Phys 2B Summer 2022

Quiz 3 Practice Solutions

Question 1:

The equivalent resistance of the bottom branch of the circuit is:

$$R_{\text{bot}} = 10\,\Omega + 20\,\Omega = 30\,\Omega$$

This is exactly the same as the resistance of the upper branch, so the current that flows through each branch will be the same.

The equivalent resistance of the entire network is:

$$R_{\text{eq}} = \left(\frac{1}{30\,\Omega} + \frac{1}{30\,\Omega}\right)^{-1} = 15\,\Omega$$

The total current that will flow is:

$$I = \frac{V}{R_{\text{eq}}} = \frac{12\,\text{V}}{15\,\Omega} = 0.8\,\text{A}$$

Now, we need to divide the total current such that the total voltage drop across each branch is the same (parallel paths must have the same voltage drops).

$$I_{\text{top}}R_{\text{top}} = I_{\text{bot}}R_{\text{bot}}$$

$R_{\text{top}} = R_{\text{bot}}$, so $I_{\text{top}} = I_{\text{bot}}$. Therefore, half of the current, $0.4\,\text{A}$, will flow through the bottom branch, meaning $\boxed{0.4\,\text{A}}$ will flow through the $10\,\Omega$ resistor.

This is choice (d).

Question 2:

The general power formula is:

$$P = IV$$

In this case, it is convenient to plug in that $I = V/R$, per Ohm's Law, since we know V and R but aren't directly told I.

$$P = \frac{V}{R}V = \frac{V^2}{R}$$

If the bulbs are attached in parallel, each feels the full voltage of the battery and will therefore have the maximum radiated power. You can either recognize that we need to multiply this power by 2 since there are two bulbs doing the same thing, or you can say that the equivalent resistance of 2 resistors in parallel is:

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

which reduces to $R/2$ in the special case $R_1 = R_2 = R$, which applies in this problem. In any case, the total radiated power is:

$$P_{\text{tot}} = \frac{V^2}{R_{\text{eq}}} = 2\frac{V^2}{R}$$

Now plug in numbers:

$$P_{\text{tot}} = 2\frac{(12.0\,\text{V})^2}{2.00\,\Omega} = \boxed{144\,\text{W}}$$

This is choice (d).

Note that the downside of doing this, from the explorers perspective, is that the battery will be depleted twice as fast as it would be for a single bulb, so it's not as though attaching light bulbs in parallel produces more light "for free." What circuit should the explorers set up if they instead want to maximize how long their battery will last, while still producing some light? The answer is NOT to use just a single bulb! How much power would this circuit radiate?

Question 3:

The (approximate) resistivity formula for conductors is:

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

We want to solve for α.

$$\alpha = \frac{\frac{\rho(T)}{\rho(T_0)} - 1}{T - T_0}$$

Now, let's read two (T, ρ) points off the plot and plug them in. Two convenient points are (reading off the plot by eye to the best of my abilities):

$$(T, \rho(T)) \approx (310\,\text{K}, 1.69 \times 10^{-8}\,\Omega\text{-m})$$

$$(T_0, \rho(T_0)) \approx (270\,\text{K}, 1.45 \times 10^{-8}\,\Omega\text{-m})$$

Plug these values in.

$$\alpha \approx \frac{\frac{1.69}{1.45} - 1}{310 - 270} \approx \boxed{0.00414}$$

The best estimate is choice (b), which is the actual plotted value, 0.0038 (the material is silver).

It's generally good practice in these kinds of questions to pick widely-separated points to minimize the relative error, but it's not necessary, and you could try out some other pair(s) to check your work. For example, another decent pair of points is:

$$(T, \rho(T)) \approx (274\,\mathrm{K}, 1.475 \times 10^{-8}\,\Omega\text{-m})$$

$$(T_0, \rho(T_0)) \approx (270\,\mathrm{K}, 1.45 \times 10^{-8}\,\Omega\text{-m})$$

This pair ultimately gives $\alpha \approx 0.00431$, consistent with our first pair.

Question 4:

The resistance formula is:

$$R = \frac{\rho L}{A}$$

We want the resistances to be the same, so:

$$\frac{\rho_{\mathrm{Al}} L}{A_{\mathrm{Al}}} = \frac{\rho_{\mathrm{Cu}} L}{A_{\mathrm{Cu}}}$$

We're told that the lengths of the wires are the same, so they will drop out. Now, solve for A_{Al}.

$$A_{\mathrm{Al}} = A_{\mathrm{Cu}} \frac{\rho_{\mathrm{Al}}}{\rho_{\mathrm{Cu}}}$$

And plug in numbers.

$$A_{\mathrm{Al}} = (1.0\,\mathrm{cm}^2) \frac{2.65 \times 10^{-8}\,\Omega\text{-m}}{1.72 \times 10^{-8}\,\Omega\text{-m}} \approx \boxed{1.54\,\mathrm{cm}^2}$$

This is choice (d).

Question 5:

Let's define I_1 to be flowing left-to-right through R_1, I_2 to be flowing right-to-left through R_2, and I_3 to be flowing upward through the central emf. Then from current conservation, we have:

$$I_1 + I_2 = I_3$$

From the left loop, we have:

$$12\,\mathrm{V} - I_1 R_1 + 9\,\mathrm{V} = 0$$

$$21\,\mathrm{V} = I_1 R_1$$

And from the right loop, we have:

$$9\,\mathrm{V} - 5\,\mathrm{V} - I_2 R_2 = 0$$

$$4\,\mathrm{V} = I_2 R_2$$

The left loop tells us I_1:

$$I_1 = \frac{21\,\mathrm{V}}{6\,\Omega} = 3.5\,\mathrm{A}$$

The right loop tells us I_2:

$$I_2 = \frac{4\,\mathrm{V}}{4\,\Omega} = 1.0\,\mathrm{A}$$

And current conservation tells us the central current, I_3:

$$I_3 = 3.5\,\mathrm{A} + 1.0\,\mathrm{A} = \boxed{4.5\,\mathrm{A}}$$

This is choice (d).

Question 6:

A discharging capacitor follows:

$$Q(t) = Q_{\max} e^{-\frac{t}{RC}}$$

We're told the time required to reach half of the initial charge, ie we're told $t_{1/2}$ such that:

$$\frac{1}{2} = e^{-\frac{t_{1/2}}{RC}}$$

We can use this to calculate RC before the dielectric is inserted, call this RC_i.

$$\ln\left(\frac{1}{2}\right) = -\frac{t_{1/2}}{RC_i}$$

$$RC_i = -\frac{t_{1/2}}{\ln\left(\frac{1}{2}\right)}$$

Plug in numbers.

$$RC_i = -\frac{2.0\,\mathrm{s}}{\ln\left(\frac{1}{2}\right)} \approx 2.885\,\mathrm{s}$$

The capacitance increases by a factor of 3.5 with the dielectric inserted, so RC does too.

$$RC_f \approx 3.5(2.885\,\text{s}) \approx 10.1\,\text{s}$$

Now, we want to use this to calculate the fraction of the charge remaining after 2.0 seconds.

$$\frac{Q(t)}{Q_{\text{max}}} = e^{-\frac{t}{RC_f}}$$

Plug in numbers.

$$\frac{Q(t)}{Q_{\text{max}}} \approx e^{-\frac{2.0\,\text{s}}{10.1\,\text{s}}} \approx \boxed{0.82}$$

This is choice (a).

Question 7:

The time required for a capacitor to discharge a given amount is proportional to its time constant, RC. Increasing R or increasing C will increase the time required to discharge to any given fraction of the initial charge.

Operation I increases C, so it would increase the time constant. I is true.

Operation II increases R, so it would increase the time constant. II is true.

Operation III doesn't change R or C. The capacitor will start out with a larger charge, but discharge more quickly as a result (it has a higher voltage, so is capable of driving a larger current), so that it reaches half of its initial charge at exactly the same time. III is false.

So, the correct choice is (d).

Question 8:

The magnitude of the magnetic field of a long current-carrying wire is:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

The net field is the sum of the wires' fields.

$$B_{\text{net}} = B_1 + B_2 = \frac{\mu_0}{4\pi}2I\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

The expression simplifies in this case because the two wires carry identical $I = 10\,\text{A}$ currents.

Wire 1 is 2.0 meters away from the point of interest, and wire 2 is 1.0 meters away.

The cross product that determines direction is $\hat{I} \times \hat{r}$, where $\hat{r}$ points from the current to the place where the field is being calculated. In this case, $\hat{I} = +\hat{y}$ and $\hat{r}$ is $+\hat{x}$ for both wires (the point of interest is in the positive x direction).

$$\hat{y} \times \hat{x} = -\hat{z}$$

Now plug in numbers with this direction information.

$$\vec{B}_{\text{net}} = (10^{-7}\text{T-m/A})2(10\,\text{A})\left(\frac{1}{2.0\,\text{m}} + \frac{1}{1.0\,\text{m}}\right)(-\hat{z}) = \boxed{(-3.0 \times 10^{-6}\,\text{T})\hat{z}}$$

This is choice (a).

Question 9:

The magnetic force on a current-carrying wire is:

$$\vec{F} = L\vec{I} \times \vec{B}$$

Due to the cross product, the force only cares about the component of $\vec{B}$ perpendicular to $\vec{I}$. In this problem, $\vec{I}$ is purely in the $+\hat{y}$ direction, so the force only cares about B_x and B_z, but $B_z = 0$, so only B_x affects the force.

$$|\vec{F}| = L|\vec{I}||B_x|$$

Now plug in numbers.

$$F = (2.0\,\text{m})(2.0\,\text{A})(2.5\,\text{T}) = \boxed{10\,\text{N}}$$

This is choice (d).

Question 10:

The magnitude of the magnetic force on a current-carrying wire is, if the field and wire are perpendicular (as they are in this problem):

$$F = ILB$$

Simply plug in numbers.

$$F = (1000\,\text{A})(25\,\text{m})(52 \times 10^{-6}\,\text{T}) = \boxed{1.3\,\text{N}}$$

This is choice (b).

Unfortunately for the scheming engineer, this is underwhelming. A human weighs something like 700 N, so a more effective and practical means of improving traction is for the driver to gain a little weight. The regulations don't bother restricting downward forces from magnetism for a reason...