Phys 2B Summer 2022

# Quiz 3 Solutions

Question 1:

Since the resistors are in series, the current through the $3.0\,\Omega$ resistor is also the current through the $9.0\,\Omega$ resistor. We can use Ohm's Law and the known current through the $3.0\,\Omega$ resistor to calculate the voltage differences across both resistors, and sum to get the voltage of the battery.

$$V = V_3 + V_9 = IR_3 + IR_9 = (1.5\,\mathrm{A})(3.0\,\Omega) + (1.5\,\mathrm{A})(9.0\,\Omega) = \boxed{18\,\mathrm{V}}$$

This is choice (e).

Question 2:

The formula for resistance is:

$$R = \frac{\rho L}{A}$$

Plug this into Ohm's Law directly.

$$I = \frac{V}{R} = \frac{VA}{\rho L}$$

Plug in numbers.

$$I = \frac{(0.015\,\mathrm{V})(5.0 \times 10^{-4}\,\mathrm{m}^2)}{(2.65 \times 10^{-8}\,\Omega\,\mathrm{m})(1.9\,\mathrm{m})} \approx 150\,\mathrm{A}$$

This is choice (e).

Yikes! It's a really bad idea to have thick uninterrupted chunks of conductor in anything that might conceivably end up with a voltage difference across it, lest it support an enormous current flow. Where might this voltage come from? Keep this in mind for our discussion of Faraday's Law in the future...

Question 3:

To be perfectly ohmic, a material must satisfy Ohm's Law:

$$V = IR$$

This means that a $V$ vs $I$ plot would be linear, with slope equal to the constant $R$. This curve is not linear, so I is false.

As $I$ increases, the wire will tend to be hotter. At higher temperature, a conductor's resistance increases (ie the slope of a $V = IR$ curve increases) and a semiconductor's resistance decreases (the slope decreases).

This curve's slope increases, so the material is probably a conductor, not a semiconductor, meaning II is true and III is false.

So, the correct choice is (b).

Question 4:

Let's define $I_1$ to be flowing left-to-right through $R_1$, $I_2$ to be flowing right-to-left through the 4 V emf, and $I_3$ to be flowing up through $R_3$.

With these definitions, the current conservation equation is:

$$I_1 + I_2 = I_3$$

The left loop is:

$$4\,\mathrm{V} = I_1 R_1 + I_3 R_3$$

And the right loop is:

$$6\,\mathrm{V} = I_3 R_3 + I_2 R_2$$

We want to find $I_1$, so let's use the current conservation equation to replace $I_2$ with $I_1$ and $I_3$ in the right loop equation, so that $I_2$ is eliminated from the system.

$$6\,\mathrm{V} = I_3 R_3 + (I_3 - I_1) R_2$$

$$6\,\mathrm{V} = I_3 (R_3 + R_2) - I_1 R_2$$

Now both loop equations depend on only $I_1$ and $I_3$. We can solve either one for $I_3$ and plug it into the other to eliminate $I_3$, and thus find the value of $I_1$. Let's start with the left loop.

$$I_3 = \frac{4\,\mathrm{V} - I_1 R_1}{R_3}$$

Now plug this into the right loop equation to eliminate $I_3$.

$$6\,\mathrm{V} = \left(\frac{4\,\mathrm{V} - I_1 R_1}{R_3}\right)(R_3 + R_2) - I_1 R_2$$

This is an equation purely for $I_1$. To simplify, it makes sense to plug in that $\frac{R_3+R_2}{R_3} = 4$ at this stage.

$$6\,\mathrm{V} = 4\,(4\,\mathrm{V} - I_1 R_1) - I_1 R_2$$

$$6\,\mathrm{V} = 16\,\mathrm{V} - I_1(4_1 + R_2)$$

$$I_1(4R_1 + R_2) = 10\,\mathrm{V}$$

$$I_1 = \frac{10\,\mathrm{V}}{4(0.25\,\Omega) + 9.0\,\Omega} = \frac{10\,\mathrm{V}}{10\,\Omega} = \boxed{1.0\,\mathrm{A}}$$

This is choice (a).

Question 5:

The charge on a discharging capacitor is given by:

$$Q(t) = Q(0)e^{-\frac{t}{RC}}$$

Clearly, $Q(0) = 10\,\mu\mathrm{C}$. Then, read off from the plot a convenient $(t, Q(t))$ pair and plug in to solve for $RC$. One such convenient pair is $Q(18\,\mathrm{s}) \approx 0.50\,\mu\mathrm{C}$.

$$\frac{0.5}{10} \approx e^{-\frac{18\,\mathrm{s}}{RC}}$$

$$0.05 \approx e^{-\frac{18\,\mathrm{s}}{RC}}$$

$$\ln(0.05) \approx -\frac{18\,\mathrm{s}}{RC}$$

$$RC \approx -\frac{18\,\mathrm{s}}{\ln(0.05)} \approx 6.0\,\mathrm{s}$$

6.0 s is in fact the plotted value.

If the resistance is $1.0\,\Omega$, then the capacitance is $6.0\,\mathrm{F}$.

This is choice (c).

Question 6:

The time required for a capacitor to charge/discharge is set by the time constant $RC$. Larger values of $RC$ mean the circuit takes a longer time to do things.

So, I and II are true. I increases the capacitance and II increases the resistance, both of which increase $RC$, making things happen more slowly.

III is also true. The capacitor initially stores

$$U(0) = \frac{Q(0)^2}{2C}$$

and eventually stores nothing when its charge is fully depleted.

$$U(\infty) = \frac{Q(\infty)^2}{2C} = 0$$

This energy needs to go somewhere, and the only option is for it to be radiated by the resistor, $P = I^2R$.

This is choice (e).

Question 7:

The value of the initial charge does not affect how long it takes to reach half of that value, only the value of $RC$ matters. The circuit that has the largest $RC$ will take longest to reach any fraction of its initial charge, including half.

The largest value of $RC$ belongs Circuit E, 4.0 ns. All other circuits are 1.0 ns or 2.0 ns.

So, the correct choice is (e).

Question 8:

The field of a long wire is:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

The field from wire 1 at the location of wire 3 is:

$$B_1 = (10^{-7}\,\text{T-m/A})\frac{2(1.0\,\text{A})}{2.0\,\text{m}} = 1.0 \times 10^{-7}\,\text{T}$$

The field from wire 2 at the location of wire 3 is:

$$B_2 = (10^{-7}\,\text{T-m/A})\frac{2(2.0\,\text{A})}{1.0\,\text{m}} = 4.0 \times 10^{-7}\,\text{T}$$

The magnetic force-per-length of a perpendicular magnetic field on a current-carrying wire is:

$$\frac{F}{L} = f = IB$$

Wires carrying currents in the same direction attract, and currents in the opposite direction repel. Force 1 will be attractive (indicate this with a minus sign, since the force points left), and force 2 will be repulsive (plus sign, force points right).

$$f_{13} = -I_3B_1 = -(3.0\,\text{A})(1.0 \times 10^{-7}\,\text{T}) = -3.0 \times 10^{-7}\,\text{N/m}$$

$$f_{23} = +I_3B_2 = (3.0\,\text{A})(4.0 \times 10^{-7}\,\text{T}) = 12 \times 10^{-7}\,\text{N/m}$$

The net force-per-length is the sum, $\boxed{f = 9.0 \times 10^{-7}\,\text{N/m to the right}}$

This is choice (d).

Question 9:

The magnetic force on a current-carrying wire is:

$$\vec{F} = L\vec{I} \times \vec{B}$$

The magnitude of the force can be expressed in terms of the angle between the current and the field:

$$F = LIB\sin\theta$$

We want to solve for $\theta$.

$$\theta = \sin^{-1}\left(\frac{F}{LIB}\right)$$

Now plug in numbers.

$$\theta = \sin^{-1}\left(\frac{2.6\,\mathrm{N}}{(3.0\,\mathrm{m})(4.0\,\mathrm{A})(0.84\,\mathrm{T})}\right) \approx \boxed{15^o}$$

This is choice (b).

Question 10:

We want:

$$\vec{B}_{\mathrm{net}} = \vec{B}_1 + \vec{B}_2 = 0$$

The two wires carry currents in the same direction, so their fields point in opposite directions only in the region between the wires. So, the 0 point is in the region between the two wires, $0 < x < 1\,\mathrm{m}$.

$$B_1 = B_2$$

$$\frac{\mu_0}{4\pi}\frac{2I_1}{r_1} = \frac{\mu_0}{4\pi}\frac{2I_2}{r_2}$$

Obviously, $r_1 = x$. We need to express $r_2$ as a positive number. In the region between the two wires,

$$r_2 = 1.0\,\mathrm{m} - x$$

is positive.

$$\frac{I_1}{x} = \frac{I_2}{1.0\,\mathrm{m} - x}$$

$$1.0\,\mathrm{m} - x = \frac{I_2}{I_1}x$$

$$1.0\,\mathrm{m} = \left(\frac{I_2}{I_1} + 1\right)x$$

$$x = \frac{1.0\mathrm{m}}{\frac{I_2}{I_1} + 1}$$

Now plug in numbers.

$$x = \frac{1.0\text{m}}{\frac{3.7\,\text{A}}{2.0\,\text{A}} + 1} \approx \boxed{0.35\,\text{m}}$$

This is choice (d).