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In this paper we demonstrate how certain very ordinary economic principles lead maximizing individuals to choose consumption-production plans that display many of the characteristics commonly associated with business cycles. Our explanation is entirely consistent with (i) rational expectations, (ii) complete current information, (iii) stable preferences, (iv) no technological change, (v) no long-lived commodities, (vi) no frictions or adjustment costs, (vii) no government, (viii) no money, and (ix) no serial dependence in the stochastic elements of the environment. We also provide a completely worked out example of the type of artificial economy we have in mind. The time-series properties of the example exhibit some major features of observed business cycles. Although this type of model may not be capable of explaining all of the regularities in actual business cycles, we believe that it provides a useful, well-defined benchmark for assessing the relative importance of factors (e.g., monetary disturbances) that we have deliberately ignored.

I. Introduction

The term "business cycles" refers to the joint time-series behavior of a wide range of economic variables such as prices, outputs, employment, consumption, and investment. In actual economies, this behav-

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ior seems to be characterized by at least two broad regularities: (1) Measured as deviations from trend, the ups and downs in individual series exhibit a considerable amount of persistence. Given that a variable is currently above (below) its time trend value, it tends to stay above (below) trend for some time. (This is a meaningful restriction only to the extent that deviations from trend form a stationary, zero-mean process.) (2) Most important, measures of various economic activities (e.g., outputs in different sectors) move together. At times when one measure is above (below) its trend, others tend also to be above (below) their trends.

These and other more specific regularities (e.g., the relative amplitudes of different series) appear to be quite general features of market economies.[1] As Lucas (1977, p. 10) argues:

> There is, as far as I know, no need to qualify these observations by restricting them to particular countries or time periods: they appear to be regularities common to all decentralized market economies. Though there is absolutely no theoretical reason to anticipate it, one is led by the facts to conclude that, with respect to the qualitative behavior of co-movements among series, *business cycles are all alike*. To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles, grounded in the *general* laws governing market economies, rather than in political or institutional characteristics specific to particular countries or periods.

In the sections that follow, we offer one explanation of the sort that Lucas suggests.

The economics literature of the last hundred years contains at least several dozen distinct and reasonably well-defined explanations of business-cycle phenomena.[2] We make no attempt to catalog these explanations, but anyone familiar with them will recognize our very deliberate attempt to illustrate our explanation in the context of a model which, by construction, minimizes the scope of these other explanations. Our basic explanation, for example, is entirely consistent with the following sorts of assumptions: (i) rational expectations; (ii) complete information; (iii) stable preferences; (iv) no technological change; (v) no long-lived commodities; (vi) no frictions or adjustment

[1] The empirical literature on business cycles is far too large to survey here. Burns and Mitchell (1946) is a classic example, however. See also Hodrick and Prescott (1981) for a recent description of some aspects of postwar U.S. business cycles.

[2] For a well-known survey, see Haberler (1963).

costs; (vii) no government; (viii) no money; and (ix) no serial dependence in stochastic elements of the environment.

Our purpose is not to dispute the explanatory power of particular hypotheses that are inconsistent with the foregoing assumptions. Instead, we simply want to focus narrowly on the joint explanatory power of two more fundamental hypotheses that are generally consistent with all business-cycle theories of which we are aware. One of these hypotheses concerns consumer preferences. The other concerns production possibilities.

The preference hypothesis is that all dated consumption goods (including leisure) that are demanded in positive amounts at any given prices are strictly normal goods at those prices. At given prices, this hypothesis implies that consumers want to "spread" any unanticipated wealth increment over both time and commodities. This is a very commonplace idea—at least with respect to the implication that consumers want to save a portion of any windfall gain. What is not so often emphasized is the implication that consumers want to allocate their incremental savings to incremental future consumption of all individual goods (including leisure) in their consumption "basket." From the consumer demand side of the market, this suggests comovement as well as persistence in desired commodity/leisure consumption series.

By itself, the preference hypothesis just described may suggest business-cycle phenomena like persistence and comovement, but it cannot explain them. Economic equilibrium always involves a confrontation between preferences and possibilities. An event that results in an unanticipated wealth increment is generally not of the sort that physically allows consumers to consume more of everything in the near future, much less more of everything now. To accommodate physical possibilities, prices will change in response to the event. Thus, the preference hypothesis, which is conditional on given prices, is not immediately applicable. Some specification of physical possibilities is required to complete the picture.

Our production possibilities hypothesis is that nontrivial capitalistic production (employment of a variety of produced inputs) is feasible and generally efficient. In this context we assume constant returns to scale, smooth substitutability of inputs, and strictly diminishing marginal productivity of any given input in any given employment. We also assume that each commodity may have many alternative uses (e.g., direct consumption and alternative employments as an input). This general description of production possibilities implies a large range of both intratemporal and intertemporal substitution opportunities.

Our preference hypothesis implies business-cycle-inducing behavior at constant exchange rates. To some extent this "smoothing" behavior will also be observed if equilibrium exchange rates do not change too much in response to exogenous output shocks. The large range of real substitution possibilities in our production hypothesis limits the size of relative price changes required to clear markets when shocks occur, and we suggest that this limit is sufficiently tight that, in equilibrium, consumers will choose to transform an unexpected increment in any particular output into increased current and future consumption of a variety of goods. Moreover, this choice is not forced by technological constraints or efficiency conditions. The same real substitution possibilities that allow consumers to smooth the effects of shocks also allow consumers to absorb efficiently the effects of shocks entirely in current consumption (which results in neither persistence nor comovement). Thus, any business-cycle regularities observed in the equilibrium of our model economy are chosen in preference to available, efficient, "no-business-cycle" alternatives.[3]

We believe that major features of observed business cycles typically will be found in the kind of model economy outlined above. If this is so, then actual business-cycle fluctuations should not be viewed entirely, if at all, as welfare-reducing deviations from "natural rate" paths of an ideally efficient Walrasian economy. By construction, no part of the behavior of prices and economic aggregates in our model can be attributed to monetary disturbances, government activity, incomplete information, biased or inefficiently formed expectations, nonmaximizing behavior ("animal spirits"), adjustment costs, or any sort of market failure. If business-cycle phenomena are present in the behavior of our model economy, they are perfectly consistent with ideal economic efficiency.

In the sections that follow, we formally define our model and present a completely worked out example with exact closed form solutions for all commodity and labor/leisure allocations, relative prices, and real interest rates. Section II contains the formal description of the model. The worked out example is contained in Sections III and IV. Section III focuses on the equilibrium quantity allocations and relative prices as functions of the current "state" of the economy. Section IV examines the stochastic behavior of the example in terms of modern time-series analysis.

[3] Black (1979) has emphasized the idea that consumers may rationally choose business-cycle plans even though efficient no-business-cycle plans are available. In his model, this is a choice of a greater degree of instability and uncertainty in return for higher expected rates of return on investment.

II. The Formal Model

The model economy we consider is populated by a single infinite-lived individual (or a constant number of identical individuals) with given initial resources, production possibilities, and tastes. The individual ("Robinson Crusoe") chooses a preferred consumption-production plan, and we interpret this plan, together with Crusoe's marginal rates of substitution, as the quantities and relative prices that evolve in a particular competitive market economy. For our purposes, this abstraction is the simplest device for generating examples of multisector rational expectations equilibria.[4]

All activities in the economy may be described as repetitions of the following one-period cycle. At the beginning of each period, Crusoe chooses (*a*) the commodity bundle to be consumed during the period, (*b*) the amount of leisure time to be consumed during the period, and (*c*) the commodity and labor inputs to various production transformations that will be completed during the period. All of these choices are constrained by the total commodity stocks available at the beginning of the period and by the (fixed) amount of time available per period (for leisure and work). During the period, various exogenous random shocks influence the production transformations. These shocks, together with input choices made at the beginning of the period, then determine the total commodity stocks that will be available at the beginning of the next period.

All commodities in the economy are produced. In general, any given commodity may be used as an input in the production of other commodities, and production of any one commodity requires positive inputs of other commodities. Thus, production is capitalistic in the sense that a variety of produced inputs are employed. Finally, we assume that all commodities are "perishable." Commodity stocks available at the beginning of a period consist entirely of "new units" produced during the previous period. In terms of the standard stock-flow relation in growth models, we are assuming a depreciation rate of 100 percent per period.[5]

[4] The model we employ is quite similar to the model described in Prescott and Mehra (1980). Their remarks (p. 1365) about the identical consumers assumption (i.e., it is not quite as restrictive as it may appear) and their treatment of the optimality of competitive equilibrium are particularly relevant. They do not, however, explicitly consider the business-cycle implications of their models.

[5] This assumption is not essential in our general model. It does, however, simplify the example worked out in Sec. III. Moreover, it allows us to emphasize some business-cycle mechanisms that are not readily explainable in terms of the stock/flow distinctions introduced by durability (e.g., Clark's [1917] famous "acceleration principle").

Mathematical Formulation

In the environment described above, Crusoe chooses his allocation plan to maximize the expected value of his utility, U, subject to resource availability and production possibilities. As viewed at time 0 ($t = 0$), we assume that U takes the form:

$$U \equiv \sum_{t=0}^{\infty} \beta^t u(C_t, Z_t), \quad 0 < \beta < 1, \tag{1}$$

where β is a discount factor, C_t is an $N \times 1$ vector of commodity consumption in period t, and Z_t is the amount of leisure time consumed in period t. The essential feature of (1) is that Crusoe's tastes are assumed to be constant over time and uninfluenced by exogenous random shocks.[6]

The production possibilities for the N commodities in the economy exhibit constant returns to scale and are represented by the following vector-valued function:

$$Y_{t+1} = F(L_t, X_t; \lambda_{t+1}), \tag{2}$$

where

Y_{t+1} ≡ an $N \times 1$ vector whose ith element, $Y_{i,t+1}$, is the total stock of commodity i available at time $t + 1$.

$F(\cdot, \cdot; \cdot)$ ≡ an $N \times 1$ vector-valued function that is concave and linearly homogeneous with respect to L_t and X_t.

L_t ≡ a vector of labor inputs allocated at time t. In the case of no joint production, L_t is an $N \times 1$ vector whose ith element, L_{it}, is the number of hours allocated at time t to the production of commodity i.

X_t ≡ a matrix of commodity inputs allocated at time t. In the case of no joint production, X_t is $N \times N$ and its i,j element, X_{ijt}, is the quantity of commodity j allocated at time t to the production of commodity i.

λ_{t+1} ≡ a random vector whose value is realized at time $t + 1$. The vector-valued stochastic process $\{\lambda_t\}$ is assumed to be an observable, time-homogeneous Markov process.[7]

[6] The preference ordering of consumption subsequences $[(C_t, Z_t), (C_{t+1}, Z_{t+1}), \ldots]$ induced by (1) does not depend on t or on consumption prior to time t. This is in contrast to other models (e.g., Kydland and Prescott 1981) where preference for current leisure depends on the amount of leisure consumed in the recent past.

[7] The notation $\{\lambda_t\}$ denotes the infinite stochastic sequence $\lambda_0, \lambda_1, \ldots, \lambda_t, \ldots$. A time-homogeneous Markov process has the property that the conditional distribution of $\lambda_{t+\tau}$ ($\tau \geq 1$) given $\lambda_t, \lambda_{t-1}, \lambda_{t-2}, \ldots$ depends only on τ and the value of λ_t. Given this, such

In the remainder of the paper we often emphasize special cases of this production technology in which (*a*) there is no joint production, (*b*) there is no technological change (the vectors in the sequence $\{\lambda_t\}$ are independent and identically distributed), and (*c*) given L_t and X_t, the elements of $Y_{t+1} = F(L_t, X_t; \lambda_{t+1})$ are independently distributed. In these special cases, business-cycle regularities are not directly imposed on the model economy by the nature of the production functions or exogenous shocks.[8]

The basic mathematical representation of the economy is completed by two resource constraints that must be satisfied at each date. With H denoting the total time available per period, labor/leisure choices are constrained by

$$Z_t + \sum_{i=1}^{N} L_{it} = H, \quad t = 0, 1, 2, \ldots . \tag{3}$$

Commodity allocation is restricted by

$$C_{jt} + \sum_{i=1}^{N} X_{ijt} = Y_{jt}, \quad j = 1, 2, \ldots, N; t = 0, 1, 2, \ldots . \tag{4}$$

Finally, the allocations made at time t (C_t, Z_t, L_t, X_t) must depend only on information observable by Crusoe at time t (e.g., outputs and shocks realized at and prior to time t).

In this model, all of Crusoe's equilibrium allocations (C_t, Z_t, L_t, X_t) and competitive relative prices at time t (including commodity-denominated wage rates) are stationary functions of the state vector $S_t \equiv (Y_t, \lambda_t)$. The relative prices are given by Crusoe's marginal rates of substitution evaluated at the quantities specified by his chosen allocation plan. Analytically, they may be expressed in terms of his marginal utility of current leisure and partial derivatives of his "current welfare function," $V(S_t)$, where[9]

processes are uniquely defined by a "one-step-ahead" conditional distribution function $G(\lambda_{t+1}|\lambda_t)$. Note, however, that our assumption does not constrain the process to be stationary or to have no drift. Moreover, the assumption does not constrain individual elements or scalar-valued functions of λ to be Markov processes. Since the production function F does not depend on the value of t per se, technological change, if any, is represented by drift and/or time-series dependence on the process $\{\lambda_t\}$. Thus, e.g., if the vectors in the sequence $\{\lambda_t\}$ are independent and identically distributed, there is no technological change.

[8] Kydland and Prescott (1981) provide an alternative model that emphasizes the potential explanatory power of both autocorrelation in exogenous shocks and persistence-inducing features of production technology (e.g., durable producer goods and multiperiod input-output lags).

[9] The vector of competitive commodity prices at time t is proportional to the gradient of $V(S_t)$ with respect to Y_t. The wage rate at time t is proportional to $(\partial/\partial Z)\{u[C(S_t), Z(S_t)]\}$.

$$V(S_t) \equiv \max E\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s, Z_s) | S_t\right]$$
$$\text{subject to equations (2), (3), and (4)} \tag{5}$$
$$= E\left\{\sum_{s=t}^{\infty} \beta^{s-t} u[C(S_s), Z(S_s)] | S_t\right\}.$$

Given the initial state of the economy, $S_0 = (Y_0, \lambda_0)$, the allocation rules and price formulae determine consumption, production inputs, and relative prices at time 0. The production function (2) and the vector shock λ_1 then determine $S_1 = (Y_1, \lambda_1)$. In this recursive manner, the intertemporal evolution of equilibrium quantities and prices is defined as a multivariate stochastic process.

A detailed examination of the equilibrium process is most easily conducted in the context of a concrete example. That is the purpose of Sections III and IV, which follow. In these sections, we illustrate the general business-cycle mechanism we have in mind and also find some apparently general relations among relative prices, interest rates, resource stocks, and input employment.

III. An Example

In this section we construct a detailed example of prices and quantities in dynamic competitive equilibrium. In order to carry out this task we make some specific assumptions regarding the form of preferences and production possibilities.

Preferences

In the example, the one-period utility, $u(C_t, Z_t)$, is of the form

$$u(C_t, Z_t) = \theta_0 \ln Z_t + \sum_{i=1}^{N} \theta_i \ln C_{it}, \tag{6}$$

where $\theta_i \geqslant 0$, $i = 0, 1, 2, \ldots, N$. In general, it is presumed that $\theta_0 > 0$. If $\theta_k = 0$ for some $k \geqslant 1$, then commodity k has no direct consumption value, but it may serve as an input in the production of other commodities.

Production Possibilities

We maintain the special assumptions of no joint production and perishable commodities. The specific production functions in the example are given by:

$$Y_{i,t+1} = \lambda_{i,t+1} L_{it}^{b_i} \prod_{j=1}^{N} X_{ijt}^{a_{ij}}, \quad i = 1, 2, \ldots, N. \tag{7}$$

The parameters b_i and a_{ij} are assumed to be nonnegative and constant over time with $b_i + \Sigma_{j=1}^{N} a_{ij} = 1$, $i = 1, 2, \ldots, N$. Except for the stochastic parameters $\{\lambda_t\}$, this is a standard Cobb-Douglas technology.[10]

Expected Utility Maximization

Subject to the production possibilities (7) and the resource constraints given by (3) and (4), Robinson Crusoe chooses a consumption-production plan at time t to maximize

$$E(U|S_t) = E\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s, Z_s)|S_t\right],$$

where $S_t = (Y_t, \lambda_t)$. Moreover, Crusoe's preferences are such that if the welfare function, $V(S_t)$, is defined as the maximum value of $E(U|S_t)$, then V and the optimal consumption-production plan are jointly the solution to

$$V(S_t) = \max \{u(C_t, Z_t) + \beta E[V(S_{t+1})|S_t]\}. \tag{8}$$

In general, functional equations like (8) are solved by "hunt and peck," or iterative procedures. In this particular example, however, dumb luck yields the following solution:

$$V(S_t) = \sum_{i=1}^{N} \gamma_i \ln Y_{it} + J(\lambda_t) + K, \tag{9}$$

where

$$\gamma_j = \theta_j + \beta \sum_{i=1}^{N} \gamma_i a_{ij}, \quad j = 1, 2, \ldots, N, \tag{10a}$$

or, equivalently, defining γ' and θ' as $1 \times N$ vectors with elements $\{\gamma_i\}$ and $\{\theta_i\}$, respectively, and A as the $N \times N$ matrix with elements $\{a_{ij}\}$,

$$\gamma' = \theta'(I - \beta A)^{-1}, \tag{10b}$$

$$J(\lambda_t) = \beta E\left[\sum_{i=1}^{N} \gamma_i \ln \lambda_{i,t+1} + J(\lambda_{t+1})|\lambda_t\right], \tag{11}$$

[10] As noted in Sec. II, the only general assumption about $\{\lambda_t\}$ is that it is a time-homogeneous Markov process. Here, it is also assumed to be strictly positive. The form in which $\lambda_{i,t+1}$ enters (7) admits a wide variety of alternative scenarios with respect to production uncertainty and neutral technological change.

and K is a constant that depends on preference and production parameters but not on Y_t or λ_t.[11]

Optimal Quantities

Optimal consumption and input quantities at time t are given by

$$C^*_{it} = \left(\frac{\theta_i}{\gamma_i}\right)Y_{it}, \quad i = 1, 2, \ldots, N, \tag{12}$$

$$Z^*_t = \theta_0\left(\theta_0 + \beta \sum_{i=1}^{N} \gamma_i b_i\right)^{-1} H, \tag{13}$$

$$X^*_{ijt} = \left(\frac{\beta\gamma_i a_{ij}}{\gamma_j}\right)Y_{jt}, \quad i, j = 1, 2, \ldots, N, \tag{14}$$

and

$$L^*_{it} = \beta\gamma_i b_i\left(\theta_0 + \beta \sum_{j=1}^{N} \gamma_j b_j\right)^{-1} H, \quad i = 1, 2, \ldots, N, \tag{15}$$

where $\{\gamma_i\}$ is given by (10a) and (10b).

The simple algebraic form of these decision rules is due in large part to the particular preferences and production possibilities assumed in the example. This simple form, however, makes it easy to analyze the qualitative features of the rules and to speculate on the generality of these features in alternative preference/production settings.

The behavior of the rules with respect to the variables and model parameters that explicitly appear in the formulae is quite natural and readily understandable. This behavior can be more or less summarized by the following two principles: (1) The portion of the total available stock of a commodity allocated to a given employment (consumption) is an increasing function of its productivity in that employment (consumption value). The same principle applies to the allocation of the time (H) available in a period. (2) The amounts of a commodity (or time) allocated to each of its productive employments and to positively valued consumption are all increasing functions of the total available amount of the commodity (or time).

[11] The validity of this solution for V may not be obvious at this point, but it can be verified in the following manner: Assume V is given by (9) and do the maximization (with respect to time t consumption and input decisions) on the right-hand side of (8). It will then be seen that the maximum on the right-hand side of (8), as a function of S_t, is given by $V(S_t)$ as defined by (9). For a related mathematical analysis, see Radner (1966). Radner assumed nonstochastic Cobb-Douglas production functions and derived optimal allocation rules for several different criteria of optimality (one of which was the discounted utility criterion used in our example). His analysis of the rules focused on their asymptotic properties and on the allocation implications of different optimality criteria.

In terms of business cycles, principle 2 is the most important. It implies that if the output of commodity i is unexpectedly high at time t, then inputs of commodity i in all of its productive employments will also be unexpectedly high at time t. Assuming that the commodity has at least several alternative employments, this not only propagates the output shock forward in time, it also spreads the future effects of the shock across sectors of the economy. At the most simplistic level of analysis, this is the primary explanation of persistence and comovement in the consumption, input, and output time series in our example.

A more remarkable aspect of the example decision rules is the absence of certain variables from their formulae. As stated in Section II, optimal decision rules in this type of model should depend only on contemporaneous values of the state vector $S_t = (Y_t, \lambda_t)$. The rules in this example conform to this principle, but they also exhibit two much more special properties. First, the allocation of any given commodity (or time) does *not* depend on the contemporaneously available amounts of *other* commodities. Second, given Y_t, none of the allocations made at time t depends on λ_t. These two properties of the decision rules are peculiar to this example, but it is nonetheless instructive to ask why they appear in this example and are not generally in the class of models outlined in Section II. Since one of the most commonly observed features of actual business cycles is the procyclical behavior of labor employment (a property *not* exhibited in this example), the following discussion focuses especially on the labor/leisure allocation rules.

Why is Crusoe's labor/leisure allocation at time t independent of both Y_t and λ_t? The idea is to state the efficiency rule—"The (discounted) marginal value product of labor in every positive employment should equal the wage rate"—in terms of utility-denominated prices and wage rates (marginal utilities of commodities and leisure). We then ask, "How do the (utility-denominated) wage rate at time t and the discounted marginal value products of labor inputs at time t depend on Y_t and λ_t?" If, for an initially optimal labor/leisure allocation, the wage rate and marginal value products of labor do not change with changes in Y_t and/or λ_t, then there is no incentive to change the allocation in response to changes in the state variables.

The wage rate at time t is just the marginal utility of leisure at time t. In any model (like our example) with an additive preference representation, this is only a function of the labor/leisure allocation at time t. Thus the issue boils down to the relation between (Y_t, λ_t) and the marginal value products of time t labor inputs.

Looking at the influence of Y_t, note that increases in Y_t generally imply increases in commodity inputs and such increases raise the marginal *physical* product of any given labor input in any given employment. For given labor inputs, however, increases in time t commodity inputs also raise time $t + 1$ outputs and hence lower time $t + 1$ (utility-denominated) commodity prices. Thus the marginal *value* products of labor inputs are subject to two opposing influences as Y_t is varied. In our example, these two opposing influences (higher marginal physical product, lower product price) exactly cancel one another, and the marginal value products of any given input (labor in particular) do not depend on the levels of other inputs. This, together with the constant availability of time per period (H), results in constant labor employment as commodity outputs fluctuate.[12]

The constancy of labor employment is certainly not a generalizable feature of our example. Judging from the example, however, general principles governing the behavior of labor employment should involve comparisons of capital/labor substitutability in production with commodity/leisure and/or present/future substitutability in consumption. (In our example, elasticities of substitution in consumption and production are identical and equal to one.) The lower the elasticity of substitution in production, the more sensitive the marginal physical products of labor are to commodity inputs. The prices of claims to future output relative to the current wage rate should be less sensitive to output shifts the greater the elasticity of substitution in consumption. Our conjecture is that if producers substitute between inputs (as relative prices change) less readily than consumers substitute between commodities and leisure and/or between present and future consumption, then equilibrium labor employment at time t will be positively associated with commodity stocks at time t. Similarly, if consumer demand for claims to future consumption is more elastic than in our example, input employment (including labor) at t will be positively associated with the conditional mean of λ_{t+1}, $E(\lambda_{t+1}|\lambda_t)$.[13]

Prices and Wage Rates

In our example, utility-denominated commodity prices at time t are given by

$$P_{it} = \frac{\partial}{\partial Y_{it}} V(S_t) = \frac{\gamma_i}{Y_{it}}, \quad i = 1, 2, \ldots, N. \tag{16}$$

[12] By a similar argument, the ex post marginal value product of labor is subject to two opposing influences as λ_{t+1} varies. These influences cancel one another in the example. Thus the conditional distribution of λ_{t+1}, which depends on λ_t, does not affect allocations at time t.

[13] Our conjectures about the equilibrium behavior of labor employment are consistent with the standard analysis of the cross-elasticity of derived demand in competitive constant returns to scale industries (e.g., Allen 1938, pp. 369–74). In future research on this topic, we hope to provide a more rigorous analysis.

The utility-denominated wage rate at time t is

$$W_t = \frac{\partial}{\partial Z_t} u(C_t^*, Z_t^*) = \left(\theta_0 + \sum_{i=1}^{N} \gamma_i b_i\right) H^{-1}. \tag{17}$$

In spite of the arbitrary choice of price level that is involved, the simplicity of these prices (and the fact that utility-denominated interest rates are constant using this numeraire—see next subsection) greatly aids analysis of the equilibrium.

Like the quantity decision rules, these prices behave (relative to one another) quite naturally with respect to the elements of Y_t and the preference and production parameters that appear in their formulae. The price of commodity i is higher (relative to other prices) the greater (relative to other commodities) is its scarcity, its productivity $\{a_{ij}\}$, and/or the preference for the commodity (θ_i).

In terms of any given commodity numeraire, the "real" wage rate (e.g., W_{it}/P_{Nt} if commodity N is the numeraire) is higher the greater are the preference for leisure (θ_0), the productivity of labor $\{b_i\}$, and the current output of the numeraire (Y_{Nt}). Thus the real wage rate and commodity output move together. As with the quantity decision rules (and for the same reasons), the fact that the time t prices and wage rate do not depend on λ_t is a peculiarity of our example.

Interest Rates

The "spot" prices discussed above specify intratemporal but not intertemporal exchange rates. Some idea of interest rates is required to complete the system.

In our example, we can observe Crusoe's demand/supply price at any time for any given type of claim to future commodities and convert that asset price to a per period compound interest rate or (if the future payoff of the asset is uncertain) expected rate of return. To illustrate this, consider a riskless claim to one unit of commodity N to be delivered at time $t + 1$. At time t, the price (denominated in commodity N) Crusoe is willing to pay for this claim is

$$\begin{aligned} \frac{\beta E[(\partial/\partial Y_{N,t+1}) V(S_{t+1}) | S_t]}{(\partial/\partial Y_{Nt}) V(S_t)} &= \beta E\left[\left(\frac{Y_{Nt}}{Y_{N,t+1}}\right) \middle| S_t\right] \\ &= \frac{1}{1 + r_{Nt}}, \end{aligned} \tag{18}$$

where r_{Nt} is the one-period (short-term) commodity N rate of interest at time t.[14]

[14] Note also that comparison of (16) and (18) shows that the utility-denominated

To pursue this illustration further, assume that the conditional distribution of $Y_{N,t+1}$, given S_t, is lognormal. Then $Y_{N,t+1}$ can be expressed as $Y_{N,t+1} = Y_{Nt}e^g$, where the conditional distribution of g, given S_t, is normal with mean $[\mu - (\sigma^2/2)]$ and variance σ^2. This implies that $E[(Y_{N,t+1}/Y_{Nt})|S_t] = e^{\mu}$ and $E[(Y_{Nt}/Y_{N,t+1})|S_t] = e^{(\sigma^2-\mu)}$. Thus

$$r_{N,t} \cong \rho + \mu - \sigma^2, \tag{19}$$

where $\rho = \beta^{-1} - 1$.

Formula (19) says that the own rate of return on a riskless claim to future delivery of a commodity is (approximately) equal to Crusoe's utility discount rate, ρ, plus the expected growth rate in availability of the commodity, μ, minus a measure, σ^2, of the uncertainty about the growth rate. The qualitative features of this characterization of r_{Nt} seem quite natural and general. The standard Fisherian model of equilibrium interest rates predicts that they will be higher (i) the greater is the impatience to consume (the greater is ρ), and (ii) the greater is expected future consumption relative to current consumption (as roughly captured by the value of μ). Finally, the more uncertain risk-averse consumers are about the future availability of a commodity, the more they are willing to pay for riskless claims to future delivery (and hence the lower the interest rate on such claims).[15]

Quantity Dynamics

The dynamic behavior of output (and hence of consumption and input employment) is implied by the production functions (7) together with the input decision rules (14) and (15). Since this is most easily expressed in terms of the logs of output, let y_t denote the $N \times 1$ vector $\{\ln Y_{it}\}$. Substituting (14) and (15) into (7) and taking logs yields

$$y_{t+1} = Ay_t + k + \eta_{t+1}, \tag{20}$$

where A is the $N \times N$ matrix $\{a_{ij}\}$, k is an $N \times 1$ vector of constants, and η_{t+1} is the $N \times 1$ stochastic vector $\{\ln \lambda_{i,t+1}\}$.

It is obvious from inspection that the matrix A is important in determining whether or not the economic time series in our example exhibit anything like business-cycle behavior. This is especially appar-

price of the asset is $\beta E(P_{N,t+1}|S_t)$. Thus, the expected utility-denominated rate of return on this (or any other) asset is $\rho \equiv \beta^{-1} - 1$, the utility discount rate.

[15] Other examples of interest rates and asset prices can be constructed by again computing Crusoe's demand/supply price for the asset as in (19). It is a straightforward exercise, for instance, to work out the term structure of our own rates of interest. For this and other examples in the context of one-commodity, N-process models of intertemporal equilibrium, see Cox, Ingersoll, and Ross (1978) and Brock (1979, 1982).

ent in the admissible case where the vectors in the sequence $\{\lambda_t\}$ are independent and identically distributed. In that case, A provides the *only* intertemporal link between deviations of outputs from their normal (expected) values.

The elements of A are elasticities of commodity outputs with respect to commodity inputs. Given constant returns to scale, these elasticities are equal to equilibrium cost shares, and thus the A matrix is an input-output matrix expressed in terms of cost shares.[16] The basic hypothesis that many commodities have many alternative productive employments is characterized in A by the presence of many columns that are relatively full of positive elements. According to (20), unexpectedly high time t output of any one of the commodities corresponding to these columns leads to increases in expected time $t + 1$ outputs of all commodities it may serve to produce. As explained earlier in the analysis of the consumption and input rules, this is the basic mechanism by which, in the language of Frisch (1933), "exterior impulses" are "propagated" through time and across commodities in the model. The importance of capitalistic production is also illustrated by considering the extreme (noncapitalistic) case where labor is the only input in production. In this case, A is a null matrix and autocorrelation in external shocks is the only possible link between Y_t and Y_{t+1}.

Preference parameters (β and θ_i, $i = 0, 1, \ldots, N$) also influence the dynamic behavior of outputs. The constant vector k in (20) is a function of these parameters. In the short run, k determines the directions in which outputs are expected to move from any given current values. In the long run (with $\{\eta_t\}$ assumed a stationary process), k determines the unconditional means or steady-state values of outputs.

The dynamic behavior of other variables can easily be derived from (20). For example, because of the form of the consumption and input allocation rules, (12) and (14), consumption and input time series mirror the behavior of the output series. Also, the dynamic behavior of utility-denominated spot prices (and hence the behavior of relative prices) is implied by the price formula (16) and the quantity dynamics (20).[17]

[16] The element a_{ij} is the equilibrium share of input j in the cost of output i, i.e., $a_{ij} = X^*_{ijt}P_{jt}/(L^*_{it}W_t + \Sigma^N_{k=1}X^*_{ikt}P_{kt})$.

[17] Specifically, combining (16) with (20) yields $p_{t+1} = Ap_t + (I - A)\alpha - k - \eta_{t+1}$, where p_t and α are, respectively, the $N \times 1$ vectors $\{\ln P_{it}\}$ and $\{\ln \gamma_i\}$. Since the wage rate (utility denominated) is constant in this price system, the price behavior characterized above is proportional to prices denominated in labor hours. Regardless of the choice of numeraire, however, the time-series behavior of relative prices is still constrained. To our knowledge the behavior of relative commodity prices over the business cycle has been much less thoroughly documented than the behavior of quantities.

Also of interest is the dynamic behavior of economic aggregates that tend to be the focus of macroeconomic theories. For example, real GNP is computed by valuing current outputs at some set of base year prices. If time b is the base year, then base year prices (from [16]) are $P_{ib} = \gamma_i / Y_{ib}$. Valuing time t output at these prices gives a measure of aggregate output, $\overline{Y}_t$, where

$$\overline{Y}_t \equiv \sum_{i=1}^{N} P_{ib} Y_{it} = \sum_{i=1}^{N} \gamma_i Y_{it} / Y_{ib}. \tag{21}$$

In logs, (21) can be rewritten as

$$\bar{y}_t \equiv \ln \overline{Y}_t = \ln \left[\sum_{i=1}^{N} \gamma_i \exp (y_{it} - y_{ib}) \right]. \tag{22}$$

The dynamic behavior of aggregate output can be analyzed using (21) or (22) and values for γ.[18]

IV. Stochastic Properties of the Example

The dynamic behavior of output is summarized by (20). In order to investigate the stochastic structure of output, we must make additional assumptions regarding the probability structure of λ_t. Thus far we have assumed only that λ_t is positive and that it follows a time-homogeneous Markov process. In order to focus on the ability of our example to generate comovements across sectors and persistence of output movements through time, we will now restrict the vectors in the sequence $\{\eta_t\} \equiv \{\ln \lambda_t\}$ to be independent and identically distributed through time and restrict the covariance matrix, $E\eta_t\eta_t' = \Sigma$, to be an identity matrix. In making these assumptions about η_t, we are intentionally placing a heavy burden on the model in terms of its ability to explain business cycles.[19] Specifically, these assumptions guarantee that any tendency for output in different sectors to move together (comovement) arises solely from the nature of the input decision rules (14) and (15) and the production technology (7), not from the existence of a common shock or shocks that are correlated

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Given the availability of price data, we believe that the investigation of relative prices represents a potentially fruitful area of empirical research.

[18] The behavior of aggregate consumption and input ("investment") allocations can be analyzed in a similar manner. Specifically, aggregate consumption is $\overline{C}_t = \Sigma_{i=1}^{N} \theta_i Y_{it} / Y_{ib}$, and aggregate gross investment is $\overline{X}_t = \Sigma_{i=1}^{N} (\gamma_i - \theta_i) Y_{it} / Y_{ib}$. Note, however, since all goods are perishable in our model, gross investment equals the total level of commodity input.

[19] This is, of course, in addition to the strong assumptions explicit in the example regarding the form of preferences and production possibilities.

across sectors. Similarly, any positive serial correlation (persistence) in output must also arise from the propagation mechanism in our model and not from serially correlated exogenous shocks.

Under the assumptions above, the system (20) becomes a well-defined, first-order vector autoregression with $E\eta_t = 0$, $E\eta_t\eta_t' = I$, and $E\eta_t\eta_s' = 0$ for $t \neq s$.[20] Stability of the system is ensured if the eigenvalues of A have modulus less than one. In our example, stability is guaranteed since A is nonnegative, and the row sums, which are one minus labor's cost share in production, are strictly less than one.

There are several different ways of writing (20) that provide additional insights into quantity dynamics. First, given stability of the system, there exists a moving average representation:

$$y_{t+1} = (I - A)^{-1}k + \eta_{t+1} + A\eta_t + A^2\eta_{t-1} + \dots . \tag{23}$$

Equation (23) demonstrates that the output of good i depends on its own contemporaneous and lagged shocks as well as the past history of shocks to all of the other sectors. Given the nature of the assumptions above about η_t, this propagation mechanism is completely summarized by A.

The steady-state (unconditional mean) value of y is given by $E(y_{t+1}) = (I - A)^{-1}k$. It is sometimes convenient to work in deviations from the steady state. We define these deviations as $\tilde{y}_{t+1} \equiv y_{t+1} - (I - A)^{-1}k$.

The reduced form of (20) in terms of $\tilde{y}$ is then $|(I - A \cdot L)|\tilde{y}_{t+1} = (I - A \cdot L)^*\eta_{t+1}$, where L is the lag operator such that $L^s x_t = x_{t-s}$, $|(I - A \cdot L)|$ is the determinant of $(I - A \cdot L)$, and $(I - A \cdot L)^*$ denotes the transposed matrix of cofactors of $(I - A \cdot L)$. The ith row of the reduced form is

$$|(I - A \cdot L)|\tilde{y}_{i,t+1} = \alpha_i'(L)\eta_{t+1} = \psi_i(L)\epsilon_{i,t+1}, \tag{24}$$

where $\alpha_i'(L)$ is a $1 \times N$ row vector with elements that are polynomials in L and represents the ith row of $(I - A \cdot L)^*$. The last equality expresses the sum of independent moving average processes as a moving average representation in one random shock, $\epsilon_{i,t+1}$.

Equation (24) implies that the individual output series generally follow autoregressions of order greater than one, as indicated by $|(I - A \cdot L)|$, with an additional moving average error structure superimposed. This result illustrates how the general class of models summarized by (20) or (23) is capable of generating business-cycle-like behavior in the individual series. By this we mean that the *expected*

[20] The assumption that $E\eta_t = E \ln \lambda_t = 0$ is made only for convenience and is not implied by the general specification. Alternatively, one can view η_t as deviations of $\ln \lambda_t$ from its mean and let the mean be incorporated in the constant term of (20).

path of y_i, if perturbed from its steady state, can display damped oscillation (with period greater than 2) around the steady state. Thus, there is nothing in the nature of the general rational expectations competitive equilibrium model we are dealing with which requires that, when perturbed from a steady state, the expected path of y_i be characterized by an immediate return to its steady state the next period or by a monotonic decay to its steady state.[21] Since most economic time series do not display pronounced peaks in their spectrum at the business-cycle frequencies, it is probably correct to say that such a condition is not a key feature of the business cycle. Nevertheless, we feel that the ability of simple rational expectations models such as ours to generate damped oscillations in the expected output path is an important and little appreciated fact.

The discussion above focuses on the wide range of serial correlation properties for output that are consistent with the equilibrium summarized by (20). A more important characteristic of business cycles appears to be the tendency of outputs in different sectors to move together. To investigate the comovements among various sectors it is necessary to investigate the properties of the system as a whole. To this end we construct an actual A matrix from an aggregated version of the 1967 input-output tables for the U.S. economy. Based on this somewhat arbitrary construct for A we investigate the stochastic properties of the vector system (20) from several perspectives. First, we characterize the system by computing the autocovariance matrices of the system. Second, we use the moving average representation (23) to compute the impulse response functions associated with each element of y. This method of characterizing the system is a useful way of summarizing the propagation mechanism implied by the model. Finally, we conduct a simulation in order to visualize a sample path from the system.

The A matrix is computed from the 23×23 input-output table for 1967. To keep the system manageable, we collapse the system to 6×6 and express entries as cost shares. The constructed A matrix is given in table 1.[22]

The typical element, $\{a_{ij}\}$, is the cost share of input j in the production of output i. For example, the (1,4) element is the cost share attributable to manufacturing in the production of agricultural com-

[21] This result also applies to any index constructed from y such as the measure of aggregate output, $\bar{y}$, of (22).

[22] The 23×23 input-output table is taken from *Historical Statistics of the United States* (1975), pp. 272–73. The cost shares for the 6×6 system are scaled up, allocating the components of values added, except labor, proportionately among the inputs. The details of these calculations are available from the authors on request.

TABLE 1

MATRIX OF COST SHARES BY INDUSTRY
(A Matrix of Eq. [20])

	1	2	3	4	5	6
1. Agriculture	.4471	.0033	.0146	.2093	.0999	.1591
2. Mining	.0000	.0935	.0427	.1744	.0549	.4854
3. Construction	.0029	.0104	.0003	.4189	.1209	.0893
4. Manufacturing	.0618	.0340	.0050	.4576	.0611	.1267
5. Transportation and trade	.0017	.0004	.0166	.1246	.1040	.3249
6. Services and miscellaneous	.0174	.0212	.0595	.1998	.0871	.3805

modities. The row sums are one minus labor's cost share and thus measure the capital intensities of the various industries.

Industry	Row Sum
1. Agriculture	.9333
2. Mining	.8509
3. Construction	.6427
4. Manufacturing	.7462
5. Transportation and trade	.5726
6. Services and miscellaneous	.7654

The equilibrium system can be summarized by

$$\begin{aligned} \tilde{y}_{t+1} &= A\tilde{y}_t + \eta_{t+1}, \\ E\eta_t &= 0,\ E\eta_t\eta_t' = \Sigma, \text{ and } E\eta_t\eta_s' = 0, \text{ for } t \neq s. \end{aligned} \tag{25}$$

Many properties of (25) are summarized by the autocovariances of y, $\Gamma_j \equiv E(\tilde{y}_t\tilde{y}_{t-j}')$ for $j = -\infty, \ldots, \infty$. Assuming that $\Sigma = I$, so that the shocks across sectors are independent and have unit variance, we see that the contemporaneous covariance matrix is $\Gamma_0 = \Sigma_{j=0}^{\infty} A^j A'^j$, and the autocovariance at lag s is $\Gamma_s = A\Gamma_{s-1} = A^s\Gamma_0$.

Table 2 presents the contemporaneous variance-covariance matrix of the logs of output along with the contemporaneous correlation matrix. Note that all sectors tend to be positively correlated with one another and thus tend to move together. The comovement implied by $\Gamma_0, \Gamma_1, \ldots$ is a property of the unconditional distribution of outputs. However, the conditional distribution of y_{t+1} given y_t has a covariance matrix identical to that of η_{t+1}, which in this case is diagonal. Thus, the unconditional behavior of output can display significant comove-

TABLE 2

A. Contemporaneous Variance-Covariance Matrix of the Logs of Output

$$\Gamma_0 = \sum_{j=0}^{\infty} A^j A'^j = [E(\tilde{y}_{it}\tilde{y}_{jt})]$$

	1	2	3	4	5	6
1. Agriculture	1.66	.421	.342	.428	.284	.387
2. Mining		1.56	.326	.389	.368	.475
3. Construction			1.35	.396	.224	.307
4. Manufacturing				1.46	.261	.361
5. Transportation and trade					1.25	.317
6. Services and miscellaneous						1.42

B. Contemporaneous Correlation Matrix of the Logs of Output

$$R_0 = [\mathrm{corr}\,(\tilde{y}_{it}, \tilde{y}_{jt})]$$

	1	2	3	4	5	6
1. Agriculture	1.00	.261	.228	.275	.197	.253
2. Mining		1.00	.224	.258	.263	.320
3. Construction			1.00	.282	.172	.222
4. Manufacturing				1.00	.193	.252
5. Transportation and trade					1.00	.239
6. Services and miscellaneous						1.00

ment even though the conditional movements in y are cross-sectionally independent.[23]

Another property of the model that is of interest is the implied variances associated with the different output series. The smallest fluctuations are exhibited by the transportation and trade sector, while the largest are found in such series as agriculture, mining, and manufacturing. These rankings are broadly consistent with an important empirical regularity of business cycles. That is, consumption goods (particularly nondurables) display much smaller fluctuations than do producer goods (which tend to be more durable). Although we have no durable goods per se in our model, the relative variances

[23] We have also analyzed the system in the frequency domain. The idea of comovement is captured by measures of coherence between series. A general feature of the model and the A matrix is the tendency for the pairwise coherences to concentrate in the low frequencies. Sargent (1979, p. 212) argues that an important feature of business-cycle phenomena is the "high pairwise coherences at low business cycle frequencies . . . ."

TABLE 3

CORRELATION MATRIX OF THE LOGS OF OUTPUT AT LAG 1

$R_1 = [\mathrm{corr}\,(\tilde{y}_{it}, \tilde{y}_{j,t-1})]$

	1	2	3	4	5	6
1. Agriculture	.559	.243	.219	.377	.251	.334
2. Mining	.206	.306	.219	.330	.223	.554
3. Construction	.172	.184	.166	.484	.226	.233
4. Manufacturing	.245	.226	.194	.529	.198	.283
5. Transportation and trade	.151	.177	.151	.247	.216	.409
6. Services and miscellaneous	.206	.236	.224	.342	.232	.476

reported in table 2 imply that agriculture, mining, and manufacturing, which are relatively important produced inputs, are, respectively, 33 percent, 25 percent, and 18 percent more volatile than transportation and trade, which are largely dominated by retail trade and consumer goods.[24]

Table 3 presents the correlation matrix of $\tilde{y}_t$ with $\tilde{y}_{t-1}$. The diagonal elements, ρ_{ii}, are the first-order autocorrelation coefficients for each series. The off-diagonal elements, ρ_{ij}, describe the correlation between the output of sector i in period t with the output of sector j in period $t - 1$. This matrix suggests how the propagation mechanism works in our model. Increases in the output of one good are translated into increases in the output of another good to the extent that the first good is an input in the production of the second good.[25]

The propagation mechanism is probably best summarized by the impulse response functions. The impulse response functions can be obtained from the moving average representation of the system:

$$\tilde{y}_{t+1} = I\eta_{t+1} + A\eta_t + A^2\eta_{t-1} + \dots . \tag{26}$$

This representation and the assumption that $E\eta_t\eta_t' = I$ enable us to use the elements of $I, A, A^2, \dots$ to trace out the response of output in each sector to a unit impulse in each of the sectors. For example, the

[24] It is also interesting to note that the coherence between agriculture and other sectors is generally lower than between manufacturing and other sectors. It turns out that the transportation and trade sector, being the least volatile and the most labor intensive, also displays less coherence with other sectors.

[25] Interesting lead-lag relationships can also be investigated using these serial correlation matrices. For example, given our model, a sector which has large correlations in its respective column relative to the correlations in its row suggests that the output in that sector acts as a leading indicator of the output in the remaining sectors. In table 3, both manufacturing and services may qualify when such a criterion is used.

sequence of (1,1) elements of I, A, A^2, etc., describes the response of output in sector 1, agriculture, to a shock in sector 1. Similarly, the sequence of (1,2) elements describes the response of agriculture to shocks in sector 2, which in this case is mining. The impulse response functions associated with a shock to each of the six sectors are plotted in figure 1. Also included in these plots is the impulse response function of aggregate output, as defined by (22).[26]

There are several features of these response functions that are important. First, the largest response is usually in the sector where the shock originates. In fact, at lag zero, this is the only sector affected. The other sectors respond only after a period of time that depends on the production technology. Second, the response in these other sectors often only gradually builds up with the peak occurring at lag 2 or 3. For example, in response to a shock in manufacturing, the output of the transportation and trade sector reaches a maximum at lag 2. These response patterns provide a detailed account of how the production technology embodied in the A matrix acts to spread the effects of any shock to other sectors, generating comovement, and to propagate the shock forward in time, generating persistence.

Third, given our A matrix, shocks to agriculture, mining, and construction have only minor impacts on the remainder of the sectors as well as on aggregate output. This is what one might expect given the small values in the first three columns of the A matrix in table 1. In other words, compared to manufacturing and services, the agricultural, mining, and construction sectors are only minor inputs into the production processes. On the other hand, the rather large output responses of all sectors to shocks in manufacturing, transportation and trade, and services highlight the central role of commodities with many productive uses in the propagation mechanism of the model.

The last phase of our investigation is to simulate the time-series behavior of the model. The analytical approaches used above to study the stochastic behavior of the system have many advantages over the simulation procedure. Nonetheless, simulation results do provide a visual impression that sometimes cannot be obtained from other means.

In figure 2, 100 values of $\tilde{y}$ are generated to simulate time paths of the system (25). These time paths most nearly correspond to the logs of output per capita in each sector expressed in deviations from their

[26] Real aggregate output is defined as in (21) with steady-state prices used as the base period prices. To construct the response function for aggregate output, values of γ are required. We computed γ according to (10b) using as θ_i's the fraction of total consumption attributable to the ith sector and assuming $\beta = 0.95$. The results are relatively insensitive to a choice of β between 0.80 and 0.99. Similar calculations could be made for aggregate consumption and gross investment.

respective steady-state values. Moreover, to keep the results in perspective, it is important to note that these simulations are generated under the assumption that $E\eta_t\eta_t' = I$ and $E\eta_t\eta_s' = 0$ for $t \neq s$. Thus, the random shocks are identically distributed and independent both cross-sectionally and through time. This assumption is probably unrealistic, which makes it inappropriate to compare these pictures with what we actually observe. Nevertheless, there is clear evidence of comovement among the different series. The behavior of aggregate output mirrors the behavior of the component series but tends to be smoother (i.e., somewhat more positively autocorrelated) and less volatile. This relative smoothness in aggregate output reflects the fact that individual sector shocks affect outputs in other sectors as they are propagated forward in time (e.g., see table 3). Thus the aggregate output measure better captures the full extent of propagation.

One aspect of the example that we have yet to investigate is the impact on the quantity dynamics of various scenarios regarding the production uncertainty and technological change summarized in the assumptions about the stochastic vectors $\{\lambda_t\}$. Thus far we have assumed that $\eta_t = \ln \lambda_t$ is independent and identically distributed (0, I). An alternative hypothesis is that λ_t represents the stock of neutral technological progress at time t. If we assume knowledge arrives in random amounts, it may be reasonable to model the elements of $\{\lambda_t\}$ as multiplicative random walks,

$$\lambda_{i,t+1} = \lambda_{it}e^{\nu_{i,t+1}}, \tag{27}$$

where we assume that ν_{t+1} is multivariate normal with mean zero and covariance matrix $\Sigma = I$. Note that these assumptions ensure that $\{\lambda_t\}$ remains a positive time-homogeneous Markov process, which thus satisfies the assumptions of Section II, and that the technological "innovations" in each sector, ν_{it}, are independent of one another so that we avoid comovements arising from common shocks.

Taking logs of (27), we get $\ln \lambda_{t+1} = \ln \lambda_t + \nu_{t+1}$ or, letting $\eta_t = \ln \lambda_t$,

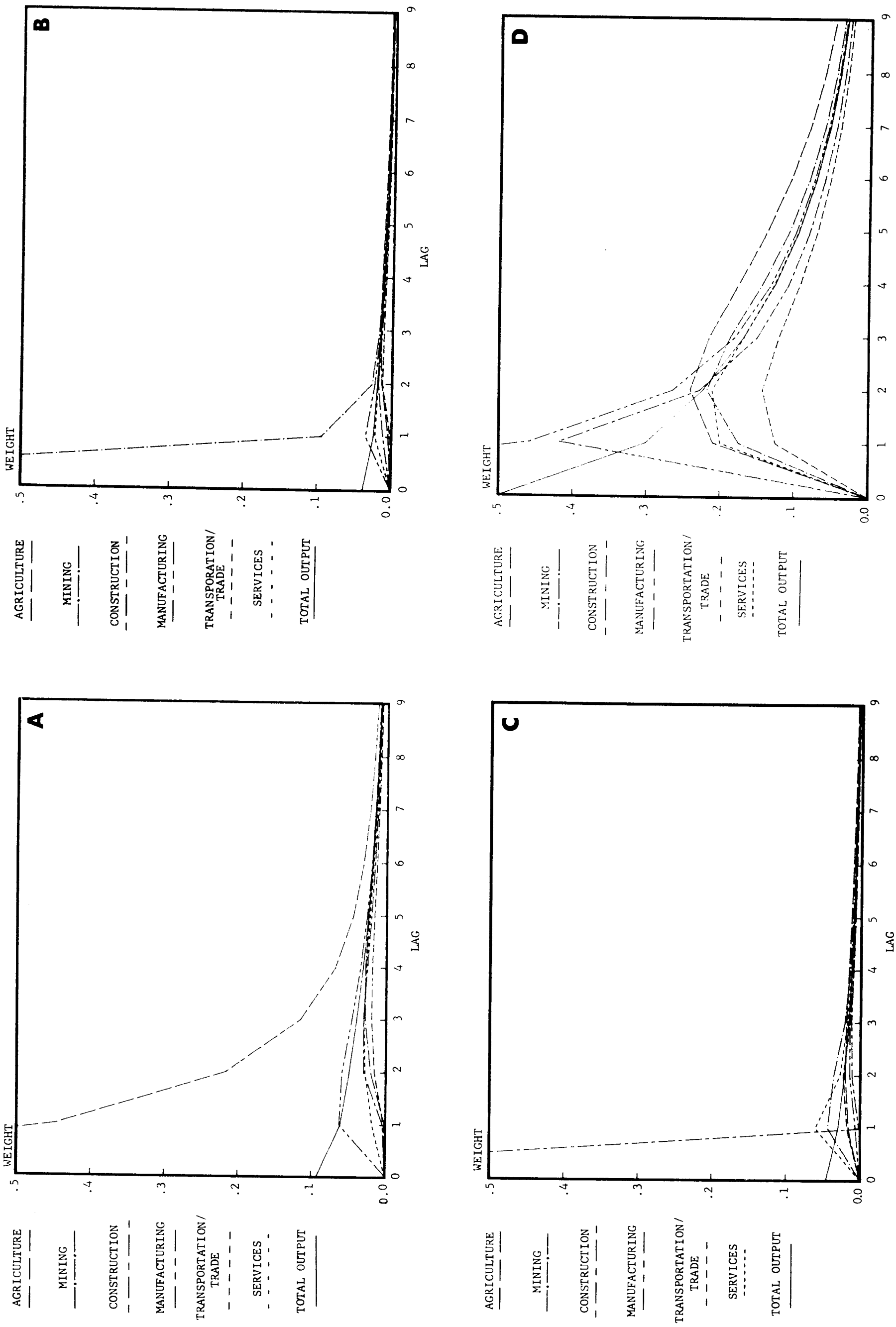
$$\eta_{t+1} = \eta_t + \nu_{t+1}. \tag{28}$$

Assuming η_t follows the random walk process of (28), we can rewrite the system (25) as

$$\Delta\tilde{y}_{t+1} = A\Delta\tilde{y}_t + \nu_{t+1}, \tag{29}$$

where $\Delta\tilde{y}_{t+1}$ is an $N \times 1$ vector of changes in the logs of outputs. Note that ν_{t+1} in (29) has the same properties as η_{t+1} in the previous analyses.

The easiest way to generate simulated values of $\tilde{y}_t$ using (29) is just


A
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MINING
CONSTRUCTION
MANUFACTURING
TRANSPORATION/ TRADE
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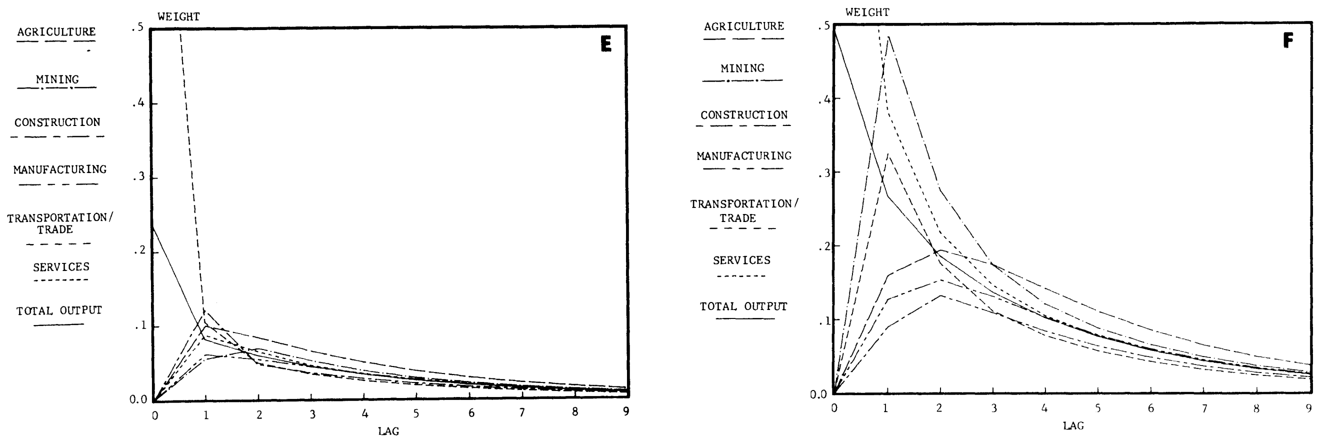


FIG. 1.—Impulse response functions. *A*, Effects of an innovation in agriculture. *B*, Effects of an innovation in mining. *C*, Effects of an innovation in construction. *D*, Effects of an innovation in manufacturing. *E*, Effects of an innovation in transportation/trade. *F*, Effects of an innovation in services.

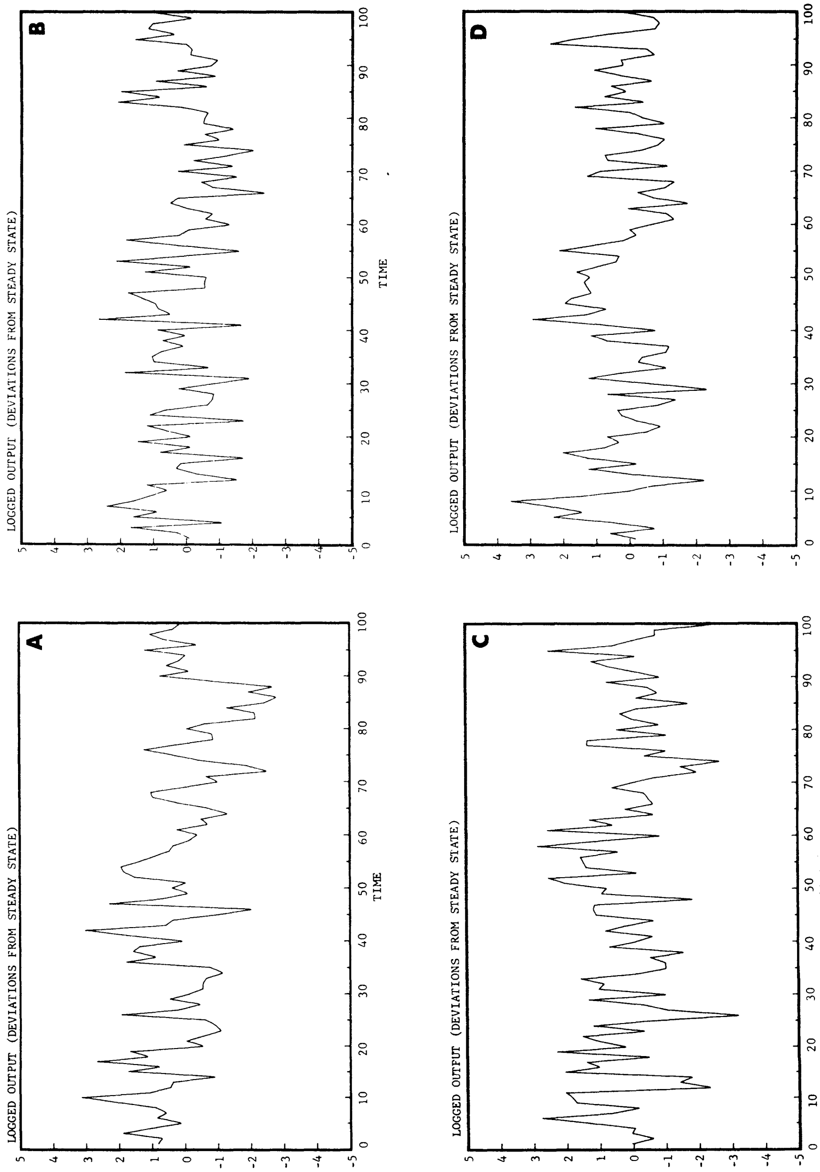


FIG. 2.—Simulation 1. *A*, Simulated output of agricultural sector. *B*, Simulated output of construction sector. *C*, Simulated output of mining sector. *D*, Simulated output of manufacturing sector.

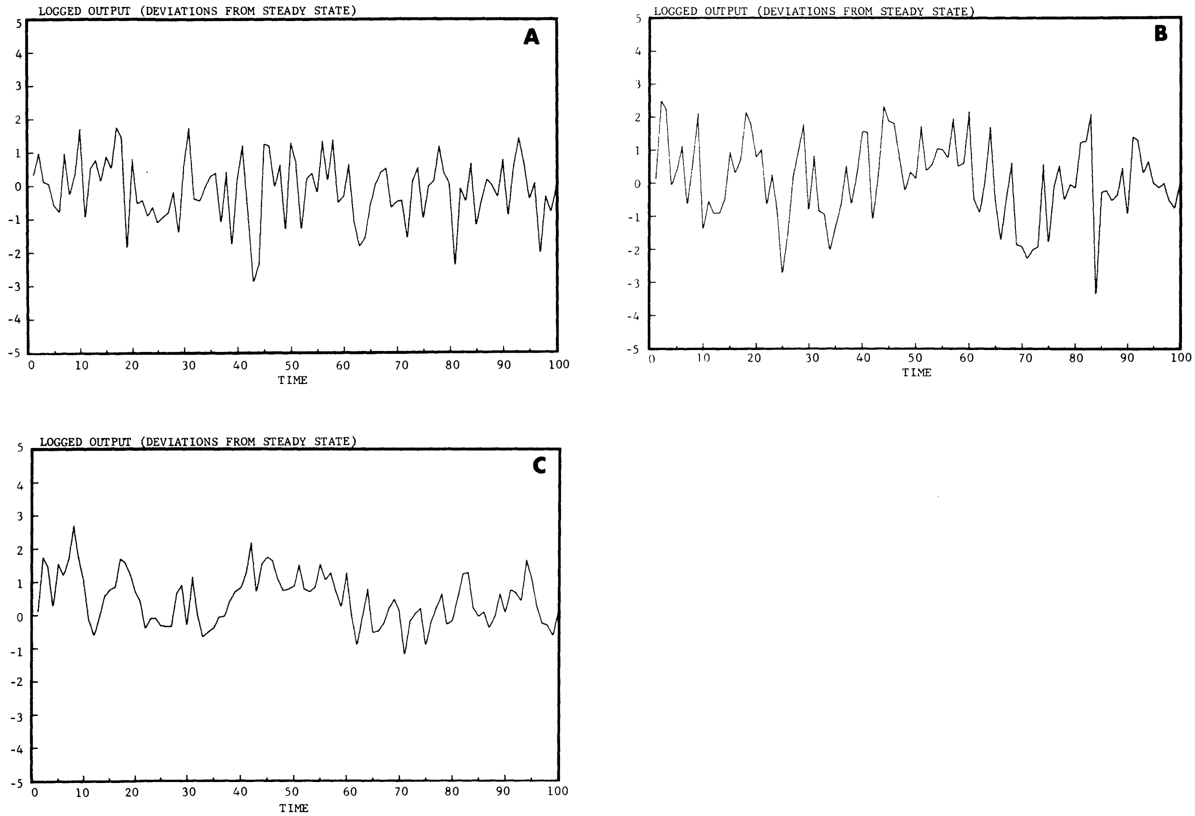
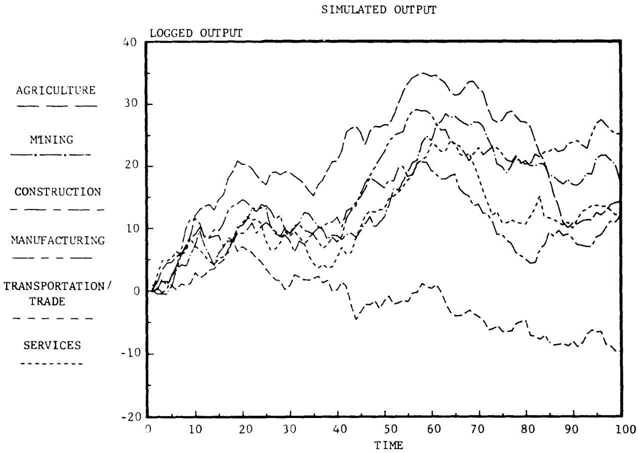


FIG. 2 (Continued).—*A*, Simulated output of transportation/trade sector. *B*, Simulated output of service sector. *C*, Simulated aggregate output.



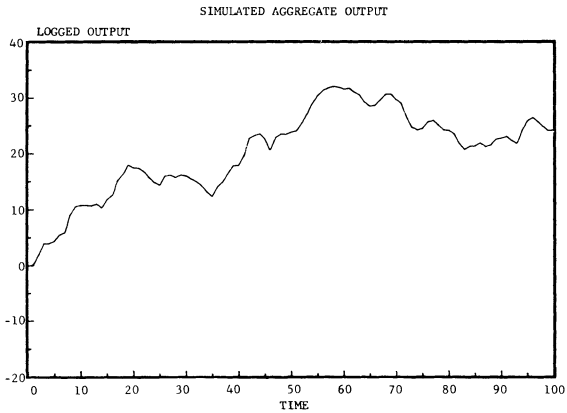


Fig. 3.—Simulation 2

to accumulate the values shown in figure 2.[27] The results of this accumulation process are shown in figure 3. The most striking characteristic of these results is the apparently high degree of comovement in these series, despite the fact that underlying innovations to each sector are independent of one another. Another way to view the high correlation among series is to realize that accumulating the simulations of figure 2 amplifies the low-frequency components of the series where the cross-sectional relationships appear to be the strongest.

[27] Under these assumptions the plots in fig. 2 can be viewed as plots of the growth rates of output per capita in each sector and as such bear a striking resemblance to actual growth rates.

The exception to this characterization appears to be transportation and trade. Although there is no reason why this must be the case, the fact that this sector is the least capital intensive (i.e., uses the fewest produced inputs) probably contributes to the sector's lower volatility and to its ability to behave more independently. Lastly, the behavior of aggregate output is plotted separately in figure 3. As might be anticipated, it behaves more smoothly than the individual series.

V. Summary and Conclusions

In this paper we demonstrated in some detail how certain very ordinary economic principles lead maximizing individuals to choose consumption-production plans that display many of the characteristics commonly associated with business cycles. In particular, if both present and future consumption are normal goods in consumer preferences, it is well known that consumers will attempt to spread over many periods the consumption effects of any unanticipated wealth increment. In a multisector model such as ours consumers will also attempt to allocate their incremental savings in a way that leads to increased consumption of many different goods. At constant relative prices, this suggests that business-cycle features like persistence and comovement are characteristics of desired consumption plans.

In general equilibrium relative prices are not constant. Prices must adjust to reflect production possibilities as well as consumer preferences. In this paper we considered a fairly "realistic" production technology that admits employment of a variety of produced inputs as well as smooth substitution possibilities. Thus, the production possibilities provide economic agents with sufficient flexibility either to absorb completely unanticipated output increments in current consumption (resulting in neither persistence nor comovement) or to spread these shocks through time and across commodities. The consumer preferences outlined above then imply that consumers will choose the latter method of absorbing output fluctuations resulting in both persistence and comovement.

In Sections III and IV we investigated a simple example that demonstrated these general principles. Of particular importance is the manner in which the model transforms and amplifies serially uncorrelated and cross-sectionally independent shocks to production in each sector into output series that exhibit positive serial correlation (persistence) and a significant amount of positive cross-sectional correlation (comovement).

It is important to emphasize that the persistence and comovement inherent in this class of models should not be confused with welfare-reducing deviations from some ideal path. Ours is a "competitive theory of economic fluctuations" such as discussed by Kydland and Prescott (1980), and thus the equilibria are Pareto optimal. Efforts to stabilize this economy can only serve to make consumers worse off.

Although equilibrium real-business-cycle models of the type we suggest are capable of generating business-cycle-like behavior, we do not claim to have isolated the only explanation for fluctuations in real activity. We do believe, however, that models of this type provide a useful, well-defined benchmark for evaluating the importance of other factors (e.g., monetary disturbances) in actual business-cycle episodes.[28]

Further research in this area is certainly called for, and we believe that progress is likely to be made on two fronts. First, empirical research is necessary to assess the extent to which simple real-business-cycle models of this type can account for the covariance structure of observed quantities and relative prices. Second, we feel that technical progress is possible in obtaining explicit solutions to models with alternative specifications of tastes and/or technology. The value of such solutions is summarized by Lucas (1977, p. 11): "One exhibits understanding of business cycles by constructing a *model* in the most literal sense: a fully articulated artificial economy which behaves through time so as to imitate closely the time series behavior of actual economies."

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[28] Some evidence on the relative importance of real vs. monetary factors in output fluctuations is presented in Nelson and Plosser (1982).

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