Phys 2B Summer 2022

Quiz 1 Practice Solutions

Question 1:

First, we need to find r^2:

$$r^2 = x^2 + y^2 = (0.15 \times 10^{-9}\,\mathrm{m})^2 + (0.40 \times 10^{-9}\,\mathrm{m})^2 \approx 1.8 \times 10^{-19}\,\mathrm{m}^2$$

This means that $r \approx 4.3 \times 10^{-10}\,\mathrm{m} = 0.43\,\mathrm{nm}$.

Now, calculate the magnitude of the force using Coulomb's Law (signs are dropped since we're only calculating the magnitude).

$$F = k\frac{q_1 q_2}{r^2} \approx (9.0 \times 10^9\,\mathrm{Nm}^2/\mathrm{C}^2)\frac{(1.6 \times 10^{-19}\,\mathrm{C})^2}{(1.8 \times 10^{-19}\,\mathrm{m}^2)} \approx 1.3 \times 10^{-9}\,\mathrm{N}$$

Finally, split this into components.

$$F_x = \frac{x}{r}F \approx \frac{0.15\,\mathrm{nm}}{0.43\,\mathrm{nm}}(1.3 \times 10^{-9}\,\mathrm{N}) \approx 0.44 \times 10^{-9}\,\mathrm{N}$$

$$F_y = \frac{y}{r}F \approx \frac{0.40\,\mathrm{nm}}{0.43\,\mathrm{nm}}(1.3 \times 10^{-9}\,\mathrm{N}) \approx 1.2 \times 10^{-9}\,\mathrm{N}$$

So, we have the absolute values of the components, but what are the signs? The electron is attracted to the proton, so the force will point down and to the left (since the electron is up and to the right from the proton). This means that both components are in fact negative.

$$\boxed{\vec{F} \approx (-0.44 \times 10^{-9}\,\mathrm{N}, -1.2 \times 10^{-9}\,\mathrm{N})}$$

This is choice (c).

Question 2:

Both charges are positive, meaning their fields both point away from them.

On the left of both charges ($x < 0$), the fields both point left, and on the right of both charges ($x > 1\,\mathrm{m}$) the fields both point right. The fields can't possibly sum to 0 in either of these regions, so the net field must be 0 at some point in the middle, $0 < x < 1\,\mathrm{m}$. So, I and III are both true.

II is false. For the field from the small charge to have the same magnitude as the field of the large charge and thus allow the net field to cancel, the point must be closer to the smaller charge, not to the larger charge.

This is choice (d).

Question 3:

The force on the proton is $F = eE$, and by Newton's 2nd Law, $F = ma$. Set these expressions for the force equal to each other.

$$ma = eE$$

We want to solve for the proton's acceleration.

$$a = \frac{eE}{m}$$

Now plug in numbers.

$$a \approx \frac{(1.6 \times 10^{-19}\,\mathrm{C})(13\,\mathrm{N/C})}{1.7 \times 10^{-27}\,\mathrm{kg}} \approx \boxed{1.2 \times 10^{9}\,\mathrm{m/s}^2}$$

This is choice (a).

Question 4:

First, we need to find r^2 for each charge.

Since it's at the origin, q_1 is 2.0 meters in the x direction and 2.0 meters in the y direction away from the point at which we want to calculate the field.

$$r_1^2 = (2.0\,\mathrm{m})^2 + (2.0\,\mathrm{m})^2 = 8.0\,\mathrm{m}^2$$

So, $r_1 = \sqrt{8.0}\,\mathrm{m}$.

Charge q_2 is located at $x = 2.0\,\mathrm{m}$, so the x distance from q_2 is 0, but the y distance is still 2.0 meters since $y_2 = 0$.

$$r_2^2 = (0)^2 + (2.0\,\mathrm{m})^2 = 4.0\,\mathrm{m}^2$$

So, $r_2 = 2.0\,\mathrm{m}$.

Now we can find the overall magnitude of the electric field from each charge.

$$E_1 = k\frac{q_1}{r_1^2} = (9.0 \times 10^9\,\mathrm{Nm^2/C^2})\frac{2.5 \times 10^{-9}\,\mathrm{C}}{8.0\,\mathrm{m}^2} \approx 2.81\,\mathrm{N/C}$$

$$E_2 = k\frac{q_2}{r_2^2} = (9.0 \times 10^9\,\mathrm{Nm^2/C^2})\frac{1.0 \times 10^{-9}\,\mathrm{C}}{4.0\,\mathrm{m}^2} \approx 2.25\,\mathrm{N/C}$$

Finding the components of E_2 is easy, since it's purely in the y direction (the x distance is 0).

$$\vec{E}_2 = (0, 2.25\,\mathrm{N/C})$$

E_1 is a little more difficult, but at least both components are the same since both distances are 2.0 meters.

$$E_{1,x} = E_{1,y} \approx \frac{2.0\,\mathrm{m}}{\sqrt{8.0}\,\mathrm{m}}(2.81\,\mathrm{N/C}) \approx 1.99\,\mathrm{N/C}$$

$$\vec{E}_1 \approx (1.99\,\mathrm{N/C}, 1.99\,\mathrm{N/C})$$

Both charges are positive, so each field will point away from its respective charge. Field 1 should therefore point up and to the right (both components should be positive), and field 2 should point purely up (positive y component). So, we don't need to adjust any signs.

Now we simply add the components, then find the magnitude of the total field vector.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (1.99\,\mathrm{N/C}, 1.99\,\mathrm{N/C}) + (0, 2.25\,\mathrm{N/C}) = (1.99\,\mathrm{N/C}, 4.24\,\mathrm{N/C})$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{(1.99\,\mathrm{N/C})^2 + (4.24\,\mathrm{N/C})^2} \approx \boxed{4.7\,\mathrm{N/C}}$$

This is choice (c).

Question 5:

First, let's determine whether the electric field will be 0 to the left of q_1, between q_1 and q_2, or to the right of q_2.

It can't be 0 in the middle, since E_1 and E_2 both point to the right in the middle region.

Since q_2 has a larger magnitude than q_1, the cancellation point must be closer to q_1 than to q_2. So, the field must be 0 in the left region, while q_2 always dominates in the right region.

In the left region, $\vec{E}_1$ points to the left (call this negative) and $\vec{E}_2$ points to the right (positive). Having reasoned out the directions of the fields, we now care only about the magnitudes of the charges (we used the signs to tell us the directions).

$$\vec{E}_1 + \vec{E}_2 = \vec{0}$$

$$-k\frac{|q_1|}{r_1^2} + k\frac{|q_2|}{r_2^2} = 0$$

$$k\frac{|q_1|}{r_1^2} = k\frac{|q_2|}{r_2^2}$$

$$\frac{|q_1|}{r_1^2} = \frac{|q_2|}{r_2^2}$$

Since q_1 is located at $x = 0$, r_1^2 is just x^2.

Since q_2 is at $x = 2.0\,\text{m}$, $r_2 = (2.0\,\text{m} - x)$ in the region on the left (x is negative in this region, so it needs to be subtracted from 2.0 m to give the full distance).

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(2.0\,\text{m} - x)^2}$$

$$|q_1|(2.0\,\text{m} - x)^2 = |q_2|x^2$$

$$|q_1|\left[(4.0\,\text{m}^2) - (4.0\,\text{m})x + x^2\right] - |q_2|x^2 = 0$$

$$(|q_1| - |q_2|)x^2 - (4.0\,\text{m})|q_1|x + (4.0\,\text{m}^2)|q_1| = 0$$

Now plug in $|q_1| = 1\,\text{C}$ and $|q_2| = 3\,\text{C}$.

$$(-2\,\text{C})x^2 - (4.0\,\text{C-m})x + (4.0\,\text{C-m}^2) = 0$$

Now plug these values into the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(4)}}{2(-2)}$$

$$x = \frac{4 \pm \sqrt{16 + 32}}{-4}$$

$$x = -\left(1 \pm \sqrt{\frac{48}{16}}\right) = -\left(1 \pm \sqrt{3}\right)$$

The correct value of x must be negative, as we reasoned out above, and the other is extraneous.

$x = -(1 + \sqrt{3})\,\text{m}$ is negative, while the other is positive.

So, $\boxed{x \approx -2.7\,\text{m}}$.

This is choice (a).

Question 6:

The electric field of a point charge has magnitude:

$$E = \frac{kq}{r^2}$$

Solve for q.

$$q = \frac{Er^2}{k}$$

We are told the electric field's components, and can use them to find the overall magnitude:

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

In this case, $E_z = 0$. Making that substitution:

$$q = \frac{\sqrt{E_x^2 + E_y^2} r^2}{k}$$

Now plug in numbers.

$$q \approx \frac{\sqrt{(14\,\mathrm{N/C})^2 + (-7.9\,\mathrm{N/C})^2}(2.0\,\mathrm{m})^2}{9.0 \times 10^9\,\mathrm{Nm}^2/\mathrm{C}^2} \approx \boxed{7.1\,\mathrm{nC}}$$

This is choice (d).

Question 7:

The electric field of an "infinite" plane of charge is:

$$E = \frac{\sigma}{2\epsilon_0}$$

The electric force felt by the proton is therefore

$$F = eE = \frac{e\sigma}{2\epsilon_0}$$

To levitate, this force must cancel the gravitational force, $F = mg$.

$$\frac{e\sigma}{2\epsilon_0} = mg$$

We want to solve for the charge-per-area σ.

$$\sigma = \frac{2\epsilon_0 mg}{e}$$

Now plug in numbers.

$$\sigma \approx \frac{2(8.85 \times 10^{-12}\,\mathrm{C}^2/\mathrm{Nm}^2)(1.7 \times 10^{-27}\,\mathrm{kg})(9.8\,\mathrm{m/s}^2)}{1.6 \times 10^{-19}\,\mathrm{C}} \approx \boxed{1.8 \times 10^{-18}\,\mathrm{C/m}^2}$$

This is choice (c).

Question 8:

The electric field of a positive line of charge points away from the line, perpendicular to it. So, north of the east-west line of charge, the field points north. (I) is true.

The electric field of a line of charge scales as $\frac{1}{r}$, not $\frac{1}{r^2}$ like the field of a point charge, so (II) is also true (twice as far away means half the strength).

The electric field of a positive line of charge points perpendicular to the line, so the force on a charge would also point perpendicularly. Charges would accelerate in the north or south direction, not east-west. (III) is false.

So, the correct choice is (d).

Question 9:

If the proton were a point charge, then 100% of its charge would be enclosed at any distance. Since the proton is instead a uniformly charged sphere (in our model; in reality, the proton's charge distribution is more like $\rho(r) \sim e^{-r}$, but that would be way too hard for a quiz!), the charge enclosed is only:

$$Q_{\text{encl}} = e\frac{V_{\text{encl}}}{V} = e\frac{r^3}{R_p^3}$$

Given the inputs of the proton, $r = \frac{1}{2}R_p$, so $\frac{r^3}{R_p^3} = \boxed{\frac{1}{8}}$. This means that $Q_{\text{encl}} = \frac{1}{8}e$.

Since only 1/8 as much charge is enclosed if the proton is a uniform sphere compared to a point charge, the field is only 1/8 as strong. This is choice (a).

This effect measurably changes the binding energies and orbitals of electrons in atoms, compared to the predictions of a model where the nucleus is a point charge.

Question 10:

Gauss' Law says that the field inside a spherically symmetric charge distribution is

$$E(r) = \frac{Q_{\text{encl}}(r)}{4\pi\epsilon_0 r^2}$$

where $Q_{\text{encl}}(r)$ is the total charge enclosed by a spherical Gaussian surface of radius r. For $E(r)$ to be a constant, $Q_{\text{encl}}(r)$ must scale as r^2 in order to cancel the r^2 in the denominator (which comes from the surface area of the Gaussian sphere). $Q_{\text{encl}}(r)$ is the spherical volume integral of the charge distribution, $\rho(r)$.

$$Q_{\text{encl}}(r) = 4\pi\int_0^r \rho(r')r'^2 \mathrm{d}r'$$

In order to produce something that scales as r^2, the integrand must scale as r. In order for the integrand to scale as r, $\rho(r)$ must scale as $\frac{1}{r}$ in order to cancel out one of the factors of r from the volume element.

$$\boxed{\rho(r) = br^{-1} \to a = -1}$$

This is choice (a).

Another perfectly fine method is to say something like “We saw in class that a linear charge distribution (ie r^1) produces an r^2 field, and that a constant charge distribution (r^0) produces a linear field (r^1), so by extrapolation a $\frac{1}{r} = r^{-1}$ charge distribution must produce a constant field (r^0), so $a = -1$.”