EOSC 213 Quiz 4 March 28th, 2019
Name: ID:

EOSC 213 - Quiz

Instructions (30 points in total)

- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 45 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Question 0: English is a funny language test

Q0 Complete the sentence with your favourite word. *If vegetarians consume vegetables, then humanitarians consume .....* [**1 point**]

Question 1: Finite-difference approximations

Let us consider the exponential function described in equation 1

$$f(x) = \exp(x). \tag{1}$$

Q1a Compute a first-order approximation to the first derivative of the exponential function at $x = 0$ ($f'(0)$) using $\Delta x = 0.2$. [**2 points**]

Q1b Compute a second-order approximation to the first derivative of the exponential function at $x = 0$ ($f'(0)$) using $\Delta x = 0.2$. [**2 points**]

Question 2: Diffusion Boundary Value Problem

The 1D transient diffusion equation with homogeneous diffusion coefficient can be written as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}. \tag{2}$$

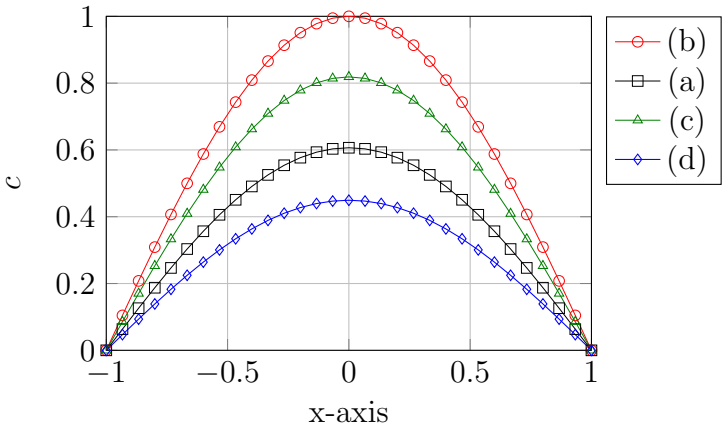
Let us consider the physical domain $x \in [-1; 1]\,\mathrm{m}$, with the specified (Dirichlet) boundary conditions $c(1,t) = c(-1,t) = 0$ (where, for example, $c(1,t)$ should be read as the concentration at $x = 1$ and all times t) and initial condition $c(x,0) = c_0 \cos\left(\frac{\pi x}{2}\right)$. Consider the following function:

$$c(x,t) = c_0 \cos\left(\frac{\pi x}{2}\right) \exp\left(-\alpha t\right) \tag{3}$$

Q2a Show that the function described in equation 3 satisfies the two boundary conditions at all times [**2 points**]

Q2b Show that the function described in equation 3 satisfies the partial differential equation 2 at every point in the domain. **Justify** your answer. [**2 points**]

Q2c Concentrations were measured at four different times and are represented in the graph below. The messy person who did the measurements does not remember at which time these measurements were taken. Can you help him, using your physical intuition? Label the 4 curves in termporal order where $t_1 < t_2 < t_3 < t_4$. [**2 points**]



Q2d What can you say about the change in the total mass in the system with time? Is that consistent with the boundary conditions? Explain. [**2 points**]

Q2e Use your physical reasoning (and equation 3) to describe the asymptotic/final solution (concentration versus x).. **[2 points]**

Question 3: Conservation equation The general conservation (or sometimes called continuity) equation says:

$$\frac{\partial \,\text{stuff}}{\partial t} = -\overrightarrow{\nabla} \cdot \overrightarrow{j}, \tag{4}$$

or, in cartesian coordinates

$$\frac{\partial \,\text{stuff}}{\partial t} = -\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z}. \tag{5}$$

$\overrightarrow{j}$ is a flux vector that describes the rate at which "stuff", the conserved quantity, fluxes (moves).

Q3a Provide two examples of what "stuff", the **conserved quantity**, could represent. **[2 points]**

Q3b Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case of the diffusion of a solute (not in a porous media) where the flux is given by Fick's law $\overrightarrow{j_{\text{diff}}} = -D\overrightarrow{\nabla} c$ **[3 points]**

Q3c What is the physical meaning of the term $\frac{\partial \,\text{stuff}}{\partial t}$ in the case of a diffusing solute? Provide a one or two sentence(s) explanation. **[2 points]**

Q3d Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case where the flux is given by advection $\overrightarrow{j_{\text{adv}}} = \overrightarrow{v}\, c$ **[3 points]**

Question 4: Python

Q4a 3 pts Consider the following dataframe **df**, with rows, columns and index given by:

date	temperature	veloc	mass	mass_tenths
2019-03-25 05:00:00+00:00	267.439863	74.146461	0.206719	0.2
2019-03-25 05:10:00+00:00	281.372218	-2.317762	0.611744	0.6
2019-03-25 05:20:00+00:00	278.318157	3.683598	0.296801	0.3
2019-03-25 05:30:00+00:00	266.754425	-83.851746	0.738440	0.7
2019-03-25 05:40:00+00:00	271.826184	-68.338026	0.879937	0.9

In the space below write python statements (single lines) that would return the following:

1. The row corresponding to March 25, 2019 at 5:30 am UCT
2. The temperature for March 25, 2019 at 5:10 am UCT
3. All temperatures in the dataframe

Q4b 3pts Write a python snippet that would use groupby to find the median velocity for all objects with `mass_tenths`=0.7 kg for this dataframe