Name: ID:

EOSC 213 - Final Exam

Instructions (XX points in total)

- Don't panic (Douglas Adams, Hitchhiker's Guide to the Galaxy)
- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 90 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Q0 [**1 point**] Complete the sentence with your favourite word. *Belgium is the most .... country in the world.*

Q1 Acidity is an important water-quality parameter at mine sites. It describes the moles of a base (typically carbonate) required to raise a water's pH to a prescribed value. Acidity is a **conserved quantity**. In practice, the units are moles of acidity per litre of water $[M/L^3]$.

In this question, you will develop a model (equations) that describe the change in acidity over time in a tailings management facility (TMF or tailings pond), under these assumptions:

1. the TMF has a total volume of $V(t)$ $[L^3]$, that changes in time in response to in- and outflows of water.
2. drainage from the pit flows into the TMF at a rate $Q_{pit}(t)$ $[L^3/T]$ with an acidity concentration of c_{pit} $[M/L^3]$, measured in units of moles acidity per litre.
3. precipitation with zero acidity $c_{precip} = 0$ $moles/l$ enters the TMF as precipitation on the surface of the TMF. Assume that the precipitation rate is given from data and can be represented in your model equations as $P(t)$, $[L/T]$ with units of mm/day. Assume that the surface area of the TMF is A $[L^2]$ with units of m^2. Assume that area **does not change** as the volume in the TMF changes (an approximation that is valid for small changes in volume).
4. evaporation removes water from the TMF at a rate that can be represented in your equation as $ET(t)$ $[L/T]$ with units of mm/day.
5. the TMF is well stirred, such that the concentration of acidity in the TMF, c_{TMF} $[M/L^3]$, measured in units of moles acidity per litre of water, is the same at all points in the pond at each instant in time.
6. water is discharged from the TMF at a rate $Q_{dis}(t)$ $[L^3/T]$ with an acidity concentration of c_{TMF}.
7. assume all other sources and sinks of water are negligible and can be ignored.

Table 1: Summary of variables for Q1

Variable	Description	Dimension
$V(t)$	Volume in the TMF	$[L^3]$
$Q_{pit}(t)$	Rate of flow from pit into TMF	$[L^3/T]$
$c_{pit}(t)$	Concentration of acidity in pit water	$[M/L^3]$
$P(t)$	Precipitation rate	$[L/T]$
$ET(t)$	Evaporation rate	$[L/T]$
A	Surface area of the TMF m	$[L^2]$
$c_{TMF}(t)$	Concentration of acidity in the TMF	$[M/L^3]$
$Q_{dis}(t)$	Rate of discharge out of TMF	$[L^3/T]$

Q1a [**4 points**] Write a mathematical expression that describes how the volume of water in the TMF changes over a time interval from time t to time $t + \Delta t$. That is, complete the following equation:

$V(t + \Delta t) - V(t) =$

Q1b [**2 points**] Write the expression from part a) as a **differential equation** governing the rate of change of the water volume in the TMF. That is, complete the following equation:

$\frac{dV}{dt} =$

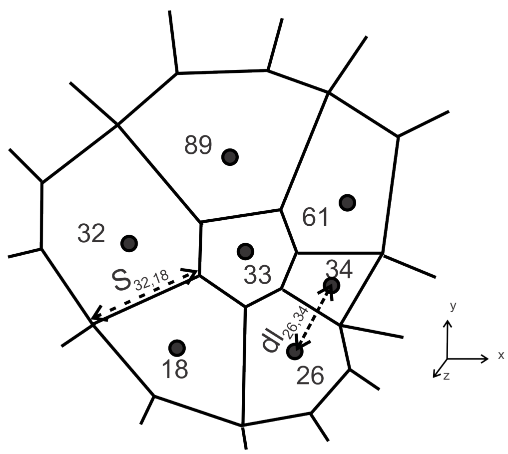
Q1c [**5 points**] Write a **differential equation**, or the **differential equations** that allow you to compute the **concentration** of acidity in the TMF $c_{TMF}(t)$ with time.

Q1d [**2 points**] Provide a brief explanation for your answer to Q1c, or if you don't think you got the answer, how you think the problem should be approached.

Q2 Finite - volume methods can be generalized from orthogonal N-S-E-W meshes to unstructured meshes. Below is a subsection from an **unstructured** finite-volume mesh. The gridblocks are labeled with their gridblock number.

[**5 points**] Write the finite-volume discrete approximation of the conservation equation for **steady-state** diffusion for gridblock 33 in terms of the concentration in gridblock 33, c_{33}, and the concentrations in the gridblocks adjacent to gridblock 33, c_i (use the numbers given in the figure). Hints:

1. The mesh is a constant thickness Δz (into the page).
2. In your equations, represent the length of the interface between two gridblocks as $S_{i,j}$ where i, j are two adjacent gridblocks that share an interface. For example, $S_{32,18} = S_{18,32}$ is the length of the interface between gridblocks 32 and 18, as indicated on the figure.
3. In your equations, represent the distance separating nodes in adjacent gridblocks as the distance $dl_{i,j}$. For example, the nodes in gridblocks are separated by a distance $dl_{34,26} = dl_{26,34}$ as inidicated on the figure.
4. The component of diffusive flux in the l direction is given by $j = -D\frac{dc}{dl}$, where dl is the distance in the l direction. This is the non-porous-media form of Fick's law.



Q3 Consider the following linear system which is the result of a finite-volume discretization of a physical problem:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ -0.005 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad Ax = B \tag{1}$$

Q3a [3 points] Write the equations given by the first, second and last rows of the linear system in terms of the variables x_1, x_2, ...

Q3b [2 points] What is the dimension of the physical system represented by this system of equations: 1, 2 or 3 dimensional? To receive credit, you must explain your answer.

Q3c [2 points] Which physical processes could be described by the linear system? To receive credit, you must explain your answer.

Q3d [2 points] Are there any source terms? If yes, specify where (and if it is a source (positive) or a sink (negative) source term).

Q3e [2 points] Describe the physical boundary conditions that are represented in this system. Refer to specific gridblock numbers.