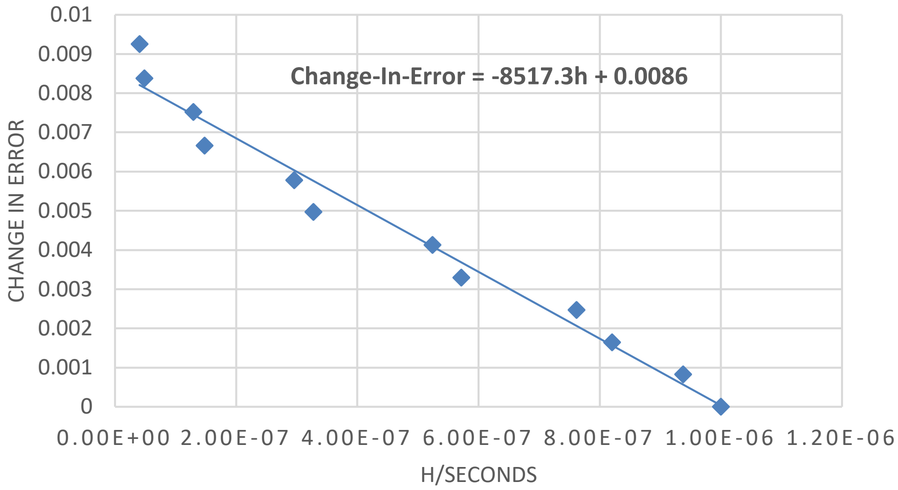
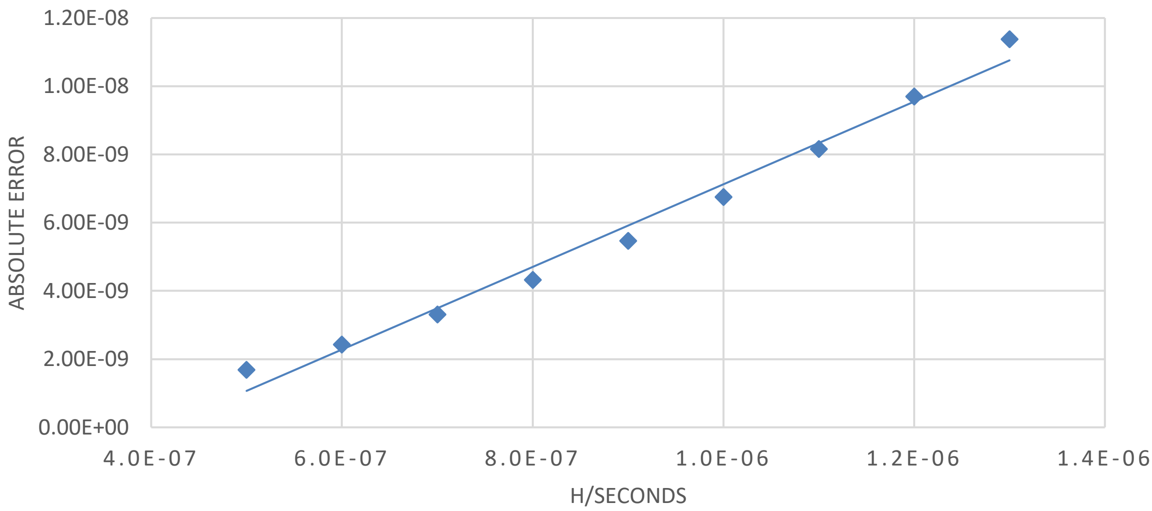
Report

The collision problem can be considered stiff if any 2 particles are close together. The meaning of close is dependent on h and the velocities seen during the simulation but for my purpose's close means $minDx \leq 0.35$. Before an iteration we check if the last iteration detected any close particles, if so we use a timestep of $\frac{h}{4}$ and Runge-Kutta to update the velocities else we use Adams-Bushford to update the velocities. In both scenarios we use explicit euler to update the distances. This method can suffer for densly packed inputs but will get more accurate results for a smaller h compared to just the explicit euler.



The change in error was computed by $y_h - y_{kh}$ for $k \in (0, 1), h = 10^{-5}s$. Since we use consistent schemes as h shrinks we get closer to the analytical solution. From the above graph we see as h decreases the distance from the original solution increases linearly implying, we linearly head towards the analytical solution, i.e. as h decreases we gain more accurate solutions at a linear rate. Therefore the converge order is $O(h)$.

Consider the simulation of 4 particles of equal mass at the corners of a regular square, it is trivial to see these particles must collide at the origin. If we measure the distance between the final collision point from the original that will be the absolute error of the simulation.



The above graph shows that the absolute error increases linearly with h which shows both that the smaller the h the more accurate the collision points. This is also experimental evidence for our scheme being consistent and that the convergence order is $O(h)$.