3. Linear Regression

Simple Linear Regression

Y = $\beta_0 + \beta_1 X_1$

- coefficients are minimized using ordinary least squares (OLS)
- **Important Assumptions**
  - assuming relationship between X and Y are linear
  - errors have constant variance (homoscedasticity)
  - errors are normally distributed with mean 0 (approximate)
  - errors independent of each other
- **Assessing model fit**
  - Residual Standard Error (RSE) = estimate of the SE of error term
    - measured in units of Y
  - R^2 statistic
    - will always increase if more variables are added

Multiple Linear Regression

Y = $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p$

- F-statistics used for MLR from the independent variables to reject null
- qualitiative predictors can be represented with dummy variables
- Polynomial regressions allow us to add non-linear relationships between predictor and response, but model overall is still linear
- potential issues (in addition to those of linear regression)
  - correlation of error terms
  - outliers
  - high-leverage point
  - collinearity/multicollinearity
    - use variance inflation factor (VIF) to solve this

Exercises

```r
#create a function that loads ISLR datasets
LoadLibraries = function (){
  library(ISLR)
  library(MASS)
  print("Libraries have been loaded")
}
LoadLibraries()
```

```
## [1] "Libraries have been loaded"
```

8

```r
slr <- lm(mpg~horsepower, data = Auto)
summary(slr)
```

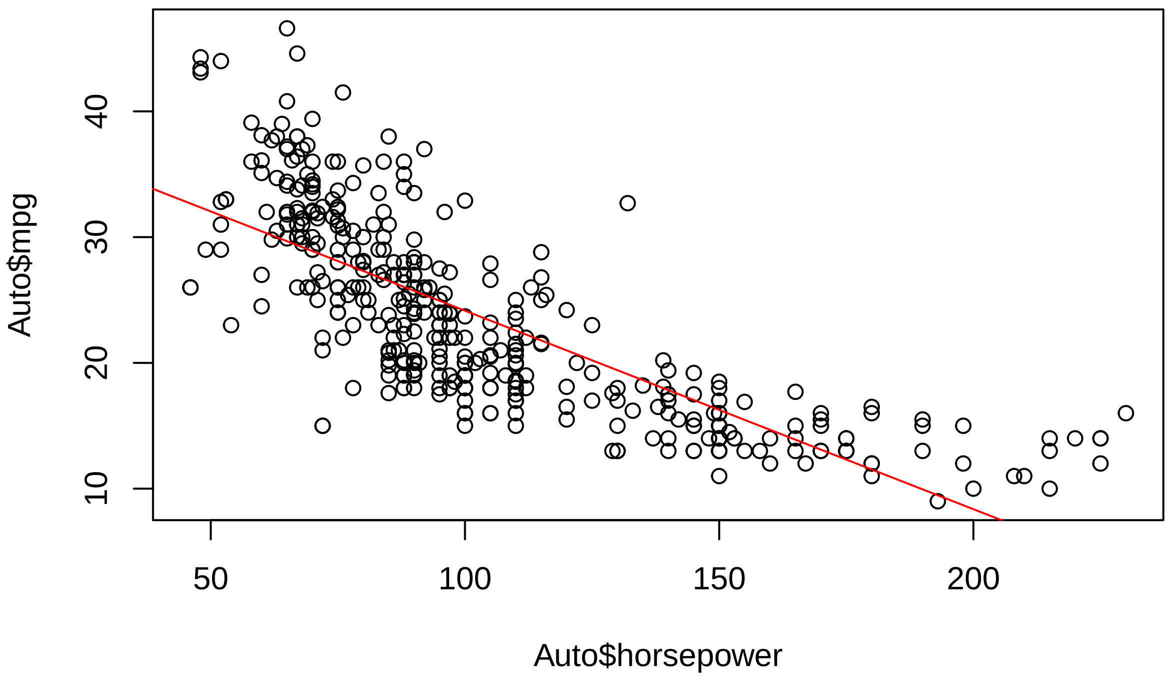
```
##
## Call:
## lm(formula = mpg ~ horsepower, data = Auto)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5710  -3.2592  -0.3435   2.7630  16.9240
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 39.935861   0.717499   55.66   <2e-16 ***
## horsepower  -0.157845   0.006446  -24.49   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.906 on 390 degrees of freedom
## Multiple R-squared:  0.6059, Adjusted R-squared:  0.6049
## F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

It seems that the predictor horsepower is significant with a negative t value and very low p value

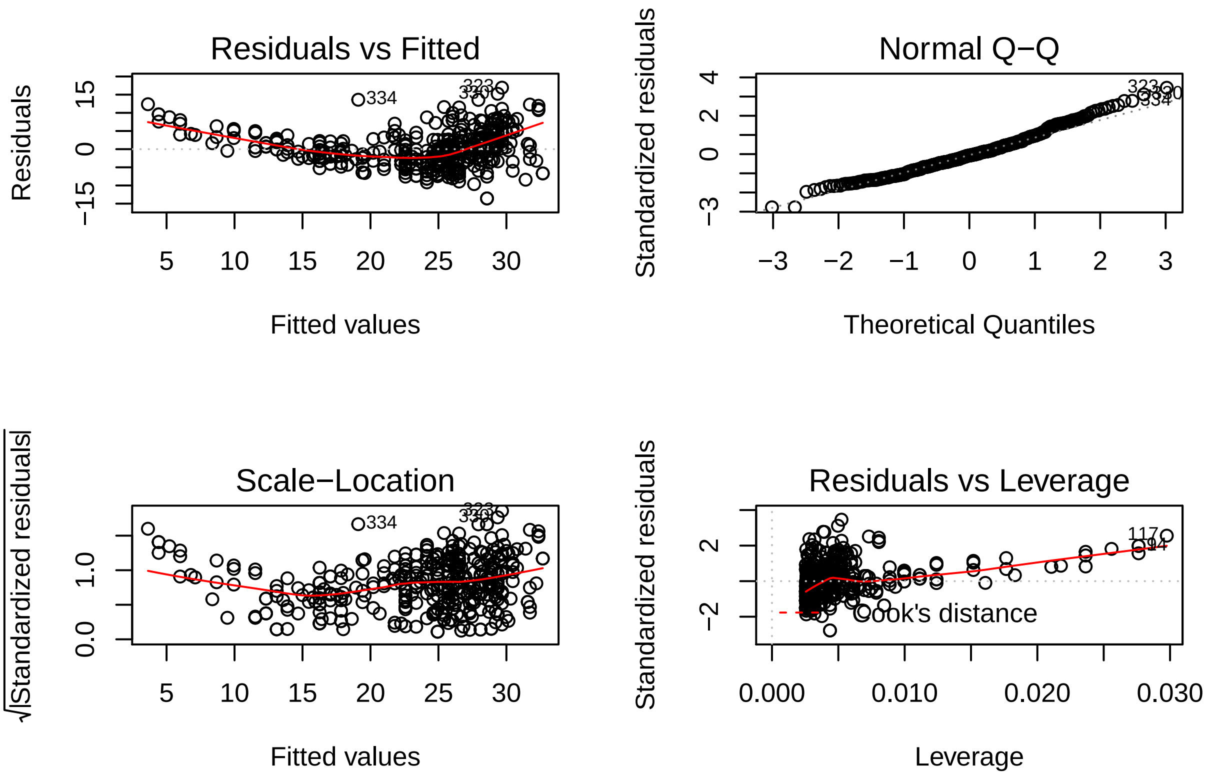
```r
predict(slr, data.frame(horsepower = 98), interval = "confidence")
```

```
##        fit      lwr      upr
## 1 24.46708 23.97308 24.96108
```

```r
plot(Auto$horsepower, Auto$mpg)
abline(reg = slr, col = "red") #regression line
```




```r
par(mfrow=c(2,2))
plot(slr) #diagnoistics plots
```



10

```
mlr <- lm(Sales~Price + Urban + US, data = Carseats)
summary(mlr)
```

```
##
## Call:
## lm(formula = Sales ~ Price + Urban + US, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9206 -1.6220 -0.0564  1.5786  7.0581
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.043469   0.651012  20.036  < 2e-16 ***
## Price       -0.054459   0.005242 -10.389  < 2e-16 ***
## UrbanYes    -0.021916   0.271650  -0.081    0.936
## USYes        1.200573   0.259042   4.635 4.86e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.472 on 396 degrees of freedom
## Multiple R-squared:  0.2393, Adjusted R-squared:  0.2335
## F-statistic: 41.52 on 3 and 396 DF,  p-value: < 2.2e-16
```

Note that both Urban and US predictors are qualitative