CSE 152A: Computer Vision

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Lecture 9: Two-View Reconstruction

Overall goals for the course

- Introduce fundamental concepts in computer vision
- Enable one or all of several such outcomes
  - Pursue higher studies in computer vision
  - Join industry to do cutting-edge work in computer vision
  - Gain appreciation of modern computer vision technologies
- Engage in discussions and interaction
- This is a great time to study computer vision!

Course Details

Course details

- Class webpage:
  - https://cseweb.ucsd.edu/~mkchandraker/classes/CSE152A/Winter2024/
- Instructor email:
  - mkchandraker@ucsd.edu
- Grading
  - 35% final exam
  - 40% homework assignments
  - 20% mid-term
  - 5% self-study exercise
  - Ungraded quizzes
- Aim is to learn together, discuss and have fun!

Course details

- TAs
  - Nicholas Chua: nchua@ucsd.edu
  - Tarun Kalluri: sskallur@ucsd.edu
  - Sreyas Ravichandran: srravichandran@ucsd.edu
- Tutors
  - Kun Wang, Kevin Chan, Zixian Wang: {kuw010, tsc003, ziw081}@ucsd.edu
- Discussion section: M 3-3:50pm
- TA office hours and tutor hours to be posted on webpage
- Piazza for questions and discussions:
  - https://piazza.com/ucsd/winter2024/cse152a

Self-Study Assignment

- Pick a technology area primarily driven by computer vision
  - Can pick one of these suggestions, or use anything else that you like

- **Virtual Reality**
  - Meta Quest Pro
  - Oculus Rift
- **Augmented Reality**
  - Microsoft Hololens
  - Magic Leap 2
- **Self-Driving**
  - Waymo
  - Tesla
- **Content Creation**
  - Adobe Photoshop
  - OpenAI Dall-E
- **Cloud Services**
  - Amazon Rekognition
  - Microsoft Azure Cognitive Services

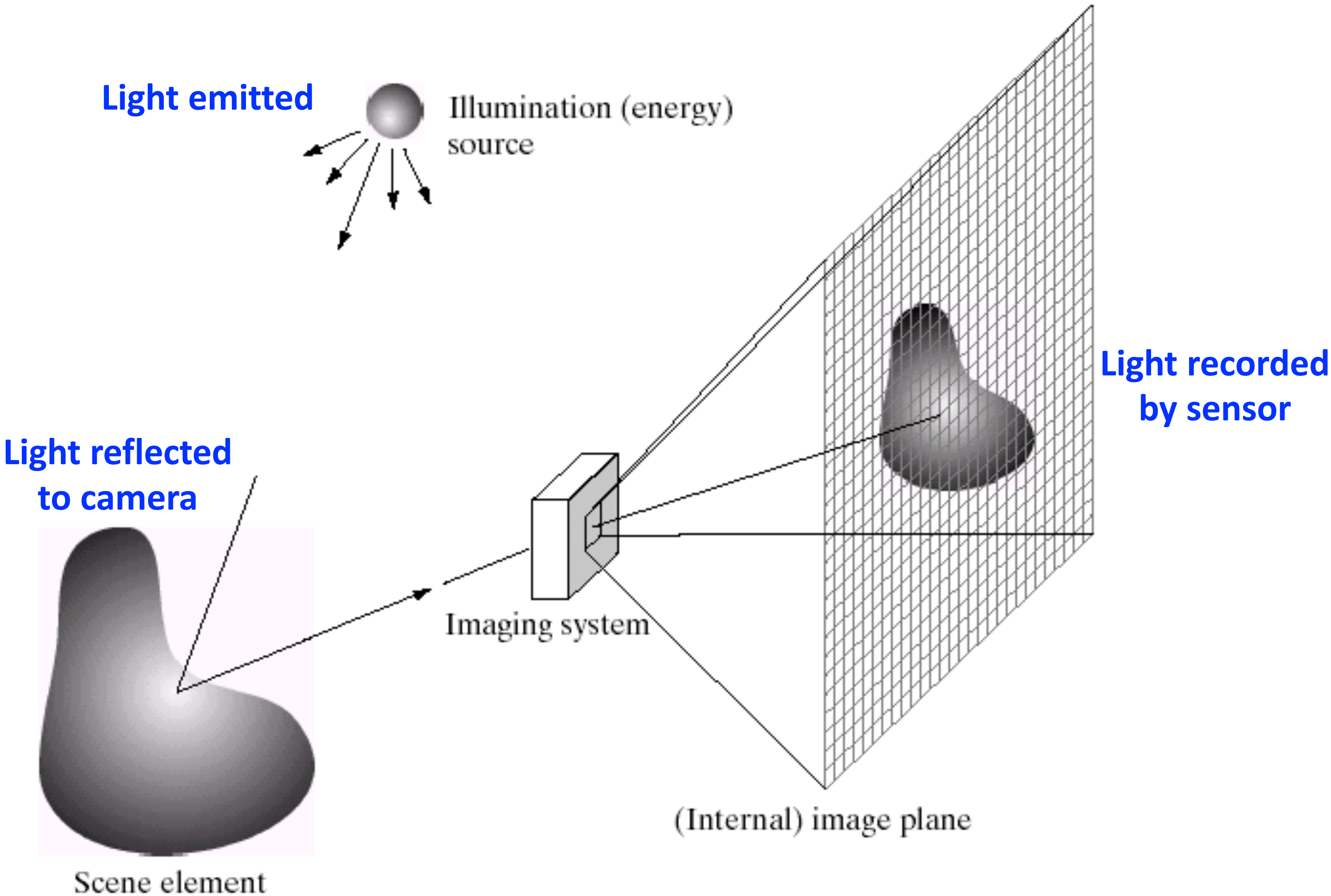
- **Sports**
  - Hawk-Eye
  - Gameface.ai
- **Face Recognition**
  - Face++
  - Apple FaceID
- **Robotics**
  - Boston Dynamics
  - iRobot Roomba
- **Space Exploration**
  - James Webb Telescope
  - Mars Rover
- **Social Media**
  - Snap
  - Instagram

Self-Study Assignment

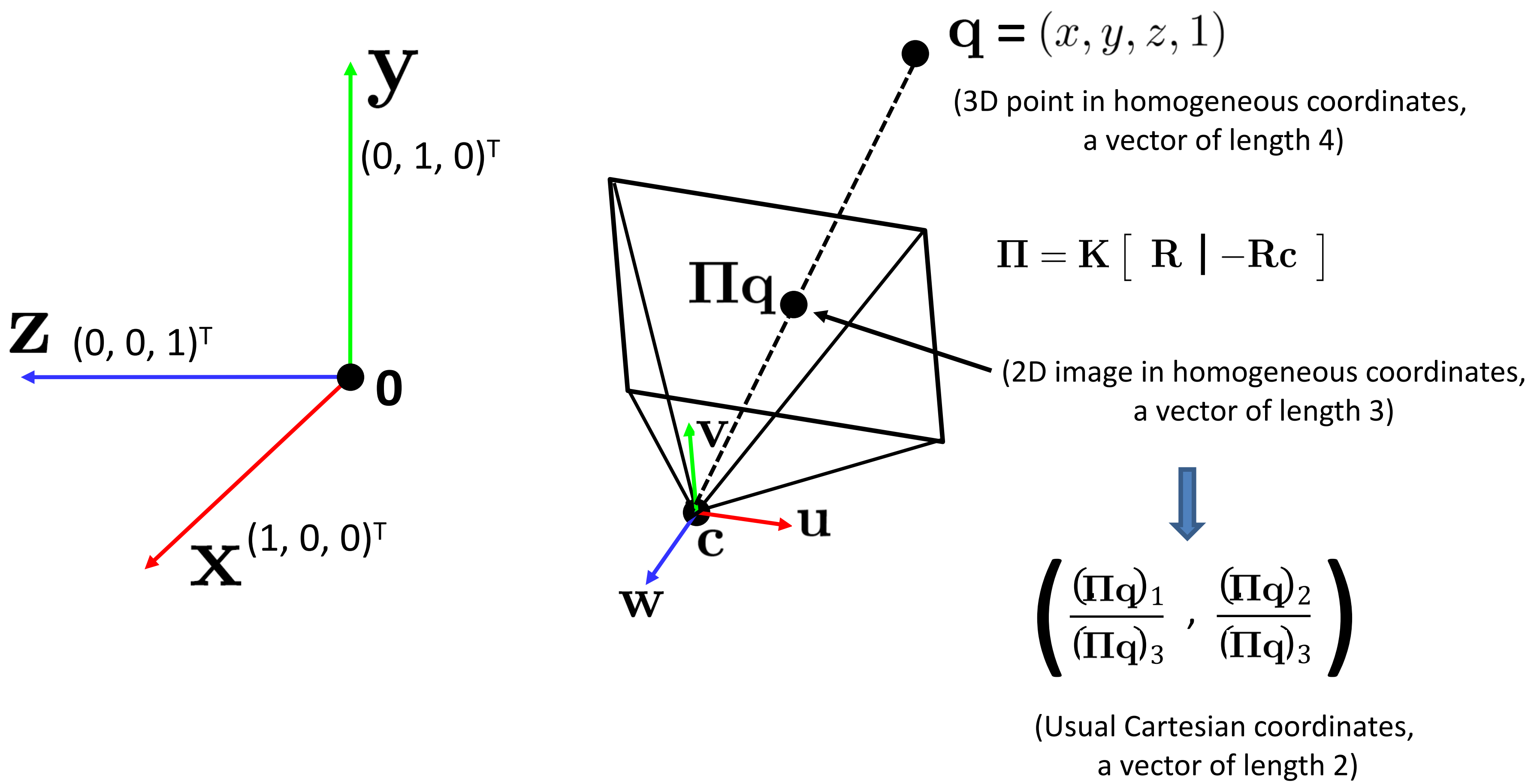
- Form teams of 4 students (mandatory, cannot be less than 4)
- Pick a technology area primarily driven by computer vision
  - Can pick one of these suggestions, or use anything else that you like
- Make a 5-slide PPT report
  - Include pictures (with citations), brief text bullet points or captions
- Prompts for each slide
  - Slide 1: Title and team members
  - Slide 2: Describe the technology and the abilities it enables
  - Slide 3: How does computer vision overcome barriers or solve needs in this technology?
  - Slide 4: How do you anticipate technology in this area will advance in the next 10 years?
  - Slide 5: What are the potential benefits and dangers from this technology in the future?
- Due date: Mar 4, 2023
- Students and instructors will vote for the top-5 studies by Mar 9
  - Top-5 studies may be presented in-class by the teams during Mar 15 lecture

Recap

Photometric: Modeling appearance

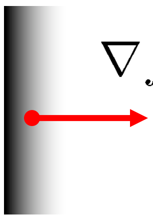


Geometric: Modeling projection

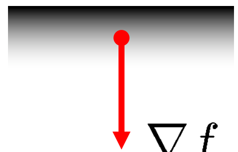


Edge Detection with Image Gradients

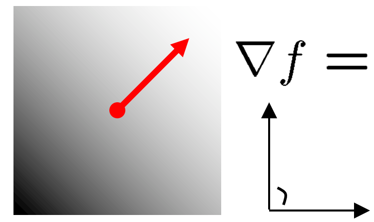
- Gradient represents direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient encodes *edge strength* and *edge direction* as

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \qquad \theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- Can efficiently compute gradient using convolutions

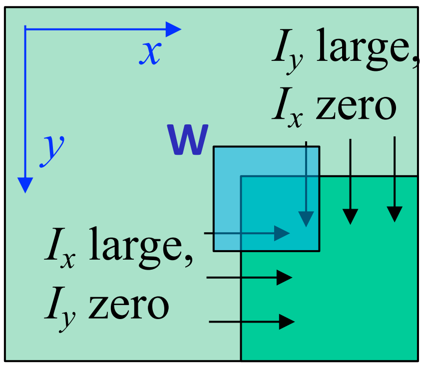
$$\boldsymbol{K}_x = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{K}_y = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

- Sobel operator is often used in practice

$$\boldsymbol{K}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} \qquad \boldsymbol{K}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Harris Corner Detector

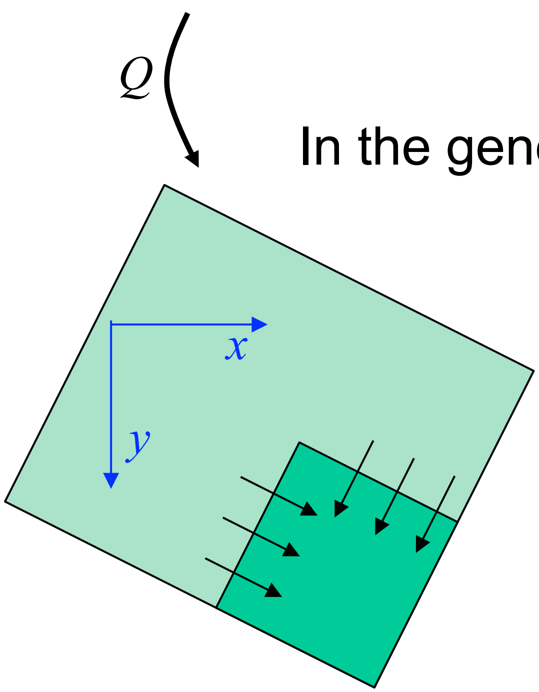
First, consider the second moment matrix for a simpler case:



Sum over a small window W around hypothetical corner

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis.



In the general case, since C is symmetric, it can be shown:

$$C = Q^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} Q$$

Rotation

Eigenvalues

If either λ close to 0, then **not** a corner, so seek locations where both large.

Simple matching methods

- SSD (Sum of Squared Differences)

$$\sum_{x,y} |W_1(x,y) - W_2(x,y)|^2$$

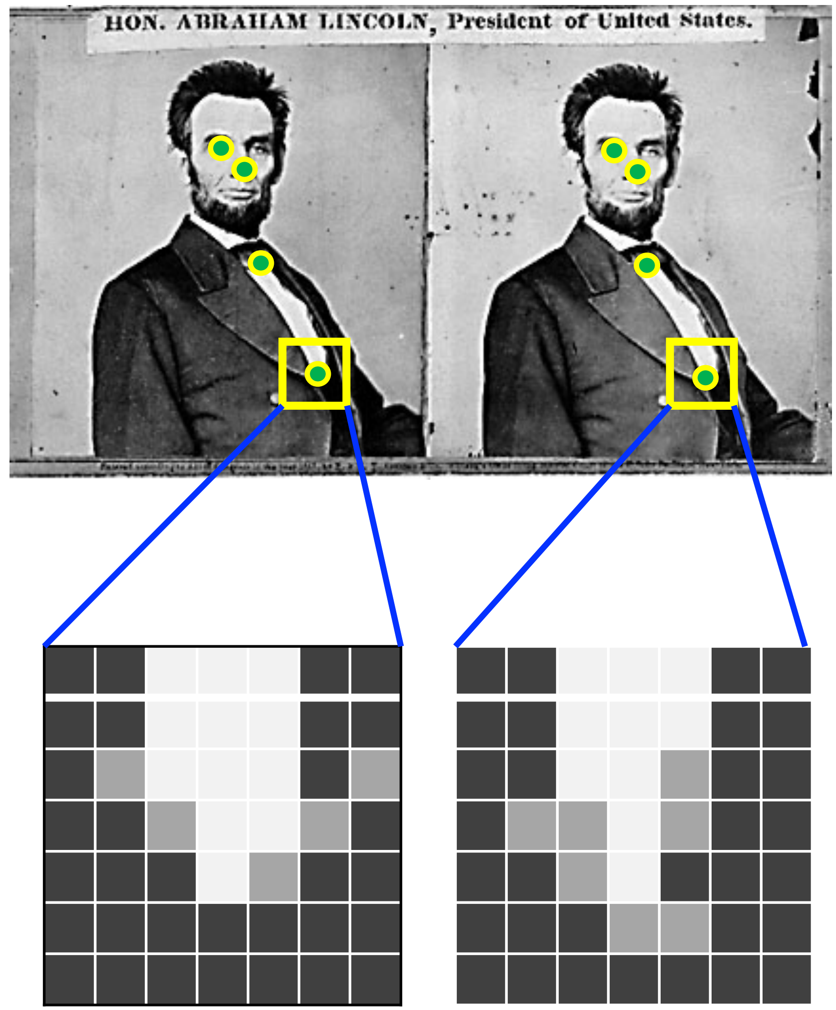
- NCC (Normalized Cross Correlation)

$$\sum_{x,y} \frac{(W_1(x,y) - \overline{W_1})(W_2(x,y) - \overline{W_2})}{\sigma_{W_1}\sigma_{W_2}}$$

$$\overline{W_i} = \frac{1}{n}\sum_{x,y} W_i \,, \quad \sigma_{W_i} = \sqrt{\frac{1}{n}\sum_{x,y}(W_i - \overline{W_i})^2}$$

(Mean) (Standard deviation)

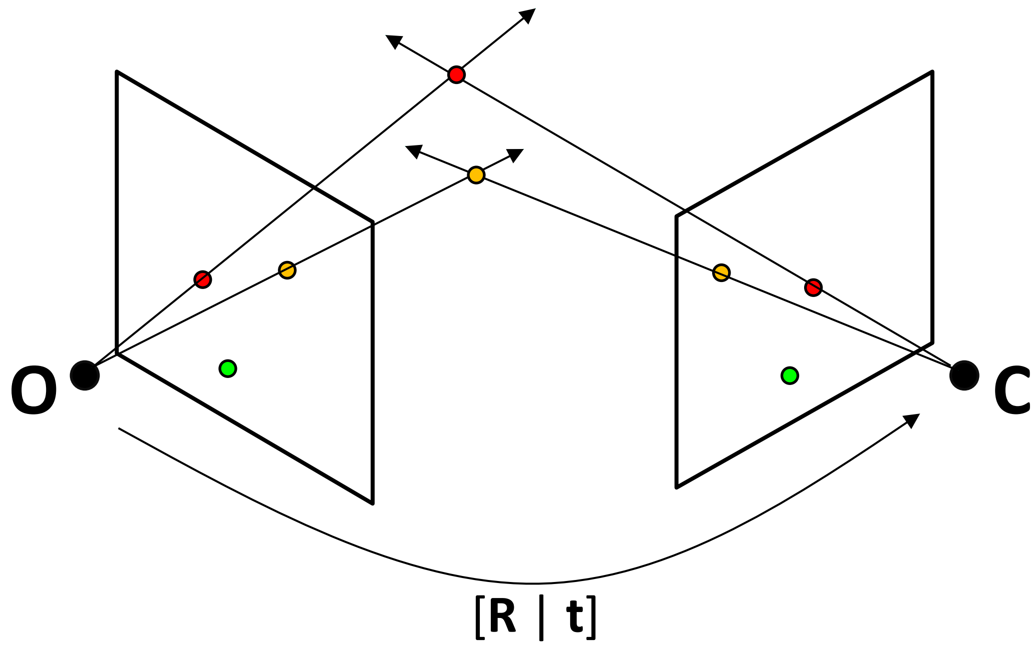
- What advantages might NCC have over SSD?



$W_1(x, y)$: k x k pixel patch in image 1

$W_2(x, y)$: k x k pixel patch in image 2

Two-View Reconstruction: Overall Idea



Step 1: Detect features in each view

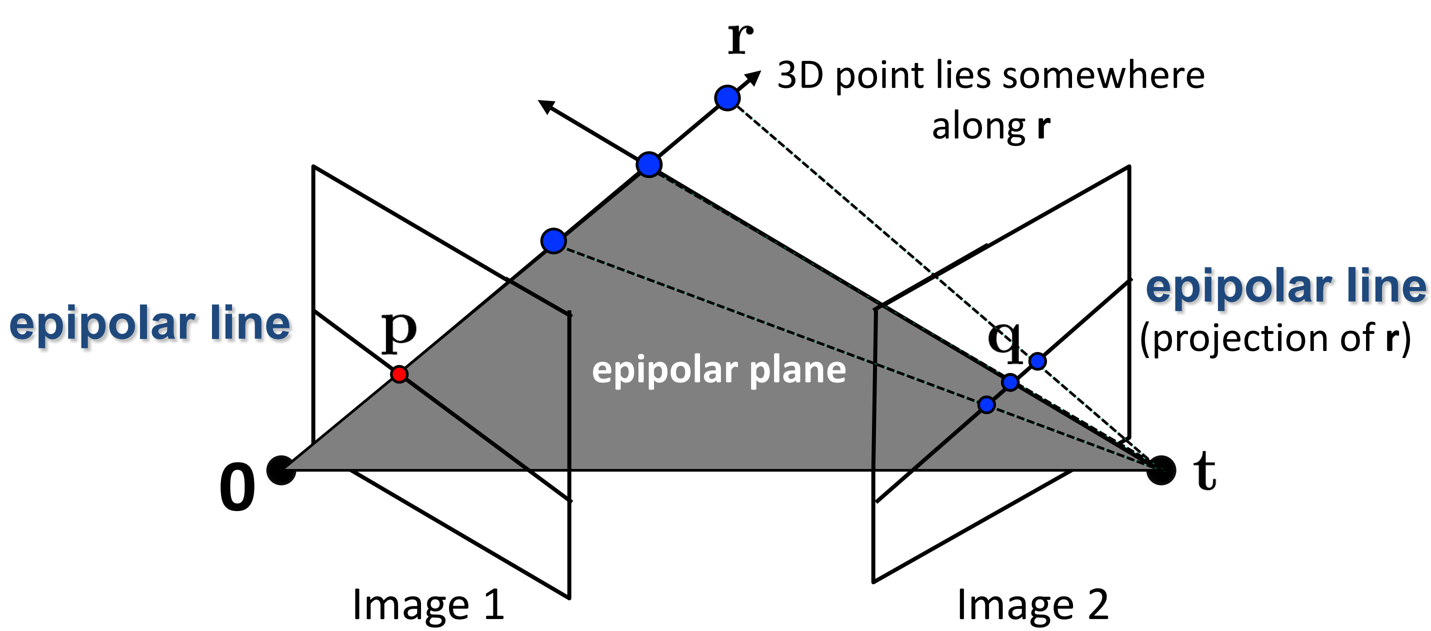
Step 2: Match features across two views

Step 3: Estimate camera rotation and translation across views

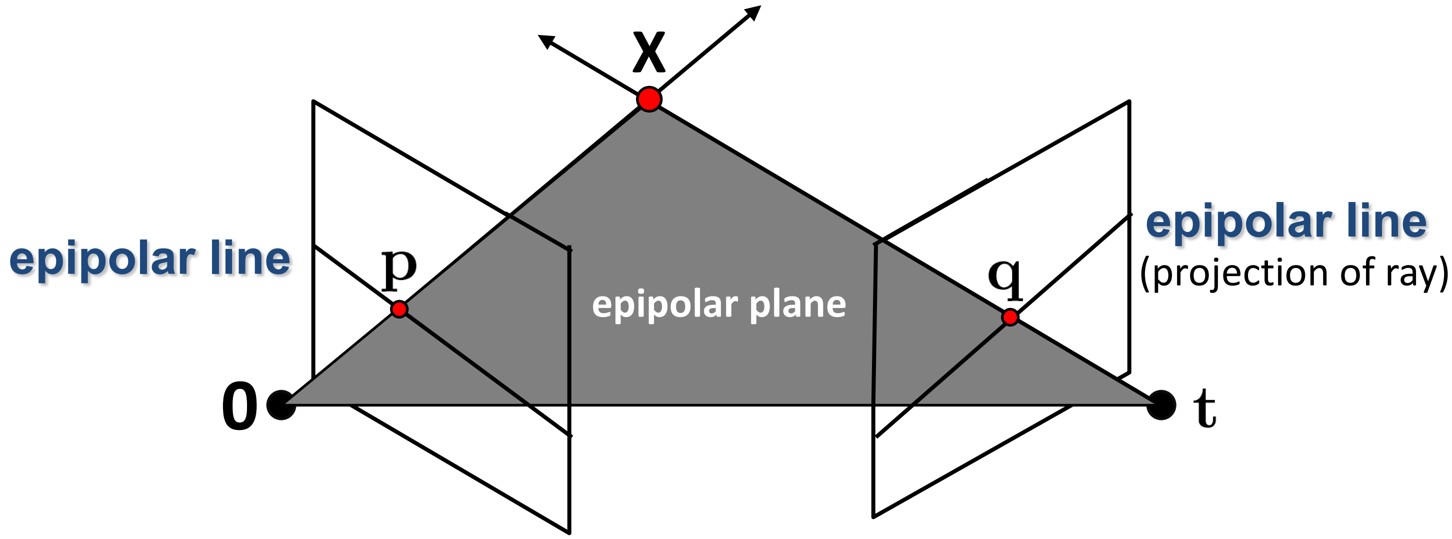
Step 4: Backproject rays from camera centers to triangulate 3D point

Two-view geometry

Corresponding point in other image is constrained to lie on a line, called the *epipolar line*.



Essential matrix



- We have: $\mathbf{q}^\top [\mathbf{t}]_\times \mathbf{R}\mathbf{p} = 0$
- Define:

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$$

Essential matrix

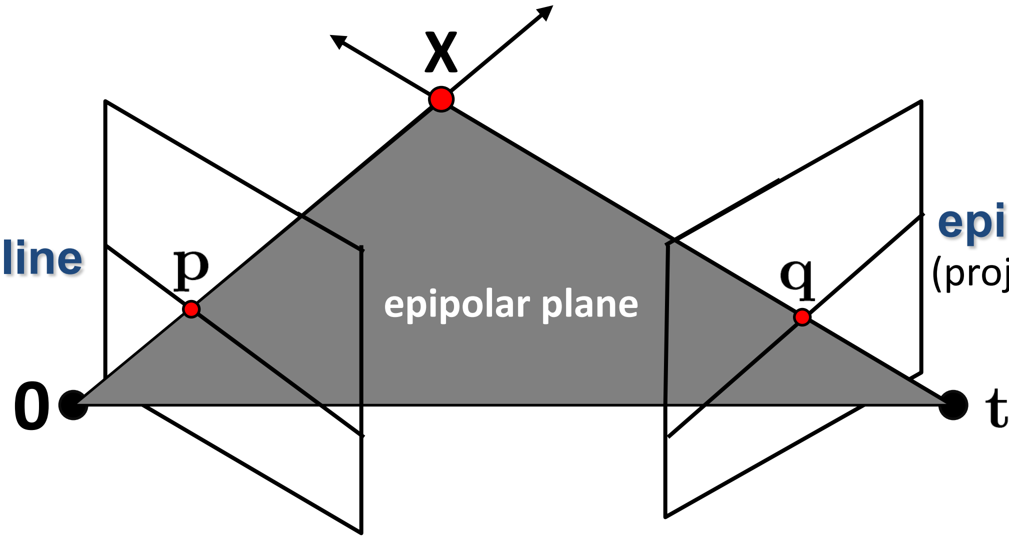
- Then, we have:

$$\boxed{\mathbf{q}^\top \mathbf{E}\mathbf{p} = 0}$$

How many degrees of freedom does **E** have?

3 variables for rotation, 3 for translation, 1 less for scale

Fundamental matrix



- Essential matrix constraint in pixel space: $(\mathbf{K}_2^{-1}\mathbf{q}')^\top \mathbf{E}(\mathbf{K}_1^{-1}\mathbf{p}') = 0$.
- Rearranging:

$$\mathbf{q}'^\top (\mathbf{K}_2^{-\top}\mathbf{E}\mathbf{K}_1^{-1})\mathbf{p}' = 0$$

- Define: $\mathbf{F} = \mathbf{K}_2^{-\top}\mathbf{E}\mathbf{K}_1^{-1}$

Fundamental matrix

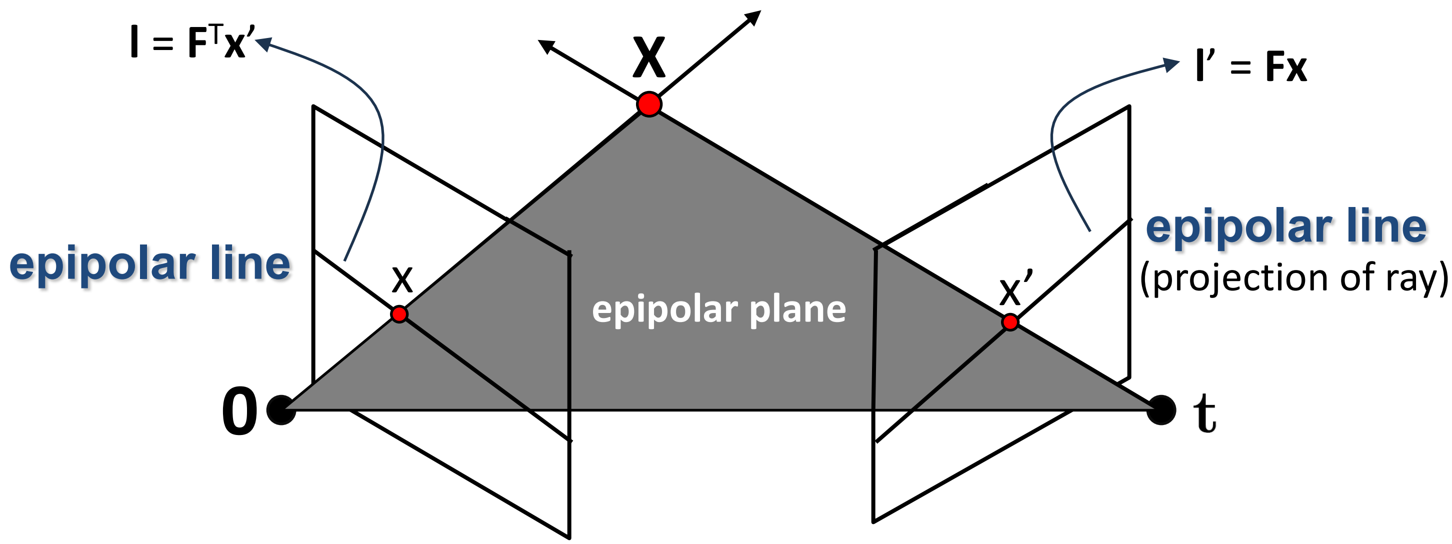
- Then, we have:

$$\boxed{\mathbf{q}'^\top \mathbf{F}\mathbf{p}' = 0}$$

How many degrees of freedom does **F** have?

9 variables, 1 less for rank-constraint, 1 less for scale

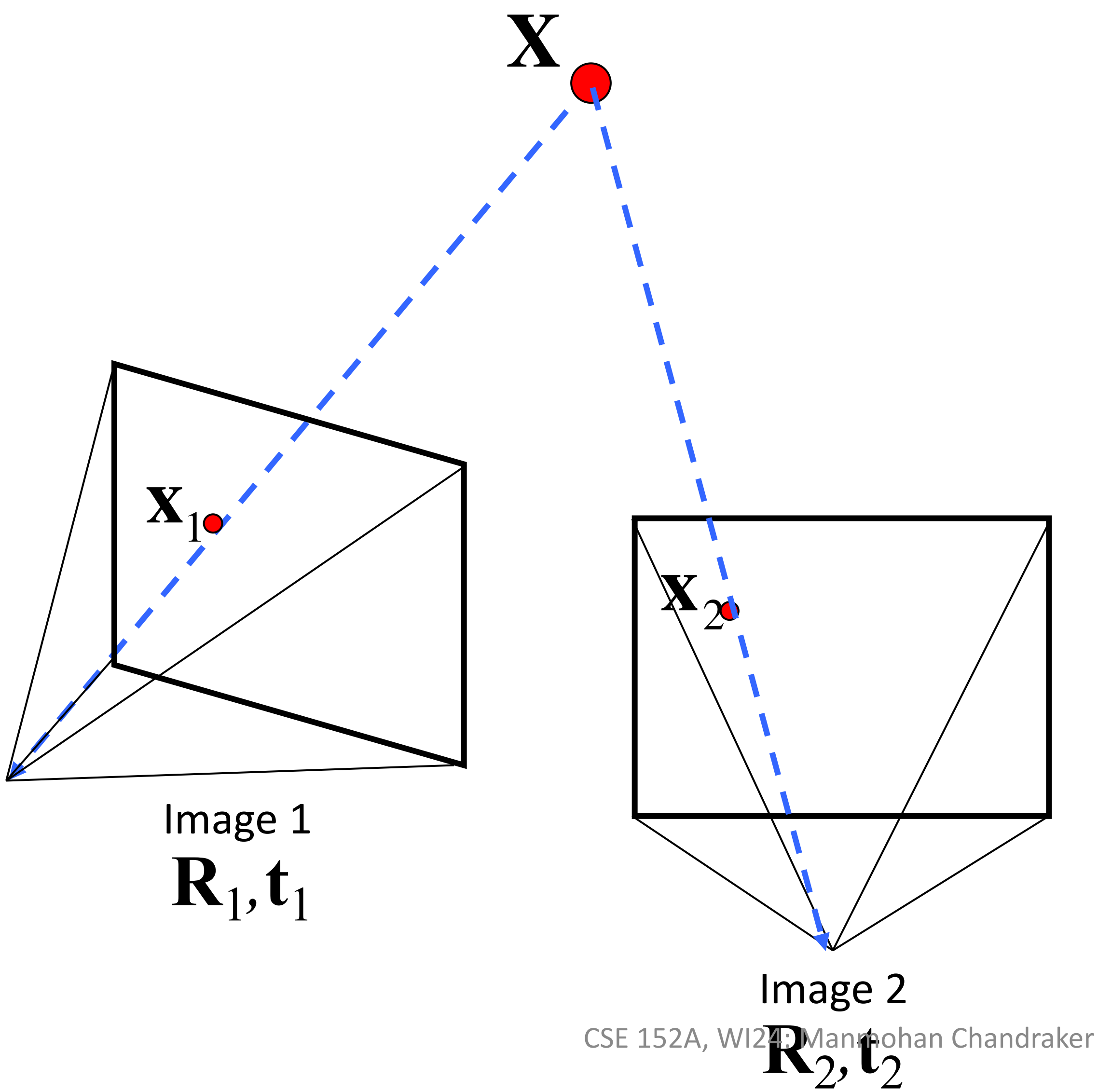
Properties of the fundamental matrix



- **Fx** is the epipolar line associated with **x**
- $\mathbf{F}^T\mathbf{x}'$ is the epipolar line associated with **x'**
- **F** is rank 2.

Fundamental Matrix

Fundamental Matrix



$$\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$$

$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

Estimating **F**

- Given just the two images, can we estimate **F**?
- Yes, with enough correspondences.

Estimating F: Direct Linear Transform

- The fundamental matrix F is defined by

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

for any pair of matches x and x' in two images.

- Let $\mathbf{x} = (u,v,1)^{\mathrm{T}}$ and $\mathbf{x}' = (u',v',1)^{\mathrm{T}}$, $\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

- Each match gives a linear equation:

$$uu' f_{11} + vu' f_{12} + u' f_{13} + uv' f_{21} + vv' f_{22} + v' f_{23} + uf_{31} + vf_{32} + f_{33} = 0$$

Direct Linear Transform Method

Given *n* point correspondences, set up a system of equations:

$$\begin{bmatrix} u_1 u_1' & v_1 u_1' & u_1' & u_1 v_1' & v_1 v_1' & v_1' & u_1 & v_1 & 1 \\ u_2 u_2' & v_2 u_2' & u_2' & u_2 v_2' & v_2 v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u_n' & v_n u_n' & u_n' & u_n v_n' & v_n v_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$.

Solving homogeneous systems

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$.
- Singular value decomposition:

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$$

$$\mathbf{U}, \mathbf{V} \text{ are rotation matrices}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} s_1 & & \\ & \ddots & \\ & & s_n \end{bmatrix}$$

- Solution **f** given by the last column of **V**.

DLT Method: Problem?

- **F** should have rank 2
- To enforce that **F** is of rank 2, **F** is replaced by **F'** that minimizes $\|\mathbf{F}^\top \mathbf{F}'\|$ subject to the rank constraint.

- This is achieved by SVD. Let $\mathbf{F} = \mathbf{U\Sigma V}^{\mathrm{T}}$, where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \text{ Let } \Sigma' = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then $\mathbf{F}' = \mathbf{U}\Sigma'\mathbf{V}^{\mathrm{T}}$ is the solution.

Direct Linear Transform Method

% Normalization on 2D points (advanced concept, implemented for you)

```
% Build the constraint matrix
   A = [x2(1,:)’.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
        x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
        x1(1,:)'             x1(2,:)'            ones(npts,1) ];

   [U,D,V] = svd(A);

% Extract fundamental matrix from the column of V
% corresponding to the smallest singular value.
   F = reshape(V(:,9),3,3)';

% Enforce rank 2 constraint
   [U,D,V] = svd(F);
   F = U * diag([D(1,1) D(2,2) 0]) * V’;
```

% Do the reverse normalization on 2D points

DLT method or 8-point algorithm

Given n point correspondences, set up a system of equations:

$$\begin{bmatrix} u_1 u_1' & v_1 u_1' & u_1' & u_1 v_1' & v_1 v_1' & v_1' & u_1 & v_1 & 1 \\ u_2 u_2' & v_2 u_2' & u_2' & u_2 v_2' & v_2 v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u_n' & v_n u_n' & u_n' & u_n v_n' & v_n v_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In practice, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$, using SVD
- For solution $\mathbf{F}_{3x3}$, impose rank(**F**) = 2 by dropping last singular value
- **F** has 7 degrees of freedom, but use $n \geq 8$ to be able to use linear method

Motion from correspondences

- Use 8-point algorithm to estimate F
- Get E from F:

$$\mathbf{F} = \mathbf{K}_2^{-\top} \mathbf{E} \mathbf{K}_1^{-1}$$

$$\mathbf{E} = \mathbf{K}_2^{\top} \mathbf{F} \mathbf{K}_1$$

- Decompose E into skew-symmetric and rotation matrices:

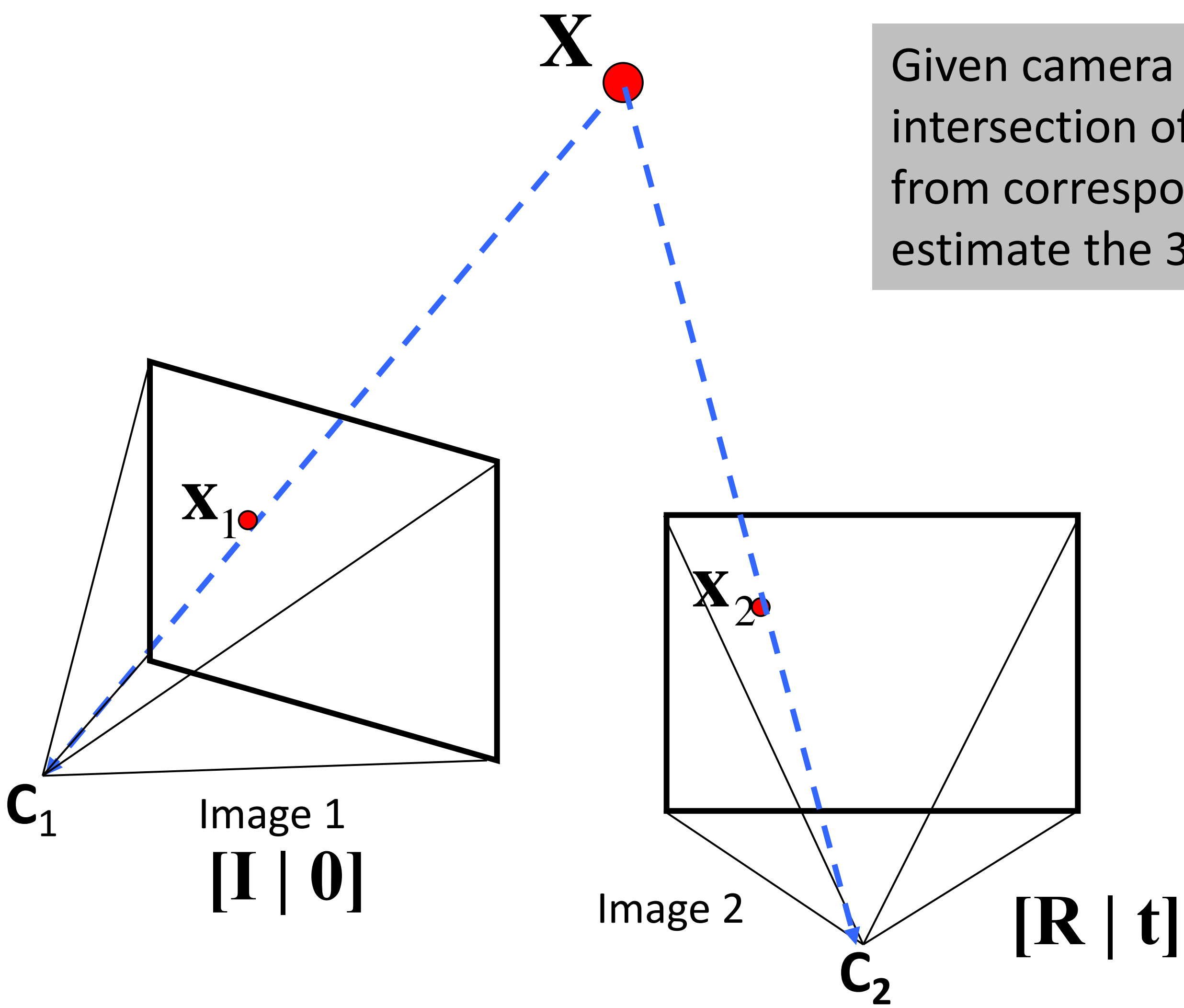
$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$

Can estimate rotation and translation from **E**

Triangulation

Given camera motion [R | t], can find intersection of back-projected rays from corresponding 2D points to estimate the 3D points

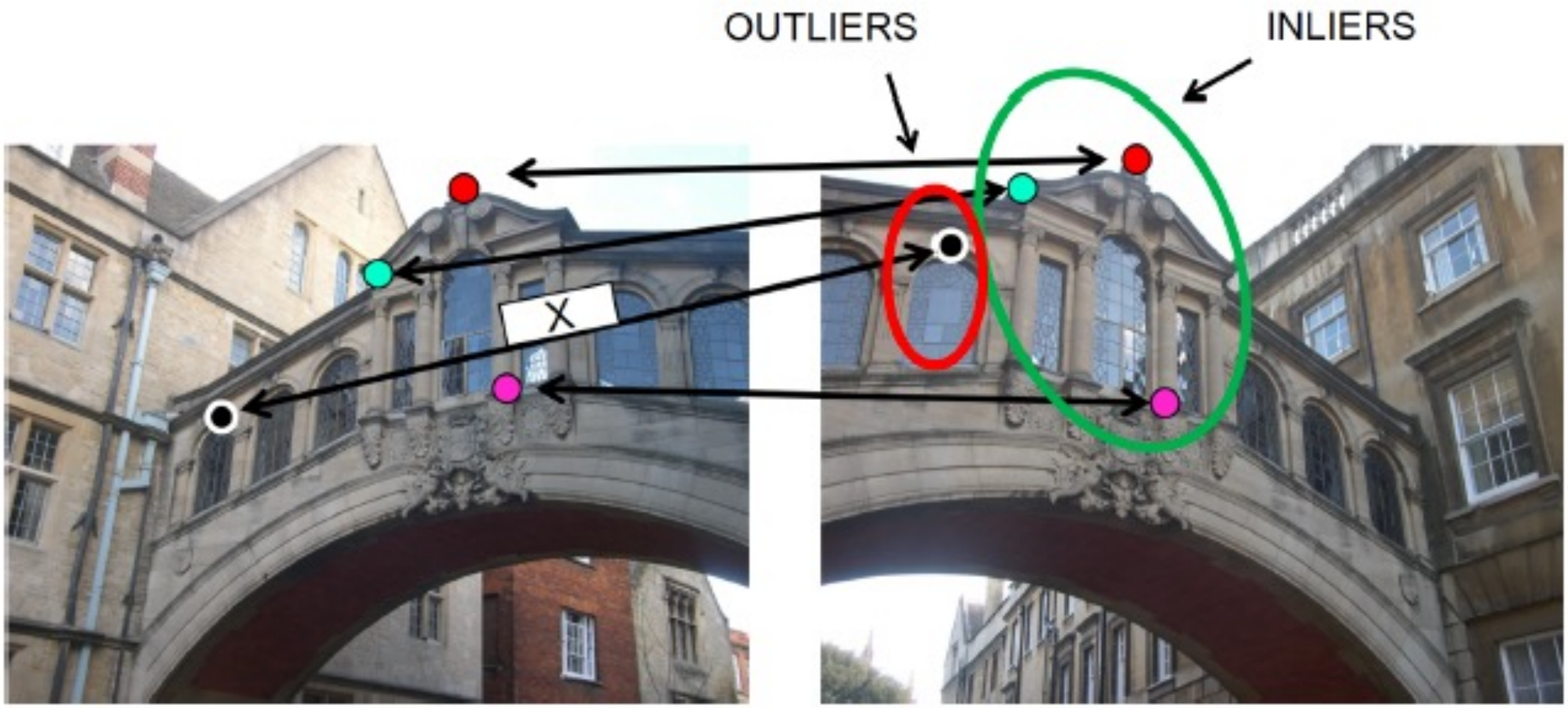
First ray: $\mathbf{C}_1 + k_1(\mathbf{x}_1^w - \mathbf{C}_1)$
Second ray: $\mathbf{C}_2 + k_2(\mathbf{x}_2^w - \mathbf{C}_2)$



DLT method or 8-point algorithm

- Pros: it is linear, easy to implement and fast
- Cons: susceptible to noise

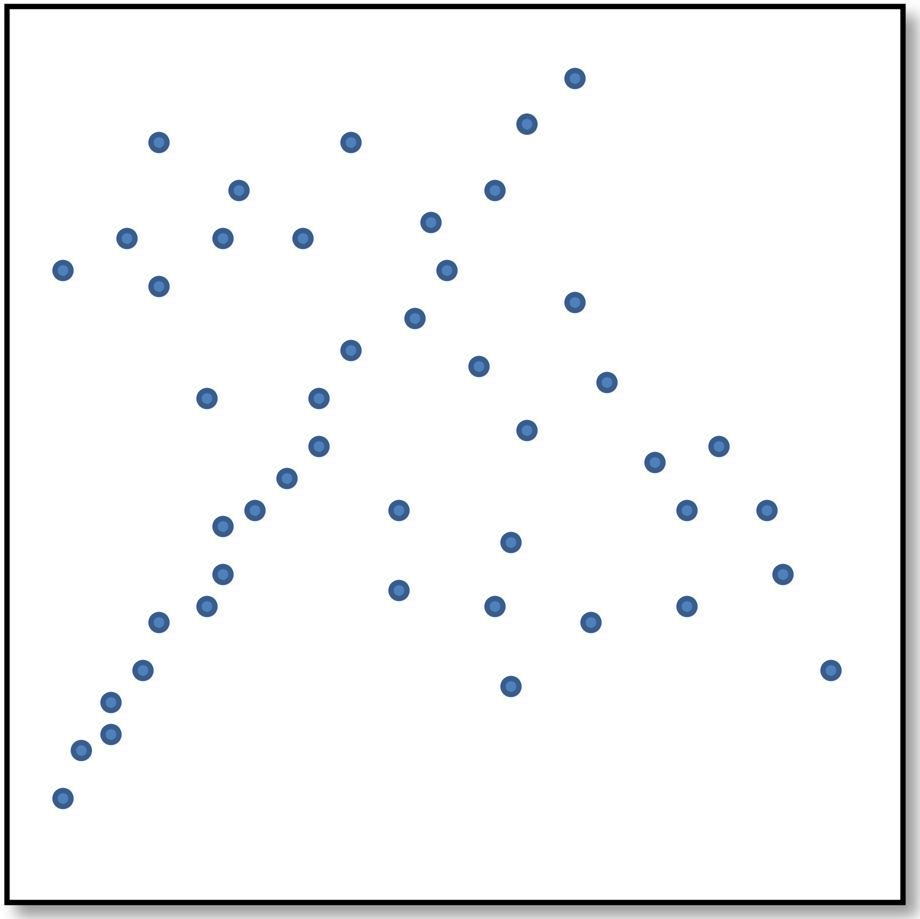
Outliers in Feature Matching



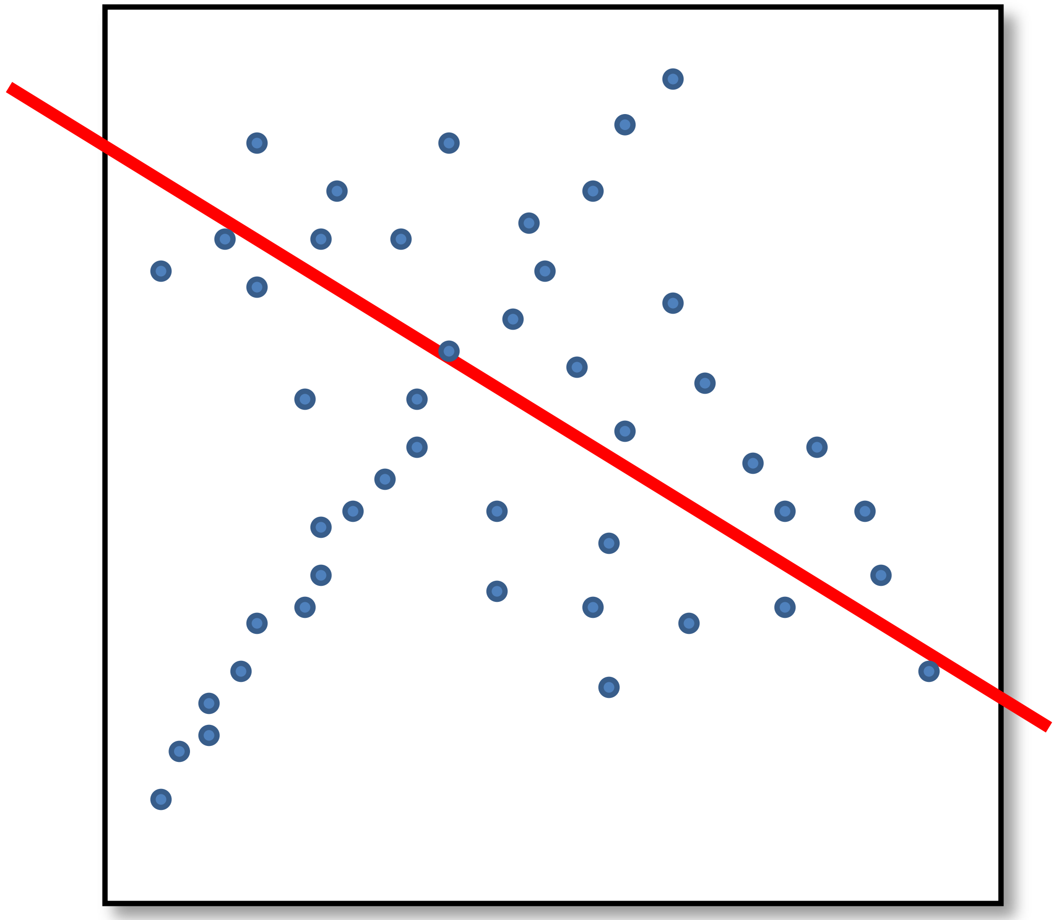
Overall plan: use the fundamental matrix as a model to remove outliers

- Points in correspondence should be consistent with some fundamental matrix
- Find the fundamental matrix with which most points are consistent (inliers)
- Remove points not consistent with the above fundamental matrix (outliers)

RANSAC: Counting Inliers

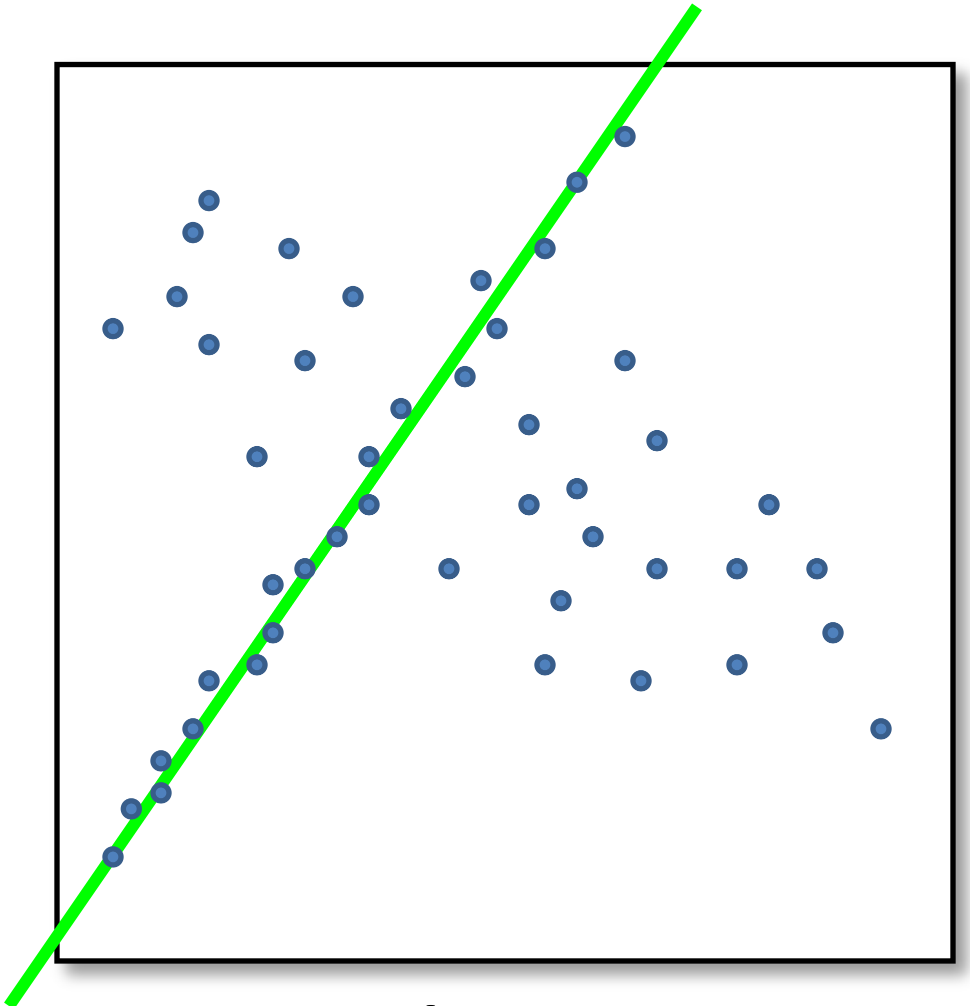


RANSAC: Counting Inliers



Inliers: 3

RANSAC: Counting Inliers

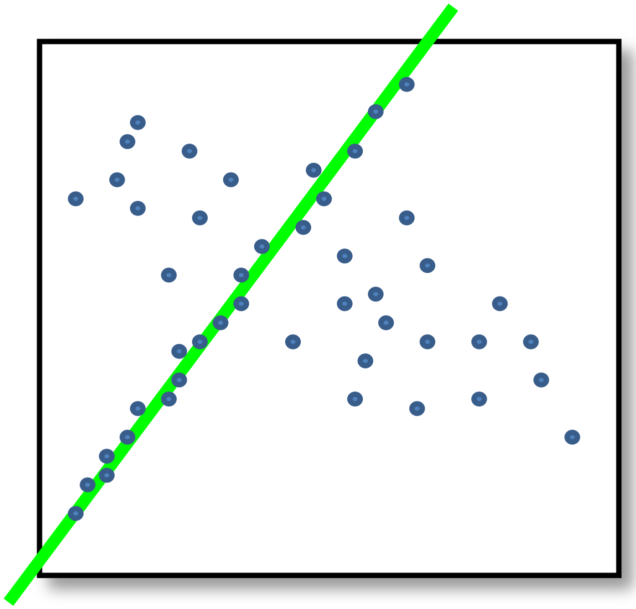


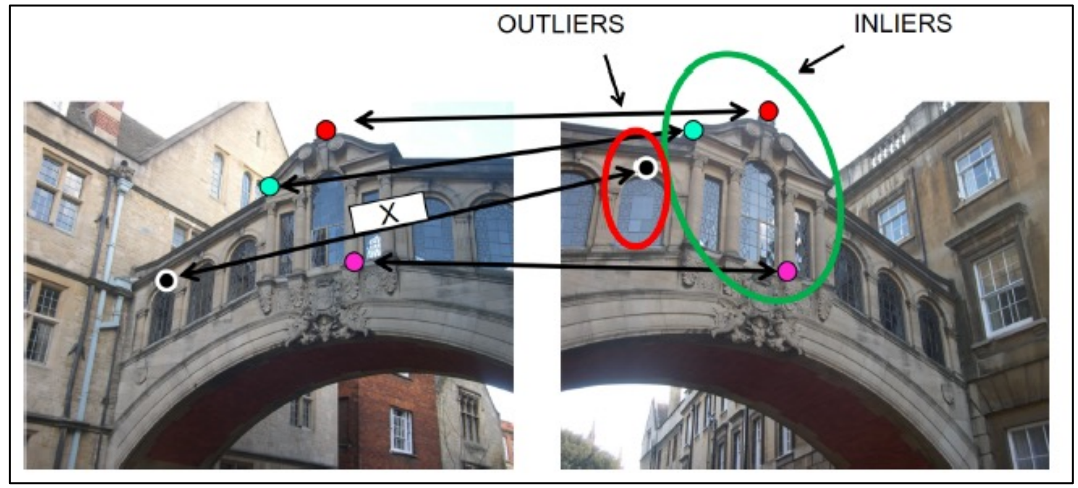
Inliers: 20

RANSAC

- General version:
  1. Randomly choose *s* samples
     - Typically *s* = minimum sample size to fit a model
  2. Fit a model (say, line) to those samples
  3. Count the number of inliers that approximately fit the model
  4. Repeat *N* times
  5. Choose the model with the largest set of inliers

RANSAC to Estimate Fundamental Matrix





- For *N* times
  - Pick 8 pairs of correspondences
  - Estimate an $\mathbf{F}$ using these 8 correspondences
  - Count number of inliers with $\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2$ close to 0
- Pick the $\mathbf{F}$ with the largest number of inliers

RANSAC

- Adaptively determine number of iterations based on outlier proportion

$$N = \frac{\log(1-p)}{\log(1-(1-\epsilon)^s)}$$

Sample size	Proportion of outliers ϵ						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Values of N for $p = 0.99$