Name: ID:

EOSC 213 - Final Exam

Instructions (51 points in total)

- Don't panic (Douglas Adams, Hitchhiker's Guide to the Galaxy)
- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 90 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.
- Use the back of a page if you run out of room, being careful to label the question with the question number you are answering.

Q0 [**1 point**] Complete the sentence with your favourite word. *Belgium is the most .... country in the world.*

Q1 **Acidity** is an important water-quality parameter at mine sites. It describes the moles of a base (typically carbonate) required to raise a water's pH to a prescribed value. Acidity is a **conserved quantity**. In practice, the units are moles of acidity per litre of water $[M/L^3]$.

In this question, you will develop a model (equations) that describe the change in acidity over time in a tailings management facility (TMF or tailings pond), under these assumptions:

1. the TMF has a total volume of $V(t)$ $[L^3]$, that changes in time in response to in- and outflows of water.
2. drainage from the pit flows into the TMF at a rate $Q_{pit}(t)$ $[L^3/T]$ with an acidity concentration of c_{pit} $[M/L^3]$, measured in units of moles acidity per litre.
3. precipitation with zero acidity $c_{precip} = 0$ $moles/l$ enters the TMF as precipitation on the surface of the TMF. Assume that the precipitation rate is given from data and can be represented in your model equations as $P(t)$, $[L/T]$ with units of mm/day. Assume that the surface area of the TMF is A $[L^2]$ with units of m^2. Assume that area **does not change** as the volume in the TMF changes (an approximation that is valid for small changes in volume).
4. evaporation removes water from the TMF at a rate that can be represented in your equation as $ET(t)$ $[L/T]$ with units of mm/day.
5. the TMF is well stirred, such that the concentration of acidity in the TMF, c_{TMF} $[M/L^3]$, measured in units of moles acidity per litre of water, is the same at all points in the pond at each instant in time.
6. water is discharged from the TMF at a rate $Q_{dis}(t)$ $[L^3/T]$ with an acidity concentration of c_{TMF}.

7. assume all other sources and sinks of water are negligible and can be ignored.

Table 1: Summary of variables for Q1

Variable	Description	Dimension
$V(t)$	Volume in the TMF	$[L^3]$
$Q_{pit}(t)$	Rate of flow from pit into TMF	$[L^3/T]$
$c_{pit}(t)$	Concentration of acidity in pit water	$[M/L^3]$
$P(t)$	Precipitation rate	$[L/T]$
$ET(t)$	Evaporation rate	$[L/T]$
A	Surface area of the TMF m	$[L^2]$
$c_{TMF}(t)$	Concentration of acidity in the TMF	$[M/L^3]$
$Q_{dis}(t)$	Rate of discharge out of TMF	$[L^3/T]$

Q1a [4 points] Write a mathematical expression that describes how the volume of water in the TMF changes over a time interval from time t to time $t + \Delta t$. That is, complete the following equation:

$V(t + \Delta t) - V(t) =$

Q1b [2 points] Write the expression from part a) as a **differential equation** governing the rate of change of the water volume in the TMF. That is, complete the following equation:

$\frac{dV}{dt} =$

Q1c [5 points] Write a **differential equation**, or the **differential equations** that allow you to compute the **concentration** of acidity in the TMF $c_{TMF}(t)$ with time.

Q1d [2 points] Provide a brief explanation for your answer to Q1c, or if you don't think you got the answer, how you think the problem should be approached.

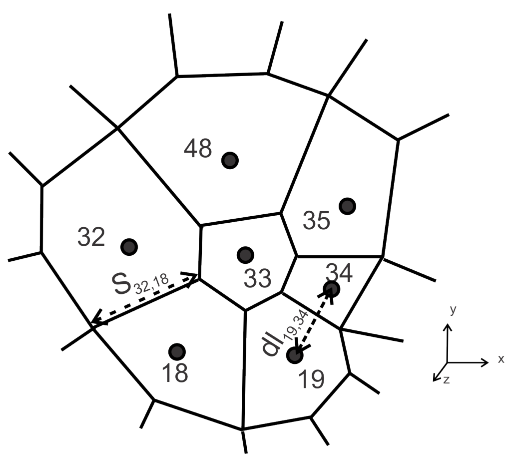
Q2 Finite - volume methods can be generalized from orthogonal N-S-E-W meshes to unstructured meshes. Below is a subsection from an **unstructured** finite-volume mesh. The gridblocks are labeled with their gridblock number. Because this is an unstructured mesh, the gridblock numbers do not follow a regular pattern as on a rectangular mesh.

Notes:

1. The mesh is a constant thickness Δz (into the page).
2. The length of the interface between two gridblocks is given by $S_{i,j}$ where i, j are two adjacent gridblocks that share an interface. For example, $S_{32,18} = S_{18,32}$ is the length of the interface between gridblocks 32 and 18, as indicated on the figure.
3. The distance separating nodes in adjacent gridblocks is given by the distance $dl_{i,j}$. For example as inidicated on the figure, the nodes in gridblocks 34 and 19 are separated by a distance $dl_{34,19} = dl_{19,34}$.

Q2a [3 points] Write a discrete appoximation for the mass flux rate by diffusion (non-porous media) between gridblocks 18 and 33, $J_{18,33}$ $[M/T]$. The component of specific diffusive flux in a direction l is given by $j = -D\frac{dc}{dl}$, where dl $[L]$ is the distance in the l direction and D $[L/T]$ is the diffusion coefficient. This is the non-porous-media form of Fick's law.

Q2b [3 points] Write the finite-volume discrete approximation of the conservation equation for **steady-state** diffusion for gridblock 33 in terms of the concentration in gridblock 33, c_{33}, the concentrations in the gridblocks adjacent to gridblock 33, c_i (use the numbers given in the figure), the geometry of the mesh given by interfacial lengths, $S_{i,j}$, distance separating nodes, $dl_{i,j}$ and the thickness of mesh Δz.



Q3 Consider the following linear system which is the result of a finite-volume discretization of a physical problem:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 0 \\ -0.005 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad Ax = B \tag{1}$$

Q3a [3 points] Write the equations given by the first, second and last rows of the linear system. Write the equations in terms of the variables x_0, x_1, $\ldots$ (eg. the pythonic convention, with zero as the first index).

Q3b [2 points] What is the dimension of the physical system represented by this system of equations: 1, 2 or 3 dimensional? To receive credit, you must explain your answer.

Q3c [2 points] Which physical processes could be described by the linear system? To receive credit, you must explain your answer.

Q3d [2 points] Are there any source terms? If yes, specify where (and if it is a source (positive) or a sink (negative) source term).

Q3e [2 points] Describe the physical boundary conditions that are represented in this system. Refer to specific gridblock numbers.

Q4 Pandas: consider the following dataframe

The following dataframe gives the temperature in deg C for 4 years, marked as either "wet" or "dry"

```python
import pandas as pd
data=[[1970,2,14,22,10,'wet'],
      [1971,4,16,24,11,'dry'],
      [1972,6,18,26,12,'wet'],
      [1973,8,12,28,13,'wet']]
df=pd.DataFrame(data,
      columns=['year','djf','mam',
               'jja','son','precip'])
df.set_index('year',inplace=True)
df.head()
```

	djf	mam	jja	son	precip
year					
1970	8	8	8	8	wet
1971	9	9	9	9	dry
1972	10	10	10	10	wet
1973	11	11	11	11	wet

Q4a [**1 point**] What does the following line print?

```python
print(df.loc[1973,'djf'] - df.loc[1970,'son']
```

Q4b 2 points

Given the dataframe above, suppose we execute the following commands

```python
wet_dry=df.groupby('precip')
precip_dict={key:the_df for key,the_df in wet_dry}
```

After execution of those two lines, what are the keys and values inside `precip_dict`? For the keys, list them explicitly. For the values give their types and a general description of the data/rows they contain.

Q4c 2 points

How would you calculate the mean temperature of the wet years given `precip_dict`? Your python doesn't have to execute, but it should unambiguously specify your intent. What is the type and content of the result returned by your code?

Q5 Classes and functions

Consider the following class:

```
import numpy as np
class hold_data:
    def __init__(self,nrows=2,ncols=2):
        self.data_array=np.zeros([nrows,ncols],dtype=np.float)

start_values = hold_data()
```

5a [1 point] Class constructor

What does the following python statement print?

`print(start_values.data_array)`

5b [2 points] Member functions

Add a member function to `hold_data` called `add_one` that adds one to every element of `start_values.data_array` and returns the new incremented array, and show how you would call it using `start_values`