

Chapter 03 Likelihood-Based Tests and Confidence Regions

Part 01

Three asymptotically equivalent tests based on likelihood:

Wald test, Likelihood Ratio test, and Score test (Lagrange multiplier test in econometrics).

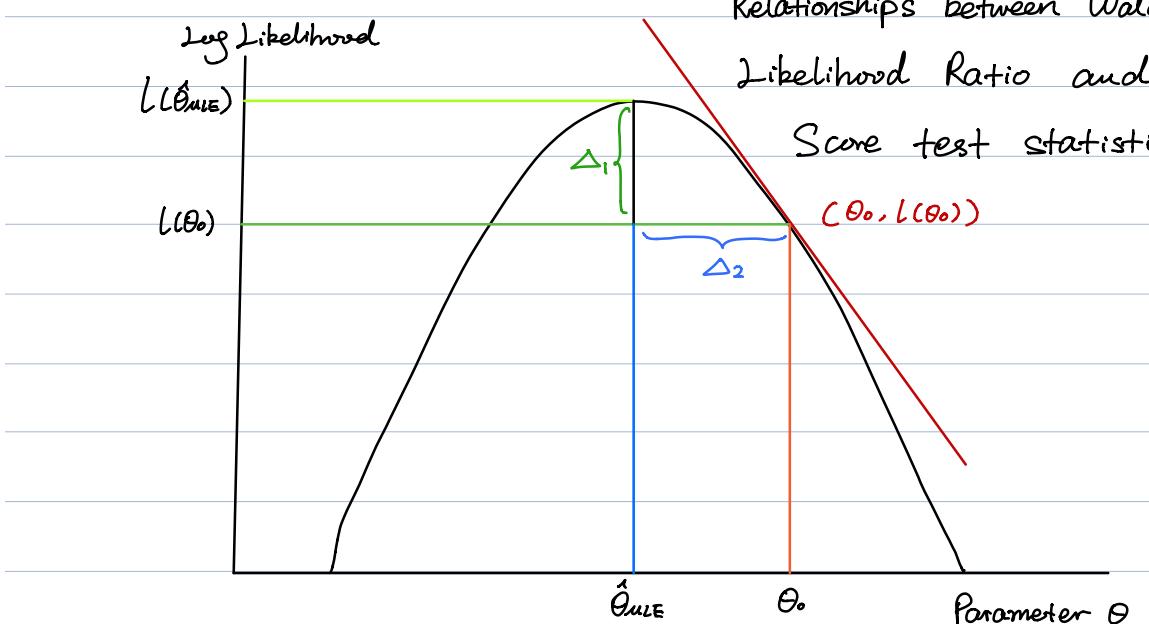
For i.i.d. data with one unknown real parameter, to test

$H_0: \theta = \theta_0$ vs $H_a: \theta \neq \theta_0$, one can use

• Wald statistic $T_W = \frac{(\hat{\theta}_{MLE} - \theta_0)^2}{[I_r(\hat{\theta}_{MLE})]^{-1}} \propto \Delta_1^2$

• Likelihood Ratio (LR) statistic $T_{LR} = -2 \frac{L(\theta_0)}{L(\hat{\theta}_{MLE})} = -2 [L(\theta_0) - L(\hat{\theta}_{MLE})] \propto \Delta_1$

• Score statistic $T_S = \frac{S^2(\theta_0)}{I_r(\theta_0)} \propto \beta^2$



The three tests are asymptotically equivalent:

- Δ_1, Δ_2 and β all increase (decrease) together.
- Under H_0 , all the three are asymptotically χ^2 distributed
- Under local alternatives ($\theta = \theta_0 + d/\sqrt{n}$), they have identical asymptotic non-central χ^2 distributions.

Difference between the three tests:

- Wald statistic is the simplest but not invariant to reparameterization, whereas the other two are.
e.g. Wald statistics for $H_0: \theta = \theta_0$ and $H_0: \theta^2 = \theta_0^2$ will be different.
- The LR statistic is more difficult to compute as it requires MLE under both H_0 and H_a , whereas the Wald and Score statistics require the MLE under the alternative and null hypothesis respectively.
- Wald and Score tests are more robust against model misspecification.