1

Which of the following statements is/are true?
A) From the Kohler equation it can be concluded that smaller solution droplets are more likely to form cloud droplets under atmospheric conditions.

TRUE. Cloud droplets are more readily formed with high solute concentrations. For a given amount of salt, small droplets maximize concentration and can thus sustain equilibrium

B) The Kelvin equation predicts that liquid water located in a small pore has a higher saturation vapor pressure than a planar or concave liquid surface at the same temperature.

TRUE? The Kelvin equation states that a concave liquid surface ($a < 0$) will have the lowest saturation vapour pressure, and a curved ($a > 0$) droplet will have the highest. The effect is most pronounced for small radii.

C) An increasing concentration of the solvent increases the equilibrium vapor pressure above an aqueous solution drop.

TRUE. As solvent increases, the behavior of the droplet approaches that of pure water i.e. higher e_{sat} than a small, solute-rich drop.

D) Of the two competing terms in the Kohler equation, the Raoult term dominates for smaller droplet radii.

TRUE. The Raoult term describes salt concentration and is inversely proportional to volume ($1/a^3$), whereas curvature effects are proportional to $1/a$. The Raoult term dominates for small a.

2

Using Figure 5.7 from the textbook for reference, what happens in each of the following scenarios?:

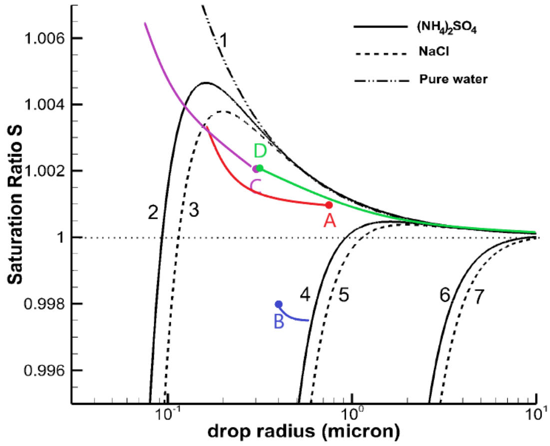

Fig. 5.7 Saturation ratio vs. drop radius for an aqueous solution drop containing various amounts of salts as determined by the Köhler equation (5.57): (1) no salt (pure water drop); (2) $(NH_4)_2SO_4$, 10^{-16} g; (3) NaCl, 10^{-16} g; (4) $(NH_4)_2SO_4$, 10^{-15} g; (5) NaCl, 10^{-15} g; (6) $(NH_4)_2SO_4$, 10^{-14} g; and (7) NaCl, 10^{-14} g.

A) A 0.7 μm droplet of NaCl, $10^{-16}g$ at S = 1.001

The droplet has less than equilibrium salt concentration, and is in the unstable regime. $\rightarrow$ It will shrink until it reaches equilibrium on curve 3

B) A 0.4 μm droplet of (NH4)2SO4, $10^{-15}g$ at S = 0.998,

The droplet has more than sufficient solute to grow $\rightarrow$ it will grow until it intersects curve 4, reaching a stable equilibrium

C) A 0.3 μm pure water droplet at S = 1.002,

Pure water droplets have no stable equilibrium points $\rightarrow$ it will shrink until it disappears

D) And a 0.3 μm droplet of NaCl, $10^{-15}g$ at S = 1.002?

The droplet has sufficient S_{vw} and size to become activated. $\rightarrow$ it will grow indefinitely until it consumes all available vapour or falls as precip

3

Consider a cloud where several solution droplets are beginning to grow and to form a population of cloud droplets as the saturation ratio is gradually increased, with temperature held constant. The solution droplets each have the same chemical composition but a broad range of sizes. Droplets with radii $r_{act} = 0.22\mu m$ are activated when the supersaturation reaches 0.35%.

A) Determine the temperature T of the cloud.

Using the Kohler equation:

$$ln\left(\frac{e_{sat,a}}{e_{sat,\infty}}\right) = \frac{A}{a} - \frac{B}{a^3}$$

$$A \approx \frac{3.3 \cdot 10^{-5}}{T}; B \approx \frac{4.3\nu m_s}{M_s}$$

and the critical radius r_{act} given by:

$$r_{act} = \sqrt{\frac{3B}{A}}$$

we can solve B in terms of A and r_{act}:

$$B = \frac{Ar_{act}^2}{3}$$

and substitute into the Kohler equation, setting $a = r_{act}$:

$$ln\left(\frac{e_{sat,a}}{e_{sat,\infty}}\right) = \frac{A}{r_{act}} - \frac{Ar_{act}2}{3r_{act}^3} = \frac{A}{r_{act}}(1 - \frac{1}{3}) = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{r_{act}T}$$

solve for T:

$$T = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{r_{act} \cdot ln\left(\frac{e_{sat,a}}{e_{sat,\infty}}\right)}$$

and plug in values $\frac{e_{sat,a}}{e_sat,\infty} = 1.0035$, $r_{act} = 2.2 \cdot 10^{-5} cm$:

$$T = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{2.2e - 5 \cdot ln\,(1.0035)}$$

$$\boxed{T = 286.2K}$$

B) Other solution droplets are activated at a supersaturation of 0.9% at the same temperature. What is the activation radius r_{act} in that case?

Re-arranging temperature equation for r_{act} and plug in $\frac{e_{sat,a}}{e_sat,\infty} = 1.009$ and $T = 286.2K$, we find:

$$T = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{r_{act} \cdot ln\left(\frac{e_{sat,a}}{e_{sat,\infty}}\right)} \rightarrow r_{act} = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{T \cdot ln\left(\frac{e_{sat,a}}{e_{sat,\infty}}\right)}$$

$$r_{act} = \frac{2}{3} \cdot \frac{3.3 \cdot 10^{-5}}{286.2K \cdot ln\,(1.009)} = \boxed{0.086\mu m}$$