Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2014	1 0 1 0 8 1 / 0 1

Title: Algorithms and Data Structures

Time Allowed:	2 hours
Examination Material Provided:	None
Additional Materials Permitted:	None
Instructions:	Answer FOUR questions. (TWO compulsory questions from Section A and TWO from Section B) ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK Calculators are NOT allowed Erasmus/Visiting students can use a dictionary

Revision:	

ED01/2014

Section A Dr. Tom Friedetzky and Dr. Matthew Johnson

Question 1

(a) Define the **linked list** data structure. **[4 Marks]**

(b) Given a linked list L, what do the following refer to?

i. L.head.data

ii. L.head.next.next

[4 Marks]

(c) Briefly explain one advantage of using a linked list rather than an array. **[3 Marks]**

(d) Write pseudo-code for inserting a piece of data d into a linked list L following a node N. **[7 Marks]**

(e) The input to the following algorithm is a linked list containing at least one node and each piece of data is an integer. Describe the output, and explain the roles of c, d and e.

Input: linked list L
Output: ???

```
c = 0
d = L.head
e = d.next
while e ≠ NULL do
  if e.data > d.data then
    c = c + 1
  end if
  d = e
  e = d.next
end while
return c
```

[7 Marks]

continued

Question 2

(a) Give formal definitions of the asymptotic classes $\mathcal{O}$ and ω. Explain the meaning behind those definitions. **[8 Marks]**

(b) Rank the following functions according to asympotic growth:

$$4^{\log n} \qquad n^{\log\log n} \qquad (\sqrt{2})^{\log n} \qquad n^{\log_3 3}$$

[6 Marks]

(c) Explain the statement of the lower bound for comparison-based sorting algorithms. Explain how BucketSort can appear to beat this bound in certain situations. **[6 Marks]**

(d) What is the difference between the binary search tree property and the min-heap property? Can the min-heap property be used to print out the keys of an n-node tree in sorted order in $O(n)$ time? Explain how or why not. **[5 Marks]**

End of Section A

Section B Dr. Tom Friedetzky and Dr. Matthew Johnson

Question 3

(a) For each of the following problems, give an algorithm that finds the desired numbers within the given amount of time. Your answers do not need to be very detailed: something such as "sort the numbers using algorithm A and return the 5th smallest as x and the 17th largest as y" is acceptable. A brief justification of the correctness of your solution is sufficient; no formal proofs are required.

i. Let S be an **unsorted** array of n distinct integers. Give an algorithm that finds the pair $x, y \in S$ that maximises $|x - y|$. Your algorithm must run in $O(n)$ worst-case time. **[2 Marks]**

ii. Let S be a **sorted** array of n distinct integers. Give an algorithm that finds the pair $x, y \in S$ that maximises $|x - y|$. Your algorithm must run in $O(1)$ worst-case time. **[2 Marks]**

iii. Let S be an **unsorted** array of n distinct integers. Give an algorithm that finds a pair $x, y \in S$ that minimises $|x - y|$, for $x \neq y$. Your algorithm must run in $O(n \log n)$ worst-case time. **[2 Marks]**

iv. Let S be a **sorted** array of n distinct integers. Give an algorithm that finds a pair $x, y \in S$ that minimises $|x - y|$, for $x \neq y$. Your algorithm must run in $O(n)$ worst-case time. **[2 Marks]**

(b) Suppose we have integer values between 1 and 1000 in a binary search tree and search for 363. Which of the following **cannot** be the sequence of keys examined? Explain your answer. **[4 Marks]**

i. 2 252 401 398 330 363

ii. 399 387 219 266 382 381 278 363

iii. 3 923 220 911 244 898 258 362 363

iv. 4 924 278 347 621 299 392 358 363

v. 5 925 202 910 245 363

this question is continued on the next page

(c) By running a breadth-first search on a directed graph G, a breadth-first tree T is obtained that contains the edges traversed during the search. Describe how the edges of G can be classified with respect to T as tree, forward, back or cross edges. **[4 Marks]**

(d) Suppose a breadth-first search is run on the directed graph given by the adjacency lists below with vertex 1 as the source. Classify each edge of the graph as a tree, forward, back or cross edge. **[6 Marks]**

$$\begin{array}{rl} 1: & 3,5 \\ 2: & 3,5 \\ 3: & 4,5 \\ 4: & 1,5,6,8 \\ 5: & 1,3 \\ 6: & 3,5,8 \\ 7: & 6 \\ 8: & 2,3,4,6,7 \end{array}$$

(e) Explain why no forward edges can be found after a breadth-first search on a graph (which might be either directed or undirected), and why no back edges can be found after a breadth-first search on an undirected graph. **[3 Marks]**

Question 4

(a) Let S be an **unsorted** array of $2n$ integers. Give an algorithm that partitions the numbers into n pairs, with the property that the partition minimises the maximum sum of any pair.

For example, say we are given the numbers $[1, 3, 5, 9]$. The possible partitions are $((1, 3), (5, 9))$, $((1, 5), (3, 9))$, and $((1, 9), (3, 5))$. The pair sums for these partitions are $(4, 14)$, $(6, 12)$, and $(10, 8)$. Thus the third partition has 10 as its maximum sum, which is the smallest maximum sum over the three partitions.

Your algorithm must run in $O(n \log n)$ worst-case time. You do **not** need to prove the correctness of your algorithm, but you should briefly justify your claims. **[6 Marks]**

(b) Explain how to build one heap that contains all elements of two given heaps with n and m elements respectively (where n and m are positive integers). Assume that the heaps are given in a tree representation, that is, each node has links to its two children. The running time of the algorithm should be $O(\log(n + m))$ in the worst case. **[6 Marks]**

(c) Define a minimum spanning tree (MST) of a connected undirected graph with weights on the edges, and briefly describe Kruskal's algorithm for finding an MST. **[3 Marks]**

(d) Describe how to adapt Kruskal's algorithm to find a spanning tree of **maximum** weight. Briefly justify your answer. **[4 Marks]**

(e) Suppose that G is a connected undirected graph with weights on the edges. Let $\{V_1, V_2\}$ be a partition of the vertices of G (every vertex of G belongs to either V_1 or V_2). Consider the set of edges E^* that each join a vertex in V_1 to a vertex in V_2. Suppose that there is an edge e in E^* that has a lesser weight than every other edge in E^*. Show that every MST of G contains e. **[6 Marks]**

continued

Question 5

(a) Consider the Stieglitz numbers S_n ($n \geq 0$ integer) defined by

$$S_n = \begin{cases} 3n & \text{for } n \leq 3, \\ 4S_{n-3} + S_{n-2} + S_{n-1} & \text{for } n \geq 4. \end{cases}$$

i. Calculate S_7. **[2 Marks]**

ii. Write pseudo-code for a **non-recursive** function that returns S_n for an integer $n \geq 0$. **[4 Marks]**

iii. Write pseudo-code for a **recursive** function that returns S_n for an integer $n \geq 0$. **[4 Marks]**

iv. How could you improve the running time of the recursive function? **[2 Marks]**

(b) Briefly explain how you might design a deterministic QuickSort with worst-case running time $O(n \log n)$. **[4 Marks]**

(c) Explain the relationship between binary search trees and sorting. **[4 Marks]**

(d) Explain how to modify InsertionSort so that it sorts its input into non-increasing order. **[5 Marks]**

END OF PAPER