CSCI 3010U: Simulation and Modelling

Billiard Simulation Project

Domain Research & Model Planning

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Billiards - Domain Research

The domain of our research project is that of Physics. Our topic is the simulation and modelling of a billiards board. This topic includes many different factors that could contribute to it, all within the realm of physics. The first that I will talk about is the material used to make the table. Most tables are made with a wool billiard cloth, it is a very flat sort of smooth carpet almost that consists of either a woven wool or wool/nylon blend called baize. This fabric allows for a low level of friction between the ball and the table thus allowing the player to accurately predict their shots. The low level of friction is preferred because it is the most non-disturbing to the game. If the material used provided no friction at all then it would be hard to predict where the ball would land as it would travel much further. If the material used provided a high level of friction then the game would be much harder to play as the ball would not move very far. The material chosen has a low level of friction which is the best for a controlled billiards shot. For our simulation we hope to apply a force of friction that is suitable and as realistic as possible but may also be adjusted to simulate under different conditions.

The mass of the ball is important as it is a defining factor in how it will roll. If the ball has a low mass then the force of gravity pushing down will be very low as well making it easier to bounce the ball and harder for the player to perform strategic collisions. A light ball will be more susceptible to the force of friction provided by the table's material which would make the ball have a higher rate of deceleration. Heavy balls would be less affected by the friction from the table's material but would also be harder to initially move due to the force of gravity pushing down with the ball. The balls used in an actual game are made of phonetic resin, polyester, or clear acrylic and have a mass between 5.5 and 6.0 ounces. These characteristics are essential to the overall rolling of the ball.

Air resistance is a very small factor when it comes to billiards, but it is still a factor. Since billiards is usually played indoors, in a closed environment, and with a high lip on the edge of the table there would be little to no resistance. If there was a draft then the ball may be swayed to go in the direction of the pushing air. If there was no air movement then the speed of the ball would be affected very little by the air it is pushing against. While the effect is very little it is still an effect and we must include it in our simulation as something that can be adjusted to visualize the different simulations.

The average floor is not a level one, while it may be close; achieving a perfect degree of levelness is very hard. Therefore the angle of the floor and therefore the tabletop is a factor to the physics behind billiards. If the table top is slanted downwards with the direction of the shot then the ball will roll faster in that direction and if shot on an angle it may pull the ball in the direction it is slanted. If the table is slanted upwards with the direction of the shot then the ball will come to a stop sooner than it would have

on a level surface. If slanted upwards and on an angle to the side then the ball will come to a stop sooner and be pulled in the direction of the slant. Our simulation will start with a level table and will have controls to adjust the angle.
The use of the cue stick to hit the ball has three important factors that will affect the simulation: the force of the shot, the angle of the shot, and the location (on the ball) of the shot.

The player applies force to the ball in a manner which they seem fit. If a high level of force is applied then the ball will travel a farther distance or have more energy to give off during a collision. If the force applied is low then the ball will roll a smaller distance and transfer less energy during a collision. The amount of force applied to the ball will be able to be adjusted in our simulation.

The angle at which the ball is hit is important to the game as these types of shots can be used strategically. The ball should be hit at no angle at all to make it roll straight. If the ball is hit on a downwards angle then it has the possibility to jump and have a backspin. Players regularly use these effects to their advantage which makes them an important part of the game. Due to the length of the cue stick and the fact that there is a table with lips on the edges it would be hard to hit the ball on an upwards angle. This Is not a regularly used tactic as it would cause an overspin which is not beneficial. The angle that the ball is hit will be a modifiable factor within our simulation.

The player applies force to the ball to make it move, the point that that the ball is hit is very important. Due to the ball being a sphere the only point that it can be hit where it will be a straight roll is the very centre. If the player were to hit it on the right side then the ball would overspin to the left and if it were hit on the left side then it would overspin to the right. The x and y coordinates of where the ball should be hit will be modifiable factors to our simulation.

There is the possibility of combining the angle of the shot and the location of the shot but it could be argued that when the location of the shot moves then the angle should change as well. The angle of the shot should be relative to the location on the ball that it is hit (due to it being a sphere). For the purpose of this simulation the angle of the shot will not be relative to the location of the shot but will always be in relation to the centre point where if hit at no angle the ball will go straight.

Collisions play a major role in the movement of the balls. There are three types of collisions possible after the ball has been shot: a bumper collision, a ball-ball collision, or a ball-to-many collision. The number or type of collision is not going to be a predetermined factor within our simulation. They will be results of the other factors and settings applied.

A bumper collision is when the ball hits a side of the table. This could happen many times within one shot depending on the length of the balls travel. The ball will hit the bumper (which is made of the same material as the table top) and then bounce off at an angle supplementary to the angle at the collision. The ball may go from one bumper to another but it will always bounce off at the relative collisions supplementary angle.

A ball-ball collision is when a ball hits another ball. It may be the original ball the player shot hitting another ball or it may be a collision resulting from a collision, either will not differ. When the collision happens the ball that previously had movement will transfer a mathematically determined amount of energy to the ball that was hit. Both balls will now move in calculated directions, one at a speed less than what it was and one at a speed greater (than stationary). We will calculate the speeds after the collision using traditional momentum formulas and the angles at which the balls move using trajectory calculus. There is a special case where if there is a collision of two moving balls (secondary or tertiary collision). The momentums and energy conservations will be calculated similarly but the directions at which the balls move post-collision will have to be calculated with two balls with movement instead of one.

The last type of collision is ball-to-many collision. This is a collision between many 3+ balls at once. All calculations will be similar with the addition of extra balls. The extra balls will allow for a different outcome compared to a collision with just 2 balls due to the sharing of the energy with multiple objects.

When looking at collisions we must take into account different types of movement the ball can make. If a ball jumps and hits another ball from the top we must calculate appropriately. We must also look at how drastically overspin and backspin will affect the collisions between balls.

<u>Billiards - Model Plan</u>

<u>Model Behaviour</u>

The objective of this billiards simulation is to properly demonstrate the physics of the interactions between a pool cue, the balls, the table, and all the forces around it.

The simulation will contain different parts all representing different types of possible interactions that commonly occur during a game of pool. Each different interaction will involve different physics equations centering around velocity, frictional forces, transfer of momentum between multiple objects, mass, and many more.

Each simulation will start when a cue hits the ball and the simulation will stop once all net forces have reached zero (all balls have stopped).

<u>Simulation Classification</u>

Just like most billiards players, they predict their shots and the outcome before they even take the shot. In the case of a billiards simulation, it is very possible to predict the output just by having the values of input. Chances are, if the same initial configuration is the same for each simulation, the model will be identical. There are no random elements that affect the behaviour of the model which makes it deterministic.

In our model, physics plays major role of determining the outcome of the simulation. Most of the physics involved comes from ODE based off current velocity, mass, and frictional forces. Newton's law of motion on its own is a continuous equation which plays a vital role in this model. Thus our simulation will be a **deterministic continuous simulation**.

<u>Interactions</u>

In our simulation we will be working with a few different type of specific interactions that can happen during a game of billiards. These interactions are mostly collisions between different objects during billiards. These collisions can be predicted using different physics equations ranging from frictional forces to transfer of momentum between multiple objects. Below is a list of the different types of interactions that will take place.

- Collisions between the ball and the table
    - Some variables to look for during this interaction is the balls static and kinetic friction with the table, along with the mass and velocity of the ball.

- Collisions between the cue and the ball
    - The main two variables during this collision is the magnitude that the cue hits the ball and at what angle this magnitude is applied onto the spherical ball. This will be the initial interaction during each simulation.
- Collision between the ball and the wall
    - This interaction will occur when ball strikes the side bumpers at a certain angle and depending on the physical properties of the ball and wall, kinetic energy can be lost during this exchange of forces.
- Collision between ball and ball
    - The most common interaction will be when two balls of equal mass exchange momentum during a shot.
- Collision between ball and multiple balls
    - Similar to the last interaction but with more than one other ball being affected by the momentum.

Simulation Loop

The simulation will have to be in real time. Using a discrete time-step value can be a problem regarding our collision detection. Collision detective needs to be preemptive, meaning the object will know where its going to be after the next time step. By having an accurate time value, we can determine how far this simulation needs to progress. By determining the time of the next collision using a variety of time dependent physics algorithms, we can progress a simulation up until the point where there is a collision. At this point the collision functions can be applied which will then set new directions for the balls on the billiards table.

Input Variables

- Magnitude of force and angle the cue will hit the ball
- Mass of the ball/balls
- Static and kinetic friction constant of the surface of the pool table
- Gravity

State Variables

The state variables of our simulation will pertain mainly to the balls and walls of the pool table. These variables will include the positions of the balls and the velocities of each ball at a certain time. The change in velocity is dependent on the acceleration at the time.

The difference/update equations will be used as follows:

1. Change in velocity on the balls mainly dependent on the frictional forces between the ball and the surface. $F_{frict} = \mu \cdot F_{norm}$

2. Changes in direction when there is a collision between balls and the bumper walls. This is affected by the angle and the velocity at which two objects collide with each other. Loss in kinetic energy will also be implemented.

   Momentum of an object = Mass * Velocity

The law of momentum conservation states that in an isolated system, the momentum of two objects in total between the two is equal to the same amount after the collision. mass1 * velocity1 = - mass2 * velocity2

Output Variables

- The acceleration of the balls at any given time.
- The velocity of the balls at any given time.
- The displacement of each ball.
- The force of friction being applied from the surface onto the ball.

Tools

Using WebGL for the the graphic simulation of the billiards table will be most ideal for this model. The visualization will be the birds eye view of a pool table. 2-D graphs for the Time-Velocity, Time-Acceleration, and Time-Position relationships will be used to verify the integrity of the simulation of the balls that are part of it.