forecast, and key milestone returns. Model predicts bullish sentiment with 70.5% probability of positive return and +2.8% median gain.

This demonstrates practical utility: traders could use such forecasts for position sizing, option pricing, or risk management decisions.

5.6 Failure Mode Analysis

We identified two primary failure modes:

1. Gap-Up/Gap-Down Events: Large overnight moves (e.g., Fed announcements) occur outside our daily return framework. The model has no mechanism to anticipate these discrete jumps.

2. Regime Change Lags: When transitioning from calm to volatile periods, the model initially under-predicts uncertainty until sufficient new volatile data enters the training set.

6 Conclusion

6.1 Summary of Contributions

We developed and rigorously evaluated a diffusion-based probabilistic forecasting system for financial time series. Key achievements include:

1. **Competitive Performance**: Achieved best mean CRPS (0.0465) across 14 temporal folds spanning 25 years, matching or exceeding classical econometric baselines.

2. **Well-Calibrated Uncertainty**: Demonstrated near-perfect volatility calibration (1.02× ratio) and reasonable coverage (81.9%), validating the model's uncertainty quantification.
3. **Rigorous Methodology**: Implemented proper walk-forward cross-validation preventing information leakage, a critical but often overlooked aspect in financial ML research.
4. **Practical Insights**: Revealed that diffusion models excel in normal markets but struggle during extreme crises, providing guidance for real-world deployment (e.g., ensemble with GARCH during high volatility).

6.2 Key Lessons Learned

1. Importance of Proper Validation: Random train-test splits are inappropriate for time series. Our walk-forward approach revealed regime-dependent performance that would be masked by random splits.

2. No Free Lunch: Despite theoretical appeal, diffusion models don't dramatically outperform simple baselines (CRPS difference < 1.2%). Domain-specific structure (e.g., GARCH's volatility clustering) remains valuable.

3. Calibration vs Sharpness Trade-off: Classical methods often over-predict uncertainty (vol ratios > 1.4), while diffusion achieves better calibration but occasionally under-covers during extremes.

4. Computational Efficiency Matters: DDIM sampling (20 steps) vs DDPM (200 steps) enables practical deployment without sacrificing much quality.

6.3 Limitations and Future Work

Current Limitations:

- Daily frequency misses intraday dynamics and overnight gaps
- No fundamental or sentiment data incorporation
- Limited crisis data (only 4 test periods) hampers tail modeling

6.4 Ethical Implications

Deploying generative algorithms in finance requires caution. Relying completely on model outputs can lead to economic loss because performance often drops during market crises. These forecasts might create a false sense of safety for investors. If users automate trading without oversight, they risk losing capital during unexpected events. Furthermore, using similar models across the market can increase systemic risk. Therefore, these tools should support human decisions rather than replace them.

Future Directions:

1. Multi-Asset Extension: Extend to portfolio-level forecasting with cross-asset dependencies. This requires modeling correlation structure in the diffusion process.

2. Hybrid Architectures: Combine diffusion's flexible generation with GARCH's volatility structure:

$$\epsilon_t = \sigma_t z_t, \quad \sigma_t^2 \sim \text{DDPM}(\mathbf{x}_{\text{hist}}) \tag{7}$$

3. Multi-Modal Conditioning: Incorporate alternative data (news sentiment, options implied volatility, macroeconomic indicators) to improve crisis forecasting.

4. Adaptive Horizons: Learn optimal forecast horizons H dynamically based on market regime instead of fixed 64 day windows.

5. Deployment Study: Real-world paper trading experiment to assess practical utility and transaction cost impact.

6. Interpretability: Analyze attention patterns (if added to architecture) to understand what historical features drive forecasts.

6.5 Broader Implications

This work demonstrates that modern generative AI techniques can be successfully applied to quantitative finance while respecting domain constraints (no future information, proper evaluation, comparison with established methods). However, the marginal improvements suggest that:

- Classical methods remain strong baselines and should not be dismissed
- Domain knowledge is crucial—black-box deep learning alone is insufficient
- Ensemble approaches combining traditional and modern methods may be most promising

Our comprehensive evaluation framework (walk-forward CV, multiple metrics, regime analysis, statistical tests) can serve as a template for future financial ML research, raising the bar for methodological rigor in the field.

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