ECE 35 Homework #5 (Spring 2023, Taur)

<u>All homework problems come from the textbook, "Introduction to Electric Circuits", by Svoboda & Dorf, 9th Edition.</u>

P 5.4-4 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-4.

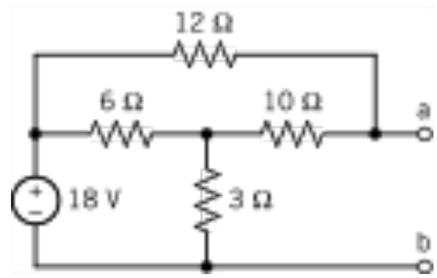
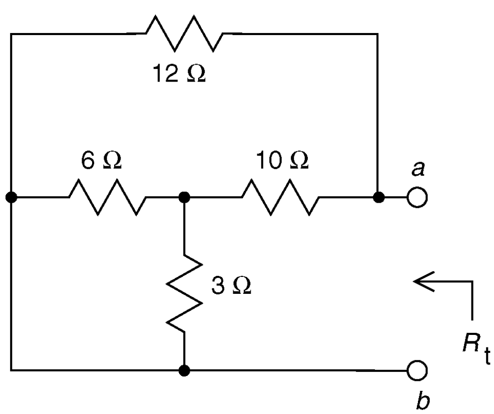
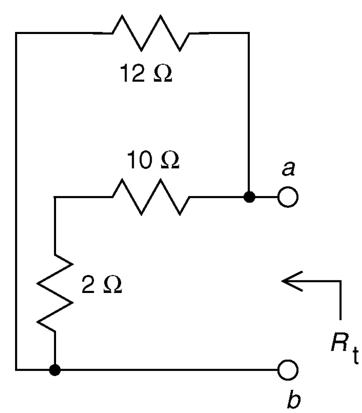


Figure P 5.4-4

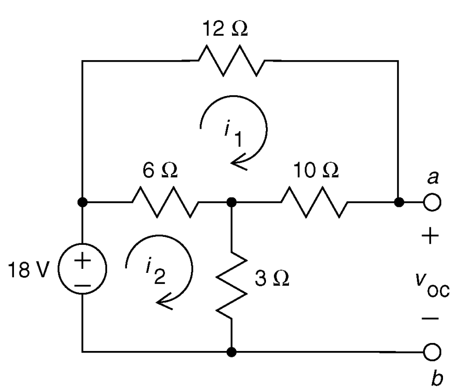
Find R_t:





$$R_t = \frac{12(10+2)}{12+(10+2)} = 6\ \Omega$$

Write mesh equations to find v_{oc}:



Mesh equations:

$$12\,i_1 + 10\,i_1 - 6\,(i_2 - i_1) = 0$$
$$6\,(i_2 - i_1) + 3\,i_2 - 18 = 0$$

$$28\,i - 6\,i$$

Finally, $v_{oc} = 3\,i_2 + 10\,i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12 \text{ V}$

P 5.4-5 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-5.

Answer: $v_{oc} = -2$ V and $R_t = -8/3\ \Omega$

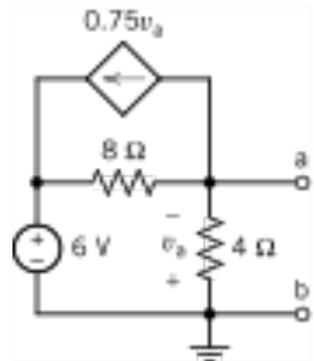
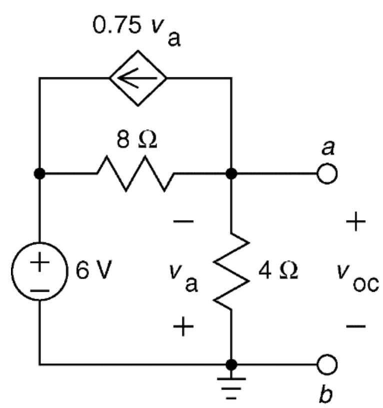


Figure P 5.4-5

Solution:

Find v_{oc}:

Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:



$$v_a = -v_{oc}$$

Apply KCL at node a:

$$-\left(\frac{6 - v_{oc}}{8}\right) + \frac{v_{oc}}{4} + \left(-\frac{3}{4}v_{oc}\right) = 0$$

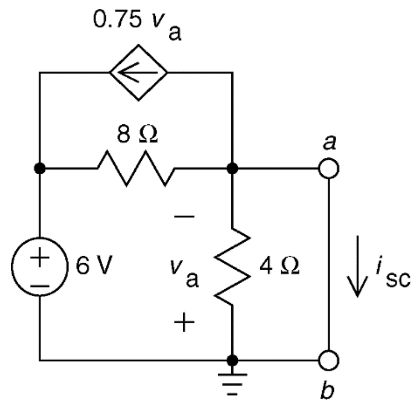
$$-6 + v_{oc} + 2\,v_{oc} - 6\,v_{oc} = 0 \quad \Rightarrow \quad v_{oc} = -2 \text{ V}$$

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces

$$v_a = 0$$

Apply KCL at node a:



$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{6}{8}=\frac{3}{4}\text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\,\Omega$$

P 5.4-10 For the circuit of Figure P 5.4-10, specify the resistance *R* that will cause current i_b to be 2 mA. The current i_a has units of amps.

Hint: Find the Thévenin equivalent circuit of the circuit connected to *R*.

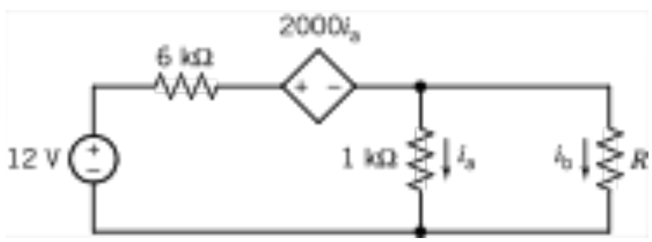
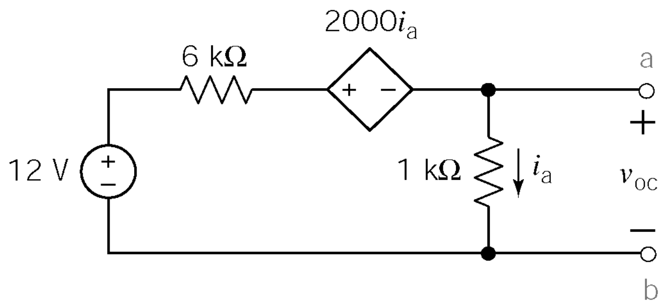
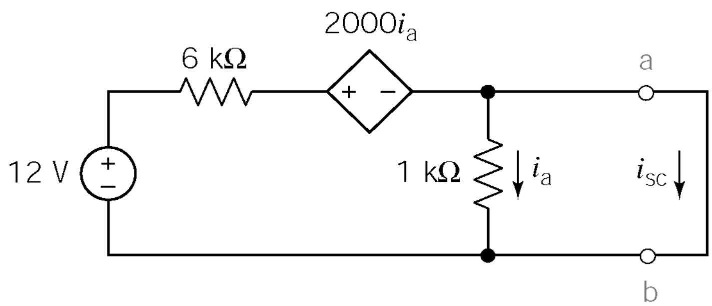


Figure P 5.4-10

Solution:

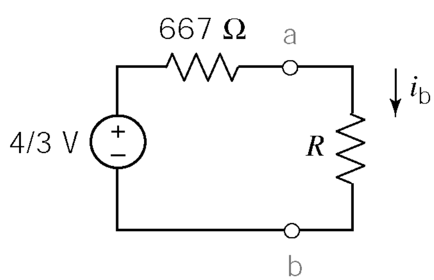


$$-12+6000\,i_a+2000\,i_a+1000\,i_a=0$$
$$i_a = 4/3000 \text{ A}$$
$$v_{oc}=1000\;i_a=\frac{4}{3}\text{ V}$$



i_a = 0 due to the short circuit

$$-12+6000\,i_{sc}=0 \;\Rightarrow\; i_{sc}=2\text{ mA}$$
$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{\frac{4}{3}}{.002}=667\ \Omega$$



$$i_b=\frac{\frac{4}{3}}{667+R}$$

i_b = 0.002 A requires

$$R=\frac{\frac{4}{3}}{0.002}-667=0$$

P 5.4-12 The circuit shown in Figure P 5.4-12 contains an adjustable resistor. The resistance R can be set to any value in the range $0 \le R \le 100$ kΩ.

(a) Determine the maximum value of the current i_a that can be obtained by adjusting R. Determine the corresponding value of R.

(b) Determine the maximum value of the voltage v_a that can be obtained by adjusting R. Determine the corresponding value of R.

(c) Determine the maximum value of the power supplied to the adjustable resistor that can be obtained by adjusting R. Determine the corresponding value of R.

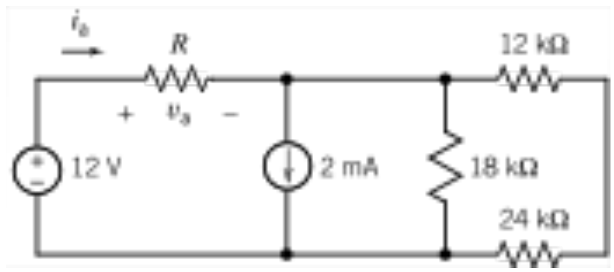
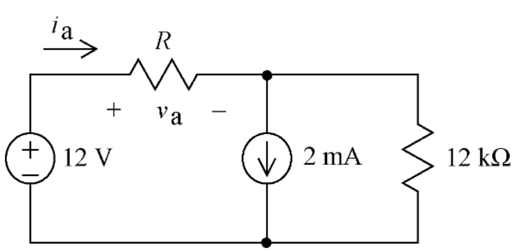
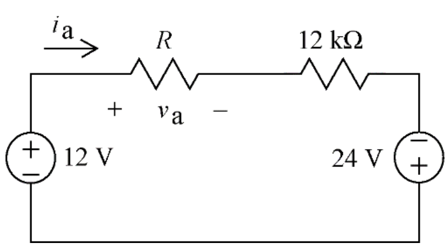


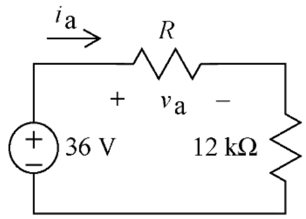
Figure P 5.4-12

Solution: Replace the part of the circuit that is connected to the variable resistor by its Thevenin equivalent circuit:



$$18\ \text{k}\Omega \,\|\, (12\ \text{k}\Omega + 24\ \text{k}\Omega) = 18\ \text{k}\Omega \,\|\, 36\ \text{k}\Omega = 12\ \text{k}\Omega$$





$$i_a = \frac{36}{R+12000} \quad \text{and} \quad v_a = \frac{R}{R+12000}36$$

$$p = i_a\, v_a = \left(\frac{36}{R+12000}\right)^2 R$$

(a) $i_a = \dfrac{36}{0+12000} = 3 \text{ mA}$ when R = 0 Ω (a short circuit).

(b) $v_a = \dfrac{10^5}{10^5+12000}36 = 32.14 \text{ V}$ when R is as large as possible, i.e. R = 100 kΩ.

(c) Maximum power is delivered to the adjustable resistor when $R = R_t = 12 \text{ k}\Omega$. Then

$$p = i_a\, v_a = \left(\frac{36}{12000+12000}\right)^2 12000 = 0.027 = 27 \text{ mW}$$

P 5.4-14 The circuit shown in Figure P 5.4-14 contains an unspecified resistance, R. Determine the value of R in each of the following two ways.

(a) Write and solve mesh equations.

(b) Replace the part of the circuit connected to the resistor R by a Thévenin equivalent circuit. Analyze the resulting circuit.

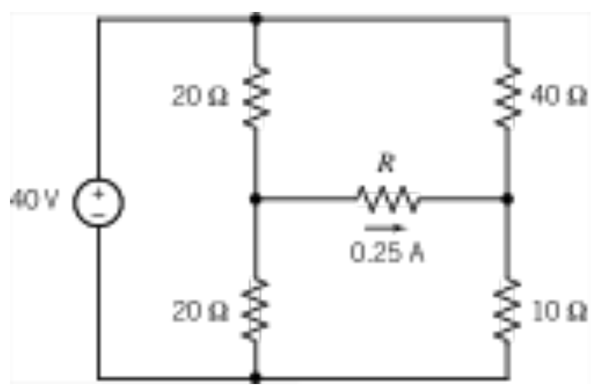
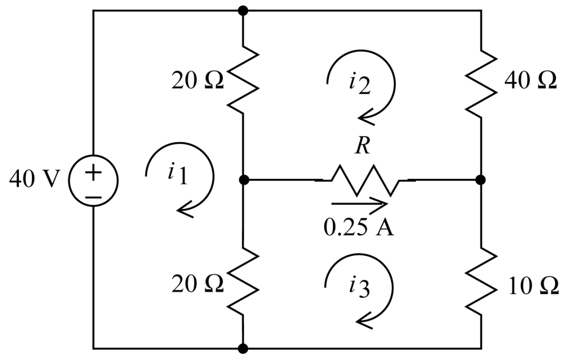


Figure P 5.4-14

Solution:
(a)



$$i_3 - i_2 = 0.25 \text{ A}$$

Apply KVL to mesh 1 to get

$$20\left(i_1 - i_2\right) + 20\left(i_1 - i_3\right) - 40 = 0$$

Apply KVL to the supermesh corresponding to the unspecified resistance to get

$$40i_2 + 10i_3 - 20\left(i_1 - i_3\right) - 20\left(i_1 - i_2\right) = 0$$

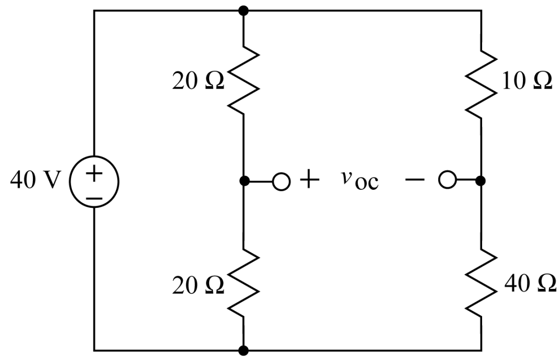
Solving, for example using MATLAB, gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 40 & -20 & -20 \\ -40 & 60 & 30 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 40 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 0.750 \\ 1.000 \end{bmatrix}$$

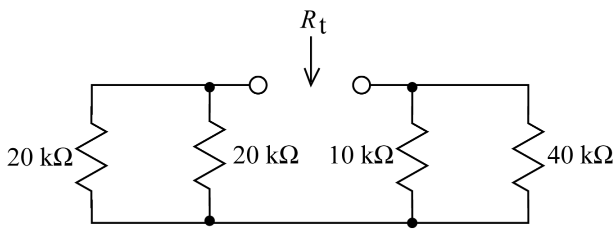
Apply KVL to mesh 2 to get

$$40i_2 + R\left(i_2 - i_3\right) - 20\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad R = \frac{20\left(i_1 - i_2\right) - 40i_2}{i_2 - i_3} = 30\ \Omega$$

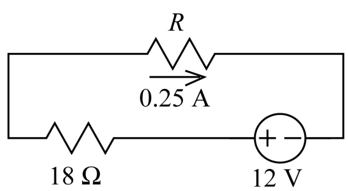
(b)



$$v_{oc} = \left(\frac{20}{20+20}\right)40 - \left(\frac{40}{10+40}\right)40 = -12 \text{ V}$$

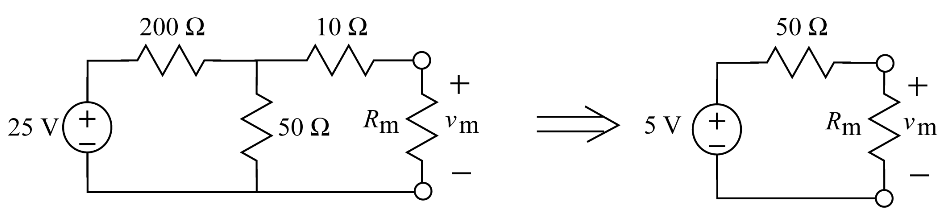


$$R_t = 18\ \Omega$$



$$0.25 = \frac{12}{18+R} \quad \Rightarrow \quad R = 30\ \Omega$$

Solution: Replace the circuit by its Thevenin equivalent circuit:



$$v_m = \left(\frac{R_m}{R_m+50}\right)5$$

(a)
$$v_{\text{mi}} = \lim_{R_{\text{m}}\to\infty} v_{\text{m}} = 5\text{ V}$$

(b) When $R_{\text{m}} = 1000\ \Omega,\ v_{\text{m}} = 4.763\text{ V}$ so

$$\%\text{ error} = \frac{5-4.762}{5}\times 100 = 4.76\%$$

(c)
$$0.02 \ge \frac{5-\left(\dfrac{R_{\text{m}}}{R_{\text{m}}+50}\right)5}{5} \quad\Rightarrow\quad \frac{R_{\text{m}}}{R_{\text{m}}+50} \ge 0.98 \quad\Rightarrow\quad R_{\text{m}} \ge 2450\ \Omega$$

P 5.4-16 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 5.4-16*a* shows a circuit with a voltmeter that measures the voltage v_m. In Figure P 5.4-16*b* the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. The voltmeter measures v_{mi}, the ideal value of v_m.

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi}$. When $R_m < \infty$, the voltmeter is not ideal and $v_m > v_{mi}$. The difference between v_m and v_{mi} is a measurement error caused by the fact that the voltmeter is not ideal.

(a) Determine the value of v_{mi}.

(b) Express the measurement error that occurs when $R_m = 1000\ \Omega$ as a percentage of v_{mi}.

(c) Determine the minimum value of R_m required to ensure that the measurement error is smaller than 2 percent of v_{mi}.

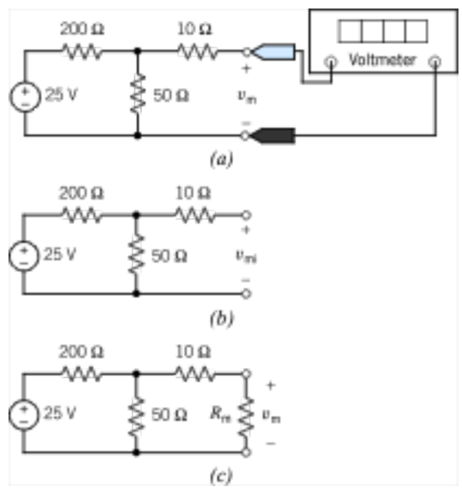


Figure P 5.4-16

P 5.5-7 Determine the value of the resistance R in the circuit shown in Figure P 5.5-7 by each of the following methods:

(a) Replace the part of the circuit to the left of terminals a–b by its the Norton equivalent circuit. Use current division to determine the value of R.

(b) Analyze the circuit shown Figure P 5.5-6 using mesh equations. Solve the mesh equations to determine the value of R.

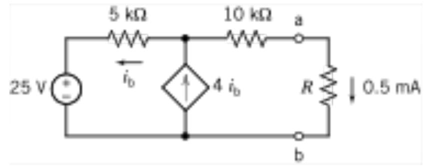
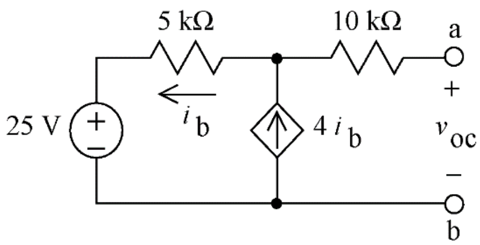


Figure P 5.5-7

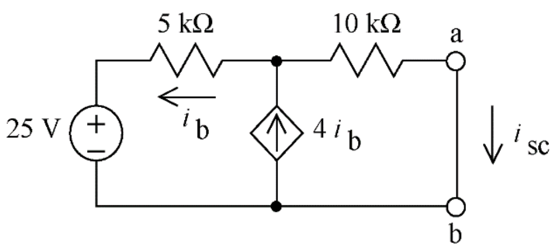
Solution: (a) Replace the part of the circuit that is connected to the left of terminals a-b by its Norton equivalent circuit:



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A. Then

$$v_{oc} = 25 + 5000\left(i_b\right) = 25 \text{ V}$$

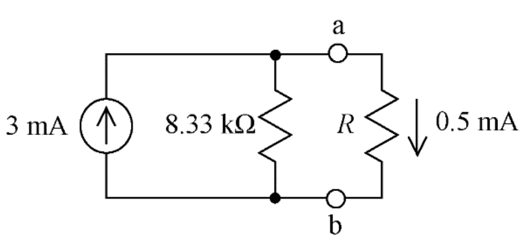
Apply KVL to the supermesh corresponding to the dependent source to get



$$-5000\, i_b + 10000\left(3\, i_b\right) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$



Then

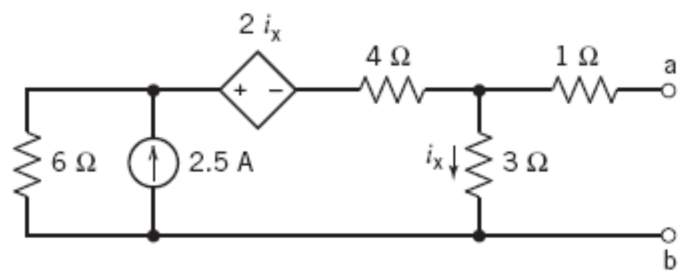
$$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\overline{3} \text{ k}\Omega$$

Current division gives

$$0.5 = \frac{8333}{R + 8333} 3 \quad \Rightarrow \quad R = 41.67 \text{ k}\Omega$$

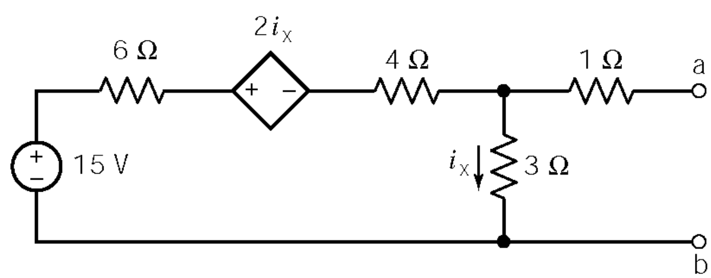
(b)

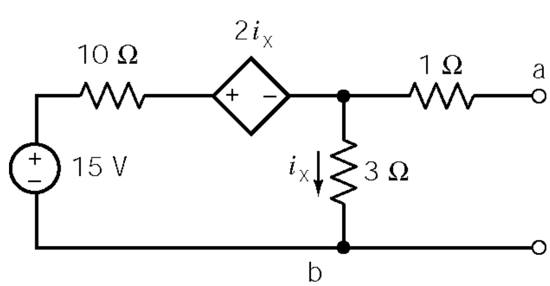
P5.5-8 Find the Norton equivalent circuit of this circuit:



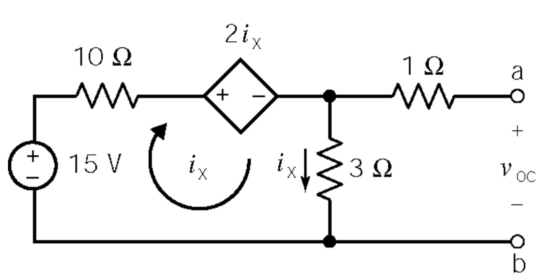
Solution

Simplify the circuit using a source transformation:



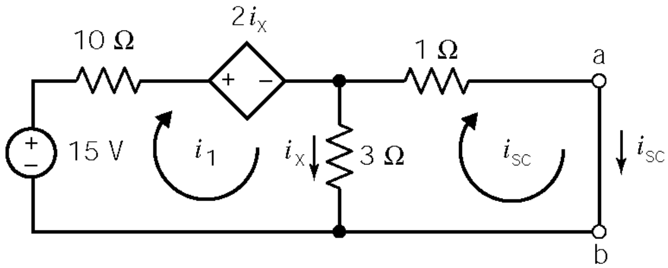


Identify the open circuit voltage and short circuit current.



Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\,i_1 - 5\,i_{sc} = 15$$

and

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\,i_{sc}$$

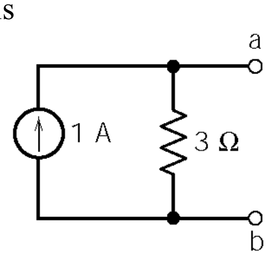
so

$$15\left(\frac{4}{3}i_{sc}\right) - 5\,i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1\text{ A}$$

The Thevenin resistance is

$$R_t = \frac{3}{1} = 3\ \Omega$$

Finally, the Norton equivalent circuit is



P 5.5-11 Determine values of R_t and i_{sc} that cause the circuit shown in Figure P 5.5-11*b* to be the Norton equivalent circuit of the circuit in Figure P 5.5-11*a*.

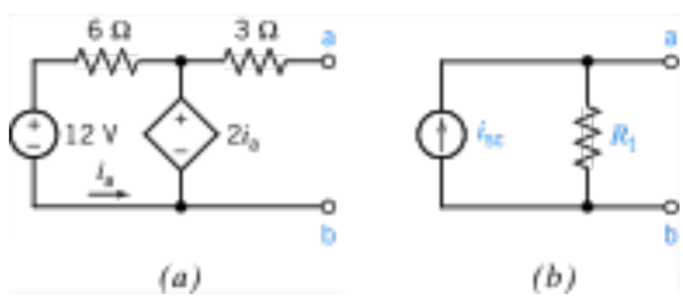
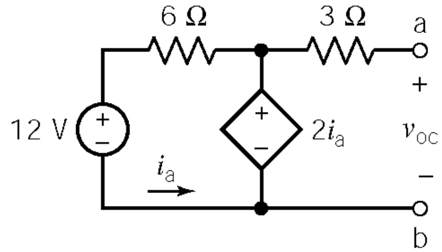


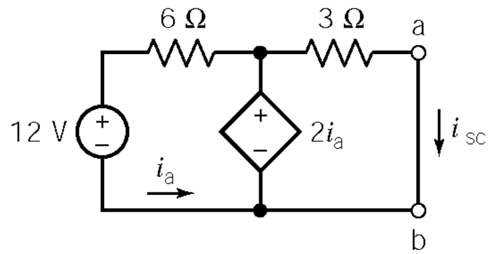
Figure P 5.5-11

Answer: $R_t = 3\ \Omega$ and $i_{sc} = -2$ A

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad\Rightarrow\quad i_a = -3 \text{ A}$$
$$v_{oc} = 2\,i_a = -6 \text{ V}$$



$$12 + 6\,i_a = 2\,i_a \quad\Rightarrow\quad i_a = -3 \text{ A}$$
$$3\,i_{sc} = 2\,i_a \quad\Rightarrow\quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\ \Omega$$

P 5.6-1 The circuit shown in Figure P 5.6-1 consists of two parts separated by a pair of terminals. Consider the part of the circuit to the left of the terminals. The open circuit voltage is v_{oc} = 8V, and the short circuit current is i_{sc} = 2A. Determine the values of

(a) The voltage source voltage, v_s, and the resistance R_2.

(b) The resistance R that maximizes the power delivered to the resistor to the right of the terminals, and the corresponding maximum power

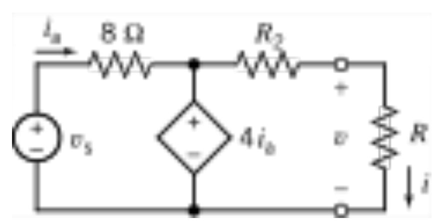
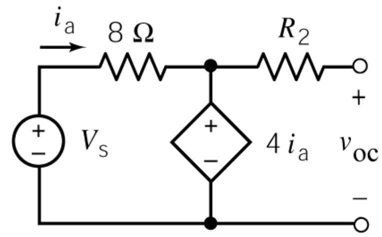


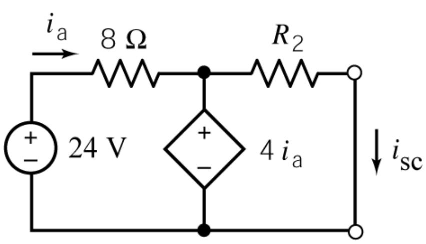
Figure P 5.6-1

Solution:

(a) The value of the current in R_2 is 0 A so $v_{oc} = 4i_a$. Then KVL gives



$$8i_a + 4i_a - V_s = 0 \quad \Rightarrow \quad V_s = 12i_a = 3(4i_a) = 3(v_{oc}) = 24 \text{ V}$$



Next, KVL gives,

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

and

$$4i_a = R_2\, i_{sc} \quad \Rightarrow \quad 4(2) = R_2(2) \quad \Rightarrow \quad R_2 = 4\ \Omega$$

(b) The power delivered to the resistor to the right of the terminals is maximized by setting R equal to the Thevenin resistance of the part of the circuit to the left of the terminals:

$$R = R_t = \frac{v_{oc}}{i_{sc}} = \frac{8}{2} = 4\ \Omega$$

Then

$$p_{max} = \frac{v_{oc}^{\,2}}{4R_t} = \frac{8^2}{4(4)} = 4 \text{ W}$$

P 5.6-5 Determine the maximum power that can be absorbed by a resistor, *R*, connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of *R*.

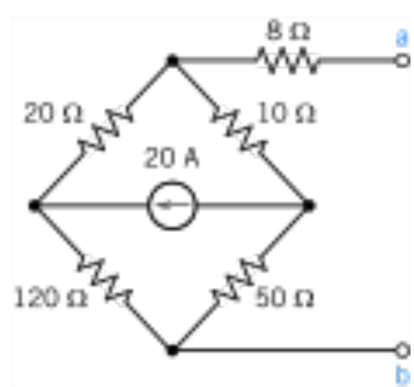
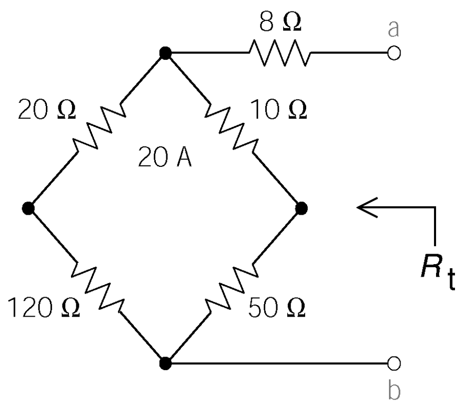
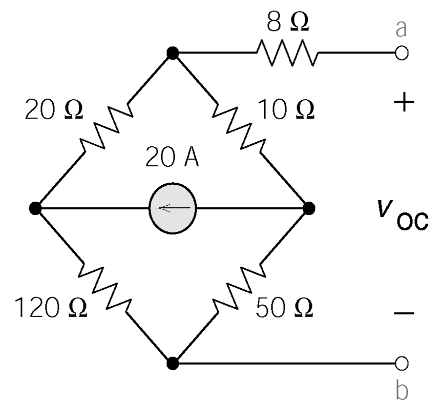


Figure P 5.6-5

Solution:





The required value of R is

$$R = R_t = 8 + \frac{(20+120)(10+50)}{(20+120)+(10+50)} = 50\ \Omega$$

$$v_{oc} = \left[\frac{170}{170+30}(20)\right]10 - \left[\frac{30}{170+30}(20)\right]50$$

$$= \frac{170(20)(10) - 30(20)(50)}{200} = \frac{4000}{200} = 20\ \text{V}$$

The maximum power is given by

$$p_{\max} = \frac{v_{oc}{}^2}{4\,R_t} = \frac{20^2}{4(50)} = 2\ \text{W}$$

P5.6-10 The part circuit shown in Figure P5.6-10*a* to left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P5.6-10*b*, will be characterized by the parameters:

$$i_{\text{sc}} = 1.5\ \text{A} \quad \text{and} \quad R_{\text{t}} = 80\ \Omega$$

(*a*) Determine the values of i_{s} and R_1.

(*b*) Given that $0 \le R_2 \le \infty$, determine the maximum value of $p = vi$, the power delivered to R_2.

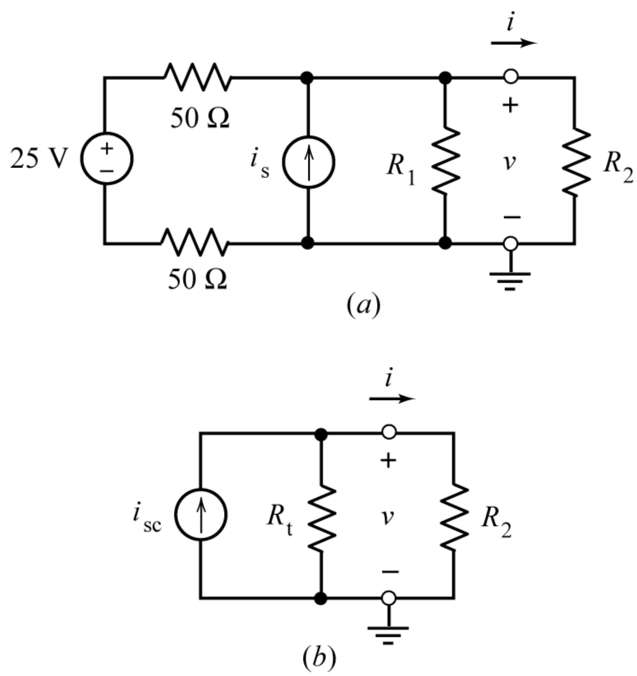
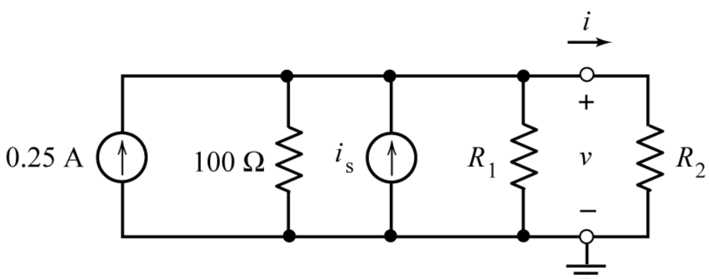
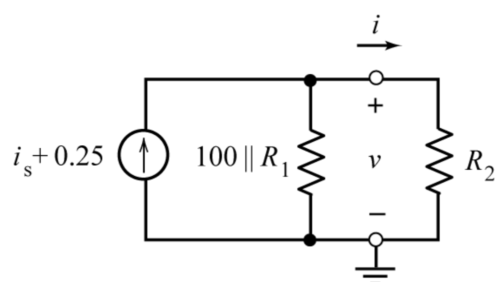


Figure P5.6-10

Solution: Two source transformations reduce the circuit as follows:





(*a*) Recognizing the parameters of the Norton equivalent circuit gives:

$$1.5 = i_{sc} = i_s + 0.25 \quad \Rightarrow \quad i_s = 1.25 \text{ A}$$ and $$80 = R_t = 100 \| R_1 = \frac{100\, R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 400 \ \Omega$$

(*b*) The maximum value of the power delivered to R_2 occurs when $R_2 = R_t = 80 \ \Omega$. Then

$$i = \frac{1}{2} i_{sc} = 0.75 \text{ A}$$ and $$p = \left(\frac{1}{2} i_{sc}\right)^2 R_t = \left(0.625^2\right) 80 = 45 \text{ W}$$