MAT1856/APM466 Assignment 1

Andrew Jhin, Student #: 1005774725

January, 2025

Fundamental Questions - 25 points

1.
   (a) Governments issue bonds instead of printing more money to finance deficits in a way that controls inflation, maintains monetary stability, and preserves investor confidence in the currency.

   (b) If investors expect a future recession, they anticipate lower interest rates, increasing demand for long-term bonds and reducing their yields.

   (c) Quantitative easing is when central banks buy financial assets to inject liquidity, which the Fed did during COVID-19 by purchasing government and mortgage-backed securities.

2. For this question, I picked 10 bonds for the 0-5 year period, aiming to have consistent 6-month gaps between them. 10 bonds were chosen to get an accurate yield curve, as having more data points increases accuracy and reduces interpolation errors. The 10 bonds chosen were:

Bond Name & ID				
CAN 1.25 Mar 25	CAN 0.5 Sep 25	CAN 0.25 Mar 26	CAN 1 Sep 26	CAN 1.25 Mar 27
CAN 2.75 Sep 27	CAN 3.5 Mar 28	CAN 3.25 Sep 28	CAN 4 Mar 29	CAN 3.5 Sep 29

Table 1: Bond Listings

The justification for these bonds being chosen is as follows:

- **Spacing of Time Between Bonds** - 6 months was the desired time between each bond maturity date to allow for consistent plotting
- **Relatively Close Coupon Prices** - For consistence sake, the aim was to have relative coupon rates since there were some outliers that had 8 percent coupon rates that would be too far from our shorter term bonds

Note: The CAN 1 Jun 27 bond was used since there was no available data within a reasonable range that was closer to Mar 27

3. The covariance matrix of a collection of stochastic processes describes how the variances of the underlying processes correlate with each other. The eigenvectors of this matrix identify the main directionality of variation, so the larger an eigenvalue, the more dominant is its direction of variations, and the eigenvalues indicate the magnitude of correlation with those directions compared to a uniform distribution.

Empirical Questions - 75 points

4.

(a) There was a consistent 6-month spread between the bonds we chose, and since each of them are semi-annual, they all lined up on the coupon payment dates. So, plotting the yield to maturity:

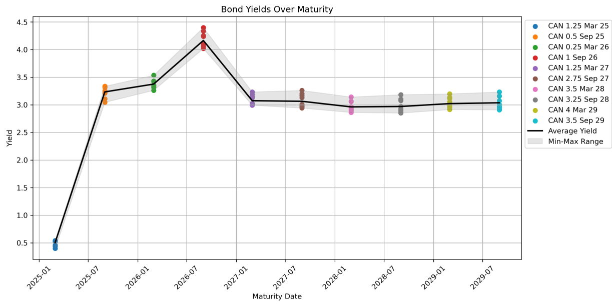

Figure 1: Yield to Maturity Plot

Python was used to first calculate the dirty price, assuming 360 days in a year and 30 days per month, with days being (6, 7, 8, 9, 10, 13, 14, 15, 16, 17):

$$DP_i = CP_i + AI = CP_i + \frac{30 \times 4 + (today_t - 1)}{360} \tag{1}$$

where DP_i is the dirty price, CP_i is the clean price, and $today_t$ represents the current day. Then, the `irr()` function in Python from the finance package in NumPy was used to calculate the yields for each bond on the respective days.

(b) For the spot rates the algorithm used to calculate the spot rates followed these steps:

i. Load the dataframe containing all the dirty prices.

ii. For bonds where maturity is less than 0.5 years, apply:

$$s_i = \frac{-1}{maturity} \times \ln\left(\frac{DP_i}{100 + \frac{coupon}{2}}\right) \tag{2}$$

iii. Use a loop to find the spot rate using the bootstrapping method, applying the equation:

$$DP_i = \sum_{t=1}^{i-1}\left[\frac{\frac{coupon_i}{2}}{(1+s_t)^t}\right] + \frac{\frac{coupon_i}{2} + 100}{(1+s_i)^i} \tag{3}$$

iv. Using the dataframe of spot rates, plot the line graph:

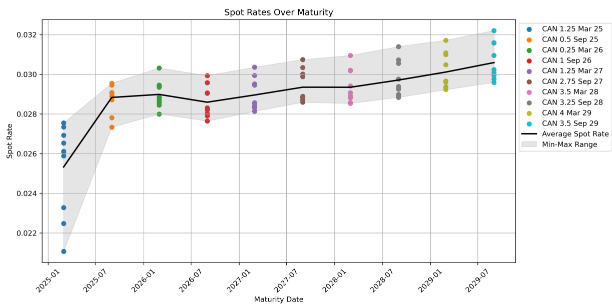

Figure 2: Spot Rate Curve

(c) For the forward rates:

   i. Load the dataframe containing all the spot rates.

   ii. There are 4 forward rates that need to be solved for so keeping the first row of spot rates fixed, iterate over the remaining 4 rows of spot rates and apply:

$$F_{t,t+n} = \left[\frac{(1+S_{t+n})^{2(t+n)}}{(1+S_t)^{2t}}\right]^{\frac{1}{2n}} - 1 \tag{4}$$

   iii. Using the dataframe of spot rates, plot the line graph:

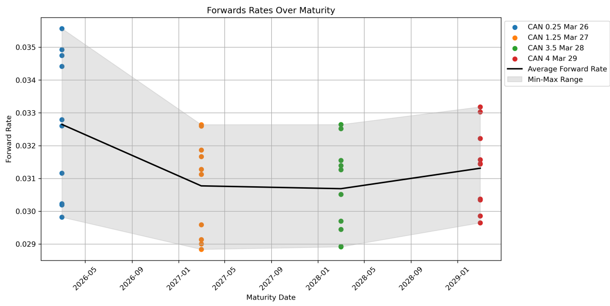

Figure 3: Spot Rate Curve

5. The covariance of the matrix for the time series of daily log-returns of yield is:

$$\begin{bmatrix} 0.004077437 & 0.000673665 & 0.000712056 & 0.000851765 & 0.000776518 \\ 0.000673665 & 0.000558963 & 0.000527314 & 0.000486263 & 0.000519789 \\ 0.000712056 & 0.000527314 & 0.000622591 & 0.000549617 & 0.000581479 \\ 0.000851765 & 0.000486263 & 0.000549617 & 0.000574818 & 0.000571132 \\ 0.000776518 & 0.000519789 & 0.000581479 & 0.000571132 & 0.000604266 \end{bmatrix}$$

The covariance of the matrix for the time series of daily log-returns of forward rates is:

$$\begin{bmatrix} 0.001484627 & 0.001031959 & 0.000722696 & 0.000764893 \\ 0.001031959 & 0.000974243 & 0.000688291 & 0.000721893 \\ 0.000722696 & 0.000688291 & 0.000638438 & 0.000619167 \\ 0.000764893 & 0.000721893 & 0.000619167 & 0.000656313 \end{bmatrix}$$

6. The eigenvalues and vectors for the yields are:

$$\begin{bmatrix} 4.91969647 \times 10^{-3} & 1.37436655 \times 10^{-3} & 8.71523958 \times 10^{-5} & 4.29969705 \times 10^{-5} & 1.38621241 \times 10^{-5} \end{bmatrix}$$

$$\begin{bmatrix} -0.87359284 & 0.48463028 & 0.02633438 & 0.03148941 & 0.01685132 \\ -0.22266596 & -0.42769879 & 0.83577749 & -0.24869117 & -0.08439374 \\ -0.23916443 & -0.47554337 & -0.07582792 & 0.82912255 & -0.15318229 \\ -0.25936877 & -0.39205680 & -0.46485783 & -0.45273127 & -0.59829834 \\ -0.25056665 & -0.44984132 & -0.28096228 & -0.21154508 & 0.78177236 \end{bmatrix}$$

The eigenvalues and vectors for the forward rates are:

$$\begin{bmatrix} 3.31329584 \times 10^{-3} & 3.26067454 \times 10^{-4} & 8.79951884 \times 10^{-5} & 2.62620281 \times 10^{-5} \end{bmatrix}$$

$$\begin{bmatrix} -0.62722243 & -0.7325974 & -0.26435017 & -0.00347434 \\ -0.52286806 & 0.14775153 & 0.83255782 & -0.10782373 \\ -0.40027975 & 0.49838802 & -0.42300722 & -0.64222301 \\ -0.41590521 & 0.4394086 & -0.24089698 & 0.75888839 \end{bmatrix}$$

References and GitHub Link to Code

https://github.com/AndrewJhin/APM466