An NBA prediction that already aged poorly

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1 Introduction

This week's problem statement, found in Zach Wissner-Gross's weekly Fiddler on the Proof column at https://thefiddler.substack.com/p/can-you-sweep-the-series, concerns a prediction made earlier this week by a TV sports personality that the Boston Celtics would defeat the New York Knicks in 5 games.

At the time of publication, this prediction had already aged poorly (which the author hints at in the problem write-up); the Knicks won the first two games of the series. For the rest of the problem, assume the math represents a hypothetical series and not the actual one.

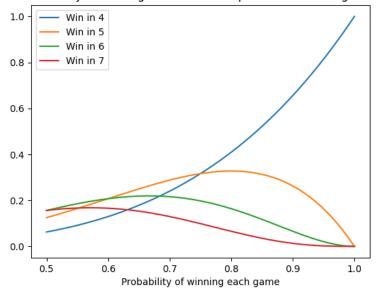
2 When is winning in 5 games most likely?

Since the series must end with a win, the probability of winning a series in exactly n games is equivalent to the probability of winning 3 out of the first n-1 games multiplied by the probability p of winning the final game. This means that the probability of winning a series in n games, where p is the probability of winning a single game, is

n	C(n-1,3)	Probability of winning in n games
4	1	p^4
5	4	$4p^4(1-p)$
6	10	$10p^4(1-p)^2$
7	20	$20p^4(1-p)^3$

A plot of these probabilities for $p \ge 0.5$ is given here. For p < 0.5, it is more likely to lose the series in n games than to win in n games for any valid given value of n.

Probability of winning the series in a specific number of games



From this plot, we can see that a, the lower bound for the region where winning in 5 games is the most likely outcome, is the non-trivial value of p where it is equally likely for a team to win in 5 games and in 6 games. Likewise, b, the upper bound for this region, is the non-trivial value of p where it is equally likely for a team to win in 5 games and in 4 games.

Therefore, a (which does not equal 0 or 1, so it is safe to divide by a or 1-a) is a solution to

$$4a^{4}(1-a) = 10a^{4}(1-a)^{2}$$
$$\frac{2}{5} = 1-a$$
$$a = \frac{3}{5}.$$

Similarly b (which is also not 0 or 1) satisfies

$$b^{4} = 4b^{4}(1 - b)$$

$$\frac{1}{4} = 1 - b$$

$$b = \frac{3}{4}.$$

3 Is a sweep more likely than a 7 game series?

Now, we assume that p is randomly selected from the uniform distribution on the interval (a, b), for the a and b found in the previous section. We want to know whether the probability of the Celtics winning in a sweep is greater than the probability of the series going to seven games, with either team winning.

The probability of the series going to 7 games, regardless of who wins, is

$$p_7 = \binom{6}{3} p^3 (1-p)^3;$$

in other words, it is the probability of each team winning exactly 3 times if they play six games. The

break-even point, where $p_4 = p^4$ is equal to p_7 , is a solution to the equation

$$p^{4} = {6 \choose 3} p^{3} (1-p)^{3}$$
$$p = 20(1-p)^{3}$$
$$0 = 20p^{3} - 60p^{2} + 61p - 20.$$

This function has one root in the interval (a,b)=(0.6,0.75). Since p_4 is increasing and p_7 is decreasing on this range, a sweep is more likely than a seven-game series when p is greater than this root. Through an implementation of Newton's method found in NewtonsMethod.py, it was found that the root is located at $p\approx 0.67658$. By plugging this value into the formula (b-p)/(b-a), we find that $p_4>p_7$ with a probability of approximately 0.48945.