Bayes rules exercises

Ex. 1:

$$P(atRed \mid seesRed) = \frac{P(seesRed \mid atRed) \cdot P(atRed)}{P(seesRed)}$$

$$P(seesRed) = P(seesRed \mid atGreen) \cdot P(aG) + P(sR \mid aR) \cdot P(aR)$$
$$= 0.2 \cdot 0.5 + 0.8 \cdot 0.5 = 0.5$$

$$\frac{0.8 \cdot 0.5}{0.5} = 0.8$$

Ex 2:

$$P(aR \mid sR) = \frac{P(sR \mid aR) \cdot P(aR)}{P(sR)} = \frac{0.8 \cdot 0.5}{0.65} \approx 0.615$$

$$P(sR) = P(sR \mid aR) \cdot P(aR) + P(sR \mid aG) \cdot P(aG)$$
$$= 0.8 \cdot 0.5 + 0.5 \cdot 0.5 = 0.65$$

Ex 3:

$$P(aA \mid sR) = \frac{P(sR \mid aA) \cdot P(aA)}{P(sR)} = \frac{0.9 \cdot 0.33}{0.367} \quad (0.81)$$

$$P(aB \mid sR) = \frac{P(sR \mid aB) \cdot P(aB)}{P(sR)} = \frac{0.1 \cdot 0.33}{0.367} \approx 0.09$$

$$P(aC \mid sR) = \frac{P(sR \mid aC) \cdot P(aC)}{P(sR)} = \frac{0.1 \cdot 0.33}{0.367} \approx 0.09$$

$P(sR)$
$P(sR \mid aA) \cdot P(aA) +$
$P(sR \mid aA') \cdot P(aA')$
$= 0.9 \cdot 0.33 + 0.1 \cdot 0.66$
$= 0.367$

Ex4:

$$P(H|R) = \frac{P(R|H) \cdot P(H)}{P(R)} = \frac{0.1 \cdot 0.6}{0.18} = 1/3$$

$$P(R) = P(R|H) \cdot P(H) + P(R|A) \cdot P(A)$$
$$= 0.1 \cdot 0.6 + 0.3 \cdot 0.4$$
$$= 0.18$$

Naive Bayes Exercise:

A1	0	1	Total
no	1	1	3
yes	2	2	4
total	3	3	6

$P(No|C_0) = 1/3$
$P(No|C_1) = 1/3$

A2	0	1	Total
P	1	2	3
Q	2	1	3
total	3	3	6

$P(Q|C_0) = 2/3$
$P(Q|C_1) = 1/3$

A3	0	1	Total
4	2	2	4
9	1	1	2
total	3	3	6

$P(4|C_0) = 2/3$
$P(4|C_1) = 2/3$

$$P(C_0), P(C_1) = 1/2$$

$P(N_0, Q, 4 \mid C_0) = 14.81\%$
$P(N_0, Q, 4 \mid C_1) = 7.41\%$

$$P(N_0, Q, 4) = P(N_0, Q, 4 \mid C_0) \cdot P(C_0) + P(N_0, Q, 4 \mid C_1) \cdot P(C_1)$$
$$= 7.4\% + 3.76\%$$
$$= 11.11\%$$

$$P(C_0 \mid N_0, Q, 4) = (P(N_0, Q, 4 \mid C_0) \cdot P(C_0)) \,/\, P(N_0, Q, 4)$$
$$= 7.41\% \,/\, 11.11\%$$
$$= 66.67\%$$

$$P(C_1 \mid N_0, Q, 4) = (P(N_0, Q, 4 \mid C_1) \cdot P(C_1)) \,/\, P(N_0, Q, 4)$$
$$= 3.71\% \,/\, 11.11\%$$
$$= 33.33\%$$

•[1]:
```
def f8(p0,p1,p2):
    pA = p0
    pBA = p1

    pB = p1*p0 + (1-p2)*(1-pA)
    pAB = (pBA * pA)/pB

    return pAB
```

[ ]:
```
def f9(p0,p1,p2):
    pA = p0
    pBA = p1
    pBnotAnot = p2

    pB = pBA * pA + (1-pBA) * (1-pA)

    pAB = (pBA * pA)/pB

    return pAB
```