Log-Log Regression

Allometric, and other scaling

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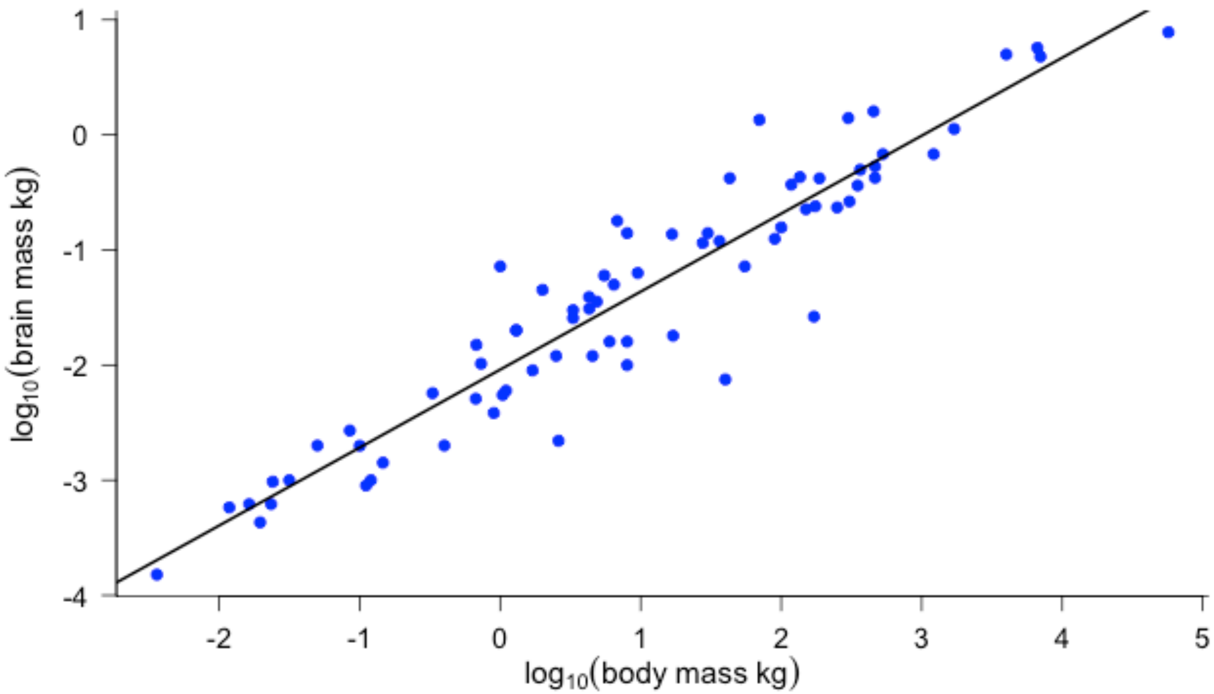
Still just linear regression

But now with log transformed data on the x and y axes



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Focus on length-mass relationships

- How does the mass of an organism scale with its length?
- Scaling is related to interesting biological and ecological processes
    - Metabolic costs
    - Transport of molecules over membranes
    - Predation or fishing / harvesting
    - Diet
    - Ecological scaling laws (abundance and biomass)

Scaling of simple shapes

- How does a cube scale with its length?
- $Mass = Density * Volume$
- Assume density stays the same no matter what size, so $Mass \propto Volume$
- $Volume = L_1 * L_2 * L_3$
- $Volume = a * L^b$
- So if the cube stays the same shape (i.e. it remains a cube)
  - How does mass change if length is doubled?
  - $Volume = 2L_1 * 2L_2 * 2L_3 = 2 * L^3$
- Isometric scaling is when the object stays the same shape as it grows or shrinks

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Spherical cows

- How does mass of a sphere change with length?
- $Volume = \frac{4}{3} * \pi * r^3$
- It is more normal to measure the length, not radius of an animal so...
- $Volume = \frac{4}{3} * \pi * \frac{L^3}{2^3}$
- $Volume = \frac{4}{3} * \pi * (\frac{L}{2})^3 = \frac{4}{3*8} * \pi * L^3$
- Again, $Volume$, and hence $Mass$ changes with $Length^3$
- So, generally we have...
- $Volume = (some\ number) * L^3$
- $Volume = a * L^b$
- Where for isometric scaling of $Volume$ (or $Mass$)

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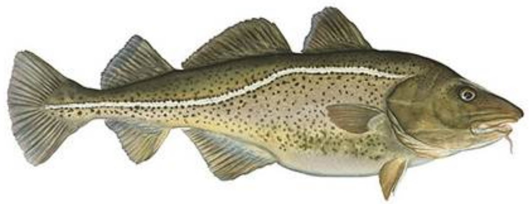
A general equation for scaling of Mass to Length

- $Mass = a * Length^b$
- Take the Log of both sides (doesnt matter what logarithmic base)
- $\log(M) = \log(a * L^b)$
- $\log(M) = \log(a) + \log(L^b)$
- $\log(M) = \log(a) + b * \log(L)$
- which is the equation of a line
- $Y = b_0 + b_1 * X$
- where... $\log(a) = b_0$
- so... $a = 10^{b_0}$ (assuming we are using logs in base 10)
- and... $b_1 = b$ and is simply the power in the allometric equation

What do these coefficients mean? - Intercept



- on a log-log scale what does the intercept mean?
- $\log(M) = b_0 + b_1 * \log(L)$
- the intercept is the coefficient, or multiplier of $Length$
- $M = 10^{b_0} * Length^{b_1}$ (again assuming base 10)
- Recall that spheres and cubes only differ in their coefficients
- $Volume_{cube} = [1] * L^3$
- $Volume_{sphere} = [\frac{4}{3*8} * \pi] * L^3$
- So, b_0, our intercept, which in the allometric equation is 10^{b_0} tells us that the shapes differ between two species, and in some ways might tell us how

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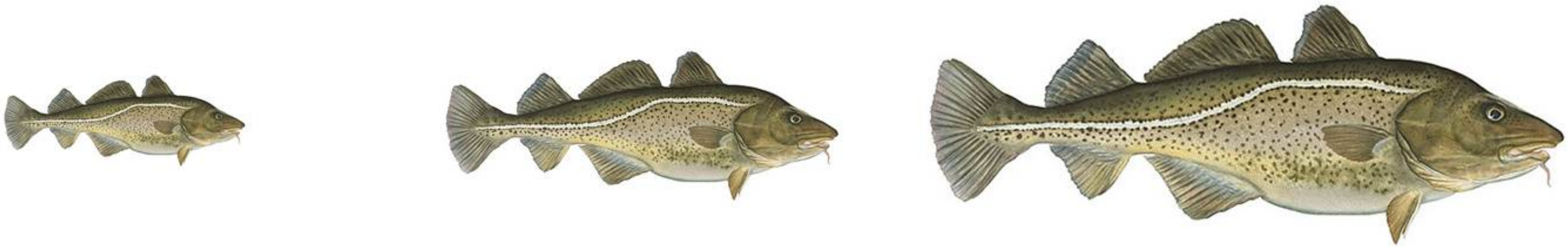
What do these coefficients mean? - Slope

If the coefficient of $\log_{10}(Length)$ is exactly 3, then the fish are growing isometrically and staying exactly the same shape. that is their width and depth is growing in proportion to their length.



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What do these coefficients mean? - Slope

If the coefficient of $\log_{10}(Length)$ is less than 3, then the fish are putting on less mass than you would predict which means their width and/or depth is not increasing in proportion to their length and they are becoming thinner as they get longer.

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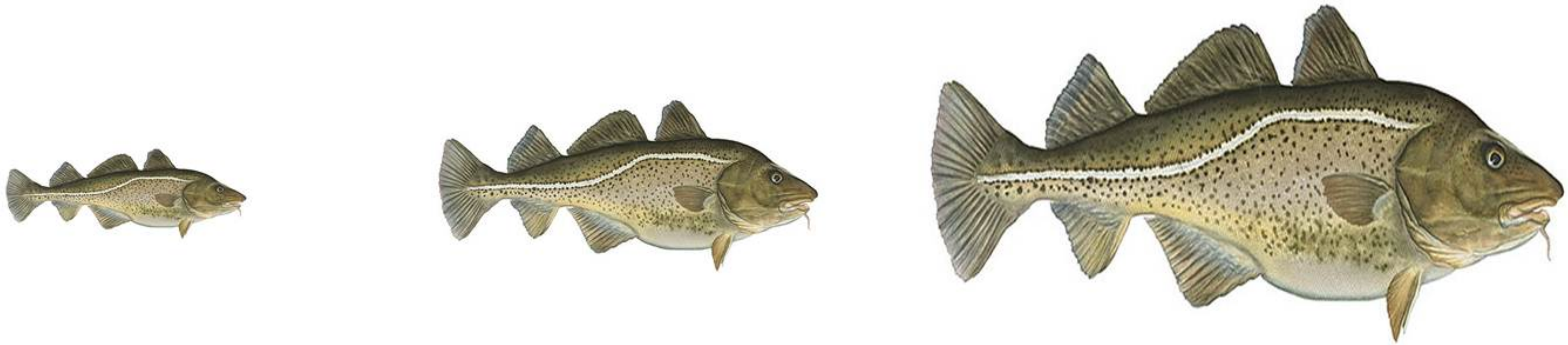
What do these coefficients mean? - Slope

If the coefficient of $\log_{10}(Length)$ is greater than 3, then the fish are putting on more mass than you would predict which means their width and/or depth is not increasing in proportion to their length and they are becoming broader as they get longer.



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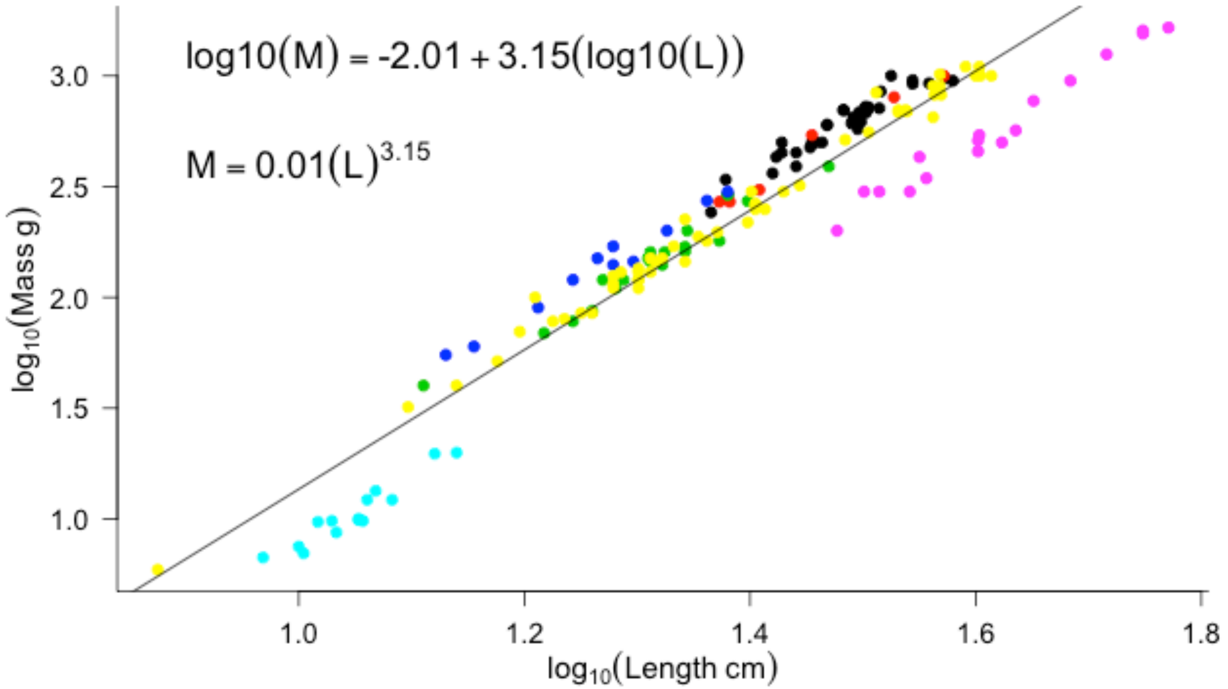
Mass - Length scaling in Finnish Fish



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Question

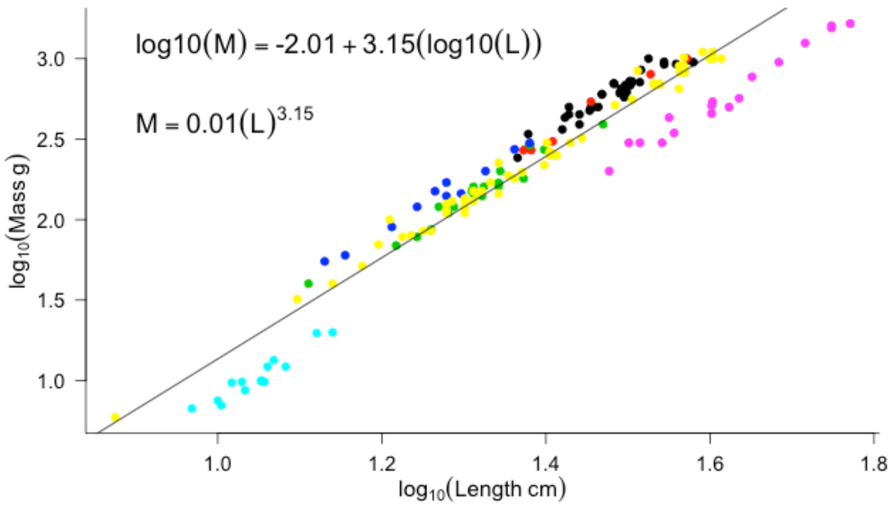
With a scaling relationship of $Mass = 0.01 * Length^{3.15}$, is mass scaling:

- Isometrically
- Faster
- Slower

Submit Show Hint Show Answer Clear

Exactly how much of an increase?

- This is just a linear equation
- $\log_{10}(M) = -2.01 + 3.15 * \log_{10}(L)$
- so a one unit increase in $\log_{10}(L)$ leads to a 3.15 unit increase in $\log_{10}(M)$
- but these are in $\log_{10}$ units...
- and in raw terms means a proportional (multiplicative) increase.



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Question

If $\log_{10}(Length)$ increases by one unit, how much does $Length$ increase by?

- + 10 grams
- + 9 grams
- 10 times larger
- 100 times larger

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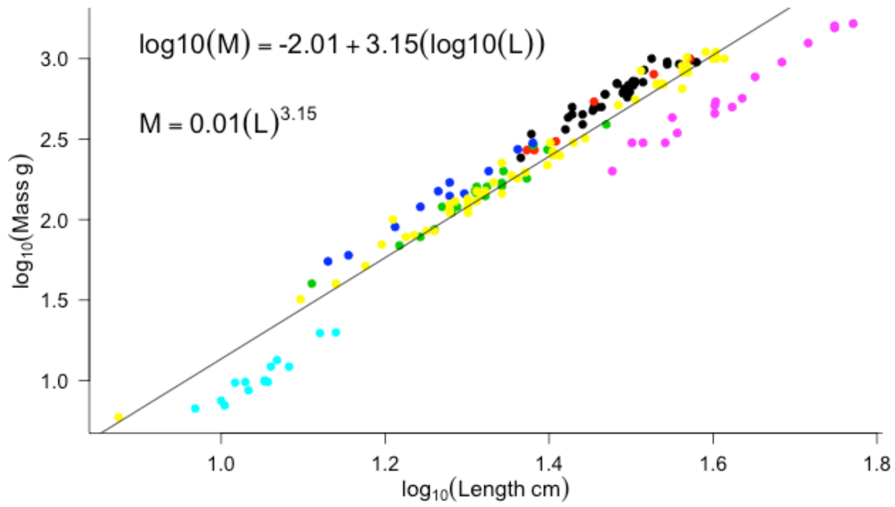
Exactly how much of an increase?

- $\log_{10}(M) = -2.01 + 3.15 * \log_{10}(L)$
- so a one unit increase in $\log_{10}(L)$ leads to a 3.15 unit increase in $\log_{10}(M)$
- which translates into: every 10-fold increase in $Length$ results in a $10^{3.15} = 1413$ proportional increase in $Mass$.
- So $L \mapsto 10 * L$ means $M \mapsto 10^{3.15} * M = 1413 * M$
- or, which might make more sense in this scale of data, a 0.1 or 10% increase in $Length$ results in a $10^{0.315} = 2.1 \approx 2$ which is an approximate doubling in $Mass$



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Return to brain size evolution

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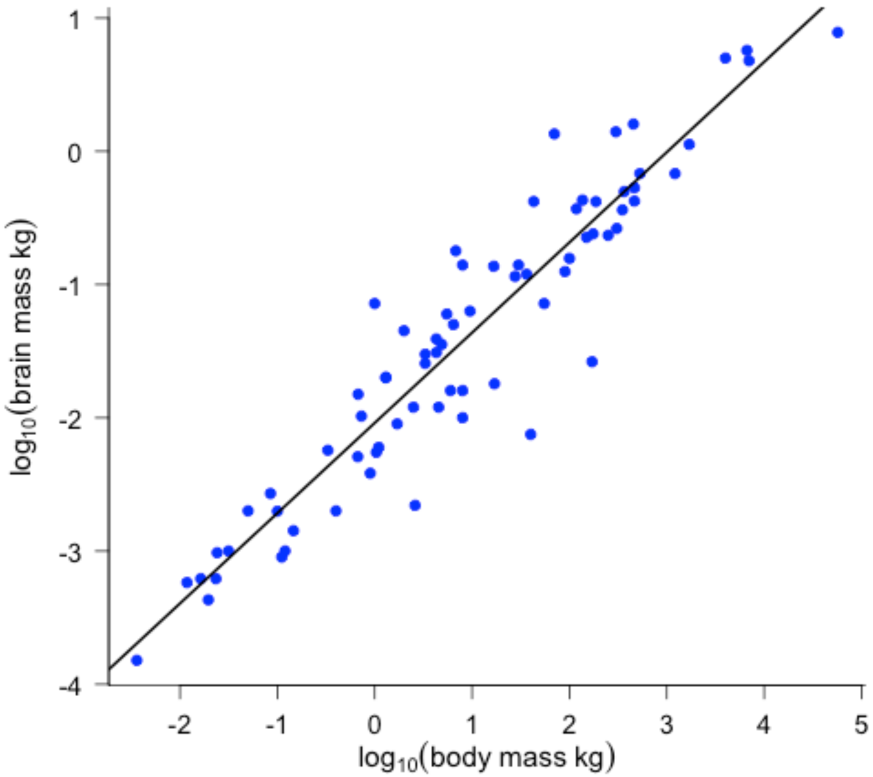
Brain - Body mass scatterplot

- Now we are not modelling $Mass \sim Length$
- but rather $Mass \sim Mass$



Question

What exponent would we expect b to take in this equation if $brain\ mass$ scales isometrically with $body\ mass$?

- 1
- 2
- 3
- 4

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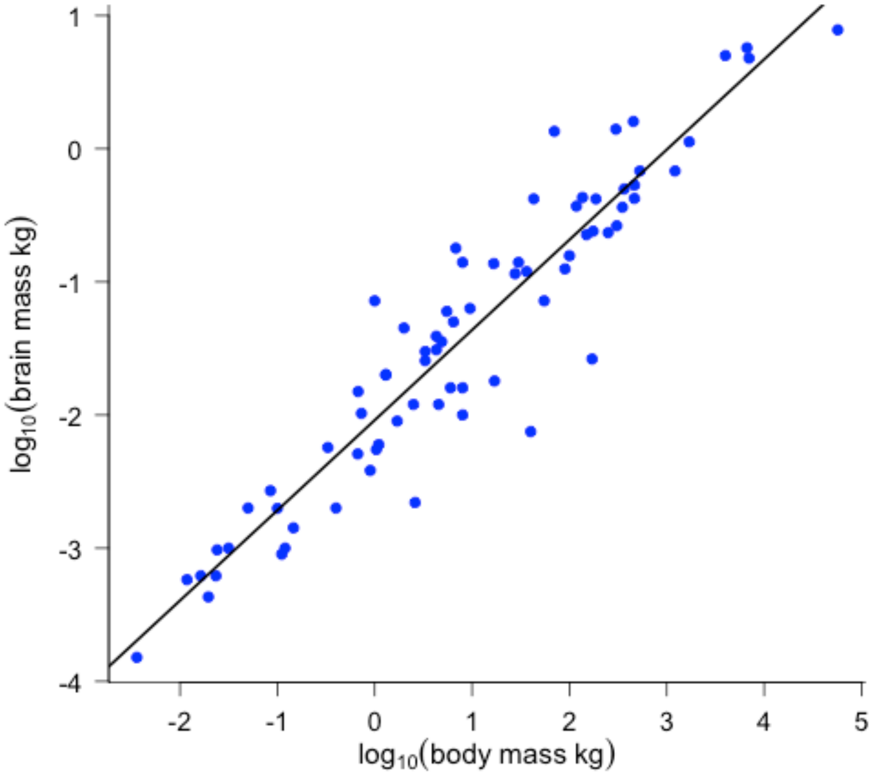
Brain - Body mass scatterplot

- So, a sensible model is $Mass_{brain} = a * Mass_{body}^1$
- which on $\log_{10}$ transformed data would mean we expect the coefficient of $\log_{10}(Mass_{body})$ to be 1.
- In reality, would you expect this value to be greater than, equal to, or less than 1?
- what are the biological reasons that might govern this relationship?



Common allometric relationships

- $Length \propto Mass^{\frac{1}{3}}$
- $SurfaceArea \propto Mass^{\frac{2}{3}}$
- $MetabolicRate \propto Mass^{\frac{3}{4}}$
- $BreathingrateorHeartrate \propto Mass^{\frac{1}{4}}$
- $Abundance \propto Mass_{body}^{\frac{3}{4}}$
  - except parasites, which are weird! (http://dx.doi.org/10.1126%2Fscience.1204337)
  - blank
- blank

test

Key learning outcomes

- Describe how change on a log scale affects the raw numbers
- Linearise the allometric equation using log transformations
- Use the equations fitted to data on log-log scale to make statements about proportional change in the response variable (y axis) as a function of proportional change in the covariates (x axis)