I Used To Rule A Company...

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1 Introduction

This week's problem statement, inspired by the now-famous affair discovered at a Coldplay concert, can be found in Zach Wissner-Gross's weekly Fiddler on the Proof column. All similarities between my name, my wife's name, the now-former Astronomer CEO's name, and his soon-to-be ex-wife's name are purely coincidental.

2 Initial problem

First, note that you have no advance knowledge of when the camera operators will show couples; you just know that 1 percent of couples will be shown on the Jumbotron. Therefore, you can assume that there is a 1 percent chance of appearing on the Jumbotron at some point during any given Coldplay concert. Since you and your partner will canoodle 50 percent of the time at the show, there is an 0.5 percent chance of being shown canoodling on-screen at any given Coldplay concert. In other words, you will be shown canoodling on the Jumbotron with a probability of 0.005, and you will avoid being shown canoodling on the Jumbotron with a probability of 0.995.

If you attend n concerts, your probability of never being shown canoodling on the Jumbotron is 0.995^n, a value which continually decreases as n increases. The largest number of concerts you can attend without having more than a 50 percent chance of being shown canoodling on the Jumbotron is $\lfloor x \rfloor$, where x is the solution to the equation

$$\begin{aligned} .995^x &= .5 && (1)\\ x \ln .995 &= \ln .5 && (2)\\ x &= \ln .5 / \ln .995 && (3)\\ x &\approx 138.3. && (4) \end{aligned}$$

Therefore, you and your partner can attend up to 138 Coldplay concerts and still have less than a 50 percent chance of being shown canoodling on the Jumbotron.

3 Expanded problem

Let $X \& Y$ be the proportions of time that each member in a given couple wants to spend canoodling during the Coldplay concert. Both X and Y are uniformly distributed from 0 to 1, with probability distribution functions

$$f_X(X) = 1 \text{ for } 0 < X < 1, \quad 0 \text{ otherwise},$$
$$f_Y(Y) = 1 \text{ for } 0 < Y < 1, \quad 0 \text{ otherwise}.$$

The variable $Z \equiv X \cdot Y$ represents the proportion of the concert that a given couple will spend canoodling. Since X and Y are independent variables, the probability distribution function of Z is

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z/x) \frac{1}{|x|}\, dx. \tag{5}$$

Since f_X and f_Y are both 0 outside of the interval $(0,1)$, we can make this integral a bit simpler. Additionally, this means that $f_Y(z/x) = 1$ when $x > z$ and $f_Y = 0$ when $x \leq z$. Also, $|x| = x$ on this domain. Therefore, on the domain of f_Z, (5) becomes

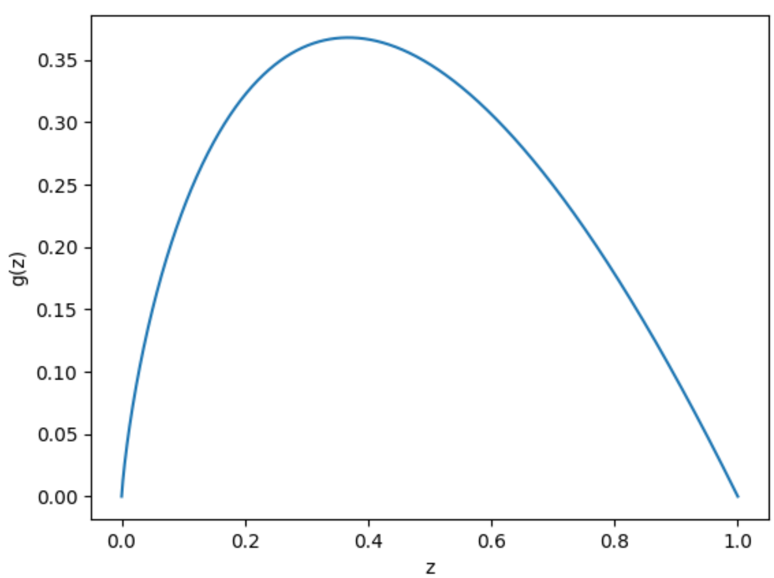
$$\begin{aligned}
f_Z(z) &= \int_0^1 f_X(x) f_Y(z/x) \frac{1}{x}\, dx && (6)\\
&= \int_0^z 1 \cdot 0 \cdot \frac{1}{x}\, dx + \int_z^1 1 \cdot 1 \cdot \frac{1}{x}\, dx && (7)\\
&= 0 + \int_z^1 \frac{1}{x}\, dx && (8)\\
&= \ln x|_z^1 && (9)\\
&= \ln 1 - \ln z && (10)\\
&= -\ln z. && (11)
\end{aligned}$$

In particular, since the domain of Z is $(0,1)$, the complete formula for f_Z is

$$f_Z(z) = -\ln z \text{ for } 0 < Z < 1, \quad 0 \text{ otherwise.} \tag{12}$$

It can be verified that $\int_0^1 f_Z(z)\, dz$ converges and equals 1, a condition that all probability distribution functions must satisfy on their domains.

As mentioned in the problem statement, each couple's likelihood of being shown canoodling on the Jumbotron is proportional to the amount of time they spend canoodling. However, there are a lot of couples canoodling for small portions of the concert and a few couples canoodling for large portions of the concert. Therefore, the value C mentioned in the problem statement, which is the most likely proportion of the concert for which a couple shown on the Jumbotron is canoodling, is the maximum of the function $g(z) = z(-\ln z)$ on the domain $(0,1)$.



The function's only critical point occurs at $g'(z) = 0$. Since $g'(z) = 1 \cdot (-\ln z) + z \cdot (-1/z) = -\ln z - 1$, we find that $g'(z) = 0$ at $\ln z = -1$, or $z = 1/e$. This is the only critical point of the function, and it is a maximum. Therefore, the most likely proportion of the concert for which any couple who appears on the Jumbotron is canoodling is $C = 1/e$.