CSE 21 Fall 2022

Homework 1 sample problems: induction, problem-solving, looking for invariants

1. Suppose you have a red cup and a green cup. At the start, the red cup contains some number r red jellybeans (and nothing else) and the green cup contains some number g green jellybeans (and nothing else). Then, in each step, you take one jellybean from each cup, mix them up, and put one jellybean back into each cup. Prove that, after any number of steps, the number of green jellybeans in the red cup is equal to the number of red jellybeans in the green cup.

2. An amoeba reproduces by splitting itself into two amoebas. This means that the original amoeba dies and its two offspring live on. Prove that if you start off with one amoeba then after any amount of time, the number of living amoebas is equal to one plus the number of amoebas who have died.

3. Suppose you are in a dark room with 100 quarters spread out on the floor. Before you go in, you know that 10 of the quarters are "heads" up and 90 are "tails" up. Your job is to enter the dark room and separate the quarters into two subsets such that each subset has the same number of "heads up" coins. (you cannot feel the texture of the coins enough to distinguish them and you cannot see.)Explain your strategy, and prove its correctness.

4. You are given an $n \times m$ jigsaw puzzle with nm pieces. Initially, none of the pieces are connected. One *move* consists of fitting together two puzzle pieces (these puzzle pieces may be alone or part of a bigger chunk.) Develop a strategy that will minimize the number of *moves* required to complete the puzzle.

5. Suppose you have a bag of 25 red marbles and 25 green marbles.

   You reach in the bag and pick two marbles at random.

   - If the two marbles are the same color (either both red or both green) then remove those two marbles and put a green marble into the bag.
   - If the two marbles are different colors (one red and one green) then remove those two marbles and put a red marble into the bag.

   Repeat this process until there is only one marble remaining. Is it more likely that the remaining marble is red or green? (Give a justification of your answer.)