Improving the Banister Fitness Fatigue Model

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What's the point?!?

After this talk, you will get

- An understanding of what performance gains from atheletic training look like
- An understanding of what models of atheletic performance are like

Background

- The Banister Fitness-Fatigue model models athletic performance as a function of training load.
- Performance can be modeled as the ability of the athlete before training begins, the positive training effects (PTE), and the negative training effects (NTE)
- Introduced in 1976 by Eric Banister
- Many applicatons

Applications

Can be applied to any sport that can be quantified. For example, it has been applied to

- Running
- Cycling
- Swimming
- Powerlifting

But not

- Football
- Soccer
- Volleyball
- Chess

Model definition

Formally, it is defined as,

$$P(n) = p_0 + \overbrace{k_1 \sum_{i=0}^{n-1} e^{-(n-i)/\tau_1} w(i)}^{\text{PTE}} - \overbrace{k_2 \sum_{i=0}^{n-1} e^{-(n-i)/\tau_2} w(i)}^{\text{NTE}}$$

where $P(n)$ is the predicted performance on day n, p_0 is the initial performance, $w(i)$ is the training load on day i.
$k_1,\ k_2,\ \tau_1$, and τ_2 are constants greater than 0.

A note on units

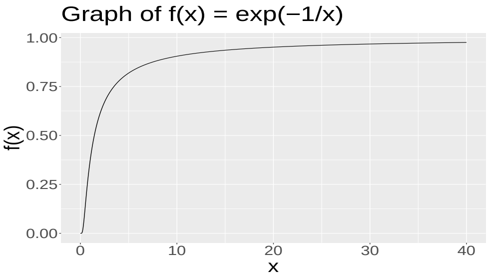
- Athletic perforamcne is measured in arbitrary units.
- Just numbers for this project
- Training load is roughly measured as the fatigue on the body, and is measured differently with different sports.
- Measuring methods include max heart rate, level of lactate in the blood, training impulse from the training regimen.

Intuition on the model

lets just look at the sum. We have

$$TE(n) \propto e^{-1/\tau} w(n-1) + e^{-2/\tau} w(n-2) + \cdots + e^{-n/\tau} w(0)$$

Under our assumption that $\tau > 0$, $e^{-1/\tau}$ is just a number between 0 and 1. Proof:



Intuition on the model, cont

Rewriting, the TE term, we get

$$TE(n) \propto (e^{-1/\tau})^1 w(n-1) + (e^{-1/\tau})^2 w(n-2) + \cdots + (e^{-1/\tau})^n w(0)$$

The k_1 and k_2 terms control the magnitude of these training effects.

Limitations of the model

We can see some of the limitations of the model.

1. It assumes that we recover the same way every day, which isn't true, since you can over train, not get enough sleep, get better at recovering over time, etc.
2. The modeling process is very specific. In practice, the model parameters depend on the person, the sport, and even the training regeme.
3. The paramters are not super interpretable; knowing just τ_1, for instance, does not tell us anything

My research tries to fix problem 1, by letting the parameters vary over time.

Application of Parameters

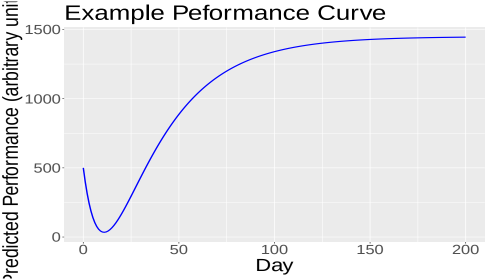
We can use the parameters to predict taper time

$$\text{Taper Time} = \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \log\left(\frac{k_1}{k_2}\right)$$

So the parameters are important!

What the model looks like

This is roughly how the plots look

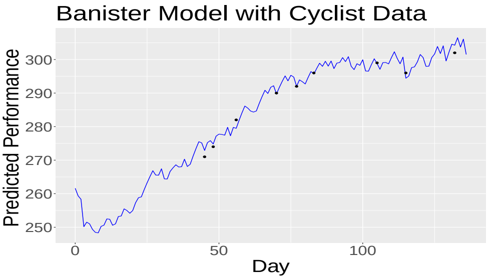


This captures a few aspects of the performance-training relationship:

- Initial negative response to the training
- A recovery and a benefit to the training
- A plateu.

First real data set

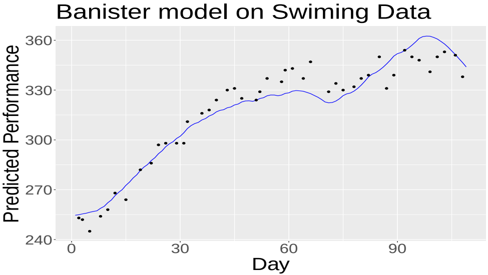
We will first see the model appled to cyclist data.



- Paramters arrived at by guess, and then optimization
- Interpolates the data points well

Real dataset 2

Applying the same model to a different dataset



We can see that the model works well in some places and misses in other places.

Time-Varying Model

We can extend this model using recursive least squares. Looking at the model:

$$P(n) = p_0 + \overbrace{k_1 \sum_{i=0}^{n-1} e^{-(n-i)/\tau_1} w(i)}^{\text{PTE}} - \overbrace{k_2 \sum_{i=0}^{n-1} e^{-(n-i)/\tau_2} w(i)}^{\text{NTE}}$$

if we are given τ_1 and τ_2, then both sums are constants, say C_1 and C_2. So then

$$P(n) = p_0 + C_1 k_1 + C_2 k_2$$

is a linear funciton of k_1 and k_2.

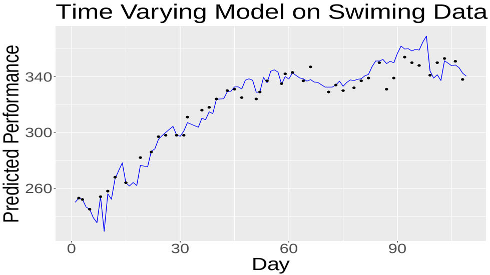
The algorithm, roughly

1. Do LS for choices of τ_1 and τ_2,
2. Choose the minimum.

My research began as implementing this in R.

Time varying model on real dataset 2

Appling to real dataset before



There is a bit of a burn-in period, but it interpolates the performance better than the previous model.

Limitations

While this does help solve (1) it introduces its own problems,

1. The parameter vary way to much
2. We cannot use this to predict the taper time
3. The burn-in period is not desirable

Conclusions

- The original model works in some cases
- The new model works in some cases but not all
- The new model cannot extract the parameters

Big Picture

- Since the origional model works well, the parameters only have to vary by a little bit.
- Implement a new slow time-varying model