

New Algorithm

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Setup

```
library(tibble)
library(ggplot2)
library(magrittr)
library(dplyr)
library(patchwork)
devtools::load_all() # loads the functions from the package
```

We are going to assume that we know p_0 exactly. That way it is either a data point in actual performance, with no cost, or we don't count it as an actual performance.

Algorithm Functions:

Helpers

```
# function inside of parameter distance
lim_func <- function(k_1, k_2, tau_1, tau_2) {
  k_1/(1-exp(-1/tau_1))-k_2/(1-exp(-1/tau_2))
}

# computes "parameter distance"
params_dist <- function(params_1, params_2) {
  # ignore params_1[[1]] since it is just the initial value
  abs(lim_func(params_1[[2]], params_1[[3]], params_1[[4]], params_1[[5]]) -
    lim_func(params_2[[2]], params_2[[3]], params_2[[4]], params_2[[5]])
  )
}

# plotting function
plot_perf <- function(observed_performance,
                      modeled_performance) {
  data <- tibble(
    "day" = c(0:length(observed_performance)),
    "pred" = modeled_performance,
    "obs" = c(NA,observed_performance)
  )
}
```

```

plot <- ggplot(data, aes(x = day)) +
  geom_point(aes(y = pred, color = "pred")) +
  geom_point(aes(y = obs, color = "obs")) +
  labs(x = "Day",
       y = "Performance",
       title = "")
plot
}

# chooses the optimal parameters from a parameter matrix
# this is fairly optimized, and is faster than an application of
# r's built in minima function
min_params <- function(params_matrix,
                       training_load,
                       obs_perf,
                       old_params,
                       lambda) {
  min_params <- old_params
  obs_indexes <- which(!is.na(obs_perf))
  n <- length(obs_indexes)
  min_cost <- sqrt(SSE(old_params, training_load, obs_perf)/n)
  for (i in 1:nrow(params_matrix)){
    params_i <- as.numeric(params_matrix[i, ])
    params_dist_i <- params_dist(params_i, old_params)
    SSE_i <- 0; cost_i <- 0
    p_0 <- params_i[[1]]
    k_1 <- params_i[[2]]; k_2 <- params_i[[3]]
    coef_1 <- exp(-1/params_i[[4]]); coef_2 <- exp(-1/params_i[[5]])
    T_1 <- 0; T_2 <- 0
    lower_index <- 1
    j <- 0
    for (new_index in obs_indexes) {
      for (t in lower_index:new_index) {
        T_1 <- coef_1*(T_1) + training_load[[t]]
        T_2 <- coef_2*(T_2) + training_load[[t]]
      }
      SSE_i <- SSE_i + (p_0 + k_1*T_1 - k_2*T_2 - obs_perf[[new_index]])^2
      j <- j + 1
      lower_index <- new_index + 1
      if(min_cost < sqrt(SSE_i/n) + lambda*params_dist_i) {
        break # goes to next set of parameters
        # for a lot of the sets of parameters,
        # the condition should trigger on the first few data points
      }
      if (j == n) {
        min_cost <- sqrt(SSE_i/n) + lambda*params_dist_i
        min_params <- params_i
      }
      else {
        # do nothing; go to the next loop
      }
    }
  }
}

```

```

}
return(list("cost" = min_cost, "opt_params" = min_params))
}

# this function creates (or calls, we will see) the parameter matrix and applies
# it to the previous function. makes things tidier in the algorithm function
update_params_opt <- function(old_params,
                              training_load,
                              sub_obs_perf,
                              bounds_type = list("test"),
                              lambda
                              ) {
  if(bounds_type == "test") {
    params_matrix <- expand.grid(p_0 = 500,
                                k_1 = c(1,2,3,4),
                                k_2 = c(2,4,6,8),
                                tau_1 = c(5:35),
                                tau_2 = c(5:35))
  }
  params_matrix <- params_matrix[params_matrix$tau_1 <= params_matrix$tau_2,]

  getting_params_output <- min_params(params_matrix,
                                       training_load,
                                       sub_obs_perf,
                                       old_params,
                                       lambda = lambda)

  out_list <- list()
  out_list$opt_params <- getting_params_output$opt_params
  out_list$cost <- getting_params_output$cost
  return(out_list)
}

```

The Important Function

```

new_pred_perf <- function(init_params,
                          training_load,
                          obs_perf,
                          bounds_type = list("test"),
                          lambda) {
  # initializations
  curr_params <- init_params
  curr_cost <- 0
  ####
  # curr_params take form c(k_1, k_2, tau_1, tau_2), to work with the time-invariant
  # functions
  ###

  matrix_params <- matrix(0, nrow = length(training_load), ncol = 5)
  colnames(matrix_params) <- c("p_0", "k_1", "k_2", "tau_1", "tau_2")
}

```

```

days <- length(training_load)
perf_out <- c(rep(curr_params[[1]], days + 1))
cost_vec <- c(rep(0, days + 1))
T_1 <- 0
T_2 <- 0

# doing the algorithm
for (i in 1:length(obs_perf)) {
  # update params if there is new information
  if (is.na(obs_perf[[i]]) == FALSE) {
    params_output <- update_params_opt(
      old_params = init_params,
      training_load = training_load,
      sub_obs_perf = obs_perf[1:i],
      # the subset up to observation
      bounds_type = bounds_type,
      lambda = lambda
    )
    curr_params <- params_output$opt_params
    curr_cost <- params_output$cost

  } else {
    } # pass
    T_1 <- exp(-1 / curr_params[[4]]) * T_1 + training_load[[i]]
    # training load index is i-1 since i starts at 2
    T_2 <- exp(-1 / curr_params[[5]]) * T_2 + training_load[[i]]
    perf_out[[i+1]] <-
      curr_params[[1]] + curr_params[[2]] * T_1 - curr_params[[3]] * T_2
    cost_vec[[i]] <- curr_cost
    #####
    # fix cost it is wrong,
    #####
    matrix_params[i, ] <- as.numeric(curr_params)
  }

#plotting
params_data <- tibble(
  "day" = 0:length(training_load),
  "k_1" = as.numeric(c(matrix_params[1, "k_1"], matrix_params[, "k_1"])),
  "k_2" = as.numeric(c(matrix_params[1, "k_2"], matrix_params[, "k_2"])),
  "tau_1" = as.numeric(c(matrix_params[1, "tau_1"], matrix_params[, "tau_1"])),
  "tau_2" = as.numeric(c(matrix_params[1, "tau_2"], matrix_params[, "tau_2"])),
  "p_0" = as.numeric(c(matrix_params[1, "p_0"], matrix_params[, "p_0"]))
)
k_plot <- ggplot(params_data, aes(x = day)) +
  geom_line(aes(y = k_1, color = "k_1")) +
  geom_line(aes(y = k_2, color = "k_2"))

tau_plot <- ggplot(params_data, aes(x = day)) +
  geom_line(aes(y = tau_1, color = "tau_1")) +
  geom_line(aes(y = tau_2, color = "tau_2"))

#outputs

```

```

out_list <- list()
out_list$performance <- perf_out
out_list$cost_vec <- cost_vec
out_list$perf_plot <- plot_perf(obs_perf, perf_out)
out_list$params_plots <- k_plot + tau_plot
return(out_list)
}

```

If `lambda` is equal to 0, then I think some weird underflow garbage happens and the output is wack, but this seems to work for very very very small values of `lambda`. I should try to fix this later.

Algorithm Pseudo-code, not complete

Inputs:

- Actual performance, vector of length n
- Training load, vector of length n
- Initial guess of parameters, vector of length 4
- p_0 , a number
- Ranges to search over for each variable, The length of these vectors multiply to ℓ

Outputs,

Algorithm:

- (1) Accept inputs
- (2) Define

- `pred_perf`, a vector of length $n + 1$, initialized with p_0
- `cur_param`, a vector of length 4, used to keep track of the parameters used on each step
- `cost`, a number to keep track of the cost function

For $i = 0, 1 \dots, n$ If Actual Performance at i is equal to `NA`, vapply `curr_param` to predicted performance recursive equation to get predicted performance set `pred_perf[[i]]` to be this predicted performance

Testing New Algorithm on simulated data

basic testing

```

set.seed(2)
days_test <- 200
perf_sim_1 <- c(500, perf_tv(p_0 = 500,
  k_1 = 1,
  tau_1 = 25,
  k_2 = 2,
  tau_2 = 10,
  days = days_test,
  training_stim = list("constant", 100))$performance
)

```

```

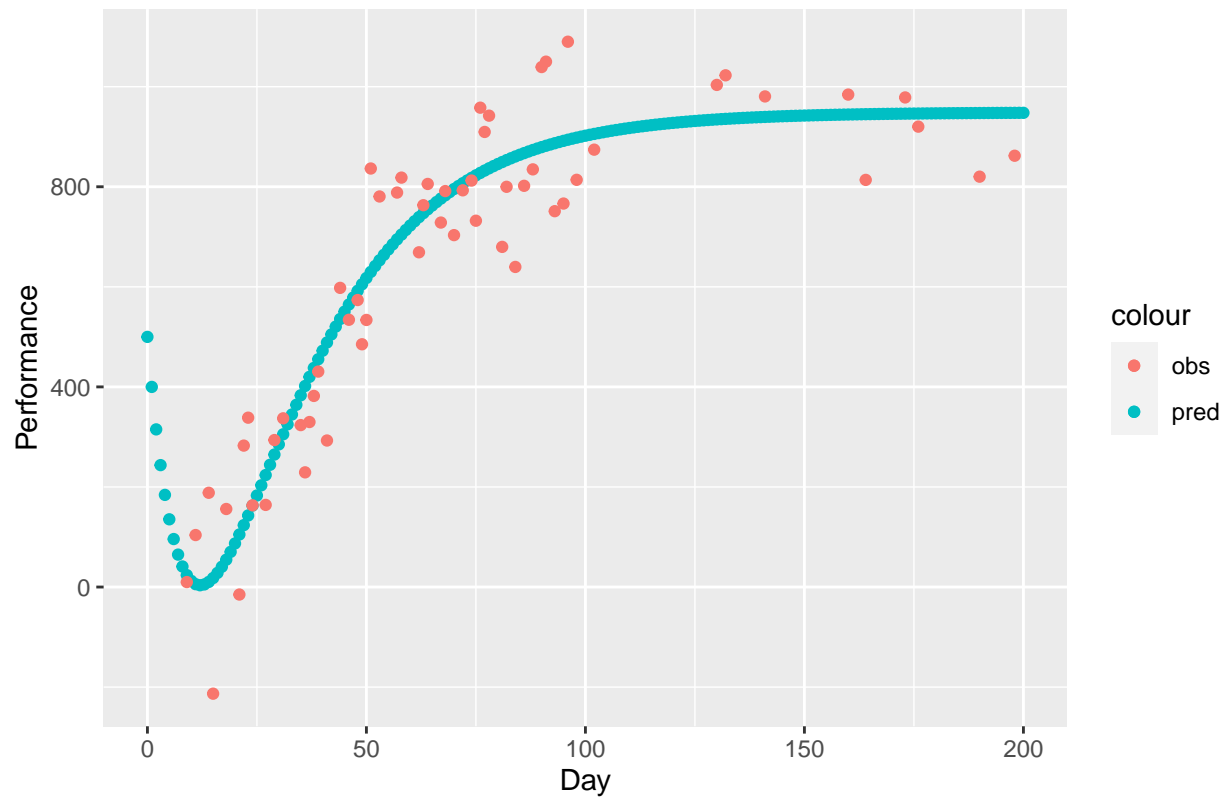
noise_fn <- function(x, sd){
  x+rnorm(1, mean = 0, sd = sd)
}

perf_sim_1 = purrr::map_dbl(perf_sim_1, noise_fn, sd = 100)

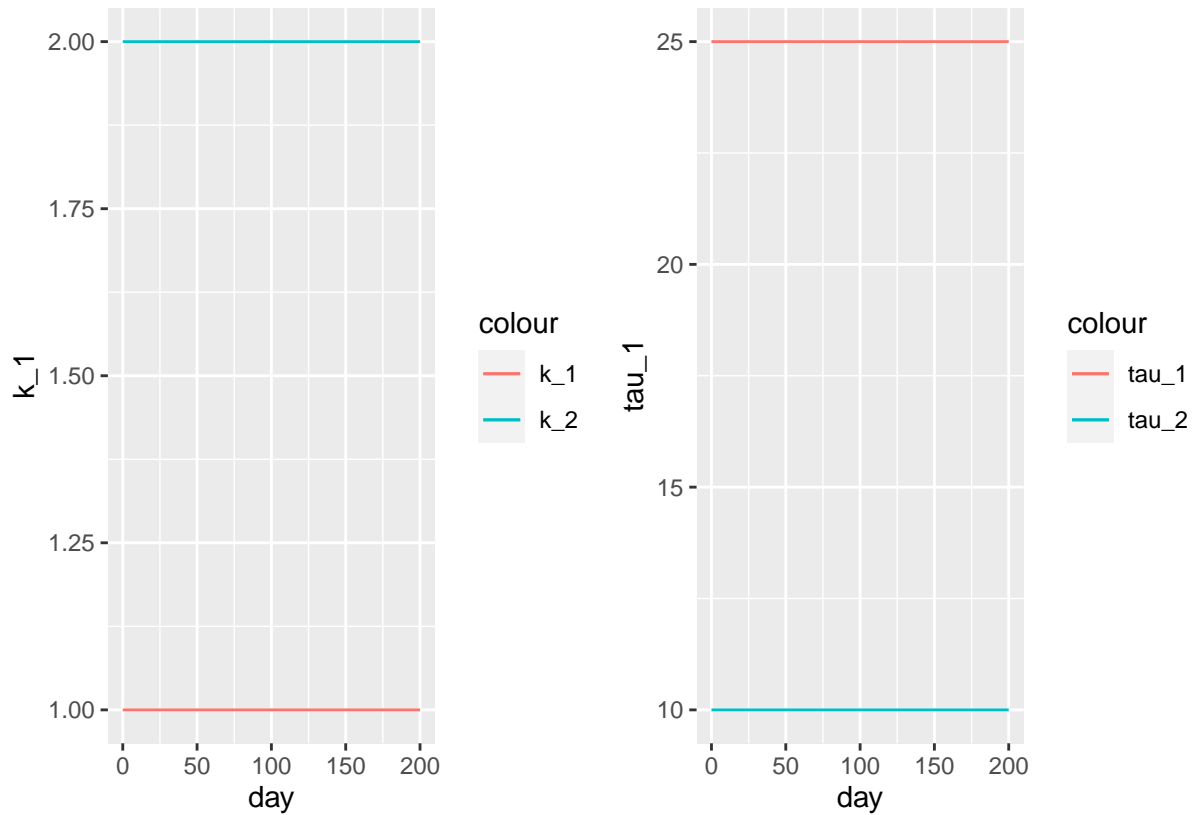
# this is one off
samp_index <- c(sample(c(1:100), 50), sample(c(101:days_test), 10))
actual_perf <- rep(NA, days_test)
for (i in samp_index) {
  actual_perf[[i]] = perf_sim_1[[i+1]]
}

model_perf <- new_pred_perf(
  c(500, 1, 2, 25, 10),
  c(rep(100, days_test)),
  actual_perf,
  lambda = 1
)
model_perf$perf_plot

```



```
model_perf$params_plots
```



More complicated simulation

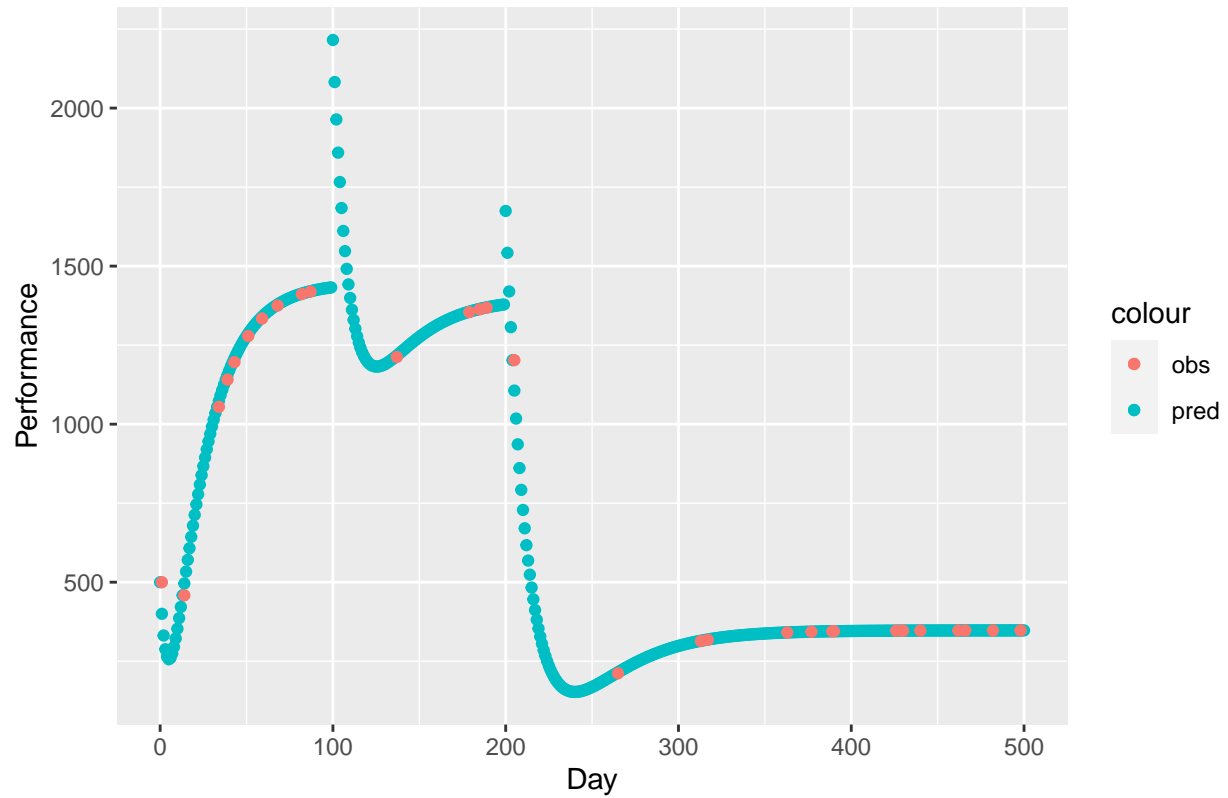
Right now, the algorithm cannot handle this case.

```
set.seed(1)
days_test <- 500
perf_sim_1 <- c(500, perf_tv(p_0 = 500,
  k_1 = c(1,2,3),
  tau_1 = c(20, 25, 30),
  k_2 = c(2, 4, 6),
  tau_2 = c(5,10, 15),
  change_days = c(100, 200),
  days = days_test,
  training_stim = list("constant", 100))$performance
)
samp_index <- c(sample(c(1:100), 10), sample(c(101:days_test), 20))
actual_perf <- rep(NA, days_test)
for (i in samp_index) {
  actual_perf[[i]] = perf_sim_1[[i]]
}

plot_check <- plot_perf(actual_perf,
  perf_sim_1) +
  labs(title = "Check that sampling from the simulation works")
plot_check
```

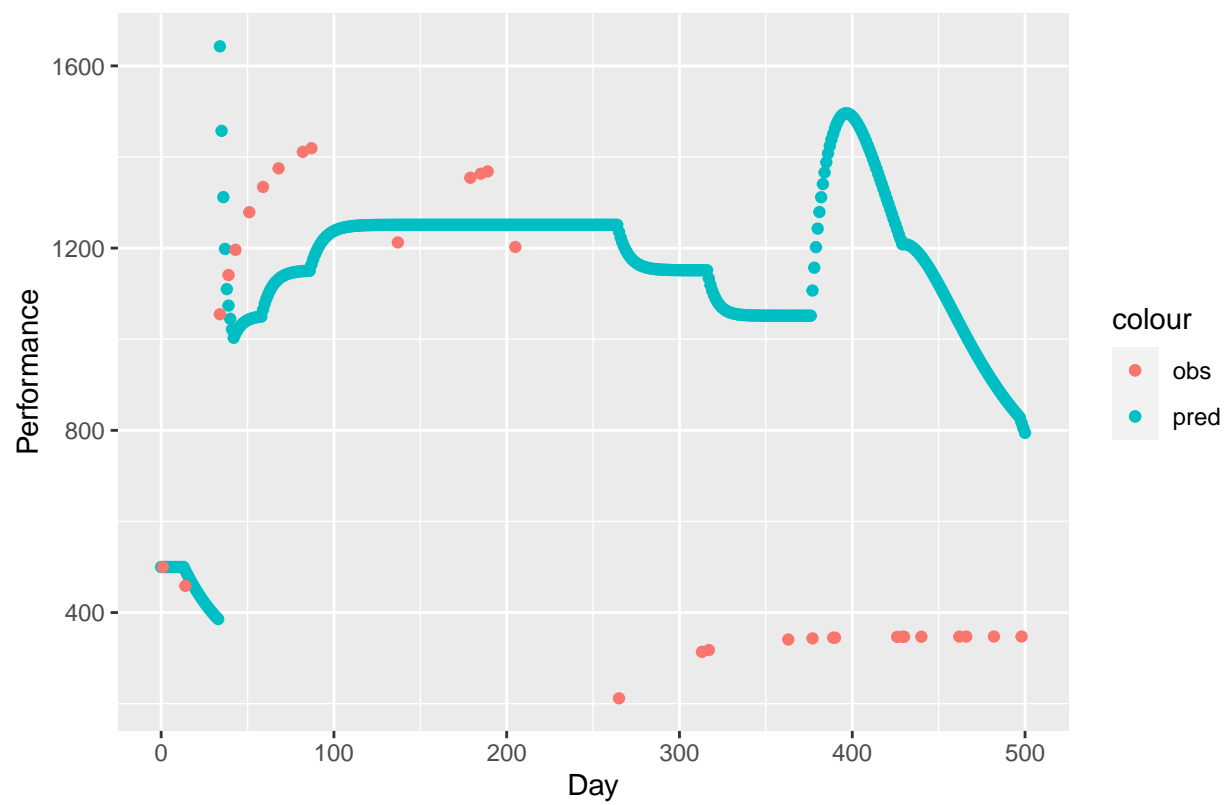
```
## Warning: Removed 471 rows containing missing values ('geom_point()').
```

Check that sampling from the simulation works

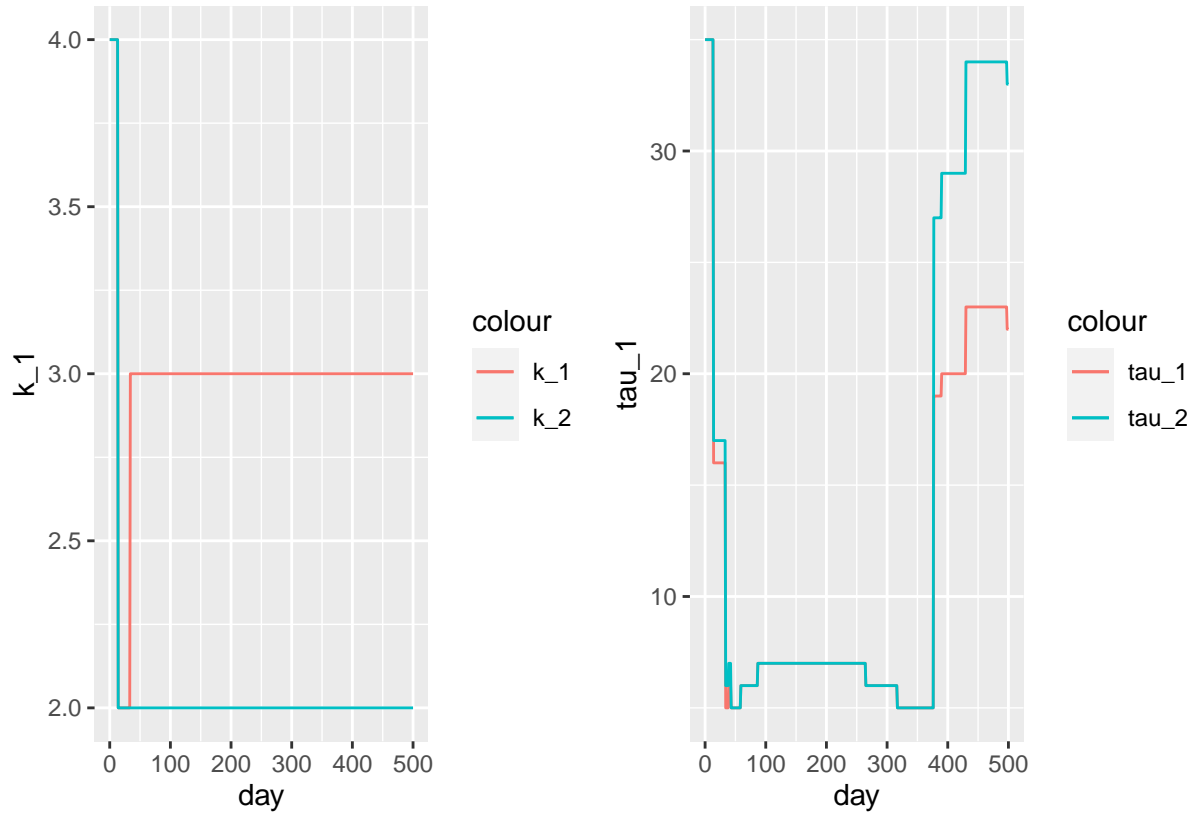


```
model_perf_2 <- new_pred_perf(  
  c(500, 1, 2, 25, 10),  
  c(rep(100, days_test)),  
  actual_perf,  
  lambda = 2  
)  
model_perf_2$perf_plot
```

```
## Warning: Removed 471 rows containing missing values ('geom_point()').
```

model_perf_2\$params_plots



It does a pretty bad job.

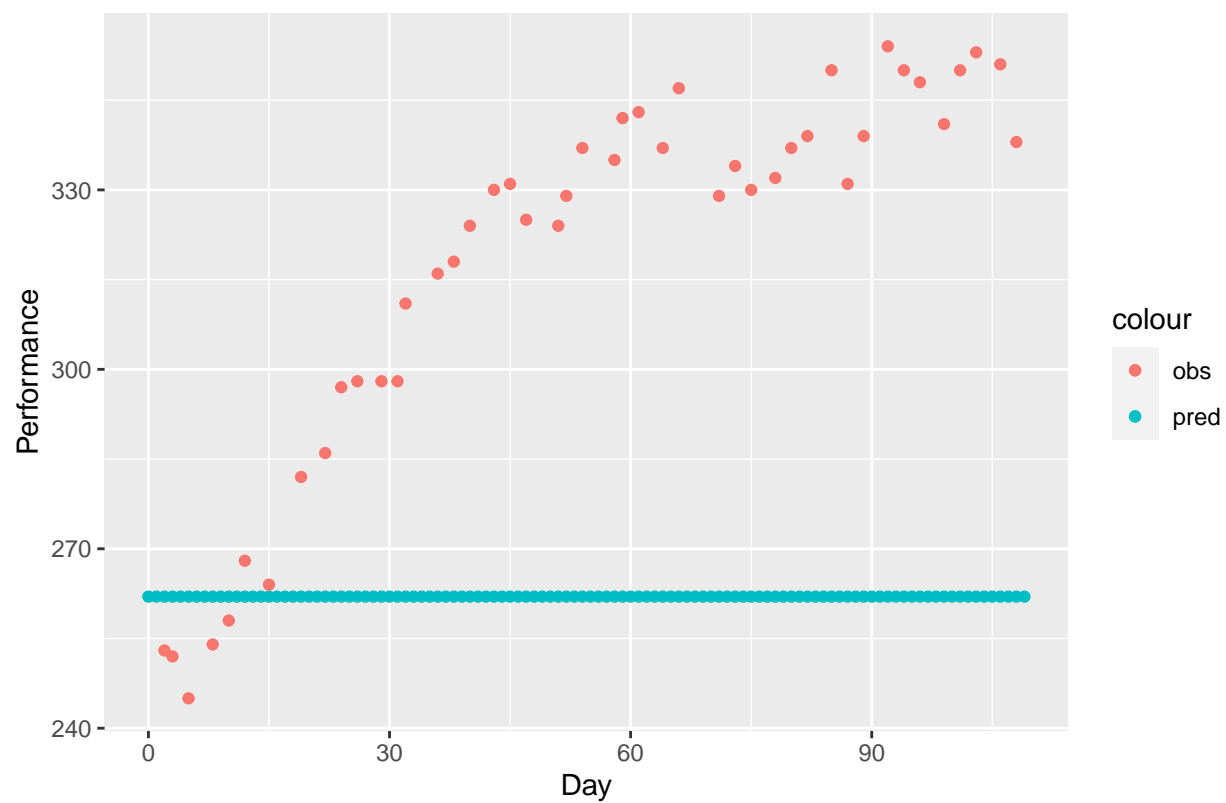
Applying to Real Data

This is a work in progress. Right now, the algorithm cannot handle this case

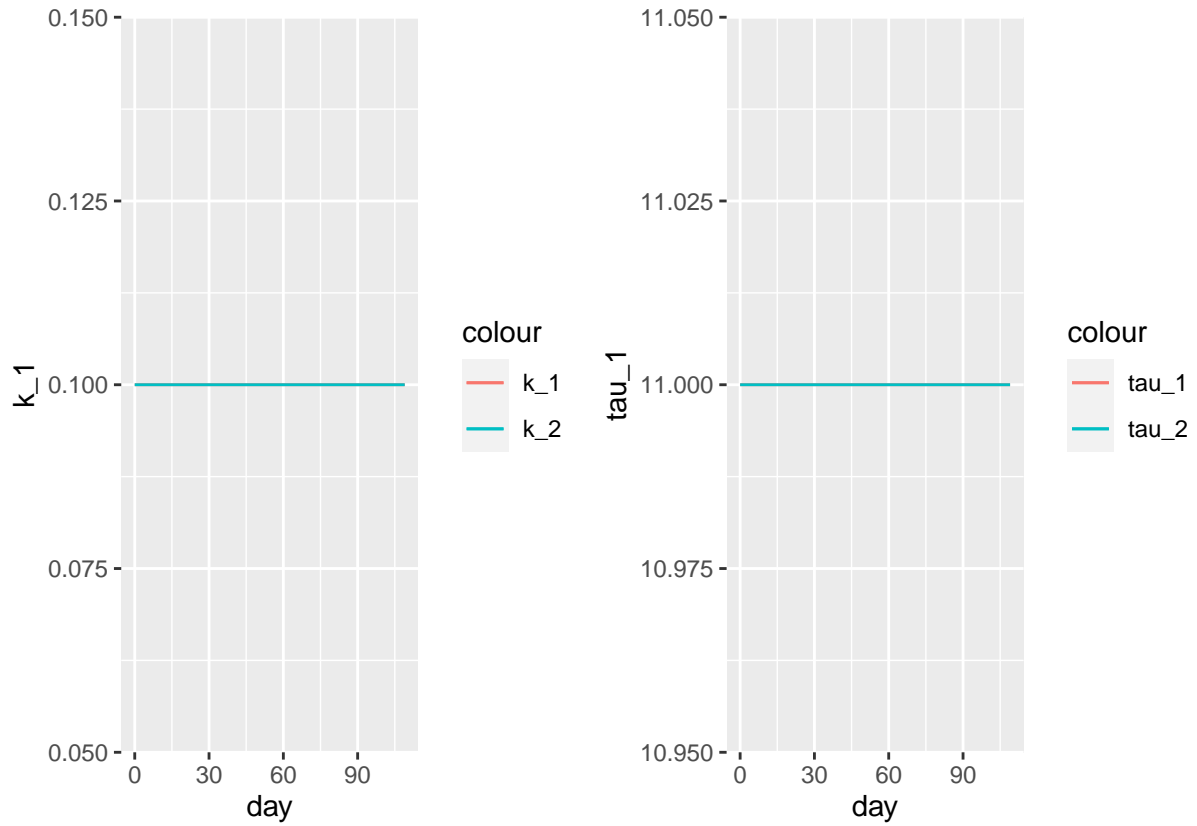
```
real_data <- data_2
day_vec_real <- real_data[[1]]
training_load_real <- real_data[[2]]
obs_perf_real <- real_data[[3]]
init_params_real <- c(262, .1, .1, 11, 11)
obs_new_alg <- new_pred_perf(
  init_params_real,
  training_load_real,
  obs_perf_real,
  lambda = .00000000005
)

obs_new_alg$perf_plot
```

```
## Warning: Removed 65 rows containing missing values ('geom_point()').
```



```
obs_new_alg$params_plots
```



Here, it does not change because the algorithm has decided that all of the options that we have told it to search over are all bad so it stuck with the initial estimate which is probably a good sign.