Homework 3

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CS 6480
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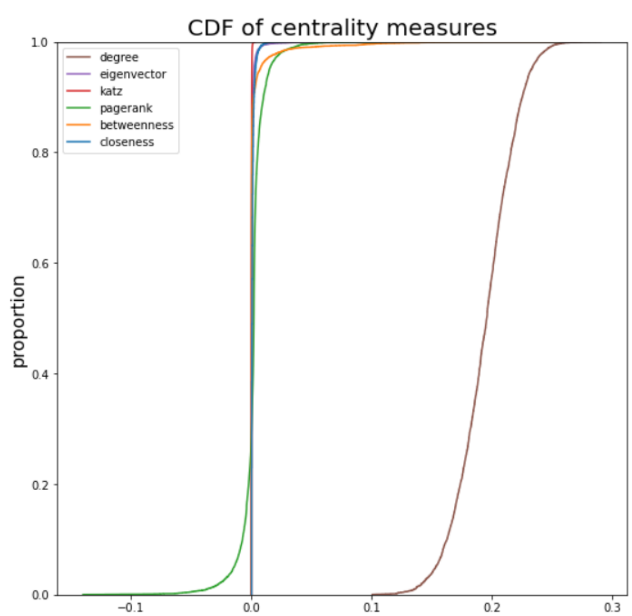
Problem 1.

a) Compute centrality measures (implemented in .ipynb file, does take about 10 minutes to run problem 1.)

b) Table shown below. Some of the centrality measures have much higher correlation than others. It would depend on how you are defining the centrality. For example degree centrality and pagerank have a high correlation of 0.954. Both of these measures depend heavily on the degree of the node and the information flow through it. I think it's interesting that betweenness and closeness have a fairly low correlation of 0.331. To me they seem like similar measures. If a node is passed through often you would think it would be relatively close to many other nodes.

	Degree	Eigenvector	Katz	Pagerank	Betweenness	Closeness
Degree	1	0.4732670047112867	-0.11117568890942629	0.9544807284809576	0.6510808438458302	0.561755442852198
Eigenvector	0.4732670047112867	1	-0.029831575894089673	0.35420636972389585	0.2227557308279146	0.2884384862425011
Katz	-0.11117568890942629	-0.029831575894089673	1	-0.100669369805698	0.03827646566067783	-0.04778079423809243
Pagerank	0.9544807284809576	0.35420636972389585	-0.100669369805698	1	0.7207075652153285	0.5029233158469362
Betweenness	0.6510808438458302	0.2227557308279146	0.03827646566067783	0.7207075652153285	1	0.3313017284435195
Closeness	0.561755442852198	0.2884384862425011	-0.04778079423809243	0.5029233158469362	0.3313017284435195	1

c) CDF plot shown below.



Problem 2.

My pagerank implementation values:

```
{'v1': 0.2221478665236697,
 'v2': 0.2048677631944891,
 'v3': 0.22763177214522773,
 'v4': 0.14711515533767663,
 'v5': 0.0990959217064639,
 'v6': 0.09914152109247303}
```

Note: I did call networkx pagerank further up in the .py file but that was just as a check for my later implementation.

Problem 3.

a) See .ipynb file (incompletely implemented)

b) The networkx betweenness centrality values for the graph are shown below. The networkx implementation must be faster as it does not use the dijkstra implementation which is O(N) = n^3.

```
{'v1': 0.0,
 'v2': 0.3666666666666667,
 'v3': 0.08333333333333333,
 'v4': 0.21666666666666667,
 'v5': 0.08333333333333333,
 'v6': 0.05}
```

Problem 4.
Largest 3 hub indexes: 9905111, 110055, 7170
Largest 3 authority indexes: 9711200, 9802150, 9802109

Hub index papers:
Title: Large N Field Theories, String Theory and Gravity
Authors: O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz

- This paper is very long and seems to be a bit of a collection on the subject. It has quite a few citations itself (on google scholar) but seems like more of a collection and reference on the topic. As a hub it links to other important authorities on the topic which makes sense.

Title: Strings, Branes and Extra Dimensions
Authors: Stefan Forste

- This paper seems again like a collection/reference paper on a topic. It has many references, which you would assume to be to authoritative papers. This one is interesting because it is not itself cited very much. I would think most of the papers that reach the top hub or authority measures would be fairly high on both, however that's what makes this interesting.

Title: D-Brane Primer
Authors: Clifford V. Johnson

- This is a collection of lecture notes with many references. Makes sense as to why it would be a hub. The lecture notes would attempt to teach on the subject and link to authorities.

Authority index papers:
Title: The Large N Limit of Superconformal Field Theories and Supergravity
Authors: Juan M. Maldacena

- This is a shorter paper but must’ve been influential on the topic. It has tens of thousands of citations on google scholar

Title: Anti De Sitter Space And Holography
Author: Edward Witten

- Interestingly, the author of the number one authority paper and some of their work is referenced in the abstract of this one. It must've been some exciting developments in the field going on in these papers. This paper also has tens of thousands of citations on google scholar.

Title: Gauge Theory Correlators from Non-Critical String Theory
Authors: S.S. Gubser, I.R. Klebanov and A.M. Polyakov

- The authority papers tend to be much shorter than high hub measure papers. They tend to introduce a new idea on a topic and that could be why they get cited so many times. There are reactions to new ideas. Also over ten thousand citations on google scholar.

Problem 5.
Describe two real-world networks wherein out-degree centrality is relevant?

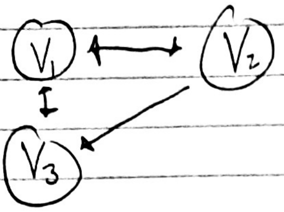
Out-degree can represent individuals that can quickly connect with the larger network. I'll explain two "real world" networks and a social media example.

- One use of a real world out degree network would be like the amber alert system. You want to push information out to many users quickly. In this case, it is to expedite finding a missing minor. Government entities that must inform the public would often have a high out degree.
- I think ads or product placement in a tv network is also a good example of high out degree. Say for a football game, many people tune into one channel that can share the advertisement with many users.
- For social media, I think of an instagram account that many people follow, so when they have a post it reaches many users.

Bonus
Written and attached on the next page.

Homework 3 bonus

Andrew Curtis



Steps: find A
compute eigenvalues of A
select largest eigen value
find eigen vector
eigen vector is eigenvector centrality

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\det|A - \lambda I| = \begin{bmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & 1 \\ 1 & 0 & 0-\lambda \end{bmatrix} = 0$$

$$-\lambda(-\lambda \cdot -\lambda - 0 \cdot 1) - 1(1 \cdot (-\lambda) - 0 \cdot 1) + 1(1 \cdot 1 - (-\lambda) \cdot 1) = 0$$

$$-\lambda^3 + 2\lambda + 1 = 0$$

$$-(\lambda+1)(\lambda^2 - \lambda - 1) = -(\lambda+1)\left(\lambda + \frac{\sqrt{5}-1}{2}\right)\left(\lambda - \frac{\sqrt{5}+1}{2}\right) = 0$$

$$\text{eigenvalues} = \left(-1, \frac{1-\sqrt{5}}{2}, \boxed{\frac{1+\sqrt{5}}{2}}\right)$$

largest value

$C_e = [x_1\ x_2\ x_3]^T$

$$\begin{bmatrix} 0-\left(\frac{1+\sqrt{5}}{2}\right) & 1 & 1 \\ 1 & 0-\left(\frac{1+\sqrt{5}}{2}\right) & 1 \\ 1 & 0 & 0-\left(\frac{1+\sqrt{5}}{2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{-\sqrt{5}-1}{2} & 0 \\ 0 & 1 & \frac{-\sqrt{5}-1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 + \left(\frac{-\sqrt{5}-1}{2}\right)x_3 = 0$$

$$x_2 + \left(\frac{-\sqrt{5}-1}{2}\right)x_3 = 0$$

$$x_3 = x_3$$

$$x_3 = \begin{pmatrix} \frac{\sqrt{5}+1}{2} \\ \frac{\sqrt{5}+1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 1.68 \\ 1.68 \\ 1 \end{pmatrix}$$

* Eigenvector Centralities
$V_1 = 1.68$ $V_2 = 1.68$ $V_3 = 1$