Phys 2B Summer 2022

Quiz 1 Solutions

Question 1:

The magnitude of the force exerted by one charge on another is:

$$F = \frac{k|q_1||q_2|}{r^2}$$

We want to solve for $|q_2|$.

$$|q_2| = \frac{r^2 F}{k|q_1|}$$

We are not told the magnitude of the force directly, but we can calculate it from the components using

$$F = \sqrt{F_x^2 + F_y^2}$$

$$|q_2| = \frac{r^2\sqrt{F_x^2 + F_y^2}}{k|q_1|}$$

Now plug in numbers.

$$|q_2| \approx \frac{(1.2\,\mathrm{m})^2\sqrt{(1.0\,\mathrm{N})^2 + (-2.6\,\mathrm{N})^2}}{(9.0 \times 10^9\mathrm{Nm}^2/\mathrm{C}^2)(6.2 \times 10^{-6}\,\mathrm{C})} \approx \boxed{72\,\mu\mathrm{C}}$$

This is choice (b).

Question 2:

Newton's 2nd Law says $F = ma$, and we can express electric forces as $F = qE$. In this case, $q = e$, the elementary charge, and m is the mass we want to find. Set these two expressions equal:

$$ma = qE$$

Solve for m and plug in that $q = e$.

$$m = \frac{eE}{a}$$

Now plug in numbers.

$$m = \frac{(1.6 \times 10^{-19}\,\mathrm{C})(1.8 \times 10^{5}\,\mathrm{N/C})}{9.3 \times 10^{10}\,\mathrm{m/s}^2} \approx \boxed{3.1 \times 10^{-25}\,\mathrm{kg}}$$

This is choice (c).

This is a little over 185 proton masses. If it's an element, it's something like tungsten or rhenium, but it could also be an ion built from multiple elements.

Question 3:

Charge is conserved, so as polystyrene strips electrons from the glass, they both build up the same magnitude of charge, with opposite signs, so the force will point upward. For two identical charges q, Coulomb's Law says:

$$F = k\frac{q^2}{r^2}$$

We want to solve for q, and set the force equal to the given gravitational force on the pellet.

$$q = r\sqrt{\frac{F}{k}}$$

Now we want the number of electrons, so divide this charge by the charge-per-electron transferred, the elementary charge.

$$N_e = \frac{r}{e}\sqrt{\frac{F}{k}}$$

Now plug in numbers.

$$N_e \approx \frac{10 \times 10^{-9}\,\mathrm{m}}{1.6 \times 10^{-19}\,\mathrm{C}}\sqrt{\frac{0.001\,\mathrm{N}}{9.0 \times 10^{9}\,\mathrm{Nm}^2/\mathrm{C}^2}} \approx \boxed{20000\,\mathrm{electrons}}$$

This is choice (a).

. . . This is not many electrons, considering that macroscopic objects have $\sim 10^{23}$ or so. Styrofoam pellets are very sticky, because electric forces are powerful.

Question 4:

The sum of the electric fields must be 0. Since we're to the right of both charges, positive charges will produce fields pointing in the positive x direction and negative charges in the negative x direction, so we don't need to do any sign adjustments. The x direction of a field happens to be the same as the sign of the charge producing it in this problem, and the x direction is the only one that matters.

$$E_1 + E_2 = 0$$

$$k\frac{q_1}{r_1^2} + k\frac{q_2}{r_2^2} = 0$$

We want to solve for q_2.

$$q_2 = -q_1\frac{r_2^2}{r_1^2}$$

Since q_1 is at the origin, r_1 is obviously 2.0 meters.

Since q_2 is instead at $x = 1.6$ meters, r_2 is only $2.0 - 1.6 = 0.4$ meters.

Now plug in numbers.

$$q_2 \approx -(2.5\,\mathrm{C})\frac{(0.4\,\mathrm{m})^2}{(2.0\,\mathrm{m})^2} \approx \boxed{-0.10\,\mathrm{C}}$$

This is choice (a).

Question 5:

I is true for an ideal dipole (no internal field), but false for a real physical dipole because of the region in between the charges. The dipole moment points from negative to positive. The field on the axis points away from the positive end, and toward the negative end, which is in the same direction as the dipole moment except in the region in between the charges of a real physical dipole. We accept both answers (b) and (d) for this question since it is reasonable to interpret I as true (ideal) or false (physical).

II is also true. The field loops from the positive end to the negative end, pointing exactly opposite the dipole moment in the perpendicular plane.

III is false. The field of a dipole is, for instance, twice as strong on the axis of the dipole as on the perpendicular plane.

So, choices (b) (physical dipole) and (d) (ideal dipole) are both acceptable.

Question 6:

The field of a dipole is $\vec{E} = -k\vec{p}/r^3$ in the plane perpendicular to the dipole. That's not 0. I is false.

The field is only zero at the center of a uniformly charged ring (ie $z = 0$), so II is false.

The field of an infinite plane is constant, $E = \frac{\sigma}{2\epsilon_0}$. Two planes with the same charge density will therefore cancel everywhere between the plates, each pointing in opposite directions. III is true.

So, only III is true. This is choice (c).

Question 7:

It doesn't matter what the unknown charge is, since the overall force is always $F = qE$; we can simply rank based on the strength of the electric field at the unknown charge's position. So long as $q \neq 0$, there is a unique largest-magnitude force.

All the known charges are the same distance from the central charge and have the same charge magnitude, so they will produce field contributions of the same magnitude; call this E.

When the charges are arranged on separate axes, their contributions sum in quadrature:

$$E_{\text{net}} = \sqrt{E_1^2 + E_2^2} = \sqrt{2}E$$

This applies to both systems B and D. The signs of the components are irrelevant.

In system C, the fields from the charges cancel, since they have the same sign but are on opposite sides of the central charge. $\vec{E}_{\text{net}} = 0$.

In system A, the fields from the charges instead sum directly:

$$E_{\text{net}} = E_1 + E_2 = 2E$$

since the charges have opposite signs and are on opposite sides of the central charge.

Since $2 > \sqrt{2}$, the correct choice is (a).

Question 8:

First, we must calculate the total charge of the sphere.

$$Q = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = (20 \times 10^{-9}\,\text{C/m}^3)\left(\frac{4}{3}\pi(0.11\,\text{m})^3\right) \approx 1.1 \times 10^{-10}\,\text{C}$$

The maximum strength of the field of a uniformly-charged sphere occurs at the surface:

$$E_{\text{max}} = \frac{kQ}{R^2}$$

Simply plug in numbers.

$$E_{\text{max}} = \frac{(9.0 \times 10^9\,\text{Nm}^2/\text{C}^2)(1.1 \times 10^{-10}\,\text{C})}{(0.11\,\text{m})^2} \approx \boxed{83\,\text{N/C}}$$

This is choice (b).

Question 9:

Gauss' Law says that the net flux through any closed surface is:

$$\Phi = \frac{Q_{\text{encl}}}{\epsilon_0}$$

The net charge enclosed by the spherical surface is $-1\,\text{C}$. The $+3\,\text{C}$ charge does not contribute anything, since it's outside. Plug in numbers.

$$\Phi = \frac{-1\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{Nm}^2} \approx \boxed{-1.1 \times 10^{11}\,\text{Nm}^2/\text{C}}$$

This is choice (e).

Question 10:

The integral to do is ($R = 0.50\,\mathrm{m}$):

$$Q = 4\pi \int_0^R \rho(r) r^2 \mathrm{d}r$$

Plug in the given $\rho(r) = br^{\frac{1}{2}}$, where $b = 1.1\,\mathrm{C/m}^{\frac{7}{2}}$ (I'm assigning these given values arbitrary variable names for now to keep the math cleaner).

$$Q = 4\pi \int_0^R (br^{\frac{1}{2}}) r^2 \mathrm{d}r = 4\pi b \int_0^R r^{\frac{5}{2}} \mathrm{d}r$$

$$Q = 4\pi b \frac{R^{\frac{7}{2}}}{\frac{7}{2}} = \frac{8}{7}\pi b R^{\frac{7}{2}}$$

Now plug the given values of R and b back in:

$$Q = \frac{8}{7}\pi (1.1\,\mathrm{C/m}^{\frac{7}{2}})(0.50\,\mathrm{m})^{\frac{7}{2}} \approx \boxed{0.35\,\mathrm{C}}$$

This is choice (b).