**Lab 1 Explained Solutions**

1) Convert the following binary numbers to decimal:

a)10110110111à 1+2+4+16+32+128+256+1024 = 1463

b) 0111101 à 1+4+8+16+32 = 61

c) 01010001à 1+16+64 = 81

2) Convert the binary numbers in the preceding problem to hex.

|  |  |  |
| --- | --- | --- |
| a) 010110110111  5 B 7 | b) 00111101  3 D | c) 01010001  5 1 |

3) Add 8000 hex and ffff hex—both two’s complement numbers—as a 16-bit adder circuit would(a). Represent the computed result using four hex digits(b). What is the sign of the computed result(c)? What is the sign of the true result(d)? Why is there a discrepancy between the computed result and the true result(e)?

(a):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Carry out à | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |
| 8000 hex à |  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| FFFF hex à | + | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 |
|  |  | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 |
|  | (b) | 7 | | | | F | | | | F | | | | | F | | | |
| (c) The left-most bit is zero, so the sign of the computed result is positive. | | | | | | | | | | | | | | | | | | |

(d) If we convert 0x8000 from its two’s complement hex form to decimal, we get -32768. Similarly, with 0xffff we get -1. Adding two negatives together SHOULD result in a negative number. The true result is negative.

(e) There is a discrepancy between the true and actual result because of the carry out in the leftmost position. 16 bits is not enough to hold -32769. The range of 16-bit two’s complement numbers is only [-32768, 32767].

4) Give the ranges of 20-bit:

a) two’s complement signed numbers *[-, -1]*

[-, -1] = [-, -1] = [-524288, 524287]

b) sign-magnitude signed numbers *[-+1, -1]*

[-, -1] = [-, -1] = [-524287, 524287]

c) unsigned numbers *[0, -1]*

[-, -1] = [0, 1048575]

5) Give the ranges of the following n-bit two’s complement numbers: *[-, -1]*

a) 5-bit

[-, -1] = [-, -1] = [-16, 15]

b) 9-bit

[-, -1] = [-, -1] = [-256, 255]

c) 11-bit

[-, -1] = [-, -1] = [-1024, 1024]

6) Convert the following decimal numbers to hex:

|  |  |
| --- | --- |
| a) 734 | |
| 0 | r1 |
| 2 | r0 |
| 2 | r1 |
| 2 | r1 |
| 2 | r0 |
| 2 | r1 |
| 2 | r1 |
| 2 | r1 |
| 2 | r1 |
| 2 | r0 |
| 2 |  |
| 1011011110  2 D E | |

|  |  |
| --- | --- |
| b) 543 | |
| 0 | r1 |
| 2 | r0 |
| 2 | r0 |
| 2 | r0 |
| 2 | r0 |
| 2 | r1 |
| 2 | r1 |
| 2 | r1 |
| 2 | r1 |
| 2 | r1 |
| 2 |  |
| 1000011111  2 1 F | |

|  |  |
| --- | --- |
| c) 2252 | |
| 0 | r1 |
| 2 | r0 |
| 2 | r0 |
| 2 | r0 |
| 2 | r1 |
| 2 | r1 |
| 2 | r0 |
| 2 | r0 |
| 2 | r1 |
| 2 | r1 |
| 2 | r0 |
| 2 | r0 |
| 2 |  |
| 100011001100  8 C C | |

7) Convert the following hex numbers to binary:

a)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 5 | | | | 6 | | | | 7 | | | | 8 | | | |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

b)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | | | | b | | | | c | | | | d | | | | e | | | | f | | | |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

c)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 4 | | | | 7 | | | | A | | | | D | | | | 4 | | | |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |

8) Convert the following hex numbers to decimal:

a) (46)16 = (4\*16­­1 + 6\*160)10 = (4\*16 + 6\*1)10 = (64 + 6)10 = (70)10

b) (bb)16 = (11\*16 + 11\*1)10 = (176 + 11)10 = (187)10

c) (dc)16 = (13\*16 + 12\*1)10 = (208 + 12)10 = (220)10

9) Add the following pairs of binary numbers. Give your answers in hex.

|  |  |  |
| --- | --- | --- |
| (a)  0111 0111 1110 1011  0000 1010 1010 1011 | (b)  0100 1100 1100 1100  0010 1010 1010 1010 | (c)  0111 0001 1100 0101  0010 1111 1100 0101 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (a) |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 |  | 1 |  | | 1 | 1 |  |
|  | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | | 1 | 1 | 0 | 1 | | 0 | 1 | 1 |
| + | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | | 0 | 1 | 0 | 1 | | 0 | 1 | 1 |
|  | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | | 0 | 0 | 1 | 0 | | 1 | 1 | 0 |
|  | 8 | | | | 2 | | | | | 9 | | | | | 6 | | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (b) |  |  |  |  | 1 |  |  |  | 1 |  | |  |  | 1 |  | |  |  |  |
|  | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | | 1 | 0 | 0 | 1 | | 1 | 0 | 0 |
| + | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | 0 | 1 | 0 | 1 | | 0 | 1 | 0 |
|  | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | | 1 | 1 | 1 | 0 | | 1 | 1 | 0 |
|  |  | 7 | | | | 7 | | | | | 7 | | | | | 6 | | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (c) |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  | 1 |  | 1 |  |
|  | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| + | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
|  | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
|  | A | | | | 1 | | | | 8 | | | | A | | | |

10) Convert the following decimal numbers to 32-bit floating-point format. Give your answers in hex.

\*I will use the first example to explain in depth the process of putting a number into floating point format. After that, I will simply provide the calculated results for each step.

a) 17.75

Step 1) Is the number positive or negative? If it is positive, the sign bit will be 1; if it is negative, the sign but will be 0. This will be used again in step 5.

17.75>0 à sign bit will be 0

Step 2) Covert the number in decimal to binary.

1. Start with what is to the left of the decimal point: (17)10 = (10001)2

|  |  |
| --- | --- |
| 0 | r1 |
| 2 | r0 |
| 2 | r0 |
| 2 | r0 |
| 2 | r1 |
| 2 |  |

1. Next do what is to the right of the decimal point: .75 = ½ + ¼ à (.75)10 = (0.11)2
2. Put this together: (17.75)10 = (10001.11)2

Step 3) Slide the decimal point over to the left and write the number in something that looks like scientific notation, only the part with the exponent will have base 2 instead of base 10. Note that everything to the right of the decimal point will be used in step 5 as the fractional portion.

10001.11 = 1.000111 x 24

Step 4) Next, take the exponent attached to the number 2 from step 3. You need to add this number to 127 and then convert to binary. This will then be used for the 8-bit adjusted exponent field.

4 + 127 = 131 à (131)10 = (1000 0011)2

\*Note: when we put 131 in binary, a helpful technique is that 131 = 128 + 3. It is easy to remember that (128)10 = (1000 0000)­2, and it is east to calculate that (3)10 = (11)2. Then we can easily add (1000 0000)­2 and (11)2 together to get our desired result.

Step 5) Now we are going to put all these pieces together:

Sign bit

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 8-bit Adjusted Exponent Field | | | | | | | | 23-bit Fractional Portion Field | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

\*Fill in zeros to the right of the fractional portion when there is extra space

Step 6) Lastly, take your binary answer from step 5 and convert to hex.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | | | | 1 | | | | 8 | | | | e | | | | 0 | | | | 0 | | | | 0 | | | | 0 | | | |

Final Answer: 418e000

b) -10.5

1) 1

2) (10.5)­10 = (1010.1)­2

3) 1010.1 = 1.0101 x 23

4) 3 + 127 = 130 à (130)­­­10 = (1000 0010)2

5) 1 10000010 01010…0

6) c128 0000

c) 1024.25

1) 0

2) (1024.25)10 = (10000000000.01)­2

3) 10000000000.01 = 1.000000000001 x 210

4) 127 + 10 = 137 à (137)10 = (1000 1001)2

5) 0 10001001 0000000000010…0

6) 4480 0800

1. 1010.75

1) 0

2) (1010.75)­10 = (1111110010.11)2

3) 1111110010.11 = 1.11111001011 x 29

4) 127 + 9 = 136 à (136)10 = (1000 1000)2

5) 0 10001000 111110010110…0

6) 447c b000