Winning a three-set tennis match while losing most of the points

Andrew Ford

July 27, 2024

1 Link to problem statement

The link to the problem statement from Zach Wissner-Gross can be found at
https://thefiddler.substack.com/p/can-you-fail-to-lose-the-tennis-match.

2 Original problem

2.1 A note on winning games or tiebreakers by two points

Since any game or tiebreaker must be won by two points, it is possible for any single game or tiebreaker to go on indefinitely. However, relative to the case where the game or tiebreaker is won by two points at the earliest opportunity, you both win and lose N extra points for some positive value of N. This brings the total proportion of points that you win closer to 50%.

Therefore, if any way to win the tennis match while losing the majority of points exists, the way where you lose the highest proportion of points will not contain any games or tiebreakers that go on longer than necessary.

Because of this, we will begin by assuming that, in the "optimal" case, your opponent scores two points in any game you win and you score zero points in any game your opponent wins. Likewise, we assume that your opponent scores five points in any tiebreaker you win and you score zero points in any tiebreaker your opponent wins.

2.2 Losing a set as badly as possible

Since this is a best of three match, you can lose one set while still winning the match. Since you're trying to win with the smallest percentage of total points won in the match, you want to lose this set as badly as possible. The obvious solution is also the best one- you can lose the set 6-0 while scoring zero points; your opponent scores 24 points.

For each game you win, your opponent could win two more points, but you would win four more. If you tie the match at 5-5, your opponent can win a 7th game, but you would still have a net gain of at least six points. Likewise, if the set goes to a tiebreaker, your opponent could win the tiebreaker 7-0, but this is not enough to offset the 12 or more points you would win relative to your opponent to win six games and force the tiebreak.

Therefore, we can assume you lose one of the first two sets 6-0, losing every point in the process. This gives your opponent 24 points while winning zero.

2.3 Eking out a win in a set

If you want to win a set while winning the lowest possible percentage of points in the set, there are three possible ways which are plausible enough to warrant consideration:

1. You win the set 6-4.

2. You win the set 7-5.

3. You win the set in a tiebreak (denoted 7-6).

If you win the set 6-4, then we assume you won 24 points and lost 12 in the six games you won, and that you won 0 points and lost 16 in the four games you lost.

If you win the set 7-5, then we assume you won 28 points and lost 14 in the seven games you won, and that you won 0 points and lost 20 in the five games you won. This results in a total of 28 points won and 34 points lost.

If you win in a tiebreak, then we assume you won 24 points and lost 12 in the six regular games you won and that you won 0 points and lost 24 in the six regular games you lost. It is also assumed that you win the tiebreak 7-5, resulting in a total of 31 points won and 41 points lost.

The following table illustrates the total number of points won and lost for each possible combination of these three options among the two sets you win, followed by the corresponding percentage of points you win in the entire match. Redundant combinations are left blank.

	6-4	7-5	7-6
6-4	48-80	52-86	55-93
7-5		56-92	59-99
7-6			62-106

Table 1: Total number of points won and lost in the two sets you win

	6-4	7-5	7-6
6-4	37.5%	37.68%	37.16%
7-5		37.84%	37.34%
7-6			36.90%

Table 2: Total number of points won in the entire match

Therefore, the lowest percentage of points you can win while still winning the match is 36.90%. This means the highest percentage of points you can lose while still winning the match is 63.10%.

3 Extra credit

To solve the extra credit, I approximated using a Monte Carlo simulation with 1,000,000 simulated tennis matches. The player who won the match won fewer points than their opponent approximately 6.3% of the time, and the two players won the same number of points approximately 1.8% of the time.