1)

159 | 186

1.62 | 1.72

1.54(1)

$$\frac{15\times1.62+1.59+18\times1.72}{34}$$

$$=\frac{1137}{680}=1.67$$

2) 405 f ^{600}

$$\frac{405}{600}\times 40=27 \qquad 40-27=13$$

3) $P(X=x)=\frac{x}{k}$ for $x=1,2,3,4$

$$P(x>2)=\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$$

$$\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}=1$$

$k=10$

4) $T\sim B(30,0.3)$

$P(7<T\leqslant 12)$

$=P(x\leqslant 12)-P(x\leqslant 7)$

$=0.915\ldots-0.28\ldots$

$=0.63$

Geometric and Negative Binomial Distribution

$\frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}$ → you roll a dice until you get a 6

For successive independent trials, each with the constant probability of success p, the number of trials needed to get one success, has a geometric distribution

$P(X=x) =$

$Geo(P)$

Binomial → number of success', fixed number trials $\{0 \text{ to } n\}$

Geometric → How many trials up to success each with probability p.

Number of events $\{1 \text{ to } \infty\}$

Cumulative: $\frac{a(1-r^n)}{1-r}$

a ~ first term

r ~ relative multiplier

$$= \frac{P(1-(1-P)^x)}{1-(1-P)^x}$$

$a = P$

$r = 1-P$

$n = x$

$$= 1-(1-P)^x$$

$(1-P)^x$ → probability a success didn't appear in the first x turns

$1-(1-P)^x$ → probability a success does occur in the first x turns

Q) $P = 0.6$ $\quad X \sim Geo(0.6)$

$P(x=5) = 0.6 \times 0.4^4 = 0.015\ldots$

$P(x \leq 5) = 1-(1-0.6)^5 = 0.98976$ (4 s.f.)

$P(x > 5) = (0.4)^5$

Assume independent and p constant

Q) $p = 0.3$ $\quad X \sim Geo(0.3)$

$P(X=4) = 0.3 \times (0.7)^3$

$= 0.1029$

$P(x<8) = P(x \leq 7) = 1-(0.7)^7 = 0.9176$

$P(x>5) = 1 - P(x \leq 5)$

$= (0.7)^5 = 0.1681$

Exercise 3A Stat1 FM

$X \sim Geo(0.15)$

a) $P(X=10) = 0.15 \times 0.85^9$

$= 0.03474$

b) $P(X \leq 7) = 1-(0.85)^7$

$= 0.6794$

c) $P(3 \leq X \leq 12)$

$= P(X \leq 12) - P(X \leq 2)$

$= 0.5803$

2a) $Y \sim Geo(0.23)$

$P(X=6) = 0.23 \times 0.77^5$

$= 0.06226$

$X \sim Geo(0.16)$

$P(x=5) = 0.16 \times 0.84^4$
$= 0.07966$ (4 s.f.)

$P(x \geq 5) = 1 - P(x \leq 4)$
$= 0.84^4$
$= 0.4979$

$P(x \leq 10) = 1 - (0.84)^{10}$
$= 0.8251$

b) $\frac{1}{0.16} = 6.25$

c) p not constant if lucky gains experience (not independent)

$E(x) = \frac{1}{p}$ $Var(x) = \frac{1-p}{p^2}$

$Y \sim Geo(p)$

$E(Y) = 2.5$

$p = \frac{1}{2.5} = 0.4$

a) 0.4

b) $\sqrt{\frac{1-0.4}{0.4^2}} = 1.936$

b) $P(x \geq 4)$
$= 0.25^3$
$= 0.1563$

10) $p = 1 - 2 \times 0.5^3$
$= \frac{3}{4}$

$X \sim Geo(\frac{3}{4})$

a) $P(x=3)$
$= \frac{3}{4} \times \frac{1}{4}^2$
$= \frac{3}{64}$

7) a) Geometric

b) p constant & independent

c) $P(x=2) = 0.16$

$p(1-p) = 0.16$

$p - p^2 - 0.16 = 0$

$p = \frac{4}{5}, \frac{1}{5}$ $p < 0.5$

$\therefore p = 0.2$

8a) 15% Blue

$$\frac{1}{0.15} = \frac{20}{3}$$

ii) $$\frac{1-p}{p^2} = \frac{1-0.15}{0.15^2}$$

$$X \sim Po(0.8)$$

$$P(X > 2)$$

$$= 1 - P(X \leq 2)$$

$$\cancel{= 1 - \frac{e^{-\lambda}\lambda^x}{x!}}$$

$$= \sum_{r=0}^{2} 1 - \frac{e^{-0.8} \times 0.8^r}{r!}$$

$$= 0.04742$$