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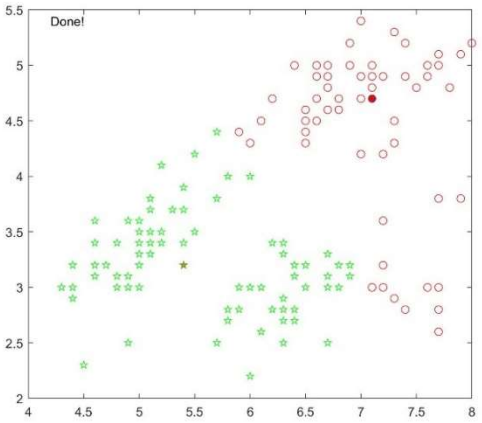
November 5, 2018

ELEC 425 – Assignment 2

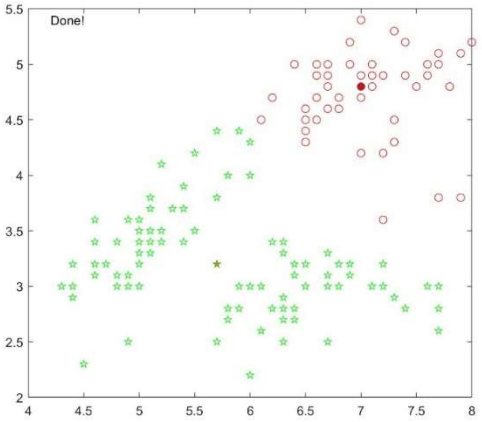
1 – Implement K-medians

2 clusters

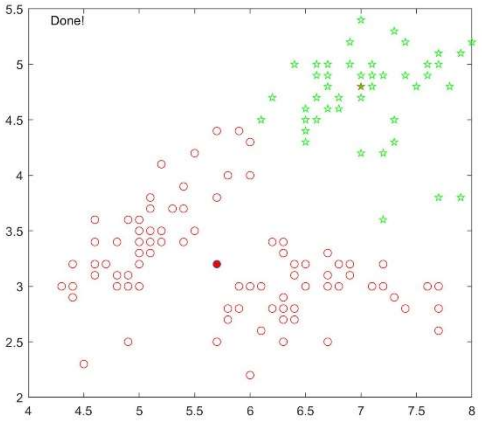
Cluster centres = [5.5, 4; 4.5, 3.2]



Cluster centres = [3.1, 5.2; 3.8, 4.2;]

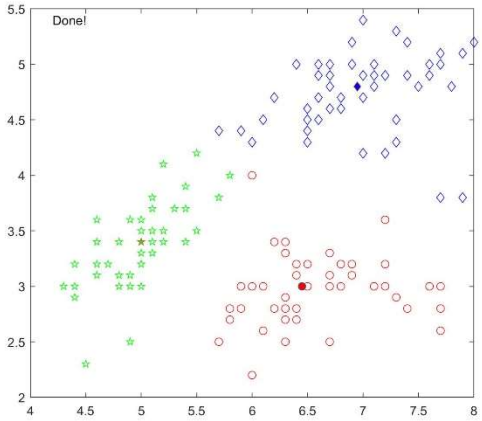


Cluster c*entres = [3, 3; 6, 6]*

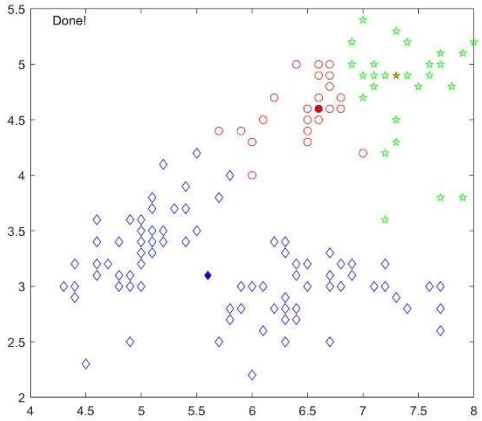


3 clusters

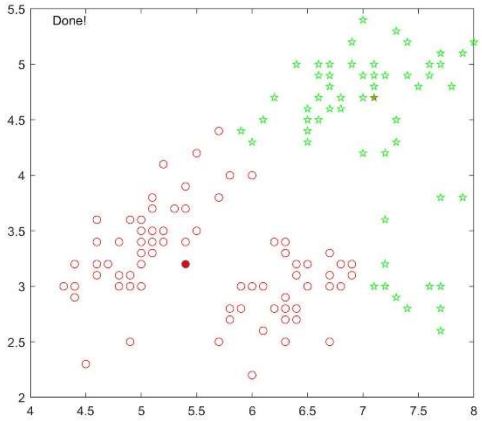
Cluster *centres = [3, 3; 4, 4; 6, 6]*



Cluster *centres = [3.5, 4.3; 3.9, 5.6; 3.4, 3.8]*

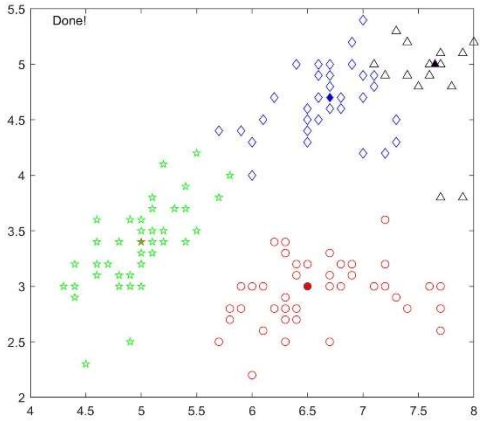


Cluster centres = [4, 5; 4.8, 5.2; 3.9, 6]

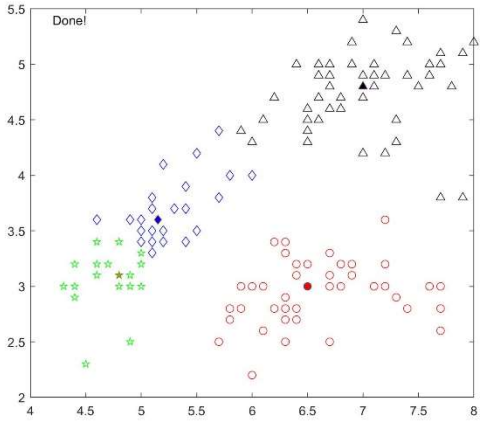


4 clusters

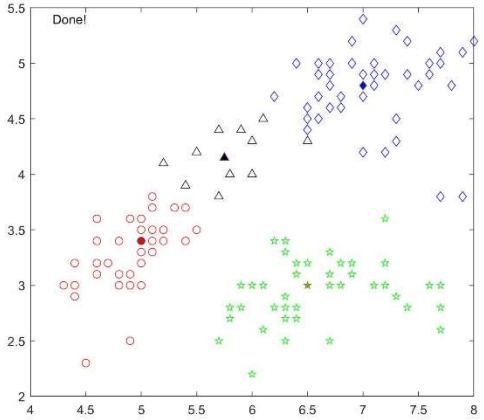
Cluster *centres = [3, 3; 4, 4; 5, 5; 6, 6]*



Cluster *centres = [6, 4; 4, 3; 5, 4; 6, 5]*



Cluster *centres = [4, 5; 8, 3; 7.6, 5; 8, 3.5]*



The relevant code can be found in *k_medians.m* (code was too large to include in this document).

2 – Prove the EM Updating Algorithm Used in K-medians

Similar to the proof that K-means minimizes the following loss function:

$$J = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{nk}\|x_n - \mu_k\|^2$$

The K-median algorithm minimizes the following loss function:

$$J = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{nk}|x_n - \mu_k|$$

The terms involving *n* are independent, so each *n* can be optimized for separately by choosing r_{nk} to be 1 for whichever value of *k* gives the minimum value of $|x_n - \mu_j|$:

$$r_{nk} = \begin{cases} 1 & if\ k = argmin_j|x_n - \mu_k| \\ & 0\ otherwise \end{cases}$$

To optimize μ_k with r_{nk} fixed, take the derivative with respect to u_k and set it to 0:

$$\frac{dJ}{d\mu_k} = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{nk} * sign[\mu_k - x_n]$$

$$0 = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{nk} * sign[\mu_k - x_n]$$

$$= (\#x_n\ smaller\ than\ \mu_k) - (\#x_n\ bigger\ than\ \mu_k)$$

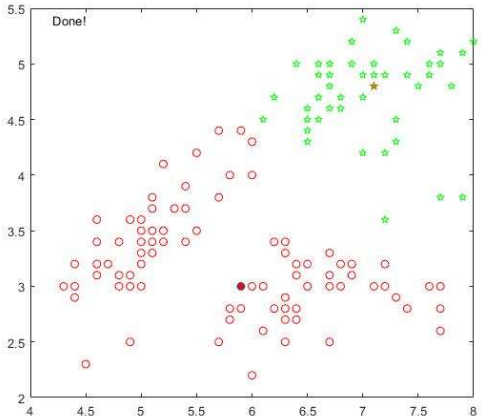
$$\hat{u}_k = median[x_1, \dots, x_n]$$

Thus, we can see that to optimize the K-median loss function, we reach the conclusion that setting μ_k to the median of all points assigned to cluster *k* optimizes this loss function. Therefore, it is proven that the K-medians algorithm minimizes that error function.

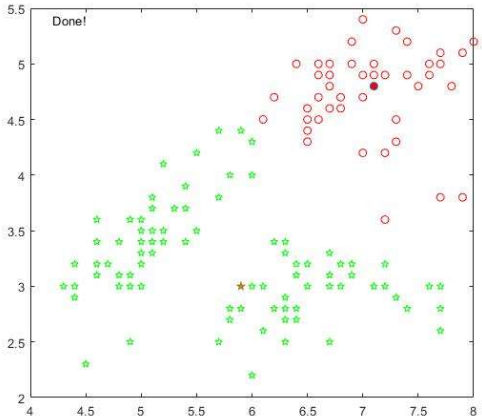
3 - K-medoids

2 clusters

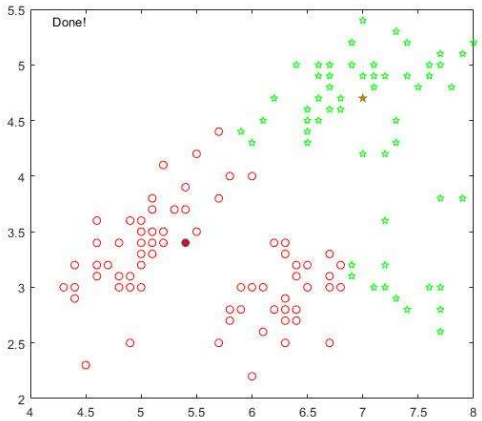
Cluster Centres = [5.9, 3.0; 7.1, 4.8]



Cluster Centres = [7.1, 4.8; 5.9, 3.0]

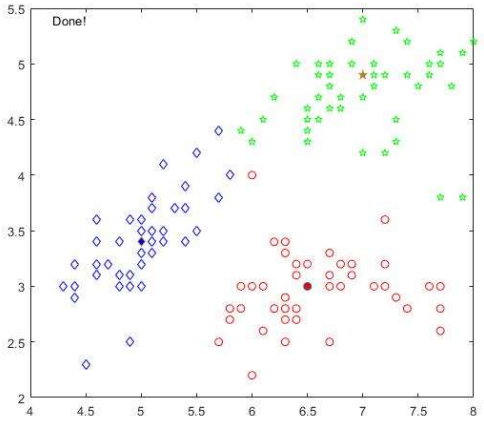


Cluster Centres = [5.4, 3.4; 7.0, 4.7]

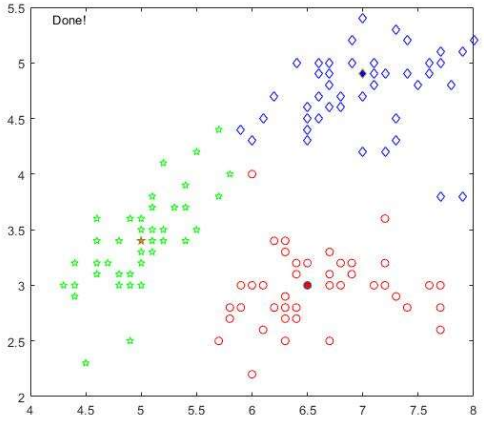


3 clusters

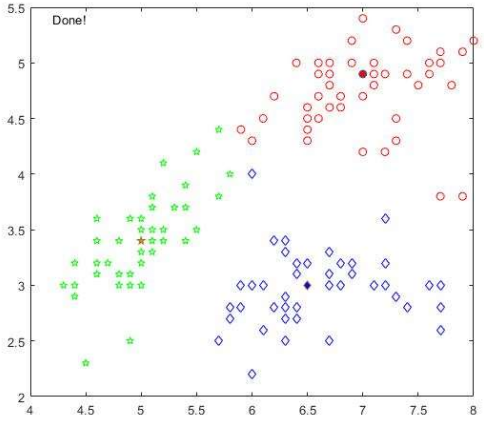
Cluster Centres = [6.5, 3.0; 7.0, 4.9; 5.0, 3.4]



Cluster Centres = [6.5, 3.0; 5.0, 3.4; 7.0, 4.9]

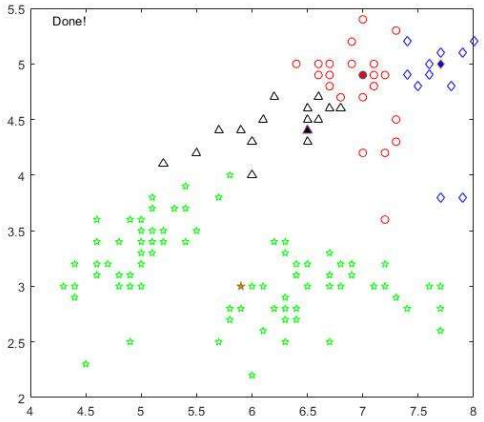


Cluster Centres = [7.0, 4.9; 5.0, 3.4; 6.5, 3.0]

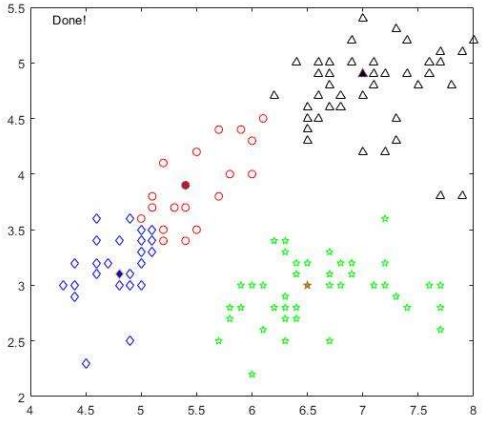


4 clusters

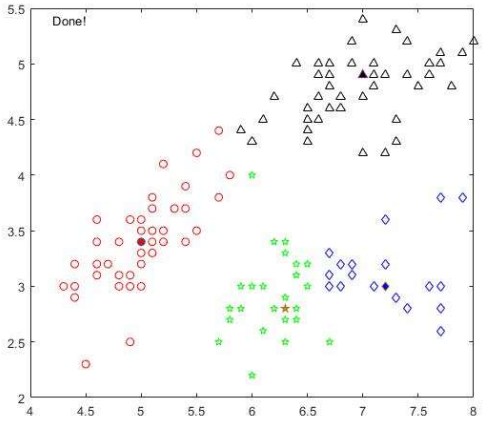
Cluster Centres = [7.0, 4.9; 5.9, 3.0; 7.7, 5.0; 6.5, 4.4]



Cluster Centres = [5.4, 3.9; 6.5, 3.0; 4.8, 3.1; 7.0, 4.9]



Cluster Centres = [5.0, 3.4; 6.3, 2.8; 7.2, 3.0; 7.0, 4.9]



The relevant code can be found in *k_medoids.m* (code was too large to include in this document).