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1.1 Systems of Linear Equations

If two systems of linear equations have exactly the same solution set, they are called equivalent. There are three basic kinds of outcomes for the solution set: 1. no solution (inconsistent) 2. one unique solution 3. infinitely-many solutions (consistent)

1.2 Row Reduction & Echelon Forms

"leading entry" = first non-zero entry in a row. Row Echelon Form. Reduced Row Echelon Form.

pivotal variables -> ex. x_1,x_2 in pivotal columns 1 & 2 | free variables -> ex. x_3,x_4 in non-pivotal columns 3 & 4

max possible # of pivotal cols = min(#cols,#rows) | if the last (augmented) col is pivotal, there is no solution. | if there are free variables, there are infinitely-many solutions. | if there are no free variables/all cols of coeff matrix (left side of dotted lines) are pivotal, there is one unique solution. | ref -> rref; solutionset = $[x_1,x_2,x_3,x_4] = [7x_2-6x_4+5;\ x_2;\ 2x_4-3;\ x_4]$ | Row Operations: R_i<->R_j; R_i/c->R_i; cR_i+R_j->R_j

$$\begin{pmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{pmatrix} \quad \begin{pmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1.3 Vector Equations

A column vector is a list of real numbers in a column.
Column vectors can be added and multiplied by scalars.

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad -3\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ -15 \end{pmatrix}$$

u + v = v + u | u + (v + w) = (u + v) + w | u + 0 = 0 + u = u | u + (-u) = (-u) + u = 0 | c(u + v) = cu + cv | (c + d)u = cu + du | c(du) = (cd)u | 1u = u | where 0: col vector w all 0 entries for R^n; and u and v are col vectors, c and d are constant scalars.

Linear Combinations = Parallelogram Law | combinations of given vectors | Span: The span of a collection of vectors is the set of all linear combinations of those vectors. | "Is $\underline{w}$ in the span of $\underline{v}_1, \underline{v}_2, \dots \underline{v}_n$?" = "Is the system whose aug. matrix is $[\underline{v}_1, \underline{v}_2, \dots \underline{v}_n \mid \underline{w}]$ consistent?" | Row Reduction -> Reduced Echelon Form; check if last column of aug. matrix is pivotal. P = inconsistent, NP = consistent.

1.4 Matrix Equation $A\underline{x}=\underline{b}$

$$\begin{pmatrix} 2 & -3 \\ 8 & 0 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} -13 \\ 32 \\ -6 \end{pmatrix} \quad 4\begin{pmatrix} 2 \\ 8 \\ -5 \end{pmatrix} + 7\begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -13 \\ 32 \\ -6 \end{pmatrix}$$

$A = [\underline{a}_1, \underline{a}_2, \dots \underline{a}_n]$ (m*n matrix) | $\underline{x} = [\underline{x}_1; \underline{x}_2; \dots \underline{x}_n]$ in R^n (col vector, n = # of col in A) | $A\underline{x} = x_1a_1 + x_2a_2 + \dots + x_na_n$ in R^n

A = (3*2); $\underline{x}$ = (2*1); $\underline{b}$ = (3*1)

Matrix multiplication is a concise way of representing linear combinations.

x = b/a *unless a = 0.

$A\underline{x}=\underline{b}$ can be solved if and only if $\underline{b}$ is a linear combination of the columns of A if $\underline{b}$ is "in" the span of the columns of A.

Can I solve $A\underline{x}=\underline{b}$ for any choice of $\underline{b}$? -> Is every vector in b "in" R^3 in the span of the columns of A? -> YES!

Let A be an m*n matrix. The following four statements are equivalent: a) $A\underline{x}=\underline{b}$ can be solved for $\underline{x}$, for any $\underline{b}$ "in" R^m.

b) Every vector $\underline{b}$ "in" R^m is a linear combination of the columns of A. c) The columns of A span R^m. d) A has a pivot in each row.

1.5 Solution Sets

A system of equations is homogeneous if it has the form $A\underline{x} = \underline{0}$ <- zero vector [0;0; … 0]

System of Equations: $3x_1 + 5x_2 - 4x_3 = 0$, $-3x_1 - 2x_2 + 4x_3 = 0$

Matrix Equation: $[3\ 5\ -4;\ -3\ -2\ 4]*[x_1\ x_2\ x_3] = [0;0]$

Vector Equations: $3x_1 + 5x_2 - 4x_3 = 0$, $-3x_1 - 2x_2 + 4x_3 = 0$

Augmented Matrix: [3 5 -4 | 0; -3 -2 4 | 0]

$$\begin{pmatrix} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & -1 & 0 \\ 1 & 0 & 1 & 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & -3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & -1 & 4 \\ 1 & 0 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & -3 & 1 \end{pmatrix}$$

Homogeneous systems always have at least one solution: $\underline{x} = \underline{0}$. But there may be more.

$[x_1;x_2;x_3] = [4/3*x_3;0;x_3]$ -> rename x_3=s -> [4/3*s;0;s] = s[4/3;0;1]

s = free parameter | sol. set = span{[4/3;0;1]} | parametric solution (in s & t) | $[x_1;x_2;x_3;x_4]$ = [-s-2t;-s+3t;s;t] = s[-1;-1;1;0] + t[-2;3;0;1] | sol. set = span{[-1;-1;1;0],[-2;3;0;1]} | Inhomogeneous systems are linear systems that are not homogeneous, i.e. $A\underline{x} = \underline{b}$ where $\underline{b} \mathrel{!}= \underline{0}$. | $[x_1;x_2;x_3;x_4]$ = [-s-2t+3;-s+3t+1;s;t] = s[-1;-1;1;0] + t[-2;3;0;1] + [3;1;0;0] | translation from the homogeneous system to inhomogeneous system.

a) Even though $A\underline{x}=\underline{0}$ is always consistent ($\underline{x}=\underline{0}$ is always in v_n), for given $\underline{b}$!=$\underline{0}$, the equation $A\underline{x}=\underline{b}$ may be inconsistent.

b) If $\mathbf{A\underline{x}=\underline{b}}$ is consistent (i.e. $\underline{b}$ is in the span of the columns of A) and if $\underline{x}=\underline{p}$ is any particular solution, then the general solution is $\underline{x}=\underline{p}+v_n$.

1.7 Linear Independence

A collection of vectors {u_1,u_2, … ,u_n} is called linearly dependent if at least one of them is in the span of the others. If a set of vectors is not linearly dependent, we call the vectors linearly independent. Vectors u_1,u_2, … ,u_n are linearly independent if and only if the vector equation $x_1u_1+x_2u_2+$ … $x_nu_n=0$ has only the trivial solution $x_1=x_2=$ … $=x_n=0$. | The columns of a matrix A are linearly independent if and only if A$\underline{x}$=$\underline{0}$ has only the trivial solution $\underline{x}$=$\underline{0}$. AKA A has a pivot position in every column. If A has more columns than rows, it's columns must be linearly dependent. | Set a given matrix equal to 0 in an aug. matrix, if all the original columns are pivotal, it is independent. | If one of the vectors is $\underline{0}$, dependent. | One vector is linearly independent if and only if it is the $\underline{0}$. | For two vectors to be linearly independent, one must be a scalar multiple of the other, therefore the two vectors must be parallel.

1.8 Linear Transformations

A matrix transformation T: R^n->R^m is a function given by matrix multiplication by some m*n matrix A. T($\underline{x}$)=A$\underline{x}$. A = [1 -5 -7; -3 7 5] -> T: $R^{3(n)}$->$R^{2(m)}$; T([x_1;x_2;x_3]) = [x_1-5x_2-7x_3;-3x_1+7x_2+5x_3] | Image of [0;½;-½] under T: [1 -5 -7; -3 7 5]*[0;½;-½] = [1;1] | Does T map to any other vector in R^3 to this vector in R^2? I.e. find all $\underline{x}$ "in" R^3 such that T($\underline{x}$)=[1;1]; A$\underline{x}$=[1;1] -> [1 -5 -7 | 1; -3 7 5 | 1] = [1 0 3 | -3/2; 0 1 2 | -½] = t[-3;-2;1] + [-3/2;-½;0] -> t must be -½ to get [3/2;1;-½] + [-3/2;-½;0] = [0;½;-½] | What is the range of T? {b "in" R^2 such that $\underline{b}$=T($\underline{x}$) for some $\underline{x}$ "in" R^3} = {$\underline{b}$ such that A$\underline{x}$=$\underline{b}$ is consistent} = R^2 every possible $\underline{b}$ because there is no contradiction (last col non-pivotal) A = [1 -3;3 5;-1 7], $\underline{b}$ = [3;2;-5] -> Range of T -> {A$\underline{x}$ such that $\underline{x}$ "in" R^3} = span{[1;3;-1],[-3;5;7]} | "projection", "ccw rotation", "shear"

1.9 The Matrix of a Linear Transformation

A function/transformation T: R^n->R^m is called linear if it respects addition and scalar multiplication. | T($\underline{u}$+$\underline{v}$) = T($\underline{u}$) +T($\underline{v}$); T(c$\underline{u}$) = cT($\underline{u}$); T($\underline{0}$) = T(0*$\underline{v}$) = $\underline{0}$; T(c_1v_1+c_2v_2+ … +c_Rv_R) = T(c_1v_1+$\underline{w}$) | Standard Basis Vectors = cols of the eye matrix "in" R^n, referred to as $\underline{e}_1$,$\underline{e}_2$, … $\underline{e}_n$ | T(x_1e_1+x_2e_2+ … +x_ne_n) = x_1T($\underline{e}_1$)+x_2T($\underline{e}_2$)+ … +x_nT($\underline{e}_n$) = R^m -> $\underline{a}_1$,$\underline{a}_2$, … $\underline{a}_n$ = [$\underline{a}_1$,$\underline{a}_2$, … $\underline{a}_n$]*[x_1;x_2; … ; x_n] | A transformation T: R^n->R^m is called onto if range(T) = R^m. I.e. for every $\underline{b}$ "in" R^m, there is at least one $\underline{x}$ "in" R^n such that T($\underline{x}$)=$\underline{b}$. **Onto if and only if every row in A is pivotal.** | One-to-one if for any $\underline{b}$ "in" R^m, there is at most one $\underline{x}$ "in" R^n such that T($\underline{x}$)=$\underline{b}$. **One-to-one if and only if every column in A is pivotal / Linearly Independent** | Every rotation is both one-to-one and onto. | T(x) = [1 1 0; 0 2 1]$\underline{x}$ (R^3->R^2): onto, but not one-to-one.

2.1 Matrix Operations

We can treat matrices as vectors and add or multiply them by scalars. | A+B=B+A; (A+B)+C=A+(B+C); A+0=A; r(A+B)=rA+rB; (r+s)A=rA+sA; r(sA) = (rs)A | Addition and scalar multiplication only works if the size makes sense. Doesn't work w/ all operations. | (T+S)($\underline{x}$) = T($\underline{x}$)+S($\underline{x}$) = A$\underline{x}$+B$\underline{x}$ = (A+B)$\underline{x}$; (rT)($\underline{x}$) = r*T($\underline{x}$) = r*A$\underline{x}$ | If T: R^n->R^m and S: R^m->R^k we can compose them R^n->R^m->R^k (S∘T)($\underline{x}$) = S(T($\underline{x}$)); S∘T: R^n->R^k; S∘T($\underline{u}$+$\underline{v}$) = S(T($\underline{u}$+$\underline{v}$)) = S(T($\underline{u}$)+T($\underline{v}$)) -> set T($\underline{u}$), T($\underline{v}$) as x, y -> S($\underline{x}$+$\underline{y}$) = S($\underline{x}$)+S($\underline{y}$) = S(T($\underline{u}$))+S(T($\underline{v}$)) = (S∘T)($\underline{u}$)+(S∘T)($\underline{v}$) | S∘T(r$\underline{v}$) = r(S∘T)($\underline{v}$) therefore S∘T is a linear transformation. | (k*m m*n) BA = B[a_1,a_2, … a_n] = [Ba_1,Ba_2, … Ba_n]

$$\begin{pmatrix}4&-2\\-3&0\\3&5\end{pmatrix}\begin{pmatrix}1&3\\2&-1\end{pmatrix}=\begin{pmatrix}0&14\\-3&-9\\13&4\end{pmatrix}=\begin{pmatrix}4\cdot 1+(-2)\cdot 2 & 4\cdot 3+(-2)(-1)\\(-3)\cdot 1+0\cdot 2 & (-3)\cdot 3+0\cdot(-1)\\3\cdot 1+5\cdot 2 & 3\cdot 3+5(-1)\end{pmatrix}$$

| AB != BA; A(BC) = (AB)C; A(B+C) = AB + AC; (B+C)A = BA + CA; (r*A)B = r(AB) = A(rB); If A is m*n, I_mA = A = AI_n I_n= n*n ID matrix; eye(n) | A$\underline{x}$=$\underline{b}$ solvable for all $\underline{b}$ -> every row is pivotal; there exists C such that AC = I | A$\underline{x}$=$\underline{b}$ has at most one solution for each $\underline{b}$ -> every column is pivotal; there exists C such that CA = I | For a square matrix: all pivotal columns <=> all pivotal rows | T: R^n->R^n is one-to-one if and only if it is onto.

2.2-2.3 Matrix Inverse

[A | I] -> rref -> [I | A^-1] | [a b | 1 0; c d | 0 1] -> multiply R_1 by c, multiply R_2 by a -> [ac bc | c 0; ac ad | 0 a] -> [ac bc | c 0; 0 ad-bc | -c a] det(A) = ad-bc; A is invertible if and only if det(A) != 0, in which case [a b; c d]^-1 = 1/(ad-bc) *[d -b; -c a]
Transpose Prop.: $(A^T)^T$ = A; $(AB)^T$ = B^TA^T; $(A+B)^T$ = A^T+B^T; $(rA)^T$ = rA^T | Inverse Prop.: $(A^{-1})^{-1}$ = A; $(AB)^{-1}$ = ($B^{-1}A^{-1}$); $(A^T)^{-1}$ = $(A^{-1})^T$

4.1 Vector Spaces & Subspaces

A "vector space" is a set of objects (which we call vectors) on which two operations are defined: addition: u,v -> u+v; scalar multiplication: λ,v -> λv for λ "in" R | P = {all polynomial functions in one variable x} space closed under addition and scalar multiplication | M_{2*3} = {2*3 matrices} is a vector space. | If v is a vector space, and w "in" v is nonempty, w is a subspace of v if w is closed under addition and scalar multiplication (in v). | w is closed under addition (in v) & w is closed under scalar multiplication (in v) & $\underline{0}$ (the zero vector in v) is in w. | {$\underline{0}$} <- the trivial subspace.

4.2 Null Space & Column Space

Given any matrix A "in" M_{m*n} there are two important subspaces: The null space of A, Nul(A) = {$\underline{x}$ "in" R^n: A$\underline{x}$ = $\underline{0}$} (= the sol. set of the homogeneous equation A$\underline{x}$=$\underline{0}$) & The column space of A, Col(A) = {A$\underline{x}$: $\underline{x}$ "in" R^n} (= span{columns of A}) | Nul(A) is a subspace of R^n; Col(A) =< R^m. | Nul(A) is a subspace of R^n & Col(A) is a subspace of R^m.

4.3 Linearly Independent Sets; Bases

Vectors v_1,v_2, … v_n in some vector space v are linearly independent if at least one of them is a linear combination of the others. There is a nontrivial linear combination of {v_1,v_2, … v_n} that = $\underline{0}$. The vectors are linearly independent if they are not linearly dependent. The only linear combination = $\underline{0}$ is the trivial one. A collection {v_1,v_2, … v_n} of vectors in H is called a basis for H if: 1. span{v_1,v_2, … v_n} = H; 2. {v_1,v_2, … v_n} is linearly independent. | Any basis of R^n consists of exactly n column vectors. The set $\{v_1, v_2, \dots v_n\}$ is a basis if and only if the matrix $[v_1\ v_2\ \dots\ v_n]$ is invertible. | If H = span$\{v_1\ v_2\ \dots\ v_n\}$, then some collection of these vectors is a basis for H. | The pivotal columns of A form a basis for Col(A). (The coefficients expressing the non-pivotal columns in terms of this basis can be read off of rref(A).) Caution: The pivotal columns of A, npot rref(A), form a basis for Col(A). Row(A) = span{rows of A} | Nul(A) = {free variables in parametric form}

4.5 Dimension

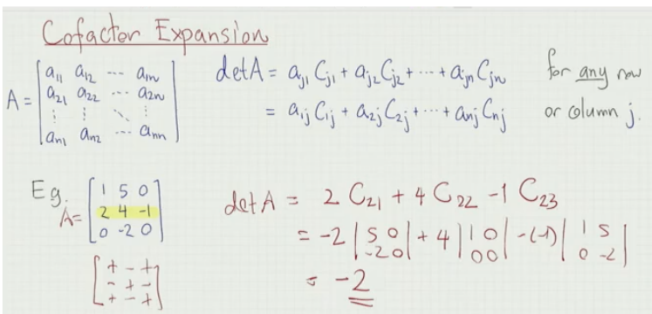
Any two bases of a vector space have the same number of vectors. | If v is a vector space, its dimension is the number of vectors in any basis.

4.4-4.5 Rank & Coordinate Systems

rank(A) = dim(Col(A)); rank = # of pivotal columns | nullity(A) = dim(Nul(A)); nullity = # of non-pivotal columns/free variables | rank(A) + nullity(A) = # of columns <- Rank Theorem | dim(Row(A)) = dim(Col(A^T)) = rank(A^T) | Row(A) = Row(rref(A)) | dim(Row(A)) = # pivotal rows in rref(A) = # pivotal 1's in A = # pivotal columns in rref(A) or A = rank(A) | If B = $\{b_1, b_2, \dots b_n\}$ is a basis for V, then the function T: V -> R^n : T(<u>v</u>) = $[\underline{v}]_B$ is a one-to-one linear transformation from V onto R^n. Such a linear transformation is called an <u>isomorphism</u>. Linear isomorphisms preserve all linear properties.

3.1-3.2 Determinants

det(A) = |A|; A is invertible if and only if det(A) != 0 in which case A^{-1} = 1/det(A) *[d -b; -c a] | Cofactor Expansion:



If A is triangular, det(A) is the product of the diagonal entries. The function det: M_n -> R is <u>not</u> linear. But it is <u>multi</u>linear. Think of det as a function of the rows. | The determinant is a linear transformation of each row separately. If A is n*n and has full rank (i.e. rank(A) = n; i.e. A is invertible) then det(A) = +- product of the pivots in row reduction. If rank(A) < n, det(A) = 0. | det(A) = 0 if and only if A is not invertible. | If solving det(A) by row reduction, if you swap adjacent rows you must not forget to multiply by (-1).

3.3 Determinants & Volume

Elementary Matrix: An elementary matrix is the result of one row operation applied to the identity matrix. | If A,B are n*n matrices, det(AB) = det(A)*det(B). | Determinants are: defined by cofactor expansion; follows row op. axioms; det(A) = 0 if and only if A is <u>not</u> invertible; det: M_n->R is multilinear; det(AB) = det(A)*det(B), det(A^n) = $(\det(A))^n$; det(A^{-1}) = 1/det(A); det(A^T) = det(A) | the sign of det[<u>a</u>,<u>b</u>] means oriented volume.

5.1 Eigenvectors & Eigenvalues

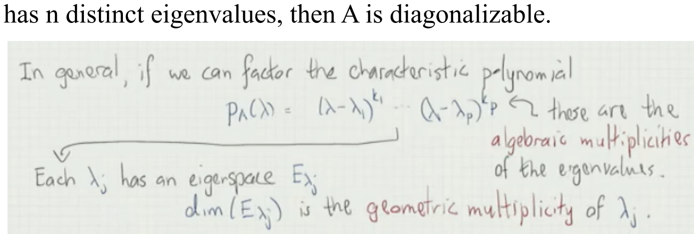
Stochastic matrix: rows sum to 1. | Let T: V->V be a linear transformation. If there is a non-zero <u>v</u> "in" V such that T(<u>v</u>) = λ<u>v</u> for some scalar λ "in" R, we call <u>v</u> an <u>eigenvector</u> of T. The scalar λ is the corresponding <u>eigenvalue</u>. | Show that 7 is an eigenvalue of A = [1 6; 5 2] -> (A-7I)<u>x</u> = <u>0</u>. [1 6; 5 2] - [7 0; 0 7] = [-6 6; 5 -5] = [1 -1; 0 0] -> $[x_1;x_2] = x_2[1;1]$ -> span{[1;1]} | If λ is an eigenvalue of A, the set of all vectors <u>v</u> "in" R^n such that A<u>v</u> = λ<u>v</u> is a subspace, called the <u>eigenspace</u> E_λ = Nul(A-λ I) | det(A-λI) = 0 | If A is an n*n matrix with n distinct eigenvalues, then there is a basis of R^n consisting of eigenvectors of A. | Eigenvectors for distinct eigenvalues are linearly independent. If A "in" M_{n*n} has n distinct eigenvalues, there is a basis of R^n consisting of eigenvectors of A. n*n matrices A and B are called similar if there is an invertible n*n matrix P with the property A = PBP^{-1} -> if A has all distinct eigenvalues, then it is similar to a diagonal matrix. Similarity is <u>not</u> the same as row equivalent. | If A and B are similar, then they have the same eigenvalues, and the eigenspaces for A have the same dimensions as the eigenspaces for B.

5.2 Characteristic Polynomial

The characteristic polynomial of an n*n matrix A is polynomial with λ found through the det(A-λI). | An n*n matrix cannot have more than n eigenvalues counted with multiplicity. | If A is an upper or lower triangular matrix, then its eigenvalues are its diagonal entries.

5.3 Diagonalization

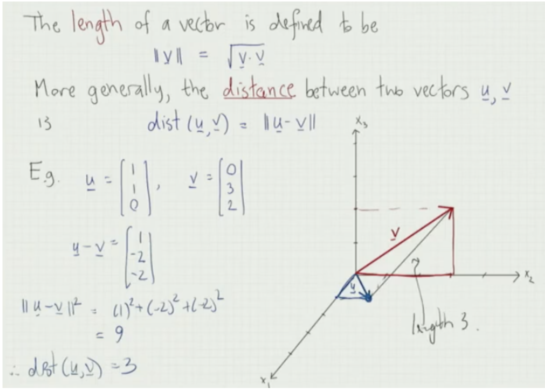
A matrix is diagonalizable if its eigenspaces span R^n. Equivalently: if there is a basis of R^n consisting of eigenvectors. If A has n distinct eigenvalues, then A is diagonalizable.

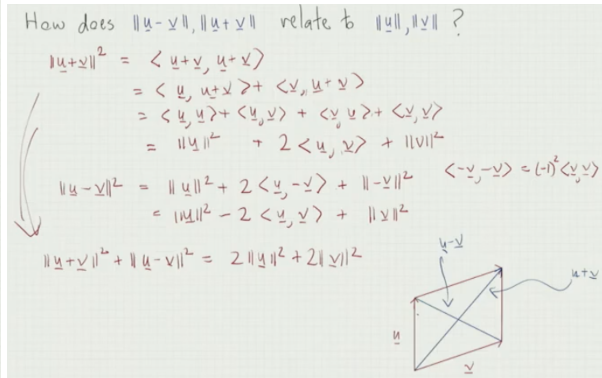


6.1,6.7 Inner Product

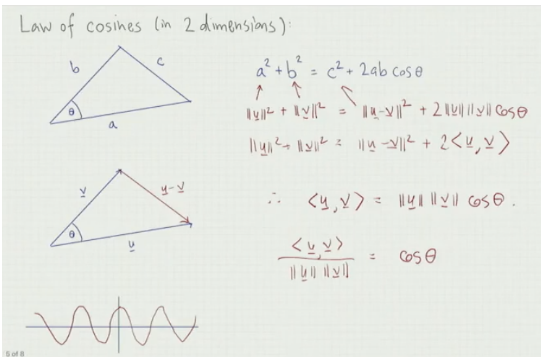
The dot product or inner product on R^n is defined by <u>u</u>*<u>v</u> = <<u>u</u>,<u>v</u>> = $\underline{u}^T\underline{v}$ = $[u_1\ u_2\ u_3][v_1;v_2;v_3] = u_1v_1+u_2v_2+u_3v_3$

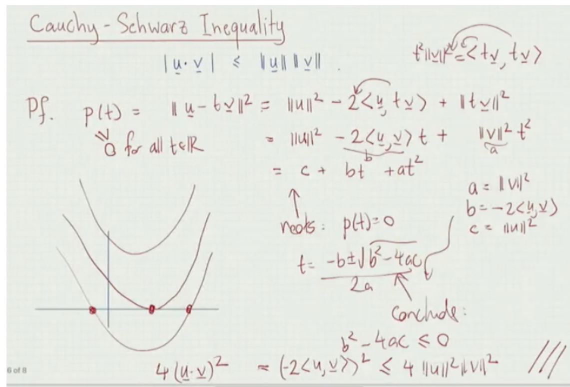
Properties: 1) $\underline{u}*\underline{v} = \underline{v}*\underline{u}$; $\langle \underline{u},\underline{v}\rangle = \langle \underline{v},\underline{u}\rangle$ 2) $\underline{u}*(\underline{v}+\underline{w}) = \underline{u}*\underline{v} + \underline{u}*\underline{w}$ 3) $\underline{u}*(c\underline{v}) = c\underline{u}*\underline{v}$ 4) $\underline{u}*\underline{u} = u_1^2+u_2^2+u_3^2 > 0$, $= 0$ if and only if $\underline{u} = 0$.



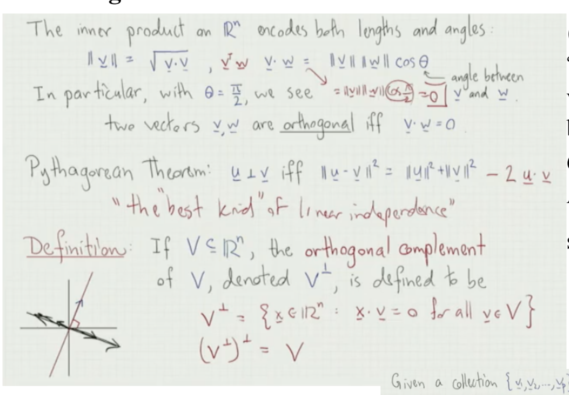


Cauchy-Schwarz Inequality: $| \underline{u}*\underline{v} | \leq \| \underline{u} \| \| \underline{v} \|$





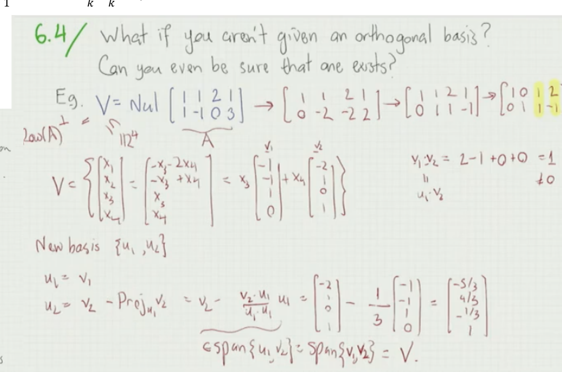
6.2 Orthogonal Sets

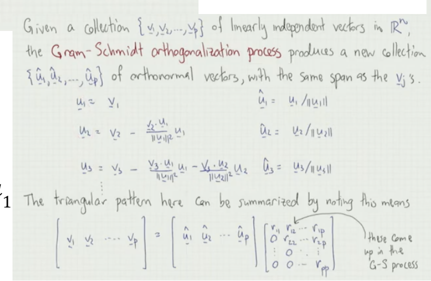


$(RowA)^{\perp} = NulA$ $(ColA)^{\perp} = NulA^{\perp}$ $V^{\perp}$ orthogonal complement of V | Let V "in" R^n be a subspace. A basis for V is called an orthogonal basis if all the basis vectors are orthogonal. It is called an orthonormal basis, if, in addition, each basis vector has length 1.

6.3 Orthogonal Projections

$A^TA = I$ if columns are orthonormal $n\times m$ $m\times n$ = the identity matrix only if a square matrix $\hat{y} = \frac{y\cdot u_1}{u_1\cdot u_1}u_1 + \frac{y\cdot u_k}{u_k\cdot u_k}u_k$ $z = y - \hat{y}$





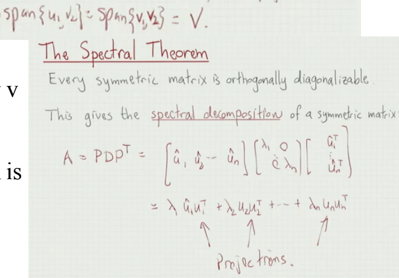
6.4 Gram-Schmidt Orthogonalization

$u_1 = v_1$

$u_2 = v_2 - Proj_{u_1}(v_2) = v_2 - \frac{v_2\cdot u_1}{u_1\cdot u_1}u_1$

$u_3 = v_3 - \frac{v_3\cdot u_1}{u_1\cdot u_1}u_1 - \frac{v_3\cdot u_2}{u_2\cdot u_2}u_2$

$\hat{u}_n = \frac{1}{\|v_n\|}v_n$ v_n is vector n $\|v_n\|$ is $\sqrt{x_1^2 + x_2^2}$ $A = QR$ $R=Q^TA = Q^{-1}A$ orthonormal basis $\{u_1,u_2,u_p\}$ for v



7.1 Spectral Theorem

$A=PDP^{-1}=PDP^T$ $A^T=(PDP^T)^T = A$ must be symmetric if it has an orthonormal basis of eigenvectors. If A is orthogonally diagonalizable then $A = A^T$, $(\lambda - \mu)u\cdot v = \lambda u\cdot v - \mu u\cdot v = Au\cdot v - u\cdot Av$