How likely is a set of cards in Risk?

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1 Introduction

This week's problem statement can be found in Zach Wissner-Gross's weekly Fiddler on the Proof column. It is about the likelihood of holding a set of cards in the board game Risk which can be traded in for more units.

2 Fiddler problem

Here, we assume that there are no wild cards; the deck contains 14 cards each from 3 groups, for a total of 42. There are two ways to get a set of three cards that can be traded in:

- All three cards are from the same group.
- All three cards are from different groups.

Consider the first case. The first card can be any of the three types. The second card has a probability of 13/41 of being the same type as the first. If the first two cards are of the same type, the third card has a 12/40 probability of being the same type as the first two.

Now consider the second case. The first card can again be any of the three types. The second card is a probability of 28/41 of being a different type from the first. If the first two cards are different types, the third card has a 14/40 probability of being the third type.

Therefore, the probability of having a set of cards which can be traded in after drawing three cards is

$$P = \frac{13}{41} \cdot \frac{12}{40} + \frac{28}{41} \cdot \frac{14}{40} \approx 0.334.$$

3 Extra credit problem

The extra credit problem introduces the two wild cards seen in an actual set of cards from a Risk set. First, note the following two facts:

Lemma 3.1. *Any group of three cards that includes a wild card is a set.*

Proof. If there are two wild cards in the group of three, then they can be assumed to match the type of the non-wild card, which makes a set.

If there is one wild card in the group of three, there are two cases. If the two non-wild cards are the same type as each other, treat the wild card as that same type to make a set. If the two non-wild cards are different types, treat the wild card as the third type to make a set. □

Lemma 3.2. *Any group of five cards contains a set of three.*

Proof. Assume that there exists a group of five cards which doesn't contain a valid set. By Lemma 3.1, this group does not contain a wild card.

Because this group of five cards cannot contain a set, it can contain at most two card types; otherwise, a set of three cards of three types would exist. By the Pigeon-Hole Principle, there must be at least three cards of at least one of the two remaining card types. This subgroup of three cards is therefore a set, contradicting the assumption that the group of five cards does not contain a tradeable set of three cards. □

Using these two lemmas to guide my strategy, I looped through all possible permutations of the first four cards drawn by a player in the Python file RiskCardSetSim.py. By Lemma 3.2, the fifth card guarantees a set, allowing me to ignore the fifth card and speed up the loop.

According to the computations in the code, it takes an average of 3.760 cards drawn to make a set.