CSE 21 Fall 2022
Homework 1
Due date: Thursday Sept 29 at 11:59 PM

INSTRUCTIONS

This homework should be done in groups of up to 4, without assistance from anyone besides the instructional staff and your group members. Homework must be submitted through Gradescope by a **single representative** of your group and received by **11:59pm** on the due date. There are no exceptions to this rule.

You will be able to look at your scanned work before submitting it. You must **type** your solutions. (hand-drawn diagrams are okay.) Your group representative can resubmit your assignment as many times as you like before the deadline. Only the most recent submission will be graded.

Students should consult their textbook, class notes, lecture slides, podcasts, group members, instructors, TAs, and tutors when they need help with homework. You may ask questions about the homework in office hours, but questions on Piazza should be private, visible only to instructors.
As a calibration assignment, this assignment and this only is mainly graded based on good-faith effort. In the future, your work in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. When asked to explain or justify, present clearly how you arrived at your conclusions and justify the correctness of your answers with mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to **convince the reader** that your results and methods are sound. We will give a hypothetical grade on this assignment in comments, based on all of these criteria, but will not make a record of this grade.

KEY CONCEPTS Induction, invariants, case analysis, logical reasoning.

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factorials-20 points The factorial function is written : $n!$ and is the product of all integers from 1 to n. Give a formula for $\sum_{i=1}^{i=n} i * (i!)$ and prove your answer correct.

(Hint: make some tables of relevant quantities for small values of i. For example, $i!$ has values $1, 2, 6, 24, 120, 720...$ $i * i!$ has values $1, 4, 18, 96, 600, 5020....$ The sum will have values $1, 5, 23....$ Use this to guess the right formula. Then use induction to prove it.)

Domino tilings- 40 points total

1. (10 pts) Consider putting down 2×1 dominos on an $n \times n$ chess board where each half of a domino exactly fills up one square of the chess board. You want to have no dominos overlap and have them fill up the entire board. For which values of n can you do this? Explain your answer carefully.
2. (10 pts) More generally, when can we cover an $n \times m$ board? Explain your answer carefully.
3. (20 pts) Now say we've removed the upper left and lower right squares of the chess board from an $n \times n$ board, for $n \geq 3$. For which values of n can we cover the remaining squares?

   (Hint: think about both the number of squares and the number of red vs. black squares)

jelly beans-20 points Say that I have a jar of red and green jelly beans with a total of 71 red beans and 100 green beans to start. I have a habit of picking two jelly beans from the jar, and if they have the same color, I eat both, but if they are different colors, I put them both back. I stop doing this when there aren't two of the same color in the jar. What is in the jar when I stop? Explain your answer.

elevators- 20 points A building has n floors, and the elevator started this morning on floor 1 and ended the day on floor k. For $1 \leq i \leq n-1$, let Up_i be the number of times the elevator went up from floor i to floor $i+1$ and $Down_i$ be the number of times the elevator went down from floor $i+1$ to floor i

(not necessarily stopping at either floor. So say the elevator went from floor 1 to floor 4, from floor 4 to floor 2, and from floor 2 to floor 5. Then we would have gone from 1 to 2 once, from 2 to 3 in both the first and last trip, from 3 to 4 in both the first and last, and from 4 to 5 in the last, so $Up_1 = 1, Up_2 = Up_3 = 2, Up_4 = 1$, and $Down_1 = 0, Down_2 = 1, Down_3 = 1, Down_4 = 0$) Give a formula for $Up_i - Down_i$ and prove that your formula is correct.