EOSC 213 Quiz 4 — March 28th, 2019

Name: — ID:

EOSC 213 - Quiz

Instructions (50 points in total)

- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 45 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Question 0: English is a funny language test

Q0 Complete the sentence with your favourite word. *If vegetarians consume vegetables, then humanitarians consume .....* [**1 point**]

Question 1: Taylor's approximation

Let us consider the exponential function described in equation 1

$$f(x) = \exp(x). \tag{1}$$

We are going to try to compute its derivative at $x = 0$ using backward, forward and centered differences. You can use a **calculator** to compute the values of $f(x)$.

Q1a Compute the derivative of the exponential function at $x = 0$ ($f'(0)$) with a **first order** approximation **and** assess your error. [**2 points**]

Q1b Compute the derivative of the exponential function at $x = 0$ ($f'(0)$) with a **second order** approximation **and** assess your error. [**2 points**]

The Taylor series of the exponential function is:

$$\begin{aligned} f(x) &= 1 + x + \tfrac{x^2}{2!} + ... + \tfrac{x^n}{n!} + ... \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \end{aligned} \tag{2}$$

Q1c Write the second order approximation of the exponential function and write the truncation error. [**2 points**]

Question 2: Diffusion

The 1D transient diffusion equation with homogeneous diffusion coefficient can be written as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}. \tag{3}$$

Let us consider the physical domain $x \in [-1; 1]\,\mathrm{m}$, with the specified boundary conditions $x(1) = x(-1) = 0$. Consider the following function:

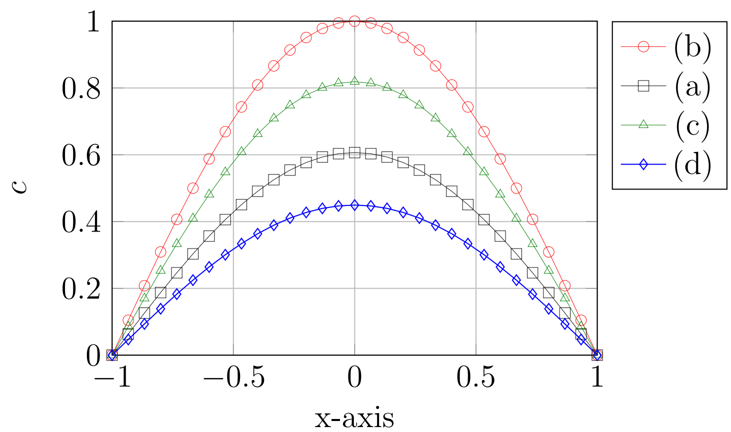
$$c(x,t) = c_0 \cos\left(\frac{\pi x}{2}\right) \exp\left(-\alpha t\right) \tag{4}$$

Q2a **Prove** that the function described in equation 4 solves the diffusion problem described by equation 3 [**2 points**]

Q2b Can you identify the value of α [**2 points**]

Q2c Show that the function described in equation 4 satisfies the boundary conditions at all times [**2 points**]

Q2d Concentrations were measured at four different times and are represented in the graph below. The messy person who did the measurements does not remember at which time these measurements were taken. Can you help him, using your physical intuition? Give the temporal sequence of the 4 curves. [**2 points**]



Q2e What can you say about the mass evolution in the system? Is that consistent with the boundary conditions? [**2 points**]

Q2f Use your physical reasoning (and equation 4) to describe the asymptotic/final solution. [**2 points**]

Question 3: Conservation equation The continuity equation says:

$$\frac{\partial \,\mathrm{Stuff}}{\partial t} = -\overrightarrow{\nabla}\overrightarrow{j}, \tag{5}$$

or, in cartesian coordinates

$$\frac{\partial \,\mathrm{Stuff}}{\partial t} = -\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z}. \tag{6}$$

$\overrightarrow{j}$ is a flux vector that describes the rate at which "stuff", the conserved quantity, fluxes (moves).

Q3a Provide two examples of what "Stuff", the **conserved quantity**, could represent. [**2 points**]

Q3b Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case of the diffusion of a solute (not in a porous media) where the flux is given by Fick's law $\overrightarrow{j_{\mathrm{diff}}} = -D\overrightarrow{\nabla}c$ [**3 points**]

Q3c What is the physical meaning of the term $\frac{\partial \,\mathrm{Stuff}}{\partial t}$ in the case of a diffusing solute. Provide a one or two sentence(s) explanation. [**2 points**]

Q3d Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case where the flux is given by advection $\overrightarrow{j_{\text{adv}}} = \vec{v}c$ **[3 points]**

Question 4: Conservation equation (2)

Consider the fluid velocity $\vec{v} = (v_x, v_y, v_z) \left[\frac{L}{T}\right]$ and the density $\rho \left[\frac{M}{L^3}\right]$, and the following equations:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho v_x}{\partial x} - \frac{\partial \rho v_y}{\partial y} - \frac{\rho \partial v_z}{\partial z}. \tag{7}$$

Q4a Is the previous equation dimensionnally homogeneous? Show why or why not. **[3 points]**

Q4b What is the expression for the flux in the x, y and z direction, *i.e.*, what are j_x, j_y and j_z? **[3 points]**

Q4c Write in words the meaning of the time derivative term $\frac{\partial \rho}{\partial t}$ **[2 points]**

Q4d Simplify this equation for the case where the fluid is incompressible. Mathematically, that is where the fluid density ρ does not vary in space or change in time. **[3 points]**

Question 5: Python

This is a question about the following code:

```
class Problem_Def:
    """
    this class holds the specifcation for the domain,
    including the value of the porosity
    """

    def __init__(self, nx, ny, poro, wx, wy):
        self.nx = nx
        self.ny = ny
        self.poro = poro
        self.wx = wx
        self.wy = wy


def get_spacing(nx=4, ny=3, poro=0.4, wx=10, wy=20):
    the_prob = Problem_Def(nx, ny, poro, wx, wy)
    delx = the_prob.wx / the_prob.nx
    dely = the_prob.wy / the_prob.ny
    return delx, dely
```

Q5a Given the code above, what does the following python statement print? [**4 points**]

```
print(f"{get_spacing(nx=6)}")
```

Q5b modify the `Problem_Def` class to incorporate `get_spacing` as an instance method [**3 points**]

That is, create a version of `Problem_Def` for which the following will work::

```
the_instance = Problem_Def()
delx, dely = the_instance.get_spacing()
```

where the new constructor has the signature::

```
def __init__(self,nx=4,ny=3,poro=0.4,wx=10,wy=20):
    ...
```

Q5c something [**3 points**]