CSE 20 Spring 2022
Midterm Practice Problems

This practice exam may help you study for the midterm on Thursday, May 5. I recommend that you also review each HW assignment (by looking at the feedback you received as well as solutions posted on Piazza), examples from class and the Canvas review quizzes (including the solutions posted on Piazza).

1. Number Systems and Base Expansion

(a) Compute the ternary (base 3) expansion of 38.

(b) Convert 41/16 to binary by using the binary point (the binary version of a decimal point.) Compute 4 bits after the binary point.

(c) Compute the product of $(6A)_{16}$ and $(11)_{16}$, without converting either number to another base.

(d) Confirm your answer for part (b) by converting $(6A)_{16}$ and $(11)_{16}$ to decimal, multiplying them, and converting the product back to base 16.

(e) How many bits will there be in the binary (base 2) expansion of 2020?

(f) Convert the straight binary number: $(110011001100)_2$ into BCD.

2. Recursively Defined Sets and Functions

Basis Step: $1 \in S_4$, $2 \in S_4$, $3 \in S_4$
Recursive Step:
If $s \in S_4$ then

- $s0 \in S_4$
- $s1 \in S_4$
- $s2 \in S_4$
- $s3 \in S_4$.

Consider the function $G : S_4 \to \mathbb{N}$ defined recursively as

Basis Step: $G(1) = 0$, $G(2) = 0$, $G(3) = 0$
Recursive Step:
If $s \in S_4$ then

- $G(s0) = G(s) + 1$
- $G(s1) = 0$
- $G(s2) = 0$
- $G(s3) = 0$

.

(a) Evaluate G on the following strings in S_4: (no justification necessary)

(i) $G(3210) =$

(ii) $G(2000) =$

(iii) $G(10101) =$

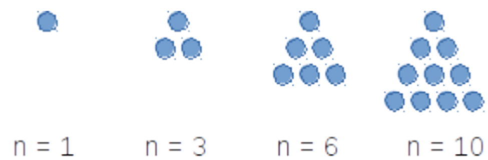
(b) Prove the statement:

$$\exists x(G(x) = 1)$$

(c) Disprove the statement:

$$\forall x(G(x) = 1)$$

3. Implementing Circuits A triangular number (or triangle number) counts the objects that can form an equilateral triangle, as in the diagram below. The n^{th} triangular number is the sum of the first n integers, as shown in the following figure illustrating the first four triangular numbers (what is the fifth one?):



Design a circuit that takes a 4-bit fixed width binary integer $x_3x_2x_1x_0$ as input, and outputs True (T or 1) if this integer is a triangular number, and False (F or 0) otherwise. You may assume that 0 is not a triangular number. (*Credit: UBC Department of Computer Science*)

4. Logical Equivalences

(a) Draw a logic circuit that uses **exactly three** gates and is logically equivalent to

$$q \leftrightarrow (p \wedge r)$$

You may (only) use AND, OR, NOT, and XOR gates.

(b) Write a compound proposition which is logically equivalent to

$$(p \oplus q) \leftrightarrow r$$

You may only use the logical operators negation ($\neg$), conjunction ($\wedge$), and disjunction ($\vee$).

(c) Find a compound proposition that is in DNF (disjunctive normal form) and is logically equivalent to

$$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r)$$

5. Logic

p is "The display is 13.3-inch"

q is "The processor is 2.2 GHz"

r is "There is at least 128GB of flash storage"

s is "There is at least 256GB of flash storage"

u is "There is at least 512GB of flash storage"

(a) Are the statements

$$p \to (r \vee s \vee u) \quad , \quad q \to (s \vee u) \quad , \quad p \leftrightarrow q \quad , \quad \neg u$$

consistent? If so, translate to English a possible assignment of truth values to the input propositions that makes all four statements true simultaneously.

(b) Consider this statement in English:

It's not the case that both the display is 13.3-inch and the processor is 2.2 GHz.

Determine whether each of the compound propositions below is equivalent to the negation of that statement, and justify your answers using either truth tables or other equivalences.

Possible compound propositions:

(I) $\neg p \vee \neg q$ (II) $\neg(p \to \neg q)$ (III) $\neg(p \wedge q)$ (IV) $(\neg p \leftrightarrow \neg q) \wedge p$

(c) Consider the compound proposition

$$(p \wedge q) \rightarrow (r \vee s \vee u)$$

Express the **contrapositive** of this conditional as a compound proposition.

Then, give an assignment of truth values to each of the input propositional variables for which the original compound proposition is True but its **converse** is False.

6. Logical equivalence

(a) Over the domain $\{1, 2, 3, 4, 5\}$ give an example of predicates $P(x), Q(x)$ which demonstrate that

$$\forall x P(x) \vee \forall x Q(x) \not\equiv \forall x(\ P(x) \vee Q(x)\ )$$

(b) Over the domain $\mathbb{R}$ give an example of predicates $P(x), Q(x)$ which demonstrate that

$$\exists x P(x) \wedge \exists x Q(x) \not\equiv \exists x(\ P(x) \wedge Q(x)\ )$$

7. Quantifiers

- Express the statement using quantifiers and the given predicates, along with any propositional logic connectives (e.g. $\wedge$, $\neg$, $\oplus$) and the arithmetic operations $+, -, *$.
- Form the negation of the statement, so that no negation operator is to the left of a quantifier.
- Express this negation you formed in simple and precise English.

a. Statement: There is a real number x, such that for all real numbers y, $x - 3y = 4$.
Allowed predicates: $Eq(a, b) =$ "$a = b$". Domain: Real numbers

b. Statement: For all real numbers x, if $x^2 < 1$, then $x > 1$.
Allowed predicates: $G(a, b) =$ "$a > b$", $Eq(a, b) =$ "$a = b$". Domain: Real numbers

c. Statement: There is a real number with no reciprocal.
Reminder: the reciprocal of a real number a is a real number b such that $ab = 1$.
Allowed predicates: $Eq(a, b) =$ "$a = b$". Domain: Real numbers

d. Statement: For all integers a, b, if a is even and b is even, then $a + b$ is even.
Note: you may NOT use div/mod for this. Evenness can be expressed using an additional quantified variable.
Allowed predicate: $Eq(a, b) =$ "$a = b$". Domain: Integers

8. Proofs

Prove the following Claim:

For all integers n, if n^2 is odd then n is odd.