CSE 101 Homework 2

Winter 2023

This homework is due on gradescope Friday January 27th at 11:59pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (The Easy Way Down, 30 points)**.** *Dave was already at the top of the ski slope by the time he realized that he wasn't prepared for it. Fortunately, the slope has many branching paths (given by a DAG G) that can be taken to the bottom, and Dave is determined to find the easiest way down. Two vertices of G are labelled TOP (Dave's current location) and BOTTOM (the place he is trying to reach). Furthermore, each edge has been assigned a difficulty rating. Dave is trying to find a path to the bottom so that the highest difficulty edge he needs to use is as small as possible.*

Give a linear time algorithm that given G, TOP, BOTTOM and the difficulty ratings finds Dave's best path.

Hint: For each vertex v, compute the maximum difficulty of the best path from TOP to v. If you do this for each v in the correct order, it is relatively straightforward.

Question 2 (Other Attempts to Find Source and Sink Components, 15 points)**.** *In class we showed that in a directed graph G the vertex with the largest postorder number is in a source SCC. Provide counter-examples to disprove the following statements:*

(a) The vertex with the smallest postorder number is always in a sink SCC. [5 points]

(b) The vertex with the largest preorder number is always in a sink SCC. [5 points]

(c) The vertex with the smallest preorder number is always in a source SCC. [5 points]

Question 3 (Max Reachable, 25 points)**.** *Let G be a directed graph where every vertex is assigned a real number. We wish to compute for each vertex v of G the largest number assigned to any vertex reachable from v.*

(a) Give a linear time algorithm for this problem if G is strongly connected. [5 points]

(b) Give a linear time algorithm for this problem if G is a DAG. [10 points]

(c) Give a linear time algorithm for this problem for a general directed graph G. (Hint: you will want to find a way to combine the previous algorithm ideas.) [10 points]

Question 4 (Line Switching, 30 points)**.** *The subway system of Graphopolis is given by an undirected graph G with the vertices representing stations and the edges representing tracks. Furthermore, the edges are partitioned into lines. Each line consists of some contiguous collection of edges and a traveller can travel from any station on a given line to any other without changing trains.*

Give an algorithm that given the graph G, a description of the lines and two stations v and w, determines the fewest number of times that a traveler would need to change trains to get from v to w. For full credit, your algorithm should be linear time.

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*