
INSTRUCTIONS
Students should feel free to discuss these problems.
KEY CONCEPTS Recursive algorithms, recurrence relations.

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1. Solve the following recurrence relations to give closed form expressions of the functions:
   (a) $A[0] = 1$, $A[n] = 2A[n-1] + 1$ for $n \geq 1$.
   (b) $B[0] = 1$, $B[n] = B[n-1] + 2n + 1$ for $n \geq 1$
   (c) $C[0] = C[1] = 1$, $C[n] = 2C[n-2]$ for $n \geq 2$.
   (d) $D[0] = 1, D[n] = nD[n-1]$ for $n \geq 1$.
   (e) $E[0] = E[1] = 1, E[n] = 2E[\lfloor n/2 \rfloor]$ for $n \geq 2$.

2. Give a recursive description of selection sort. You can use a sub-routine $ArgMinimum(A[1..n])$ which returns the location of a minimum value in array $A[1..n]$ and takes linear time $O(n)$.

   Prove this recursive algorithm correctly sorts the input from smallest to largest. Give a recurrence relation for its time , and solve the recurrence relation to give the time up to order.

3. Describe how to use MergeSort to solve the Element Distinctness problem. How much time total does this take, in order notation?

4. Design a recursive algorithm for the following problem: The input is an array of integers $A[1..n]$ so that $A[1] > A[n]$. The problem is to find an integer I so that $1 \leq I \leq n-1$ and $A[I] > A[I+1]$, i.e., a particular place where the list is not sorted. Prove that your algorithm is correct, and give a time analysis up to orderusing a recurrence relation.

5. Consider the following recursive algorithm to take a binary integer $b_{n-1}..b_0$ representing the integer $\sum_{i=0}^{i=n-1} b_i 2^i$ to decimal. It uses a sub-procedure *Add* that adds two decimal numbers.

   Convert$[b_{n-1}..b_0]$
   (a) If $n = 1$ return b_0.
   (b) $X = Convert(b_{n-1}..b_1)$
   (c) $Y = Add(X, X)$
   (d) $ReturnAdd(Y, b_0)$

   Prove that this algorithm returns the integer whose binary representation is the input. Give a time analysis using recurrence relations assuming *Add* is constnt time. Then give a time analysis assuming that *Add* takes linear time in terms of the number of decimal digits in the two inputs. (You will first have to think about how many digits is the decimal representation on an integer that takes n bits in binary.)