

# Supporting Information for “Sea-surface temperature patterns, radiative cooling, and hydrological sensitivity”

Andrew I. L. Williams<sup>1,2</sup> & Nadir Jeevanjee<sup>3</sup>

<sup>1</sup>Program in Atmospheric and Oceanic Sciences, Princeton University

<sup>2</sup>Scripps Institution of Oceanography, University of California San Diego

<sup>3</sup>Geophysical Fluid Dynamics Laboratory

## Contents of this file

1. Text S1
2. Text S2
3. Figures S1 to S5

### 1. Text S1 - list of models used

The 8 CMIP5 models we use in the main text are: MRI-CGCM3, MPI-ESM-MR, IPSL-CM5A-LR, bcc-csm1-1, CNRM-CM5, MPI-ESM-LR, CanAM4, HadGEM2-A.

The 7 CMIP6 models we use in the main text are: HadGEM3-GC31-LL, IPSL-CM6A-LR, MRI-ESM2-0, CNRM-CM6-1, CanESM5, CESM2, MIROC6.

## 2. Text S2 - a note on OLS regression

In the main text, we discuss the relationship between the total  $\Delta\eta_{\text{LW,clear}}$  and its tropical  $\Delta\eta_{\text{LW,clear}}^{\text{trop}}$  and extratropical  $\Delta\eta_{\text{LW,clear}}^{\text{extratrop}}$  contributions. Here we provide a short note to explain why the coefficients in Eq. (4) take on the values they do ( $\alpha \approx 0.01 \text{ mm day}^{-1} \text{ K}^{-1}$  and  $\beta \approx 0.35$ ). For brevity, we drop the  $\Delta$ s and the  $(\cdot)_{\text{LW,clear}}$  subscripts in this section.

Because the tropics ( $\pm 30^\circ$ ) cover half of the surface area of the Earth, we can write

$$\eta = \frac{1}{2} (\eta^{\text{trop}} + \eta^{\text{extratrop}}). \quad (1)$$

This would imply a regression slope in Fig. 2a of  $\frac{1}{2}$ , and an intercept which is half the multi-model mean value of  $\eta^{\text{extratrop}}$  (about  $\approx 0.0055 \text{ mm day}^{-1} \text{ K}^{-1}$ ). The reason that our best fit estimates of the slope and intercept are biased compared to this simple estimate is because there is a negative correlation between the tropical and extratropical contributions to  $\Delta\eta_{\text{LW,clear}}$ . This biases the slope to be somewhat smaller than  $1/2$ , and because the slope and intercept of an ordinary least squares regression are inversely related, this also means that the intercept is biased high compared to our simple estimate. We now show this mathematically.

Assuming that our regressions yield noiseless estimates of these terms for the 15 CMIP5/6 models, the OLS slope between  $\eta$  and  $\eta^{\text{trop}}$  can be written as:

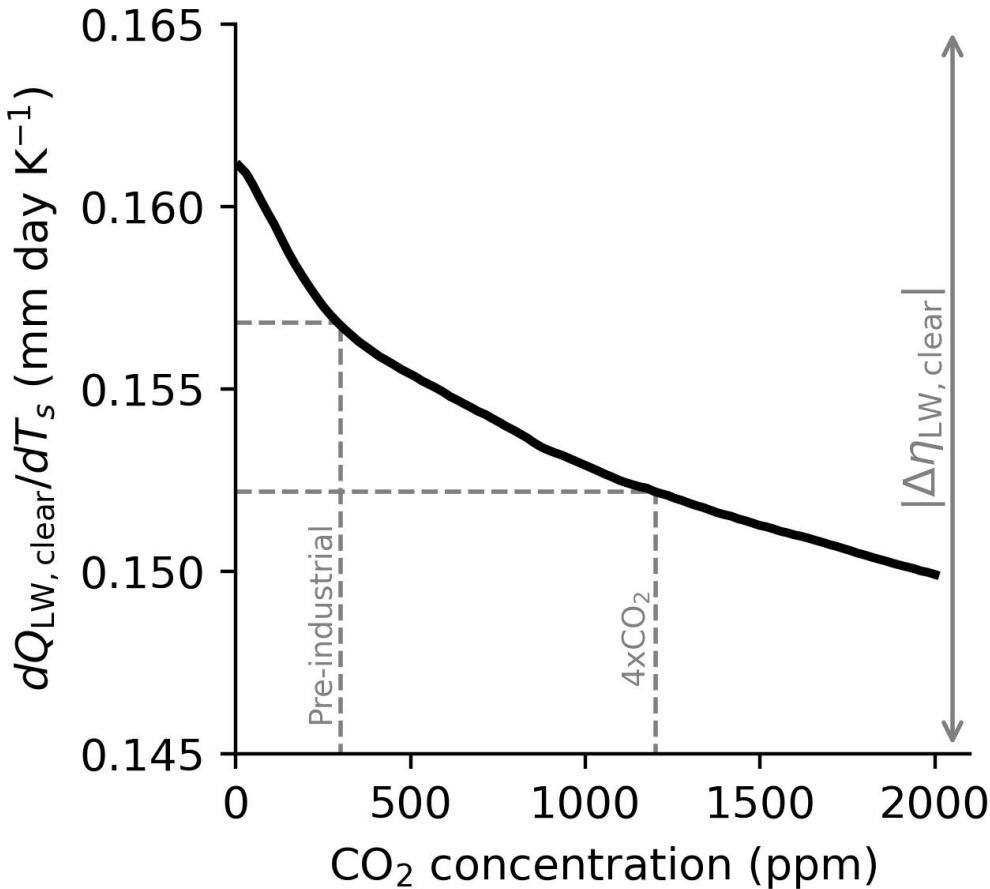
$$\frac{d\eta}{d\eta^{\text{trop}}} = \frac{\text{cov}(\eta, \eta^{\text{trop}})}{\text{var}(\eta^{\text{trop}})} \quad (2)$$

where cov is the covariance and var is the variance. Substituting in Eq. (1) and expanding yields

$$\frac{d\eta}{d\eta^{\text{trop}}} = \frac{\frac{1}{2}\text{var}(\eta^{\text{trop}}) + \frac{1}{2}\text{cov}(\eta^{\text{extratrop}}, \eta^{\text{trop}})}{\text{var}(\eta^{\text{trop}})} = \frac{1}{2} \left( 1 + \frac{\text{cov}(\eta^{\text{extratrop}}, \eta^{\text{trop}})}{\text{var}(\eta^{\text{trop}})} \right). \quad (3)$$

Equation (3) makes it clear that the regression slope between global and tropical  $\eta$  will only be  $= 1/2$  if the tropical and extratropical components are uncorrelated (i.e.,  $\text{cov}(\eta^{\text{extratrop}}, \eta^{\text{trop}}) = 0$ ). However, across the CMIP5/6 simulations there is a slight anti-correlation between the tropical and extratropical components of  $\Delta\eta_{\text{LW,clear}}$  (Fig. S2), which makes the slope less than  $1/2$ .

Furthermore, in OLS regressions the slope ( $\hat{\beta}$ ) and intercept ( $\hat{\alpha}$ ) are related by ( $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ ). Hence, the fact that the regression slope is ‘biased low’ by the covariance between tropical and extratropical components also explains why the intercept is ‘biased high’ compared to our simple estimate of  $\frac{1}{2}\overline{\eta^{\text{extratrop}}}$ .



**Figure S1.** *Changes in longwave cooling with temperature are somewhat dependent on baseline CO<sub>2</sub> concentrations.* Curve is calculated using single-column simulations with the RRTMG radiation scheme; the temperature profile is a moist adiabat at  $T_s=300\text{K}$  up to an isothermal stratosphere of 220K. Relative humidity is set to be a vertically-uniform value of 70%. For each CO<sub>2</sub> concentration, we run a control simulation and one with surface temperatures perturbed by 5K, then difference the total clear-sky longwave cooling in each run to get  $dQ_{\text{LW, clear}}/dT_s$ . The vertical extent of the plot equals the multi-model mean value of  $\Delta\eta_{\text{LW, clear}}$ ; the masking effect of CO<sub>2</sub> is insufficient to fully explain the value of  $\Delta\eta_{\text{LW, clear}}$ .

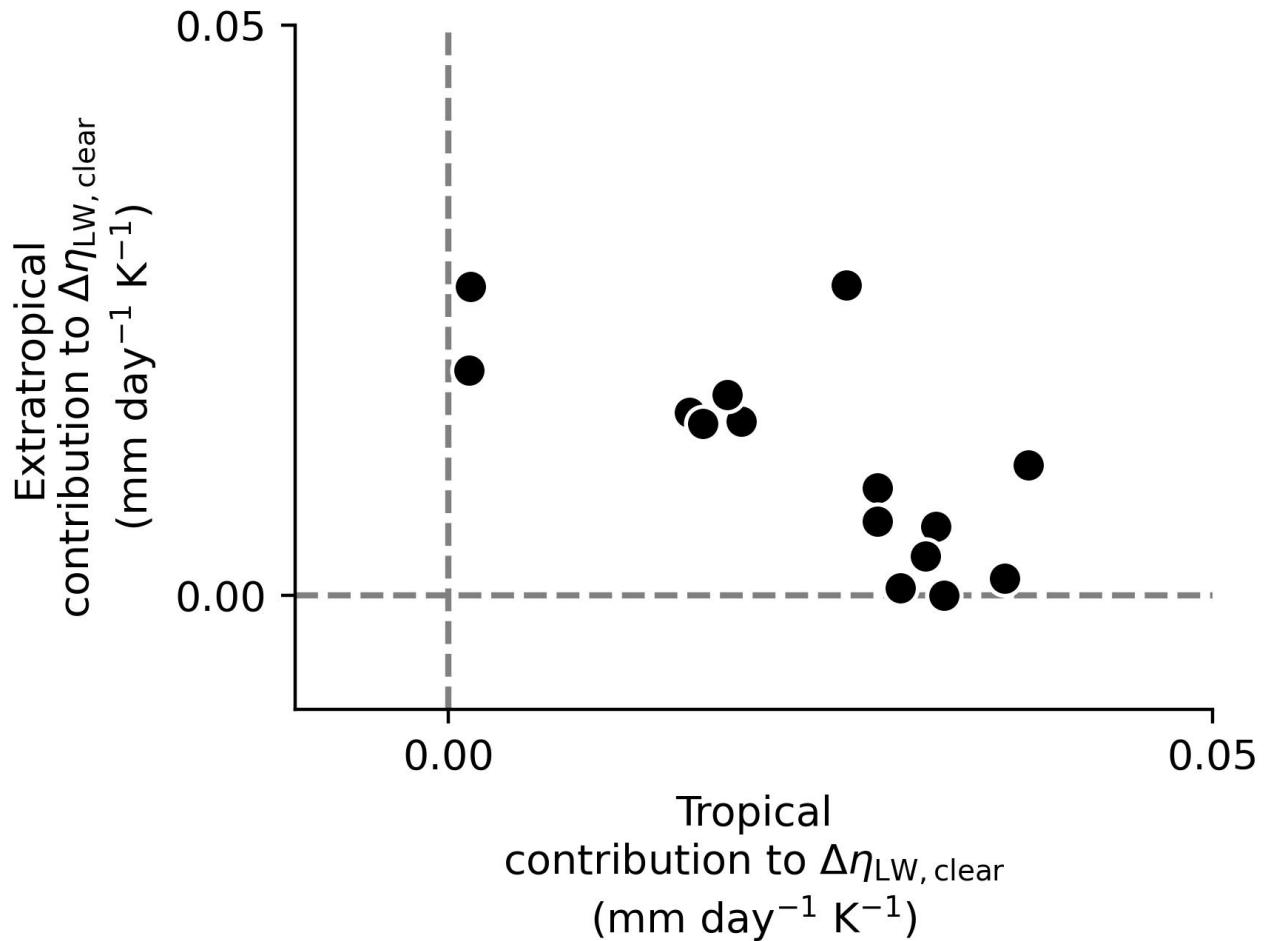
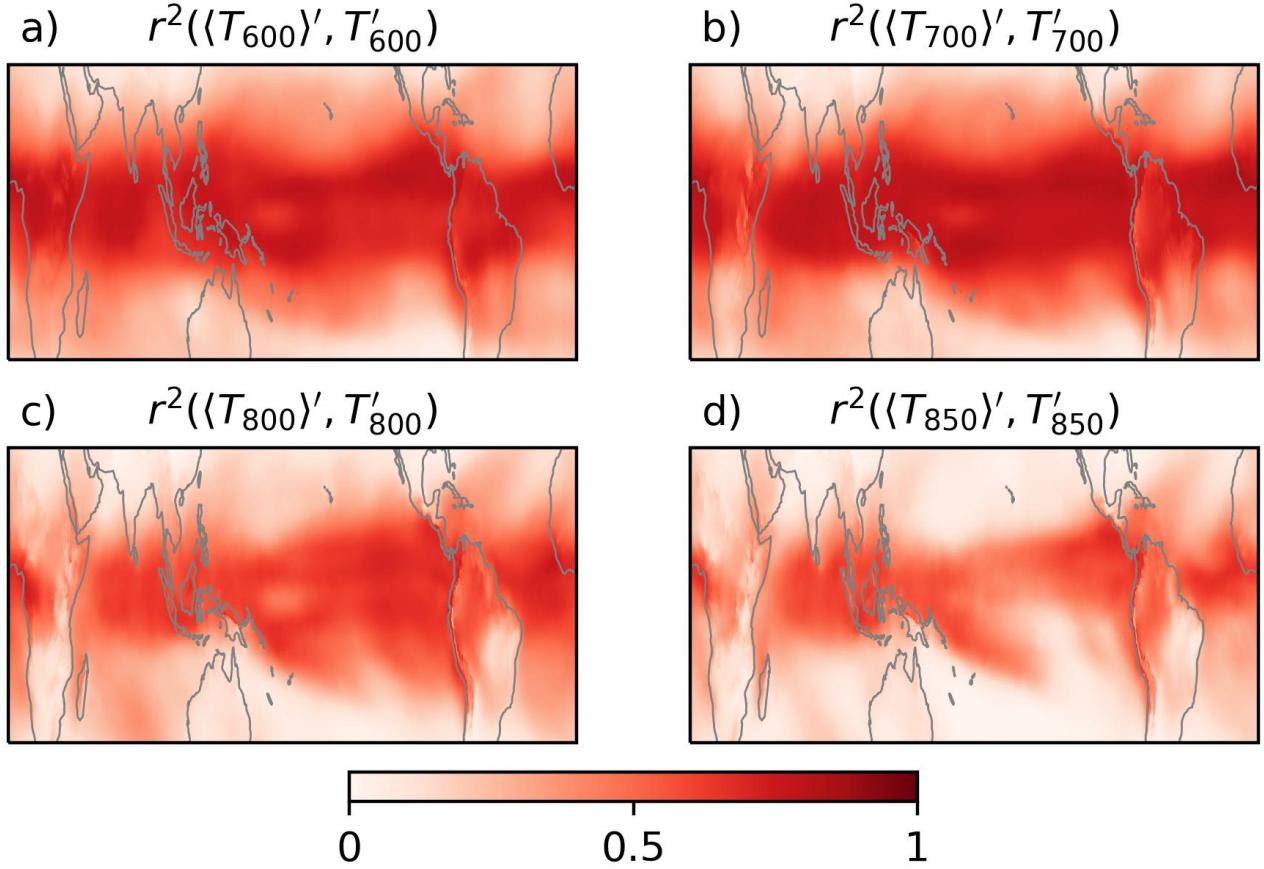
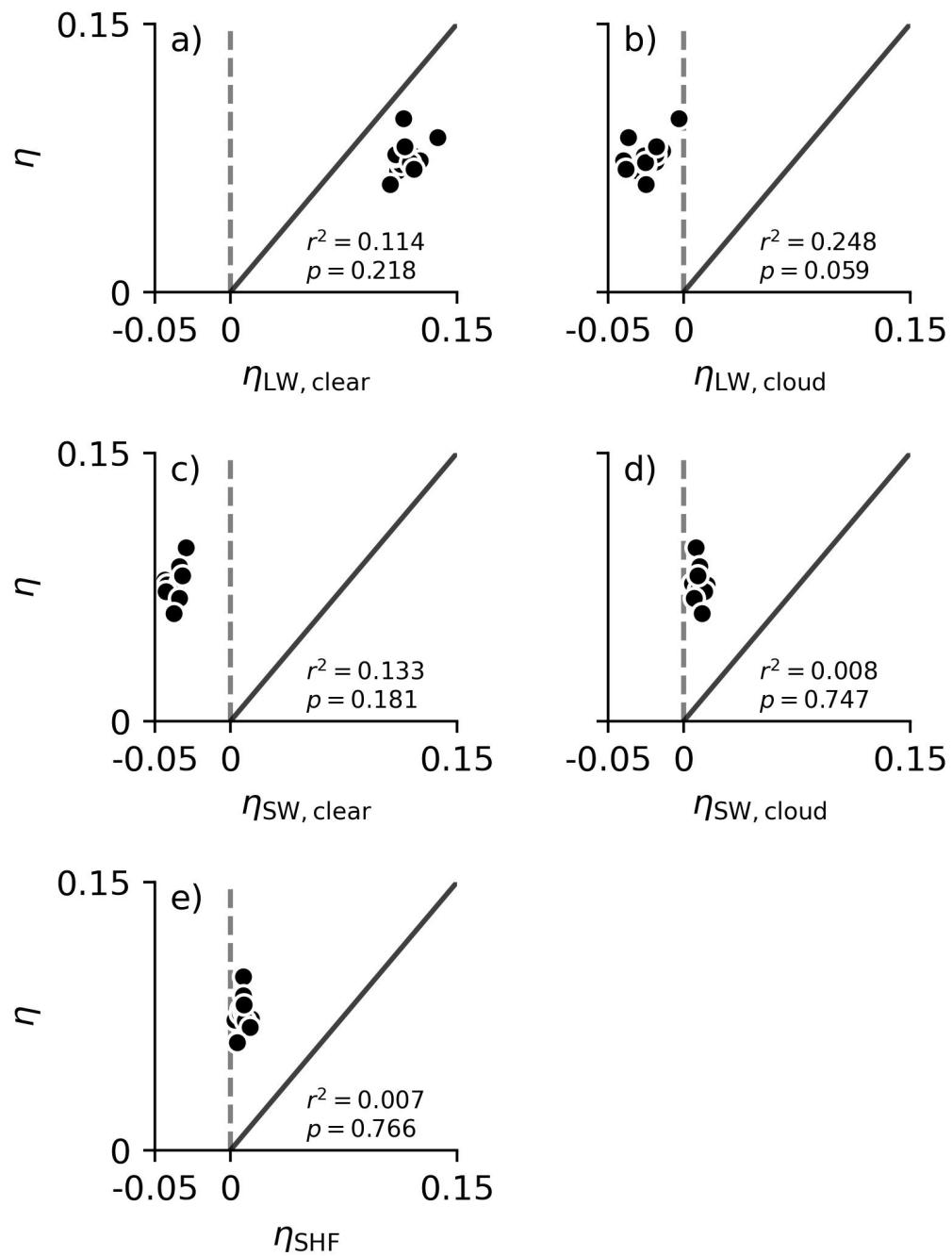


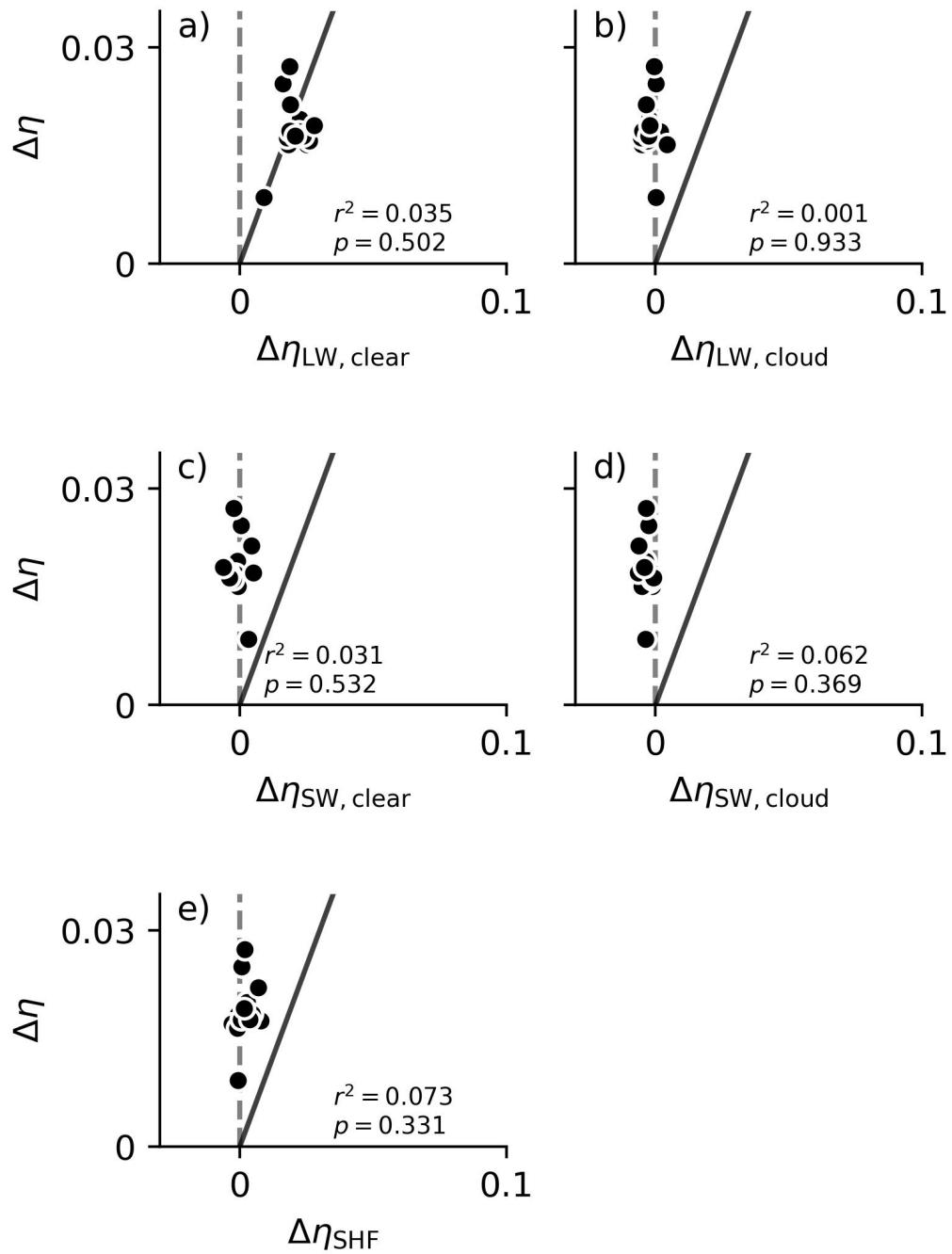
Figure S2. *Tropical and extratropical contributions to  $\Delta\eta_{\text{LW,clear}}$  are anti-correlated.*



**Figure S3. *The weak-temperature gradient assumption becomes valid at pressures of around 700hPa.*** A spatial coherence analysis of monthly, tropical temperatures from ERA5 reanalysis (1940-2024). Each panel shows the square of the Pearson correlation coefficient between the tropically-averaged temperature and the local at that level. All quantities are de-seasonalized before performing the analysis. High correlations throughout the tropics (as in (a) and (b)) indicate that tropical-mean temperatures explain much of the variance in local tropical temperatures and thus that the weak-temperature gradient (WTG) assumption is reasonably valid. The correlations become weaker poleward of 20°, but at 700hPa and above, around 30% of the variance in local temperatures is still explained by the tropical-mean even near the edge of the tropics.



**Figure S4. *Inter-model spread in  $\eta_{\text{abrupt}}$  is not captured by any of the individual energy budget contributions.*** Scatter plots of  $\eta_{\text{abrupt}}$  against its contributions from different components of the atmospheric energy budget.



**Figure S5.** *Inter-model spread in  $\Delta\eta$  is not captured by longwave, clear-sky cooling, but the mean value is.* Scatter plots of  $\Delta\eta$  against its contributions from different components of the atmospheric energy budget.