

HW2

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3.2)

```
y <- 57
n <- 100
theta0 <- seq(.1, .9, by = .1)
n0 <- c(1,2,8,16,32)

mat <- matrix(0, length(n0), length(theta0)) #create empty matrix to store prob values for post. beta d
rownames(mat) <- c(1,2,8,16,32)
colnames(mat) <- c(seq(.1, .9, by = .1))
#compute a and b given all combination theta0 and n0 values
for(i in 1:length(n0)){
  for (j in 1:length(theta0)){
    a = n0[i] * theta0[j]
    b = n0[i] * (1 - theta0[j])
    #with our prior being a beta dist. for theta, our posterior is also beta(a+y,b+n-y) for theta 0 <= thet
    mat[i,j] <- 1 - pbeta(.5, a + y, b + n - y)} #posterior probs that theta >0.5 for each value of a and
  print(a)
```

```
## [1] 28.8
```

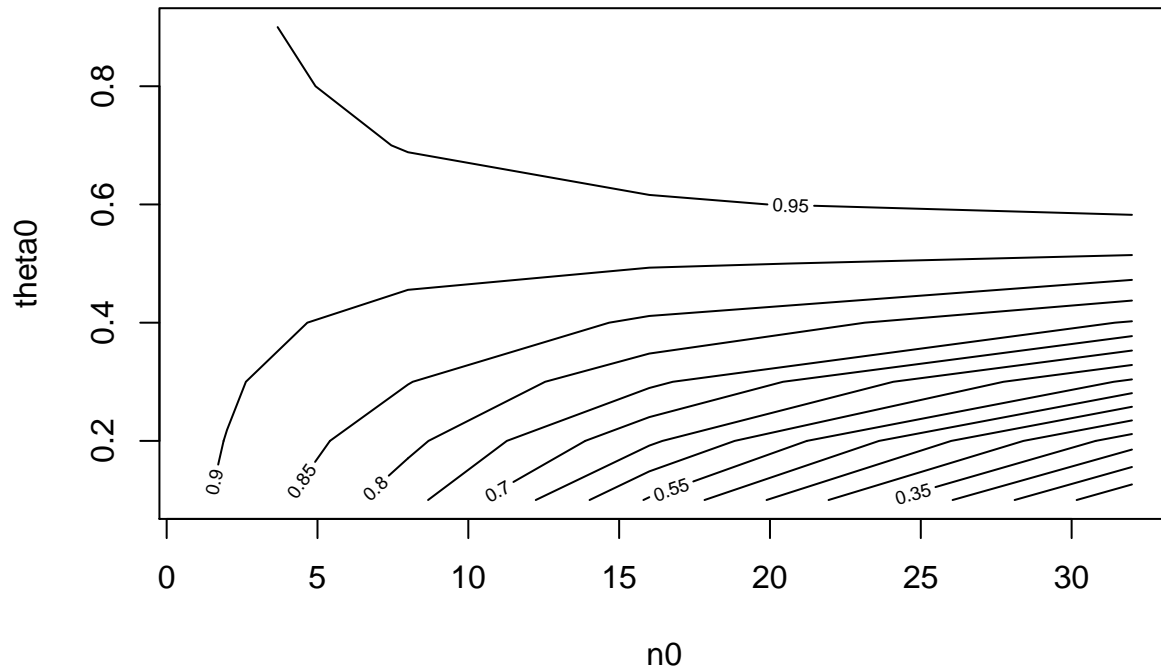
```
print(b)
```

```
## [1] 3.2
```

```
print(mat)
```

```
##           0.1           0.2           0.3           0.4           0.5           0.6           0.7
## 1  0.9067174 0.9100204 0.9132361 0.9163656 0.9194100 0.9223703 0.9252477
## 2  0.8914671 0.8987201 0.9056150 0.9121591 0.9183604 0.9242272 0.9297689
## 8  0.7686623 0.8130640 0.8517895 0.8847656 0.9121799 0.9344278 0.9520516
## 16 0.5445167 0.6591158 0.7606645 0.8430584 0.9042520 0.9458344 0.9716716
## 32 0.1554088 0.3248808 0.5417239 0.7465826 0.8894420 0.9628561 0.9905703
##           0.8           0.9
## 1  0.9280434 0.9307587
## 2  0.9349949 0.9399155
## 8  0.9656774 0.9759580
## 16 0.9863403 0.9939428
## 32 0.9982198 0.9997538
```

```
contour(n0, theta0, mat, xlab = 'n0', ylab = 'theta0', levels = seq(.1,1, by =.05))
```



From the output above, we can see that the values of a and b are 28.8 and 3.2 respectively for all the combinations of θ_0 and n_0 . The values for $Pr(\theta > 0.5 | \sum Y_i = 57)$ are also computed in the matrix above.

Based on the data that $\sum_{i=1}^{100} Y_i = 57$, the matrix probability, and posterior beta, we can see that the individuals who expect a prior $\theta < 0.5$ would have around a 0.9 certainty that the posterior θ is greater than 0.5. As the sample size increases however, the posterior belief for these individuals decreases. For those who expect the prior $\theta > 0.5$, they have around a 0.95 certainty that the posterior probability is greater than 0.5 regardless of the prior sample size.

3.3a)

```
#95% Posterior CI's for part a)
```

```
CIa.theta <- qgamma( c(.025,.975), 237,20); CIa.theta
```

```
## [1] 10.38924 13.40545
```

```
CIb.theta <- qgamma( c(.025,.975), 125,14); CIb.theta
```

```
## [1] 7.432064 10.560308
```

3.3b)

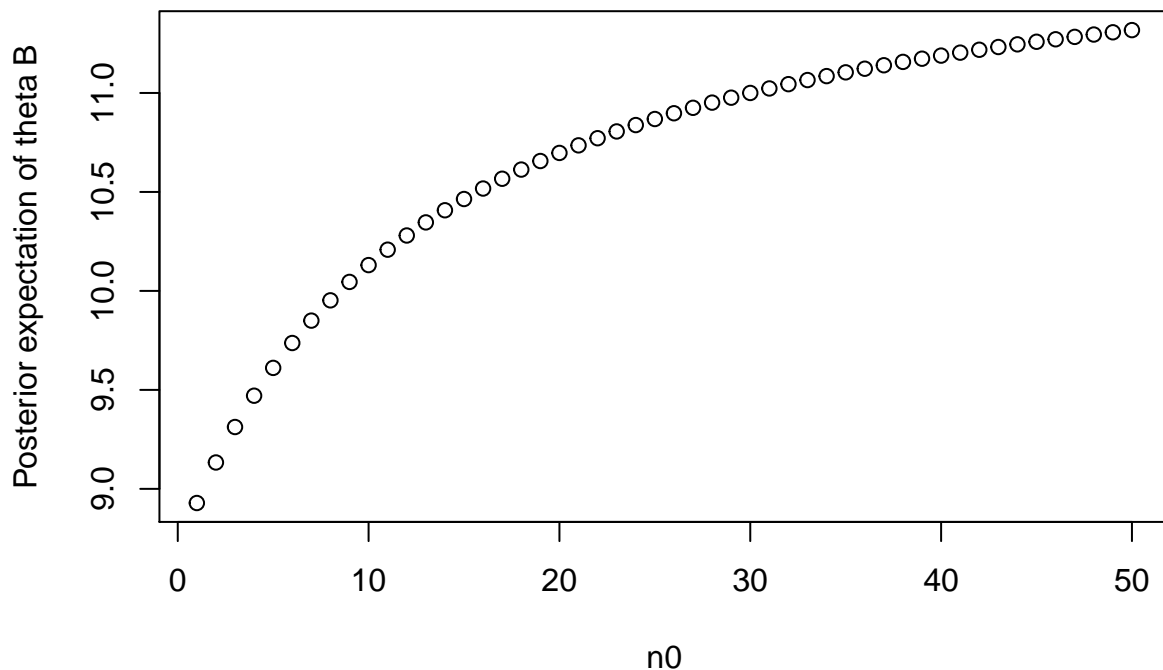
```
n0 <- c(seq(1,50, by = 1))
```

```
#posterior expectation of theta b given n0 values
```

```
expect.b <- (12*n0+113)/(n0+13); expect.b
```

```
## [1] 8.928571 9.133333 9.312500 9.470588 9.611111 9.736842 9.850000
## [8] 9.952381 10.045455 10.130435 10.208333 10.280000 10.346154 10.407407
## [15] 10.464286 10.517241 10.566667 10.612903 10.656250 10.696970 10.735294
## [22] 10.771429 10.805556 10.837838 10.868421 10.897436 10.925000 10.951220
## [29] 10.976190 11.000000 11.022727 11.044444 11.065217 11.085106 11.104167
## [36] 11.122449 11.140000 11.156863 11.173077 11.188679 11.203704 11.218182
## [43] 11.232143 11.245614 11.258621 11.271186 11.283333 11.295082 11.306452
## [50] 11.317460
```

```
plot(expect.b, xlab = "n0", ylab = "Posterior expectation of theta B")
```



We can see the posterior expected values of θ_B given n_0 values from the plot above. Since the expected value of θ_A is around 11.8, in order for the posterior expectation of θ_B to be close to that of θ_A , we need to have the prior belief that n_0 is larger than 40 so that the posterior mean of θ_B can be close to the posterior mean of θ_A ; it seems that the expectation of θ_B reaches the expectation of θ_A asymptotically from the plot.

c) Since it is stated that mice B and related to mice A, then it would not make sense to state that θ_A and θ_B are independent. Because the prior information about mice B comes from information from mice A, it wouldn't make sense to have $p(\theta_A, \theta_B) = p(\theta_A) * p(\theta_B)$. Instead, we should use $p(\theta_A, \theta_B) = p(\theta_A) * p(\theta_B | \theta_A)$.

3.9a)

A conjugate prior of the Galenshore distribution can also be the Galenshore distribution itself, although it seems that we can also use the Gamma distribution.

```
dgalenshore <- function(x, a, theta){
  if (any(x <= 0) || any(a <= 0) || any(theta <= 0))
    stop("Invalid parameters")
  else {
    return((2/gamma(a)) * theta^(2*a) * x^(2*a-1) * exp(-theta^2 * x^2))
  }
}

x <- seq(0.01, 3, 0.01)
plot(x, dgalenshore(x, 1, 1), type = "l", xlab = "y", ylab = "Galenshore Density", lwd = 2, ylim = c(0, 2))
lines(x, dgalenshore(x, 2,1), type = "l", col = "blue", lwd = 2, lty = 2)
lines(x, dgalenshore(x, 1,2), type = "l", col = "red", lwd = 2, lty = 3)
lines(x, dgalenshore(x, 2,2), type = "l", col = "purple", lwd = 2, lty = 4)
legend("topright", c("Galen(1,1)", "Galen(2,1)", "Galen(1,2)", "Galen(2,2)"), col = c("black", "blue", "red", "purple"),
```

