CSE 167 (WI 2025) Exercise 2

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Exercise 2.1

Problem

Two bases $(\vec{a}_1, \vec{a}_2)$ and $(\vec{b}_1, \vec{b}_2)$ are related as:

$$\vec{b}_1 = -\vec{a}_1 + 2\vec{a}_2, \quad \vec{b}_2 = 3\vec{a}_1 - 5\vec{a}_2$$

A vector $\vec{v}$ is given in the $(\vec{a}_1, \vec{a}_2)$ basis as:

$$\vec{v} = \vec{a}_1 - \vec{a}_2$$

Find the coefficients x and y such that:

$$\vec{v} = x\vec{b}_1 + y\vec{b}_2$$

Solution

1. **Substitute** $\vec{b}_1$ and $\vec{b}_2$ into the equation for $\vec{v}$:

$$\vec{v} = x(-\vec{a}_1 + 2\vec{a}_2) + y(3\vec{a}_1 - 5\vec{a}_2)$$

2. **Expand**:

$$\vec{v} = (-x + 3y)\vec{a}_1 + (2x - 5y)\vec{a}_2$$

3. **Equate coefficients** with $\vec{v} = \vec{a}_1 - \vec{a}_2$:

$$\begin{aligned} -x + 3y &= 1 \quad (\text{coefficient of } \vec{a}_1) \\ 2x - 5y &= -1 \quad (\text{coefficient of } \vec{a}_2) \end{aligned}$$

4. **Solve the linear system**:
   - Multiply $-x + 3y = 1$ by 2 and add it to $2x - 5y = -1$:

$$-2x + 6y + 2x - 5y = 2 - 1 \implies y = 1$$

   - Substitute $y = 1$ into $-x + 3y = 1$:

$$-x + 3(1) = 1 \implies -x + 3 = 1 \implies x = 2$$

5. **Final Answer**:

$$x = 2, \quad y = 1$$

Exercise 2.2

Problem

Provide an example of two square matrices A and B such that $AB \neq BA$.

Solution

Let:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Compute AB:

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Compute BA:

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Therefore:

$$AB \neq BA$$