UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2018
Linear Systems Fundamentals

FINAL EXAM

You are allowed two 2-sided sheets of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME ______________________________

Signature ______________________________

Your signature affirms that you have completed this exam on your own and in accordance with the ECE 101 honor code.

Student ID Number ______________________________

Problem	Weight	Score
1	20 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
6	20 pts	
Total	120 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Tables 3.1, 3.2, 4.1, 4.2, 5.1, 5.2, 9.1, and 9.2 are attached.
Good luck!

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Problem 1 [Do the Convolution] (20 points, 10pts each part)

Assume you are given a discrete-time LTI system consisting of the cascade shown below:

$$x[n] \longrightarrow \boxed{h_1[n]} \xrightarrow{z[n]} \boxed{h_2[n]} \longrightarrow y[n]$$

The two impulse responses are given by $h_1[n] = 2^n u[n]$ and $h_2[n] = 3^n u[n]$, respectively.

The input signal is given by $x[n] = \delta[n] - 2\delta[n-1]$.

(a) Determine the output $y[n]$. [Hint: determine $z[n]$ first].

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Problem 1 [Do the Convolution] (cont.)

(b) Consider the same system, shown again below,

$$x[n] \longrightarrow \boxed{h_1[n]} \xrightarrow{z[n]} \boxed{h_2[n]} \longrightarrow y[n]$$

where, again $h_1[n] = 2^n u[n]$ and $h_2[n] = 3^n u[n]$.

Determine the overall impulse response $h[n]$.

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Problem 2 [DT Fourier Series] (20 points, 10pts each part)

Let $x[n]$ be the periodic discrete-time signal with fundamental period $N = 4$ defined by:

$$x[n] = \begin{cases} 1, & n = 0, 1 \\ -1, & n = 2, 3. \end{cases}$$

(a) (i) (2 points)

Sketch $x[n]$ precisely.

(ii) (8 points)

Determine the DT Fourier series coefficients a_k, $k = 0, 1, 2, 3$ of the signal $x[n]$. Write your answers in the box below. **Justify your answers.**

$$\boxed{\begin{aligned} a_0 &= \\ a_1 &= \\ a_2 &= \\ a_3 &= \end{aligned}}$$

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Problem 2 [DT Fourier Series] (cont.)

(b) Consider the signal $x_1[n] = \frac{1}{2}(x[n] - x[n-1])$, where $x[n]$ is the signal in part (a).

(i) (2 points)
Sketch $x_1[n]$ precisely.

(ii) (3 points)
Let b_k be the Fourier series coefficients of $x_1[n]$. Express b_k as a function of a_k, the Fourier series coefficients of $x[n]$.

(iii) (5 points)
The signal $x_1[n]$ satisfies $x_1[n] = \cos(\omega n)$ for some $\omega \in [0, 2\pi)$. Determine ω. Then determine the values b_k, $k = 0, 1, 2, 3$. Write your answers in the box below. **Justify your answers.**

$$\boxed{\begin{aligned} \omega &= \\ b_0 &= \\ b_1 &= \\ b_2 &= \\ b_3 &= \end{aligned}}$$

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Problem 3 [DTFT] (20 points)

Consider a causal and stable LTI system whose input $x[n]$ and output $y[n]$ are related by the first-order difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

(a) (6 pts)

Determine the frequency response $H(e^{j\omega})$ of the system.

Justify your answer.

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Problem 3 [DTFT] (cont.)

(b) (6 pts)

Determine the impulse response $h[n]$ of the system. Justify your answer.

(c) (4 pts)

Evaluate the following integral.

$$\frac{1}{2\pi}\int_0^{2\pi} H(e^{j\omega})e^{j2\omega}d\omega.$$

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Problem 3 [DTFT] (cont.)

(d) (4 pts)

Evaluate the following integral.

$$\frac{1}{2\pi}\int_0^{2\pi}\left|H(e^{j\omega})\right|^2 d\omega.$$

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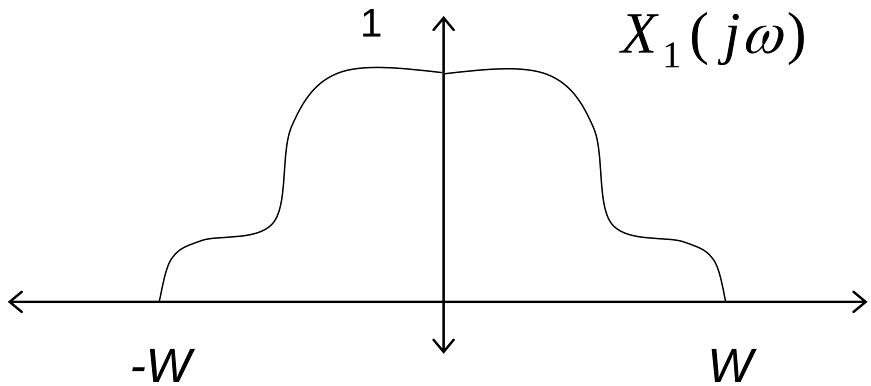
Problem 4 [Amplitude Modulation]

Let $x_1(t)$ be a continuous-time signal bandlimited to $[-W, W]$, with Fourier transform $X_1(j\omega)$ shown below.



You wish to transmit the signal using amplitude modulation, i.e., by modulating the signal with a carrier signal $c(t) = \cos(\frac{3W}{2}t)$. Just as you are about the transmit the modulated signal, a saboteur who wishes to prevent a successful transmission, secretly adds a scrambling signal $x_2(t)$, also band-limited to $[-W, W]$, but mistakenly modulated with the carrier signal $s(t) = \sin(\frac{3W}{2}t)$. The signal actually transmitted is therefore:

$$y(t) = x_1(t)c(t) + x_2(t)s(t).$$

Note: The following identities may be helpful to you in solving this problem:

$$\begin{aligned}
\cos^2(\theta) &= \frac{1 + \cos(2\theta)}{2} \\
\sin^2(\theta) &= \frac{1 - \cos(2\theta)}{2} \\
\cos(\theta)\sin(\theta) &= \frac{\sin(2\theta)}{2}
\end{aligned}$$

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Problem 4 [Amplitude Modulation](cont.)

(a) (4 pts)

Express the Fourier transform $Y(j\omega)$ of the output signal in terms of the transforms $X_1(j\omega)$ and $X_2(j\omega)$ of the signals $x_1(t)$ and $x_2(t)$.

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Problem 4 [Amplitude Modulation](cont.)

(b) (4 pts)

At the receiving end of the transmission, you have a demodulation circuit that first produces the signal $z(t) = y(t)c(t)$ with Fourier transform $Z(j\omega)$.

Express the Fourier transform $Z(j\omega)$ of the output signal in terms of the transforms $X_1(j\omega)$ and $X_2(j\omega)$ of the signals $x_1(t)$ and $x_2(t)$.

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Problem 4 [Amplitude Modulation](cont.)

(c) (4 pts)

The signal $z(t)$ is then input to a low-pass filter with high frequency cutoff at $\frac{3W}{2}$ and gain 2.

Show that the output $y_1(t)$ of the low-pass filter is equal to $x_1(t)$, the desired signal.

(d) (2 pts)

How would you recover $x_2(t)$ from $y(t)$? (You'll want to show it to the saboteur, of course.)

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Problem 4 [Amplitude Modulation](cont.)

(e) (4 pts)

Annoyed at being outsmarted, the saboteur decides to ruin your next transmission by applying to your modulated signal $x_1(t)c(t)$ a high-pass filter with low frequency cutoff $\frac{3W}{2}$ and unity gain. The resulting transmitted signal is denoted $v(t)$.

Sketch precisely the Fourier transform $V(j\omega)$ of the transmitted signal $v(t)$. Express $V(j\omega)$ in terms of $X_1(j\omega)$.

(f) (2 pts)

How would you recover $x_1(t)$ from $v(t)$?

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Problem 5 [Sampling Theory] (20 pts)

(a) (10 points)

Let $x(t)$ be a continuous-time band-limited signal with

$$X(j\omega) = 0 \text{ for } |\omega| > W$$

For each of the signals below, indicate the condition that, according to the sampling theorem, the sampling frequency ω_s should satisfy so that the signal can be reconstructed from its samples, and also indicate the corresponding condition on the sampling period T.

Justify your answers by explicit reference to the Fourier Transforms $Y(j\omega)$ of the signals.

(i) $y(t) = x(t/3)$.

(ii) $y(t) = x(t)x(-t)$.

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Problem 5 (cont.)

(b) (10 points)

Consider the signal $x(t) = \cos(2\pi t)$. Let $x_p(t)$ be obtained by impulse-train sampling $x(t)$ with sampling frequency $\omega_s = 3\pi$.

(i) Let $X_p(j\omega)$ be the Fourier transform of the signal $x_p(t)$. Sketch $X_p(j\omega)$ precisely in the frequency range $[-6\pi, 6\pi]$.

(ii) Suppose $x_p(t)$ is passed through a low-pass filter with gain $T = 2\pi/\omega_s = 2/3$ and cutoff frequency $\omega_c = \omega_s/2 = 3\pi/2$. Determine the output $y(t)$ of the low-pass filter and determine whether aliasing occurred.

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Problem 6

(a) (10 points)

Let S be a stable LTI system defined by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 2x(t)$$

Let $H(j\omega)$ be the frequency response of the system.

(i) Draw the pole-zero plot and sketch precisely the magnitude of the frequency response $\left|H(j\omega)\right|$.

(ii) Is the frequency response lowpass, highpass, bandpass, or all-pass? Indicate your answer below by checking the appropriate box.

Low-pass	High-pass	Band-pass	All-pass
☐	☐	☐	☐

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Problem 6 (cont.)

(b) (10 points) Let S be a **causal** LTI system with system function

$$H(s) = \frac{s-2}{s^2 - 2s - 8}.$$

Determine the ROC and the impulse response $h(t)$.

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