ECE 101: Linear Systems Fundamentals

Spring 2020 - Lecture 3

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Today's topics

- Discrete-time unit impulse signal
- Discrete-time unit step signal

Signals and Systems (2th Edition): section 1.4.1

Unit Step and Unit Impulse Signals

Discrete-Time: Unit Step

- We define the discrete-time unit step function as:

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

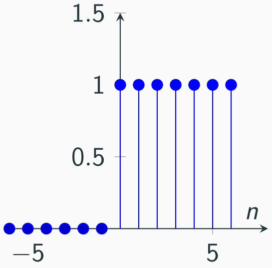

Figure 1: Plot of step signal $u[n]$

Discrete-Time: Unit Impulse

- We define the discrete-time unit impulse as:

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases} = u[n] - u[n-1]$$

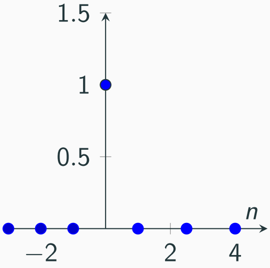

Figure 2: Plot of unit impulse signal $\delta[n]$

Discrete-Time: Unit Impulse

- Relations between $u[n]$ and $\delta[n]$

  - $\delta[n] = u[n] - u[n-1]$

  - $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

  - $u[n] = \sum_{k=-\infty}^{\infty} u[k]\delta[n-k].$

Properties of $\delta[n]$

Sampling Property

$$x[n]\delta[n-k] = x[k]\delta[n-k].$$

- By the definition of $\delta[n]$, $\delta[n-k] = 1$ if $n = k$, and 0 otherwise. Therefore,

$$\begin{aligned} x[n]\delta[n-k] &= \begin{cases} x[n], & n = k \\ 0, & n \neq k \end{cases} \\ &= x[k]\delta[n-k]. \end{aligned}$$

Properties of $\delta[n]$

- Example of sampling property: When $k = 0$, we have $x[n]\delta[n] = x[0]\delta[n]$!

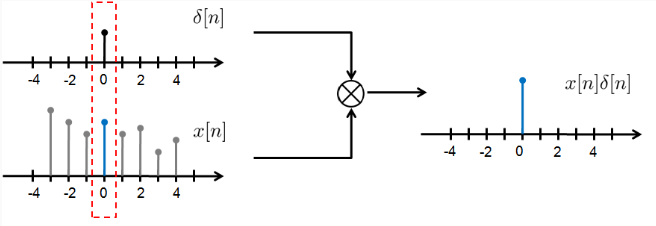

Figure 3: Example of sampling property of $\delta[n]$

Properties of $\delta[n]$

Sifting Property

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0].$$

Proof.

- The sifting property follows from the sampling property:

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] &= \sum_{n=-\infty}^{\infty} x[k]\delta[n-n_0] \\
&\overset{(a)}{=} x[n_0]\left(\sum_{n=-\infty}^{\infty} \delta[n-n_0]\right) \\
&\overset{(b)}{=} x[n_0].
\end{aligned}$$

$\square$

Properties of $\delta[n]$

Representation Property

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

Proof.

- Consider the sampling property:

$$x[n]\delta[n-k] = x[k]\delta[n-k].$$

- Instead of summing over n (shifting property), sum over k:

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = \sum_{k=-\infty}^{\infty} x[n]\delta[n-k] = x[n].$$

$\square$

Representation property of $\delta[n]$

- The representation property shows that every discrete-time signal x[n] can be represented as a linear combination of shifted and unit impulses.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

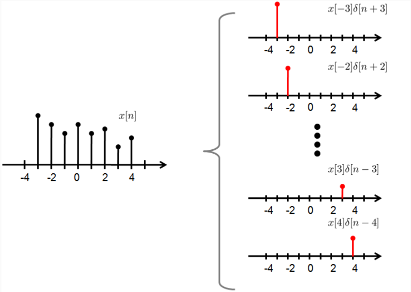

Figure 4: Representing a signal, $x[n]$, using a train of impulses, $\delta[n-k]$