

# F09 Ordered pairs

Course in Semantics · Ling 531 / 731  
University of Kansas

Just as we can list the members of a set, we can list the members of a function. Each member is an **ordered pair**. An ordered pair is a pair of items that is ordered.

Sets are not ordered.

$$\{a, b\} = \{b, a\}$$

But ordered pairs are strict about this.

$$\langle a, b \rangle \neq \langle b, a \rangle$$

If we take a function and plug something in, we get one item in return.

$$g : \{ x \mid x \text{ is a boy band} \} \rightarrow \{ x \mid x \text{ is a country} \}$$
$$g(x) = x\text{'s country of origin}$$

One Direction	→	UK
98 Degrees	→	UK
Backstreet Boys	→	US
Menudo	→	Puerto Rico
Westlife	→	Ireland
New Kids on the Block	→	US

We can write the function as a set of ordered pairs. On the left is a member of the domain, on the right is the member of the range the function maps it to.

$$g = \{ \langle \text{One Direction, UK} \rangle, \langle \text{98 Degrees, UK} \rangle, \langle \text{Backstreet Boys, US} \rangle, \langle \text{Menudo, Puerto Rico} \rangle, \langle \text{Westlife, Ireland} \rangle, \langle \text{New Kids on the Block, US} \rangle \}$$

Can we abstract over this set? Yes we can! Essentially, the expression denotes a relation between two sets known as a **Cartesian product**.

Let  $B = \{ x \mid x \text{ is a boy band} \}$  and

let  $C = \{ x \mid x \text{ is a country} \}$

In that case,  $g = B \times C$  (READ: *g is the Cartesian product of sets B and C*)

$$B \times C = \{ \langle x, y \rangle \mid x \in B \text{ and } y \in C \}$$

(READ: *The Cartesian product of B and C is the set of ordered pairs  $\langle x, y \rangle$  such that x is in B and y is in C.*)

That is, it's a set of ordered pairs, each of whose first member is a boy band, and each of whose second is a country.