Lab Write-Up Week 7

Abbe Theory of Image Formation (II)

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Modulation Transfer Function

In order to graph the MTF of the system a series of Ronchi rulings were imaged and the contrast was measured. The Objective BFP diameter was 3mm, and the aperture stop was closed down for the coherent MTF and opened to have an equivalent NA for the Incoherent MTF.

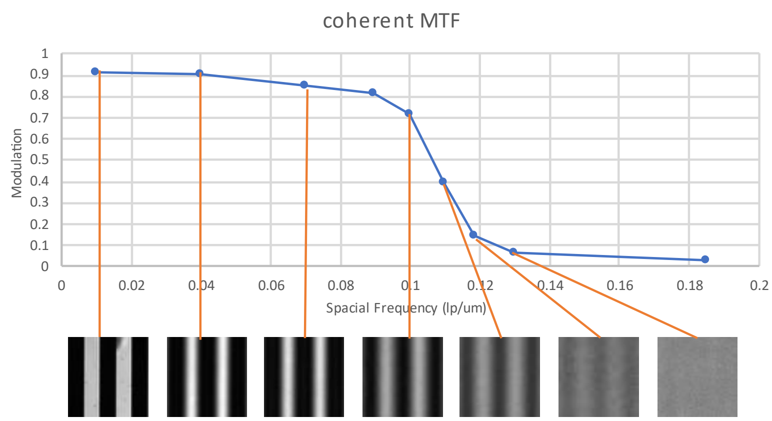


Figure 1. Graph of the coherent case MTF. Objective NA 0.06. Aperture stop closed to minimum diameter (NA approx. 0). All images taken with 32 ms exposure.

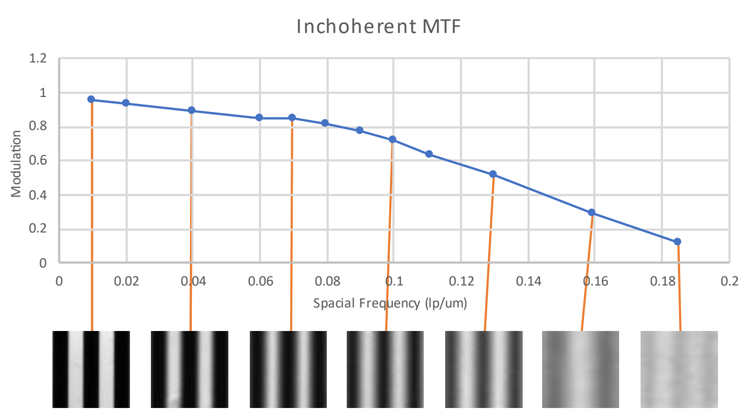


Figure 2. Graph of the incoherent case MTF. Objective NA 0.06. Condenser NA 0.06. All images taken with 2.4 ms exposure. Graph of MTF appears to show a semi-coherent case, as the graph begins flat, before dropping down. This signifies that the Condenser NA is actually less than 0.06.

The maximum spatial frequency is known by the equation:

Equation 1. $k = \frac{(NA_{objective} + NA_{condenser})}{\lambda}$

For the incoherent condition: $k = \frac{(0.06)}{0.528\ um} = 0.11\ cycles/um$

For the coherent condition: $k = \frac{(0.06+0.06)}{0.528\ um} = 0.22\ cycles/um$

The Maximum Spatial Frequencies calculated match the graphs of the MTFs. For the incoherent case, 0.11 is the middle of the transient where contrast is lost. In a perfectly incoherent system, this would be a vertical step exactly at 0.11.

For the incoherent case, the graph does not reach 0, as there was not a small enough Raleigh resolution target to image. From the graph, it is expected that the max spatial frequency is approximately 0.2.

Slanted Edge MTF

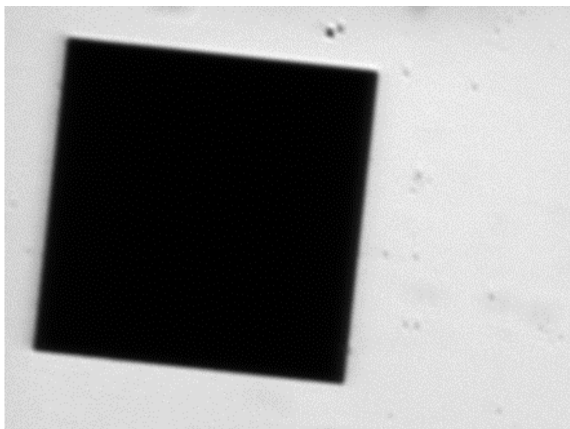


Figure 3. Image of the reference black using incoherent illumination. Exp 0.808 ms. BFP 6.5mm diameter. Objective NA = Condenser NA = 0.15.

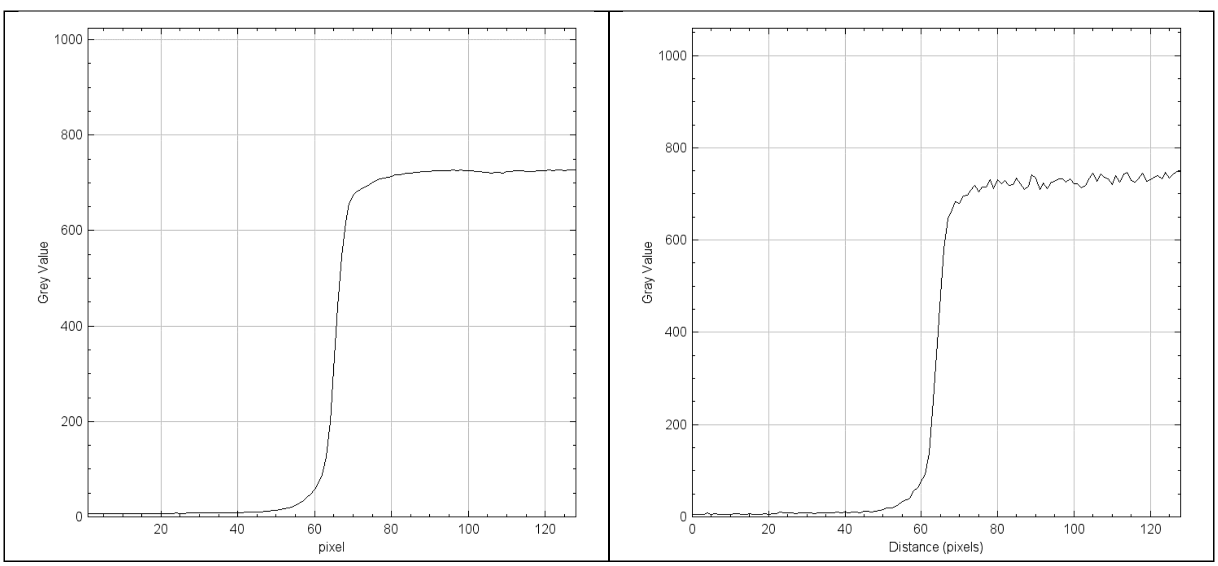


Figure 4. (Left) Edge Spread Function plot of Figure 3. (Right): Line Plot through the right edge of reference square in Figure 3.

The Edge Spread Function and line plot of the edge look similar. The line plot has extra noise, as it is take over a single row of pixels, whereas the ESF is taken over a much larger area of the edge, reducing noise.

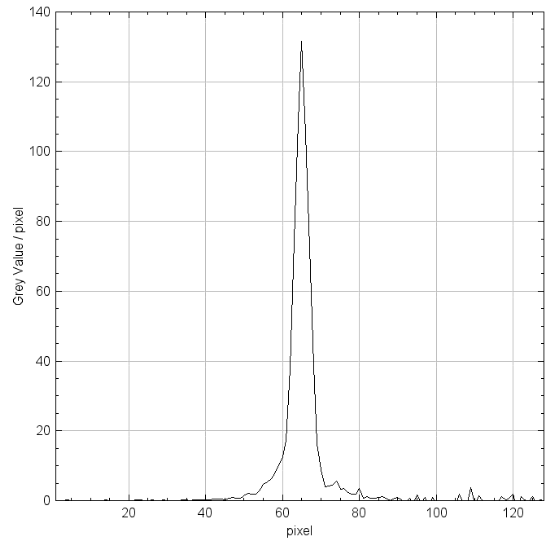


Figure 5. LSF of Figure 3. The FWHM of the line is 5px (2.88 um).

With the width of the line in pixels, the resolution of the system can be found as

$$\sigma = line\ width * \frac{camera\ px\ spacing}{M} = 5px * \frac{3.45um}{6} = 2.88\ um$$

The Rayleigh resolution of the system is defined by:

$$\sigma = \frac{1.22\lambda}{NA_{obj} + NA_{cond}} = \frac{1.22 * 0.528um}{0.15 + 0.15} = 2.15\ um$$

As the line being image, being the derivative of the squares edge, is very narrow and can be considered as a column of point sources. As such, the with of the line when imaged should be equal to the diameter of an airy disk from a PSF. Therefore, the line with is the resolution of the system. The error between the measured resolution and the Rayleigh resolution is 34%.

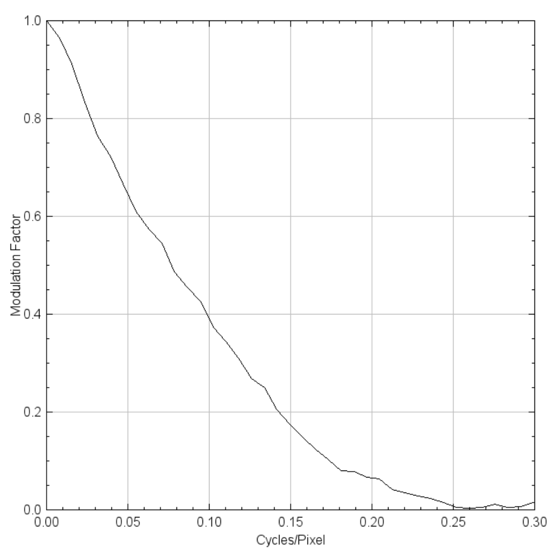


Figure 6. MTF of incoherent illumination. Cycles/pixel = 0.575 Cycles/um. Cutoff frequency is 0.26 cycles/px.

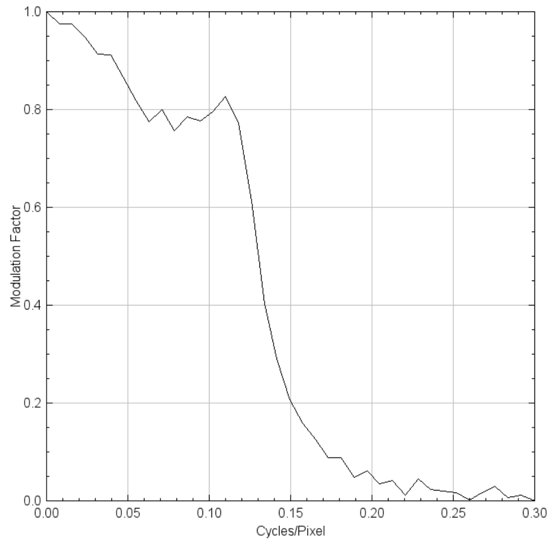


Figure 7. MTF of coherent illumination. Cycles/pixel = 0.575 Cycles/um. Cutoff frequency is 0.14 cycles/px

The cutoff frequency with incoherent illumination is expected to be:

$$\frac{NA}{\lambda} = \frac{0.13 + 0.13}{0.528um} = 0.49\ cycles/um$$

$$0.49\frac{cycles}{um} * \frac{3.45\ um}{6} = 0.28\ cycles/px$$

The cutoff frequency shown by Figure 6 is 0.26um which suggests the condenser NA is slightly smaller than the objective NA.

The cutoff frequency with coherent illumination is expected to be:

$$\frac{NA}{\lambda} = \frac{0.13}{0.528um} = 0.25\ cycles/um$$

$$0.25\frac{cycles}{um} * \frac{3.45\ um}{6} = 0.14\ cycles/px$$

The cutoff frequency shown by Figure 7, is roughly between 0.12 and 0.15 um. A cutoff frequency of 0.14 cycles/px matches the data collected. The cutoff in coherent illumination is expected to be a brick wall type filter, but as due to the minimum condenser aperture diameter being limited to approximately 1mm, the system exhibits partial coherence, and the cutoff resembles a slope.

Out of Focus MTF

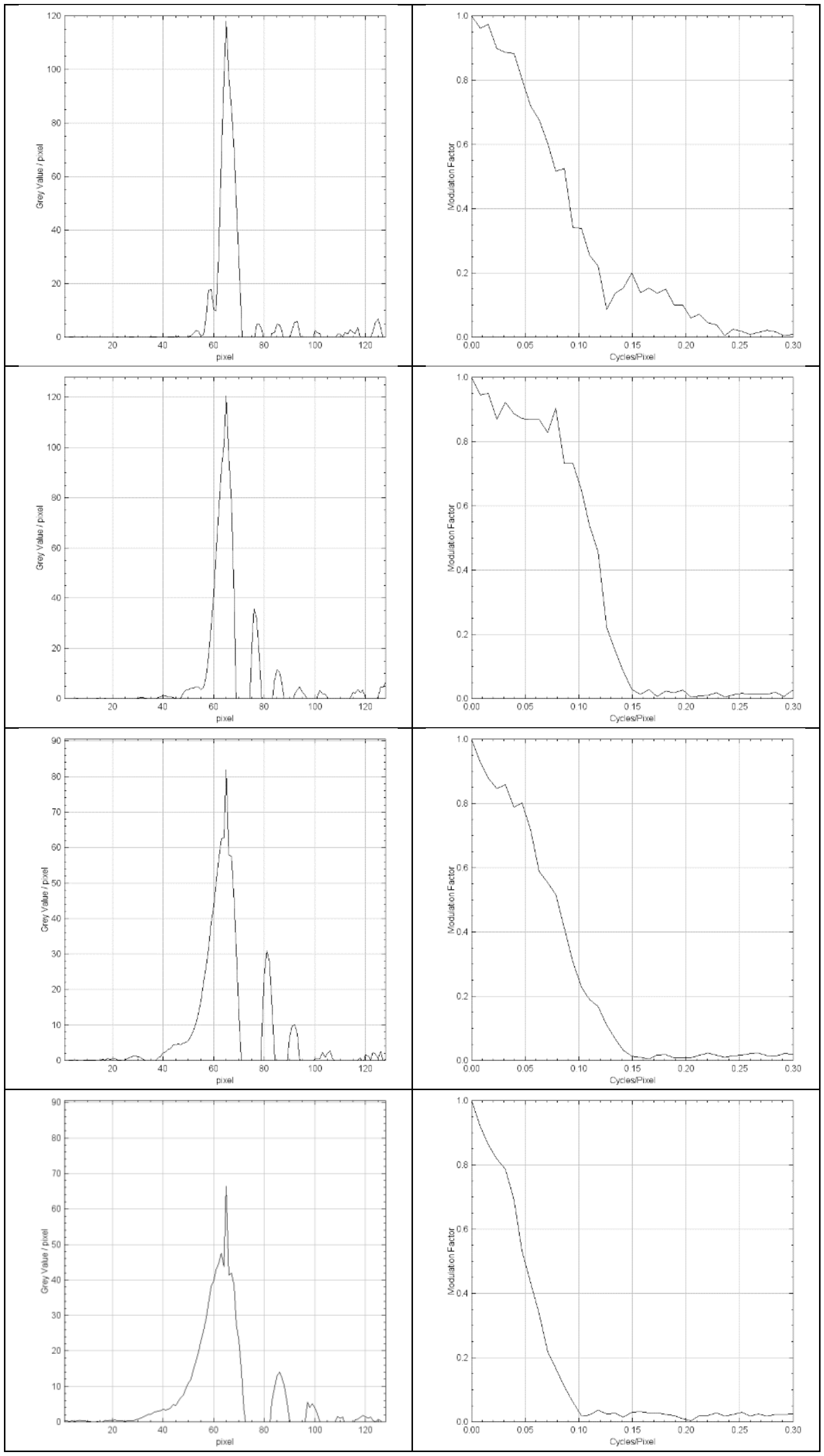


Figure 8. From Top to Bottom: In focus, 50um out of focus, 100um out of focus, 150um out of focus. On left side are LSF, and on the right are the corresponding MTF. LSF width can be seen increase and MTF cutoff decreasing as the image loses focus.

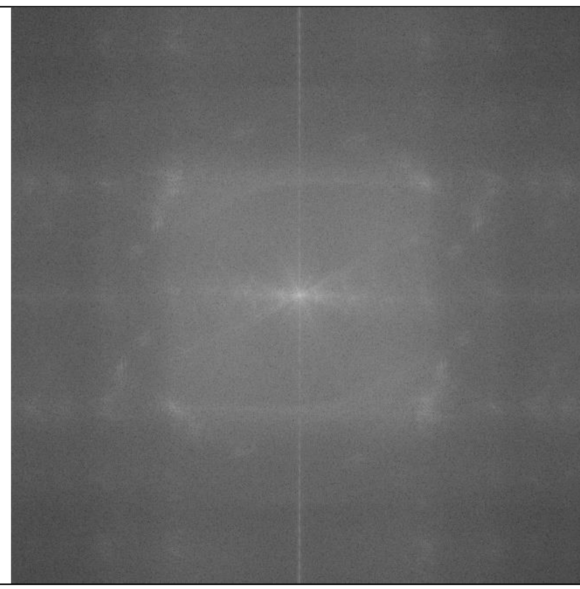
dist. from focus	FWHM (px)	Cutoff (cycle/px)	Cutoff (cycle/um)
0um	6	0.24	0.42
50um	8	0.17	0.30
100um	10	0.14	0.24
150um	11	0.1	0.17

Table 1. Measurement of the line spread function width and the cutoff frequency for each of samples in Figure 8.

As the image becomes out of focus, the width of the LSF increases, and the cutoff frequency decrease. As the cutoff frequency decreases, finer details will no longer be visible in the image, and the image will only be composed of the low frequency features. This would cause the width of the LSF to increase.

This corresponds to an increase in the airy disk diameter from a PSF.

Fast Fourier Transform



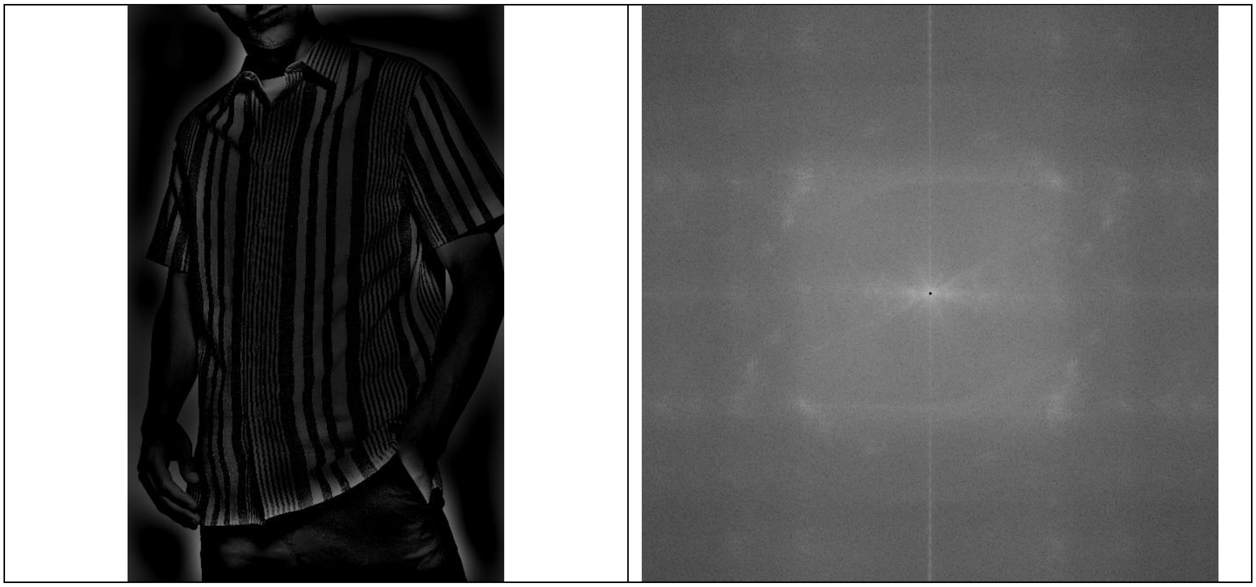


Figure 9. (Top Left) Black and White image of a man wearing a stripped shirt. (Top Right): FFT transform of image. (Bottom Right): Darkfield image create by Inverse FFT of bottom right. (Bottom Right): FFT image, with a small black dot applied to the center of the image.

The Fourier Transform of an image is equivalent to what one would see at the BFP of the objective lens, As with darkfield image, applying a mask digitally to the 0^{th} order of the Fourier Transform, and then applying the inverse transformation create a darkfield image.

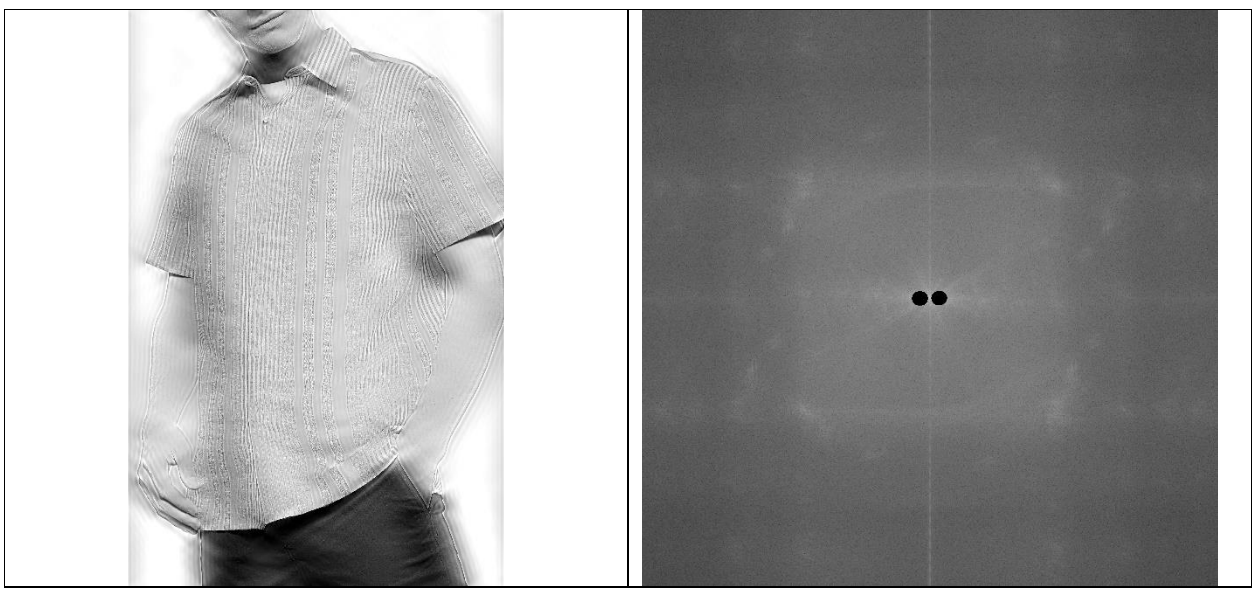


Figure 10. Another example of masking parts of the Fourier Transform, removing low frequency vertical details.

In Figure 10, the FFT has been masked along the horizontal axis, close to the 0^{th} order. This blocks the low frequency information of vertical features. Thus, the stripes of the shirts blend and become one color. High frequency information is still preserved, which appears as ripples and random noise patterns where the stripes used to be.