F09 Ordered pairs

Course in Semantics · Ling 531 / 731
University of Kansas

Just as we can list the members of a set, we can list the members of a function. Each member is an **ordered pair**. An ordered pair is a pair of items that is ordered.

Sets are not ordered.

$\{a, b\} = \{b, a\}$

But ordered pairs are strict about this.

$\langle a, b\rangle \neq \langle b, a\rangle$

If we take a function and plug something in, we get one item in return.

g : { x | x is a boy band } → { x | x is a country }
g(x) = x's country of origin

$$\begin{bmatrix} \text{One Direction} & \rightarrow & \text{UK} \\ \text{98 Degrees} & \rightarrow & \text{UK} \\ \text{Backstreet Boys} & \rightarrow & \text{US} \\ \text{Menudo} & \rightarrow & \text{Puerto Rico} \\ \text{Westlife} & \rightarrow & \text{Ireland} \\ \text{New Kids on the Block} & \rightarrow & \text{US} \end{bmatrix}$$

We can write the function as a set of ordered pairs. On the left is a member of the domain, on the right is the member of the range the function maps it to.

g = { ⟨One Direction, UK⟩, ⟨98 Degrees, UK⟩, ⟨Backstreet Boys, US⟩, ⟨Menudo, Puerto Rico⟩, ⟨Westlife, Ireland⟩, ⟨New Kids on the Block, US⟩ }

Can we abstract over this set? Yes we can! Essentially, the expression denotes a relation between two sets known as a **Cartesian product**.

Let B = { x | x is a boy band } and
let C = { x | x is a country }

In that case, $g = B \times C$ (READ: *g is the Cartesian product of sets B and C*)

$B \times C = \{ \langle x, y\rangle \mid x \in B \text{ and } y \in C \}$
(READ: *The Cartesian product of B and C is the set of ordered pairs ⟨x,y⟩ such that x is in B and y is in C.*)

That is, it's a set of ordered pairs, each of whose first member is a boy band, and each of whose second is a country.