

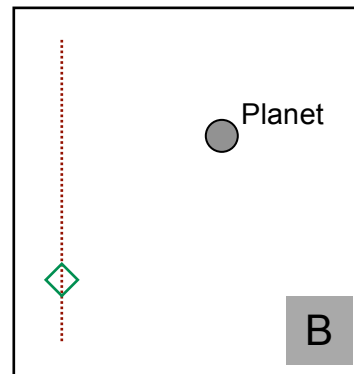
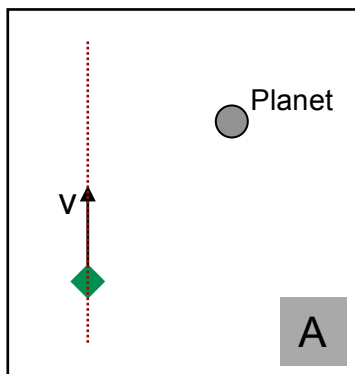
## **EOSC 211: Some Numerical Integration - Assignment 2 background**

### **Movies**

Sketch how you think the spacecraft's flight path and speeds in the two other trajectories will compare with the one shown.

### **Sketching out the Problem:**

1. sketch the direction of the force and acceleration on the spacecraft in A.



2. sketch the spacecraft's position, velocity and acceleration at some time  $t = \Delta t$  later in B.

### **Calculations:**

We will use the initial conditions from Part 5 of the assignment:

$$s_{x0} = -3050 \text{ km}$$

$$s_{y0} = -3 * R_{\text{merc}}$$

$$v_{y0} = 7 \text{ km/s in the positive y-direction}$$

$$v_{x0} = 0$$

The values for the mass and radius of Mercury and the gravitational constant,  $G$ , are

$$M_{\text{merc}} = 3.3 \times 10^{23} \text{ kg}$$

$$R_{\text{merc}} = 2440 \text{ km}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

**Step 1:** Fill in the initial conditions – ie the x-y coordinates of speed and position of the spacecraft at time  $t=0$  in cols 5-8 of row 1 of the table. Calculate,  $s$  the distance of the spacecraft from the planet, and  $s^2$ .

t (s)	$\Delta v_x$ (m/s)	$\Delta v_y$ (m/s)	$\Delta s_x$ (m)	$\Delta s_y$ (m)	$v_x$ (m/s)	$v_y$ (m/s)	$s_x$ (m)	$s_y$ (m)	$s^2$ (m)	a (m/s <sup>2</sup> )	$a_x$ (m/s <sup>2</sup> )	$a_y$ (m/s <sup>2</sup> )
0	XXX	XXX	XXX	XXX								
60												
120												

**Step 2:** Calculate the magnitude of the acceleration,  $a$ , on the spacecraft at time,  $t=0$  due to the planet and add it to the table above. Resolve the acceleration into its x- and y- coordinates and fill these in ( $a_x$  and  $a_y$ ).

**Step 3:** If we assume the acceleration is constant over a time interval  $\Delta t$ , then after the time  $\Delta t$  there is a change in velocity due to this acceleration. This is in the direction of the acceleration vector so the easiest thing is to work in terms of the x- and y- components of the change in velocity.

Write down the equations for the x- and y- components of the *change in velocity* in terms of the x- and y- components of acceleration and the time interval  $\Delta t$ .

$$\Delta v_x = \underline{\hspace{2cm}}$$

$$\Delta v_y = \underline{\hspace{2cm}}$$

Similarly there is a change in position, which depends both on the velocity at the beginning of the time interval and on the acceleration:

$$\Delta s_x = v_x \Delta t + \frac{1}{2} a_x \Delta t^2$$

$$\Delta s_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$$

Fill in these changes in velocity and position (occurring after 60 seconds) in row 2, columns 1-4 of the table.

**Step 4:** The new velocity after a time  $\Delta t$  is the initial velocity plus the change in velocity.

So in our table above we can calculate the x- and y-components of velocity at  $t=60$  seconds using

$$v_x^{t=60} = v_x^{t=0} + \Delta v_x$$

$$v_y^{t=60} = v_y^{t=0} + \Delta v_y$$

and the x- and y- components of position at  $t= 60$  seconds using

$$s_x^{t=60} = s_x^{t=0} + \Delta s_x$$

$$s_y^{t=60} = s_y^{t=0} + \Delta s_y$$

Calculate the new  $v_x$ ,  $v_y$ ,  $s_x$ ,  $s_y$  and fill in columns 5-8 of the second row of the table.

**Step 5:** You can now see that you can essentially repeat steps 2-4 for each successive 60 seconds of the spacecraft's trajectory using Steps 2-4 above. To calculate the entire spacecraft trajectory you would repeat steps 2-4 until you have reached a time  $t = t_{\text{final}}$  (given as 40 minutes in part 5 of the assignment). **Hint:** If you do this by hand (correctly!) for the first 3 rows given you can of course check your MATLAB code using this table....