ECE 35 Homework #7 (Spring 2023, Taur)

<u>All homework problems come from the textbook, "Introduction to Electric Circuits", by Svoboda & Dorf, 9th Edition.</u>

P 8.3-3 The circuit shown in Figure P 8.3-3 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, for $t > 0$.

Answer: $v(t) = -6 + 18e^{-6.67t}$ V for $t > 0$

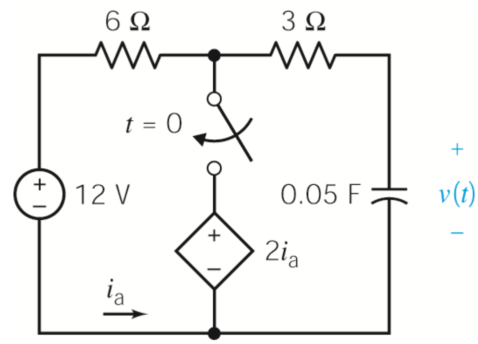
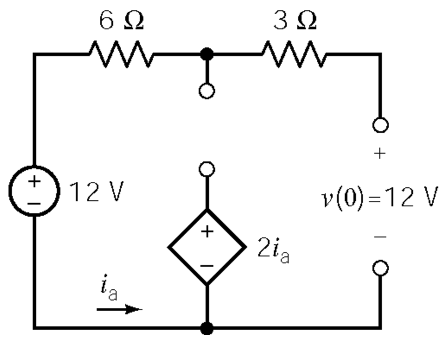
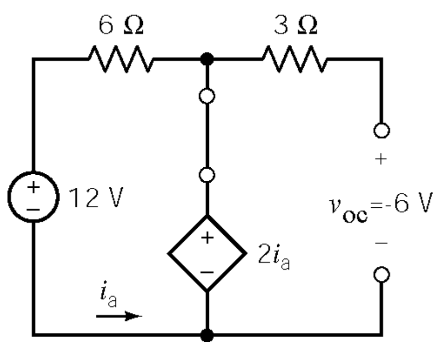


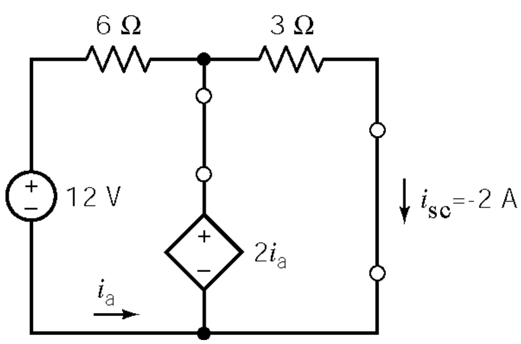
Figure P 8.3-3

Solution: Before the switch closes:



After the switch closes:





Therefore $$R_t = \frac{-6}{-2} = 3\ \Omega \quad \text{so} \quad \tau = 3(0.05) = 0.15\ \text{s}.$$

Finally,

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = -6 + 18\, e^{-6.67t}\ \text{V} \quad \text{for } t > 0$$

P 8.3-10 A security alarm for an office building door is modeled by the circuit of Figure P 8.3-10. The switch represents the door interlock, and v is the alarm indicator voltage. Find $v(t)$ for $t > 0$ for the circuit of Figure P 8.3-10. The switch has been closed for a long time at $t = 0^-$.

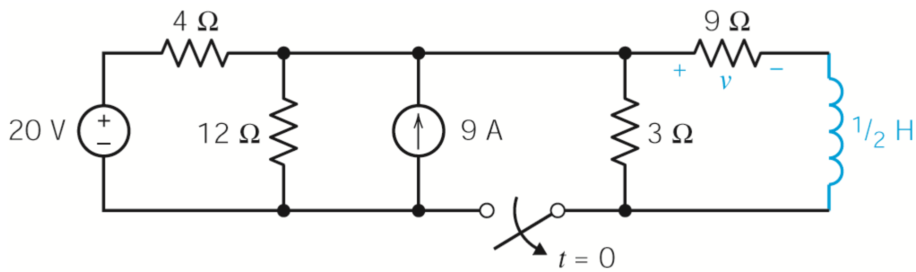
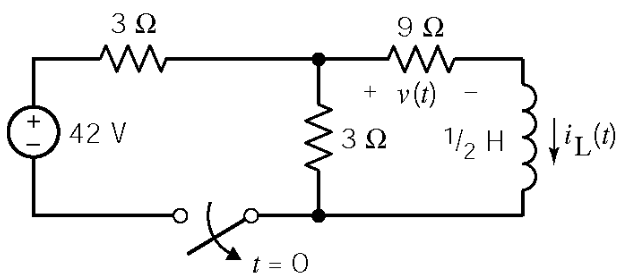
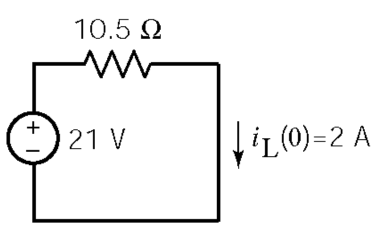


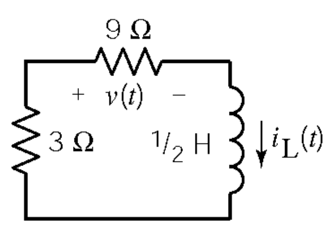
Figure P 8.3-10

Solution: First, use source transformations to obtain the equivalent circuit



for $t < 0$: for $t > 0$:





So $i_L(0) = 2\text{ A},\ I_{sc} = 0,\ R_t = 3+9 = 12\ \Omega,\ \tau = \dfrac{L}{R_t} = \dfrac{\frac{1}{2}}{12} = \dfrac{1}{24}\text{ s}$

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9\, i_L(t) = 18\, e^{-24t} \quad t>0$

P 8.3-13 The circuit shown in Figure P 8.3-13 is at steady state when the switch opens at time $t = 0$. Determine $v(t)$ for $t \geq 0$.

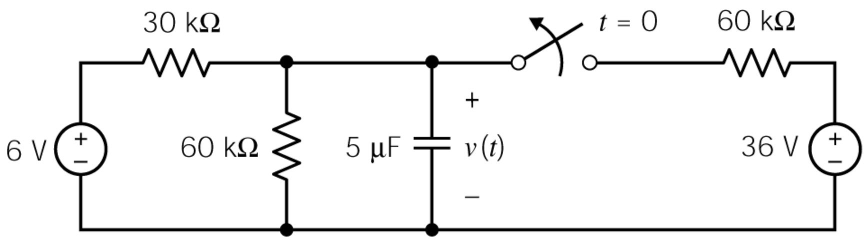
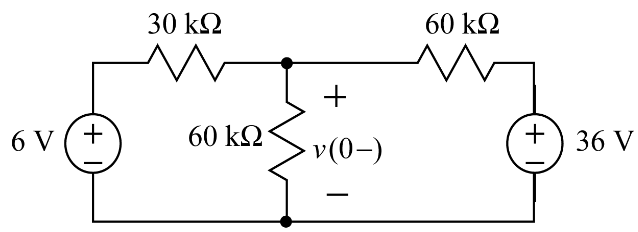


Figure P 8.3-13

Solution: Before $t = 0$, with the switch closed and the circuit at the steady state, the capacitor acts like an open circuit so we have

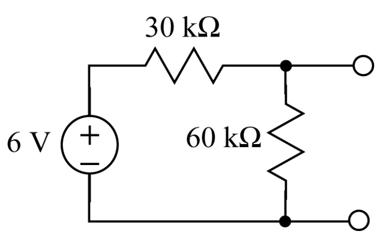


Using superposition

$$v(0-) = \frac{60 \| 60}{30 + (60 \| 60)} 6 + \frac{60 \| 30}{60 + (60 \| 30)} 36 = \left(\frac{1}{2}\right) 6 + \left(\frac{1}{4}\right) 36 = 12 \text{ V}$$

The capacitor voltage is continuous so $v(0+) = v(0-) = 12 \text{ V}$.

After $t = 0$ the switch is open. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor:



$$v_{oc} = \frac{60}{60 + 30} 6 = 4 \text{ V}$$

$$R_t = 30 \| 60 = 20 \text{ k}\Omega$$

The time constant is $\tau = R_t C = (20 \times 10^3)(5 \times 10^{-6}) = 0.1$ s so $\frac{1}{\tau} = 10 \ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = (v(0+) - v_{oc}) e^{-t/\tau} + v_{oc} = (12 - 4) e^{-10t} + 4 = 4 + 8 e^{-10t} \text{ V} \quad \text{for } t \ge 0$$

P 8.3-14 The circuit shown in Figure P 8.3-14 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

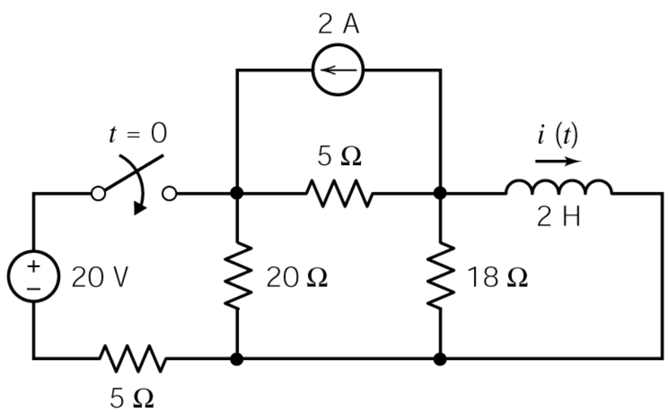
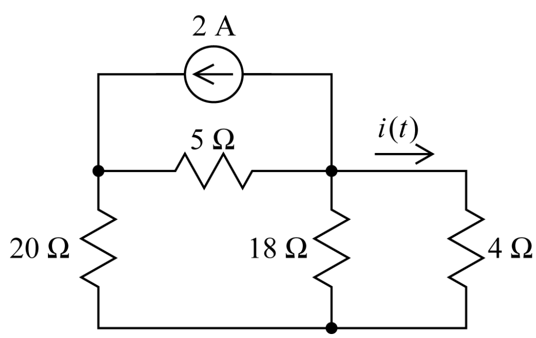


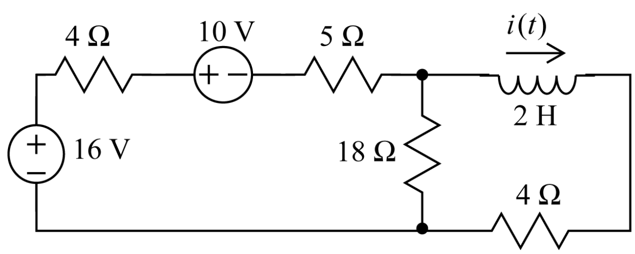
Figure P 8.3-14

Solution: Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have

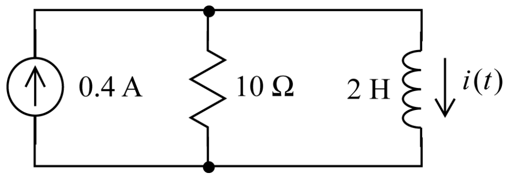


$$i(t) = -\frac{18}{4+18}\left[\frac{5}{5+20+(18 \| 4)}2\right] = 0.29 \text{ A}$$

After $t = 0$, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit. First, performing a couple of source transformations reduces the circuit to



Next, replace the series voltage sources by an equivalent voltage source, replace the series resistors by an equivalent resistor and do a couple of source transformations to get



so $\tau = \frac{2}{10} = 0.25 \quad \Rightarrow \quad \frac{1}{\tau} = 5\frac{1}{s}$

The current is given by $i(t) = [0.29 - 0.4]e^{-5t} + 0.4 = 0.4 - 0.11e^{-5t}$ A for $t \ge 0$

P 8.3-19 The circuit shown in Figure P 8.3-19 is at steady state before the switch closes. Find $v(t)$ for $t \ge 0$.

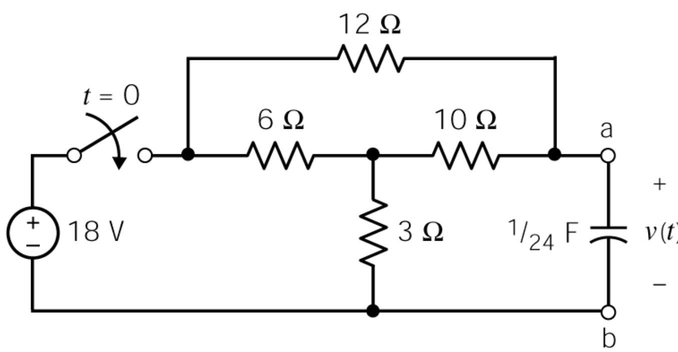
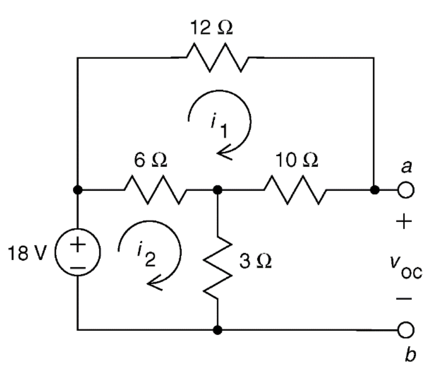


Figure P 8.3-19

Solution: Before the switch closes $v(t) = 0$ so $v(0+) = v(0-) = 0$ V.

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of the terminals $a - b$.

Write mesh equations to find v_{oc}:



Mesh equations:

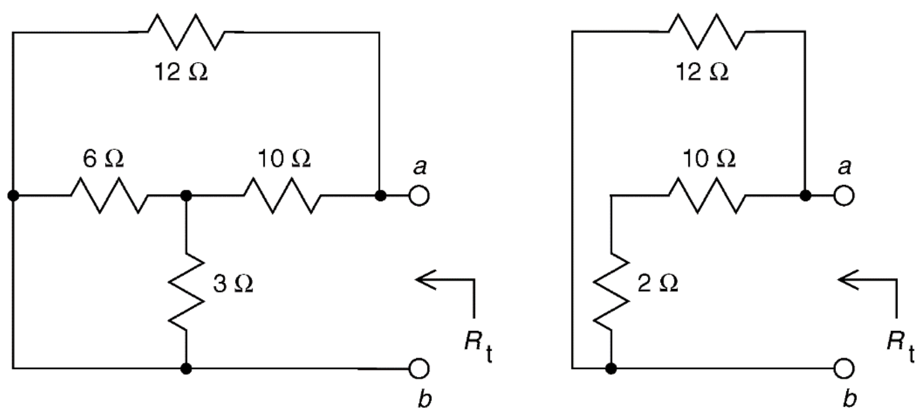
$$12\,i_1 + 10\,i_1 - 6\,(i_2 - i_1) = 0$$
$$6\,(i_2 - i_1) + 3\,i_2 - 18 = 0$$

$$28\,i_1 = 6\,i_2$$
$$9\,i_2 - 6\,i_1 = 18$$

Using KVL,

$$v_{oc} = 3\,i_2 + 10\,i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12\text{ V}$$

Find R_t:



$$R_t = \frac{12\,(10+2)}{12+(10+2)} = 6\ \Omega$$

Then

$$\tau = R_t C = 6\left(\frac{1}{24}\right) = \frac{1}{4}\text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

and

$$v(t) = \left(v(0+) - v_{\text{oc}}\right)e^{-t/\tau} + v_{\text{oc}} = (0-12)e^{-4t} + 12 = 12\left(1-e^{-4t}\right)\text{ V} \quad \text{for } t \ge 0$$

P 8.3-20 The circuit shown in Figure P 8.3-20 is at steady state before the switch closes. Determine $i(t)$ for $t \geq 0$.

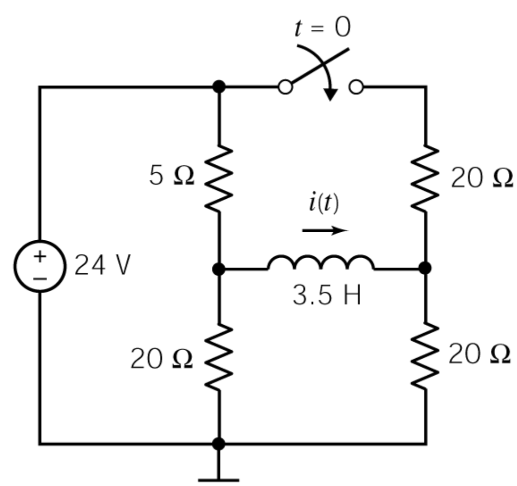
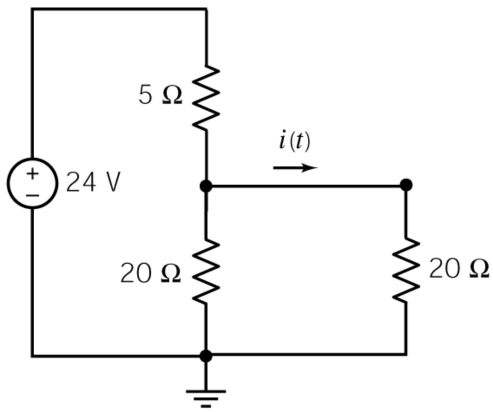


Figure P 8.3-20

Solution: Before the switch closes the circuit is at steady state so the inductor acts like a short circuit. We have

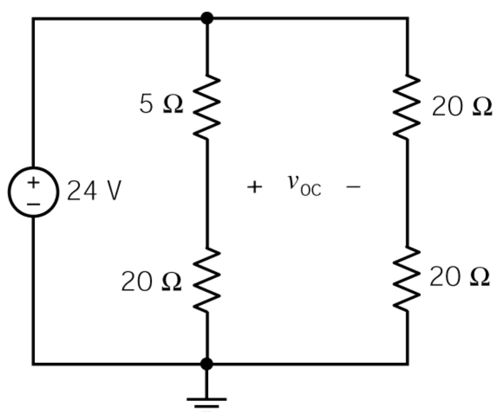


$$i(t) = \frac{1}{2}\left(\frac{24}{5+(20 \parallel 20)}\right) = 0.8 \text{ A}$$

so

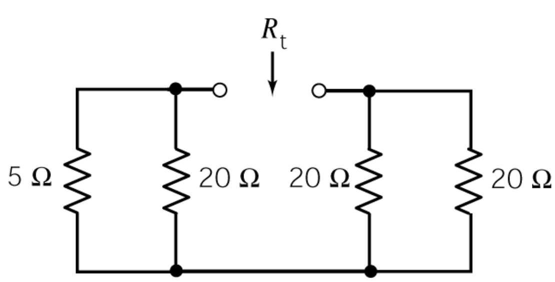
$$i(0+) = i(0-) = 0.8 \text{ A}$$

After the switch closes, find the Thevenin equivalent circuit for the part of the circuit connected to the inductor.



Using voltage division twice

$$v_{oc} = \left(\frac{20}{25} - \frac{1}{2}\right)24 = 7.2 \text{ V}$$



$$R_t = (5 \parallel 20) + (20 \parallel 20) = 14\ \Omega$$

$$i_{sc} = \frac{v_{oc}}{R_t} = \frac{7.2}{14} = 0.514 \text{ A}$$

Then

$$\tau = \frac{L}{R_t} = \frac{3.5}{14} = \frac{1}{4} \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 4 \ \frac{1}{\text{s}}$$

and

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0.8 - 0.514)e^{-4t} + 0.514 = 0.286e^{-4t} + 0.514 \text{ A} \quad \text{for } t \geq 0$$

P 8.3-22 The circuit shown in Figure P 8.3-22 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

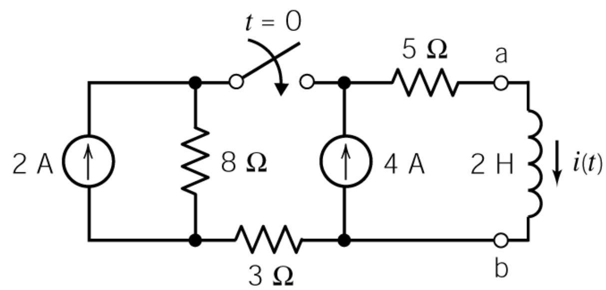
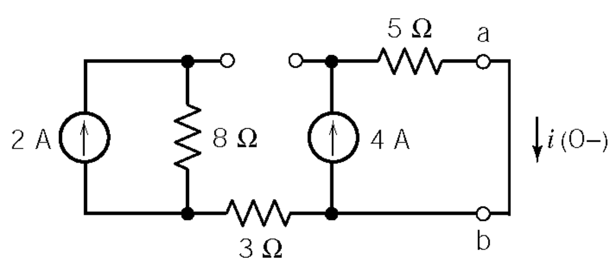


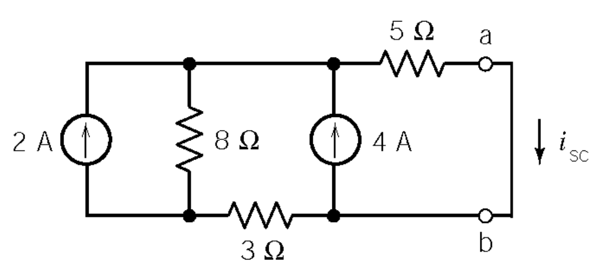
Figure P 8.3-22

Solution: Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have



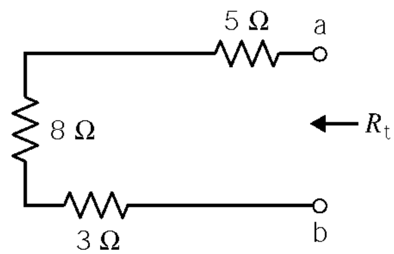
$$i(0+) = i(0-) = 4 \text{ A}$$

After $t = 0$, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit.



Using superposition, the short circuit current is given by

$$i_{sc} = \left(\frac{8}{8 + (5 + 3)}\right)2 + \left(\frac{3 + 8}{(3 + 8) + 5}\right)4 = 3.75 \text{ A}$$



so

$$R_t = 8+3+5 = 16\ \Omega$$

$$\tau = \frac{2}{16} = 0.125\text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 8\ \frac{1}{\text{s}}$$

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (4-3.75)e^{-8t} + 3.75 = 3.75 - 0.25\,e^{-8t}\ \text{A} \quad \text{for } t \ge 0$$

P 8.4-1 The circuit shown in Figure P 8.4-1 is at steady state before the switch closes at time t = 0. The switch remains closed for 1.5 s and then opens. Determine the capacitor voltage, $v(t)$, for $t > 0$.

Hint: Determine $v(t)$ when the switch is closed. Evaluate $v(t)$ at time $t = 1.5$ s to get $v(1.5)$. Use $v(1.5)$ as the initial condition to determine $v(t)$ after the switch opens again.

Answer:

$$v(t) = \begin{cases} 5 + 5e^{-0.5t}\ \text{V} & \text{for } 0 < t < 1.5\text{ s} \\ 10 - 2.64e^{-2.5(t-1.5)}\ \text{V} & \text{for } 1.5\text{ s} < t \end{cases}$$

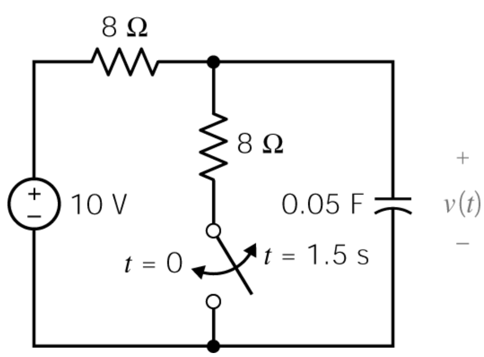
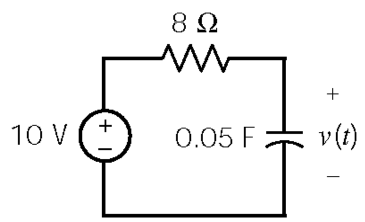


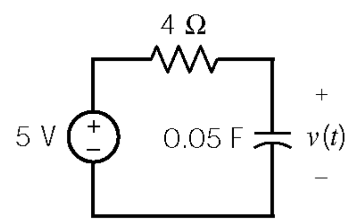
Figure P 8.4-1

Solution:

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $v(0) = 10$ V. For $0 < t < 1.5$s, $v_{oc} =$ 5 V and $R_t = 4\ \Omega$ so $\tau = 4 \times 0.05 = 0.2$ s . Therefore

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 5 + 5e^{-5t} \text{ V} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At t =1.5 s, $v(1.5) = 5 + 5e^{-0.05(1.5)} = 5$ V .

For 1.5 s $< t$, $v_{oc} = 10$ V and $R_t = 8\ \Omega$ so $\tau = 8 \times 0.05 = 0.4$ s .

Therefore

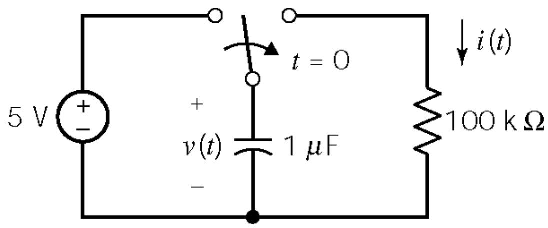
$$v(t) = v_{oc} + (v(1.5) - v_{oc})\, e^{-(t-1.5)/\tau} = 10 - 5\, e^{-2.5(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$v(t) = \begin{cases} 5 + 5\, e^{-5t} \text{ V} & \text{for } 0 < t < 1.5 \text{ s} \\ 10 - 5\, e^{-2.5(t-1.5)} \text{ V} & \text{for } 1.5 \text{ s} < t \end{cases}$$

P 8.4-4 An electronic flash on a camera uses

Solution:



$v(0) = 5\ \text{V},\quad v(\infty) = 0$ and

$\tau = 10^5 \times 10^{-6} = 0.1\ \text{s}$

$$\therefore\quad v(t) = 5\,e^{-10\,t}\ \text{V}\quad \text{for } t > 0$$

$$2.5 = 5\,e^{-10\,t_1} \qquad \underline{t_1 = 0.0693\ \text{s}}$$

$$i(t_1) = \frac{v(t_1)}{100\times 10^3} = \frac{2.5}{100\times 10^3} = \underline{25\ \mu\text{A}}$$

P 8.6-11 The voltage source voltage in the circuit shown in Figure P 8.6-11 is

$$v_s(t) = 5 + 20u\,(t)$$

Determine $i(t)$ for $t \geq 0$.

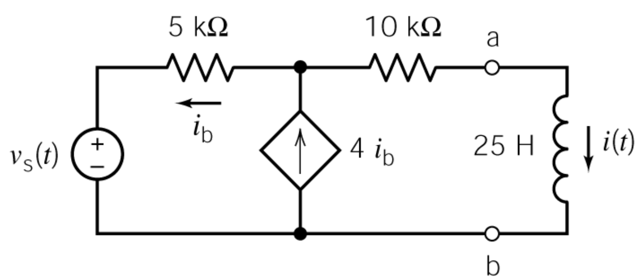
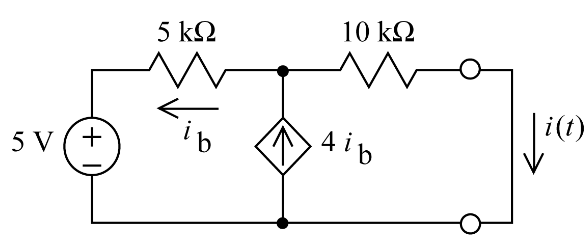


Figure P 8.6-11

Solution:

For $t > 0$ the circuit is at steady state so the inductor acts like a short circuit:



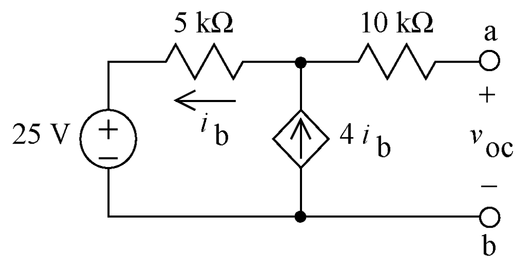
Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000 i_b + 1000(3 i_b) - 5 = 0 \quad \Rightarrow \quad i_b = 0.2\ \text{mA}$$

Apply KVL to get

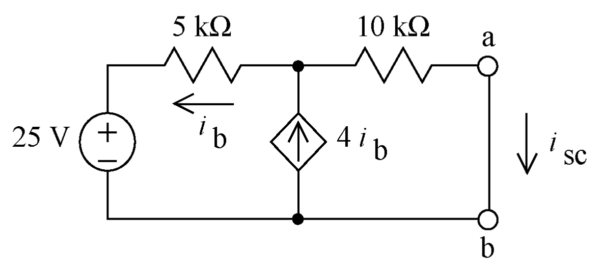
$$i(t) = 3 i_b = 0.6\ \text{mA}$$

For t > 0, find the Norton equivalent circuit for the part of the circuit that is connected to the inductor.



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A. Then

$$v_{oc} = 25 - 5000\left(i_b\right) = 25 \text{ V}$$



Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_b + 10000\left(3\, i_b\right) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$

Then

$$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\bar{3} \text{ k}\Omega$$

Then

$$\tau = \frac{25}{8333} = 3 \text{ ms}$$

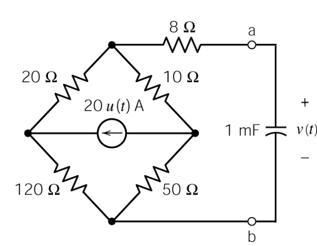
So

$$\frac{1}{\tau} = 333 \ \frac{1}{\text{s}}$$

Now

$$i(t) = \left[0.6 - 3\right]e^{-333t} + 3 = 3 - 2.4e^{-333t} \ \text{ mA} \quad \text{for } t \ge 0$$

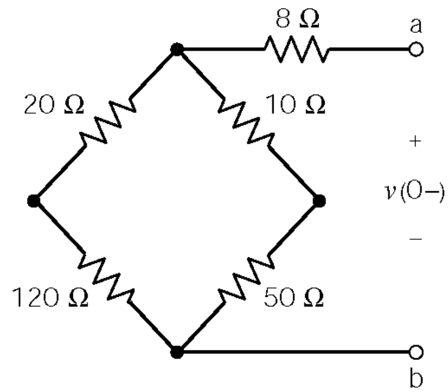
P 8.6-15 Determine $v(t)$ for $t \ge 0$ for the circuit shown in Figure P 8.6-15.



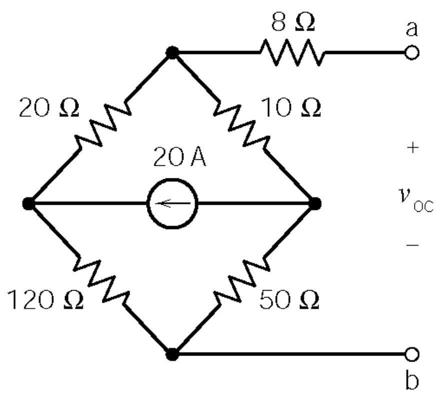
Solution:

When $t<0$ and the circuit is at steady state, the capacitor acts like an open circuit. The 0 A current source also acts like an open circuit. The initial condition is

$$v(0+)=v(0-)=0\text{ V}$$



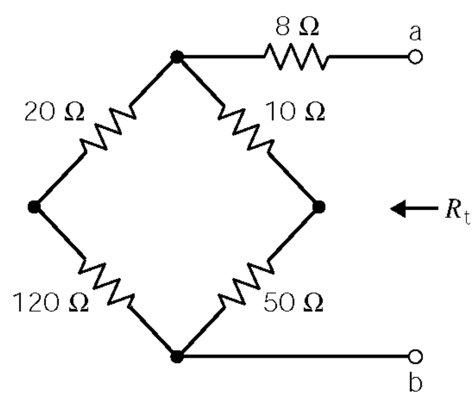
For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



$$v_{oc}=\left[\frac{170}{170+30}(20)\right]10-\left[\frac{30}{170+30}(20)\right]50$$

$$=\frac{170(20)(10)-30(20)(50)}{200}=\frac{4000}{200}=20\text{ V}$$

$$R_t=8+\frac{(20+120)(10+50)}{(20+120)+(10+50)}=50\ \Omega$$



The time constant is $\tau = R_t C = (50)(10^{-3}) = 0.05$ s so $\frac{1}{\tau} = 20 \ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (0-20)e^{-20t} + 20 = 20\left(1 - e^{-20t}\right) \text{ V} \quad \text{for } t \ge 0$$

P8.6-19 Determine the current $i_o(t)$ in the circuit shown in Figure P8.6-19.

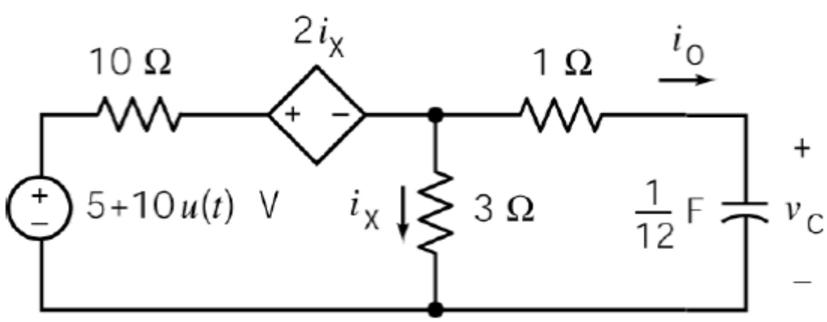


Figure P8.6-19

Solution: This is a first order circuit containing a capacitor. First, determine $v_C(t)$.

Consider the circuit for time *t* < 0.

Step 1: Determine the initial capacitor voltage.

The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 5 V, a constant.

The capacitor will act like an open circuit.

P 8.6-23 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-23.

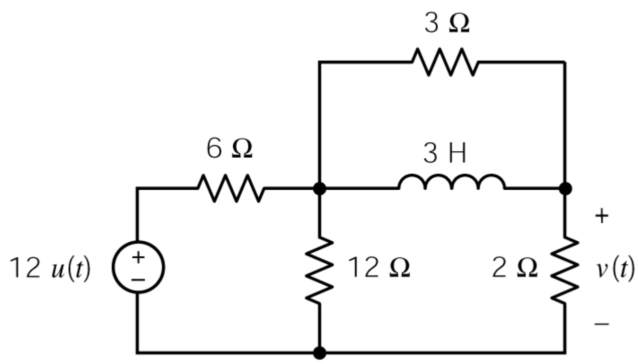
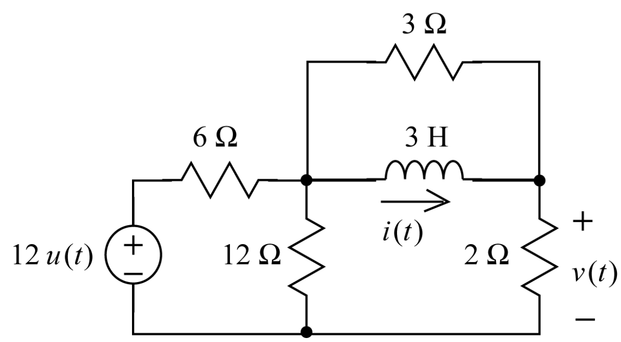
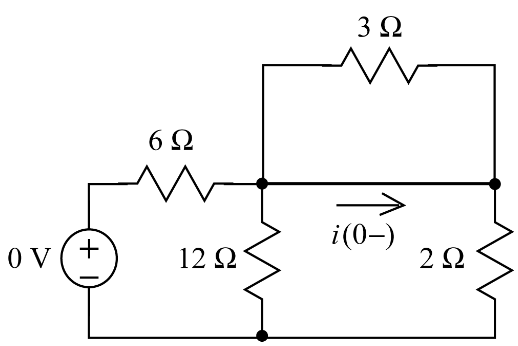


Figure P 8.6-23

Solution: The resistor voltage, $v(t)$, may not be continuous at time $t = 0$. The inductor will be continuous. We will find the inductor current first and then find $v(t)$. Label the inductor current as $i(t)$.

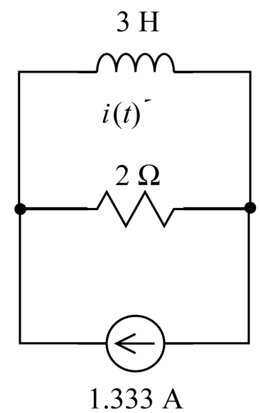




For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = 0 \text{ A}$$

For $t > 0$ use source transformations to simplify the part of the circuit connected to the inductor until is a Norton equivalent circuit.



Recognize that

$$R_t = 2\ \Omega \quad \text{and} \quad i_{sc} = 1.333\ \text{A}$$

The time constant is

$$\tau = \frac{L}{R_t} = \frac{3}{2} \quad \Rightarrow \quad \frac{1}{\tau} = 0.667\ \frac{1}{\text{s}}$$

Then

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = 1.333\left(1 - e^{-0.667t}\right)\ \text{A} \quad \text{for } t \ge 0$$

Returning to the original circuit we see that

$$\frac{v(t)}{2} = i(t) + \frac{3\frac{d}{dt}i(t)}{3} = i(t) + \frac{d}{dt}i(t)$$
$$= 1.333\left(1 - e^{-0.667t}\right) + (-0.667)(1.333)\left(-e^{-0.667t}\right) = 1.333 - 0.4439e^{-0.667}$$

Finally

$$v(t) = 2.667 - 0.889e^{0.667t}\ \text{V} \quad \text{for } t > 0$$