Phys 2B Summer 2022

# Quiz 4 Practice Solutions

Question 1:

The magnetic field of a solenoid is:

$$B = \mu_0 n I$$

Simply plug in numbers.

$$B = (4\pi \times 10^{-7}\,\text{T-m/A})(1300\,\text{m}^{-1})(490\,\text{A}) \approx \boxed{0.80\,\text{T}}$$

This is choice (c).

Question 2:

The orientation energy of a magnetic dipole is:

$$U = -\vec{\mu} \cdot \vec{B}$$

The magnetic dipole in this problem points down, since by the right hand rule a clockwise current viewed from above makes a downward magnetic dipole. That means that it is parallel to the vertical component of the magnetic field (not antiparallel), ie $\vec{\mu} \cdot \vec{B}$ is positive, so $U$ is negative.

Since the orientation energy is a dot product, it cares only about the parallel component of the field, ie the vertical component, so:

$$U = -\mu B_V$$

The magnitude of the dipole is $\mu = AI$. Putting everything together:

$$U = -(AI)B_V = -AIB_V$$

Plug in numbers.

$$U = -(1.3\,\text{m}^2)(2.0\,\text{A})(35 \times 10^{-6}\,\text{T}) = \boxed{-9.1 \times 10^{-5}\,\text{J}}$$

This is choice (c).

Question 3:

Since the field is perpendicular to the velocity and is holding the charge in circular motion:

$$\frac{mv^2}{r} = qvB$$

We want to solve for $m$.

$$m = \frac{qBr}{v}$$

Now plug in numbers.

$$m = \frac{(2 \times 1.6 \times 10^{-19}\,\mathrm{C})(0.20\,\mathrm{T})(0.14\,\mathrm{m})}{44000\,\mathrm{m/s}} \approx \boxed{2.0 \times 10^{-25}\,\mathrm{kg}}$$

This is about 120 proton/neutron masses, so the ion is probably doubly-ionized Tin-120 (very plausible).

This is choice (e).

Question 4:

The force is given by:

$$\vec{F} = q\vec{v} \times \vec{B}$$

The cross product only cares about perpendicular components, so the electron's $v_x$ is irrelevant because the field is purely in the $x$ direction. Since the electron's perpendicular velocity is in the $y$ direction and the field is in the $x$ direction, the cross product will point in the $\hat{y} \times \hat{x} = -\hat{z}$ direc tion.

$$\vec{F} = -qv_yB_x\hat{z}$$

Now plug in numbers.

$$\vec{F} = -(-1.6 \times 10^{-19}\,\mathrm{C})(1.0\,\mathrm{m/s})(1.5\,\mathrm{T})\hat{z} \approx \boxed{(2.4 \times 10^{-19}\,\mathrm{N})\hat{z}}$$

This is choice (d).

Question 5:

Faraday's Law says:

$$\mathcal{E}_{\mathrm{ind}} = -\frac{\mathrm{d}(\vec{B} \cdot \vec{A})}{\mathrm{d}t} = -\frac{\mathrm{d}(BA\cos\theta)}{\mathrm{d}t}$$

In this case, both $B$ and $A$ are constant, but $\cos\theta$ is not.

$$\mathcal{E}_{\mathrm{ind}} = -BA\frac{\mathrm{d}(\cos\theta)}{\mathrm{d}t}$$

$$\mathcal{E}_{\text{ind}} = -BA\left(-\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right) = \omega BA\sin\omega t$$

The question wants the maximum value, which occurs when the sine is 1.

$$\mathcal{E}_{\text{ind,max}} = \omega BA$$

Now plug in numbers.

$$\mathcal{E}_{\text{ind,max}} = (160\,\text{rad/s})(4.0\,\text{T})(\pi(1.2\,\text{m})^2) \approx \boxed{2900\,\text{V}}$$

This is choice (e).

Question 6:

Ampere's Law says:

$$BL = \mu_0 I_{\text{encl}}$$

We want to draw a circular loop, so $L = 2\pi R$. Since the current is uniformly distributed:

$$I_{\text{encl}} = I\frac{A_{\text{encl}}}{A} = I\frac{\pi r^2}{\pi R^2} = I\frac{r^2}{R^2}$$

Plug these in.

$$B(2\pi r) = \mu_0\left(I\frac{r^2}{R^2}\right)$$

$$B = 2\frac{\mu_0}{4\pi}\frac{r}{R^2}I$$

Plug in numbers.

$$B = 2(10^{-7}\,\text{T-m/A})\frac{0.0022\,\text{m}}{(0.0056\,\text{m})^2}(85\,\text{A}) \approx 1.2\times10^{-3}\,\text{T}$$

To be concrete, let's imagine that the conventional current is flowing into the page. Then this field circles clockwise, and electrons are moving out of the page.

Again to be concrete, consider an electron toward the top of the wire. Here, the magnetic field points to the right, so $\vec{v}\times\vec{B}$ points up, *away from the center of the wire.*

$$\vec{F} = q\vec{v}\times\vec{B}$$

Since the electron has a negative charge, the force instead points down, *toward the center of the wire*, meaning in the $-\hat{r}$ direction. We would get the same result for a positive charge with the opposite velocity, and if we considered charges on the right, or left, or bottom of the wire. There's always a force toward the center. Now plug in numbers.

$$\vec{F} \approx (1.6\times10^{-19}\,\mathrm{C})(4.5\times10^{-3}\,\mathrm{m/s})(1.2\times10^{-3}\,\mathrm{T})(-\hat{r}) \approx \boxed{8.6\times10^{-25}\,\mathrm{N\,toward\,the\,center}}$$

This is choice (b).

So, why don't direct currents cluster near the center of wires? The answer is: Like charges repel each other electrically. The magnetic force wants the charges to cluster near the center, but the electric force wants the charges to spread apart as much as possible. For realistic direct currents, the electric effect is powerful enough to prevent the magnetic clustering effect. (Test it out yourself. What's the electric repulsion between two electrons that are separated by $\sim 1\,\mathrm{mm}$? Compare that to the magnetic force above.)

Question 7:

First, combine the inductances into a single equivalent inductance. The inductors are in series, so they just add:

$$L_{\mathrm{eq}} = L_1 + L_2 = 10\,\mathrm{mH} + 3.3\,\mathrm{mH} = 13.3\,\mathrm{mH}$$

The angular frequency of an LC circuit is:

$$\omega = \frac{1}{\sqrt{LC}}$$

Plug in numbers.

$$\omega = \frac{1}{\sqrt{(13.3\times10^{-3}\,\mathrm{H})(3.0\times10^{-6}\,\mathrm{F})}} \approx \boxed{5000\,\mathrm{rad/s}}$$

This is choice (d).

Question 8:

The total energy in an $LC$ circuit is:

$$U = U_C + U_L = \frac{1}{2}\frac{1}{C}Q^2 + \frac{1}{2}LI^2$$

We want to solve for $L$.

$$2U - \frac{Q^2}{C} = LI^2$$

$$L = \frac{2U - Q^2/C}{I^2}$$

Now plug in numbers.

$$L = \frac{2(4.0\,\mathrm{J}) - (0.018\,\mathrm{C})^2/(45\times10^{-6}\,\mathrm{F})}{(97\,\mathrm{A})^2} \approx \boxed{8.5\times10^{-5}\,\mathrm{H}}$$

This is choice (b).

Question 9:

The absolute value of the current in an $LC$ circuit that begins with the capacitor maximally charged is:

$$|I(t)| = |Q(0)\omega\sin(\omega t)|$$

where $\omega = 1/\sqrt{LC}$. Simply plug in numbers.

$$|I(t)| \approx (220\times10^{-9}\,\mathrm{C})\left(\frac{1}{\sqrt{(44\times10^{-12}\,\mathrm{H})(86\times10^{-9}\,\mathrm{F})}}\right)\sin\left[\left(\frac{1}{\sqrt{(44\times10^{-12}\,\mathrm{H})(86\times10^{-9}\,\mathrm{F})}}\right)(2.9\,\mathrm{s})\right]$$

$$I(t) \approx \boxed{36\,\mathrm{A}}$$

If you solve for $\omega$ first and then plug it in, make sure to keep many, many digits. The sine depends very sharply on the precise value of $\omega$, so a rounded value can lead to a very different answer than the correct one.

This is choice (c).

Question 10:

The central equation of a transformer is:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

where the primary is the input and the secondary is the output. In this case:

$$\frac{V_P}{V_S} = \frac{6}{500}$$

Any coil setup with $N_P/N_S = 6/500$ works. Of the options given, only choice (c), $\boxed{120/10000 = 12/1000 = 6/500}$, is valid.