CSE 101 Homework 0

Winter 2023

This homework is due on gradescope Friday January 13th at 11:59pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Program Runtimes, 20 points)**.** *Consider the following two programs:*

```
Alg1(n)
  for i = 1 to n^3
    for j = 1 to n
      Print(j)
```

and

```
Alg2(n)
  for i = 1 to n^3
    if i <= n
      for j = 1 to n
        Print(j)
```

For each of these programs give the asymptotic runtime as $\Theta(f(n))$ for some function f and justify your work.

Question 2 (Asymptotic Comparisons, 20 points)**.** *Sort the following functions of n in terms of their asymptotic growth rates. In particular, ones should go later in the list if they are larger when sufficiently large values of n are used as inputs. Which of these functions have polynomial growth rates? Remember to justify your answers.*

- $a(n) = 2^{\sqrt{\log(n)}}$
- $b(n) = 2^{\sqrt{n}}$
- $c(n) = 10^{10^{10}} n^{0.01}$
- $d(n) = 6^{\log_2(n)}$
- $e(n) = n(1000 + \sqrt{n})(1000 + n)$

Question 3 (Graph Coloring, 30 points)**.** *Let G be a finite graph with maximum degree at most d (that is no vertex is connected to more than d other vertices). Show that each vertex of G can be assigned an integer in $\{1, 2, \ldots, d+1\}$ so that no two adjacent vertices are assigned the same integer. Hint: Use induction on the number of vertices.*

Question 4 (Recurrence Relation, 30 points)**.** *Suppose that you have a function $T(n)$ defined by $T(1) = 1$ and*

$$T(n) = T(n-1) + n$$

for $n > 1$.

(a) Prove by induction that $T(n) = n(n+1)/2$. [15 points]

(b) Consider the following "proof" that $T(n) = O(n)$ (note that this contradicts part (a)):

We proceed by strong induction on n. Clearly $T(1) = O(1)$, which gives us our base case. If we assume that $T(n) = O(n)$, then $T(n+1) = T(n) + (n+1) = O(n) + O(n) = O(n)$. This completes our inductive step and proves that $T(n) = O(n)$ for all n.

What is wrong with the above proof? (Hint: Consider what the implied constant in the O term would be.) [15 points]

Question 5 (Extra credit, 1 point)**.** *Approximately how much time did you spend working on this homework?*