CSE 101

Please take a copy of the Syllabus

CSE 101: Introduction to Algorithms

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Course webpage:
http://cseweb.ucsd.edu/~dakane/CSE101/

Office Hours Schedule:
https://tinyurl.com/y3r9j45k

FinAid Survey

- Please remember to fill out the financial aid survey for this course on canvas.

Practice Quiz

What is your favorite number?

(A) 0

(B) e

(C) π

(D) 17

(E) 101

Introduction

- What kinds of problems will we consider in this course?
- Fibonacci numbers.
- Asymptotic Runtimes.
- Levels of algorithm design.

Straightforward Programming Problems

- Display text
- Copy a file
- Count number of occurrences of a given word

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Each has a straightforward algorithm that is hard to improve upon.

Algorithms Problems

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- Find the best pairing of students to dorm rooms
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These problems are

- Well defined mathematically
- Still not easy to solve

AI Problems

- Image recognition
- Game playing
- Understanding natural language

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- Understanding natural language

For these problems, much of the difficulty is in formalizing exactly what you are trying to do.

This Class

- We will focus on algorithms problems

Problem: Fibonacci Numbers

Definition:

The Fibonacci numbers are the sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55,…

Defined by

$F_0 = F_1 = 1$

$F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$

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Problem: Given n, compute F_n.

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Essentially turned definition into an algorithm.

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$$T(n) = \begin{cases} 2 & \text{if } n \le 1 \\ T(n-1)+T(n-2)+3 & \text{else} \end{cases}$$

Question: Runtime

If your computer executes a billion lines of code per second, approximately how long does it take to compute F(100)?

A) A millisecond
B) A second
C) A year
D) 100,000 years
E) Forever

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$T(100) \approx 2.87 \cdot 10^{21}$
At a billion lines of code per second, this is just over 90,000 years.

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Too many recursive calls.

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$\boxed{F(5)}$

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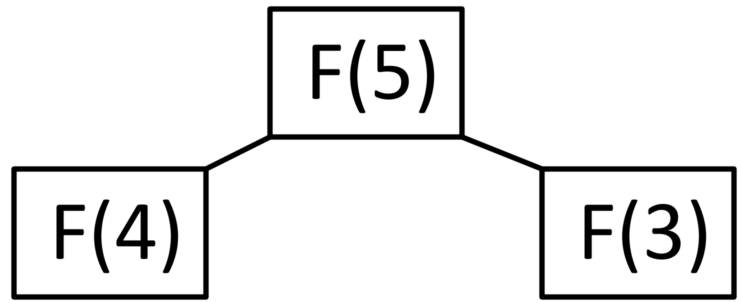

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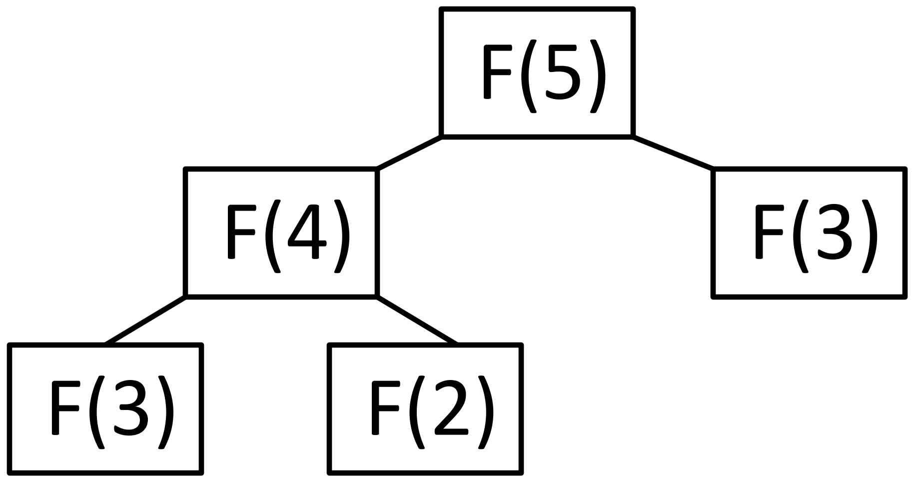

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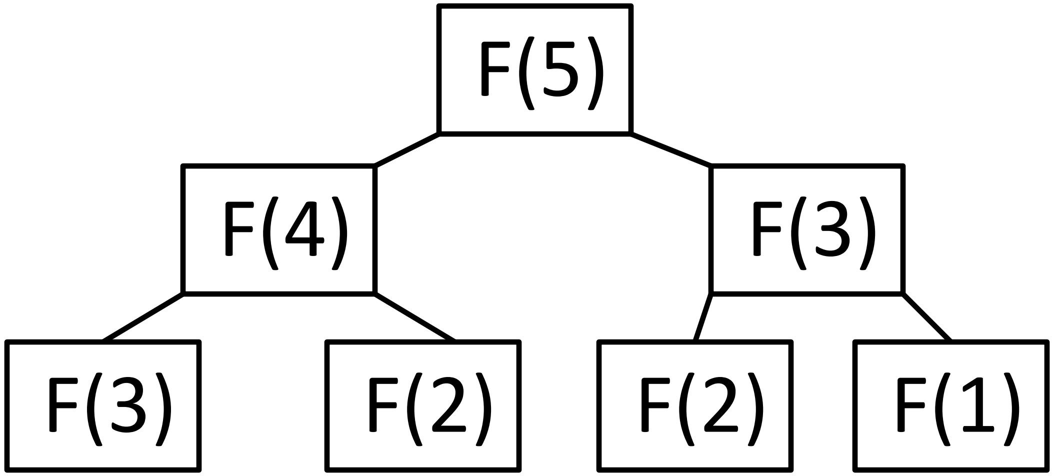

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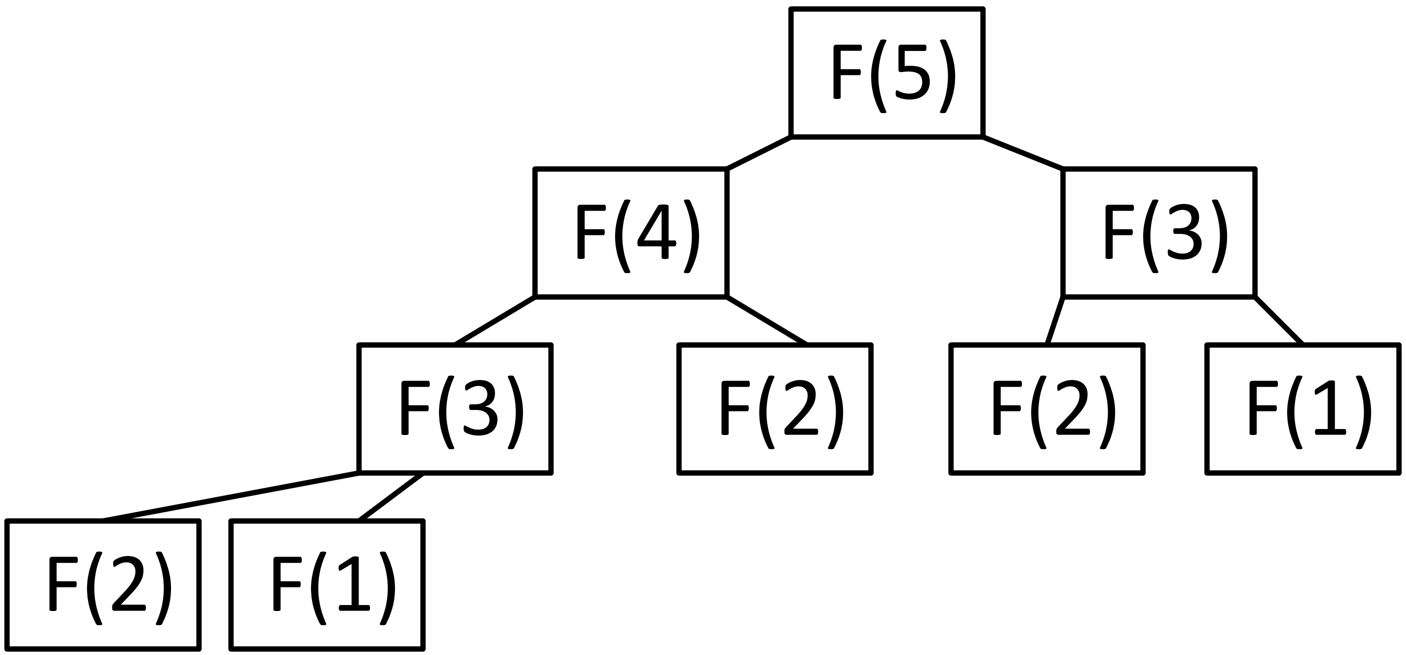

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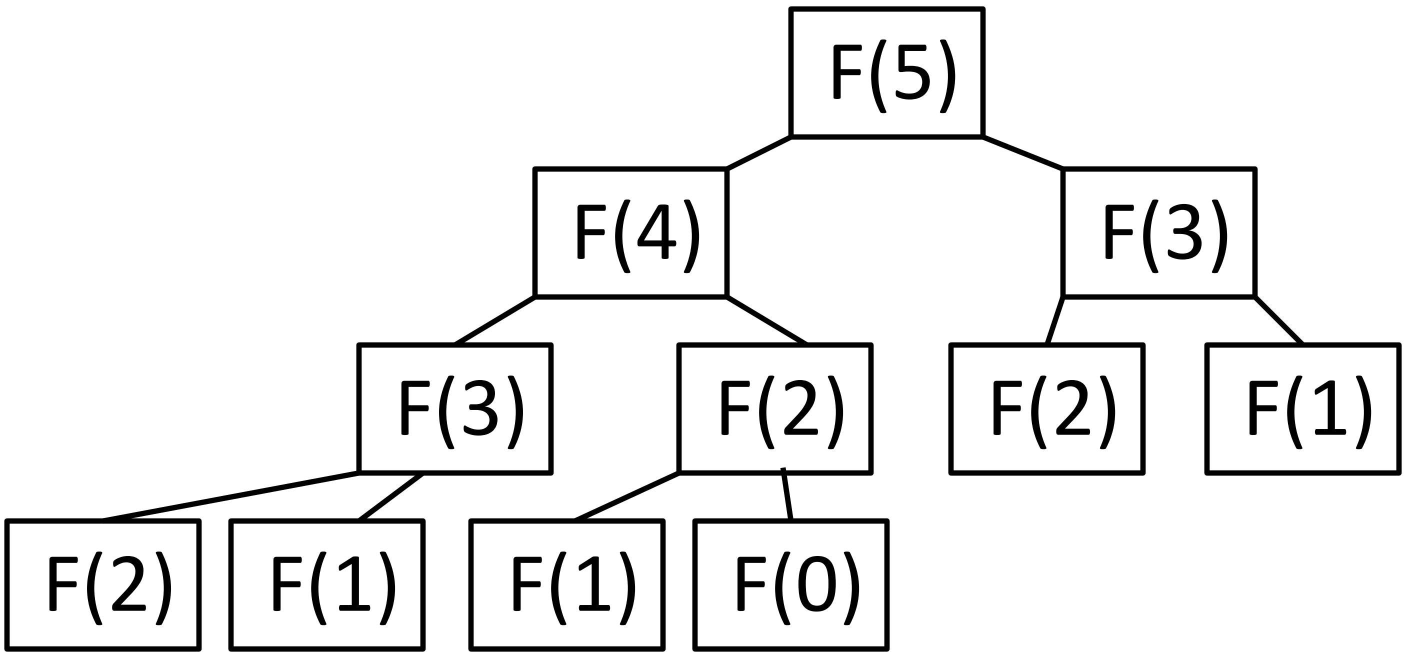

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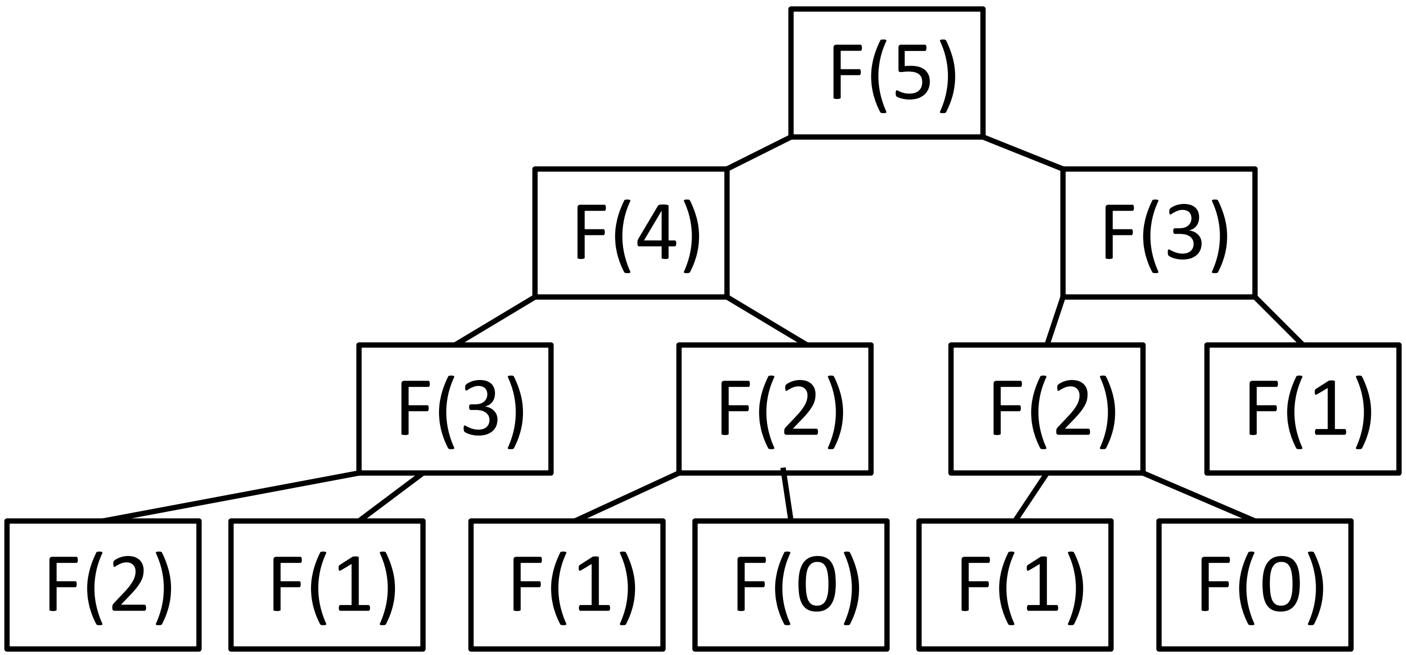

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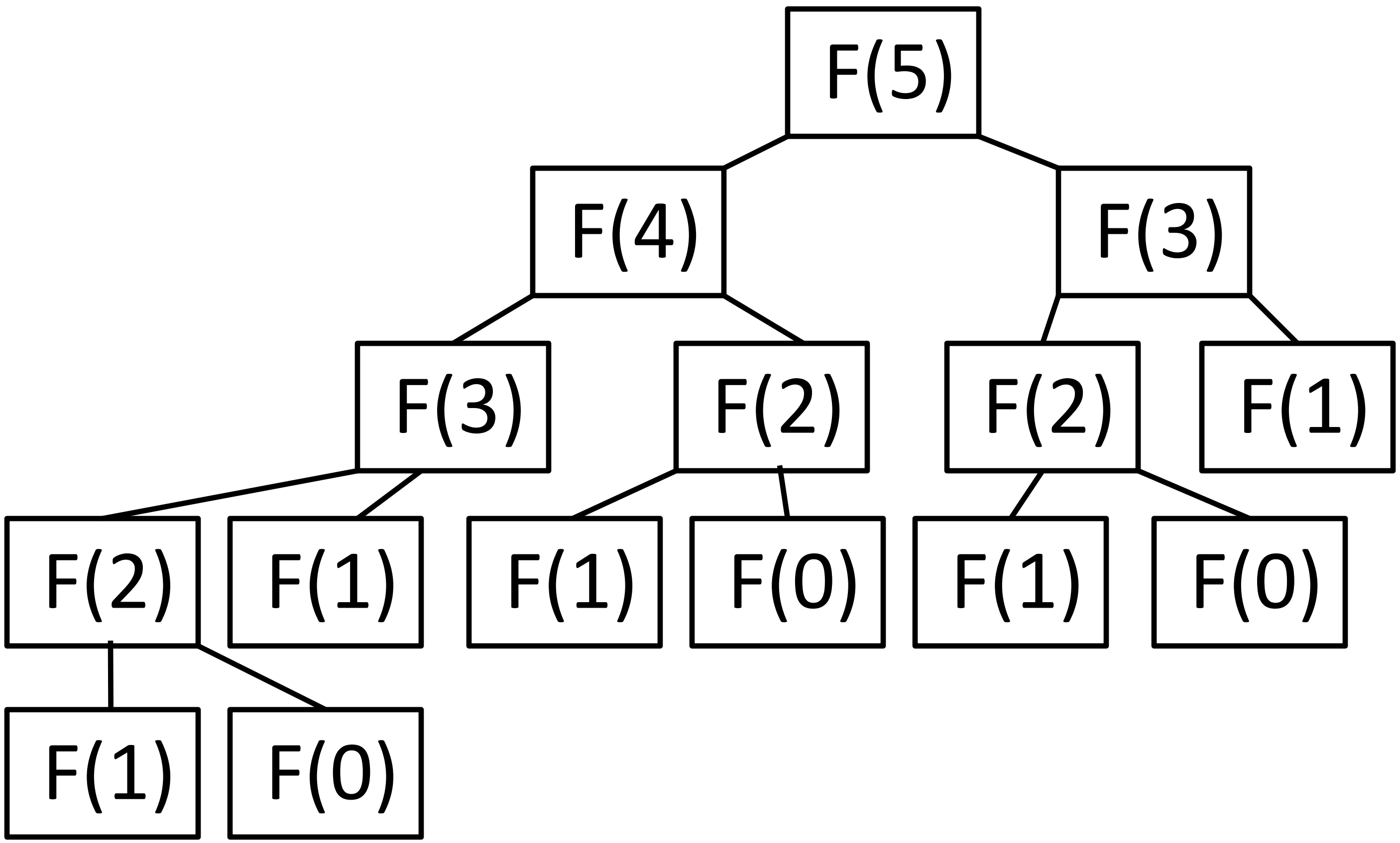

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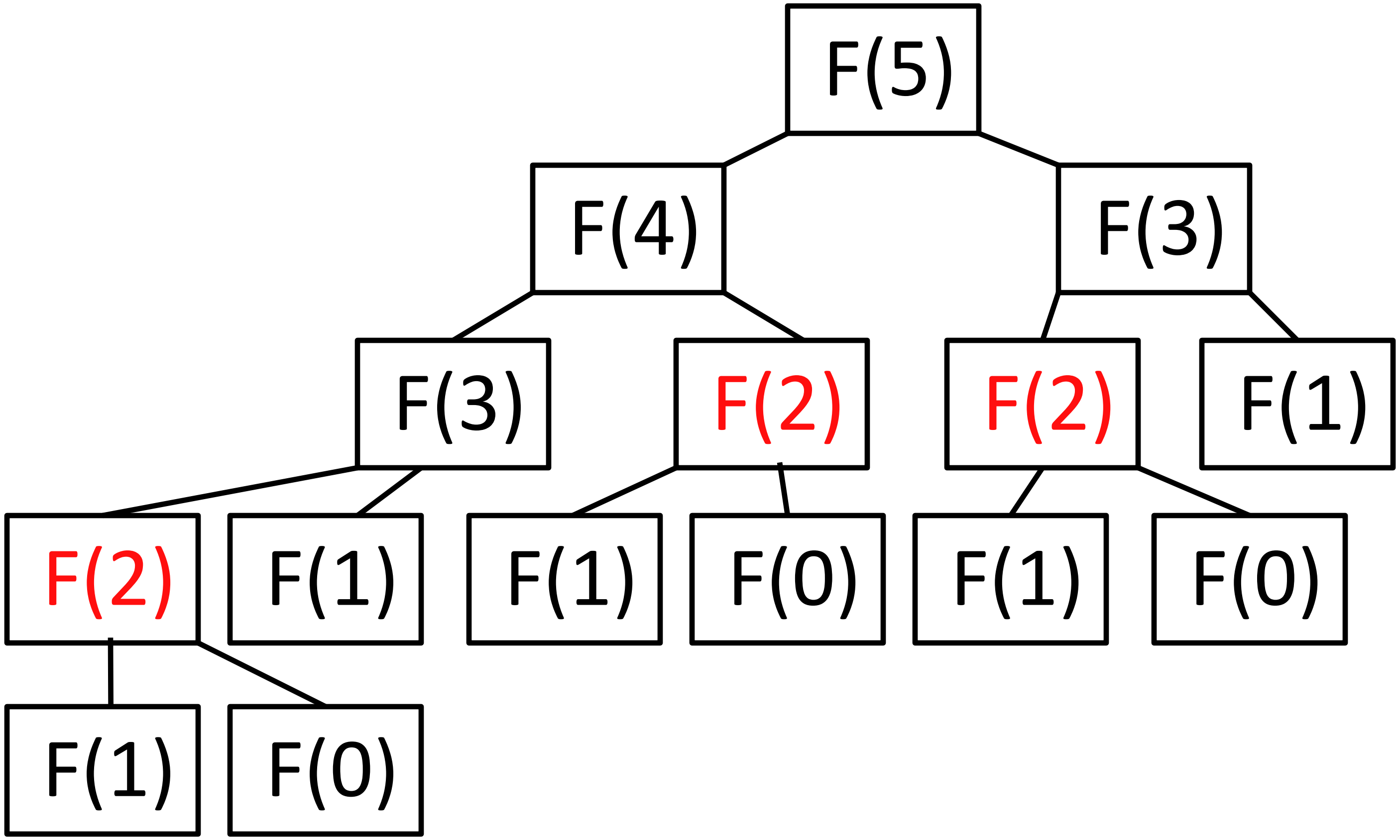

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As long as you have records of the previous answers, you can just write down the next one.

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Simulate the above idea using array A to store values $A[n] = F_n$.

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The power of algorithms: Sometimes the right algorithm is the difference between something working and not finishing in your lifetime.

Question: Runtime

Is $T(n) = 2n+1$ a good description of this program's runtime?

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A) Yes

B) No

Discussion of Runtimes

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Is $T(n) = 2n+1$ really an accurate description of that program's runtime?

- Is initializing an array one operation or several?
- What about adding large integers?
- Should we count machine ops?
  - Doesn't this depend on implementation?

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Too much to consider for every analysis

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- These issues usually just constant factors.
- If we analyze runtime in a way that *ignores* constant factors (like big-O), we don't have to deal with them.
- But ignoring constant factors 1 second is the same as 100,000 years.
- On the other hand, we can still compare things *asymptotically*. A $\Theta(n)$ algorithm will beat an $\Theta(n^2)$ algorithm for n large enough.

Advantages and Disadvantages of Asymptotic Analysis

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Because of this, this class will almost exclusively use big-O analysis.