Phys 2B Summer 2022

Final Exam Practice Solutions

Question 1:

The damping paramemter ζ for an RLC circuit is:

$$\zeta = \frac{R/2L}{\omega_0} = \frac{R}{2L}\sqrt{LC} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

For critical damping, $\zeta = 1$. We want to solve for R.

$$R = 2\sqrt{\frac{L}{C}}$$

Now plug in numbers.

$$R = 2\sqrt{\frac{81 \times 10^{-9}\,\mathrm{H}}{12 \times 10^{-9}\,\mathrm{F}}} \approx \boxed{5.2\,\Omega}$$

This is choice (d).

Question 2:

Note: I would not normally include a question that cannot be solved if you aren't able to answer another question, to avoid magnifying a single mistake. Because this is the practice quiz and this pair of questions is good RLC circuit practice, I'm making an exception.

If the circuit is critically damped ($\zeta = 1$), then the damped frequency is 0, and the $Q(t)$ equation is purely a decaying exponential.

$$Q(t) = Q(0)e^{-\frac{R}{2L}t}\cos(0) = Q(0)e^{-\frac{R}{2L}t}$$

Our condition is that $Q(t)/Q(0) = 0.12$.

$$0.12 = e^{-\frac{R}{2L}t}$$

Solve for t.

$$\ln(0.12) = -\frac{R}{2L}t$$

$$t = -\frac{2L}{R}\ln(0.12) = -\tau\ln\left(\frac{Q(t)}{Q(0)}\right)$$

(Note that the last equality is a general formula for any final-to-initial value ratio for critically damped anything.)

Plug in numbers.

$$t \approx -\frac{2(81 \times 10^{-9}\,\mathrm{H})}{5.2\,\Omega} \ln(0.12) \approx \boxed{66\,\mathrm{ns}}$$

This is choice (e).

Question 3:

The damped angular frequency of an RLC circuit is:

$$\omega_{\mathrm{damped}} = \omega_0\sqrt{1-\zeta^2} = \frac{1}{\sqrt{LC}}\sqrt{1-\left(\frac{R}{2}\right)^2\frac{C}{L}}$$

Now plug in numbers.

$$\omega_{\mathrm{damped}} = \frac{1}{\sqrt{(45 \times 10^{-9}\,\mathrm{H})(6.2 \times 10^{-9}\,\mathrm{F})}}\sqrt{1-\left(\frac{1.7\,\Omega}{2}\right)^2\frac{6.2 \times 10^{-9}\,\mathrm{F}}{45 \times 10^{-9}\,\mathrm{H}}} \approx \boxed{5.7 \times 10^{7}\,\mathrm{rad/s}}$$

This is choice (d).

Question 4:

Like for Questions 1 and 2: I would not normally include a question that cannot be solved if you aren't able to answer another question, to avoid magnifying a single mistake. Because this is the practice quiz and this pair of questions is good RLC circuit practice, I'm making an exception.

The charge on the capacitor in an RLC circuit is:

$$Q(t) = Q(0)e^{-\frac{t}{\tau}}\cos(\omega_{\mathrm{damped}}t)$$

We already calculated ω_{damped} in the previous problem. We only need to calculate τ.

$$\tau = \frac{2L}{R}$$

Plug in numbers.

$$\tau = \frac{2(45\,\mathrm{nH})}{1.7\,\Omega} \approx 52.9\,\mathrm{ns}$$

Expressed in terms of ns, the damped angular frequency is:

$$\omega_{\mathrm{damped}} = 0.057\,\mathrm{rad/ns}$$

Plug these numbers in.

$$Q(9.0\,\mathrm{ns}) \approx (2.2\,\mu\mathrm{C})e^{-\frac{9.0\,\mathrm{ns}}{52.9\,\mathrm{ns}}}\cos\left[(0.057\,\mathrm{rad/ns})(9.0\,\mathrm{ns})\right] \approx \boxed{1.6\,\mu\mathrm{C}}$$

This is choice (e).

Question 5:

The circuit goes through many oscillations before the oscillations die out (so it's not critically damped or overdamped), but the oscilations are decreasing in amplitude over time (so it's not undamped). The circuit is underdamped.

This is choice (b).

Question 6:

Maxwell's Correction to Ampere's Law says:

$$BL = \mu_0 \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

In this case, the area component of $\Phi_E = EA$ isn't changing, so

$$\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = A\frac{\mathrm{d}E}{\mathrm{d}t}$$

Substituting this in, we have:

$$BL = \mu_0 \epsilon_0 A \frac{\mathrm{d}E}{\mathrm{d}t}$$

We want to solve for the rate of change of the electric field.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{BL}{\mu_0 \epsilon_0 A}$$

The Amperian loop to draw in this case is a circle with radius equal to the radius of the plates, so $L = 2\pi R$.

The area is $A = \pi R^2$.

Making these substitutions:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{B(2\pi R)}{\mu_0 \epsilon_0 (\pi R^2)} = \frac{2B}{\mu_0 \epsilon_0 R}$$

Now plug in numbers.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{2(22 \times 10^{-6}\,\mathrm{T})}{(4\pi \times 10^{-7}\,\mathrm{T\text{-}m/A})(8.85 \times 10^{-12}\,\mathrm{F/m})(0.025\,\mathrm{m})} \approx \boxed{1.6 \times 10^{14}\,(\mathrm{N/C})/\mathrm{s}}$$

This is choice (e).

Question 7:

The correct choice is (a).

Question 8:

The magnetic field in a light wave is related to the electric field in the same wave by:

$$B = \frac{E}{c}$$

Plug in numbers.

$$B = \frac{90\,\mathrm{N/C}}{3.0 \times 10^{8}\,\mathrm{m/s}} = \boxed{0.30\,\mu\mathrm{T}}$$

This is choice (b).

Question 9:

The energy of a photon is:

$$E = \frac{hc}{\lambda}$$

$$E \approx \frac{(6.63 \times 10^{-34}\,\mathrm{J\text{-}s})(3.0 \times 10^{8}\,\mathrm{m/s})}{555 \times 10^{-9}\,\mathrm{m}} \approx \boxed{3.6 \times 10^{-19}\,\mathrm{J}}$$

This is choice (c).

Question 10:

The radiative electric field scales as $1/r$, not $1/r^2$.

$$\frac{E_{\text{surface}}}{E_{\text{Earth}}} = \frac{r_{\text{Earth}}}{r_{\text{surface}}}$$

$$E_{\text{surface}} = E_{\text{Earth}} \frac{r_{\text{Earth}}}{r_{\text{surface}}}$$

Plug in numbers.

$$E_{\text{surface}} = (1.2 \times 10^{-3}\,\mathrm{N/C}) \frac{9.5 \times 10^{17}\,\mathrm{m}}{7.0 \times 10^{8}\,\mathrm{m}} \approx \boxed{1.6 \times 10^{6}\,\mathrm{N/C}}$$

This is choice (a).

Question 11:

First, let's determine whether the electric field will be 0 to the left of q_1, between q_1 and q_2, or to the right of q_2.

It can't be 0 in the middle, since E_1 and E_2 both point to the right in the middle region.

Since q_2 has a larger magnitude than q_1, the cancellation point must be closer to q_1 than to q_2. So, the field must be 0 in the left region, while q_2 always dominates in the right region.

In the left region, $\vec{E}_1$ points to the left (call this negative) and $\vec{E}_2$ points to the right (positive). Having reasoned out the directions of the fields, we now care only about the magnitudes of the charges (we used the signs to tell us the directions).

$$\vec{E}_1 + \vec{E}_2 = \vec{0}$$

$$-k\frac{|q_1|}{r_1^2} + k\frac{|q_2|}{r_2^2} = 0$$

$$k\frac{|q_1|}{r_1^2} = k\frac{|q_2|}{r_2^2}$$

$$\frac{|q_1|}{r_1^2} = \frac{|q_2|}{r_2^2}$$

Since q_1 is located at $x = 0$, r_1^2 is just x^2.
Since q_2 is at $x = 2.0\,\mathrm{m}$, $r_2 = (2.0\,\mathrm{m} - x)$ in the region on the left (x is negative in this region, so it needs to be subtracted from 2.0 m to give the full distance).

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(2.0\,\mathrm{m} - x)^2}$$

$$|q_1|(2.0\,\mathrm{m} - x)^2 = |q_2|x^2$$

$$|q_1|\left[(4.0\,\mathrm{m}^2) - (4.0\,\mathrm{m})x + x^2\right] - |q_2|x^2 = 0$$

$$(|q_1| - |q_2|)x^2 - (4.0\,\mathrm{m})|q_1|x + (4.0\,\mathrm{m}^2)|q_1| = 0$$

Now plug in $|q_1| = 1\,\mathrm{C}$ and $|q_2| = 3\,\mathrm{C}$.

$$(-2\,\mathrm{C})x^2 - (4.0\,\mathrm{C\text{-}m})x + (4.0\,\mathrm{C\text{-}m}^2) = 0$$

Now plug these values into the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(4)}}{2(-2)}$$

$$x = \frac{4 \pm \sqrt{16 + 32}}{-4}$$

$$x = -\left(1 \pm \sqrt{\frac{48}{16}}\right) = -\left(1 \pm \sqrt{3}\right)$$

The correct value of x must be negative, as we reasoned out above, and the other is extraneous.

$x = -(1+\sqrt{3})\,\mathrm{m}$ is negative, while the other is positive.

So, $\boxed{x \approx -2.7\,\mathrm{m}}$.

This is choice (a).

Question 12:

If the proton were a point charge, then 100% of its charge would be enclosed at any distance. Since the proton is instead a uniformly charged sphere (in our model; in reality, the proton's charge distribution is more like $\rho(r) \sim e^{-r}$, but that would be way too hard for a quiz!), the charge enclosed is only:

$$Q_{\mathrm{encl}} = e\frac{V_{\mathrm{encl}}}{V} = e\frac{r^3}{R_p^3}$$

Given the inputs of the proton, $r = \frac{1}{2}R_p$, so $\frac{r^3}{R_p^3} = \boxed{\frac{1}{8}}$. This means that $Q_{\mathrm{encl}} = \frac{1}{8}e$.

Since only 1/8 as much charge is enclosed if the proton is a uniform sphere compared to a point charge, the field is only 1/8 as strong. This is choice (a).

This effect measurably changes the binding energies and orbitals of electrons in atoms, compared to the predictions of a model where the nucleus is a point charge.

Question 13:

Gauss' Law says that the field inside a spherically symmetric charge distribution is

$$E(r) = \frac{Q_{\mathrm{encl}}(r)}{4\pi\epsilon_0 r^2}$$

where $Q_{\mathrm{encl}}(r)$ is the total charge enclosed by a spherical Gaussian surface of radius r. For $E(r)$ to be a constant, $Q_{\mathrm{encl}}(r)$ must scale as r^2 in order to cancel the r^2 in the denominator (which comes from the surface area of the Gaussian sphere). $Q_{\mathrm{encl}}(r)$ is the spherical volume integral of the charge distribution, $\rho(r)$.

$$Q_{\mathrm{encl}}(r) = 4\pi\int_0^r \rho(r')r'^2\mathrm{d}r'$$

In order to produce something that scales as r^2, the integrand must scale as r. In order for the integrand to scale as r, $\rho(r)$ must scale as $\frac{1}{r}$ in order to cancel out one of the factors of r from the volume element.

$$\boxed{\rho(r) = br^{-1} \rightarrow a = -1}$$

This is choice (a).

Another perfectly fine method is to say something like "We saw in class that a linear charge distribution (ie r^1) produces an r^2 field, and that a constant charge distribution (r^0) produces a linear field (r^1), so by extrapolation a $\frac{1}{r} = r^{-1}$ charge distribution must produce a constant field (r^0), so $a = -1$."

Question 14:

The electric field is the negative gradient of the potential. In this case, the potential depends only on x, so:

$$\vec{E}(x) = -\left(\frac{\mathrm{d}V}{\mathrm{d}x}\right)\hat{x} = -(2.80\,\mathrm{V/m^2})x\hat{x}$$

Now plug in $x = 15.0\,\mathrm{m}$:

$$\vec{E}(x) = (-2.80\,\mathrm{V/m^2})(15.0\,\mathrm{m})\hat{x} = \boxed{(-42.0\,\mathrm{V/m})\hat{x}}$$

This is choice (a).

Question 15:

As discussed in class, a general solution to these problems is:

$$Q_{12} = \frac{Q_{\mathrm{initial}}}{1 + \frac{C_9}{C_{12}}}$$

We can get Q_{initial} from the defining equation of capacitance:

$$Q = CV = (12\,\mathrm{nF})(9.0\,\mathrm{V}) = 108\,\mathrm{nC}$$

Plugging in numbers:

$$Q_{12} = \frac{108\,\mathrm{nC}}{1 + \frac{9\,\mathrm{nF}}{12\,\mathrm{nF}}} \approx \boxed{62\,\mathrm{nC}}$$

This is choice (b).

This implies that $Q_9 \approx 46\,\mathrm{nC}$. We can check that these charges will produce the same voltages using $V = Q/C$.

$$V_{12} \approx \frac{62\,\mathrm{nC}}{12\,\mathrm{nF}} \approx 5.2\,\mathrm{V}$$

$$V_9 \approx \frac{46\,\mathrm{nC}}{9\,\mathrm{nF}} \approx 5.1\,\mathrm{V}$$

Our answer checks out, to within rounding error.

Question 16:

The resistance formula is:

$$R = \frac{\rho L}{A}$$

We want the resistances to be the same, so:

$$\frac{\rho_{\mathrm{Al}} L}{A_{\mathrm{Al}}} = \frac{\rho_{\mathrm{Cu}} L}{A_{\mathrm{Cu}}}$$

We're told that the lengths of the wires are the same, so they will drop out. Now, solve for A_{Al}.

$$A_{\mathrm{Al}} = A_{\mathrm{Cu}} \frac{\rho_{\mathrm{Al}}}{\rho_{\mathrm{Cu}}}$$

And plug in numbers.

$$A_{\mathrm{Al}} = (1.0\,\mathrm{cm}^2) \frac{2.65 \times 10^{-8}\ \Omega\text{-m}}{1.72 \times 10^{-8}\ \Omega\text{-m}} \approx \boxed{1.54\,\mathrm{cm}^2}$$

This is choice (d).

Question 17:

Let's define I_1 to be flowing left-to-right through R_1, I_2 to be flowing right-to-left through R_2, and I_3 to be flowing upward through the central emf. Then from current conservation, we have:

$$I_1 + I_2 = I_3$$

From the left loop, we have:

$$12\,\mathrm{V} - I_1 R_1 + 9\,\mathrm{V} = 0$$

$$21\,\mathrm{V} = I_1 R_1$$

And from the right loop, we have:

$$9\,\mathrm{V} - 5\,\mathrm{V} - I_2 R_2 = 0$$

$$4\,\mathrm{V} = I_2 R_2$$

The left loop tells us I_1:

$$I_1 = \frac{21\,\mathrm{V}}{6\,\Omega} = 3.5\,\mathrm{A}$$

The right loop tells us I_2:

$$I_2 = \frac{4\,\mathrm{V}}{4\,\Omega} = 1.0\,\mathrm{A}$$

And current conservation tells us the central current, I_3:

$$I_3 = 3.5\,\mathrm{A} + 1.0\,\mathrm{A} = \boxed{4.5\,\mathrm{A}}$$

This is choice (d).

Question 18:

The magnetic force on a current-carrying wire is:

$$\vec{F} = L\vec{I} \times \vec{B}$$

Due to the cross product, the force only cares about the component of $\vec{B}$ perpendicular to $\vec{I}$. In this problem, $\vec{I}$ is purely in the $+\hat{y}$ direction, so the force only cares about B_x and B_z, but $B_z = 0$, so only B_x affects the force.

$$|\vec{F}| = L|\vec{I}||B_x|$$

Now plug in numbers.

$$F = (2.0\,\mathrm{m})(2.0\,\mathrm{A})(2.5\,\mathrm{T}) = \boxed{10\,\mathrm{N}}$$

This is choice (d).

Question 19:

Since the field is perpendicular to the velocity and is holding the charge in circular motion:

$$\frac{mv^2}{r} = qvB$$

We want to solve for m.

$$m = \frac{qBr}{v}$$

Now plug in numbers.

$$m = \frac{(2 \times 1.6 \times 10^{-19}\,\mathrm{C})(0.20\,\mathrm{T})(0.14\,\mathrm{m})}{44000\,\mathrm{m/s}} \approx \boxed{2.0 \times 10^{-25}\,\mathrm{kg}}$$

This is about 120 proton/neutron masses, so the ion is probably doubly-ionized Tin-120 (very plausible).

This is choice (e).

Question 20:

The total energy in an LC circuit is:

$$U = U_C + U_L = \frac{1}{2}\frac{1}{C}Q^2 + \frac{1}{2}LI^2$$

We want to solve for L.

$$2U - \frac{Q^2}{C} = LI^2$$

$$L = \frac{2U - Q^2/C}{I^2}$$

Now plug in numbers.

$$L = \frac{2(4.0\,\mathrm{J}) - (0.018\,\mathrm{C})^2/(45 \times 10^{-6}\,\mathrm{F})}{(97\,\mathrm{A})^2} \approx \boxed{8.5 \times 10^{-5}\,\mathrm{H}}$$

This is choice (b).