ASSIGNMENT 1 CSCI 3070

Asymptotic Notations, Divide and Conquer, Heap

OCTOBER 11, 2020

OTU
Andrew Murdoch
100707816

Andrew Murdoch
100707816

Assignment 1

Part 1:

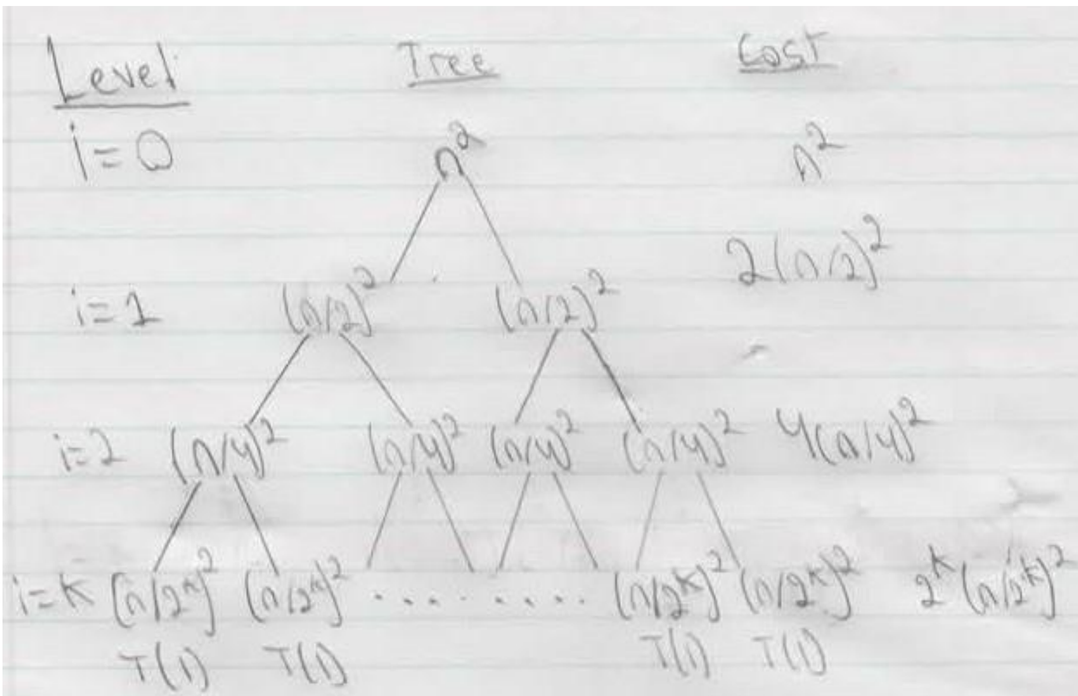
a) T(n) = 2T(n/2) + 3n + 7 (Master Theorem)

a = 2 b = 2 $\log_2 2 = 1$ $K = 1$

Since both $\log_2 2$ and K are equal and p = 0, case 2 of master theorem applies.

Therefor T $\in \Theta(n \log n)$.

b) T(n) = 7T(n/2) + n^2 (Recursion Tree)



$\sum_{i=0}^{\log_2 n} 2^i (\frac{n}{2^i})^2$

$let\ 2^i = a$

$= \sum_{i=0}^{\log_2 n} a(\frac{n}{a})^2$

$= \sum_{i=0}^{\log_2 n} \frac{n^2}{a}$ = $\sum_{i=0}^{\log_2 n} \frac{n^2}{2^i}$ = $\sum_{i=0}^{\log_2 n} n^2 (\frac{1}{2^i})^i$ = $\sum_{i=0}^{\log_2 n} n^2 (\frac{1}{2^i})^i$ = $n^2 \frac{(\frac{1}{2})^0 - (\frac{1}{2})^{\log_2 n+1}}{1-(\frac{1}{2})}$

$Note\colon \sum_{i=m}^{n} ar^i = \sum_{i=m}^{n} a\frac{r^m - r^{n+1}}{1-r}\ \ where\ r \neq 1$

= $2n^2(1 - \left(\frac{1}{2}\right)^{\log_2 n}\left(\frac{1}{2}\right))$ = $2n^2(1 - (\frac{1}{n})\left(\frac{1}{2}\right))$ = $2n^2(1 - \frac{1}{2n})$= $2n^2 - \frac{2n^2}{2n}$ = $2n^2 - n$ = O(n^2)

Andrew Murdoch
100707816

c) T(n) = T(n/2) + T(n/4) + T(n/8) + n (Substitution method with guess of O (n log n))

T(n) = $c(\left(\frac{n}{2}\right)\lg\left(\frac{n}{2}\right)+\left(\frac{n}{4}\right)\lg\left(\frac{n}{4}\right)+\left(\frac{n}{8}\right)\lg\left(\frac{n}{8}\right))$

T(n) = $c(\left(\frac{n}{2}\right)(\lg n-\lg 2)+\left(\frac{n}{4}\right)(\lg n-\lg 4)+\left(\frac{n}{8}\right)(\lg n-\lg 8))$

T(n) = $c(\left(\frac{n}{2}\right)(\lg n-1)+\left(\frac{n}{4}\right)(\lg n-2)+\left(\frac{n}{8}\right)(\lg n-3))$

T(n) = $c(\frac{\mathrm{n}}{2\mathrm{logn}}-\frac{n}{2}+\frac{n}{4\log n}-\frac{n}{2}+\frac{n}{8\lg n}-\frac{3n}{8})$

T(n) = $cn(\frac{1}{2\mathrm{logn}}-\frac{1}{2}+\frac{1}{4\log n}-\frac{1}{2}+\frac{1}{8\lg n}-\frac{3}{8})$

T(n) = $cn(\frac{1}{2\mathrm{logn}}+\frac{1}{4\log n}+\frac{1}{8\lg n}-\frac{11}{8})$

T(n) = $c(\frac{7}{8n(\log n)}-\frac{11}{8n})$ "n log n grows faster than n"

Therefor T(n) = O(n log n)

d) T(n) = 4T(n/16) + $\sqrt{n}$ (Master Theorem)

a = 4   b = 16 $\log_{16} 4 = 1/2$ $K = 1/2$

Since both $\log_2 2$ and K are equal and p = 0, case 2 of master theorem applies.

Therefor T ∈ $\Theta(\sqrt{n}\ \log n)$.

Part 2:

Category	Functions
Constant	
Linear	$2^{\log_2 n}$, 18n
Polynomial	n^2, $3n^2+7n+15$, $\sqrt{n^3}$, $n^3-\log n$, n^3, $\frac{3}{4}n^4$
Exponential	2^n, 4^n, $n^{71}+5^n+17n$
Factorial	n!
Logarithmic	$\log_{10} n$, $3\log_2 n$,