SIMULATING THE GROWTH OF A BLACK HOLE USING VARIOUS METHODS

by

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ABSTRACT

This thesis explores the evolution of a black hole within a globular cluster (spherical system) and various methods were performed to analyze differences in black hole growth within a spherical system. These methods were comprised of analytical solutions and N-body simulations, multiple parameters had to be adjusted and optimized within the N-body simulation. Three main methods of black hole growth were examined: adiabatic, fixed and dynamic. From these methods properties of the system were measured such as: energy profiles, density profiles, the position of the BH from the centre of the system and other various properties as to gain a better understanding of how a black hole interacts within a large spherical system of particles over time. The black hole was tracked and had its position plotted with varying masses to see how this might impact its motion. It is suggested that the motion of the black hole is reduced with an increase in mass, reduced meaning the motion does not go as far out into the system and is dampened out faster. It was also investigated to see if there were any major differences between the fixed growth and dynamic growth, as it was predicted that a dynamic black hole will have a lower central density than its fixed counterpart due to particles within the system wanting to achieve energy equipartition. It was found that the dynamic growth did have an overall lower central density than its fixed growth counterpart. This is due to the black hole carving out a central region in the system and as it encounters nearby particles it ejects them from this region.

ACKNOWLEDGEMENTS

I'd like to thank my thesis supervisor and professor Joseph MacMillan for helping me along the way in this project. I wouldn't have gotten as far as I did without his help.

I'd also like to thank my peer Kinza Bahktiar for assistance along the way.

Thank you to my family and friends for supporting me throughout the years.

AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of the University of Ontario Institute of Technology. The work is original except where indicated by special reference in the text and no part of the dissertation has been submitted for any other degree. Any views expressed in the dissertation are those of the author and in no way represent those of the University of Ontario Institute of Technology. The thesis has not been presented to any other University for examination either in Canada or elsewhere.

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1. INTRODUCTION

1.1 Globular Clusters and Black Holes

This thesis will be examining how a black hole (BH) grows inside a dense system of stellar objects. In order to begin exploring this topic some introduction is needed. First by introducing the globular cluster (GC) it is a very densely packed system of stars. GCs contain no gas, dust or young stars, the clusters themselves appear to be very long lived and dynamically stable. Our Milky Way contains about 200 GCs and each cluster can have anywhere from 10^4 to 10^6 stars (Binney & Tremaine, 1987). A GC is a near spherical system, it is spherical-like due to the strong gravitational attraction between stars and this tight binding of gravity gives the cluster a high density towards its centre. It has long been suspected that intermediate mass black holes (IMBH) grow and reside in dense stellar systems and IMBHs typically have a mass of around 100 to 10,000 solar masses (Baumgardt *et al.*, 2015). GCs are generally very old and could possibly be the relics of the formation of a galaxy itself and they are found in the halos of galaxies. A GC central density is much higher than most galaxies, a typical value is $10^4 M_\odot\ pc^{-3}$, compared to our solar neighborhood density $0.05 M_\odot\ pc^{-3}$.

The GC 47 Tucanae is located roughly 4001 pc from Earth and is about 36.8 pc across, refer to fig 1.1. A beautiful and very bright cluster it is the second brightest after Omega Centauri, what is quite peculiar about 47 Tucanae is new evidence arising which suggests that the cluster may be harboring an IMBH.

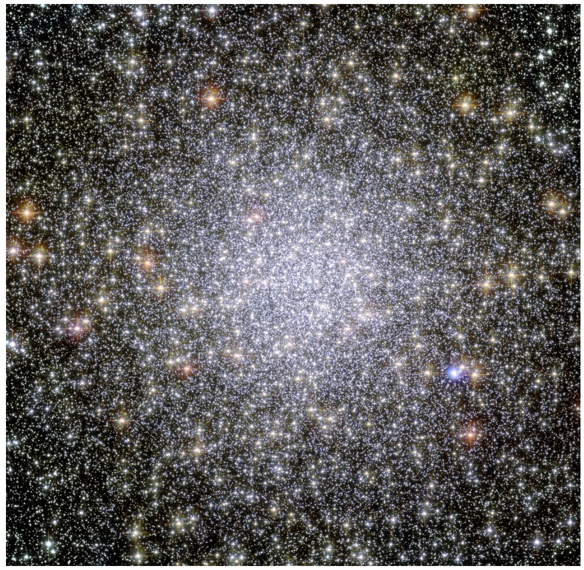

Figure 1.1: 47 Tucanae, observed by the NASA/ESA Hubble Space Telescope (2015).

A BH is a region in space-time where the effects of gravity are so strong that once too close to it nothing can escape its grasp, not even light (Wald, 1997). Light cannot escape because the escape velocity from the BH at this point exceeds the speed of light. This boundary of no escape is called the Event Horizon, because light cannot escape if an object was to fall past this zone while being observed, the object would simply appear to freeze in time as the light being reflected off the object can no longer be seen. Einstein's theory of general relativity predicts that an extremely dense mass can deform space-time creating a BH. BHs that have stellar mass are usually formed by the collapse of massive stars, these can continue to grow by absorbing nearby matter or even merging with another BH. There is a general consensus that the centre of most galaxies harbor a super massive black hole. Using the Schwarzschild radius equation, one can calculate what radius an object must be compressed to for a BH to be created.

$$r_s = \frac{2GM}{c^2}. \tag{1.1}$$

G is the gravitational constant, M is the object mass, and c is the speed of light. To put this in perspective, take an average human with a weight of 70 kg.

$$r_s = \frac{2(6.674 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}})(70\,\mathrm{kg})}{(3 \times 10^8\,\mathrm{m\,s^{-1}})^2} \tag{1.2}$$

$$r_s \approx 1 \times 10^{-25}\,\mathrm{m} \tag{1.3}$$

This means an average human will need to be compressed down to a size smaller than a proton. If done though you would have created your very own BH!

IMBHs strongly affect the spatial distribution of stars in GCs. Massive stars sink into the centre more efficiently in order to achieve energy equipartition. However, as stars sink closer to the centre they are scattered by the BH (Bülent *et al.*, 2017). We will keep this important detail in mind for later. Bülent and his team primarily observed velocity dispersion profiles and density profiles with respect to the distance from the centre of their system. They ran dynamical N-body simulations of isolated star clusters which evolved under the influence of stellar evolution and two-body relaxation. They ran their simulations up to a time of 11.75 Gyr to match the age of 47 Tucanae. An important point to note is that they did not take primordial binaries into account as Bülent states they do not play a notable role in the final density profile.

The opposing paper to Bülent's, (Mann *et al.*, 2018) analyzes stellar proper motions in the core of 47 Tucanae to explore the possibility of an IMBH influence on the stellar dynamics of the cluster. They use short wavelength photometry to gain a very clear view of stellar motions into the centre of the crowded core. A density distribution is taken as a central IMBH point mass added to a combination of King profiles. They conclude that concentrated binaries and stellar-mass BHs produce a sufficient velocity dispersion signal in the core as to make an IMBH unnecessary to fit observations.

In this thesis the features that will be examined are density and energy profiles of the system. It is very interesting to see two teams debate on the existence of this IMBH. Attempting to try and contribute to their discussion would be outside the scope of this Thesis project so I will be strictly analyzing the behaviour of BHs and how they affect a cluster of bodies. No photometry will be carried out in this project everything will be done using N-body methods. I want to simulate the growth of a BH in a large collection of particles over a long period of time, in order to see if I can replicate behaviours of a BH that occur in nature. Such as star scattering when energy equipartition is reached , energy equipartition can be thought of as stars with high kinetic energy transferring their kinetic energy to stars with less kinetic energy (Binney & Tremaine, 1987). If the system forms by violent relaxation the position and velocity distributions of the stars will be independent of mass, meaning that the more massive stars will have a higher kinetic energy and as a consequence the massive stars will sink to the centre of the system as they transfer away their kinetic energy.

1.2 Adiabatic Growth

Imagine a pendulum inside a box support and it is oscillating back and forth, if you were to violently shake the support in a jerky manner the bob will swing around very chaotically. However, if you gently and smoothly move the support the pendulum will continue to swing in a smooth way. This gradual change of the external conditions defines an adiabatic process (Griffiths, 2005). Meaning that in this isolated spherical system we can assume slow BH growth which also leads to a slowly varying potential energy. The method of producing a BH using adiabatic growth has been done by (MacMillan, 2001). This method is different from the methods that shall be used in this thesis because the adiabatic growth method is not a particle simulation but rather an algorithm of solving equations. It involves the use of a distribution function which

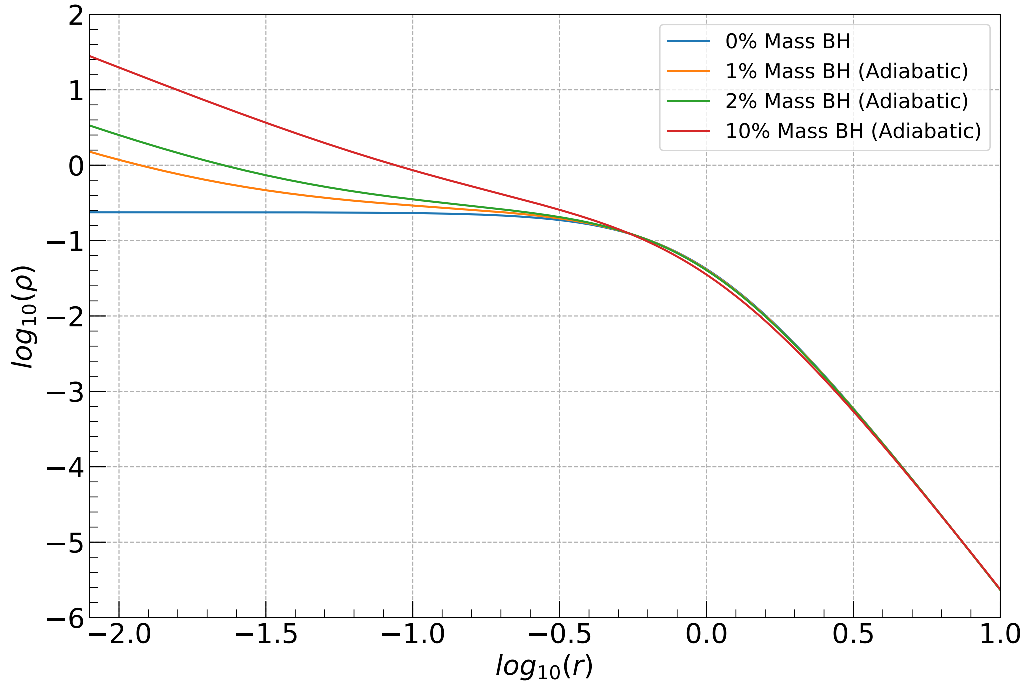

Figure 1.2: Adiabatic growth of a BH with varying final masses. The final mass being a percentage of the system's total mass.

is quite complex and will not be expanded on further. After obtaining this distribution function the potential energy of the system can be calculated.

$$U(r) = U(r) + \frac{GM_{BH}}{r} \tag{1.4}$$

G is the gravitational constant, M_{BH} is the final mass of the BH and r is the radius of the system.

In Figure 1.2 this density profile is a snapshot of the system when the BH has reached its final mass, all of the subsequent density profiles are as well the final snapshot of the system. The 0% mass BH represents a Plummer Sphere, which is the spherical system where all the particles are located and all the interactions between the particles take place. A Plummer Sphere is a model for a density distribution and gravitational potential distribution in a spherical halo of matter. The density and gravitational potential distributions of Plummer Sphere are,

$$\rho(r) = \frac{3M}{4\pi\epsilon^3}\frac{1}{(1+\frac{r^2}{\epsilon^2})^{5/2}}, \tag{1.5}$$

$$\Phi(r) = \frac{-GM}{\sqrt{r^2+\epsilon^2}}. \tag{1.6}$$

Where M is the mass of a particle in the system, r is the position of the particle from the centre of the system and epsilon is the softening length. The simulations ran in this project spanned 26.4 simulation time units to be exact, which equates to 10 dynamical times, 1 dynamical time is enough time for a particle in the system to go from one end of the Plummer Sphere to another and back. This allows the system to feel the effects of the BH. The 1%, 2% and 10% mass BH plots are similar in the sense that they are Plummer Spheres but contain a BH with a higher final mass and as such their central densities are significantly higher. The 1% mass BH would represent the Plummer Sphere but harbouring a BH near its centre with 1% of the entire Plummer Sphere's mass.

1.3 N-Body Methods

An N-body simulation approximates the motion of particles, often specifically particles that interact with one another through some types of physical forces. Using this broad definition, the types of particles that can be simulated using N-body methods are quite significant, ranging from celestial bodies to individual atoms in a gas cloud. These simulations can be useful for observing non-linear structure formation such as galaxy formation, or in the case of this project, the dynamical evolution of a stellar cluster. There are two popular forms of N-body methods, tree methods, and particle mesh. The method used in this project is the tree method, this will be expanded on more, particle mesh is another interesting method but will not be further examined. However, the reader is encouraged to look into this other powerful method of performing N-body simulations.

The most obvious way to perform an N-body simulation is through direct integration of the Newtonian gravitational force equation, where the total force on each particle is the superposition of the forces imparted by each of the other particles as seen in equation 1.7,

$$\sum_{i \neq n}^{N} \frac{G m_n m_i}{r_{ni}^2} \hat{r}_{ni}. \tag{1.7}$$

This method is known as the brute force method, the only approximation being made is that during the time dt, the acceleration of each particle is constant. The major shortcoming of this algorithm is the asymptotic computational time, for very accurate results the brute force method gives the most correct results. Unfortunately, it is very slow when the number of particles is large. A common way to improve the computational time is to implement a tree algorithm known as the Barnes-Hut algorithm, named after its creators.

Until recently the gravitational N-body problem has been modelled numerically either by direct integration, in which the computation needed increases as N^2, N representing the number of particles. In 1986 a novel method of directly calculating the force on N bodies that grows only as Nlog(N) was developed (Barnes & Hut, 1986). The technique uses a tree-structured hierarchical subdivision of space into cubic cells, this method is called the tree method. In order to decrease the number of force calculations necessary per time step, one can begin to think about neglecting individual bodies that are far away from the body whose net force we are interested in calculating. Since the gravitational force decreases as $\frac{1}{r^2}$, bodies that become substantially far enough from any body become significantly less important to its subsequent motion. The Barnes-Hut algorithm provides a systematic way to define "far-away" along with providing a method for approximating the force due to far-away bodies. An oct-tree data structure is used to represent the position of the bodies in the system. A finite region of space is divided into octants, each of which will hold a number of bodies. Each of these subspaces is assigned a mass, which is defined to be located at the center of mass of the bodies in that octant with a mass equal to their sum. Then, each octant that contains more than one body is again subdivided into eight smaller octants, each of which has a mass as previously defined. This process is continued recursively until each octant contains either zero or one bodies. A visual representation of this data structure is shown in Figure 1.3 in 2D.

Recall that for the brute-force algorithm, each time-step took a time that was proportional to the number of bodies squared, N^2. For the Barnes-Hut algorithm, each body calculates a force with on average log(N) other bodies in the tree. The overall time for one time-step in the Barnes-Hut algorithm is proportional to Nlog(N). Asymptotically, the difference between the run times is substantial.

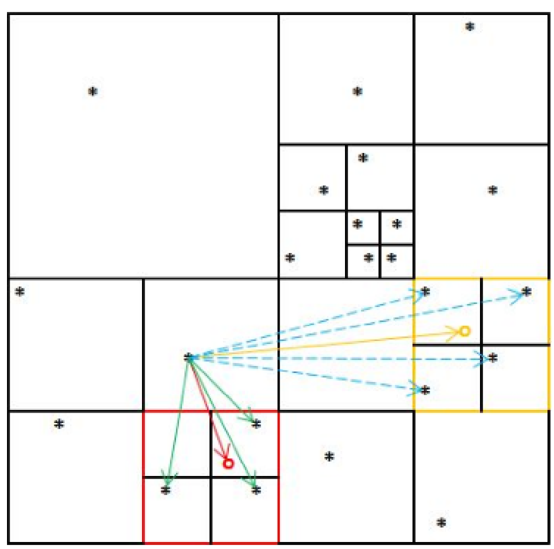

Figure 1.3: An example of a tree method dividing up particles into cells.

2. FIXED BLACK HOLE GROWTH

2.1 DNC

To simulate the GC we use a collisionless N-body code developed by (Stiff, 2003). The code is an algorithm written in C, David Stiff named it DNC, Dave's N-body Code. It is an N-body simulation of N particles using the tree method described earlier. First a spherical system must be generated, the particles are then placed inside the system and their accelerations are calculated, the particles are then moved forward in time. A large amount of time was put into finding optimal parameters before introducing a black hole to this system. Conservation of energy is necessary in order to have a system that can correspond to something real and physical.

Many runs were done with 10,000 particles, the parameters being adjusted were the time step, to have a good resolution of the system, the other parameter is the softening length. Because this code is collisionless a softening length must be in place to avoid nonphysical interactions between particles. If particles get very close with no softening length in place, the particles accelerations can spike, causing particles to be ejected from the system. With the softening length in place it can allow particles to get close to each other without the non-physical interaction of them suddenly obtaining an extremely high acceleration. Once the flattest energy plot was obtained by modifying the parameters multiple times, the total number of the particles were increased to 100,000. More runs were done again at this level to ensure the energy of the system was as flat as possible. Finally N was then increased to 1,000,000.

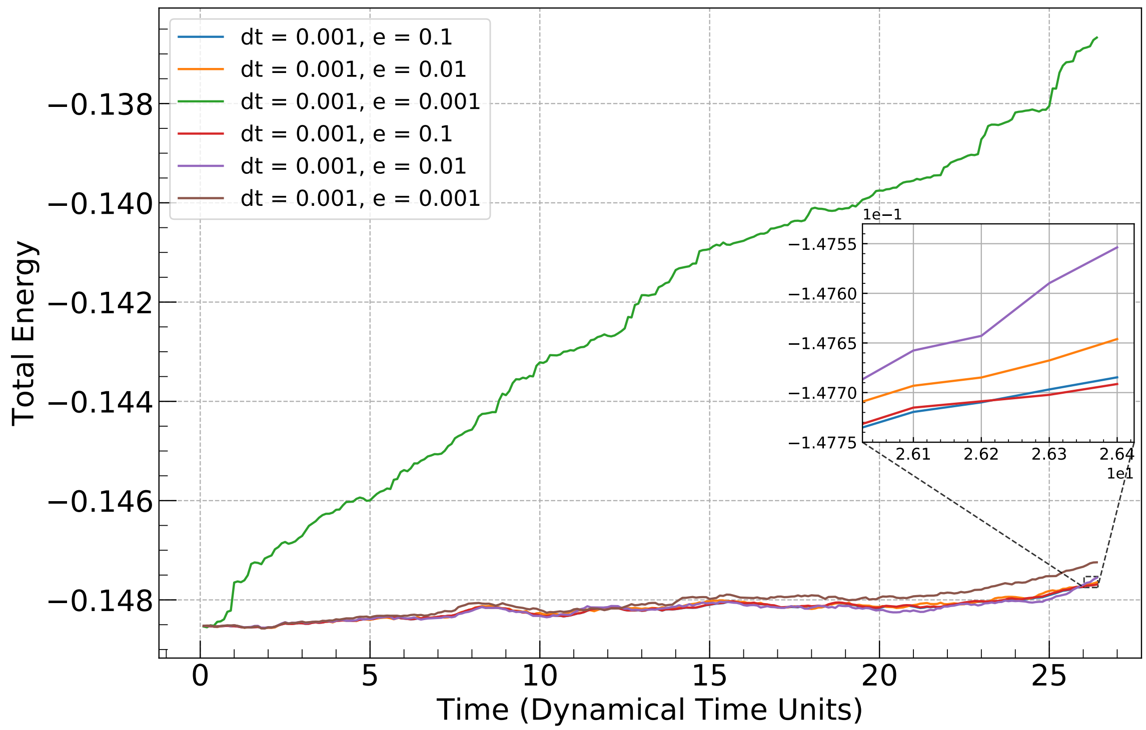

Figure 2.1: 10,000 particle simulation runs all with varying parameters.

Above is Figure 2.1 and it shows some of the runs that were carried out. The parameters with the lowest change in energy were with a time step of 0.001 and a softening length of 0.1. However this is only for 10,000 particles, so if N were to be increased the energy difference may not hold. More runs were carried out in order to test these chosen parameters. You can see that there is very little energy change for all the runs except for the one outlier. Because all the lowest changes in energy had a time step of 0.001 I wanted to test runs at that time step but now with varying softening length to see if it had an impact on the energy of the system. N was then increased to 100,000 and the time step kept at 0.001 but now with varying softening lengths.

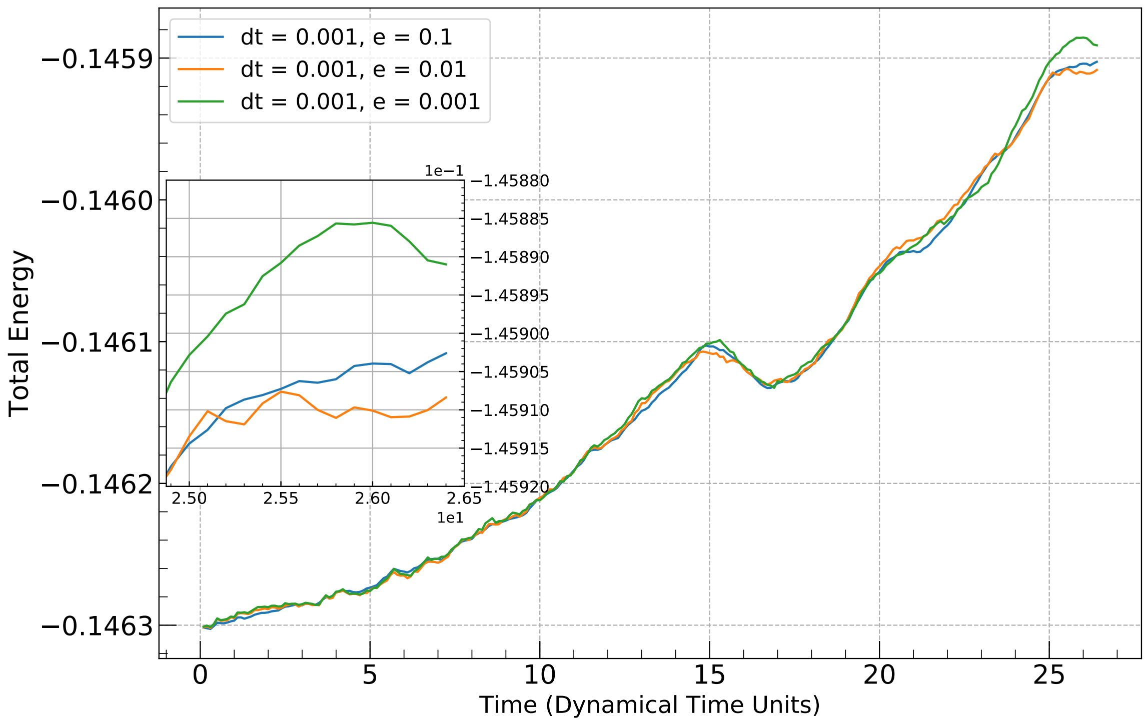

Figure 2.2: 100,000 particle simulation runs with varying softening lengths.

Now examining Figure 2.2 the energy profiles are very similar and there is quite little difference between the parameters. Over a very long period of time the energy changes by around 0.0004 simulation energy units, an incredibly small amount. The time step of 0.001 and softening length of 0.01 were chosen for the 1,000,000 particle runs as it had the overall lowest change in energy albeit very insignificant relative to the other sets of parameters. This is where examining the energy of the system ends and now I can begin to explore properties of the BH and any impacts it has on the system, or how the system may affect the BH.

2.2 Modifications to DNC

In order to run this simulation with a BH, several modifications had to be done to the program. Initially DNC would take the number of particles you pass to it, perform the acceleration computations needed to evolve the system forward and that was it. There are a couple ways a fixed BH could be added, it could be treated as a particle or a potential. In this case a potential was the method taken, it is much simpler to treat the BH as a potential at the centre of the system compared to treating it as a particle.

This builds a basic model for how the BH grows and how the system is affected. The position of this "fixed BH" was set at the origin of the system and with every time step its mass would grow by some fixed amount. As this potential grows, the particles accelerations are recalculated every time step in order to see how the system is being affected by the growing potential. After the simulation is done running there are many output files to be analyzed, the energy of the system can be plotted, the density profile, it is also possible to track the position of the BH, however in this instance not very useful, it will be more useful later on in the dynamic case.

Below is the method used to calculate the acceleration of all the particles in the system based on how they are affected by the BH. x,y,z are the positions of each particle, m is the BH's mass, these equations are run through many times in a loop to keep updating each particle's behaviour based on the current size of the BH.

$$a_x = \frac{-mx}{r^3}. \tag{2.1}$$

$$a_y = \frac{-my}{r^3}. \tag{2.2}$$

$$a_z = \frac{-mz}{r^3}. \tag{2.3}$$

Where r is the radius of the BH and epsilon is the softening length.

$$r = \sqrt{x^2 + y^2 + z^2 + \epsilon^2}. \tag{2.4}$$

These equations were implemented into the DNC's main loop.

2.3 Results

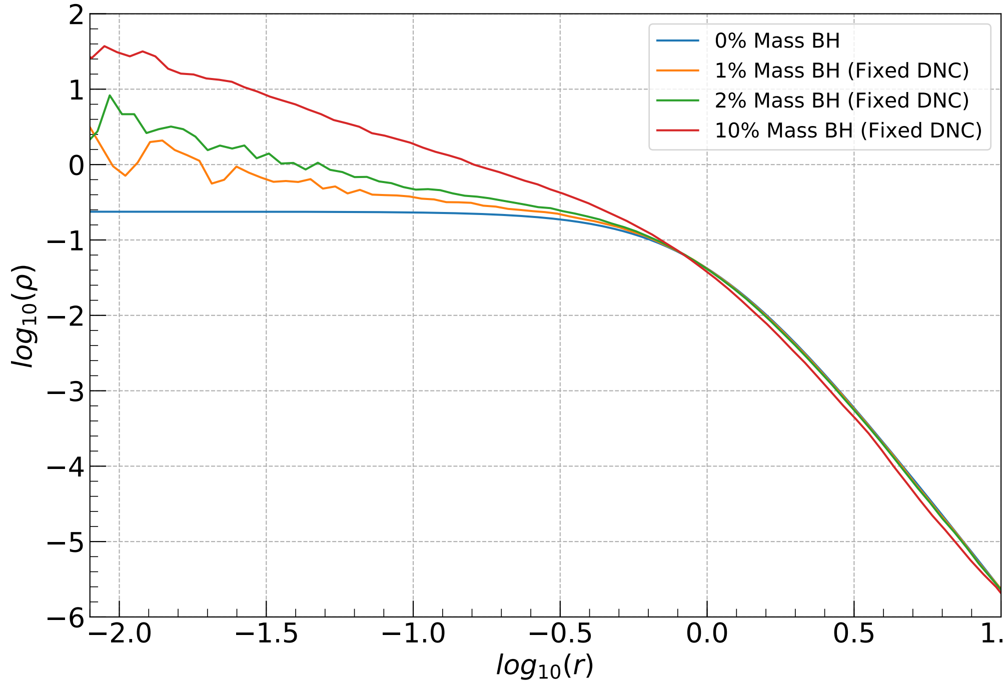

Figure 2.3: Density profiles of the fixed BH simulation runs.

In Figure 2.3, 1% mass BH means 1% of the total system's mass, the total system being the entire Plummer Sphere and all of the particles contained within. The density profile demonstrates that there is a relationship between the central density and final mass of the BH, if the BH has a larger final mass then it will correspondingly have a larger central density. Another feature to be noticed is that the higher the BH's mass the farther out into the system it affects other particles, such as the 10% mass BH, its density profile takes a while longer to taper off and overlap onto the other density profiles. Nothing non-physical appears to be going on here, relative to the dynamic case the fixed case is much simpler, so by adding onto the complexity of the system hopefully the realistic effects of a BH can be observed.

3. DYNAMIC BLACK HOLE GROWTH

3.1 Further Modifications to DNC

To get to the dynamic case the DNC code will again need to be modified to allow the BH to be treated as an actual particle and give it the ability to move throughout the system. This was the main challenge of this project. Various parts of the code will be changed and even the addition of new scripts will be implemented to make this possible.

The equations that were implemented in the previous case were removed, this returned DNC back to its original state. It is again now a code that generates a Plummer Sphere and calculates the forces and accelerations of each particle per iteration. But now a BH must be added to the system and be treated as a dynamical particle, what is an efficient way of tackling this problem? Each particle in this system is automatically assigned a particle ID of 0, this can be cleverly taken advantage of.

By writing a new script that appends a particle to the system, this new particle's ID can be overwritten to have a different and unique ID, as such this particle was given an ID of 1. This new script is not very complicated but it does require knowledge of how classes and objects work in Python, as the system's class is being used to append a new particle object to the system. The new particle is appended very close to the origin's position (0,0,0).

The system now has 1,000,000 particles all with an ID of 0 and one particle with an ID of 1, 1,000,001 particles in total. A completely different and unique particle now exists in the system that can be manipulated and called explicitly with ease. DNC's main loop was then modified so that this particle with an ID of 1, have its mass increase by some amount dM,

$$dM = \frac{M_{BH}}{stopStep}. \tag{3.1}$$

The final mass of the BH is passed to DNC as an argument and also the time step to stop at, stopStep. If say our time step was 0.001, to reach time units 26.4, the stopStep will be 26,400. In the case of having a BH with a final mass of 10% of the system's mass, dM will have the following value,

$$dM = \frac{0.1}{26,400} = 3.78 \times 10^{-6}\,. \tag{3.2}$$

This results in the BH having a linear growth per time step. There is now a dynamic particle in the system which grows as time evolves and data can be pulled from the system as the program loops, specifically the BH's mass, position and velocity. This is exactly what was implemented into the main loop, DNC was again modified to write all these values to a new file while the simulation runs. Using this data of the BH, its whole path throughout the simulation can be plotted. There is quite a bit of data that can be analyzed, just from knowing the position of the BH, its distance from the centre of the system can be calculated, and as well the change in distance over time.

Would the BH travel very chaotically and have huge changes in position during the entire simulation? This is another interesting question that can be answered. After all, the behaviour of BH's are being explored here. The purpose here is to ask questions and try to answer them. Another interesting question is, how far out does the BH go into the system? Would its mass affect this? These questions will

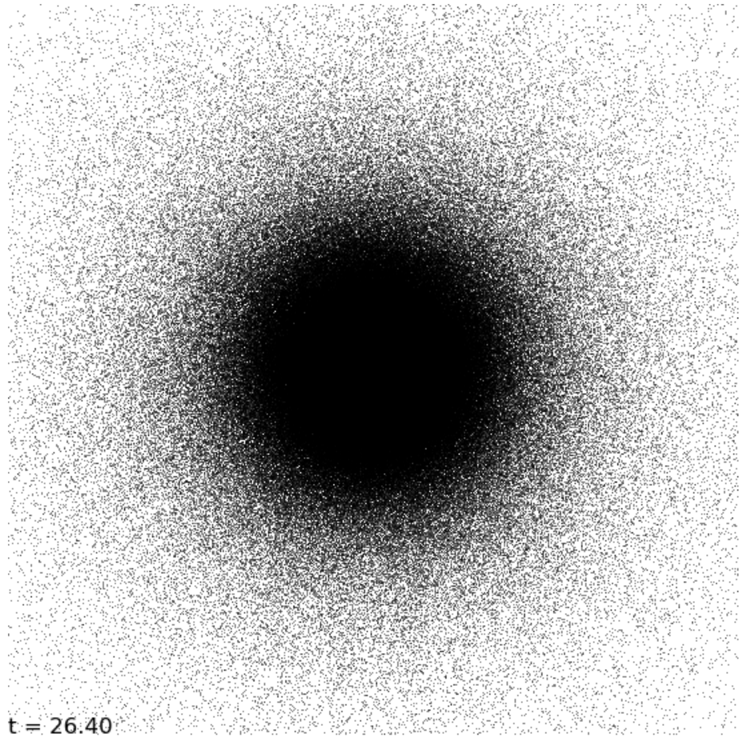

Figure 3.1: A snapshot of the final output file of a 1,000,001 particle run with BH having a final mass of 10% of the system's total mass.

be addressed in the coming section. Using the output files from DNC, the density profiles of the system can be plotted, not only can these be obtained but a density distribution can be produced as well, showing the smooth transitions of low to high density regions. Finally an animation can be produced from the output data files, it captures the particles in motion, intuitively the system starts out dispersed, by the end of the animation the centre is much denser and darker, indicating a significant increase in central density.

Each and every one of those particles in Figure 3.1 represents a small patch of phase space. Similar to the fixed BH case, in the dynamic case, four runs were performed. Starting with no BH in the system for the first run, a BH is then introduced and with each successive run the BH's final mass is increased up to a maximum mass of 10%. Darker regions indicate high density, or more "particles".

3.2 Results

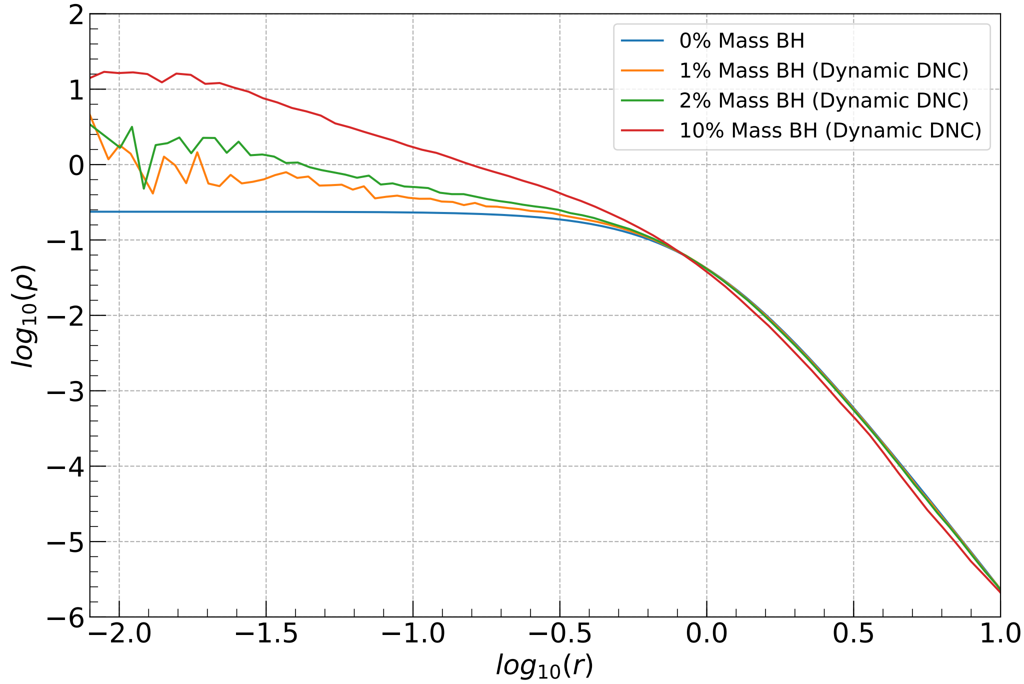

Figure 3.2: Density profiles of the dynamic BH simulation runs.

The density profiles of the dynamic system in Figure 3.2 look very similar to the fixed system. With careful observation there is a key subtle difference. Recall earlier in the paper when it was stated that stars which sink closer to the centre of the system will be scattered by the BH. This is important to note, as it describes a real feature of BHs. This key difference cannot be seen unless the dynamic profiles are compared to the fixed profiles. This however will be done in the next chapter. For now, the questions which were asked in the previous section will be answered.

For the case for the 10% mass BH in Figure 3.3 it travels out a distance of around 0.02 simulation distance units.

$$log_{10}(0.02) \approx -1.70. \tag{3.3}$$

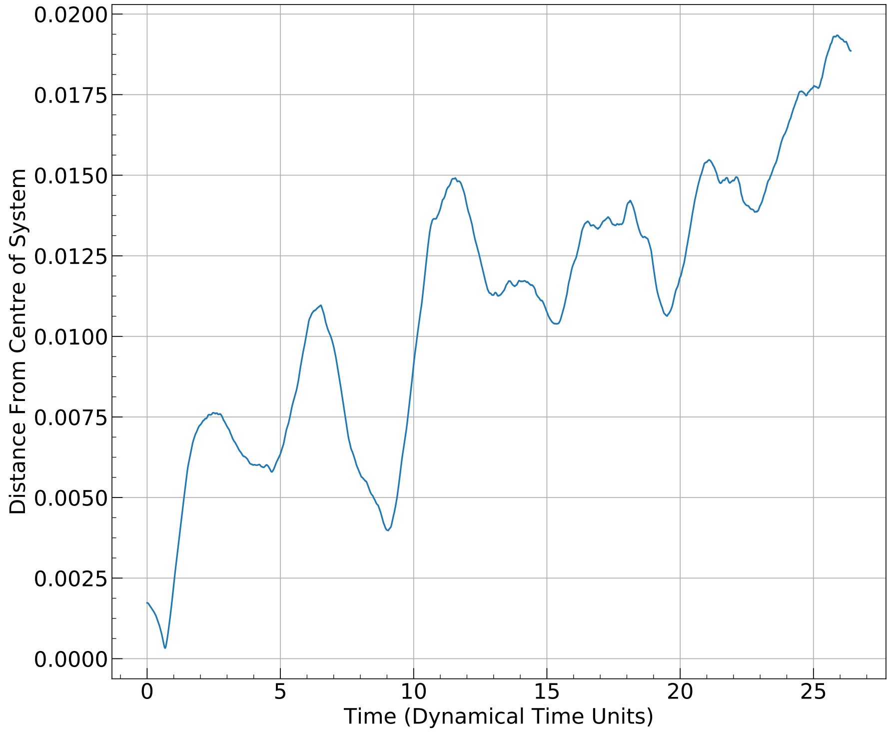

Figure 3.3: The BH's distance from the centre of the system. BH with a final mass of 10%.

Refer back to Figure 3.2. This value of -1.70 is roughly as far as the 10% mass BH travels out to and the density profile begins to drop around this value, where as values lower than it have a much more stable and consistent density. Another feature of Figure 3.3 is of the change in difference of position. Notice as time goes on, the extreme turning points begin to dampen out, it appears that the BH is slowing down. However to be sure of this a difference in position must be shown. A script I wrote to calculate distance from the centre of the system will only need to be modified slightly and then this feature can be plotted.

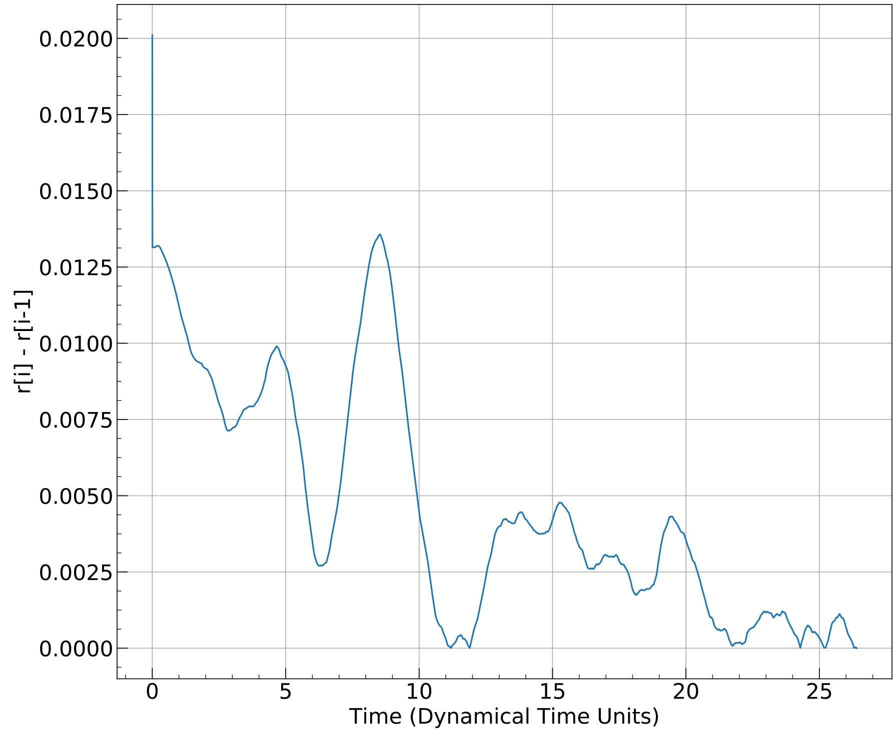

Figure 3.4: The difference in position of the 10% mass BH.

Here in Figure 3.4 it is very clear to see that in fact the BH does dampen out and it moves less chaotically over time. The data being calculated for this plot is as follows,

$$r[i] - r[i-1] = \sqrt{(x[i] - x[i-1])^2 + (y[i] - y[i-1])^2 + (z[i] - z[i-1])^2}. \tag{3.4}$$

This is being iterated through in a script where i is the index of iteration the loop is on. The position is always being compared to the previous instance of position for the BH. As the curve in Figure 3.4 begins to have less extreme turning points this indicates a dampening effect and that the BH is beginning to relax and settle. Would the mass affect this behaviour?

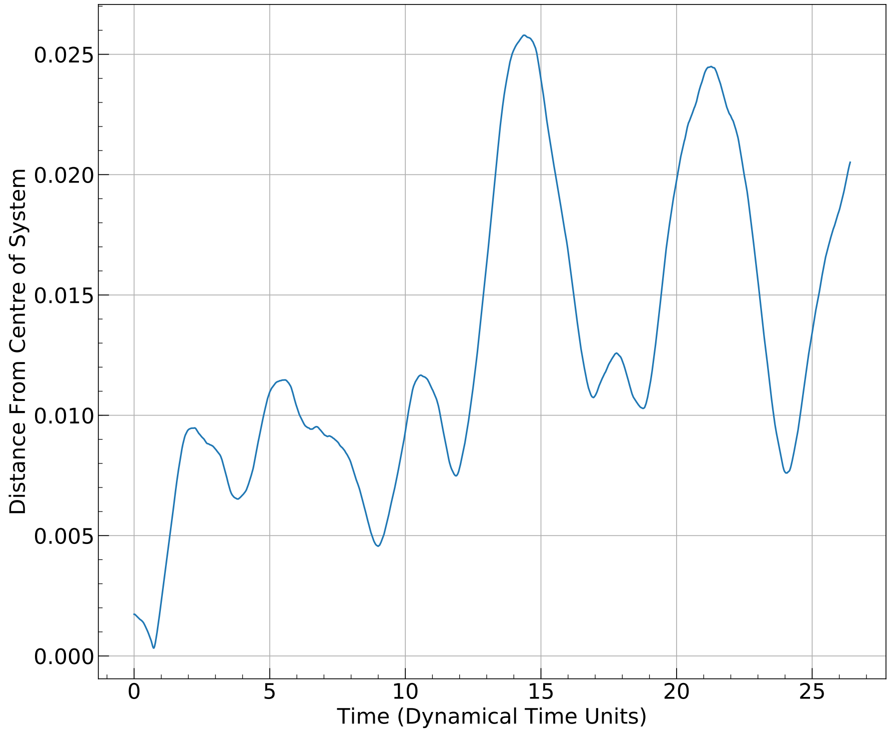

Figure 3.5: The BH's distance from the centre of the system. BH with a final mass of 1%.

In the case of the 1% mass BH it travelled farther away from the centre of the system than the 10% mass BH.

$$log_{10}(0.025) \approx -1.60. \tag{3.5}$$

Again refer back to Figure 3.2. The 1% mass BH should only have a strong attraction effect on the particles up to around -1.60, then the density begins to decrease as the distance from the centre of the system is increased. Intuitively this BH's difference in position should also dampen out over time, however it appears to being do so much slower than its 10% counterpart. The lower the BH's mass the farther out it can travel but it has less of an effect on distant particles.

3.3 Motion of Black Hole

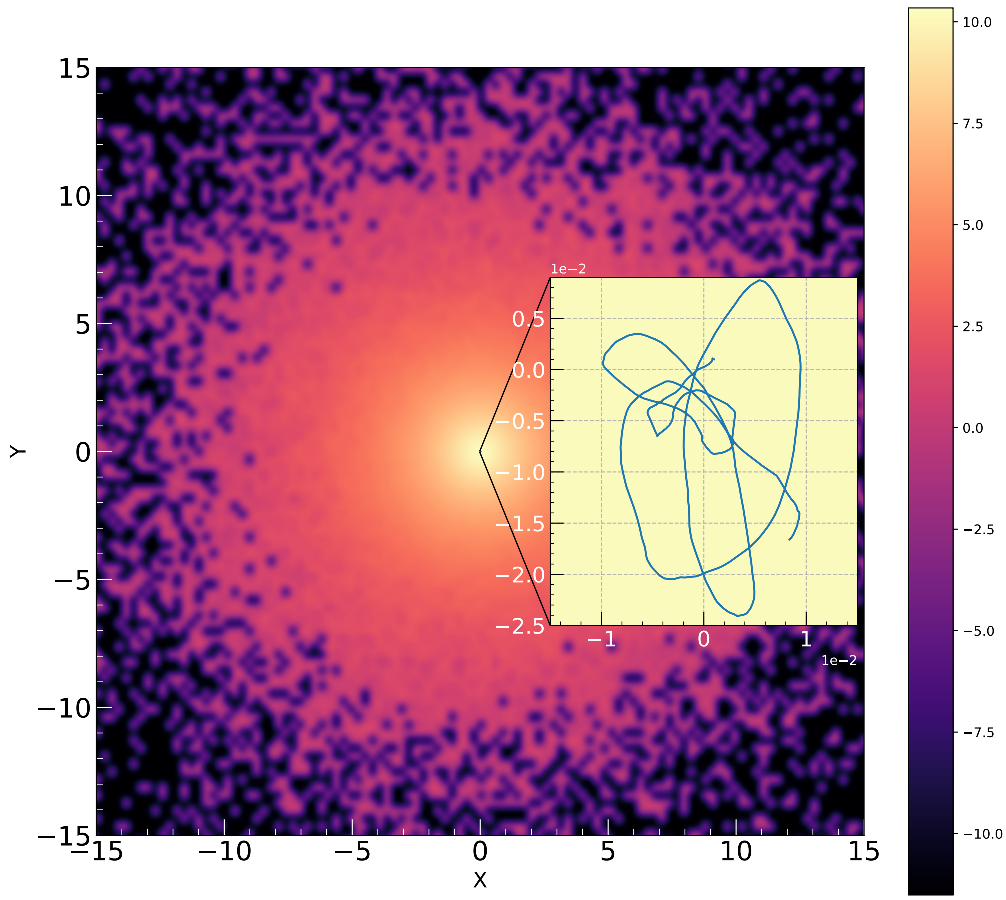

Figure 3.6: A density distribution image of the system containing a 1% mass BH.

Similar to Figure 3.1, Figure 3.6 is a density distribution of a snapshot of the system. This particular figure is a distribution of the system containing a 1% mass BH. An inset plot has been placed zooming far into the core of the system. It is showing the BH's path it travelled during the simulation in the x-y plane. The colour bar on the right indicates density of the system. Black indicates very low density whereas pale yellow indicates very high density.

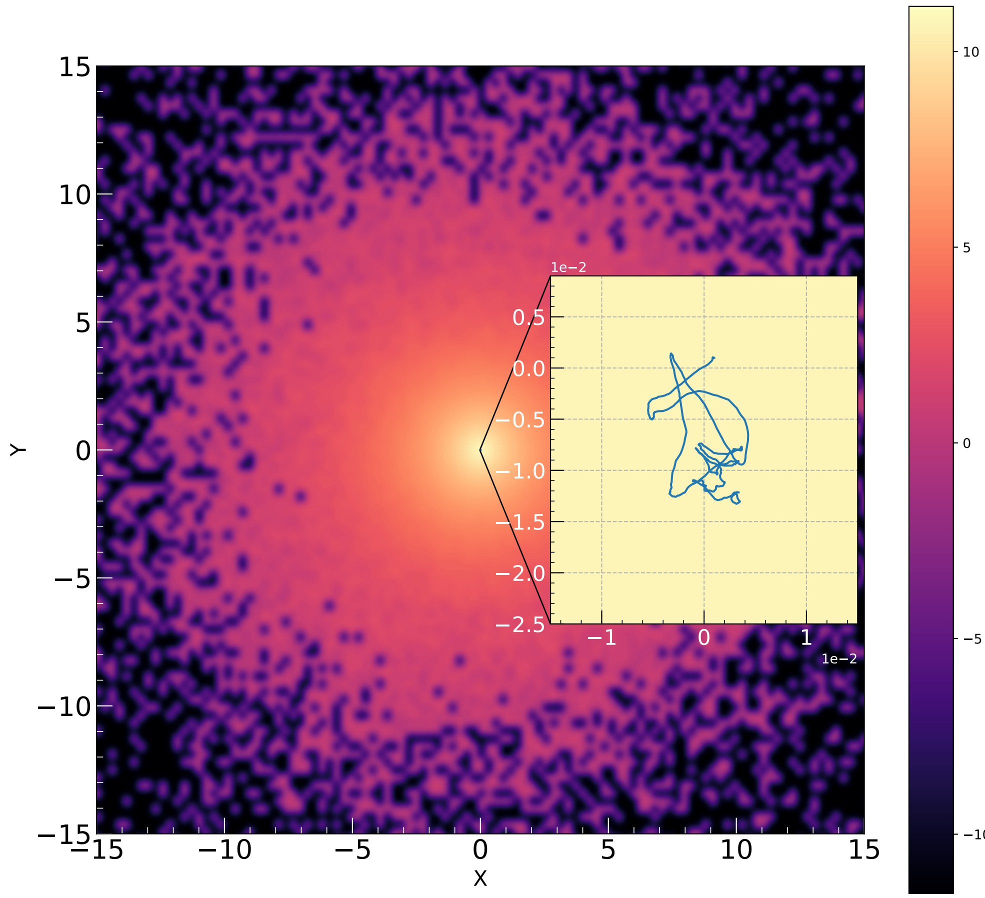

Figure 3.7: A density distribution image of the system containing a 10% mass BH.

Interestingly in Figure 3.7 as the BH's mass increases, its path tends to become more chaotic while dampening out quicker. Axes limits were kept constant to see effects of the increase in mass on the distance travelled from the centre of the system. Recall that as time went on the difference in position would decrease thus its movement would relax. Look closely at the inset plot, wherever there are huge long distances covered is likely where the BH began its walk. Where there appears to be a lot of dense scribbling is likely where its walk ended.

4. ANALYSIS

4.1 Adiabatic Growth vs Fixed Growth

As seen if Figure 4.1 the adiabatic growth curve and fixed growth curve follow similar features however there are noticeable bumps in the fixed growth and this is due to the fact that it is a particle simulation and there is some level of noise involved. On average the fixed growth has a higher central density than the adiabatic growth and it follows the trend of the adiabatic growth curve. There is a more significant discrepancy in the 10% mass BH plot, the fixed growth is notably higher than the adiabatic growth for quite some time until all the curves overlap.

In Figure 4.2 we can see a similar story only slightly different. Unlike the fixed growth the central density for the dynamic growth is not higher on average compared to the adiabatic growth until we simulate a BH with a final mass of 10%. Then again the same discrepancy pops out, the dynamic growth then has a greater central density compared to the adiabatic growth. Why might this be the case? Also note that in the 10% mass case the adiabatic theory suggests that the BH should have a higher central density than the dynamic counterpart. This will be important to keep in mind for section 4.3.

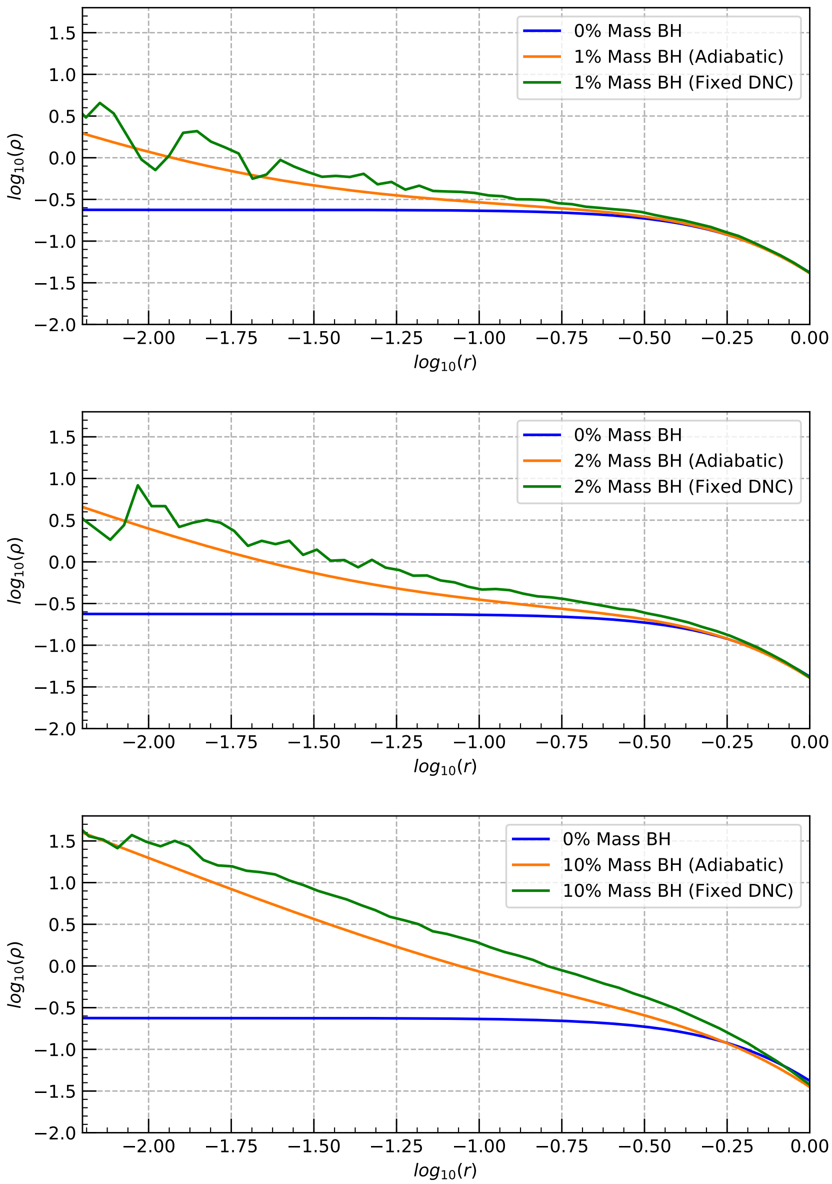

Figure 4.1: Density profiles of Adiabatic growth vs Fixed growth.

4.2 Adiabatic Growth vs Dynamic Growth

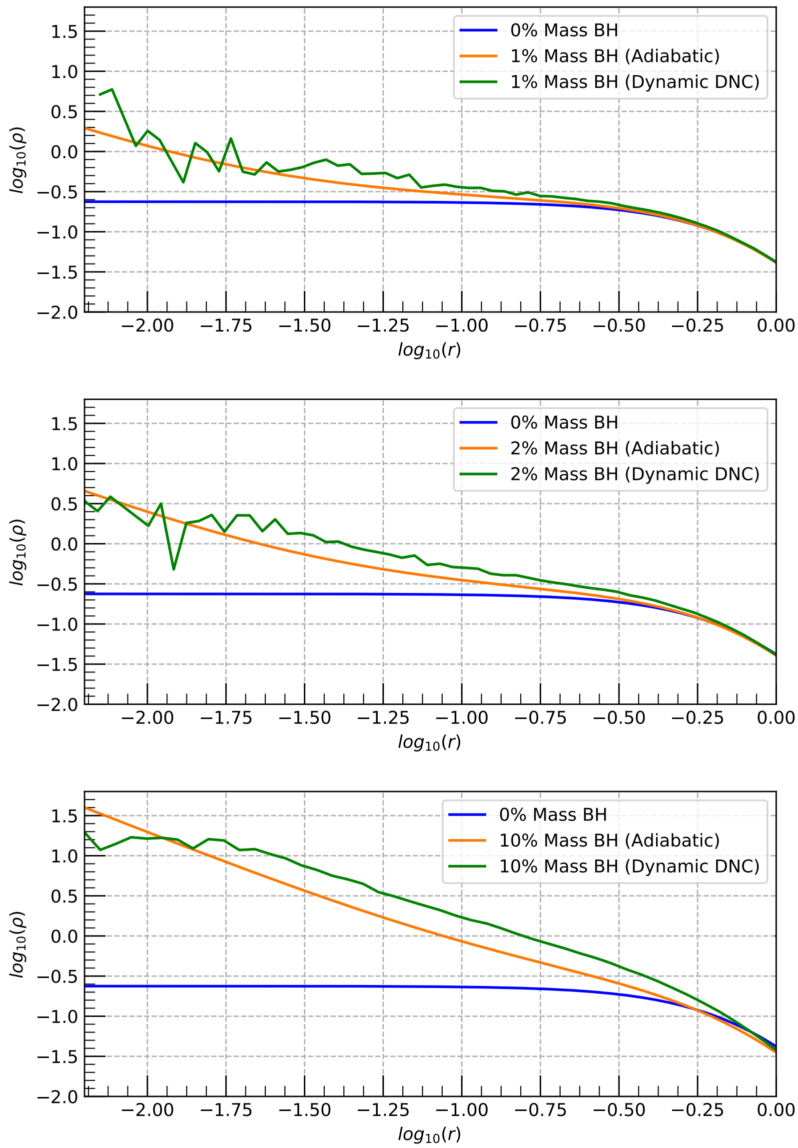

Figure 4.2: Density profiles of Adiabatic growth vs Dynamic growth.

4.3 Fixed Growth vs Dynamic Growth

Here is a very important finding, one that confirms a statement made earlier. Observe the difference in density between the dynamic and fixed 2% mass profiles in Figure 4.3. The fixed is slightly higher than the dynamic, this indicates that the fixed BH system is able to accrete more particles to its centre than its counterpart. Why would the dynamic BH not be able to accrete as many particles, or more, than the fixed BH? Would this suggest a fixed BH has an advantage over the dynamic BH, if examining which is a better candidate for accreting a greater central density within the system?

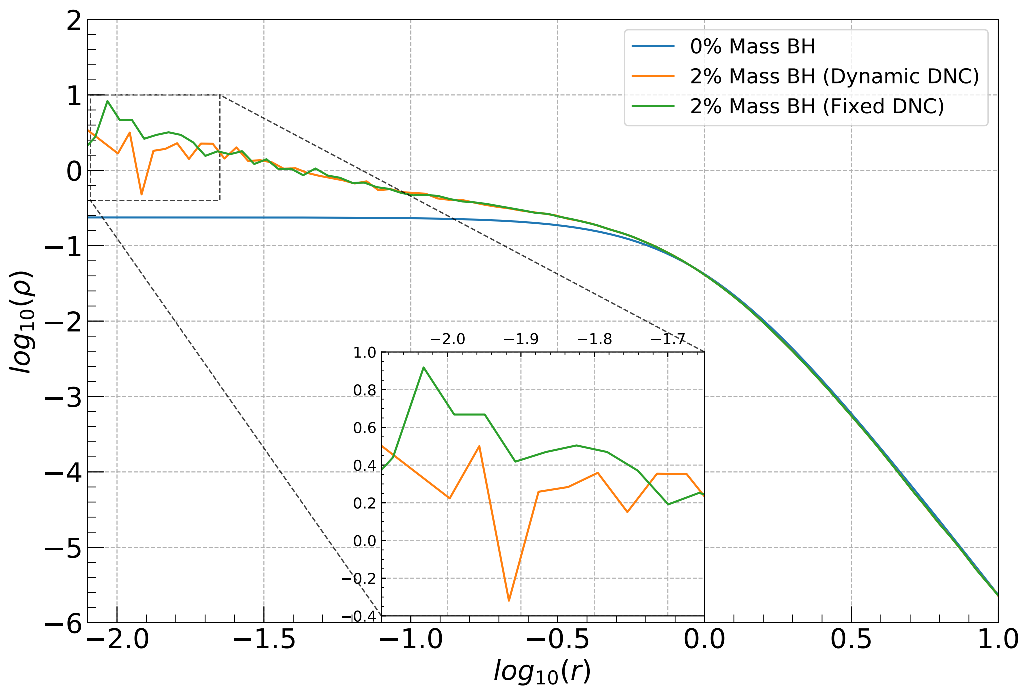

Figure 4.3: Comparison of fixed growth and dynamic growth of a 2% mass BH.

The density profile between dynamic and fixed systems is very similar for the most part, they both reach far out into the system, in terms of attracting particles. The important thing to notice is the difference in central density. In the presence of a very large BH this characteristic becomes much more noticeable, such as in Figure 4.4. The dynamic system has a significantly lower central density than the fixed system. This is likely due to the scattering of particles which fall too close to the BH. The two-body encounters between the BH and particles will slingshot them away from the centre. This is why the dynamic system has a flatter and lower central density than the fixed system. This is a major result of this project, demonstrating with an N-body simulation a behaviour of BHs in nature, that they eject particles near them carving out a section for themselves and cause a lower density in that region.

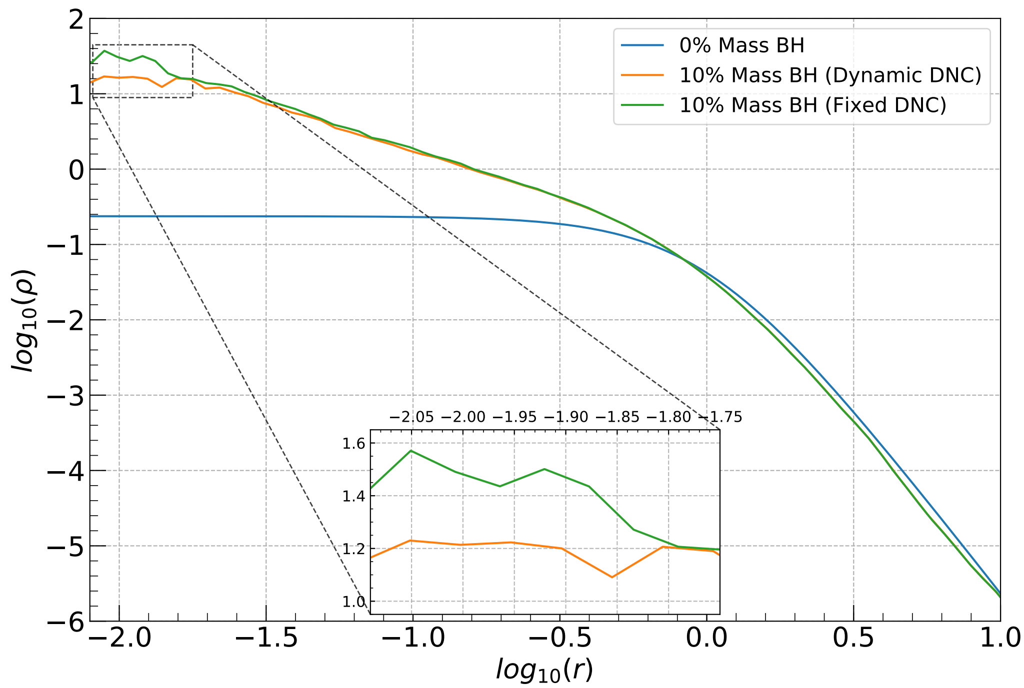

Figure 4.4: Comparison of fixed growth and dynamic growth of a 10% mass BH.

5. CONCLUSIONS

This thesis has explored the behaviours of GC evolution although not directly with observational evidence but rather with N-body simulations. This was done under the assumption the BH originates near the centre of the system and grows linearly with time. Three methods of growing BHs have been compared, adiabatic growth (slow growth), fixed growth (BH stays at the origin) and dynamic growth (BH is free to move). The adiabatic growth was carried out by (MacMillan, 2001) and he was able to provide me with output data to compare my methods with. After obtaining my own data from DNC various properties of the system were explored.

I began with the optimizing of time step and softening length parameters in order to achieve the lowest change in energy within the system. Energy profiles were plotted to confirm which parameters led to the highest amount of energy conservation. This effort may have not made such a significant impact on the system as the differences between the parameters were only slightly varying, however I felt it was necessary to carry this out. This was done at N particles of 10,000 initially as simulations run with this amount of particles were fast and easy to analyze the effects of the parameters. Once good parameters were found N particles were increased to 100,000 and again more runs were carried out to find the next good set of parameters. Afterwards N particles were increased to 1,000,000 to have a good resolution of the system. With the output data from these energy conserved runs I could now analyze the data.

To begin the analysis I plotted the BH position from the centre of the system against time to see exactly how far out the BH would go and if its mass would affect this. By comparing a low mass BH to a high mass BH I found that the greater the mass the less distance it travelled from the centre, not only that but it would also tend to dampen out quicker. In Figures 3.6 and 3.7 you can see the difference in distance from the centre of the system, the tracked position is placed on top of a density distribution of the system. To show that the BH would relax faster at a higher mass I calculated the change in position as a function of time and plotted the data in Figure 3.4. It showed that the higher mass BH relative to the lower mass BH dampens out faster.

I then looked at the density profiles of the system and it was found that larger BH final masses corresponded to a larger central density but also it took longer for the density to taper off as the larger the BH the greater its reach, affecting particles farther out into the system. This was true in all different methods of growing the BH. After plotting the density profiles of each different method of growth I wanted to compare them to see any key differences in the methods. Adiabatic growth was first compared to the the fixed and dynamic growth methods and it was found that the fixed growth central density was significantly higher than the adiabatic method. The dynamic method had on average a higher central density compared to the adiabatic method as seen in Figures 4.1 and 4.2. Why do my two methods have a higher central density compared to the adiabatic method?

One possible answer to this question is by analyzing the comparison of density profiles between the fixed and dynamic growth methods. By observing Figure 4.3 we can see that the fixed central density is greater than the dynamic central density however this feature is greatly emphasized in Figure 4.4 due to the BH being an order of magnitude larger. The reason for this increase in fixed central density, or decrease in dynamic central density, depending on the perspective at hand, is that in the dynamic growth method's case the BH is free to move. This means that as the BH encounters other nearby particles in two-body interactions it effectively throws them away from the centre. This creates a “carving out” effect as the BH carves out a section for itself near the centre where less particles will be located. This was an important result to find as this is a natural feature of BHs.

5.1 Future Work

It should be kept in mind that these methods of growth, in particular the adiabatic growth method, apply only to spherical systems. Replacing spherical symmetry for another form of symmetry in order to simulate non-spherical systems may be possible but this has yet to be done and could be a potential future project. Other interesting areas to explore could be multiple BHs, how would having two or more BHs affect the evolution of the GC? Depending on where they originate and how quickly they grow in mass could lead to seeing some very interesting features, this is another potential project. For my project I would have liked to explore more in depth how the BH travels, if its just random chaotic motion or if there is a method to predict where the BH walks based on what its final mass will be and where it originates within the system. A final potential project idea is to build an entirely new algorithm as the one used in this project is collisionless hence why a softening length was needed. This is a huge project and would take a lot of dedication and work but it would be interesting to see the comparison of collisional with collisionless BH dynamics. It could potentially better our understanding of the dynamics of BH motion and why they behave particular ways in their motion.

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