1 Original problem

After each team gets its guaranteed possession, there are three possible scenarios: Your team wins, your team loses, and the game is tied. The probability of the second team matching your team's score is 1/3, and due to the symmetry of the problem, the probability your team has won at this point and the probability your team has lost at this point are also both 1/3.

Assume that the game is still tied, and whichever team scores next wins. Since you get the ball first, there is a 2/3 chance of you winning immediately. The probability of you winning on your third possession (including the initial guaranteed one) is $\frac{1}{9} \cdot \frac{2}{3}$, and in general, the probability of your team winning at any point once the first score ends the game is

$$\frac{2}{3} + \frac{1}{9} \cdot \frac{2}{3} + \left(\frac{1}{9}\right)^2 \frac{2}{3} + \ldots = \frac{2}{3}\left(\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n\right) = \frac{2}{3}\frac{1}{1-1/9} = \frac{2}{3} \cdot \frac{9}{8} = \frac{3}{4}.$$

This means the total probability of your team winning is

$$P(\text{win}) = P(\text{early win}) + P(\text{later win}|\text{early tie}) = \frac{1}{3} + \frac{1}{3} \cdot \frac{3}{4} = \frac{7}{12}.$$

2 Extra credit

In the extra credit, both teams have the option of adopting either the original strategy or a different strategy which gives a 1/2 chance of scoring a touchdown and a 1/2 chance of scoring a field goal.

Note first that, if a team is ever in a position where they win instantly after scoring, there is no reason to adopt this second strategy. This can apply anytime after both teams have taken one possession or it can apply to your opponents on their first possession if your team previously failed to score. This also means that the calculations for the sudden-victory portion apply again in the extra credit; in other words, it is again true that

$$P(\text{later win}|\text{early tie}) = \frac{3}{4}.$$

The opponent's strategy will depend on your own point total in your first possession. From there, we can work backwards to determine your team's optimal strategy.

Case 1: You fail to score. In this case, the next team who scores wins; your opponent will win with probability 3/4 and you win with probability 1/4.

Case 2: You kick a field goal. If the opponent attempts the original strategy, they have a 1/3 chance of scoring a touchdown and winning immediately, a 1/3 chance of failing to score, and a 1/3 chance of tying the game. Your probability of winning in this case would be 7/12. If your opponent attempts the new strategy, they have a 1/2 chance of winning immediately and a 1/2 chance of losing immediately. They would obviously prefer this strategy, so your chance of winning the game if you kick a field goal on your first possession is therefore 1/2.

Case 3: You score a touchdown. A field goal does not help your opponent here, so they take the new strategy and tie the game with probability 1/2; you win immediately if they fail to score. The probability of you winning the game is therefore $1/2 + (1/2)(3/4) = 7/8$.

Given these three cases, you can now decide which strategy is optimal. If you take the original strategy, your probability of winning is

$$P(\text{win}) = \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{7}{8} = \frac{13}{24}.$$

If you take the new strategy, your probability of winning is

$$P(\text{win}) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{7}{8} = \frac{9}{16}.$$

Based on this result, your team can maximize its probability of winning by going for the touchdown immediately, and will win with probability 9/16 in that case.