ECE 35 Homework #3 (Spring 2023, Taur)

<u>All homework problems come from the textbook, "Introduction to Electric Circuits", by Svoboda & Dorf, 9th Edition.</u>

P 4.2-2 Determine the node voltages for the circuit of Figure P 4.2-2.

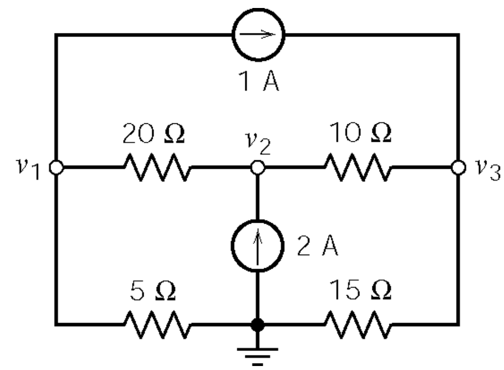


Figure P 4.2-2

Solution:

KCL at node 1:
$$\frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = 0 \quad \Rightarrow \quad 5\,v_1 - v_2 = -20$$

KCL at node 2:
$$\frac{v_1 - v_2}{20} + 2 = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad -v_1 + 3\,v_2 - 2\,v_3 = 40$$

KCL at node 3:
$$\frac{v_2 - v_3}{10} + 1 = \frac{v_3}{15} \quad \Rightarrow \quad -3\,v_2 + 5\,v_3 = 30$$

Solving gives $v_1 = 2$ V, $v_2 = 30$ V and $v_3 = 24$ V.

P4.3-3. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-3.

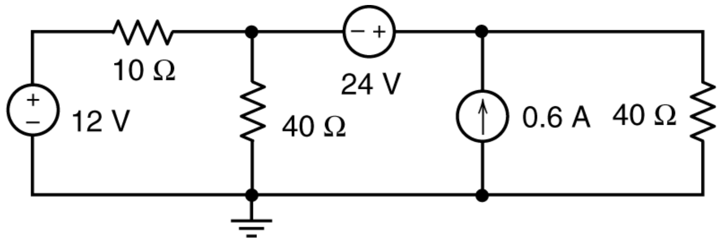
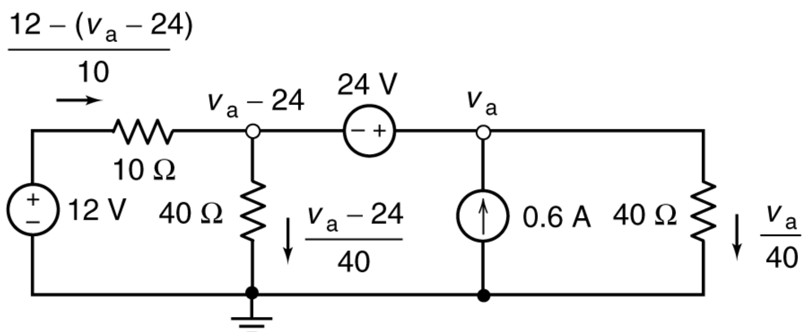
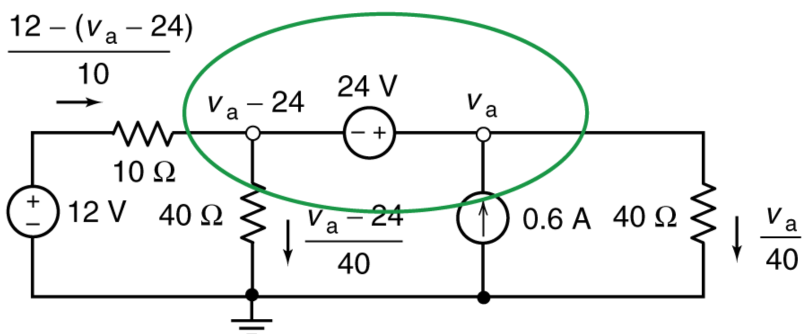


Figure P4.3-3

Solution: First, label the node voltages, and express the resistor currents in terms of the node voltages.



Identify the super node corresponding to the 24 V source



Apply KCL to the super node to get

$$\frac{12-(v_a-24)}{10}+0.6=\frac{v_a-24}{40}+\frac{v_a}{40} \quad\Rightarrow\quad 196=6v_a \quad\Rightarrow\quad v_a=32\text{ V}$$

The 12 V source supplies $12\left(\frac{12-(v_a-24)}{10}\right)=12\left(\frac{12-(32-24)}{10}\right)=4.8\text{ W}$

The 24 V source supplies $24\left(-0.6+\frac{v_a}{40}\right)=24\left(-0.6+\frac{32}{40}\right)=4.8\text{ W}$

The current source supplies $0.6v_a=0.6(32)=19.2\text{ W}$

P 4.3-7 Determine the values of the node voltages, v_1 and v_2, in Figure P 4.3-7. Determine the values of the currents i_a and i_b.

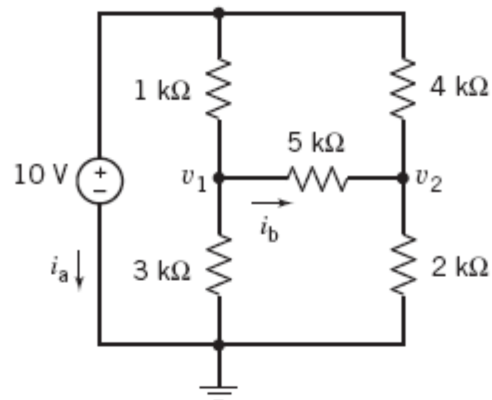
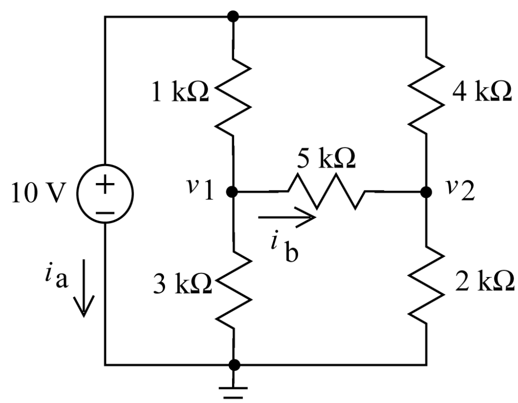


Figure P 4.3-7

Solution:



Apply KCL at nodes 1 and 2 to get

$$\frac{10-v_1}{1000}=\frac{v_1}{3000}+\frac{v_1-v_2}{5000} \quad \Rightarrow \quad 23v_1-3v_2=150$$

$$\frac{10-v_2}{4000}+\frac{v_1-v_3}{5000}=\frac{v_3}{2000} \quad \Rightarrow \quad \text{-}4v_1+19v_3=50$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 23 & -3 \\ -4 & 19 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 150 \\ 50 \end{bmatrix} \quad \Rightarrow \quad v_1=7.06 \text{ V and } v_1=4.12 \text{ V}$$

Then

$$i_b=\frac{v_1-v_2}{5000}=\frac{7.06-4.12}{5000}=0.588 \text{ mA}$$

Apply KCL at the top node to get

$$i_a = \frac{v_1 - 10}{1000} + \frac{v_2 - 10}{4000} = \frac{7.06 - 10}{1000} + \frac{4.12 - 10}{4000} = -4.41 \text{ mA}$$

P 4.3-9 Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

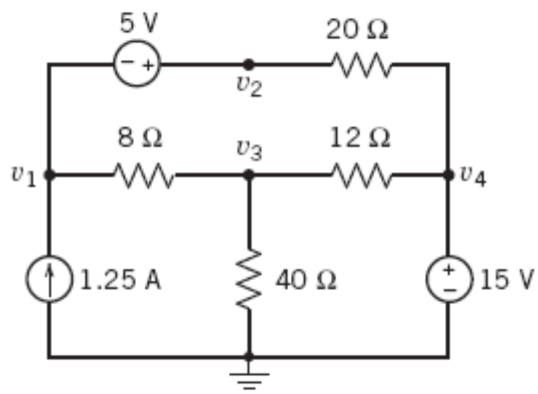


Figure P 4.3-9

Solution:

Express the voltage source voltages as functions of the node voltages to get

$$v_2 - v_1 = 5 \text{ and } v_4 = 15$$

Apply KCL to the supernode corresponding to the 5 V source to get

$$1.25 = \frac{v_1 - v_3}{8} + \frac{v_2 - 15}{20} = 0 \quad \Rightarrow \quad 80 = 5v_1 + 2v_2 - 5v_3$$

Apply KCL at node 3 to get

$$\frac{v_1 - v_3}{8} = \frac{v_3}{40} + \frac{v_3 - 15}{12} \quad \Rightarrow \quad -15v_1 + 28v_3 = 150$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 2 & -5 \\ -15 & 0 & 28 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 80 \\ 150 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 22.4 \\ 27.4 \\ 17.4 \end{bmatrix}$$

So the node voltages are:

$$v_1 = 22.4 \text{ V}, \; v_2 = 27.4 \text{ V}, \; v_3 = 17.4 \text{ V, and } v_4 = 15$$

P 4.3-12 Determine the values of the node voltages of the circuit shown in Figure P 4.3-12.

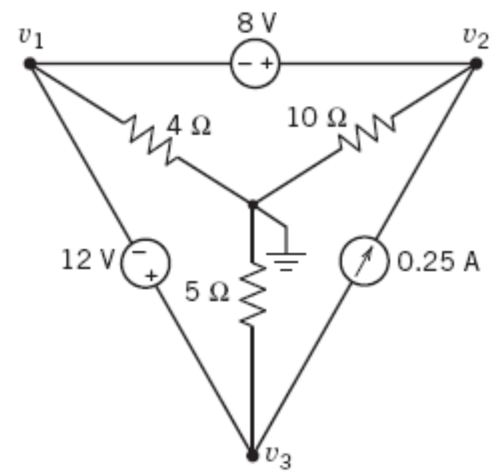


Figure P 4.3-12

Solution:
Express the voltage source voltages in terms of the node voltages:

$$v_2 - v_1 = 8 \quad \text{and} \quad v_3 - v_1 = 12$$

Apply KVL to the supernode to get

$$\frac{v_2}{10} + \frac{v_1}{4} + \frac{v_3}{5} = 0 \qquad \Rightarrow \qquad 2v_2 + 5v_1 + 4v_3 = 0$$

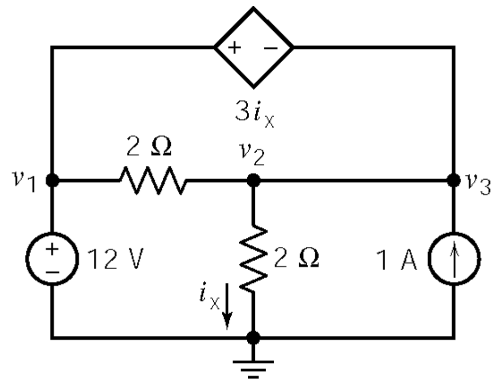
so

$$2(8 + v_1) + 5v_1 + 4(12 + v_1) = 0 \qquad \Rightarrow \qquad v_1 = -\frac{64}{11} \text{ V}$$

The node voltages are

$$v_1 = -5.818 \text{ V}$$
$$v_2 = 2.182 \text{ V}$$
$$v_3 = 6.182 \text{ V}$$

Solution:



First, express the controlling current of the CCVS in terms of the node voltages: $i_x = \frac{v_2}{2}$

Next, express the controlled voltage in terms of the node voltages:

$$12 - v_2 = 3i_x = 3\frac{v_2}{2} \Rightarrow v_2 = \frac{24}{5} \text{ V}$$

P 4.4-8 Determine the value of the power supplied by the dependent source in Figure P 4.4-8.

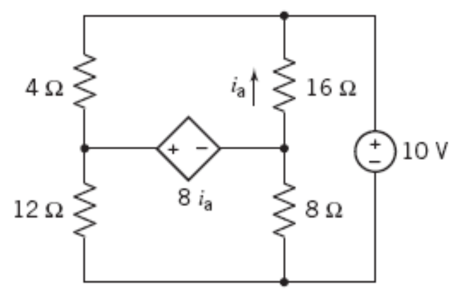


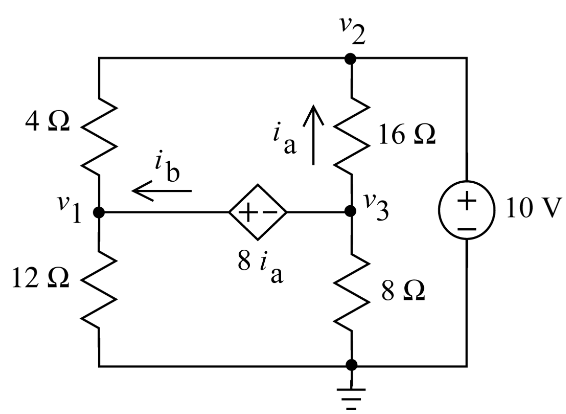
Figure P 4.4-8

Solution:

Label the node voltages.

First, v_2 = 10 V due to the independent voltage source. Next, express the controlling current of the dependent source in terms of the node voltages:

$$i_a = \frac{v_3 - v_2}{16} = \frac{v_3 - 10}{16}$$



Now the controlled voltage of the dependent source can be expressed as

$$v_1 - v_3 = 8\, i_a = 8\left(\frac{v_3 - 10}{16}\right) \quad \Rightarrow \quad v_1 = \frac{3}{2} v_3 - 5$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{v_1 - v_2}{4} + \frac{v_1}{12} + \frac{v_3 - v_2}{16} + \frac{v_3}{8} = 0$$

Multiplying by 48 and using v_2 = 10 V gives

$$16v_1 + 9v_3 = 150$$

Substituting the earlier expression for v_1

$$16\left(\frac{3}{2} v_3 - 5\right) + 9v_3 = 150 \quad \Rightarrow \quad v_3 = 6.970 \text{ V}$$

P 4.4-11 **Determine the power supplied by the dependent source in the circuit shown in Figure P 4.4-11.**

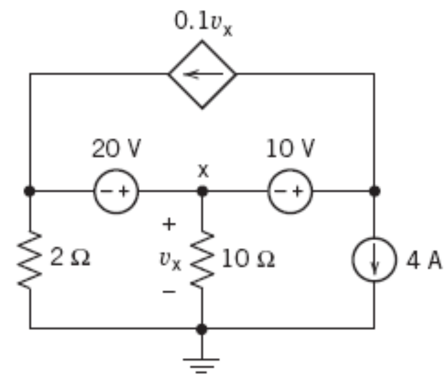


Figure P 4.4-11

Solution:

This circuit contains two ungrounded voltage sources, both incident to node x. In such a circuit it is necessary to merge the supernodes corresponding to the two ungrounded voltage sources into a single supernode. That single supernode separates the two voltage sources and their nodes from the rest of the circuit. It consists of the two resistors and the current source. Apply KCL to this supernode to get

$$\frac{v_x - 20}{2} + \frac{v_x}{10} + 4 = 0 \quad \Rightarrow \quad v_x = 10 \text{ V}$$

The power supplied by the dependent source is

$$(0.1\, v_x)(-30) = -30 \text{ W}$$

P 4.5-2 The values of the mesh currents in the circuit shown in Figure P 4.5-2 are

$i_1 = 2$ A, $i_2 = 3$ A, and $i_3 = 4$ A.

Determine the values of the resistance R and of the voltages v_1 and v_2 of the voltage sources.

Answers: $R = 12\ \Omega$, $v_1 = -4$ V, and $v_2 = -28$ V

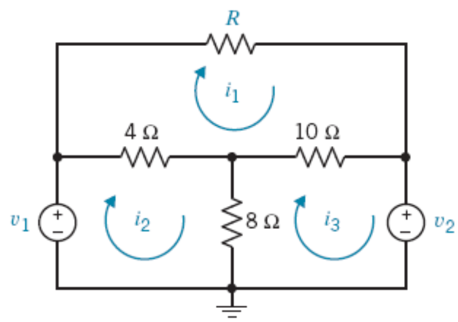


Figure P 4.5-2

Solution:
The mesh equations are:

Top mesh: $$4\,(2-3)+R(2)+10\,(2-4)=0$$

so $R = 12\ \Omega$.

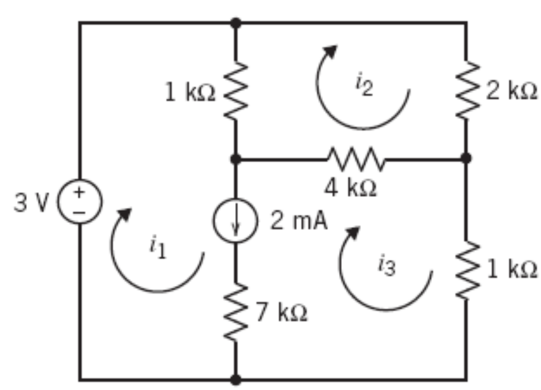
Bottom, right mesh: $$8\,(4-3)+10\,(4-2)+v_2=0$$

so $v_2 = -28$ V.

Bottom left mesh $$-v_1+4\,(3-2)+8\,(3-4)=0$$

so $v_1 = -4$ V.

P 4.6-8 Determine values of the mesh currents, i_1, i_2, and i_3, in the circuit shown in Figure P 4.6-8.



Solution: Use units of V, mA and kΩ. Express the currents to the supermesh to get

$$i_1 - i_3 = 2$$

Apply KVL to the supermesh to get

$$4(i_3 - i_3) + (1)i_3 - 3 + (1)(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 - 5i_2 + 5i_3 = 3$$

Apply KVL to mesh 2 to get

$$2i_2 + 4(i_2 - i_3) + (1)(i_2 - i_1) = 0 \quad \Rightarrow \quad (-1)i_1 + 7i_2 - 4i_3 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -5 & 5 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

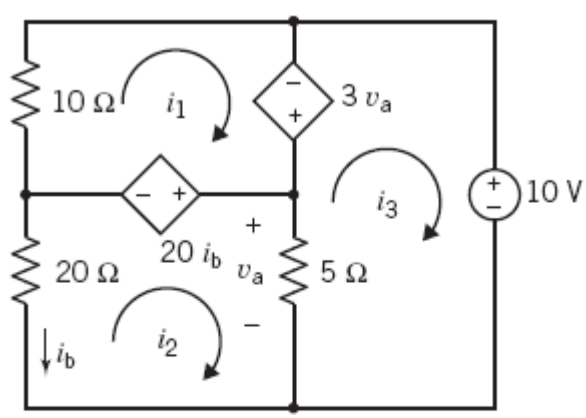


Figure P 4.7-7

Solution:
Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5(i_2 - i_3) \text{ and } i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\ i_b = -20\ i_2 \quad \text{and} \quad 3\ v_a = 15\left(i_2 - i_3\right)$$

Apply KVL to the meshes

$$-15\left(i_2 - i_3\right) + \left(-20\ i_2\right) + 10\ i_1 = 0$$
$$-\left(-20\ i_2\right) + 5\left(i_2 - i_3\right) + 20\ i_2 = 0$$
$$10 - 5\left(i_2 - i_3\right) + 15\ \left(i_2 - i_3\right) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -1.25\ \text{A},\ i_2 = +0.125\ \text{A, and}\ i_3 = +1.125\ \text{A}$$

P 4.7-11 Determine the values of the mesh currents of the circuit shown in Figure P 4.7-11.

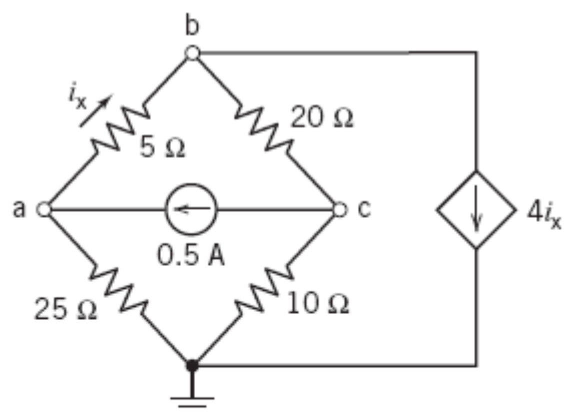
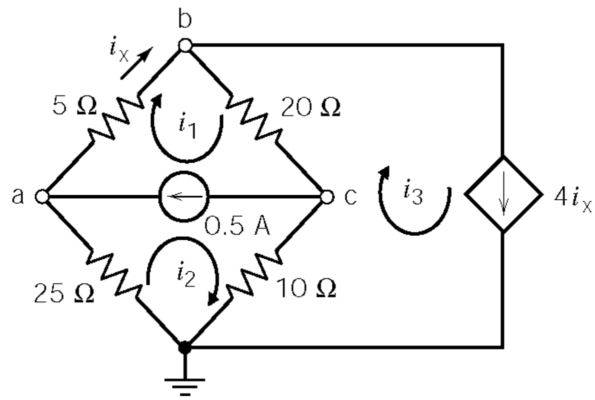


Figure P 4.7-11

Solution:

Label the mesh currents.



Express i_x in terms of the mesh currents:

$$i_x = i_1$$

Express $4i_x$ in terms of the mesh currents:

$$4i_x = i_3$$

Express the current source current in terms of the mesh currents to get:

$$0.5 = i_1 - i_2 \quad \Rightarrow \quad i_2 = i_x - 0.5$$

Apply KVL to supermesh corresponding to the current source to get

$$5i_1 + 20\left(i_1 - i_3\right) + 10\left(i_2 - i_3\right) + 25i_2 = 0$$

Substituting gives

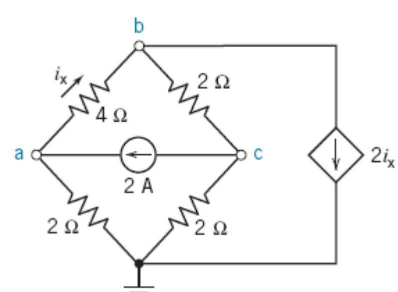
$$5i_x + 20\left(-3i_x\right) + 10\left(i_x - 0.5 - 4i_x\right) + 25\left(i_x - 0.5\right) = 0 \quad \Rightarrow \quad i_x = -\frac{35}{120} = -0.29167$$

So the mesh currents are

$$i_1 = i_x = -0.29167 \quad \text{A}$$
$$i_2 = i_x - 0.5 = -0.79167 \quad \text{A}$$
$$i_3 = 4i_x = -1.1667 \quad \text{A}$$

P 4.11-2 An old lab report asserts that the node voltages of the circuit of Figure P 4.10-2 are

$v_a = 4$ V, $v_b = 20$ V, and $v_c = 12$ V.



Solution:

Apply KCL at node *a*:

$$-\left(\frac{v_b - v_a}{4}\right) - 2 + \frac{v_a}{2} = 0$$

$$-\left(\frac{20-4}{4}\right) - 2 + \frac{4}{2} = -4 \neq 0$$

The given voltages do not satisfy the KCL equation at node *a*. They are **not correct.**