UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2020
Linear Systems Fundamentals

MIDTERM EXAM

Instructions

- Open Canvas website, no electronics for problem solving.
- Tables 3.1 and 3.2 from the textbook are attached to the exam.
- No collaboration or external help of any kind, either in person or via web, phone, or any other device, is allowed. You may be asked to explain your answers over a Zoom call when the exam is graded.
- Time allowed: 2.5 consecutive hours (from 5pm PST Thursday to 11:59pm PST Friday)
- Write your solutions in the applicable space in the exam. **Justify your answers.** If extra pages are needed to show all of your work, use the scratch pages at the end of the exam and submit them.
- Upload your solutions via Gradescope

PRINT YOUR NAME ______________________________

Student ID Number ______________________________

Signature ______________________________

Your signature confirms that you have completed this exam on your own and in accordance with the Academic Integrity Agreement.

Problem	Weight	Score
1	16 pts	
2	24 pts	
3	30 pts	
4	30 pts	
Total	100 pts	

Good luck!

Name/Student ID: ____

Problem 1 (16 points: 2 parts, 8 points each

(a) (8 pts)

Let $x(t) = t\,(u(t) - u(t-1))$. Define $y(t) = x((-3t) - 2)$.

Describe how to obtain $y(t)$ from $x(t)$ by the following rules:

Approach 1:

First shift $x(t)$ by $b =$ __________
($b > 0$ means shift right by $b < 0$ means shift left by $|b|$)

Then (circle one) **stretch** or **compress** the signal by $a =$ __________
($a < 0$ means flip; $|a| \geq 1$ is the stretch or compress factor)

or

Approach 2:

First (circle one) **stretch** or **compress** the signal by $c =$ __________
($c < 0$ means flip; $|c| \geq 1$ is the stretch or compress factor)

Then shift $x(t)$ by $d =$ __________
($d > 0$ means shift right by $d < 0$ means shift left by $|d|$)

Name/Student ID: ____________________

Problem 1 (cont.)

(b) (8 pts) Let $x(t) = t\,(u(t) - u(t-1))$. Define $y(t) = x((-3t) - 2)$.

Sketch precisely $y(t)$.

Name/Student ID: ______________________________

Problem 2 (24 points: 3 parts, 8 points each)

Let $x[n] = e^{j\frac{\pi}{3}(n-1)} + \sum_{k=-\infty}^{\infty} \delta[n-6k]$.

(a) (8 pts)

Determine the fundamental period of $x[n]$.

Name/Student ID: ______________________________

Problem 2 (cont.)

Let $x[n] = e^{j\frac{\pi}{3}(n-1)} + \sum_{k=-\infty}^{\infty} \delta[n-6k]$.

(b) (8 pts)

Determine if $x[n]$ is even, odd, or neither.

Name/Student ID: ______________________________

Problem 2 (cont.

Let $x[n] = e^{j\frac{\pi}{3}(n-1)} + \sum_{k=-\infty}^{\infty} \delta[n-6k]$.

(c) (8 pts)

Determine the discrete-time Fourier series (DTFS) coefficients of $x[n]$.

Name/Student ID: ______________________________

Problem 3 (30 points: 3 parts, 10 points each)

Consider the discrete-time linear time-invariant (LTI) system S with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1].$$

(a) (10 pts)

Determine the step response $s[n]$ of the system S, and sketch $s[n]$ precisely in the interval $-3 \leq n \leq 3$.

Name/Student ID: ______________________________

Problem 3 (cont.)

Consider the discrete-time linear time-invariant (LTI) system S with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1].$$

(b) (10 pts)

Indicate whether the system S satisfies each listed property.

Justify your answers with *specific* reference to the system S.

True	**False**	
☐	☐	Memoryless
☐	☐	Causal
☐	☐	Stable

Name/Student ID: ______________________________

Problem 3 (cont.)

Consider the discrete-time linear time-invariant (LTI) system S with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1].$$

(c) (10 pts)

The system S is invertible. Determine the impulse response $g[n]$ of the inverse system.

Name/Student ID: ______________________________

Problem 4 (30 points: 3 parts, 10 points each)

The signal $x[n] = 1 + \cos(\frac{\pi}{2}n)\sin(\frac{\pi}{3}n)$ is passed through the discrete-time filter whose frequency response is given by:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{2\pi}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(a) (10 pts)

Determine the discrete-time Fourier series (DTFS) of the signal $x[n]$.

Name/Student ID: ______________________________

Problem 4 (cont.)

The signal $x[n] = 1 + \cos(\frac{\pi}{2}n)\sin(\frac{\pi}{3}n)$ is passed through the discrete-time filter whose frequency response is given by:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{2\pi}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(b) (10 pts)

Determine the output signal $y[n]$ produced by the input signal $x[n]$. Write it in a form that does not involve complex exponential signals.

Name/Student ID: ________________________________

Problem 4 (cont.)

The signal $x[n] = 1 + \cos(\frac{\pi}{2}n)\sin(\frac{\pi}{3}n)$ is passed through the discrete-time filter whose frequency response is given by:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{2\pi}{3} \\ 0, & \text{otherwise.} \end{cases}$$

(c) (10 pts)

Let b_k denote the DTFS of the output signal $y[n]$. Determine the value of b_{11}, b_{18}, and b_{52} (not the rock band) . Express your answers numerically, as well as in terms of $b_0, b_1, \ldots, b_N$ where N is the fundamental period of $y[n]$.

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	$\begin{matrix} a_k \\ b_k \end{matrix}$
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t)$, $\alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\} \\ \mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{matrix} \mathcal{Re}\{a_k\} \\ j\mathcal{Im}\{a_k\} \end{matrix}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{array}{l} x[n] \\ y[n] \end{array}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{array}{l} a_k \\ b_k \end{array}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix} \text{viewed as periodic} \\ \text{with period } mN \end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix} \text{finite valued and periodic only} \\ \text{if } a_0 = 0 \end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [\text{x[n] real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [\text{x[n] real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} \lvert x[n] \rvert^2 = \sum_{k=\langle N \rangle} \lvert a_k \rvert^2$$