EOSC 213 Final Quiz April 18^{th}, 2019
Name: ID:

EOSC 213 - Quiz

Instructions (XX points in total)

- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 60 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Question 0: Roger joke

Q0 Complete the sentence with your favourite word. *Belgium is the most ... country in the world* [**1 point**]

Question 1: ODE and finite-difference approximations

Q1a Write a discrete approximation of $\frac{d^2y}{dx^2}$ [**2 points**]

Q1b Write a first order discrete approximation of $\frac{dy}{dx}$ [**2 points**]

Q1c Write a second order discrete approximation of $\frac{dy}{dx}$ [**2 points**]

Let us consider the following differential problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 5y &= 5x^2 + 2x \\ y(0) &= 2 \\ y(3) &= 5 \end{cases} \tag{1}$$

Q1d Write the differential problem in a discrete approximation (using x_i where i refers the center of the gridblocks and Δx is the distance between these centers) [**2 points**]

A friend of yours used a computational method to solve the differential problem. Here are the results:

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	2	1.25	1	1.25	2	3.25	5

Q1e Verify that the system of equations you wrote at Q1d is approximatively solved by the values given in the previous table. [**4 points**]

Q1f Prove that the function $y_1(x) = x^2 - 2x + 2$ is a solution to the differential problem and that it satisfies the boundary conditions [**4 points**]

Q1g Give your friend feedback on her/his answer based on your previous answers. [**2 points**]

Q1d Using the answers of Q1a and Q1c, can you comment on the validity of his numerical answer? [**2 points**]

Q1b Compute a second-order approximation to the first derivative of the function $f(x)$ at $x = 2$. [**2 points**]

Question 2: Diffusion Boundary Value Problem

Consider the two following matrixes and the linear system:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ -0.005 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad Ac = B \tag{2}$$

Q2a Write the system of equations described by these matrixes [**2 points**]

Q2b How many dimensions (1D, 2D, 3D) are modelled by these matrixes? [**2 points**]

Q2c Which physical process is described by these matrixes? [**2 points**]

Q2d Are there any source terms? If yes, specify where (and if it is a source (positive) or a sink (negative) source term). If no, justify why. [**3 points**]

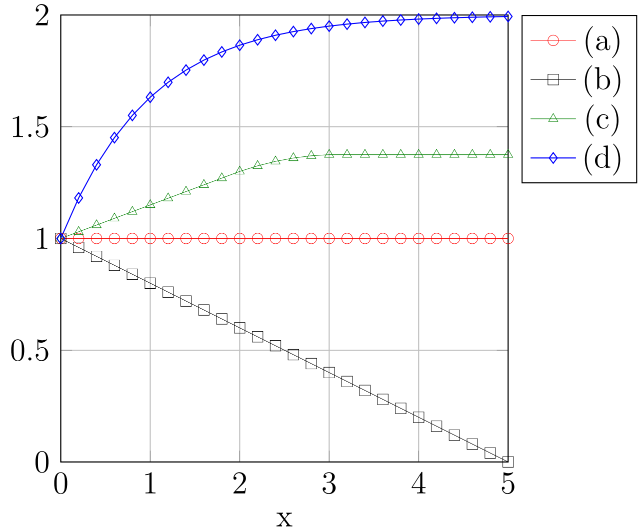
Q2e Is this describing a steady-state or a transient equation? [**2 points**]

Q2f Describe the boundary conditions (mathematically and physically). [**4 points**]

Q2g Write the PDE equation associated to this problem [**2 points**]

Q2h Describe the link between the PDE and the discretized system of equations? Specifically, link the PDE terms to the equations you have given in Q2a. [**2 points**]

Q2i Identify which of these 4 graphs is the asymptotic (final) solution to the presented problem. Give justifications as why the other ones are not compatible with the given system. If you don't know which one, try to describe the different results (what boundary conditions, ...). We assume that the medium is homogeneous: diffusion coefficient is the same everywhere. [**2 points**]


2
1.5
1
0.5
0
0
1
2
3
4
5
x
(a)
(b)
(c)
(d)