F15: Formalizing definiteness

Ling 331 / 731
Spring 2016

- We saw how the **truth-conditional meaning** of definiteness involves reference and a presupposition of uniqueness
- We know the **syntactic structure** of the determiner phrase
- We have a theory of **semantic types**

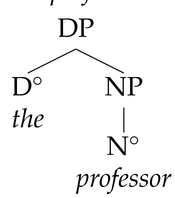
We can formalize the meaning of the definite determiner by bringing the three together

1 Syntax

The DP Hypothesis (Abney 1987) has been widely accepted in syntactic theory, as it offers a much clearer and fundamentally simpler means of building nominal expressions.

Essentially, the noun is not the head of a nominal expression; the determiner is. The determiner takes the NP as its complement.[1]

(1) a. *the professor*

b. 

Armed with this structure, we can start building the meaning.

2 Type theory

We'll use type theory to help build the meaning.

We have the meaning of *professor*— it's the characteristic function of the set of professors.

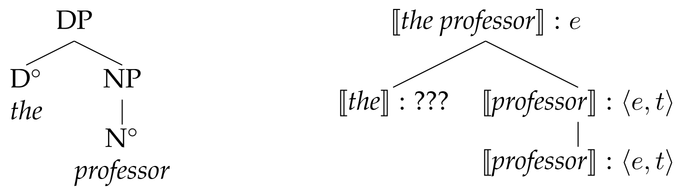
(2) $[\![\textit{professor}]\!] = \lambda x \in D_e.\ \text{professor}(x) : \langle e, t\rangle$

We use D_e as the set of individuals (things of type e)

If we build a meaning tree out of (1b), we see that we have the types of all of the nodes but one.

[1]Granted, an old type structure like $[_{NP}$ Det N $]$ would have the same semantics, given what we've seen so far. One interesting question is whether it would in general.

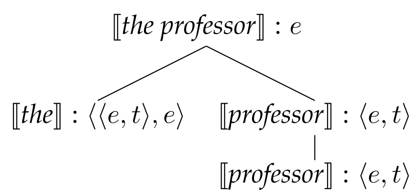
(3)



We can use type theory to figure out what the type of $[\![\textit{the}]\!]$ must be, using a logical argument.

- $[\![\textit{The}]\!]$ and $[\![\textit{professor}]\!]$ are sisters, so given our rules of composition so far, they must combine by Functional Application.
- FA involves taking one sister and plugging it into the other.
- The semantic type of a function is the type created by the ordered pair of the type of its input, and the type of its output. (Like the type of $[\![\textit{professor}]\!]$, which is $\langle e, t\rangle$).
- The type of $[\![\textit{the professor}]\!]$ is e; it denotes an individual…in this case, me.
- So, the output of FA involving $[\![\textit{the}]\!]$ and $[\![\textit{professor}]\!]$ is of type e.
- The output of $[\![\textit{professor}]\!]$ is of type t.
- This mismatch means that $[\![\textit{professor}]\!]$ is not the function that takes *the* as its argument.
- Instead, it must be the other way around.
- The input to $[\![\textit{the}]\!]$ is of type $\langle e, t\rangle$.
- The output to $[\![\textit{the}]\!]$ is of type e.
- Thus, $[\![\textit{the}]\!]$ must be of type $\langle\langle e, t\rangle, e\rangle$

(4)



This means that $[\![\textit{the}]\!]$ is a function from the set of things of type $\langle e, t\rangle$ to the set of things of type e.

Things of type e are called **individuals**.
Things of type $\langle e, t\rangle$ are called **properties** of individuals.

Thus, $[\![\textit{the}]\!]$ is a function from properties to individuals. If you give it a property, it gives you an individual.

The question becomes: How do we link the property to the individual?

3 Truth conditions

We can use the truth conditions we discussed to link the input property to the output individual.

$$[\![\textit{the}]\!] \text{ involves } \begin{cases} \text{reference} \\ \text{presupposition of existence} \\ \text{presupposition of uniqueness in a context} \end{cases}$$

(5) $[\![$*the professor*$]\!]$ = Andrew McKenzie = $[\![$*Andrew McKenzie*$]\!]$

(6) $[\![$*the professor knows semantics*$]\!]$ = $[\![$*Andrew McKenzie knows semantics*$]\!]$

$[\![$*the*$]\!]$ takes a set of individuals, and picks out the only one among them with a particular property.

(7) $[\![$*the professor knows semantics*$]\!]$ = the only person in the relevant context who is a professor knows semantics

So, how do we formalize that?

4 Formalizing meaning

To formalize the meaning of an expression, we need to know:

1. What kind of semantic object the meaning is
2. What the truth conditions of the meaning are

When we say 'kind of semantic object', that refers to our ontology. Instead of a single domain D, we can break D down into different sets of objects.

(8) **Ontology** so far

set	type	description	formal value
D_t	t	the set of truth values	{ 1, 0 }
D_e	e	the set of individuals (entities)	{ x | x is an individual }
$D_{\langle e,t \rangle}$	$\langle e, t\rangle$	the set of properties (of individuals)	{ f | f : $D_e \to D_t$ }

4.1 The kind of object

We used the structure and the type theory to figure out what kind of semantic object $[\![$*the*$]\!]$ has to be. It has to be a function that takes a property of individuals and gives you an individual. That is, it's a function from $D_{\langle e,t \rangle}$ to D_e.

$$[\![\textit{the}]\!] = \underset{\text{func. from}}{f:}\ \underset{\text{domain}}{\underline{\ D_{\langle e,\, t\rangle}\ }}\ \underset{\text{to}}{\rightarrow}\ \underset{\text{range}}{\underline{\ D_e\ }}$$

$$\text{for any } \underset{\text{object}}{\underline{\qquad\qquad}}\ \underset{\text{in}}{\in}\ \underset{\text{domain}}{\underline{\qquad\qquad}},$$

$$\underset{\text{func. applied to object}}{f(\underline{\qquad\qquad})}\ \underset{\text{eq.}}{=}\ [\ \underset{\text{meaning}}{\underline{\qquad\qquad\qquad\qquad}}\ ]$$

The sorts of objects in $D_{\langle e,\, t\rangle}$ are functions, so we can put a variable in their stead. Usually we use f as a variable for functions. But we can't use f because we're already using it. So we'll use g.

$$[\![\textit{the}]\!] = \underset{\text{func. from}}{f:}\ \underset{\text{domain}}{\underline{\ D_{\langle e,\, t\rangle}\ }}\ \underset{\text{to}}{\rightarrow}\ \underset{\text{range}}{\underline{\ D_e\ }}$$

$$\text{for any } \underset{\text{object}}{\underline{\quad g \quad}}\ \underset{\text{in}}{\in}\ \underset{\text{domain}}{\underline{\ D_{\langle e,\, t\rangle}\ }},$$

$$\underset{\text{func. applied to object}}{f(\underline{\quad g \quad})}\ \underset{\text{eq.}}{=}\ [\ \underset{\text{meaning}}{\underline{\qquad\qquad\qquad\qquad}}\ ]$$

Now, we get to the meaning. That's part two: What are the truth condtions? The meaning is reference, but there's also presupposition. How do we formalize presupposition?

4.2 Formalizing Presupposition

Presuppositions are propositions that the meaning of our expression depends on. That is, our expression means something *only if* the presupposition is true.[2] There are several ways we can do this, but we will stick with restricting the domain of the function.

Why? If the expression's presupposition fails, the expression is uninterpretable. It has no meaning. Formally, if we put the presupposition as a domain condition, we limit the domain to only those objects where the presupposition holds[3]. If the presupposition does not hold of an object, then the function cannot select it. It cannot select anything, and so the function remains unsaturated. The expression remains unsaturated, and is thus uninterpretable.

Okay. So what is our presupposition/domain restriction here? There are two:

- Existence: There is an $x \in D_e$ with the property of the NP

[2] Independently, uttering the expression is felicitous only if its presuppostions are true *and if* everyone involved also knows its presuppositions to be true.

[3] A preposition holds of a world where it is true

- Uniqueness: There is only one $x \in D_e$ with the property of the NP

Wait a second. If that's our presupposition, then $[\![$*the professor*$]\!]$ presupposes that there is only one individual in all the world that is a professor. That is blatantly false, as we saw. There is only one in the salient context, though.

How do we formalize a context? We could try to specify that context directly, as more recent theories do. But to do that here requires us to introduce tons of background we have not gotten to yet. So let's not.

Instead, we can choose a subset of individuals that fits the context. So instead of picking 'this classroom', we pick out the set of individuals in this classroom. In that set, there is only one professor, and that's how $[\![$*the professor*$]\!]$ picks out me. In another classroom, though, it will pick out someone else.

(9) Let C_e be a contextually salient subset of D_e.

Armed with C_e, we can rewrite our presuppositions.

- Existence: There is an $x \in C_e$ with the property of the NP
- Uniqueness: There is only one $x \in C_e$ with the property of the NP

These can be added as a domain restriction. (We can simply put Uniqueness because it entails Existence)[4]

$$[\![\textit{the}]\!] = \underset{\text{func. from}}{f:} \; \underset{\text{domain}}{\underline{\quad D_{\langle e,\,t\rangle} \quad}} \; \underset{\text{to}}{\rightarrow} \; \underset{\text{range}}{\underline{\quad D_e \quad}}$$

$$\text{for any } \underset{\text{object}}{\underline{\quad g \quad}} \; \underset{\text{in}}{\in} \; \underset{\text{domain}}{\underline{\quad D_{\langle e,\,t\rangle} \quad}}, \; \underset{\text{domain restriction}}{\text{such that there is only one } x \in C_e \text{ such that } g(x) = 1,}$$

$$\underset{\text{func. applied to object}}{f(\underline{\quad g \quad})} \; \underset{\text{eq.}}{=} \; [\; \underset{\text{meaning}}{\underline{\qquad\qquad\qquad}} \;]$$

So what is the meaning? The unique individual that fits the description.

$$[\![\textit{the}]\!] = \underset{\text{func. from}}{f:} \; \underset{\text{domain}}{\underline{\quad D_{\langle e,\,t\rangle} \quad}} \; \underset{\text{to}}{\rightarrow} \; \underset{\text{range}}{\underline{\quad D_e \quad}}$$

$$\text{for any } \underset{\text{object}}{\underline{\quad g \quad}} \; \underset{\text{in}}{\in} \; \underset{\text{domain}}{\underline{\quad D_{\langle e,\,t\rangle} \quad}}, \; \underset{\text{domain restriction}}{\text{such that there is only one } x \in C_e \text{ such that } g(x) = 1,}$$

$$\underset{\text{func. applied to object}}{f(\underline{\quad g \quad})} \; \underset{\text{eq.}}{=} \; [\; \underset{\text{meaning}}{\underline{\text{the unique } x \in C_e \text{ s.t. } g(x) = 1}} \;]$$

[4] Existence does not entail Uniqueness. Question: How come the entailment only goes one way?

So now, if anyone asks you what *the* means, you can tell 'em:

$[\![\textit{The}]\!]$ denotes the (partial) function from properties to individuals such that for any property where there is only one individual in the salient context with that property, the function returns the unique indvidual in the salient context with that property. For all other properties, it is undefined.

Easy! :)

5 Into the lambda calculus

Now that we have our function, we can convert it to the lambda-calculus, using the same technique as before. Let's review that technique.

$$[\![\textit{prof.}]\!] = \underset{\text{func. from}}{f:} \; \underset{\text{domain}}{\underline{\;D_e\;}} \; \underset{\text{to}}{\rightarrow} \; \underset{\text{range}}{\underline{\;D_t\;}}$$

$$\text{for any } \underset{\text{object}}{\underline{\;x\;}} \; \underset{\text{in}}{\in} \; \underset{\text{domain}}{\underline{\;D_e\;}},$$

$$\underset{\text{func. applied to object}}{f(\underline{\;x\;})} \; \underset{\text{eq.}}{=} [\; \underset{\text{meaning}}{\underline{\;1 \text{ iff } x \text{ is a professor}\;}} \;]$$

Place the parts like so...

(10) $[\![\textit{prof.}]\!] = \lambda x \in D_e.\ 1 \text{ iff } x \text{ is a professor}$

Then we can abbreviate the value condition.

(11) $[\![\textit{prof.}]\!] = \lambda x \in D_e.\ \text{professor}(x)$

Let's do the same here, with the same parts.

$$[\![\textit{the}]\!] = \underset{\text{func. from}}{f:} \; \underset{\text{domain}}{\underline{\;D_{\langle e,t\rangle}\;}} \; \underset{\text{to}}{\rightarrow} \; \underset{\text{range}}{\underline{\;D_e\;}}$$

$$\text{for any } \underset{\text{object}}{\underline{\;g\;}} \; \underset{\text{in}}{\in} \; \underset{\text{domain}}{\underline{\;D_{\langle e,t\rangle}\;}}, \; \underset{\text{domain restriction}}{\text{such that there is only one } x \in C_e \text{ such that } g(x) = 1,}$$

$$\underset{\text{func. applied to object}}{f(\underline{\;g\;})} \; \underset{\text{eq.}}{=} [\; \underset{\text{meaning}}{\underline{\;\text{the unique } x \in C_e \text{ s.t. } g(x) = 1\;}} \;]$$

(12) $[\![\textit{the}]\!] = \lambda g \in D_{\langle e,\, t\rangle}.$ the unique $x \in C_e$ such that $g(x) = 1$

We can abbreviate the value condition with the **iota-operator**. Introduced by Bertrand Russell for his notion of definiteness, this operator is now typically used with Frege's notion of definiteness in mind. (We're using Frege's notion).

(13) **Iota-operator**
$\iota x[\ g(x)\ ]$ = the unique x such that $g(x) = 1$

If we apply the ι-operator to our case here:

(14) $[\![\textit{the}]\!] = \lambda g \in D_{\langle e,\, t\rangle}.\ \iota x[\ g(x)\ ]\ :\ \langle\langle e, t\rangle, e\rangle$

And there we have it!

6 Wrap-up

We have used the syntax, the type theory, and the truth conditions of *the* to develop a formalized meaning. This formalization will allow us to build compositional structures that show how the meaning of *the* contributes to the meaning of propositions it is in.

It will also allow us to make precise predictions about what *the* means, and about what kinds of solutions we can imagine to handle cases where the meaning of definiteness is present, but *the* is not.