

## Chapter 03 Likelihood-Based Tests and Confidence Regions

### Part 01

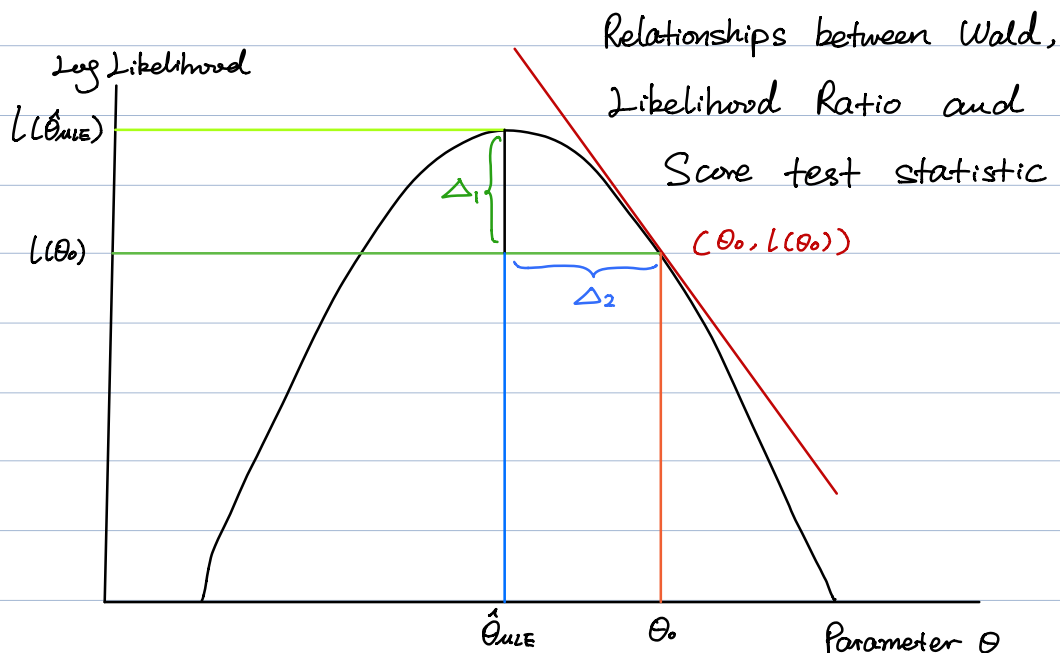
Three asymptotically equivalent tests based on likelihood:  
Wald test, Likelihood Ratio test, and Score test (Lagrange multiplier test in econometrics).

For i.i.d. data with one unknown real parameter, to test  
 $H_0: \theta = \theta_0$  vs  $H_a: \theta \neq \theta_0$ , one can use

- Wald statistic  $T_W = \frac{(\hat{\theta}_{MLE} - \theta_0)^2}{[I_T(\hat{\theta}_{MLE})]^{-1}} \propto \Delta^2$

- Likelihood Ratio (LR) statistic  $T_{LR} = -2 \frac{L(\theta_0)}{L(\hat{\theta}_{MLE})} = -2 [L(\theta_0) - L(\hat{\theta}_{MLE})] \propto \Delta_1$

- Score statistic  $T_S = \frac{S^2(\theta_0)}{I_T(\theta_0)} \propto \Delta_2$



The three tests are asymptotically equivalent:

- $\Delta_1$ ,  $\Delta_2$  and  $\beta$  all increases (decrease) together.
- Under  $H_0$ , all the three are asymptotically  $\chi^2_i$  distributed
- Under local alternatives ( $\theta = \theta_0 + d/\sqrt{n}$ ), they have identical asymptotic non-central  $\chi^2$  distributions.

Difference between the three tests:

- Wald statistic is the simplest but not invariant to reparameterization, whereas the other two are.

e.g. Wald statistics for  $H_0: \sigma = \sigma_0$  and  $H_0: \sigma^2 = \sigma_0^2$  will be different.

- The LR statistic is more difficult to compute as it requires  $\hat{\theta}_{MLE}$  under both  $H_0$  and  $H_a$ , whereas the Wald and Score statistics require the MLE under the alternative and null hypothesis respectively.

- Wald and Score tests are more robust against model misspecification.