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Dimensionless Equations: fixed B

$$\frac{\partial N}{\partial t} = \nabla^2 (DN) + \frac{\partial}{\partial E}[bN] + Q$$

$N(r, E, t)$

$Q(r, E)$, units are $1/t$ and let $Q(r,E) = E^{-\gamma_0} q(r) Q_0$, Q_0 has units $\frac{1}{tE}$

$D(E) = D_0 d(E)$

$b(E) = \beta E^2$ where $\beta = 8\times 10^{-17}\left(U_{rad} + 6\times 10^{11} \frac{B^2}{8\pi}\right) \mathrm{GeV}^{-1} \mathrm{s}^{-1}$

from Berezhinski et. al. 1990.
B in G, U_{rad} in eV cm^{-3}, E in GeV

Choose scalings for dimensionless variables: (prime means no units)

$t \to t_0 t'$ so $\frac{\partial}{\partial t} \to \frac{1}{t_0}\frac{\partial}{\partial t'}$

$r \to r_0 r'$ so $\frac{\partial}{\partial r} \to \frac{1}{r_0}\frac{\partial}{\partial r'}$

$E \to E_0 E'$ so $\frac{\partial}{\partial E} \to \frac{1}{E_0}\frac{\partial}{\partial E'}$

Then on substitution we obtain, for 1D model in r only:

$$\frac{1}{t_0}\frac{\partial N}{\partial t'} = \frac{D_0}{r_0^2} d(E') \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial N}{\partial r'}\right) + \frac{\beta E_0^2}{E_0}\frac{\partial}{\partial E'}\left(E'^2 N\right) + Q_0 E_0^{-\gamma_0} q(r') E'^{-\gamma_0}$$

Thus,

$$\frac{\partial N}{\partial t} = \frac{D_0 t_0}{r_0^2} d(E) \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) + \beta E_0 t_0 \frac{\partial}{\partial E}\left(E^2 N\right) + Q_0 t_0 E_0^{-\gamma_0} q(r) E^{-\gamma_0}$$

where all variables are now dimensionless.

Choose Q_0 so that $Q_0 t_0 E_0^{-\gamma_0} = 1$

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Values of dimensionless parameters

We have $\Theta = \frac{D_0 t_0}{r_0^2}$ and $\Phi = \beta E_0 t_0$

Let $E_0 = 1\,\mathrm{GeV}$, $r_0 = 10\,\mathrm{kpc}$, $t_0 = 10^9\,\mathrm{yr}$, $D_0 = 10^{28}\,\mathrm{cm^2/s}$

$$\Theta = \frac{10^{28} \times 3\times10^7 \times 10^9}{(3\times10^{22})^2} = \frac{3\times10^{44}}{9\times10^{44}} = \frac{1}{3}$$

$$\Phi = 8\times10^{-17}\left(1\,\mathrm{eV} + \frac{6\times10^{11}\times(10^{-5})^2}{8\pi}\right)\times 1 \times 3\times10^7 \times 10^9$$

$$= 8\times10^{-17}(1+2.4)\times 3\times10^{16} = 8 \quad \text{for } B = 10\,\mu G$$

For energy dependent diffusion:

$d(E)$ is used in dimensionless E

for example:

$$d(E') = \begin{cases} D_0, & E' \leq E/E_0 \\ D_0^2, & E' > E/E_0 \end{cases}$$

Equation to solve is:

$$\frac{\partial N}{\partial t} = \Theta\, d(E) \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) + \Phi \frac{\partial}{\partial E}\left(E^2 N\right) + q(r) E^{-\gamma_0}$$

all variables, (r, t, E) are scaled (dimensionless)

$q(r)$ is proportional to $SFR(r)$

Dimensionless Equation: variable B

As before, but now we keep $B(r)$ in the energy loss term and make it dimensionless.

$$b(r,E) = \beta(B)\, E^2$$

where $\beta = 8\times 10^{-17}\left[U_{rad} + 6\times 10^{11}\dfrac{B^2(r)}{8\pi}\right]$ $GeV^{-1}\, s^{-1}$

$= \alpha + \xi B^2(r)$ with $\alpha = 8\times 10^{-17}$
$\xi = 2\times 10^{-6}$

Same scaling as before, but also $B \to B_0 B'$

$$\frac{1}{t_0}\frac{\partial N}{\partial t'} = \frac{D_0}{r_0^2} d(E')\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial N}{\partial r'}\right) + \left(\frac{\alpha E_0^2}{E_0} + \xi \frac{E_0^2 B_0^2 B'^2}{E_0}\right)\frac{\partial}{\partial E'}\left(E'^2 N\right) + Q_0 E_0^{-\gamma_0} q(r') E'^{-\gamma_0}$$

$$\frac{\partial N}{\partial t} = \frac{D_0 t_0}{r_0^2} d(E)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) + \alpha t_0 E_0 \frac{\partial}{\partial E}\left(E^2 N\right) + \xi t_0 E_0 B_0^2 B(r)\frac{\partial}{\partial E}\left(E^2 N\right)$$
$$+ Q_0 E_0^{-\gamma_0} t_0\, q(r) E^{-\gamma_0}$$

As before choose Q_0 so $Q_0 t_0 E_0^{-\gamma_0} = 1$

$\Theta = \dfrac{t_0 D_0}{r_0^2} = \dfrac{1}{3}$ for $E_0 = 1\,GeV$, $r_0 = 10\,kpc$, $t_0 = 10^9\,yr$, $D_0 = 10^{28}\,cm^2/s$

Now $\Psi = \alpha t_0 E_0 = 8\times 10^{-17} \times 1\times 10^9 \times 3\times 10^7 = 2.4$

and $\Sigma = \xi t_0 E_0 B_0^2 = 2\times 10^{-6} \times 10^9 \times 3\times 10^7 \times 1 \times (10^{-5})^2 = 6$

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Equation to solve for variable $B(r)$

$$\frac{\partial N}{\partial t} = \Theta\, d(E)\, \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial N}{\partial r}\right) + \Phi \frac{\partial}{\partial E}\left(E^2 N\right) + \Sigma B(r) \frac{\partial}{\partial E}\left(E^2 N\right)$$

$$+ q(r) E^{-\gamma_0}$$

all variables (r, t, E, B) are scaled (dimensionless)

$q(r)$ is proportional to $SFR(r)$.