Phys 2B Summer 2022

Quiz 2 Practice Solutions

Question 1:

Conductors in equilibrium CAN have a net charge. The only restriction is that this charge must reside on the surface of the conductor. I is false.

Conductors in equilibrium CANNOT have a nonzero net electric field. If they did, then charges would move inside them in response to the electric force produced, and so would they not be in equilibrium! II is true.

Conductors in equilibrium CAN have a nonzero net potential. The potential inside a conductor in equilibrium must be constant, but it need not be constant at 0. III is false.

So, the correct choice is (b).

Question 2:

The potential due to a point charge is:

$$V = k\frac{q}{r}$$

In this case, there are two identical point charges, both protons with charge e, each $r = 1\,\mathrm{m}$ away from the point of interest, so:

$$V_{\mathrm{net}} = 2k\frac{e}{r}$$

Now plug in numbers.

$$V_{\mathrm{net}} = 2(9.0 \times 10^{9}\,\mathrm{Nm^2/C^2})\frac{(1.6 \times 10^{-19}\,\mathrm{C})}{(1.0\,\mathrm{m})} \approx \boxed{2.9\,\mathrm{nV}}$$

This is choice (d).

Question 3:

The potential due to a point charge is:

$$V = \frac{kq}{r}$$

We want to find the point where:

$$V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 0$$

r_1 is obviously just x, since q_1 is at the origin.

r_2 needs to account for q_2 being at $x = 1\,\mathrm{m}$. Since r_2 needs to be positive, we should write this as $r_2 = 1\mathrm{m} - x$, which will be positive since we are looking for a point between the charges, $0 < x < 1\,\mathrm{m}$.

$$\frac{q_1}{x} + \frac{q_2}{1\,\mathrm{m} - x} = 0$$

Unlike the field version of this problem, we don't need to worry about direction.

We want to solve for x.

$$q_1(1\,\mathrm{m} - x) + q_2 x = 0$$

$$q_1(1\,\mathrm{m}) - (q_1 - q_2)x = 0$$

$$x = \frac{q_1(1\,\mathrm{m})}{q_1 - q_2}$$

Now plug in numbers.

$$x = \frac{2.0\,\mathrm{C}(1\,\mathrm{m})}{2.0\,\mathrm{C} - (-3.7\,\mathrm{C})} = \frac{2}{5.7}(1\,\mathrm{m}) \approx \boxed{0.35\,\mathrm{m}}$$

This is choice (d).

Question 4:

The electric field is the negative gradient of the potential. In this case, the potential depends only on x, so:

$$\vec{E}(x) = -\left(\frac{\mathrm{d}V}{\mathrm{d}x}\right)\hat{x} = -(2.80\,\mathrm{V/m^2})x\hat{x}$$

Now plug in $x = 15.0\,\mathrm{m}$:

$$\vec{E}(x) = (-2.80\,\mathrm{V/m^2})(15.0\,\mathrm{m})\hat{x} = \boxed{(-42.0\,\mathrm{V/m})\hat{x}}$$

This is choice (a).

Question 5:

There are a few ways to think about this problem. One is simply to know the formula for the potential of a ring:

$$V(z) = \frac{kQ}{(z^2 + R^2)^{\frac{1}{2}}}$$

The center of the ring is at $z = 0$, so:

$$V_{\text{center}} = \frac{kQ}{R}$$

An alternative way to think about this problem is to realize that the potential doesn't care about direction, so all of the charges on the ring summed together are equivalent to a single point charge one ring radius away, when viewed from the center of the ring. That thinking would give you the same point charge-like formula above. In any case, the energy required is:

$$U = qV = eV = \frac{keQ}{R}$$

In this case, $q = e$ since the charge is a proton. Now plug in numbers.

$$U = \frac{(9.0 \times 10^9\,\mathrm{Nm^2/C^2})(1.6 \times 10^{-19}\,\mathrm{C})(8.0 \times 10^{-9}\,\mathrm{C})}{0.2\,\mathrm{m}} \approx \boxed{5.8 \times 10^{-17}\,\mathrm{J}}$$

This is choice (c).

Question 6:

The parallel-plate capacitance formula is:

$$C = \frac{\epsilon A}{d}$$

Pulling the plates farther apart means increasing d, thereby decreasing C. The defining equation of capacitance is:

$$Q = CV$$

The voltage difference across the capacitor is set by the battery, which never changes, so V is constant and I is true, but since C decreases, Q must also decrease proportionally. That means II is false.

One way to express the energy stored by a capacitor is:

$$U = \frac{1}{2}CV^2$$

As above, V is constant, but C decreases, so U decreases and III is false.

The correct choice is therefore (a), I only.

If the capacitor were disconnected from the battery first, then instead the charge would be constant (the capacitor needs to be connected to the battery for its charge to be adjusted) and the voltage would be allowed to change (the capacitor's voltage would no longer be maintained by the battery).

Question 7:

If the field is approximately constant, then

$$\Delta V = Ed$$

where d is the thickness of the material. Solve for d:

$$d = \frac{\Delta V}{E}$$

$\Delta V = 5 \times 10^5\,\mathrm{V}$, and the material will undergo dielectric breakdown and become a conductor if the field reaches $1.2 \times 10^8\,\mathrm{V/m}$.

$$d > \frac{5 \times 10^5\,\mathrm{V}}{1.2 \times 10^8\,\mathrm{V/m}} \approx \boxed{4.2\,\mathrm{mm}}$$

If d is any smaller than this, then the voltage will change so much across so small a distance (ie the field = voltage slope will be so strong) that the material will undergo breakdown.

This is choice (d).

Incidentally, 500 kV is the voltage of modern power lines in southern California.

Question 8:

As discussed in class, a general solution to these problems is:

$$Q_{12} = \frac{Q_{\text{initial}}}{1 + \frac{C_9}{C_{12}}}$$

We can get Q_{initial} from the defining equation of capacitance:

$$Q = CV = (12\,\mathrm{nF})(9.0\,\mathrm{V}) = 108\,\mathrm{nC}$$

Plugging in numbers:

$$Q_{12} = \frac{108\,\mathrm{nC}}{1 + \frac{9\,\mathrm{nF}}{12\,\mathrm{nF}}} \approx \boxed{62\,\mathrm{nC}}$$

This is choice (b).

This implies that $Q_9 \approx 46\,\mathrm{nC}$. We can check that these charges will produce the same voltages using $V = Q/C$.

$$V_{12} \approx \frac{62\,\mathrm{nC}}{12\,\mathrm{nF}} \approx 5.2\,\mathrm{V}$$

$$V_9 \approx \frac{46\,\mathrm{nC}}{9\,\mathrm{nF}} \approx 5.1\,\mathrm{V}$$

Our answer checks out, to within rounding error.

Question 9:

First, we can reduce the capacitors in parallel in the center:

$$C_{\text{eq}} = 1.5\,\mu\text{F} + 2.5\,\mu\text{F} = 4.0\,\mu\text{F}$$

After this replacement, we have three capacitors in series.

$$C_{\text{eq}} = \left(\frac{1}{4.0\,\mu\text{F}} + \frac{1}{4.0\,\mu\text{F}} + \frac{1}{3.0\,\mu\text{F}}\right)^{-1} = \boxed{1.2\,\mu\text{F}}$$

This is choice (b).

Question 10:

The power formula is:

$$P = IV$$

We want to solve for I.

$$I = \frac{P}{V}$$

Now plug in numbers.

$$I = \frac{1000\,\text{W}}{110\,\text{V}} \approx 9.1\,\text{A}$$

The total charge is this current, multiplied by the time over which the current flowed:

$$Q = IT \approx (9.1\,\text{C/S}(35\,\text{s}) \approx 318\,\text{C}$$

Now, convert this to the number of electrons using the elementary charge.

$$N_e = \frac{Q}{e} \approx \frac{318\,\text{C}}{1.6\times 10^{-19}\,\text{C/electron}} \approx \boxed{2.0\times 10^{21}\,\text{electrons}}$$

This is choice (e).