Mathematical Anomalies of Bowling

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1 Introduction

This week's problem statement can be found in Zach Wissner-Gross's weekly Fiddler on the Proof column at https://thefiddler.substack.com/p/can-you-hack-bowling

2 Original Problem

What is the fewest number of pins you can knock over to achieve 100 points?

In bowling, clustering strikes together is the fastest way to rack up points. Therefore, it makes sense to start with a streak of strikes. For example, if you bowl four strikes to start off a game, you get the following scores:

Frame	1	2	3	4
Result	X	X	X	X
Score	30	30	$20 + a$	$10 + a + b$

Here, a and b represent the scores of the next two rolls. It's tempting to continue rolling strikes (easier said than done), but your score is already $90 + 2a + b$. By knocking over 5 pins on your next roll, you can reach 100. After that, you can roll gutter balls for the rest of the game to reach a score of 100 while knocking over only 45 pins.

3 Extra Credit Problem

After a game of bowling where you and your opponent both knock down an identical number of pins, your scores are very far apart. In fact, your opponent noticed that this difference is the largest possible difference between scores where both players knocked down the same number of pins. What is that difference?

To solve this problem, I will first find an upper bound for this difference, and then I'll find a case of this upper bound being reached.

3.1 Finding an upper bound

Let N be the total number of pins you knocked down (and also the number your opponent knocked down). Assume that you are the winner, and your opponent lost. Since every pin knocked down is worth at least one point, your opponent's score is at least N.

For frames 1-9, let p_i be the number of pins you knocked down and q_i be the number of pins your opponent knocked down, where i is the frame number. Typically, after a strike, the bonus points associated with each of the next two rolls are associated with the frame where the strike occurred, but I'll associate them with the frame where the rolls occur for the purposes of this computation.

Additionally, for a fixed p_i, there is never any benefit to knocking over more pins on the second roll- strikes are always better than spares, and there are situations where a first roll might count for a previous strike or spare but the second roll doesn't. Therefore, I will assume that p_i is the score from the first roll, and the second roll of a frame (if it happens) knocks over 0 pins.

For the tenth frame, define p_{10}, p_{11}, and p_{12} as the number of pins knocked over out of the first, second, and third sets of ten pins respectively. If there is no second or third set of pins awarded in the tenth frame, define p_{11} and/or p_{12} as zero as appropriate. Likewise, define q_{10}, q_{11}, and q_{12} in the same way for your opponent.

Then $0 \leq p_i, q_i \leq 10$ for all i. The maximum number of points you could score in each frame is as follows:

i	Maximum score	Minimum score	Difference
1	p_1	p_1	0
2	$2p_2$	p_2	p_2
3-10	$3p_i$	p_i	$2p_i$
11	$2p_{11}$	p_{11}	p_{11}
12	p_{12}	p_{12}	0

Some of these maxima are attainable only if you bowl strikes on the previous two frames. For the purposes of establishing an upper bound, we can assume this.

Since each p_i is an integer from 0 to 10 inclusive, an upper bound for the maximum difference is 180: 10 each from p_2 and p_{11}, and 20 each from $p_3, \ldots, p_{10}$. Therefore, it is never possible for two opponents' scores to be separated by more than 180 points if they knock over the same number of pins.

3.2 Achieving the upper bound

To achieve this upper bound of 180, you would have to bowl strikes in your first ten frames- this is necessary to get the maximum number of repeats from $p_2, \ldots, p_{11}$ as defined above. This already implies you must knock over at least 100 pins. Because of this, it's worth examining how many pins your opponent can knock over without double-counting any toward their final score.

Since your opponent would have to knock over a lot of pins without getting a strike or spare, assume your opponent knocks over 9 pins in each of the first 9 frames. This accounts for 81 pins.

For the tenth frame, note that q_{11} and q_{12} count toward the score only as bonuses applied to previous strikes and spares. Therefore, if your opponent bowls three strikes in frame 10, they knock over 30 pins and score 30 points. This means it's possible for your opponent to knock over 111 pins and score 111 points.

This is enough pins for you to get strikes on frames 1-10, and then another strike on your second roll of frame 10. Finally, to finish off the game, you can only knock over one pin. This gives you 291 points, exactly 180 more than your opponent.

In this scenario, you missed out on a perfect game on your final roll, but this level of mathematical glory might be even more rare.