EOSC 213 - Quiz

Instructions (20 points in total)

- Read the examination before beginning.

- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).

- You have exactly 30 minutes for the examination.

- Be as precise and clear as possible.

- This is a closed book examination.

- If you get stuck, make an assumption, state what it is and try to carry on.

Question 1: Taylor's approximation
Let us consider the exponential function 2

$$f(x) = \exp(x). \tag{1}$$

We are going to try to compute its derivative at $x = 0$ using backward, forward and centered differences. You can use a **calculator** to compute the values of $f(x)$.

Q1a Compute the derivative of the exponential function at $x = 0$ ($f'(0)$) with a **first order** approximation **and** assess your error.

Q1b Compute the derivative of the exponential function at $x = 0$ ($f'(0)$) with a **second order** approximation **and** assess your error.

The Taylor series of the exponential function is:

$$\begin{aligned} f(x) &= 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ... \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \end{aligned} \tag{2}$$

Q1c Write the second order approximation of the exponential function and write the truncation error.

Question 2: Diffusion

The 1D transient diffusion equation with homogeneous diffusion coefficient can be written as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}. \tag{3}$$

Let us consider the phyiscal domain $x \in [-1; 1]$ m, with the specified boundary conditions $x(1) = x(-1) = 0$. Consider the following function:

$$c(x,t) = c_0\cos\left(\frac{\pi x}{2}\right)\exp\left(-\alpha t\right) \tag{4}$$

Q2a Show that the function described in equation 4 solves the diffusion problem described by equation 3 [**2 points**]

Q2b Can you identify the value of α [**2 points**]

Q2c Show that the function described in equation 4 satisfies the boundary conditions at all times [**2 points**]

Q2d Concentrations were measured at three different times and are represented in the graph below. The messy person who did the measurements does not remember at which time these measurements were taken. Can you help him, using your physical intuition? Give the temporal sequence of the 4 curves. [**2 points**]
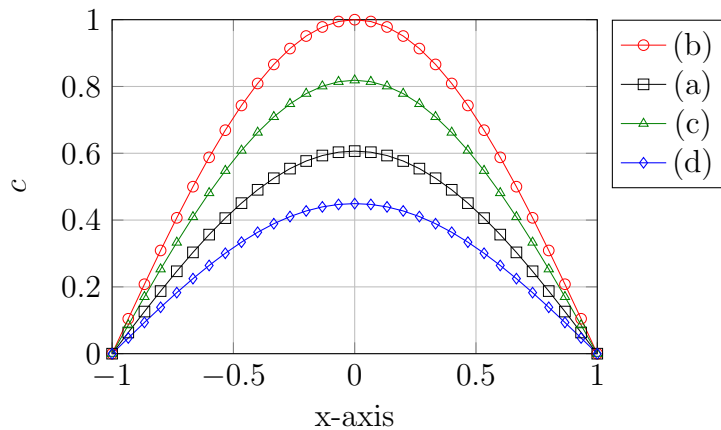
Q2e What can you say about the mass evolution in the system? Is that consistent with the boundary conditions?

Q2f Use your physical reasoning (and equation 4) to describe the asymptotic/final solution.

Question 3: Conservation equation The continuity equation says:

$$\frac{\partial \text{Stuff}}{\partial t} + \text{div} \, \vec{j} = 0 \tag{5}$$

Q3a Use this equation to describe the advective movement of solutes

Python Questions
This is a question about the following code:

```python
class Problem_Def:
    """
    this class holds the specifcation for the domain,
    including the value of the porosity
    """

    def __init__(self, nx, ny, poro, wx, wy):
        self.nx = nx
        self.ny = ny
        self.poro = poro
        self.wx = wx
        self.wy = wy


def get_spacing(nx=4, ny=3, poro=0.4, wx=10, wy=20):
    the_prob = Problem_Def(nx, ny, poro, wx, wy)
    delx = the_prob.wx / the_prob.nx
    dely = the_prob.wy / the_prob.ny
    return delx, dely
```

Q9 Given the code above, what does the following python statement print? [**2 points**]

```python
print(f"{get_spacing(nx=6)}")
```

Q10 modify the `Problem_Def` class to incorporate `get_spacing` as an instance method [**2 points**]
That is, create a version of `Problem_Def` for which the following will work::

```python
the_instance = Problem_Def()
delx, dely = the_instance.get_spacing()
```

where the new constructor has the signature::

```python
def __init__(self,nx=4,ny=3,poro=0.4,wx=10,wy=20):
    ...
```