Phys 2B Summer 2022

Quiz 4 Solutions

Question 1:

The magnetic dipole orientation energy is:

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

Changing from antiparallel to the field to parallel means changing the angle from π radians to 0.

$$\Delta U = U_f - U_i = -\mu B \cos 0 + \mu B \cos \pi = -\mu B - \mu B = -2\mu B$$

The minus sign means that the energy is released. Now plug in numbers.

$$\Delta U = -2(24\,\mathrm{J/T})(54 \times 10^{-6}\,\mathrm{T}) \approx \boxed{2.6\,\mathrm{mJ\,released}}$$

This is choice (b).

Question 2:

The magnetic force is serving as a centripetal force in this problem, so:

$$qvB = \frac{mv^2}{r}$$

We want to solve for r.

$$r = \frac{mv}{qB}$$

The two isotopes differ only in mass, so the difference is:

$$\Delta r = r_{11} - r_{10} = \frac{v}{qB}(m_{11} - m_{10})$$

Plug in numbers.

$$\Delta r \approx \frac{(1.24 \times 10^5\,\mathrm{m/s})}{(5 \times 1.60 \times 10^{-19}\,\mathrm{C})(0.550\,\mathrm{T})}(1.83 - 1.66) \times 10^{-26}\,\mathrm{kg} \approx \boxed{0.479\,\mathrm{mm}}$$

This is choice (c).

Question 3:

The energy per volume in a magnetic field is:

$$u = \frac{1}{2}\frac{1}{\mu_0}B^2$$

The magnetic field of a solenoid is $B = \mu_0(N/L)I$, so:

$$u = \frac{1}{2}\frac{1}{\mu_0}(\mu_0(N/L)I)^2 = \frac{\mu_0 N^2 I^2}{2L^2}$$

Now we must multiply by the solenoid's volume to get the total energy.

$$U = uV = uAL = u(\pi R^2)L = \frac{\mu_0 N^2 I^2}{2L^2}\pi R^2 L = \frac{\pi\mu_0 N^2 I^2 R^2}{2L}$$

Now plug in numbers.

$$U = \frac{\pi(4\pi \times 10^{-7}\,\text{T-m/A})(800)^2(90\,\text{A})^2(0.028\,\text{m})^2}{2(3.5\,\text{m})} \approx \boxed{2.3\,\text{J}}$$

This is choice (d).

You could also treat the solenoid as an inductor, calculating its inductance and applying $U_L = \frac{1}{2}LI^2$.

Question 4:

In this problem, the magnetic field is changing and the area is constant, so Faraday's Law says:

$$|\mathcal{E}| = A\frac{\mathrm{d}B}{\mathrm{d}t}$$

From the plot, we can calculate the magnetic field's time derivative (ie the slope) using:

$$\frac{\Delta B}{\Delta t} = \frac{B_2 - B_1}{t_2 - t_1}$$

The slope is constant, so the average slope is equal to the slope at any given moment, including $t = 2.0\,\text{s}$.

Some convenient points are $(t_1, B_1) = (0\,\text{s}, 2\,\text{T})$ and $(t_2, B_2) = (1\,\text{s}, 4\,\text{T})$

$$\frac{\Delta B}{\Delta t} = \frac{4\,\text{T} - 2\,\text{T}}{1\,\text{s} - 0\,\text{s}} = 2.0\,\text{T/s}$$

To get the current, divide by the resistance.

$$I = \frac{A\frac{\mathrm{d}B}{\mathrm{d}t}}{R}$$

Now plug in numbers.

$$I = \frac{(2.5\,\mathrm{m}^2)(2.0\,\mathrm{T/s})}{8.9\,\Omega} \approx \boxed{0.56\,\mathrm{A}}$$

This is choice (b).

Question 5:

Motional emf is given by (for perpendicular motion):

$$\mathcal{E} = vLB$$

Plug in numbers.

$$\mathcal{E} = (0.070\,\mathrm{m/s})(150\,\mathrm{m})(2.0 \times 10^{-3}\,\mathrm{T}) \approx \boxed{21\,\mathrm{mV}}$$

This is choice (c).

Question 6:

The force felt by a proton at the edge is:

$$F = qvB$$

since the magnetic field and proton's velocity are perpendicular. If we imagine that the beam is moving away from us, "into the page", then the field is clockwise, and the proton's velocity is into the page. $\vec{v} \times \vec{B}$ points inward, toward the center of the beam.

This is choice (b).

Question 7:

The motional emf is:

$$|\mathcal{E}| = vLB$$

To find the current that will flow in the rod, divide by the resistance:

$$I = \frac{vLB}{R}$$

The current in the rod is perpendicular to the field, so the force can be found with only multiplication:

$$F = ILB = \frac{vLB}{R}LB = \frac{vL^2B^2}{R}$$

Now plug in numbers.

$$F = \frac{(3.4\,\mathrm{m/s})(0.96\,\mathrm{m})^2(5.0\,\mathrm{T})^2}{13\,\Omega} \approx \boxed{6.0\,\mathrm{N}}$$

There are several ways to work out the direction. Probably the simplest is to note that opposing the change means producing a force down, since the rod is moving up.

This is choice (e).

Question 8:

The period of an LC circuit is:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

We can read the period from the plot, for instance by counting the time between two crests. $T = 6.0\,\text{ps}$.

Solve for L.

$$L = \left(\frac{T}{2\pi}\right)^2 \frac{1}{C}$$

Plug in numbers.

$$L = \left(\frac{6.0 \times 10^{-9}\,\text{s}}{2\pi}\right)^2 \frac{1}{24 \times 10^{-9}\,\text{F}} \approx \boxed{0.038\,\text{nH}}$$

This is choice (a).

Question 9:

By energy conservation:

$$U_{C,\text{max}} = U_{L,\text{max}}$$

$$\frac{1}{2}\frac{Q_{\text{max}}^2}{C} = \frac{1}{2}LI_{\text{max}}^2$$

$$I_{\text{max}} = \frac{1}{\sqrt{LC}}Q_{\text{max}}$$

Doubling Q_{max} doubles I_{max} if nothing else changes, so I is true.

Large inductance L makes I_{max} smaller, so II is false.

Large capacitance C also makes I_{max} smaller, so III is also false.

The correct choice is therefore (a), I only.

Question 10:

It is most natural to apply energy conservation. The maximum energy on the capacitor is equal to the sum of the energy on the capacitor and the energy on the inductor at any time.

$$\frac{1}{2}\frac{Q_{\text{max}}^2}{C} = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

We want to solve for I.

$$I = \frac{1}{\sqrt{LC}}\sqrt{Q_{\max}^2 - Q^2}$$

Plug in numbers.

$$I = \frac{1}{\sqrt{(1.2 \times 10^{-9}\,\mathrm{H})(86 \times 10^{-6}\,\mathrm{F})}}\sqrt{(190 \times 10^{-9}\,\mathrm{C})^2 - (-148 \times 10^{-9}\,\mathrm{C})^2} \approx \boxed{0.37\,\mathrm{A}}$$

This is choice (c).