ECE 35 Homework #6 (Spring 2023, Taur)

All homework problems come from the textbook, "Introduction to Electric Circuits", by Svoboda & Dorf, 9th Edition.

P 7.2-2 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the current, $i(t)$, when the capacitance is $C = 0.125$ F and the voltage is $v(t) = 12\cos(2t + 30°)$ V.

$$\frac{d}{dt}A\cos(\omega t+\theta) = -A\sin(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta)$$
$$= -A\omega\sin(\omega t+\theta)$$
$$= A\omega\cos\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$$

Hint:

Answer: $i(t) = 3\cos(2t + 120°)$ A

Solution:

$$i(t) = C\frac{d}{dt}v(t) = \frac{1}{8}\frac{d}{dt}12\cos\left(2t+30^\circ\right) = \frac{1}{8}(12)(-2)\sin\left(2t+30^\circ\right) = 3\cos\left(2t+120^\circ\right.$$

P 7.2-12 The capacitor voltage in the circuit shown in Figure P 7.2-12 is given by $v(t) = 12 - 10e^{-2t}$ V for $t \geq 0$. Determine $i(t)$ for $t > 0$.

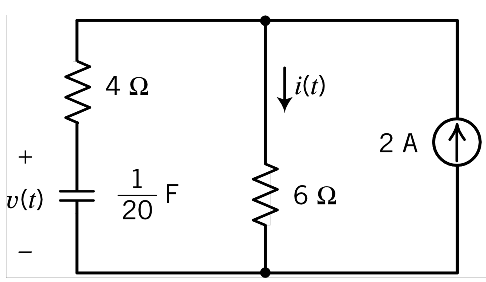


Figure P 7.2-12

Solution:

P 7.2-13 The capacitor voltage in the circuit shown in Figure P 7.2-13 is given by

$$v(t) = 2.4 + 5.6e^{-5t} \text{ V} \quad \text{for} \quad t \geq 0$$

Determine $i(t)$ for $t > 0$.

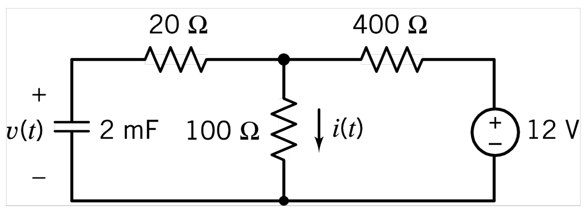
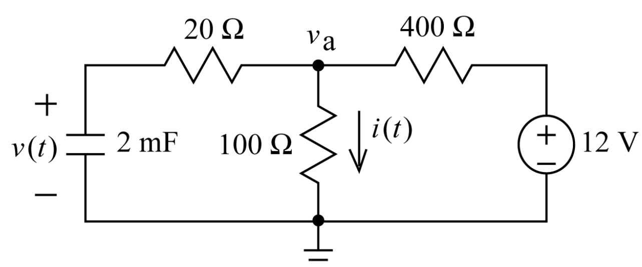


Figure P 7.2-13

Solution:

We'll write and solve a node equation. Label the node voltages as shown. Apply KCL at node *a* to get



$$\frac{v(t) - v_a}{20} = \frac{v_a}{100} + \frac{v_a - 12}{400} \quad \Rightarrow \quad v_a = \frac{20v(t) + 12}{25}$$

So

$$v_a = 2.4 + 4.48e^{-5t} \text{ V} \quad \text{for } t > 0$$

Then

$$i(t) = \frac{v_a}{100} = 24 + 44.8e^{-5t} \text{ mA} \quad \text{for } t > 0$$

P 7.3-6 The initial capacitor voltage of the circuit shown in Figure P 7.3-6 is $v_c(0^-) = 3$ V. Determine (a) the voltage $v(t)$ and (b) the energy stored in the capacitor at $t = 0.2$ s and $t = 0.8$ s when

$$i(t) = \begin{cases} 3e^{5t} \text{ A} & 0 < t < 1 \\ 0 & t \geq 1\text{s} \end{cases}$$

Answer:

(a) $18e^{5t}$ V, $0 \leq t < 1$

(b) $w(0.2) = 6.65$ J and $w(0.8) = 2.68$ kJ

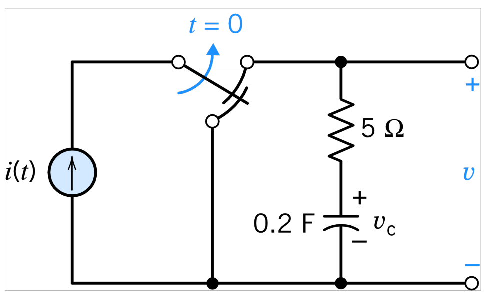


Figure P 7.3-6

Solution:

We have $v(0^+) = v(0^-) = 3$ V

$$v_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + v_c(0) = 5\int_0^t 3\,e^{5t}\,dt + 3 = 3\left(e^{5t}-1\right)+3 = 3\,e^{5t} \text{ V},\ \ 0<t<1$$

a)

$$v(t) = v_R(t)+v_c(t) = 5\,i(t)+v_c(t) = 15\,e^{5t}+3\,e^{5t} = \underline{18\,e^{5t} \text{ V}},\ \ 0<t<1$$

b)

$$W(t) = \frac{1}{2}Cv_c^2(t) = \frac{1}{2}\times 0.2\left(3e^{5t}\right)^2 = 0.9e^{10t} \text{ J} \ \Rightarrow\ \begin{cases} W(t)\big|_{t=0.2s} = \underline{6.65 \text{ J}} \\ W(t)\big|_{t=0.8s} = \underline{2.68 \text{ kJ}} \end{cases}$$

P 7.4-6 Determine the value of the equivalent capacitance, C_{eq}, in the circuit shown in Figure P 7.4-6.

Answer: $C_{eq} = 10$ F

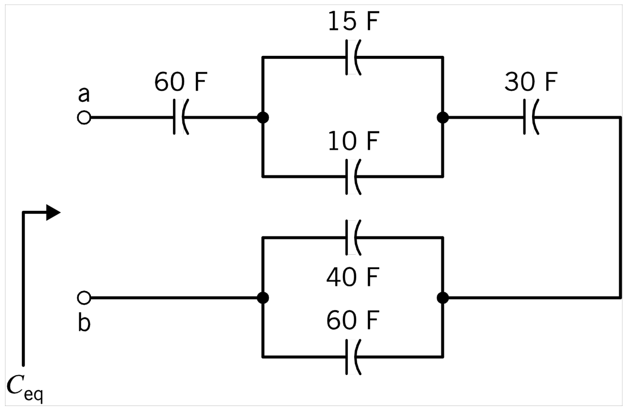


Figure P 7.4-6.

Solution:

$$C_{eq} = \frac{1}{\dfrac{1}{60}+\dfrac{1}{15+10}+\dfrac{1}{30}+\dfrac{1}{40+60}} = 10 \text{ F}$$

P 7.4-8 The circuit shown in Figure P 7.4-8 is at steady state before the switch opens at time $t = 0$.

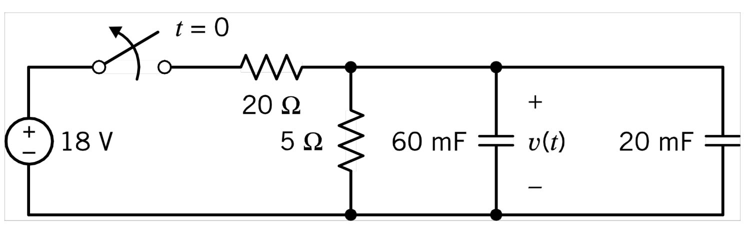


Figure P 7.4-8

The voltage $v(t)$ is given by

$$v(t) = \begin{cases} 3.6\,\text{V} & \text{for } t \le 0 \\ 3.6e^{-2.5t} & \text{for } t \ge 0 \end{cases}$$

(a) Determine the energy stored by each capacitor before the switch opens.

(b) Determine the energy stored by each capacitor 1 s after the switch opens.

The parallel capacitors can be replaced by an equivalent capacitor.

(c) Determine the energy stored by the equivalent capacitor before the switch opens.

(d) Determine the energy stored by the equivalent capacitor 1 s after the switch opens.

Solution:

(a) The energy stored in the 60 mF capacitor is $w_1 = \frac{1}{2}(0.060)3.6^2 = 0.3888\ \text{W}$ and the energy stored in the 20 mF capacitor is $w_2 = \frac{1}{2}(0.020)3.6^2 = 0.1296\ \text{J}$.

(b) One second after the switch opens, the voltage across the capacitors is $3.6e^{-2.5} = 0.2955\ \text{V}$.

Then $w_1 = 2.620\ \text{mJ}$ and $w_2 = 0.873\ \text{mJ}.$

Next $C_{\text{eq}} = 0.06 + 0.02 = 80\ \text{mF}.$

(c) $w_{\text{eq}} = \frac{1}{2}(0.08)3.6^2 = 0.5184\ \text{J} = w_1 + w_2$

(d) $w_{\text{eq}} = \frac{1}{2}(0.08)(0.2955)^2 = 3.493\ \text{mJ} = w_1 + w_2$

P 7.5-12 The inductor current in the circuit shown in Figure P 7.5-12 is given by

$$i(t) = 6 + 4e^{-8t}\ \text{A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

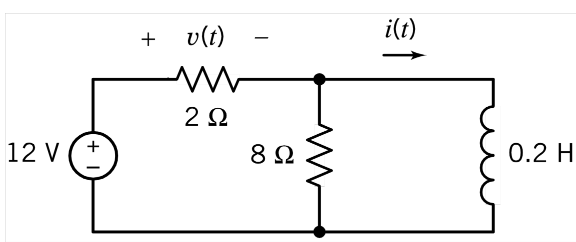
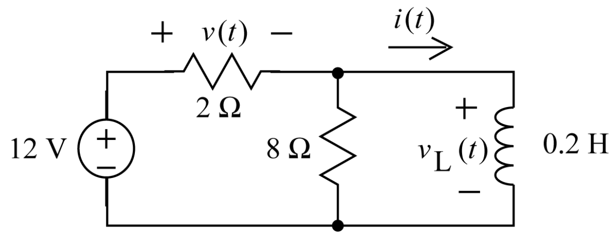


Figure P 7.5-12

Solution:

$$v_L(t) = 0.2\frac{d}{dt}i(t)$$
$$= -6.4e^{-8t} \text{ V} \quad \text{for } t > 0$$



Use KVL to get

$$v(t) = 12 - \left(-6.4e^{-8t}\right) = 12 + 6.4e^{-8t} \text{ V} \quad \text{for } t > 0$$

P 7.5-13 The inductor current in the circuit shown in Figure P 7.5-13 is given by

$$i(t) = 5 - 3e^{-4t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

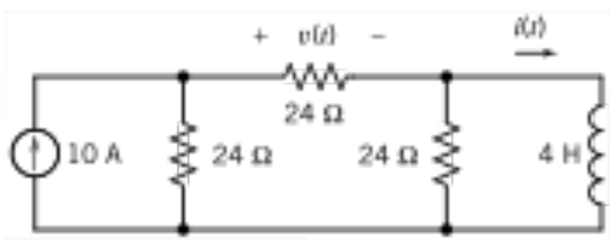
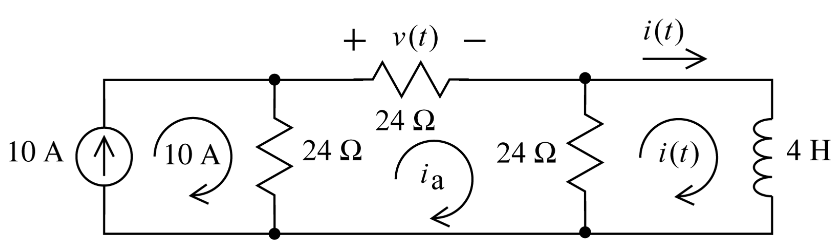


Figure P 7.5-13

Solution:



We'll write and solve a mesh equation. Label the meshes as shown. Apply KVL to the center mesh to get

$$24i_a + 24\left(i_a - i(t)\right) + 24\left(i_a - 10\right) = 0 \quad \Rightarrow \quad i_a = \frac{i(t)+10}{3} = 5 - e^{-4t} \text{ A} \quad \text{for } t > 0$$

Then

$$v(t) = 24i_a = 120 - 24e^{-4t} \text{ V} \quad \text{for } t > 0$$

P 7.6-3 The voltage, $v(t)$, across a 25-mH inductor used in a fusion power experiment is

$$v(t) = \begin{cases} 0 & t \le 0 \\ 6\cos 100t & t \ge 0 \end{cases}$$

where the units of time are s and the units of voltage are V. The current in this inductor is zero before the voltage changes at $t = 0$. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.

Hint: 2(cos A)(sin B) = sin(A + B) + sin(A – B)

Answer: $p(t) = 7.2 \sin 200t$ W and $w(t) = 3.6[1 - \cos 200t]$ mJ

Solution:

$$i(t) = \frac{1}{25\times10^{-3}}\int_0^t 6\cos 100\tau \, d\tau + 0 = \frac{6}{(25\times10^{-3})(100)}\left[\sin 100\tau \,\Big|_0^t\right] = 2.4\sin 100t$$

$$p(t) = v(t)\, i(t) = (6\cos 100\,t)(2.4\sin 100t) = 7.2\,[\,2(\cos 100\,t)(\sin 100\,t)\,]$$
$$= 7.2\,[\sin 200\,t + \sin 0] = 7.2\sin 200\,t$$

$$W(t) = \int_0^t p(\tau)\, d\tau = 7.2\int_0^t \sin\, 200\tau\, d\tau = -\frac{7.2}{200}\left[\cos\, 200\tau\,\Big|_0^t\right]$$
$$= 0.036[1 - \cos 200t]\text{ J} = 36\,[1 - \cos 200t\,]\text{ mJ}$$

P 7.7-8 The circuit shown in Figure P 7.7-8 is at steady state before the switch closes. The inductor currents are both zero before the switch closes ($i_1(0) = i_2(0) = 0$).

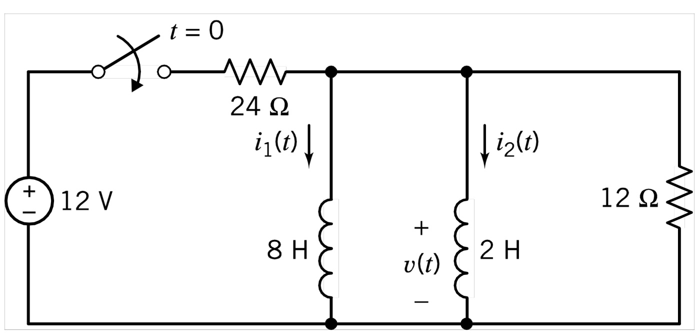


Figure P 7.7-8

The voltage $v(t)$ is given by

$$v(t) = \begin{cases} 0\,\text{V} & \text{for } t < 0 \\ 4e^{-5t}\ \text{V} & \text{for } t > 0 \end{cases}$$

(a) Determine the inductor currents, $i_1(t)$ and $i_2(t)$, for $t \geq 0$.

(b) Determine the energy stored by each inductor 200 ms after the switch closes.

The parallel inductors can be replaced by an equivalent inductor.

(c) Determine the current in the equivalent inductor, directed downward, for $t \geq 0$.

(d) Determine the energy stored by the equivalent inductor 200 ms after the switch closes.

Solution:

(a)
$$i_1(t) = \frac{1}{8}\int_0^t 4e^{-5\tau}\,d\tau + 0 = \frac{1}{-10}\left(e^{-5t} - 1\right) = 0.1\left(1 - e^{-5t}\right)\ \text{A} \quad \text{for } t \geq 0$$

$$i_2(t) = \frac{1}{2}\int_0^t 4e^{-5\tau}\,d\tau = 0.4\left(1 - e^{-5t}\right)\ \text{A} \quad \text{for } t \geq 0$$

(b) When t = 0.25, $i_1(0.2) = 0.1\left(1 - e^{-1}\right) = 63.2\ \text{mA}$ and $i_2(0.2) = 0.4\left(1 - e^{-1}\right) = 252.8\ \text{mA}$ so the energy stored by the 8 H inductor is $w_1 = \frac{1}{2}(8)0.0632^2 = 16.0\ \text{mJ}$ and the energy stored by the 2 H indictor is $w_2 = 63.9\ \text{mJ}$.

(c)
$$L_{eq} = \frac{8.2}{8+2} = 1.6\ \text{H}$$

$$i(t) = \frac{1}{1.6}\int_0^t 4e^{-5\tau}\,d\tau = 0.5\left(1 - e^{-5t}\right)\ \text{A} \quad \text{for } t \geq 0$$

(d) When t = 0.2 s, $i(0.2) = 0.5\left(1 - e^{-1}\right) = 316\ \text{mA}$ so the energy stored by the equivalent inductor is

$$w = \frac{1}{2}(1.6)0.316^2 = 79.9\ \text{mJ} = w_1 + w_2$$.

P 7.8-8 The circuit shown in Figure P 7.8-8 is at steady state when the switch opens at time $t = 0$. Determine $v_1(0-)$, $v_1(0+)$, $i_2(0-)$, $i_2(0+)$, $i_3(0-)$, $i_3(0+)$, $v_4(0-)$, and $v_4(0+)$.

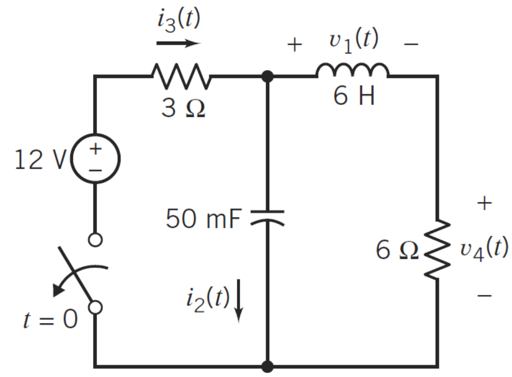
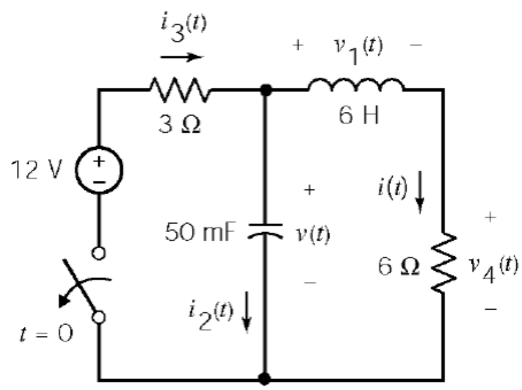


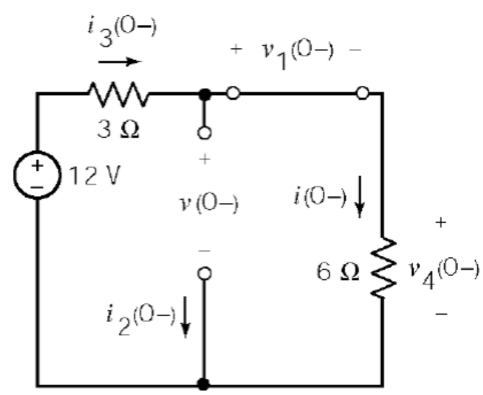
Figure P 7.8-8

Solution: The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch closed and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



$$i_3(0-) = i(0-) = \frac{12}{9} = 1.33 \text{ A}$$

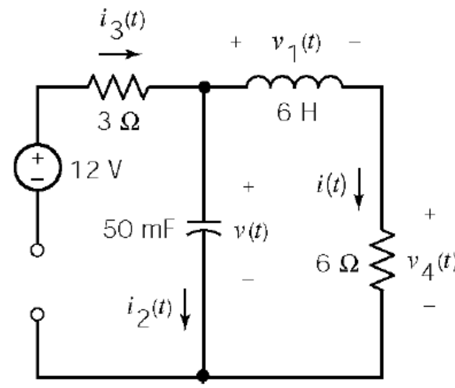
$$v_4(0-) = v(0-) = 6i(0-) = 8 \text{ V}$$

$$v_1(0-) = 0 \text{ V and } i_2(0-) = 0 \text{ A}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+) = v(0-) = 8 \text{ V} \text{ and } i(0+) = i(0-) = 1.33 \text{ A}$$

After the switch opens the circuit looks like this:



From KCL:

$$i_3(t)=0 \text{ A and } i_2(t)=-i(t)$$

From KVL:

$$v_1(t)+6\,i(t)=v(t)$$

From Ohm's Law:

$$v_4(t)=6\,i(t)$$

At $t=0+$

$$i_3(0+)=0 \text{ A and } i_2(0+)=-i(0+)=-1.33 \text{ A}$$

$$v_1(0+)=v(0+)-6\,i(0+)=8-6(1.333)=0 \text{ V}$$

$$v_4(0+)=6\,i(0+)=8 \text{ V}$$

P 7.8-10 The circuit shown in Figure P 7.8-10 is at steady state when the switch closes at time $t=0$. Determine $v_1(0-)$, $v_1(0+)$, $i_2(0-)$, and $i_2(0+)$.

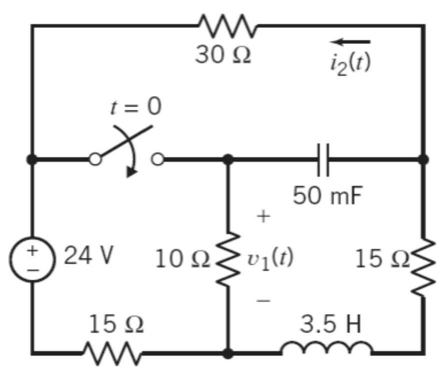
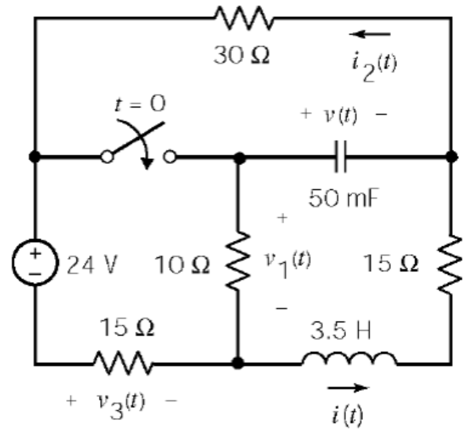


Figure P 7.8-10

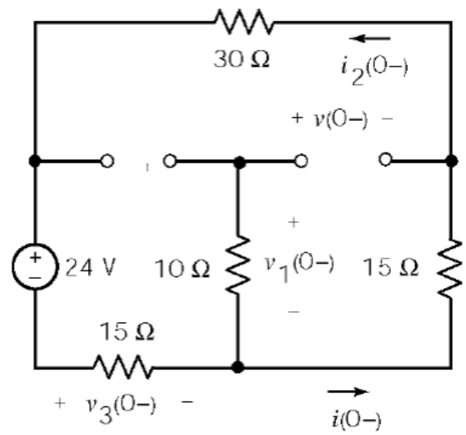
Solution:

The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



$$i_2(0-) = i(0-) = \frac{24}{60} = -0.4 \text{ A}$$

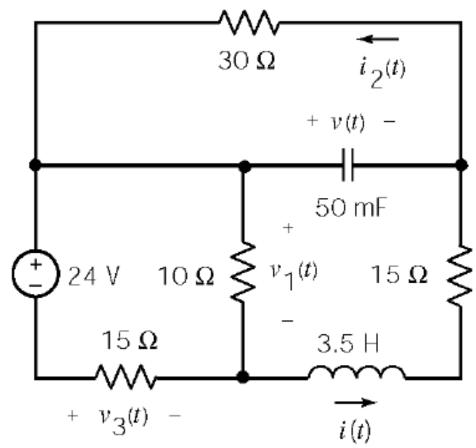
$$v_1(0-) = 0 \text{ V}$$

$$v(0-) - 15\,i(0-) = v_1(0-) \quad \Rightarrow \quad v(0-) = -6 \text{ V}$$

$$v_3(0-) = 15\,i(0-) = -6 \text{ V}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+) = v(0-) = -6 \text{ V} \text{ and } i(0+) = i(0-) = -0.4 \text{ A}$$



After the switch closes the circuit looks like this:

From Ohm's Law:

$$i_2(t) = -\frac{v(t)}{30}$$

From KVL:

$$v_1(t) = v_3(t) + 24$$

From KCL:

$$\frac{v_1(t)}{10} + \frac{v_3(t)}{15} = i(t)$$

At $t = 0+$

$$i_2(0+) = -\frac{v(0+)}{30} = 0.2 \text{ A}$$

$$\left.\begin{aligned} v_1(0+) &= v_3(0+) + 24 \\ \frac{v_1(0+)}{10} + \frac{v_3(0+)}{15} &= i(0+) \end{aligned}\right\} \Rightarrow v_1(0+) = 7.2 \text{ V and } v_3(0+) = -16.8 \text{ V}$$

P7.8-11

The circuit shown in Figure 7.8-11 has reached steady state before the switch opens at time $t = 0$. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

Answers: $i_L(0-) = 1.25$ A, $v_C(0-) = 20$ V, $v_R(0-) = -5$ V and $v_R(0+) = -4$ V

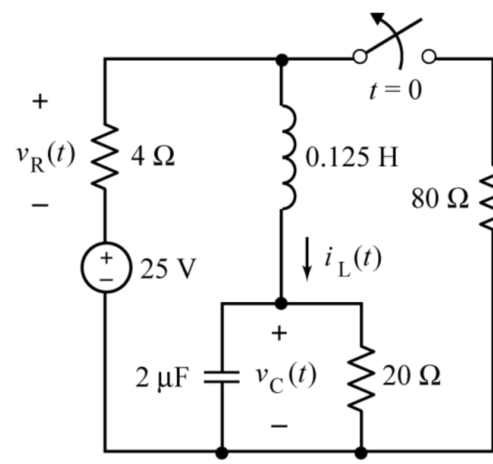
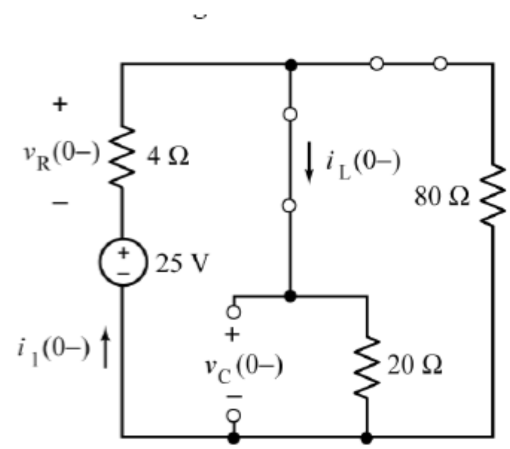


Figure 7.8-11

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.



At t=0−, **the capacitor acts like an open circuit** and **the inductor acts like a short circuit**. From the circuit

$$i_1(0-) = \frac{25}{4+(20 \| 80)} = \frac{25}{4+16} = 1.25 \text{ A},$$

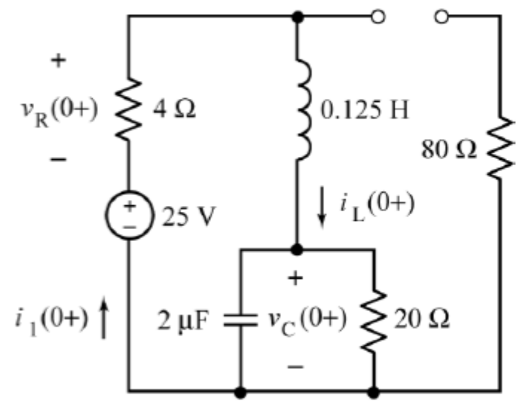
$$i_L(0-) = \left(\frac{80}{20+80}\right) i_1(0-) = 1 \text{ A},$$

$$v_C(0-) = 20 i_L(0-) = 20 \text{ V}$$

and

$$v_R(0-) = -4 i_1(0-) = -5 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so



$$v_C(0+) = v_C(0-) = 20 \text{ V} \text{ and}$$

$$i_L(0+) = i_L(0-) = 1 \text{ A}$$

Apply KCL at the top node to see that

$$i_1(0+) = i_L(0+) = 1 \text{ A}$$

From Ohm's law

$$v_R(0+) = -4 i_1(0+) = -4 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)