

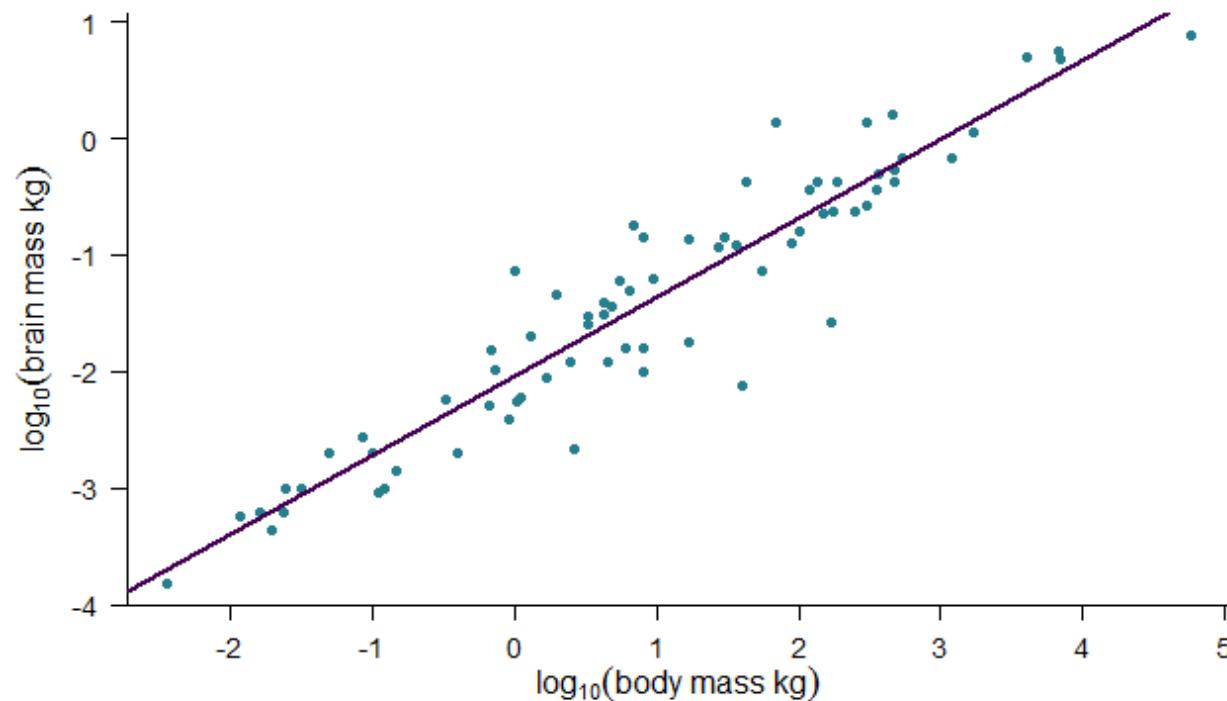
Log-Log Regression

Allometric, and other scaling

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Still just linear regression

But now with log transformed data on the x and y axes



Focus on length-mass relationships

- How does the mass of an organism scale with its length?
- Scaling is related to interesting biological and ecological processes
 - Metabolic costs
 - Transport of molecules over membranes
 - Predation or fishing / harvesting
 - Diet
 - Ecological scaling laws (abundance and biomass)

Scaling of simple shapes

- How does a cube scale with its length?
- $Mass = Density * Volume$
- Assume density stays the same no matter what size, so $Mass \propto Volume$
- $Volume = L_1 * L_2 * L_3$
- $Volume = a * L^b$
- So if the cube stays the same shape (i.e. it remains a cube)
 - How does mass change if length is doubled?
 - $Volume = 2L_1 * 2L_2 * 2L_3 = 2 * L^3$
- Isometric scaling is when the object stays the same shape as it grows or shrinks

Spherical cows

- How does mass of a sphere change with length?
- $Volume = \frac{4}{3} * \pi * r^3$
- It is more normal to measure the length, not radius of an animal so...
- $Volume = \frac{4}{3} * \pi * \frac{L^3}{2^3}$
- $Volume = \frac{4}{3} * \pi * (\frac{L}{2})^3 = \frac{4}{3*8} * \pi * L^3$
- Again, $Volume$, and hence $Mass$ changes with $Length^3$
- So, generally we have...
- $Volume = (some\ number) * L^3$
- $Volume = a * L^b$
- Where for isometric scaling of $Volume$ (or $Mass$) with $Length$, $b = 3$

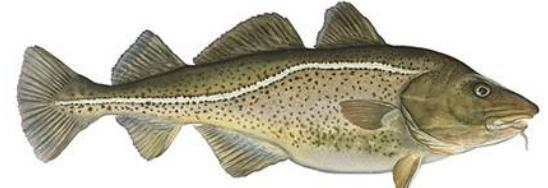


A general equation for scaling of Mass to Length

- $Mass = a * Length^b$
- Take the Log of both sides (doesnt matter what logarithmic base)
- $\log(M) = \log(a * L^b)$
- $\log(M) = \log(a) + \log(L^b)$
- $\log(M) = \log(a) + b * \log(L)$
- which is the equation of a line
- $Y = b_0 + b_1 * X$
- where... $\log(a) = b_0$
- so... $a = 10^{b_0}$ (assuming we are using logs in base 10)
- and... $b_1 = b$ and is simply the power in the allometric equation

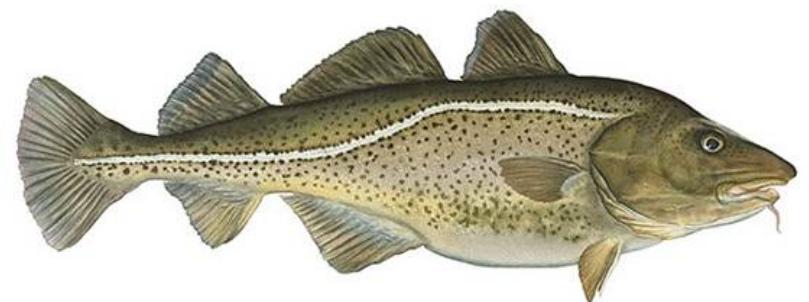
What do these coefficients mean? - Intercept

- on a log-log scale what does the intercept mean?
- $\log(M) = b_0 + b_1 * \log(L)$
- the intercept is the coefficient, or multiplier of *Length*
- $M = 10^{b_0} * Length^{b_1}$ (again assuming base 10)
- Recall that spheres and cubes only differ in their coefficients
- $Volume_{cube} = [1] * L^3$
- $Volume_{sphere} = [\frac{4}{3*\pi}] * \pi * L^3$
- So, b_0 , our intercept, which in the allometric equation is 10^{b_0} tells us that the shapes differ between two species, and in some ways might tell us how



What do these coefficients mean? - Slope

If the coefficient of $\log_{10}(Length)$ is exactly 3, then the fish are growing isometrically and staying exactly the same shape. that is their width and depth is growing in proportion to their length.

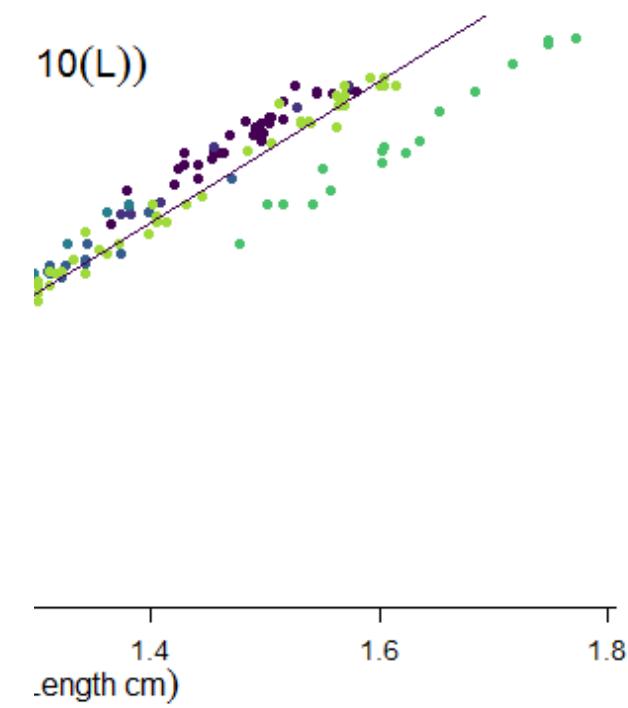


What do these coefficients mean? - Slope

If the coefficient of $\log_{10}(Length)$ is less than 3, then the fish are putting on less mass than you would predict which means their width and/or depth is not increasing in proportion to their length and they are becoming thinner as they get longer.



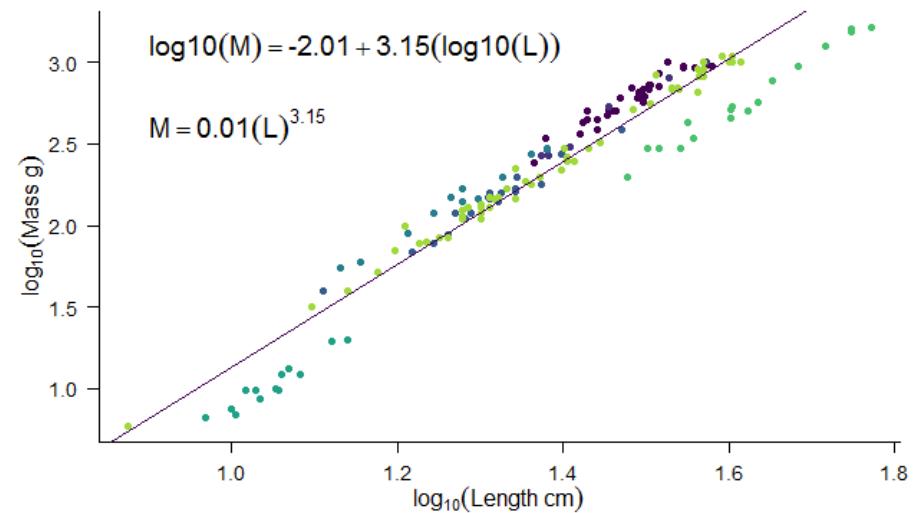
Inish Fish



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Exactly how much of an increase?

- This is just a linear equation
- $\log_{10}(M) = -2.01 + 3.15 * \log_{10}(L)$
- so a one unit increase in $\log_{10}(L)$ leads to a 3.15 unit increase in $\log_{10}(M)$
- but these are in \log_{10} units...
- and in raw terms means a proportional (multiplicative) increase.



Question

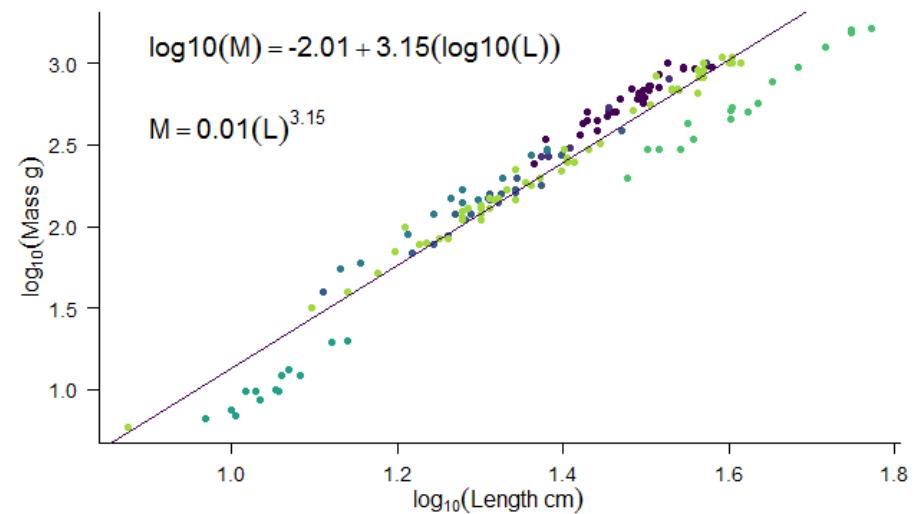
If $\log_{10}(Length)$ increases by one unit, how much does $Length$ increase by?

- + 10 grams
- + 9 grams
- 10 times larger
- 100 times larger

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Exactly how much of an increase?

- $\log_{10}(M) = -2.01 + 3.15 * \log_{10}(L)$
- so a one unit increase in $\log_{10}(L)$ leads to a 3.15 unit increase in $\log_{10}(M)$
- which translates into: every 10-fold increase in *Length* results in a $10^{3.15} = 1413$ proportional increase in *Mass*.
- So $L \mapsto 10 * L$ means
 $M \mapsto 10^{3.15} * M = 1413 * M$



- or, which might make more sense in this scale of data, a 0.1 or 10% increase in *Length* results in a $10^{0.315} = 2.1 \approx 2$ which is an approximate doubling in *Mass*

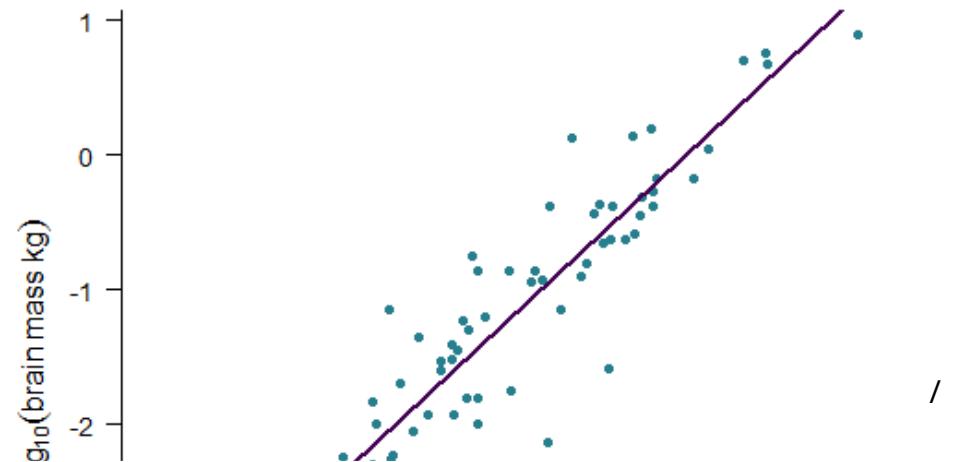
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Return to brain size evolution



Brain - Body mass scatterplot

- Now we are not modelling $Mass \sim Length$
- but rather $Mass \sim Mass$



Question

What exponent would we expect b to take in this equation if *brain mass* scales isometrically with *body mass*?

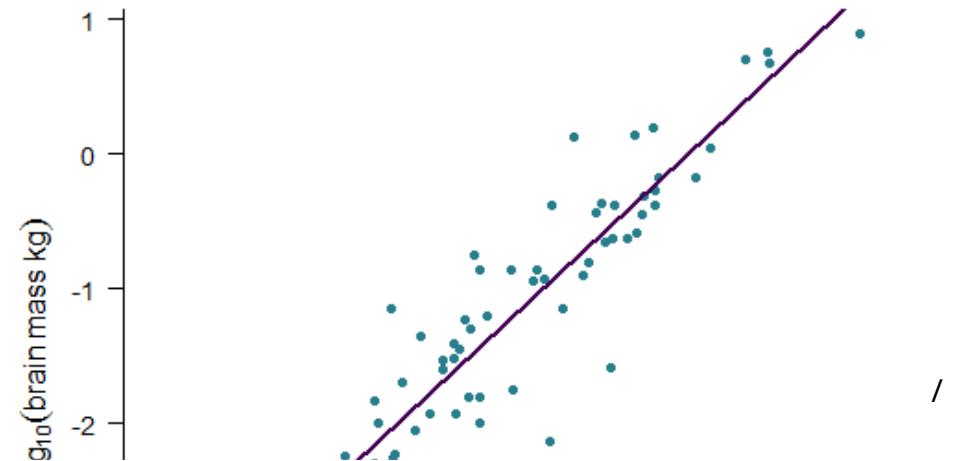
- 1
- 2
- 3
- 4

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Brain - Body mass scatterplot

- So, a sensible model is

$$Mass_{brain} = a * Mass_{body}^1$$
- which on \log_{10} transformed data would mean we expect the coefficient of $\log_{10}(Mass_{body})$ to be 1.
- In reality, would you expect this value to be greater than, equal to, or less than 1?



Common allometric relationships

- $Length \propto Mass^{\frac{1}{3}}$
- $SurfaceArea \propto Mass^{\frac{2}{3}}$
- $MetabolicRate \propto Mass^{\frac{3}{4}}$
- $Breathingrate\text{ or }Heartrate \propto Mass^{\frac{1}{4}}$
- $Abundance \propto Mass_{body}^{\frac{3}{4}}$
 - except parasites, which are weird! (<http://dx.doi.org/10.1126%2Fscience.1204337>)

