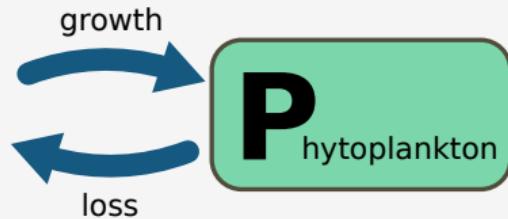
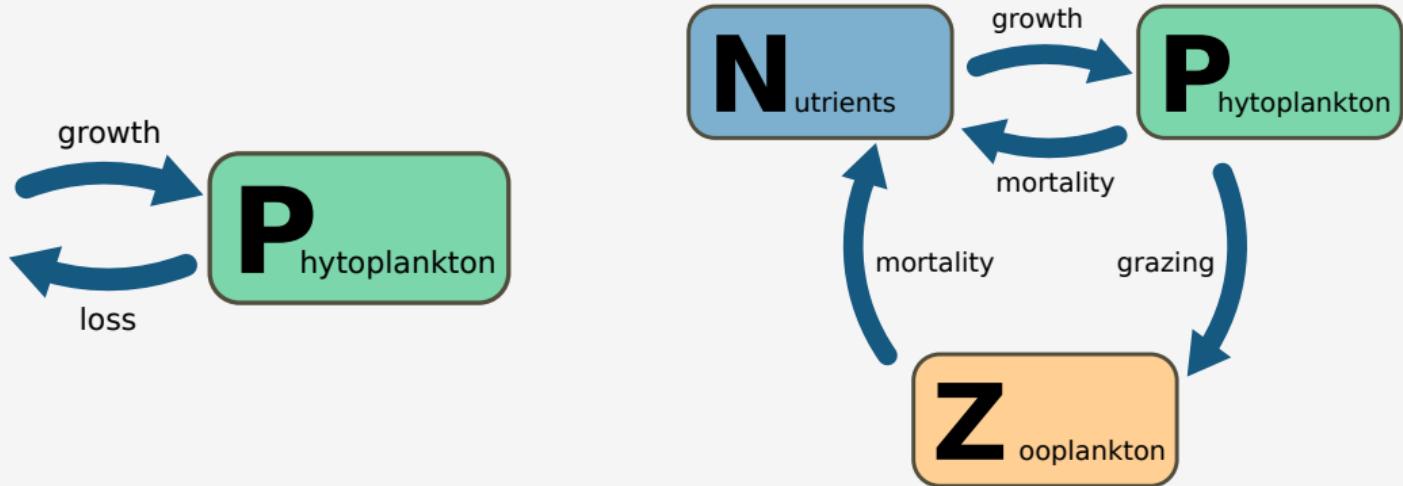


from P-growth to NPZ



from Pgrowth to NPZ



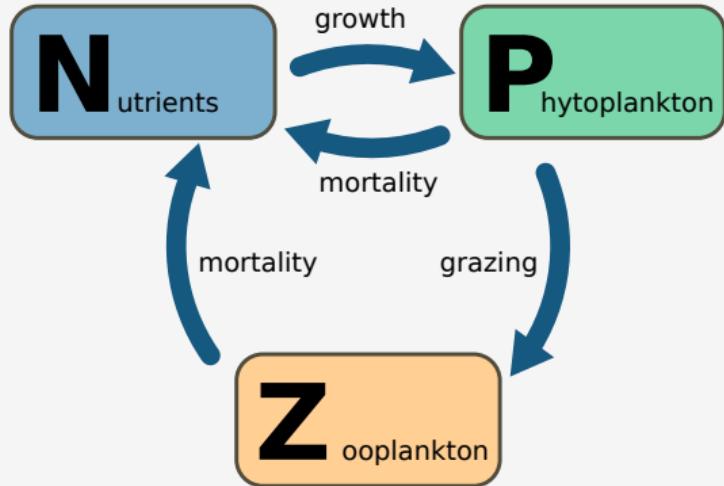
from Pgrowth to NPZ

growth: $\theta_{\text{vmax}} \frac{N}{N + \theta_{\text{nuthalfsat}}} P$

grazing: $\theta_{\text{graz}} P Z$

P mortality: $\theta_{\text{mort_p}} P$

Z mortality: $\theta_{\text{mort_z}} Z^2$

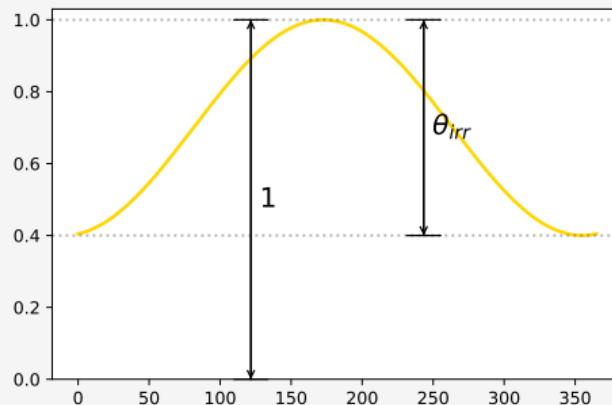


$$\begin{aligned}\frac{\partial N}{\partial t} &= -\theta_{\text{vmax}} \frac{N}{N + \theta_{\text{nuthalfsat}}} P + \theta_{\text{mort_p}} P + \theta_{\text{mort_z}} Z^2 \\ \frac{\partial P}{\partial t} &= \theta_{\text{vmax}} \frac{N}{N + \theta_{\text{nuthalfsat}}} P - \theta_{\text{graz}} P Z - \theta_{\text{mort_p}} P \\ \frac{\partial Z}{\partial t} &= \theta_{\text{graz}} P Z - \theta_{\text{mort_z}} Z^2\end{aligned}$$

adding light

- Modulate phytoplankton growth by annual light cycle (modeled to roughly represent day length in the northern hemisphere, with a minimum on Dec 21).

$$1 + 0.5 \cdot \left(\theta_{\text{irr}} \cdot \sin \left(\frac{2\pi}{365} (t - 81.25) \right) - \theta_{\text{irr}} \right) \quad \text{with } \theta_{\text{irr}} \in [0, 1]$$



- growth: $\theta_{\text{vmax}} \left(1 + 0.5 \cdot \left(\theta_{\text{irr}} \cdot \sin \left(\frac{2\pi}{365} (t - 81.25) \right) - \theta_{\text{irr}} \right) \right) \frac{N}{N + \theta_{\text{nuthalfsat}}} P$
- 6 parameters in total: $\theta_{\text{vmax}}, \theta_{\text{nuthalfsat}}, \theta_{\text{graz}}, \theta_{\text{mort_p}}, \theta_{\text{mort_z}}$ and θ_{irr} which we will all estimate in Stan

Stan functions block

```
functions {
    real[] npz(real t,           // time
               real[] x,         // state
               real[] theta,     // parameters
               real[] x_r,       // fixed real data (empty)
               int[] x_i) {      // fixed integer data (empty)

    /*
    guide to theta:
    theta[1]:  vmax          maximum growth rate in Michaelis-Menten formulation
    theta[2]:  nuthalfsat    nutrient half-saturation in Michaelis-Menten formulation
    theta[3]:  graz           zooplankton grazing rate
    theta[4]:  mort_p         phytoplankton mortality rate
    theta[5]:  mort_z         zooplankton mortality rate
    theta[6]:  irr            light amplitude
    */

    real n = fmax(0.0, x[1]);
    real p = fmax(0.0, x[2]);
    real z = fmax(0.0, x[3]);

    real light = 1.0 - 0.5*(theta[6]*sin(pi()*((t-81.25)/182.5)) - theta[6]);
    real growth = theta[1]*n/(theta[2]+n) * light * p;
    real grazing = theta[3]*p*z;
    real ploss = theta[4]*p;
    real zloss = theta[5]*z*z;

    return {-growth+ploss+zloss, growth-grazing-ploss, grazing-zloss};
}
}
```

goal: estimate

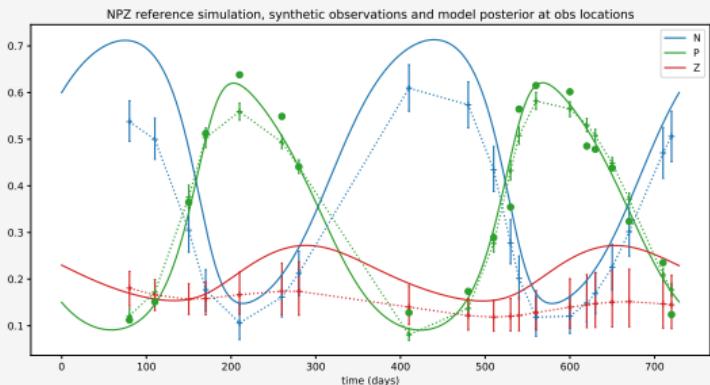
- parameters
- initial conditions
- observation error for each observed variable

```
parameters {
    real<lower=0> vmax;
    real<lower=0> nuthalfsat;
    real<lower=0> graz;
    real<lower=0> mort_p;
    real<lower=0> mort_z;
    real<lower=0,upper=1> irr;
    real<lower=0> x0[3];           // initial conditions
    real<lower=0> sigma[nobsvar]; // obs error
}
transformed parameters {
    real theta[6] = {vmax,nuthalfsat,graz,mort_p,mort_z,irr};
    real x[nobs, 3] = integrate_ode_rk45(npz, x0, 0, tobs, theta,
                                         rep_array(0.0, 0), rep_array(0, 0),
                                         1e-5, 1e-4, 1e4);
}
model {
    vmax      ~ normal(0.1, 0.1);
    nuthalfsat ~ uniform(0.0, 1.0);
    graz       ~ normal(0.01, 0.01);
    mort_p     ~ normal(0.01, 0.01);
    mort_z     ~ normal(0.01, 0.01);
    irr        ~ uniform(0.0, 1.0);
    x0[1:3]    ~ normal(0.1, 0.1);
    for (iobs in 1:nobs){
        obs[iobs,] ~ normal(x[iobs,iobsvar], sigma);
    }
}
```

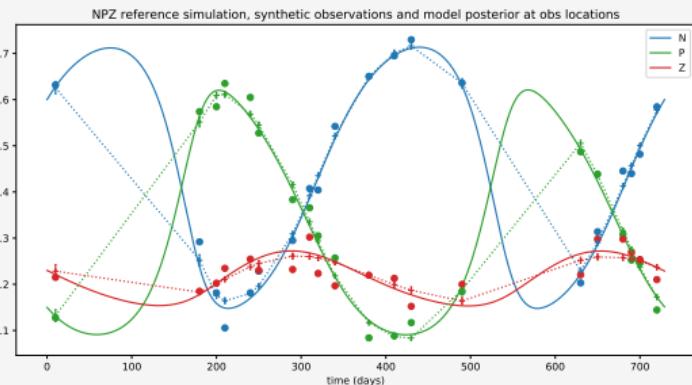
- **choose:** Turn all 3 variables into observed variables OR select only one observed variable.
- Examine the effect on the output.

exercise

```
iobsvar = np.array([i_p])
sigma = np.array([0.03])
```

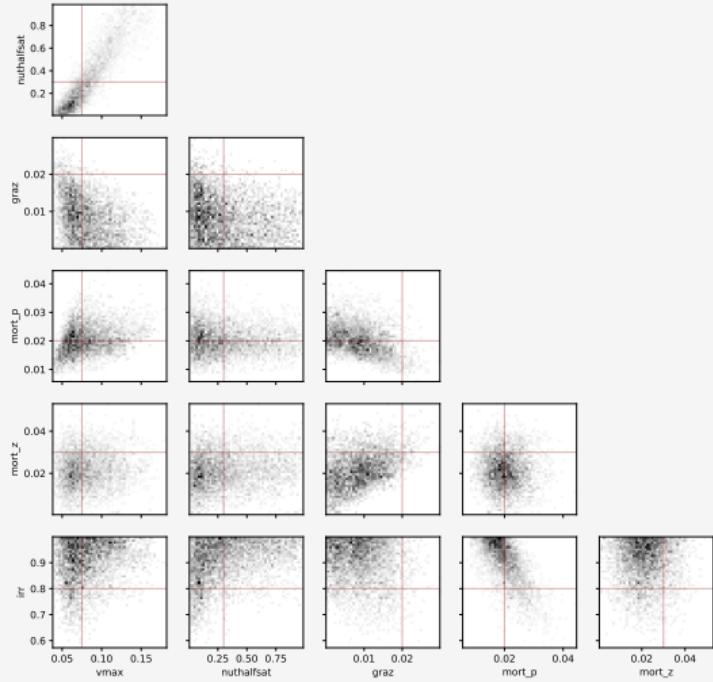


```
iobsvar = np.array([i_n,i_p,i_z])
sigma = np.array([0.03,0.03,0.03])
```

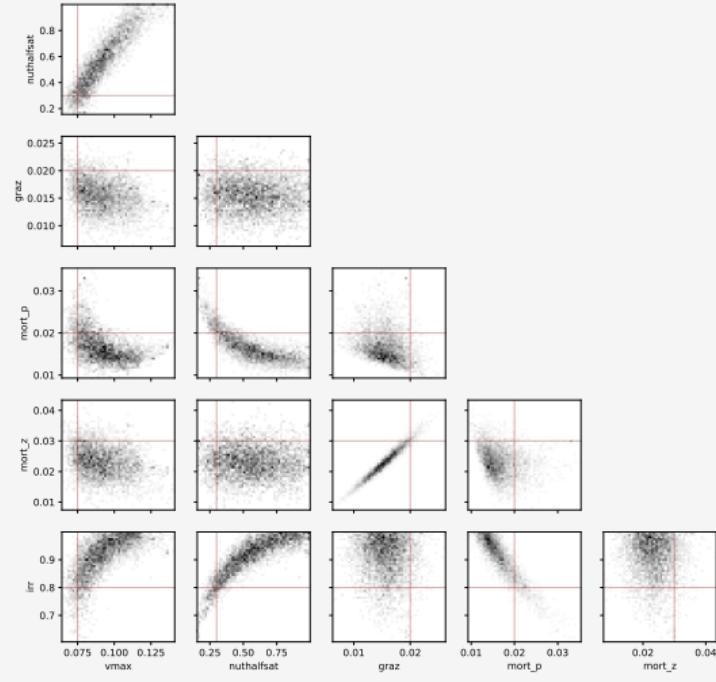


exercise

```
iobsvar = np.array([i_p])
sigma = np.array([0.03])
```



```
iobsvar = np.array([i_n,i_p,i_z])
sigma = np.array([0.03,0.03,0.03])
```



adding real data: Narragansett Bay data set

adding real data: Narragansett Bay data set

<https://web.uri.edu/gso/research/plankton/>

The screenshot shows the Graduate School of Oceanography website. At the top, there's a banner with a blue gradient background featuring a photograph of red flowers in the foreground and a lighthouse in the distance across the water. The University of Rhode Island logo is in the top left, and social media icons are in the top right. Below the banner, the text "Graduate School of Oceanography" is displayed. The main navigation menu includes links for Home, About, People, Academics, Research and Outreach (which is underlined), News and Events, and Contact. Below the menu, a large section title "The Narragansett Bay Long-Term Plankton Time Series" is centered. Underneath this title is a horizontal navigation bar with links: PLANKTON TIME SERIES OVERVIEW (underlined), WEEKLY SAMPLING PROCEDURE, DATA, RESOURCES, REFERENCES, and CONTACT. A large image at the bottom of the page shows a coastal scene with houses on a rocky shore and a boat in the water. Overlaid on this image is a circular inset showing a microscopic view of plankton cells.

THE UNIVERSITY OF RHODE ISLAND

Graduate School of Oceanography

URI > Graduate School of Oceanography > Research and Outreach > The Narragansett Bay Long-Term Plankton Time Series

Home About People Academics **Research and Outreach** News and Events Contact

The Narragansett Bay Long-Term Plankton Time Series

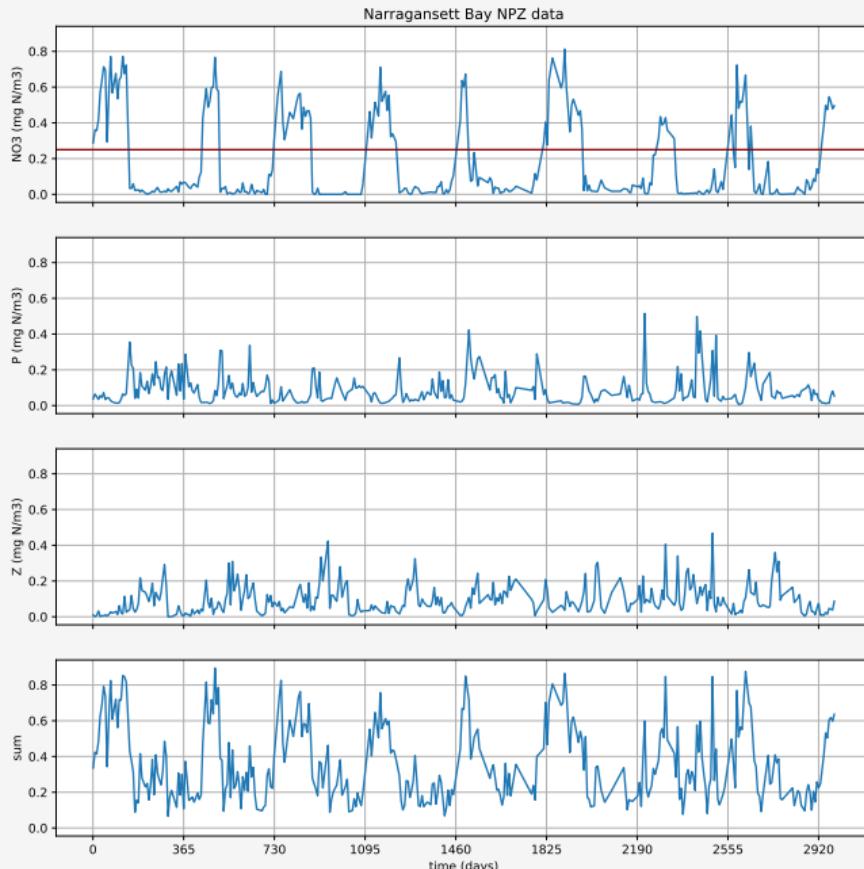
PLANKTON TIME SERIES OVERVIEW WEEKLY SAMPLING PROCEDURE DATA RESOURCES REFERENCES

CONTACT

One of the world's longest-running plankton time series

J. Paul Mattern NPZ introduction

adding real data: Narragansett Bay data set



allow influx and loss of N in the model

- introduce a background concentration for each variable and allow mixing with that background concentration
- have a winter period for each year with stronger mixing and different background concentrations
- estimate start and duration of winter period for each year (initial guess is based on data)
- estimate background concentrations for winter and regular period (same for each year)

add supply (nudging/relaxation) term:

$$\frac{\partial N}{\partial t} = \dots + \theta_{\text{mix}} \cdot (\theta_{\text{bgconc_n}} - N)$$

- The prior distributions for the parameter are all quite “tight”, because the model and Stan can be quite sensitive to the parameter choice.
- Modify one of the priors and examine the effects on the prior model solution. Be sure to modify the prior in the first part of the notebook before the data is being fit.
- If you feel adventurous, modify the prior in the second piece of Stan code as well and run it with fitting data.

```
// priors for parameters
vmax      ~ normal(0.15, 0.03);
nuthalfsat ~ normal(0.17, 0.04);
graz       ~ normal(0.15, 0.04);
mort_p     ~ normal(0.02, 0.01);
mort_z     ~ normal(0.02, 0.005);
bgconc_n1 ~ normal(0.01, 0.001); // (regular)
bgconc_n2 ~ normal(0.66, 0.08); // (winter)
bgconc_p1 ~ normal(0.11, 0.01); // (regular)
bgconc_p2 ~ normal(0.05, 0.005); // (winter)
bgconc_z1 ~ normal(0.09, 0.01); // (regular)
bgconc_z2 ~ normal(0.05, 0.03); // (winter)
mix1      ~ normal(0.01, 0.03); // (regular)
mix2      ~ normal(0.19, 0.02); // (winter)
```

exercise

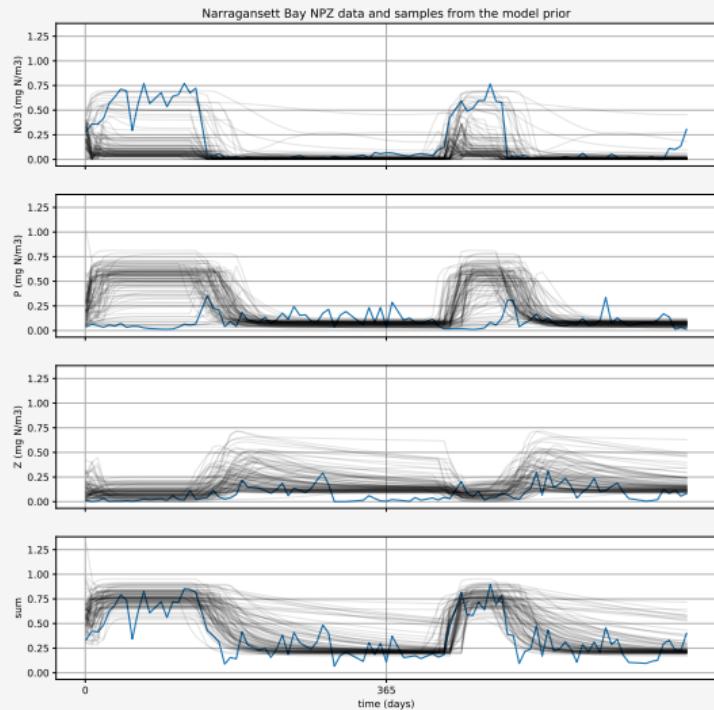
going from

```
vmax ~ normal(0.15, 0.03);
```

to

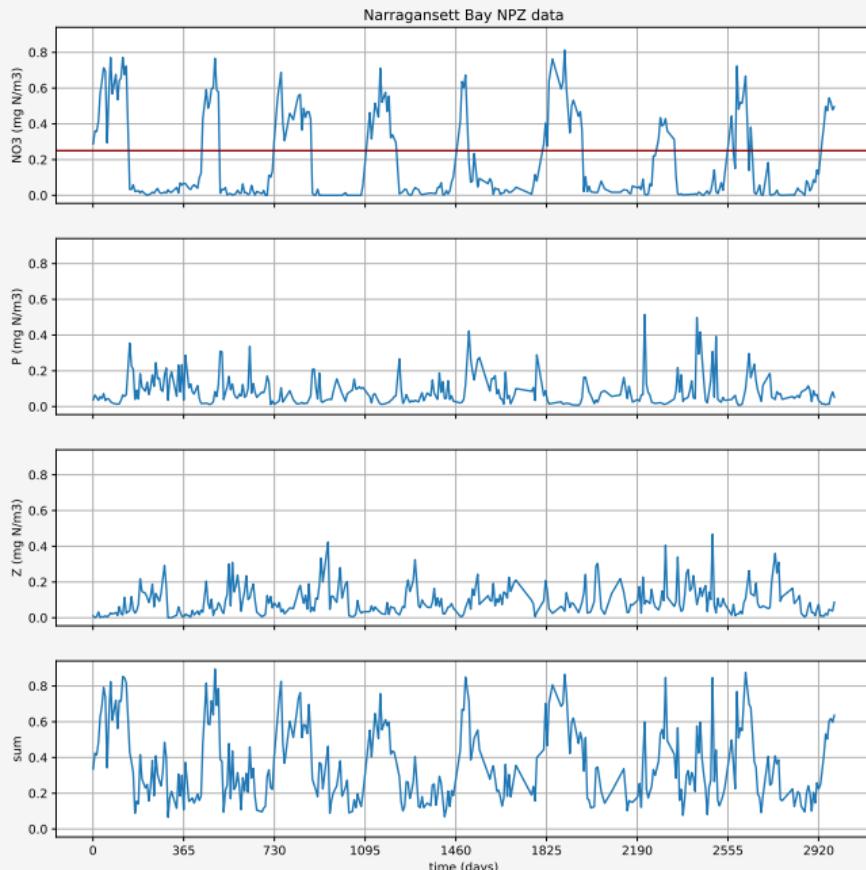
```
vmax ~ normal(0.5, 0.5);
```

results in

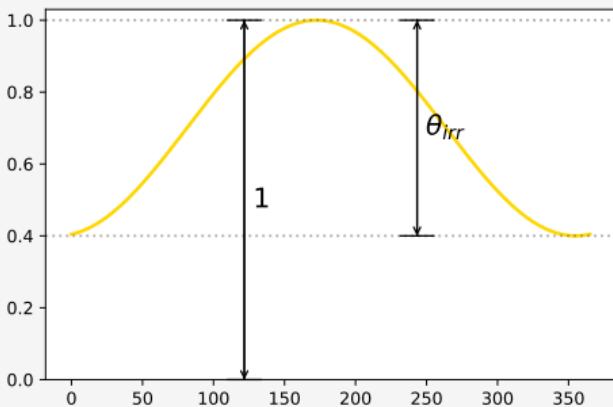


- ① develop a different model for supply and loss of N
- ② add in light limitation
- ③ come up with priors for the winter period start and duration without an informed guess
- ④ modify the NPZ model formulation (unobserved N contained in a detritus variable, light limitation in winter?)
- ⑤ add new observations (there are more in the Narragansett Bay dataset)

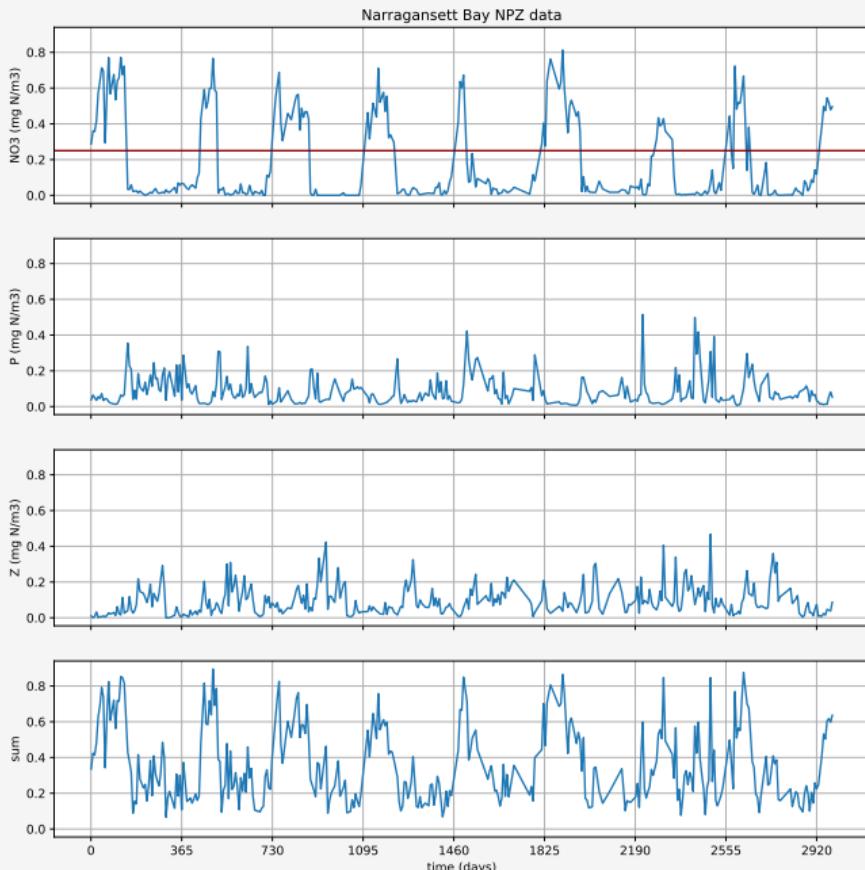
idea: develop a different model for supply and loss of N



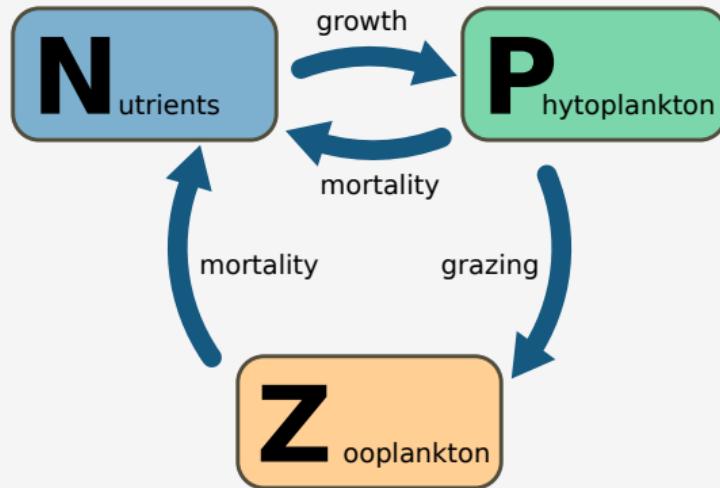
idea: add in light limitation



idea: come up with priors for the winter period start and duration without an informed guess



idea: modify the NPZ model formulation (unobserved N contained in a detritus variable, light limitation in winter?)



candidates:

- other nutrients like phosphorous
- ...

