A Fisher Ideal Take on Fiscal Debt

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August 18, 2023

Abstract

This paper derives a Fisher ideal quantity index of US Treasury debt.

Keywords: Fisher Ideal Index, Aggregation Theory, Fiscal Debt

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1 Introduction

The fiscal response to the COVID crisis in the US and the ensuing rise of inflation and interest rates has brought the concept of "fiscal capacity" back into the spotlight. Despite growing debt levels, borrowing costs trended downward from the 1980s until the US financial crisis in the late 2000s. All of this suggests that there must be more to the fiscal-capacity discussion than just the stock of outstanding principal values. If that additional component could be discovered, could it also shed light on the connection between debt and the macroeconomy, like the fiscal theory of the price level (FTPL)?

Krishnamurthy and Vissing-Jorgensen (2012), Vissing-Jorgensen and Krishnamurthy (2013), and Nagel (2016)—to name just a few—have established that the value of fiscal debt in the US is much more than the sum of its outstanding stock of principal. Caballero, Farhi and Gourinchas (2017) build on this idea to explain historically low borrowing costs in the face of historically high debt levels. More recently, Brunnermeier, Merkel and Sannikov (2022) applied these concepts to the fiscal capacity and the FTPL literatures. They show that the demand side of this debt market raises the borrowing limits of the fiscal authority because the additional debt provides safety or transaction services, which I'll refer to more generally as monetary services.[1] Theoretically, they show that the inclusion of these monetary services disrupt the traditional FTPL channel (see Leeper, 1991, as a seminal example) and, along with the empirical results of Jiang *et al.* (2019), suggest that this channel is important in the valuation of Treasury debt. This paper explores this monetary service channel in more depth, aiding in our understanding of debt dynamics and their impact on the economy as a whole.

The purpose and contribution of this paper is two-fold. First, I measure the monetary

[1] These monetary services generally include the aforementioned transactions services as well as other attributes such as liquidity, safety, use as collateral, etc.

services established as theoretically important in the literature. This measure provides additional insight not only into how debt and its dynamics impact the greater economy, but also how global events impact the monetary services of outstanding fiscal debt. If fiscal monetary services do provide additional fiscal capacity and influence inflation dynamics as Brunnermeier *et al.* (2022) theorize, then understanding the extent of the impact is vital to policy makers. Second, I provide both theoretical and empirical evidence of a fiscal inflation channel through the rate premiums (liquidity and/or safety) created by those monetary services. That is, while the inflation is derived from an externality of fiscal policy, it is not though the standard channels predicted by the FTPL. Showing that fiscal debt influences inflation through its monetary properties suggests that these two theories may not be as different as suspected and provides a potential bridge between the two. Together, the results within this paper provide a contribution to the monetary aggregation, fiscal capacity, and inflation literatures.

The first result of this paper is the derivation of the underlying theory for the quantity index of interest. As is noted in the extensive monetary aggregation literature (e.g. Barnett, 1978, 1980; Barnett and Serletis, 2000), a statistical index used to track a true aggregate like the one considered here must be derived from an optimizing agent. Therefore, I consider a partial equilibrium model of the representative household which incorporates the monetary services of short- and long-term fiscal debt. The resulting holding-period user costs of these securities include both the respective coupon payments as well as the expected future capital gains. I then expand the standard government budget constraint to show that the value of these fiscal monetary services (the quantity index multiplied by its price dual) wholly adds to the fiscal capacity of the government.[2] Having shown the potential importance of these fiscal monetary services, the natural

[2] This is identical to the exercise conducted by Brunnermeier *et al.* (2020), though I abstract from the bubble term to focus solely on the monetary services component.

next step is to assess how this quantity index has evolved over time.

In evaluating the index and comparing it to the ubiquitous simple sum aggregate, I then derive the growth rate of the monetary services provided by the fiscal authority. Isolating the growth of these fiscal monetary services is calculated as the growth rate of the Fisher ideal index—which incorporates both the monetary services and quantities—less that of the simple sum aggregate—which is a pure quantity measurement. Fluctuations in this growth rate, and the historical events surrounding them, suggest that I am indeed capturing what is intended. For example, I find a sharp and sustained rise in the growth of fiscal monetary services throughout the period encompassing the American Recovery and Reinvestment Act of 2009 and the European debt crisis that followed shortly thereafter. I also find a sharp contraction in these fiscal monetary services during the "dash for cash" liquidity squeeze seen in the early months of the 2020 pandemic. Having this measure of fiscal monetary services provides a new data point in evaluating the impact of fiscal deficits and debt on the economy at large.

What kind of monetary services do Treasury securities provide? Generally, they are considered to be both safe and liquid. Since the aggregation technique used here does not separate the two, I test these aspects empirically. I find that Treasury securities do reduce the price of safety in the market, but that they—at best—have no impact on liquidity and may even reduce liquidity in the market. While Treasury bills are considered to be nearly as liquid as any other asset out there, the full portfolio of Treasury debt doesn't necessarily share that attribute. And since each issuance of new Treasury debt extracts reserves and currency from the market, the net impact is likely negative. Re-evaluating the impact with a segmented portfolio confirms this. I find that the positive impact on liquidity from bills to not be statistically significant, while notes/bonds reduce liquidity in the market. Both are found to increase safety in the market, however. Thus, while we consider Treasury debt to be safe and liquid overall, it seems that the monetary services provided center around safety.[3]

Lastly, I show that an increase in fiscal monetary services has a positive, persistent, and statistically significant impact on the inflation rate. Perhaps the most important result in this paper, a one-percent increase generates an elevated inflation rate that peaks between four and five basis points and lasts for ten months. Put another way, a one-time increase in fiscal monetary services causes a permanent increase in the price level. This result is considered economically significant since, while the shock of interest is only one percentage point, the average growth rate in the sample is approximately 2.5 percent and frequently rises into the 5-10 percent range. Thus, sudden changes in the monetary services provided by the fiscal authority have the potential to dramatically influence price level dynamics. A small-scale New Keynesian model with debt in the utility function, however, can predict this inflationary impact without the need for the FTPL. Instead, it is derived from the impact of these monetary services on the rate premiums. Thus, to paraphrase: *inflation is always and everywhere a joint monetary-fiscal phenomenon.*

The remainder of this paper is organized as follows. Section 2 presents the underlying index number theory and considers some of the hurdles with applying it to the available data. Section 3 develops the theory to motivate a proper statistical index number. Section 4 derives the Fisher ideal index that tracks the true aggregate of marketable fiscal debt. Section 5 presents the Fisher ideal index and compares it to the ubiquitous simple sum aggregate. Section **??** constructs a small-scale New Keynesian model to explore the interaction between monetary and fiscal policy when both short- and long-term government debt provide monetary services. Section **??** then empirically tests the predictions of the theoretical model and the ability of this metric to forecast inflationary dynamics. Section **??** concludes.

[3] The potential capture of Treasuries providing monetary services in their use as collateral is also explored in Appendix **??**, though those results are restricted by sample size and less conclusive.

2 Motivation

Measuring the monetary services of fiscal debt specifically is a novel concept, but the idea of considering the monetary services of financial assets in general is not new. In this section I briefly touch on the monetary aggregation literature, motivate the need for a new measure of fiscal debt, and consider the hurdles in applying the theory to this data specificially.

2.1 Bonds Market Segmentation

Fiscal debt is ubiquitously measured as the sum of outstanding principal values. Simple-sum aggregates come with an assumption that the underlying assets are perfect substitutes. Signs of *imperfect* substitutability would include a yield-to-maturity (YTM) spread between assets that are identical in everything but name. Figure 1 shows the median yields to maturity of Treasury notes and Treasury bonds with roughly the same time to maturity for the 1970–2020 period. That is, the time series compare the YTM of a Treasury note that matures within t periods with a Treasury bond that also matures with t periods. Technically speaking, the only difference between these securities on the secondary market is the label attached to their initial maturity. So we should expect that a note and bond would be perfect substitutes and thus carry the same YTM.

Amihud and Mendelson (1991) find that the YTM spread between notes and bills is driven by relative liquidity. Using monthly data on Treasury securities identified at the CUSIP level, I extend their work to empirically test this claim across the three common types of nominal securities. The data are first separated by their type label: bills, notes, and bonds. Then, for every month in the Oct 1996–Dec 2020 subsample period, each "senior" security (bond/note) is matched to a "junior" security (note/bill) that matures

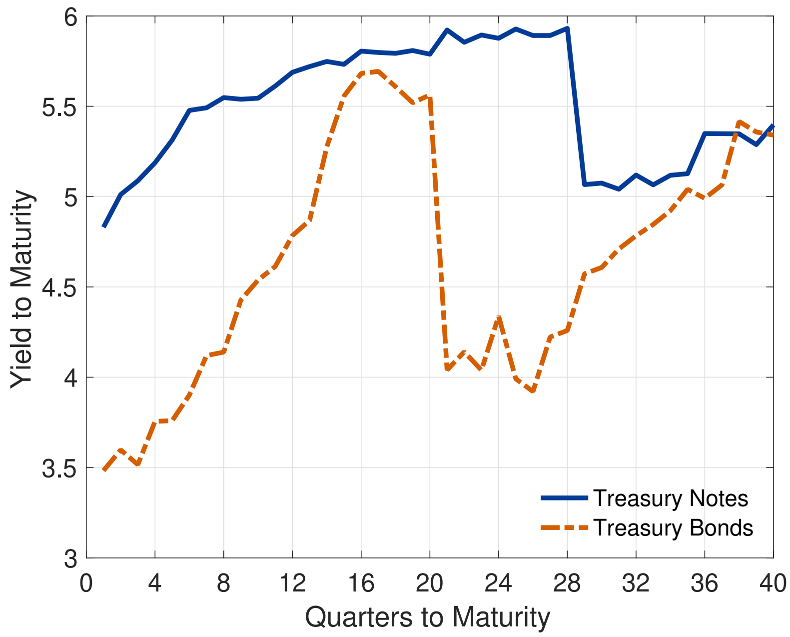

Figure 1: Median Treasury Bond and Note Yields with Same Time to Maturity (1970-2020)

within one day of the respective note, allowing for exact matches.[4] Securities without a match are removed from the analysis. The securities' YTM spreads are then regressed against the spreads in their relative bid-ask ratios, which are calculated as

$$\text{relative bid-ask ratio} = \frac{\text{ask price} \ - \ \text{bid price}}{\text{ask price} \ + \ \text{accrued interset}} \times 100.$$

An increase in the bid–ask spread implies a relative decrease in the liquidity of the "senior" security. I also control for the spread in their respective coupon rates, a general time to maturity (months/years) for any term effects, and the expected state of the

[4] The October 1996 cutoff for this analysis comes from the source data in the CRSP database. Prior to October 16, 1996, CRSP data on bid and ask prices were sourced from the FRBNY; while sourced from GovPX and its aquiring institutions since then. According to the CRSP documentation: "The FRBNY described its listed bid price as '...the most widely quoted price from the range of quotations received'. The ask price was determined by the FRBNY based on what they expect a typical bid-ask spread to be. The rule used to make this derivation was not public domain. GovPX described its listed bid and ask prices as the 'best price'. To determine their 'best price' they observe the prices from the five inter-dealer brokers and report the bid and ask prices that produce the smallest bid-ask spread." Use of latter period data ensures that these irregularities in the data do not interfere with the analysis.

economy at the time through the 10yr–2yr yield curve spread.[5] Table 1 presents the results for the notes–bills, bonds–notes, and bonds–bills analyses.

Matching the findings of Amihud and Mendelson (1991), the differences in note and bill YTM can be explained by their relative liquidities. That is, bills benefit from a liquidity premium and cannot be considered as perfect substitutes for notes, and vice versa.

The results of the bonds–notes and bonds–bills analyses paint bonds primarily as a savings vehicle in financial markets. Neither the bond's coupon rate nor the expected state of the economy impact its YTM spread versus bills, suggesting that bonds are largely purchased to hold. Additionally, a decrease in the liquidity of bonds relative to bills causes the YTM spread to decrease, opposite of the notes result. A plausible explanation for this is that the demand for bonds reduces the YTM relative to bills, but that these bonds are purchased for savings purposes and are more likely held to maturity, reducing the liquidity of the market overall. The positive, yet not statistically significant bonds-notes liquidity result also suggests that bonds occupy a part of the market that is less concerned with the liquidity of the investment and more about the safety and return.

Overall, these results confirm the generally accepted theory that bills are primarily used as a liquidity hedge while bonds are primarily a savings vehicle. Notes seem to fall somewhere in the middle, being more sensitive to market liquidity relative to bills, and more sensative to differences in return relative to bonds. These results also confirm that, when aggregating, none of these assets should be treated as perfect substitutes for the others.

[5] Note that the coupon rate for bills is zero. The 10yr–2yr spread is used because it is a spread between two notes, attempting to get at the general expectations of the future economy without crossing the asset types in the process. Standard errors are heteroskedasitcally robust (HC1).

Table 1: Relative Liquidity and Yield to Maturity[a]

	Notes–Bills[b]	Bonds–Notes	Bonds–Bills[b]
Relative Bid-Ask Spread	0.4163**	0.0981	−0.1424**
	(0.173)	(0.074)	(0.074)
Coupon Rate Spread	0.0210***	0.0170***	0.0076
	(0.001)	(0.005)	(0.013)
Months to Maturity	−0.0152***		−0.1028***
	(0.002)		(0.018)
Years to Maturity		−0.0049***	
		(0.001)	
10y-2y Spread	0.0234***	−0.0124***	−0.0484
	(0.003)	(0.002)	(0.033)
Constant	0.0076	−0.0554**	−0.3244**
	(0.010)	(0.030)	(0.154)
Observations	2250	7430	78
R-Squared	0.207	0.505	0.399
F-statistic	99.48	207.2	20.19

[a] The dependent variable is the yield-to-maturity spread between securities matched by thier respective days to maturity. Standard errors are heteroskedasticity robust (HC1). Designations $***$, $**$, and $*$ represent results that are statistically significant at the one, five, and ten percent levels, respecitivly.
[b] Analyses considering Treasury bills are limited to those securities with six months to maturity or less. This ensures that both securities have no other payments remaining until the maturity date.

2.2 Finding the Proper Quantity Index

Given that these Treasury securities are not perfect substitutes, the next step is selecting the best quantity index for proper aggregation. Christensen *et al.* (1971) show that the homogenous tanslog function

$$\ln f(x_t) = \alpha_0 + \sum_{n=1}^{N} \alpha_n \ln x_t^n + \frac{1}{2} \sum_{j,k=1}^{N} \gamma_{j,k} \ln x_t^j \ln x_t^k,$$

can provide a second-order approximation to an unknown, arbitrary twice-continuously-differentiable linear homogenous aggregator. Here, $\sum_{n=1}^{N} \alpha_n = 1$, $\gamma_{j,k} = \gamma_{k,j}$, and $\sum_{k=1}^{N} \gamma_{j,k} = 0$ for $j \in [1, N]$. Boisvert (1982) explains that the translog function is is more flexible than the Cobb-Douglas or constant elasticity of substitution (CES) functional forms by allowing the partial elastiticies of substituion between the assets to vary. He also goes on to explain that this functional form can be viewed as a) an exact aggregator function, b) a second-order Taylor approximation to a general, but unknown aggregator function, or c) a second-order approximation to a CES function. These properties make the translog function the perfect starting point in deriving the best quantity index.

Diewert (1976, Section 2) shows that the Törnqvist-Theil Divisia quantity index

$$\ln M_t^d = \ln M_{t-1}^d + \sum_{n=1}^{N} \frac{1}{2}(s_{n,t} + s_{n,t-1}) \left(\ln x_t^n - \ln x_{t-1}^n\right),$$

can be derived directly from a translog aggregator function. Here, M_t^d is the level of the Divisia monetary aggregate, x_t^n is the nominal value of asset n, and

$$s_{n,t} = \frac{\eta_t^n x_t^n}{\boldsymbol{\eta}_t \mathbf{x}'_t}$$

corresponds to the weight on the marginal change in financial asset n. Here, $\mathbf{x}_t$ and $\boldsymbol{\eta}_t$ are $N \times 1$ vectors of nominal quantities and user costs, respectively. This type of quantity index is ubiquitous in the monetary aggregation literature.

One drawback of the Törnqvist-Theil Divisia index is that it does not handle assets coming into and out of the market ($x_t^n = 0$ for some n, t) very well. Given the frequent changes in Treasury policy and the potential categorizations I will use, zero values are likely to be commonplace in this analysis. When calculating general monetary aggregates, this problem does not arise often and is managed by imputing a reservation price and switching to a Fisher ideal index

$$M_t^f = M_{t-1}^f \left[\frac{(\boldsymbol{\eta}_t \mathbf{x'}_t)(\boldsymbol{\eta}_{t-1} \mathbf{x'}_t)}{(\boldsymbol{\eta}_t \mathbf{x'}_{t-1})(\boldsymbol{\eta}_{t-1} \mathbf{x'}_{t-1})} \right]^{\frac{1}{2}} \tag{1}$$

for those time periods. Diewert (1976) shows that this quantity index is also superlative, with Diewert (1978) and Dumagan (2002) showing that the two indexes will appriximate each other numerically and mathematically, respectively. Additionally, Diewert (1976) shows that the ideal index is derived from a quadratic mean of order 2 quantity index, which corresponds to the assumed translog aggregator function and is even recommended as the preferred index to use (Diewert, 1976, Section 5). Therefore, a Fisher ideal index is used in the remaining analysis.

3 The User Cost of Treasury Securities

As Barnett (1980) outlines, aggregation theory relies on known, exact functional forms with estimable parameters. The functions of interest are typically utility and production functions, which are often impossible to know. This is why I rely on statistical index numbers, whose theory only relies on the existence of maximizing behavior. From this optimizing behavior we can derive the user costs of the underlying components and bypass the unknown parameters because the resulting index numbers are not dependent on any specialized properties of the aggregator function. That is, while I'm not deriving the true aggregate itself, the resulting quantity index will track the true aggregate. A complete guide to index number theory and its application to monetary aggregation can be found in Barnett and Serletis (2000).

A proper quantity index requires a price that is derived from an optimizing agent.

Thus, motivating the measurement of marketable government debt requires a partial equilibrium model that derives both the period-by-period user cost of holding government debt as well as the budgetary constraints the fiscal authority encounters. This model incorporates both short- and long-term debt issued by the government, as well as an alternative long-term asset that acts as a pure savings vehicle and can be used as the benchmark asset. In this model, I refer to the alternative asset as "capital," though it could also be motivated in some other fashion.

3.1 Long-Term Asset Dynamics

The long-term government bonds and capital both evolve in a similar fashion. As described by Krause and Moyen (2016), each period new nominal long-term government bonds $B_t^{L,n}$ are issued, which are added to the stock of outstanding long-term debt B_t^L. A portion $\alpha \in (0, 1)$ of the previous period's stock of long-term bonds mature, while the remaining $(1 - \alpha)$ remain in the stock of outstanding long-term debt. The maturity of these bonds is therefore $1/\alpha$. Together, the stock of outstanding long-term debt evolves such that

$$B_t^L = (1 - \alpha)B_{t-1}^L + B_t^{L,n}. \tag{2}$$

The average nominal interest rate paid on the stock of outstanding long-term debt r_t^L evolves such that

$$r_t^L B_t^L = (1 - \alpha)r_{t-1}^L B_{t-1}^L + r_t^{L,n} B_t^{L,n}, \tag{3}$$

where $r_t^{L,n}$ is the interest rate on newly-issued long-term bonds.

Capital is structured in a similar fashion, with the total stock of outstanding capital evolving according to

$$K_t = (1 - \delta)K_{t-1} + I_t, \tag{4}$$

where I_t is new investment in capital and $\delta \in (0,1)$ designates the portion of the outstanding stock of capital that matures each period. This implies that the maturity of capital is $1/\delta$. The average interest rate paid on the capital stock outstanding R_t is derived by

$$R_t K_t = (1-\delta)R_{t-1}K_{t-1} + R_t^n I_t, \tag{5}$$

where R_t^n is the interest rate on newly-issued capital.

3.2 Representative Household

The representative household in this model works l_t hours each period for nominal wage W_t. It also earns income through it's previous investments in capital $(\delta + R_{t-1})K_{t-1}$, short-term (one-period) bonds $(1+r_t)B_{t-1}$, long-term bonds $(\alpha + r_{t-1}^L)B_{t-1}^L$, and profits from a continuum of intermediate goods-producing firms $\int_0^1 \Pi_t(s)\mathrm{d}s$.[6] This income is spread between a real consumption good c_t at price p_t as well as new investments in nominal short-term bonds B_t, long-term bonds $B_t^{L,n}$, capital I_t, and a nominal lump sum tax $P_t\tau_t$. Combined, the household's period-by-period budget constraint can be expressed as

$$\begin{aligned} B_t + B_t^{L,n} + p_t c_t + I_t = (\delta + R_{t-1})K_{t-1} + (1+r_{t-1})B_{t-1} + \\ (\alpha + r_{t-1}^L)B_{t-1}^L + W_t l_t + \int_0^1 \Pi_t(s)\mathrm{d}s - P_t\tau_t. \end{aligned} \tag{6}$$

The household objective is to maximize utility over the real consumption good, the

[6] The δ considered here can deviate from that in (4) if depreciation is considered. In this present study, however, we assume zero deprecation and treat "capital" as an alternative long-term security similar to the long-term government bond. Additionally, the inclusion/exclusion of the corporate profit term does not alter the results of this partial equilibrium setup, but would be needed in a full, general equilibrium presentation.

real monetary services provided by its portfolio of government bonds, and leisure

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v\left(m_t\right) + x(1-l_t) \right\}, \tag{7}$$

where $u(\cdot)$, $v(\cdot)$, and $x(\cdot)$ are increasing, concave functions.[7] Real monetary services are approximated using a second-order translog function

$$\ln m_t = \theta_0 + \theta_s \ln b_t + \theta_L \ln b_t^L + \frac{1}{2}\omega_{s,s} \left(\ln b_t\right)^2 + \frac{1}{2}\omega_{L,L} \left(\ln b_t^L\right)^2 + \omega_{s,L} \ln b_t \ln b_t^L. \tag{8}$$

Solving the household's problem results in the following dynamic conditions:

$$1 + \frac{\gamma_{2,t}}{b_t} \left(\theta_s + \omega_{s,s} \ln b_t + \omega_{s,L} \ln b_t^L\right) = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1+r_t}{\pi_{t+1}} \right], \tag{9}$$

$$\begin{aligned} 1 + \frac{\gamma_{2,t}}{b_t^L} \left(\theta_L + \omega_{L,L} \ln b_t^L + \omega_{s,L} \ln b_t\right) + \gamma_{4,t} \left(r_t^L - r_t^{L,n}\right) \\ = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \left\{ 1 + r_t^L + (1-\alpha)\gamma_{4,t+1} \left(r_t^L - r_{t+1}^{L,n}\right) \right\} \right], \end{aligned} \tag{10}$$

and

$$1 + \gamma_{3,t} \left(R_t - R_t^n\right) = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t + (1-\delta)\gamma_{3,t+1} \left(R_t - R_{t+1}^n\right) \right\} \right], \tag{11}$$

The solution technique to the household's problem can be found in Appendix A. Here, $\mu_{1,t} = u'(c_t)$, $\gamma_{2,t} = v'(\cdot)/u'(\cdot)$, and $\gamma_{3,t}$ and $\gamma_{4,t}$ are the relative prices of the long-term assets B_t^L and K_t, respectively. These prices evolve according to

$$\gamma_{3,t} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \left\{ 1 + (1-\delta)\gamma_{3,t+1} \right\} \right], \tag{12}$$

[7] The assumption of additive separability in c_t and m_t is stronger than the needed weakly separable assumption for this analysis. This is for better tractability and does not change the result in the end.

and

$$\gamma_{4,t} = \beta \mathbb{E}_t \left[\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}} \left\{ 1 + (1-\alpha)\gamma_{4,t+1} \right\} \right]. \tag{13}$$

The user costs of the short- and long-term bonds are

$$\eta_t = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]}, \tag{14}$$

and

$$\eta_t^L = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t^{L,n} + (1-\alpha)\gamma_{4,t+1} \Delta r_{t+1}^{L,n} \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]}, \tag{15}$$

respectively. The derivations of these user costs can be found in Appendix B.

4 Data and Methodology

In this section, I construct the Fisher ideal quantity index described in Section 3 from data compiled by the Center for Research in Securities Prices on marketable Treasury securities. Non-marketable debt is typically held by intergovernmental agencies, and is therefore not typically considered to be a significant burden.[8] The Treasury began its "regular and predictable" debt issuance campaign in the 1970s and data regarding the spot and forward interest rates goes back to January 1977. Therefore the variables constructed here will cover the February 1977 to December 2020 period.

The first issue that needs to be addressed is essentially analogous to new-product

[8] The inclusion of only publicly-held, marketable US Treasury debt would be the optimal choice here, but data restrictions keep me from assessing this. For instance, the CRSP data doesn't include the amount of each T-bill issuance held publicly. Another option could be to exclude the issuances held by the Federal Reserve, since that is the primary non-public holder of US Treasuries, but the SOMA data only goes back to 2003. Future research could apply this methodology to the shortened timeframe.

bias. As (1) shows, the calculation of the growth rates requires both current and lagged quantities and user costs. If I were to treat each issuance as a separate $m_{i,t}$, there would be distortions at issuance and maturity months. For example, in the issuance month of $m_{i,t}$, the preceding month's yield $r_{i,t-1}$ of that particular issuance doesn't exist, which means its user cost $\eta_{i,t-1}$ also doesn't exist. Therefore, an assumption would need to be made regarding this lagged user cost. Feenstra (1994) and others have outlined a theoretically justified solution to this problem in which the price is set at its reservation level, where the quantity demanded would equal zero in the preceding month.[9] It's also well documented that on-the-run bonds sell at a premium over their off-the-run counterparts, suggesting $\eta_{i,t-1} > \eta_{i,t}$, but by how much? Estimating the reservation price of each issuance in each month over the sample period would seem to be too technically burdensome. With how distortionary this bias can be, some adjustments and/or assumptions are needed.

To reduce the magnitude of this issue, I cluster the assets into groups that are most likely to be perfect substitutes in any given month. While there may be discrepancies in on-the-run/off-the-run securities, they should average out within the clusters. Based on the work of Amihud and Mendelson (1991) as well as the descriptive statistics in Figures 1 and **??**, the securities need to be separated across their designation of *bills*, *notes*, and *bonds*. Since 1997, the Treasury has issued Treasury Inflation-Protected Securities (TIPS), which yield a real rate of interest instead of the traditional nominal yield. These have been shown to be less liquid than their nominal counterparts, so I create two additional categories of *TIPS notes* and *TIPS bonds*. Lastly, as these securities mature, their yields tend to fall with the trend of the constant maturity yield curve, reflecting the changing monetary services they provide. Therefore, each of these five categories

[9] The Center for Financial Stability, which publishes data on Divisia monetary aggregates, delays the inclusion of new monetary assets for a few months. In this situation, however, the regular issuance and maturity of short-term debt make this strategy problematic.

needs to be further segmented by their time to maturity. Since bonds and longer notes are issued on a quarterly basis, it's common to have individual months where there are no securities of a certain type.[10] A fine-grained segmentation such as this would again be subject to a large amount of new-product bias, so I define each sub-category as the quarters-to-maturity. That is, for securities that will mature over the next one-to-three months, they are categorized has maturing within one quarter. A thirty-year threshold on bonds suggests 120 quarters-worth of categories, but there were a series of longer-term bonds that were issued between 1953 and 1965, which could impact the first years of the analysis. Therefore, the number of quarters-to-maturity categories is set at 160, or forty years. Overall, when accounting for both the types of securities and the quarters to maturity, there are 800 categories.[11]

The next step in this process is to identify the proper interest rates from the theory above. Since the user costs represent the holding period return and incorporate the interest rates paid on newly-issued securities (r_t and $r_t^{L,n}$) and not on the average interest rate paid out on outstanding debt (r_t^L), the most accurate interest rates to consider are the coupon rates. Additionally, the term $(1-\alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n}$ represents the expected capital gains of the long-term bond. To account for these expectations, I use the current price of the bonds adjusted for its maturity, multiplied by the difference between the one-month-ahead forward rate and spot rate for each bond's particular month and maturity. For TIPS, I consider the real spot and forward yield curves in the calculation of expected capital gains. While forward rates incorporate more than just future expectations of current spot rates, the impact of any term premium looking only one month ahead

[10] These notes and bonds are typically subject to reopenings in the months following the initial issuance, but the maturity dates are unchanged. For example, if a 30-year bond is issued in February, a reopening of that issuance may be offered in March, but the time-to-maturity on that reopening is 29 years, 11 months.

[11] Note that, based on (1), baskets that are consistently empty do not impact the measurement, allowing me to keep it consistent across the types of securities. So while there are 800 total baskets, a large number of them will not contribute to the measurement, but it does provide the flexibility needed for the aggregate over the almost fifty-year analysis.

should be negligible.

The rate considered for each basket of securities is the quantity-weighted average over the issuances therein, though a further assumption is needed to address missing values/new-product bias. As discussed above, even the broader categories used here do not ensure that there are no issues with new-product bias. Since most of the empty categories in this situation are simply a cluster of maturing debt through time, and not new issuances of debt, I use the linear interpolation of the rates across the maturities instead of calculating a reservation rate. This ensures that I'm capturing the true shape of the yield curves used.

The benchmark rate used here is the maximum holding-period return of the baskets of securities considered each month, plus twenty-five basis points.[12] This is similar to the strategy used by the Federal Reserve in their calculation of Divisia monetary aggregates and is common in other derivations of user costs.

5 Results

5.1 The Measure

The month-over-month growth rates of both the true aggregate and its simple sum counterpart are constructed from the data. This indirect approach to the aggregates' levels is necessary as there is no initial value from which to begin the calculation of (1). Simply taking the logarithm of (1) yields the growth-rate variation used. The general levels of the growth rates are similar, so I present their spread in Figure **??**. To aid

[12] Previous iterations of this measurement attempted to use the Baa Corporate Bond yield, which would have accounted for both the liquidity and safety attributes of Treasury securities. However, even the addition of 200 basis points did not ensure that all user costs were positive across the sample period, especially in the volatile years of 1979–1981 and long periods of time between 2014 and 2016. Adjustments to create that kind of spread for the early years also had the drawback of washing out the differences in user costs in the later parts of the sample period.

in the visual analysis, the 12-month moving average (centered on the seventh month) spread of these growth rates is superimposed. As can be seen, the true aggregate tends to grow faster than the simple sum aggregate, but there are also periods of relatively slow growth.

Since the true aggregate incorporates both the quantities of the underlying assets and the monetary services they provide, the spread intuitively reveals the growth of these monetary services alone. That is, I am essentially factoring-out the growth that comes purely from the issued principal values. The spread in Figure **??**, therefore, represents the growth of fiscally-provided monetary services. This figure shows us that the Treasury generally adds to the stock of monetary services over time, with notable exceptions in the late-1990s when the government was running a fiscal surplus. In that instance, the growth rates suggest that the monetary services provided were shrinking along with the principal values.

Using these growth rate spreads, I derive an index that tracks the stock of monetary services, shown in Figure **??**. Multiple theories have claimed that Treasury securities provide services above and beyond their principal value, but this is the first attempt to measure such services. The base month considered here is August 1989 due to the relative flatness of the yield curve at that time. When the yield curve is flat, the differences in the user costs are minimal, suggesting that the underlying assets are perceived as near-perfect substitutes and collapsing the Fisher ideal index into a simple sum aggregate.

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A Representative Household Solution

Combining (2) and (3)

$$\left(r_t^L - r_t^{L,n}\right) B_t^L = (1-\alpha)\left(r_{t-1}^L - r_t^{L,n}\right) B_{t-1}^L, \tag{A.1}$$

(4) and (5)

$$\left(R_t - R_t^n\right) K_t = (1-\delta)\left(R_{t-1} - R_t^n\right) K_{t-1}, \tag{A.2}$$

and (6), (2), and (4)

$$\begin{aligned} B_t + B_t^L - (1-\alpha)B_{t-1}^L + p_t c_t + K_t - (1-\delta)K_{t-1} &= (\delta + R_{t-1})K_{t-1} \\ &+ (1+r_{t-1})B_{t-1} + (\alpha + r_{t-1}^L)B_{t-1}^L + W_t l_t + \int_0^1 \Pi_t(s)\mathrm{d}s - P_t\tau_t. \end{aligned} \tag{A.3}$$

A.1 Household's Problem

(7) subject to (8), (A.1), (A.2), and (A.3). Choosing $\left\{B_t, B_t^L, c_t, K_t, R_t, r_t^L, l_t, M_t\right\}$

A.2 Bellman Equation

$$
\begin{aligned}
&\mathbb{V}\big(B_{t-1}, B^L_{t-1}, K_{t-1}, R_{t-1}, r^L_{t-1}\big) \\
&\quad = \max\Bigg\{u(c_t) + v\left(\frac{M_t}{P_t}\right) + x(l_t) + \beta\mathbb{E}_t\big[\mathbb{V}\big(B_t, B^L_t, K_t, R_t, r^L_t\big)\big] \\
&\quad + \frac{\mu_{1,t}}{p_t}\Big[(\delta + R_{t-1})K_{t-1} + (1 + r_{t-1})B_{t-1} + (\alpha + r^L_{t-1})B^L_{t-1} + W_t l_t \\
&\quad + \int_0^1 \Pi_t(s)\mathrm{d}s - P_t\tau_t - B_t - B^L_t + (1-\alpha)B^L_{t-1} - p_t c_t - K_t + (1-\delta)K_{t-1}\Big] \\
&\quad + \frac{\mu_{2,t}}{p_t}\left[\theta_0 + \theta_s \ln b_t + \theta_L \ln b^L_t + \frac{1}{2}\omega_{s,s}\left(\ln b_t\right)^2 + \frac{1}{2}\omega_{L,L}\left(\ln b^L_t\right)^2 + \omega_{s,L}\ln b_t \ln b^L_t - \ln m_t\right] \\
&\quad + \frac{\mu_{3,t}}{p_t}\Big[(1-\delta)\left(R_{t-1} - R^n_t\right)K_{t-1} - \left(R_t - R^n_t\right)K_t\Big] \\
&\quad + \frac{\mu_{4,t}}{p_t}\Big[(1-\alpha)\left(r^L_{t-1} - r^{L,n}_t\right)B^L_{t-1} - \left(r^L_t - r^{L,n}_t\right)B^L_t\Big]\Bigg\}
\end{aligned} \tag{A.4}
$$

A.3 First Order Conditions

$$u'(c_t) = \mu_{1,t} \tag{A.5}$$

$$v'\left(m_t\right) = \mu_{2,t} \tag{A.6}$$

$$x'(1 - l_t) = -\mu_{1,t}w_t \tag{A.7}$$

$$\beta\mathbb{E}_t\big[\mathbb{V}_1\left(B_t, B^L_t, K_t, R_t, r^L_t\right)\big] = \frac{\mu_{1,t}}{p_t} + \mu_{2,t}\left[\theta_s\frac{1}{B_t} + \omega_{s,s}\ln b_t\frac{1}{B_t} + \omega_{s,L}\ln b^L_t\frac{1}{B_t}\right] \tag{A.8}$$

$$\beta\mathbb{E}_t\big[\mathbb{V}_2\left(B_t, B^L_t, K_t, R_t, r^L_t\right)\big] = \frac{\mu_{1,t}}{p_t} + \mu_{2,t}\left[\theta_L\frac{1}{B^L_t} + \omega_{L,L}\ln b^L_t\frac{1}{B^L_t} + \omega_{s,L}\ln b_t\frac{1}{B^L_t}\right] \tag{A.9}$$

$$\beta\mathbb{E}_t\big[\mathbb{V}_3\left(B_t, B^L_t, K_t, R_t, r^L_t\right)\big] = \frac{\mu_{1,t}}{p_t} + \frac{\mu_{3,t}}{p_t}\left(R_t - R^n_t\right) \tag{A.10}$$

$$\beta\mathbb{E}_t\big[\mathbb{V}_4\left(B_t, B^L_t, K_t, R_t, r^L_t\right)\big] = \frac{\mu_{3,t}}{p_t}K_t \tag{A.11}$$

$$\beta\mathbb{E}_t\big[\mathbb{V}_5\left(B_t, B_t^L, K_t, R_t, r_t^L\right)\big] = \frac{\mu_{4,t}}{p_t}B_t^L \tag{A.12}$$

A.4 Bienveniste-Scheinkman Conditions

$$\mathbb{V}_1\left(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L\right) = \frac{\mu_{1,t}}{p_t}(1+r_{t-1}) \tag{A.13}$$

$$\mathbb{V}_2\left(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L\right) = \frac{\mu_{1,t}}{p_t}(1+r_{t-1}^L) + \frac{\mu_{4,t}}{p_t}(1-\alpha)\left(r_{t-1}^L - r_t^{L,n}\right) \tag{A.14}$$

$$\mathbb{V}_3\left(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L\right) = \frac{\mu_{1,t}}{p_t}(1+R_{t-1}) + \frac{\mu_{3,t}}{p_t}(1-\delta)\left(R_{t-1} - R_t^n\right) \tag{A.15}$$

$$\mathbb{V}_4\left(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L\right) = \left(\frac{\mu_{1,t}}{p_t} + (1-\delta)\frac{\mu_{3,t}}{p_t}\right)K_{t-1} \tag{A.16}$$

$$\mathbb{V}_5\left(B_{t-1}, B_{t-1}^L, K_{t-1}, R_{t-1}, r_{t-1}^L\right) = \left(\frac{\mu_{1,t}}{p_t} + (1-\alpha)\frac{\mu_{4,t}}{p_t}\right)B_{t-1}^L \tag{A.17}$$

A.5 Optimality Conditions

$$u'(c_t) = \mu_{1,t} \tag{A.18}$$

$$v'\left(\frac{M_t}{p_t}\right) = \mu_{2,t} \tag{A.19}$$

$$x'(1-l_t) = -\mu_{1,t}w_t \tag{A.20}$$

$$1 + \frac{\gamma_{2,t}}{b_t}\left(\theta_s + \omega_{s,s}\ln b_t + \omega_{s,L}\ln b_t^L\right) = \beta\mathbb{E}_t\left[\frac{\mu_{1,t+1}}{\mu_{1,t}}\frac{1+r_t}{\pi_{t+1}}\right], \tag{A.21}$$

$$\begin{aligned} 1 + \frac{\gamma_{2,t}}{b_t^L}\left(\theta_L + \omega_{L,L}\ln b_t^L + \omega_{s,L}\ln b_t\right) + \gamma_{4,t}\left(r_t^L - r_t^{L,n}\right) \\ = \beta\mathbb{E}_t\left[\frac{\mu_{1,t+1}}{p_{t+1}}(1+r_t^L) + \frac{\mu_{4,t+1}}{p_{t+1}}(1-\alpha)\left(r_t^L - r_{t+1}^{L,n}\right)\right] \end{aligned} \tag{A.22}$$

$$\frac{\mu_{1,t}}{p_t} + \frac{\mu_{3,t}}{p_t}\left(R_t - R_t^n\right) = \beta\mathbb{E}_t\left[\frac{\mu_{1,t+1}}{p_{t+1}}(1+R_t) + \frac{\mu_{3,t+1}}{p_{t+1}}(1-\delta)\left(R_t - R_{t+1}^n\right)\right] \tag{A.23}$$

$$\frac{\mu_{3,t}}{p_t}K_t = \beta\mathbb{E}_t\left[\frac{\mu_{1,t+1}}{p_{t+1}} + (1-\delta)\frac{\mu_{3,t+1}}{p_{t+1}}\right]K_t \tag{A.24}$$

$$\frac{\mu_{4,t}}{p_t}B_t^L = \beta\mathbb{E}_t\left[\frac{\mu_{1,t+1}}{p_{t+1}} + (1-\alpha)\frac{\mu_{4,t+1}}{p_{t+1}}\right]B_t^L \tag{A.25}$$

B Derivation of the User Costs

Combining (11) and (12) yields:

$$1 = \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right] \tag{B.1}$$

Substituting this for the 1 in (9) and rearranging yields the marginal benefit/marginal cost equilibrium:

$$\frac{\gamma_{2,t}}{b_t^L} \left(\theta_L + \omega_{L,L} \ln b_t^L + \omega_{s,L} \ln b_t \right) = \beta \mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right] \tag{B.2}$$

Now dividing both sides by (B.1) converts the right side to the standard user cost form

$$\frac{\gamma_{2,t}}{b_t^L} \left(\theta_L + \omega_{L,L} \ln b_t^L + \omega_{s,L} \ln b_t \right) = \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n + \delta * - \delta - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]} \tag{B.3}$$

Decoupling $\gamma_{2,t}$ shows that the right hand side is the marginal cost of holding the short-term asset, expressed in terms of utility.

$$v'(m_t) \frac{\left(\theta_L + \omega_{L,L} \ln b_t^L + \omega_{s,L} \ln b_t \right)}{b_t^L} = u'(c_t) \frac{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n - r_t \right\} \right]}{\mathbb{E}_t \left[\frac{\mu_{t+1}}{\mu_t} \frac{1}{\pi_{t+1}} \left\{ 1 + R_t^n - (1-\delta)\gamma_{3,t+1} \Delta R_{t+1}^n \right\} \right]} \tag{B.4}$$

An assumption of risk neutrality or perfect certainty effectively implies that that that $\frac{\mu_{1,t+1}}{\mu_{1,t}} \frac{1}{\pi_{t+1}}$ is independent of the one-period returns in brackets, simplifying this expression to

$$\eta_t = \frac{R_t^n - (1-\delta)\mathbb{E}_t[\gamma_{3,t+1} \Delta R_{t+1}^n] - r_t}{1 + R_t^n - (1-\delta)\mathbb{E}_t[\gamma_{3,t+1} \Delta R_{t+1}^n]}, \tag{B.5}$$

where η_t equals the left-hand side of (B.3) and represents the user cost of holding the short term-asset for one period. See Barnett, Liu and Jensen (1997) for an in-depth exploration of monetary index number theory under risk.

Beginning the same procedure from (10) will provide the analogous user cost for a long-term security

$$\eta_t^L = \frac{\mathbb{E}_t\left[\frac{\mu_{t+1}}{\mu_t}\frac{1}{\pi_{t+1}}\left\{R_t^n - (1-\delta)\gamma_{3,t+1}\Delta R_{t+1}^n - r_t^{L,n} + (1-\alpha)\gamma_{4,t+1}\Delta r_{t+1}^{L,n}\right\}\right]}{\mathbb{E}_t\left[\frac{\mu_{t+1}}{\mu_t}\frac{1}{\pi_{t+1}}\left\{1 + R_t^n - (1-\delta)\gamma_{3,t+1}\Delta R_{t+1}^n\right\}\right]}, \tag{B.6}$$

and again assuming independence as above yields

$$\eta_t^L = \frac{R_t^n - (1-\delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n] - r_t^{L,n} + (1-\alpha)\mathbb{E}_t[\gamma_{4,t+1}\Delta r_{t+1}^{L,n}]}{1 + R_t^n - (1-\delta)\mathbb{E}_t[\gamma_{3,t+1}\Delta R_{t+1}^n]}. \tag{B.7}$$

Since we're dealing with long-term assets here, the period-by-period user cost incorporates both the expected one-period payouts as well as the expected capital gains/loses. The capital gains are incorporated via the expected change in the one-period payouts, scaled by the expected future price of the asset.

C The Full Model

The contributions of the model are primarily derived from the household's problem. Here, however, I outline the remainder of the model for further analysis.

C.1 The Representative Household

The initial setup allowed the various utility functions to remain flexible. Here I assign a logarithmic functional form to consumption $u(c_t) = \ln c_t$ and constant relative risk aversion (CRRA) functions to real monetary services $v(m_t) = \frac{m_t^{1-\nu}-1}{1-\nu}$ and leisure hours $x(1-l_t) = \chi\frac{(1-l_t)^{1+\psi}-1}{1+\psi}$.

C.2 Final Goods-Producing Firm

A final goods-producing firm acts as a retailer, aggregating the intermediate goods $i \in [0,1]$ into a consumable bundle with production CES production function

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}}\mathrm{d}i\right]^{\frac{\theta}{\theta-1}}. \tag{C.1}$$

This firm operates in a perfectly competitive market, choosing the quantity of each intermediate good $y_t(i)$ $\forall i$ that maximizes profits

$$\Pi_t^f = p_t y_t - \int_0^1 p_t(i)y_t(i)\mathrm{d}i. \tag{C.2}$$

Doing so leads to the standard demand function for the intermediate goods

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\frac{1}{\theta}} y_t \ \forall i. \tag{C.3}$$

C.3 Intermediate Goods Producing Firms

The intermediate goods-producing firms operate in a monopolistically-competitive environment, hiring labor $l_t(i)$ and utilizing a linear production technology $y_t(i) = z_t l_t(i)$, where z_t is a common technology that follows the stationary auto-regressive process

$$\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_t^z, \tag{C.4}$$

where $\rho_z \in (0, 1)$ and $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$. In maximizing their profits, the intermediate firms face a Rotemburg quadratic cost of price adjustment $\phi \left(\frac{p_t(i)}{\pi_t p_{t-1}(i)} - 1 \right)^2 y_t$, measured in term of the final output. Choosing the price of its respective intermediate good to maximize profits subject to the demand of the final-goods firm and assuming a symmetric equilibrium yields the a linearized Phillips curve relationship

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{\theta}{\phi} (\tilde{w}_t - \tilde{z}_t), \tag{C.5}$$

where the tilde denotes the variable's percent deviation from its steady state value.

C.4 Fiscal and Monetary Policy

The government's budget constraint is straight-forward in that the fiscal authority takes in revenue via a lump sum tax τ_t and borrowing at both short B_t and long $B_t^{L,n}$ maturities. Real spending g_t is considered to be exogenous

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_t^g, \tag{C.6}$$

where $\rho_g \in (0, 1)$, g is the steady state value of real government spending, and $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$. Together with the law of motion for long-term debt (2), the real fiscal budget constraint is

$$g_t + \frac{F_t}{p_t} + (1 + r_{t-1})\frac{B_{t-1}}{p_t} + (1 + r^L_{t-1})\frac{B^L_{t-1}}{p_t} = \tau_t + \frac{F_{t-1}}{p_t} + \frac{B_t}{p_t} + \frac{B^L_t}{p_t} \tag{C.7}$$

C.5 Equilibrium Conditions and Calibration

In equilibrium, $k_t = 0$ and $f_t = f$ for all t and $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation rate.

Calibration of this model starts with a standardized steady state aggregate output $y = 1$ and debt-GDP ratio of one-hundred percent ($b+b^L = y$). The steady state value of currency is derived from the ratio of total public federal debt to the monetary base, which is 8.13 during the 1982–2022 period, suggesting $f = 0.12$. The weight on currency in the monetary aggregate $\lambda_1 = 0.74$ is calculated from the relative user costs on the monetary assets. The average 10-year treasury constant maturity rate during this same period is about 5.73 percent, which is considered as the steady state value of r^L_t. The weights on short-term and long-term debt are calibrated such that a 1.25 percentage point spread exists between the short-term and long-term rates, which is roughly in line with the average 10yr–3m yield curve spread during the 1982–2022 period. This yields values of $\lambda_2 = 0.1089$ and $\lambda_3 = 0.1511$. The parameter governing the elasticity of substitution σ, is set at 0.40. Belongia and Ireland (2014) uses 0.50 for the elasticity across currency and deposits. Assuming currency and deposits are closer to perfect substitutes than long- and short-term debt this would be the upper bound on the analogous parameter in this model. The assumption of 10-year bonds above implies $\alpha = \delta = 0.025$.

The discount factor $\beta = 0.9624$ matches a two percent inflation target $\pi = 1.02$ and the spread on the benchmark rate $R - r^L = 0.0025$ as assumed in the derivation of the monetary aggregate in Section 4. I also follow Belongia and Ireland (2014) in setting $\theta = 6$ and $\phi = 50$, which imply a 20 percent markup over the intermediate goods and full price adjustment over a period of 3.75 quarters. For the baseline analysis, I assume no smoothing to the policy rates $\rho_\tau = \rho_r = 0$, and policy values of $\rho_\pi = 1.5$, $\rho_y = 0$, and $\rho_b = 0.5$ to ensure determinacy when evaluating impulse responses.