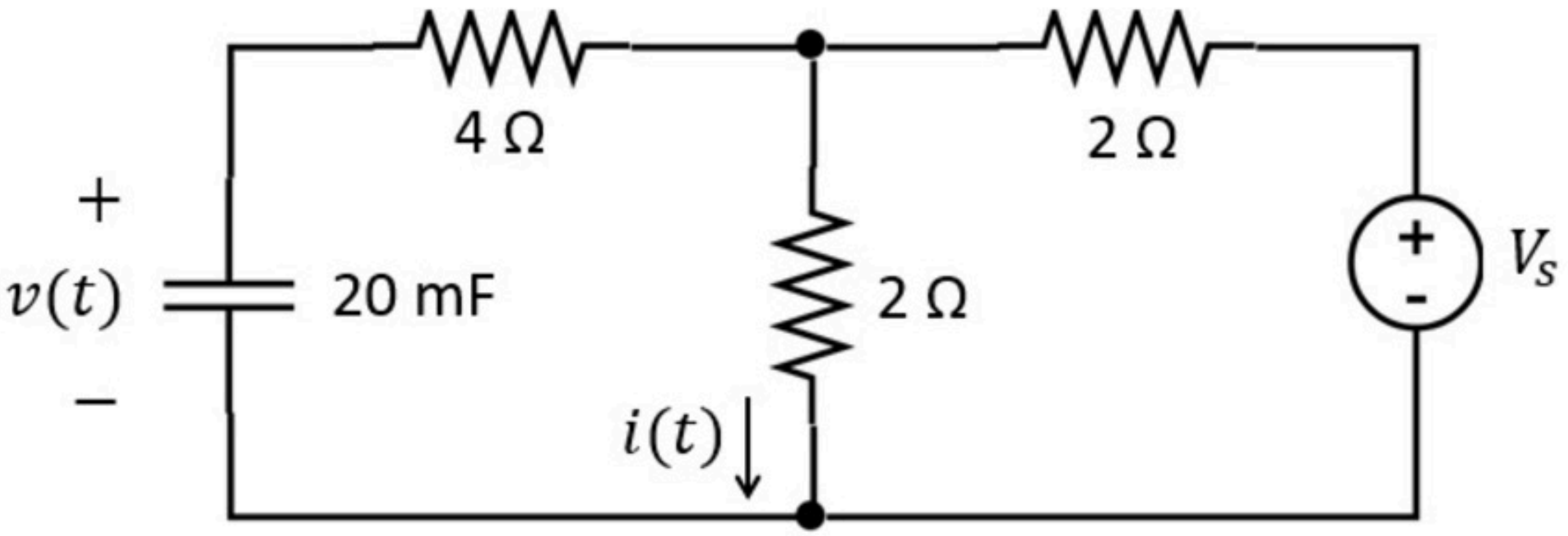
First order circuits 001

Problem has been graded.

Given a voltage $v(t)$, find the current $i(t)$.

$$v(t) = A_1 + B_1 \cdot e^{-10t}$$

$$i(t) = A_2 + B_2 \cdot e^{-10t}$$



Given Variables:

A1 : 10 V

B1 : 10 V

Vs : 20 V

Calculate the following:

A2 (A) :

5

B2 (A) :

1

Given a voltage $v(t)$, find the current $i(t)$.

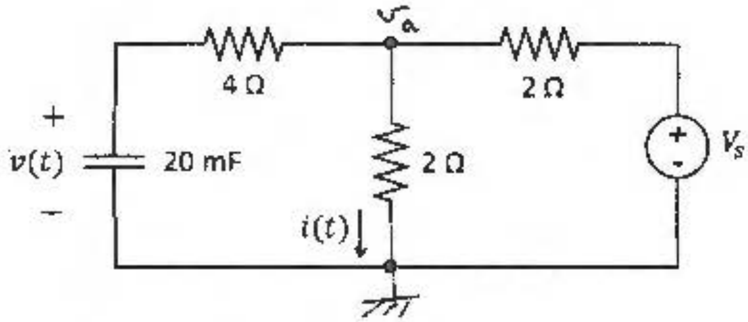
$$v(t) = A_1 + B_1 \cdot e^{-10t}$$

$$i(t) = A_2 + B_2 \cdot e^{-10t}$$

A1 : 10 V

B1 : 10 V

Vs : 20 V



NODAL

$$\frac{v_a - v}{4} + \frac{v_a - V_s}{2} + \frac{v_a}{2} = 0$$

$$v_a - v + 2(v_a - V_s) + 2v_a = 0$$

$$5v_a = v + 2V_s$$

$$v_a = \frac{v}{5} + \frac{2V_s}{5}$$

$$i = \frac{v_a}{2} = \frac{v}{10} + \frac{V_s}{5} = 1 + e^{-10t} + 4$$

$$\boxed{A_2 = 5\text{ A}}$$

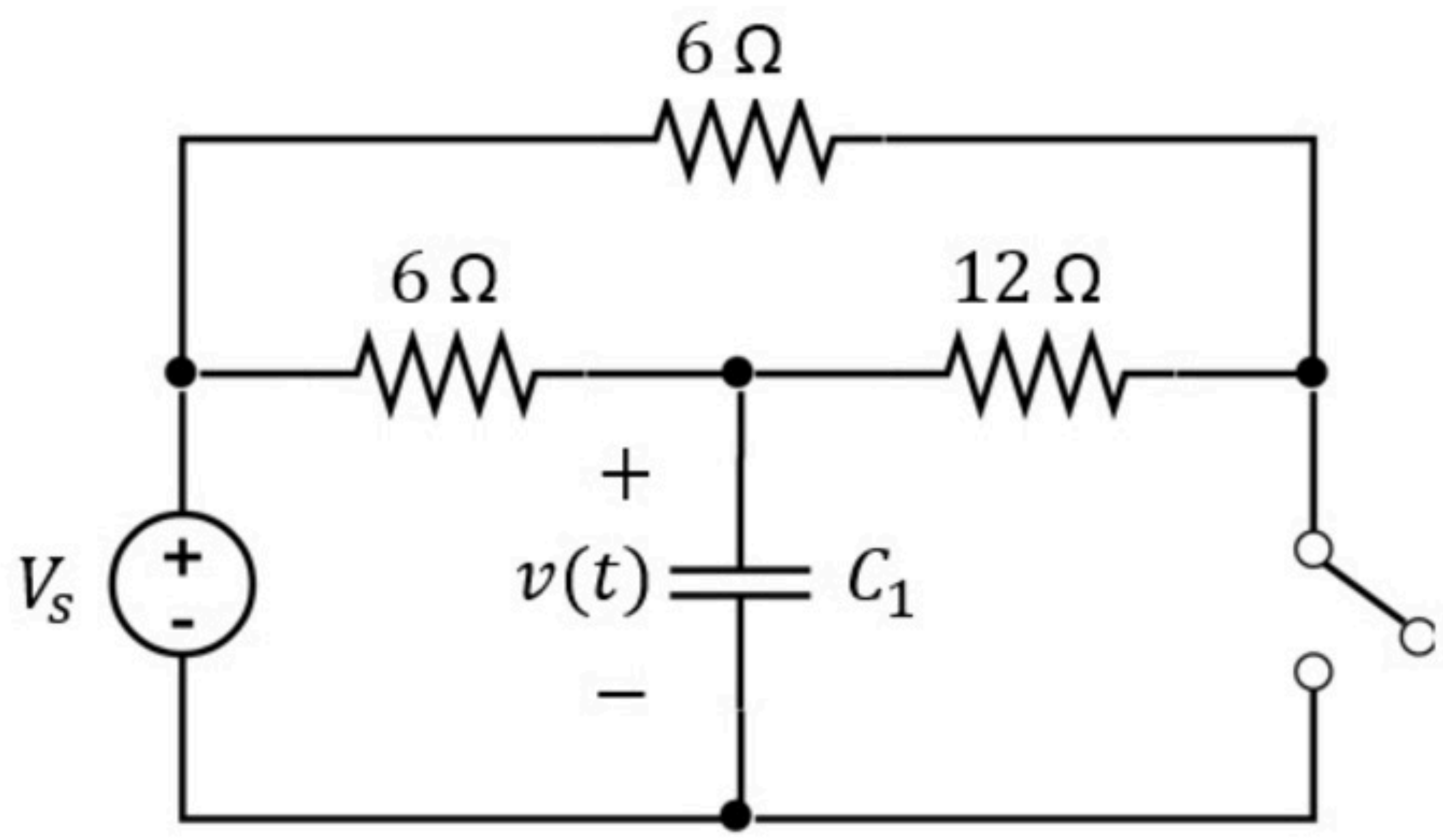
$$\boxed{B_2 = 1\text{ A}}$$

First order circuits 002

Unlimited Attempts.

a) Find the time constant τ_1 and the steady state capacitor voltage $v_1 = v(\infty)$ when the switch is open.

b) Find the time constant τ_2 and the steady state capacitor voltage $v_2 = v(\infty)$ when the switch is closed.



Given Variables:

Vs : 30 V

C1 : 8 nF

Calculate the following:

tau1 (ns) :

36

v1 (V) :

30

tau2 (ns) :

32

v2 (V) :

20

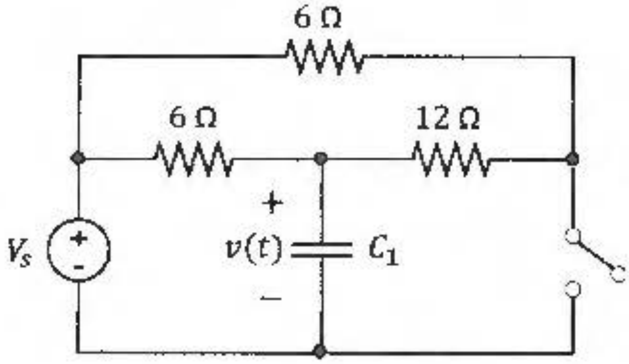
Hint: The circuit is different in the two cases

a) Find the time constant τ_1 and the steady state capacitor voltage $v_1 = v(\infty)$ when the switch is open.

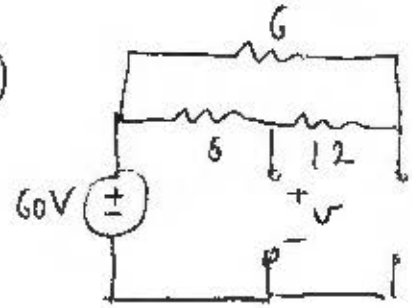
b) Find the time constant τ_2 and the steady state capacitor voltage $v_2 = v(\infty)$ when the switch is closed.

Vs : 60 V

C1 : 6 nF

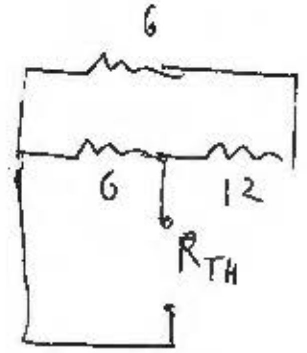


(a)



→ No CURRENTS

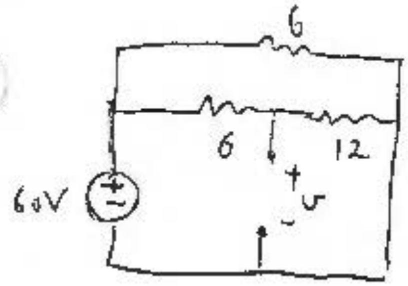
$\boxed{v_1 = 60V}$



$\tau_1 = R_{TH} \cdot C_1 \Rightarrow \boxed{\tau_1 = 27\ ns}$

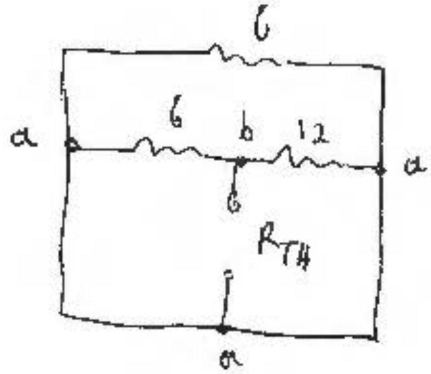
$R_{TH} = 6 \,//\, 18 = \frac{9}{2}\ \Omega$

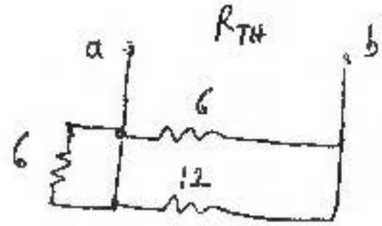
(b)



$v = 60 \cdot \frac{12}{6+12} = 40$

$\boxed{v_2 = 40V}$





$R_{TH} = 6 \,//\, 12 = 4\ \Omega$

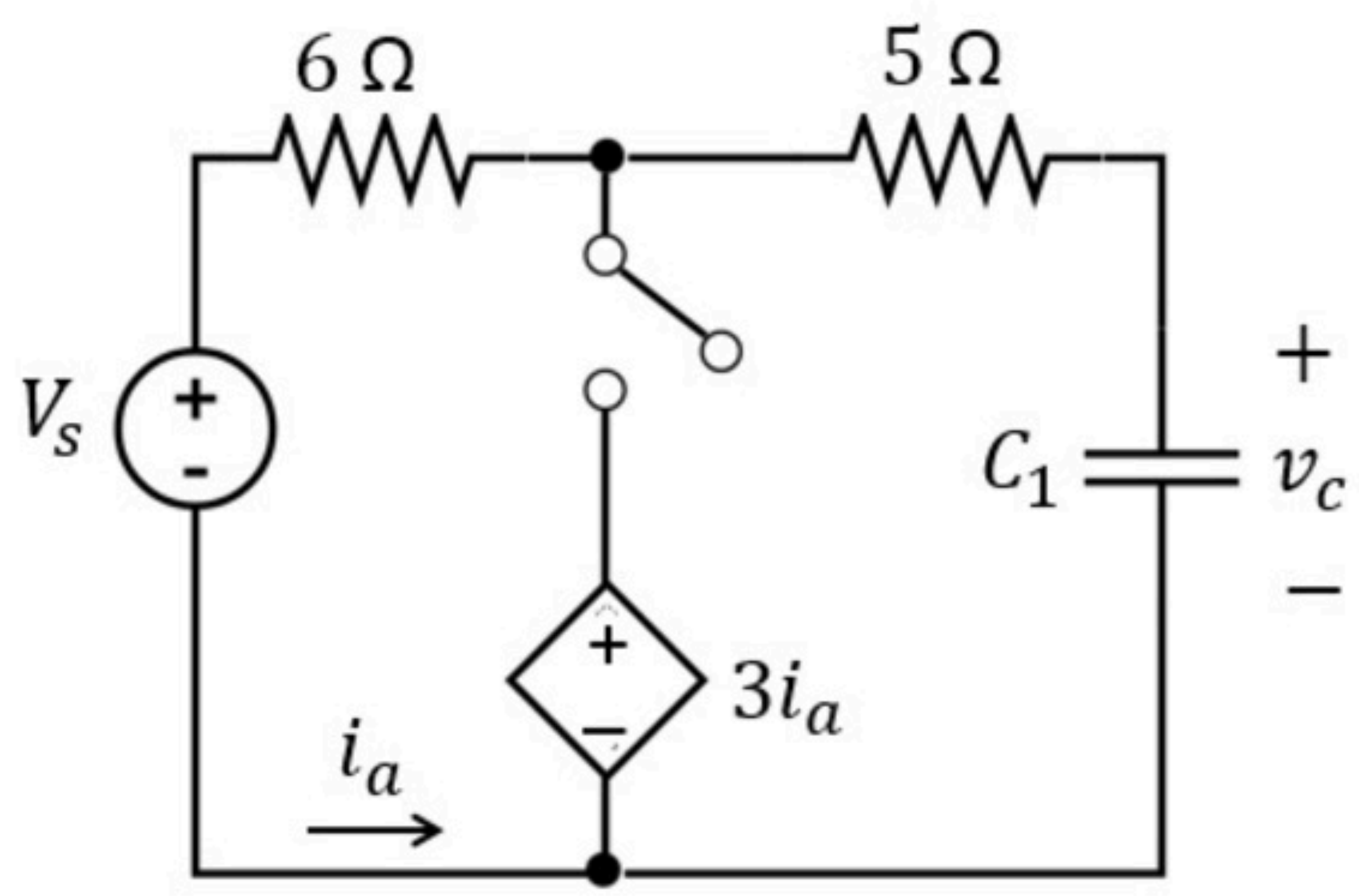
$\tau_2 = R_{TH} \cdot C_1 \Rightarrow \boxed{\tau_2 = 24\ ns}$

First order circuits 003

Problem has been graded.

The switch has been open for a long time before it closes at time $t = 0$.

Find the capacitor voltage $v_c = A + B \cdot e^{-t/\tau}$ for $t > 0$.



Given Variables:

Vs : 15 V

C1 : 0.1 nF

Calculate the following:

A (V) :

-15

B (V) :

30

tau (ns) :

0.5

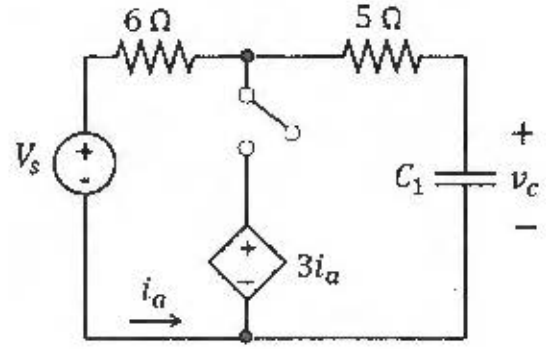
Hint: First find the capacitor voltage at t = 0-. Note where A and B are.

The switch has been open for a long time before it closes at time $t = 0$.

Find the capacitor voltage $v_c = A + B \cdot e^{-t/\tau}$ for $t > 0$.

Vs : 30 V

C1 : 0.2 nF



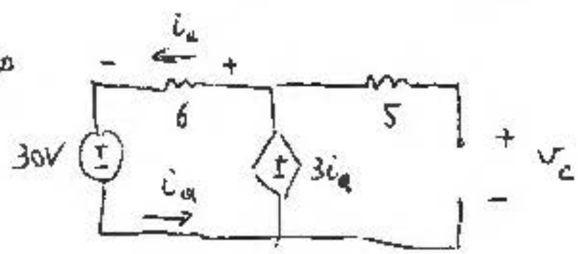
(a) $t = 0^-$

$v_c(0^-) = 30V$

$\Downarrow$

(b) $v_c(0^+) = 30V$

(c) $t = \infty$



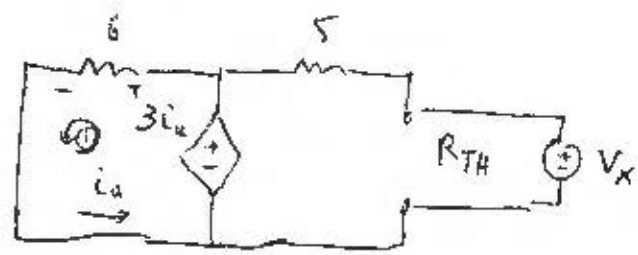
$30 + 6i_a - 3i_a = 0$

$3i_a = -30$

$i_a = -10$ A

$\Rightarrow v_c(\infty) = 3i_a = -30V$

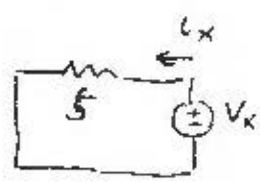
(d)



KVL ①: $3i_a - 6i_a = 0$

$\Rightarrow i_a = 0$

$\Rightarrow$



$\frac{V_x}{5} = i_x \Rightarrow R_{TH} = 5\,\Omega$

$\boxed{\tau = RC = 1\,ns}$

$A = v_c(\infty) \Rightarrow \boxed{A = -30V}$

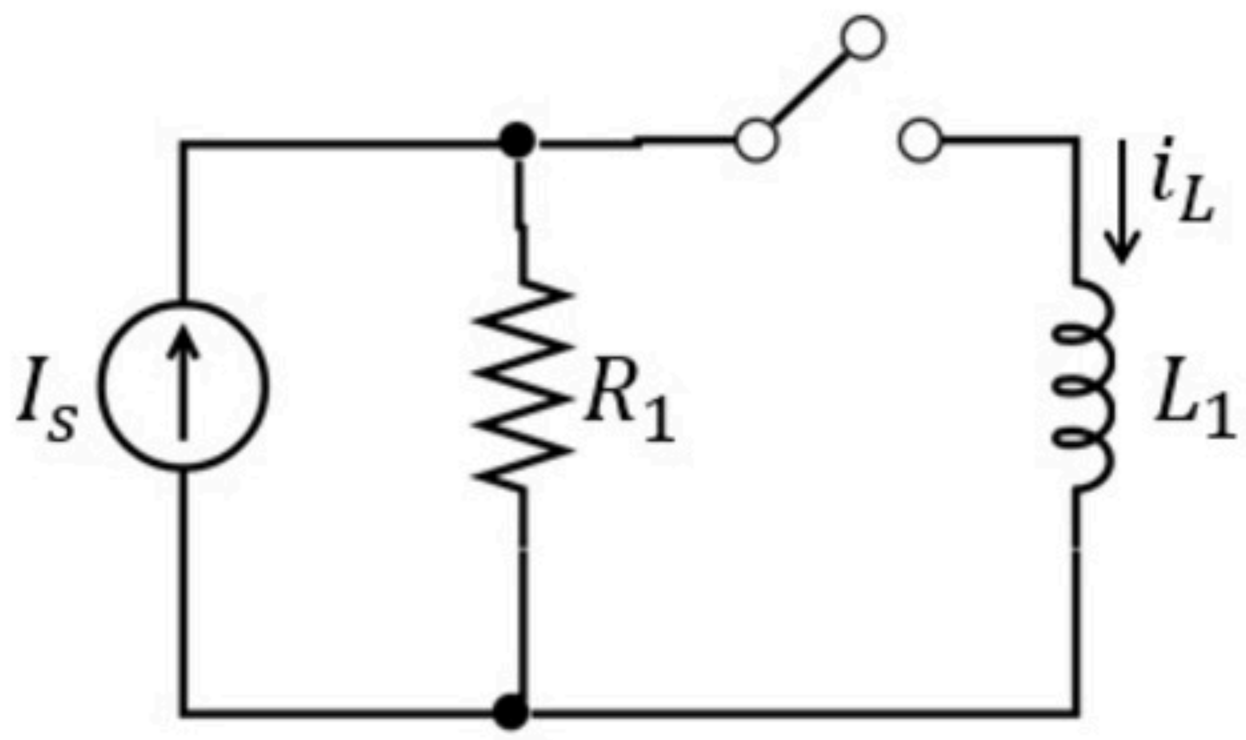
$A + B = v_c(0^+) \Rightarrow \boxed{B = 60V}$

First order circuits 004

Problem has been graded.

The switch closes at time $t = 0$.
Find the current i_L for $t > 0$:

$$i_L(t) = A \cdot e^{-t/\tau} + B$$



Given Variables:

Is : 2 A

R1 : 10 kohm

L1 : 10 mH

Calculate the following:

A (A) :

-2

B (A) :

2

tau (ms) :

0.001

Hint: What is the current iL for t < 0?

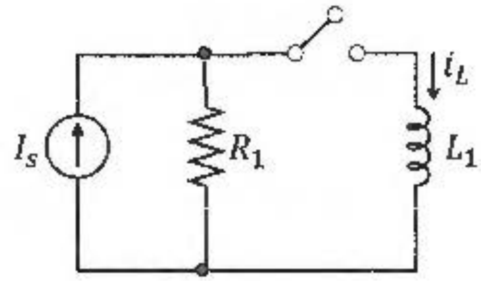
The switch closes at time $t = 0$.
Find the current i_L for $t > 0$:

$$i_L(t) = A \cdot e^{-t/\tau} + B$$

Is : 2 A

R1 : 3 kohm

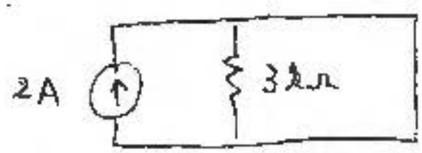
L1 : 30 mH



(a) $t = 0^-$: $i_L = 0$

(b) $t = 0^+$: $i_L(0^+) = i_L(0^-) = 0\,A$

(c) $t = \infty$:



$i_L(\infty) = 2\,A$

(d)

$R_{TH} = 3\,k\Omega$

$\tau = \dfrac{L_1}{R_{TH}} = \dfrac{30 \cdot 10^{-3}}{3 \cdot 10^{3}} = 10 \cdot 10^{-6}\,s$

$\boxed{\tau = 0.01\ ms}$

$B = i_L(\infty) = 2A \Rightarrow \boxed{B = 2A}$

$A + B = i_L(0^+) = 0A \Rightarrow \boxed{A = -2A}$

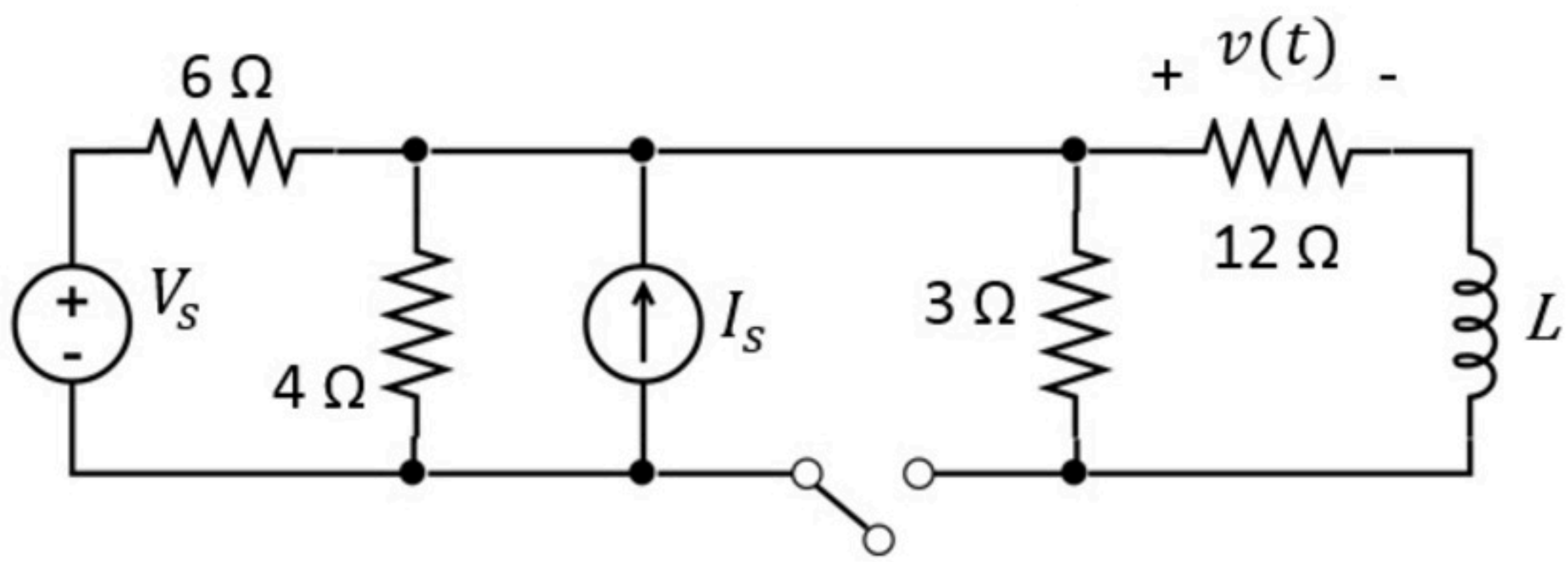
$$i_L(t) = 2 - 2e^{-\frac{t}{0.01\,ms}}\ A$$

$$i_L(t) = 2\left(1 - e^{-\frac{t}{0.01\,ms}}\right)\ A$$

First order circuits 005

Problem has been graded.

The switch has been closed for a long time before it opens at $t = 0$. After the switch opens, find the resistor voltage $v(t) = A \cdot e^{-t/\tau} + B$.



Given Variables:

Vs : 28 V

Is : 2 A

L : 1.5 mH

Calculate the following:

A (V) :

8

B (V) :

0

tau (ms) :

0.1

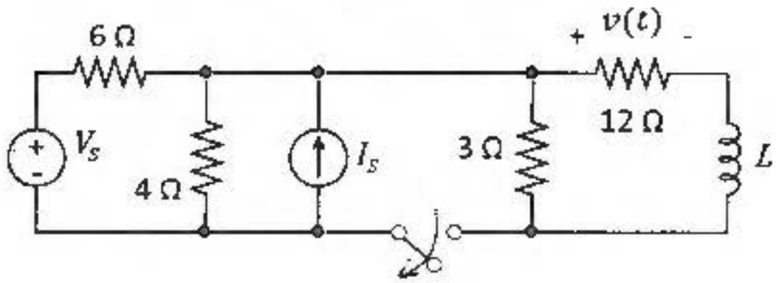
Hint: What is the current iL for t < 0?

The switch has been closed for a long time before it opens at $t = 0$. After the switch opens, find the resistor voltage $v(t) = A \cdot e^{-t/\tau} + B$.

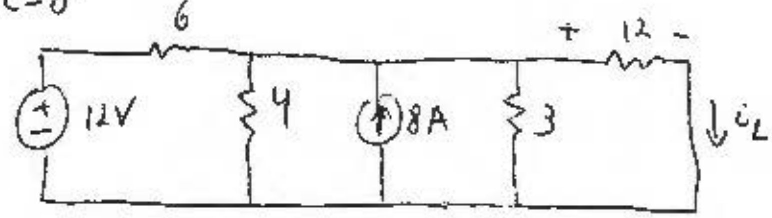
Vs : 12 V

Is : 8 A

L : 15 mH



(a) $t = 0^-$



$i_L(0^-) = 1\,A \qquad v(0^-) = 12\,V$

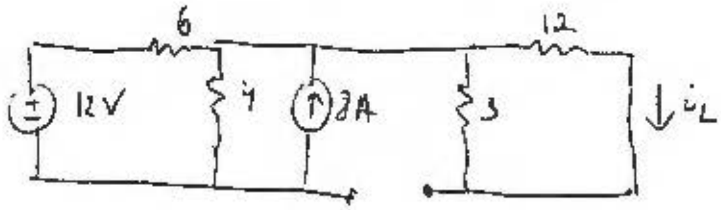
SUPERPOSITION

$$i_{L_1} = \frac{12}{6 + 4\|3\|12} \cdot \frac{4\|3}{4\|3 + 12} = \frac{1}{5}\,A$$

$$i_{L_2} = \frac{6\|4\|3}{6\|4\|3 + 12} \cdot 8 = \frac{4}{5}\,A$$

(b) $t = 0^+$: $i_L(0^+) = 1\,A \Rightarrow v(0^+) = 12\,V$

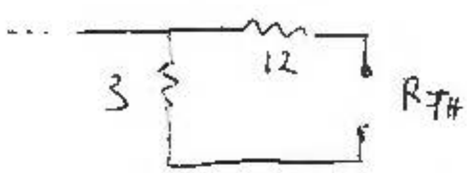
(c)



$i_L(\infty) = 0\,A$

$v(\infty) = 0\,V$

(d)



$R_{TH} = 15\,\Omega$

$$\tau = \frac{L}{R_{TH}} = \frac{15 \cdot 10^{-3}}{15} = 10^{-3}\,s$$

$\boxed{\tau = 1\,ms}$

$B = v(\infty) \Rightarrow \boxed{B = 0\,V}$

$A + B = v(0^+) \Rightarrow \boxed{A = 12\,V}$

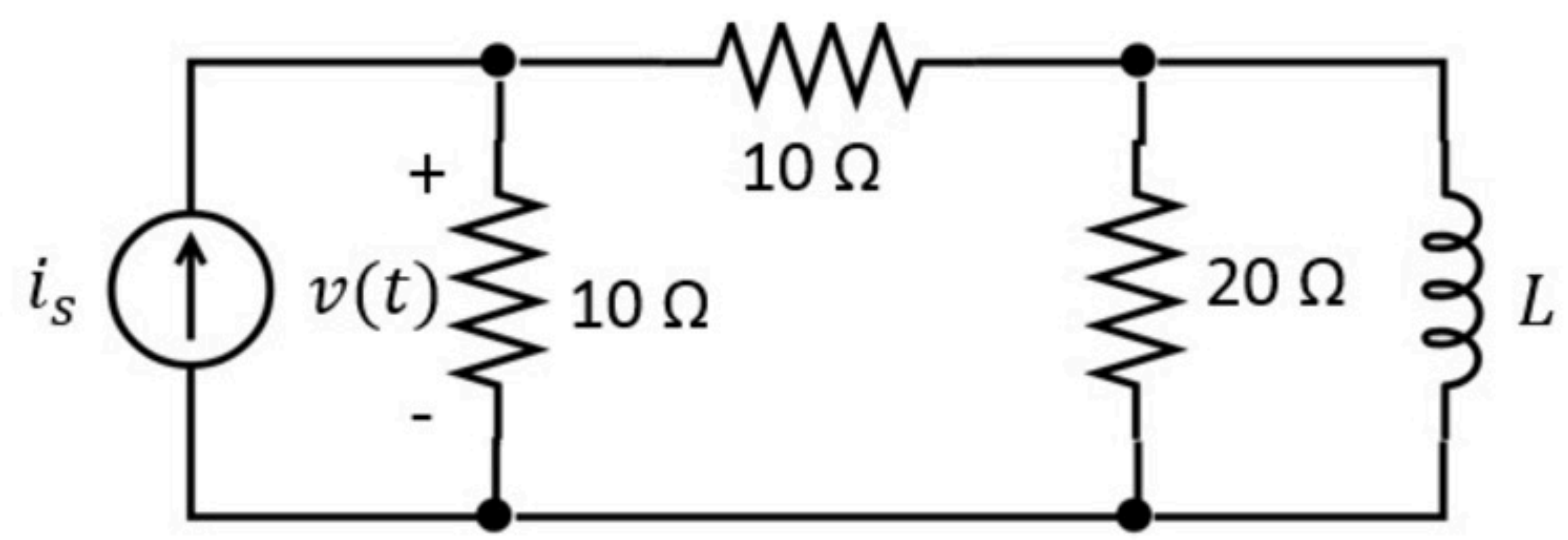
First order circuits 006

Problem has been graded.

When $t < 0$, $i_s = I_0$

When $t > 0$, $i_s = I_1$

Find $v(t) = A \cdot e^{-t/\tau} + B$ for $t > 0$



Given Variables:

I0 : 2 A

I1 : 10 A

L : 0.25 uH

Calculate the following:

A (V) :

20

B (V) :

50

tau (ns) :

25

Hint: What is the inductor current for t < 0?

When $t < 0$, $i_s = I_0$

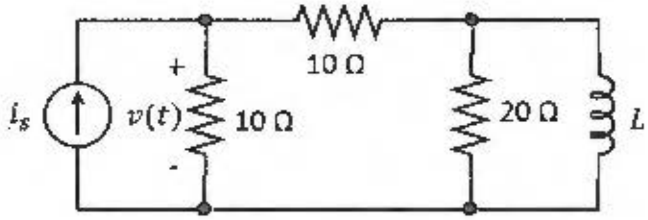
When $t > 0$, $i_s = I_1$

Find $v(t) = A \cdot e^{-t/\tau} + B$ for $t > 0$

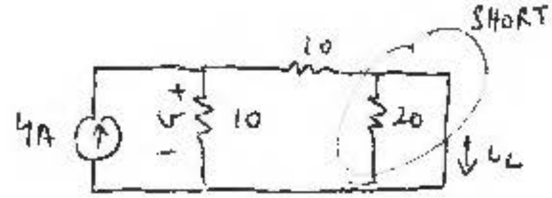
I0 : 4 A

I1 : 6 A

L : 0.25 uH



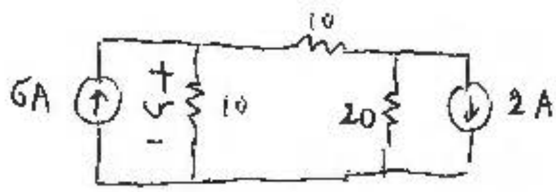
(a) $t = 0^-$



$v(0^-) = (4A) \cdot (10 // 10) = 20V$

$i_L(0^-) = (4A) \frac{10}{10+10} = 2A$

(b) $t = 0^+$



$i_L(0^+) = 2A$

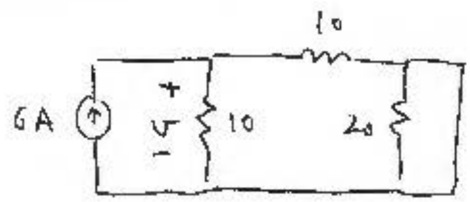
SUPERPOSITION:

$v_1 = 6 \cdot (10 // 30) = 6 \cdot \frac{30}{4} = 45V$

$v_2 = (-2) \cdot (20 // 20) \cdot \frac{1}{2} = -10V$

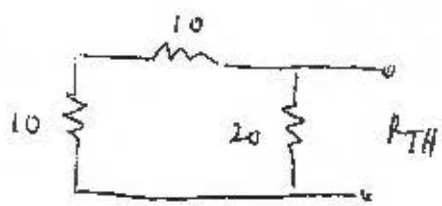
$v(0^+) = v_1 + v_2 = 35V$

(c) $t = \infty$



$v(\infty) = 6 \cdot (10 // 10) = 30V$

(d)



$R_{TH} = 20 // 20 = 10\,\Omega$

$\tau = \frac{L}{R_{TH}} = \frac{0.25 \cdot 10^{-6}}{10} = 25 \cdot 10^{-9}$

$\boxed{\tau = 25 \text{ ns}}$

$B = v(\infty) \Rightarrow \boxed{B = 30V}$

$A + B = v(0^+) \Rightarrow \boxed{A = 5V}$

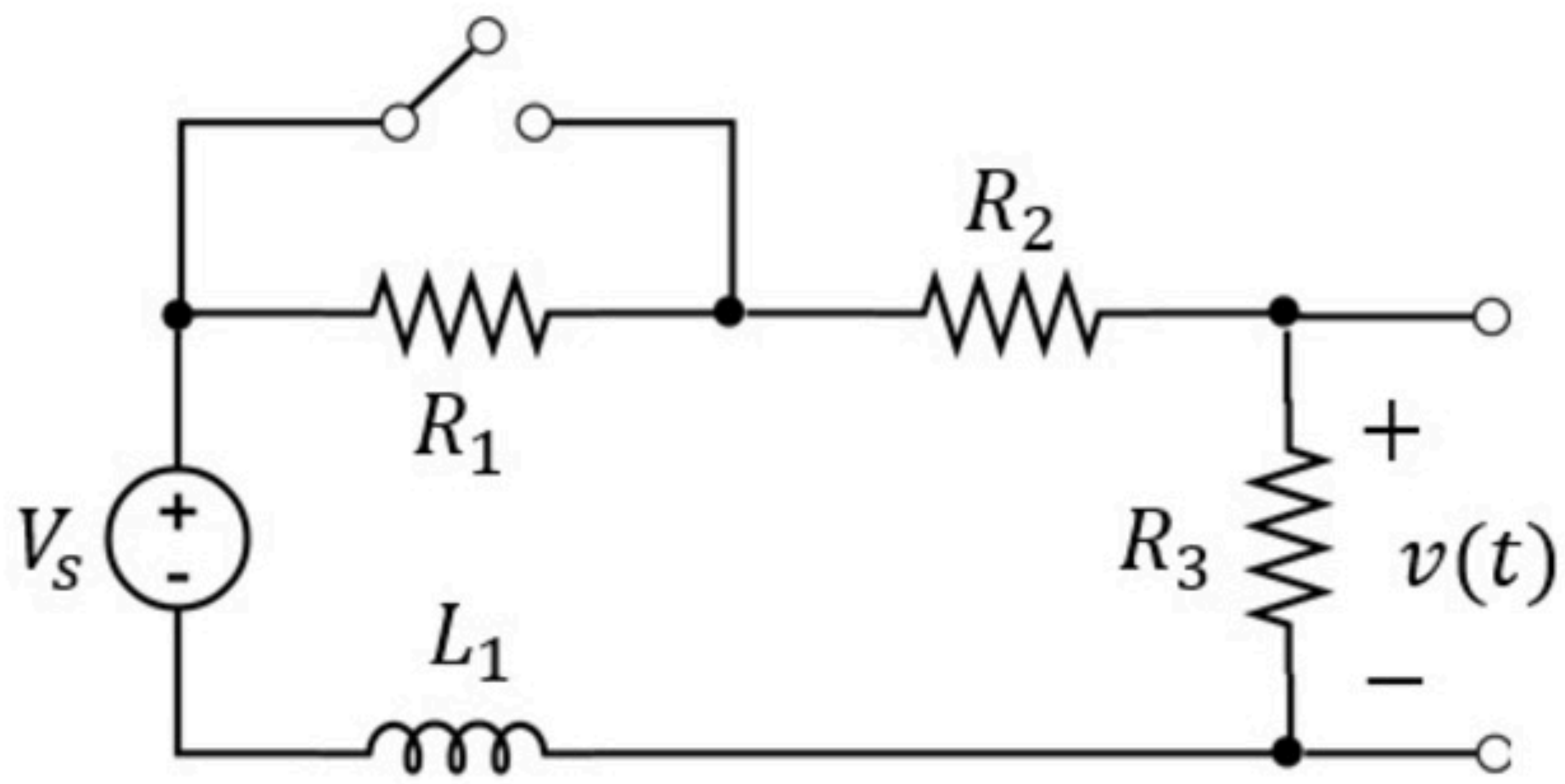
First order circuits 007

Problem has been graded.

The switch closes at time $t = 0$ and we measure

$$v(t) = 24 - 12e^{-t/2\,\mu s}\ \text{V} \qquad \text{for } t > 0$$

Find the values of R_1, R_2, and L_1.



Given Variables:

Vs : 48 V

R3 : 12 kohm

Calculate the following:

R1 (ohm) :

24000

R2 (ohm) :

12000

L1 (H) :

0.048

Hint: Consider v(inf) first. Mind the units.

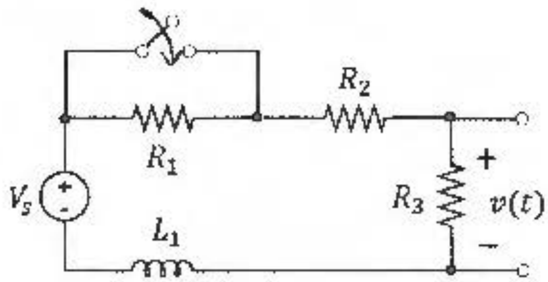
The switch closes at time $t = 0$ and we measure

$$v(t) = 24 - 12e^{-t/2\mu s}\ \text{V} \qquad \text{for } t > 0$$

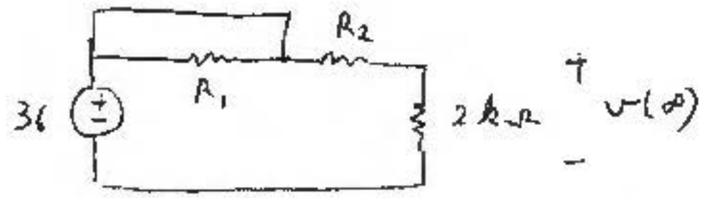
Vs : 36 V

R3 : 2 kohm

Find the values of R_1, R_2, and L_1.



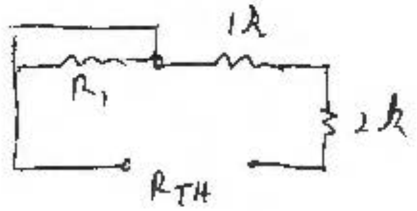
ⓐ $t = \infty$: $v(\infty) = 24V$.



$$v(\infty) = 36 \cdot \frac{2k}{2k + R_2}$$

$\Rightarrow 24 R_2 = 36 \cdot 2k - 24 \cdot 2k \Rightarrow R_2 = 1\,k\Omega \Rightarrow \boxed{R_2 = 1000\,\Omega}$

ⓑ



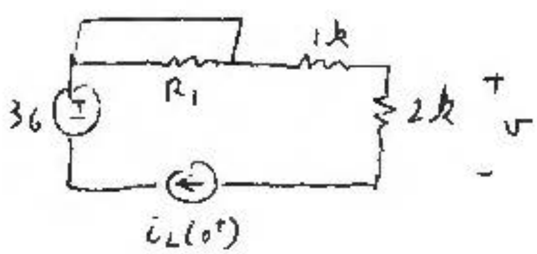
$R_{TH} = 3\,k\Omega$

$\tau = 2\,10^{-6}\,s$
$= \frac{L_1}{R_{TH}}$

$\Rightarrow L_1 = \tau \cdot R_{TH} = 2 \cdot 10^{-6} \cdot 3\,10^{3} = 6 \cdot 10^{-3}$

$\boxed{L_1 = 0.006\ H}$

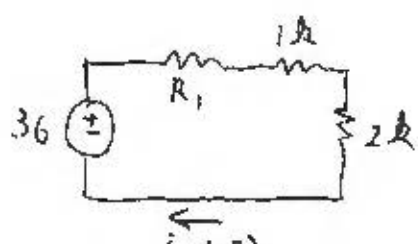
ⓒ $t = 0^+$. $v(0^+) = 12V$



$v(0^+) = i_L(0^+) \cdot 2k$

$\Rightarrow i_L(0^+) = \frac{12}{2k} = 6\ mA$

ⓓ $t = 0^-$:



$i_L(0^-) = i_L(0^+) = 6\ 10^{-3}$

$= \frac{36}{R_1 + 1k + 2k}$

$R_1 = \frac{36}{6 \cdot 10^{-3}} - 1k - 2k \qquad R_1 = 3k$

$\boxed{R_1 = 3000\,\Omega}$

First order circuits 008

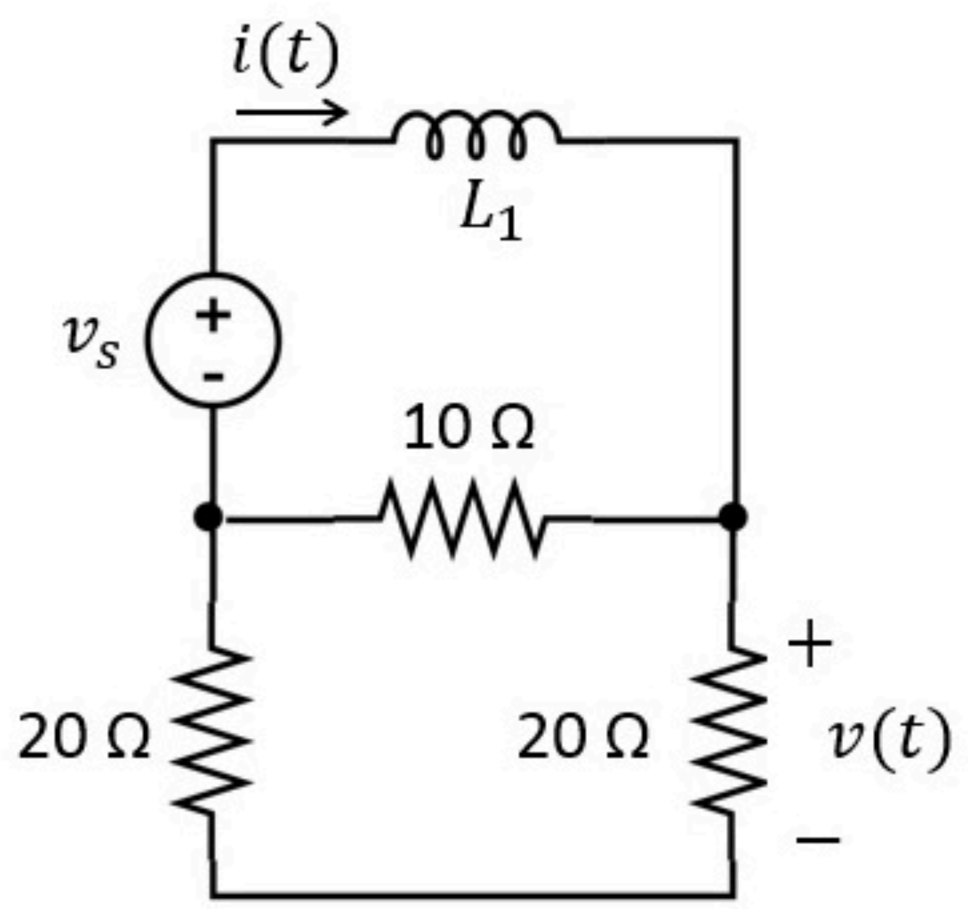
Unlimited Attempts.

When $t < 0$, $v_s = V_0$

When $t > 0$, $v_s = V_1$

Find $i(t) = A_1 + B_1 \cdot e^{-t/\tau 1}$ for $t > 0$

and $v(t) = A_2 + B_2 \cdot e^{-t/\tau 2}$ for $t > 0$



Given Variables:

V0 : 24 V

V1 : 32 V

L1 : 4 mH

Calculate the following:

A1 (A) :

4 ✓

B1 (A) :

-1 ✓

tau1 (ms) :

0.5 ✓

A2 (V) :

16 ✓

B2 (V) :

-4 ✓

tau2 (ms) :

0.5 ✓

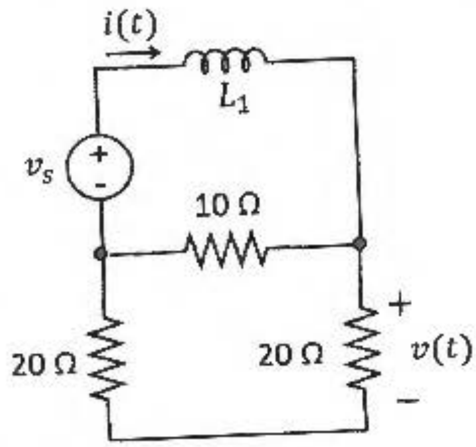
Hint: What is the current iL for t < 0? Note where A and B are.

When $t < 0$, $v_s = V_0$ — V0 : 40 V

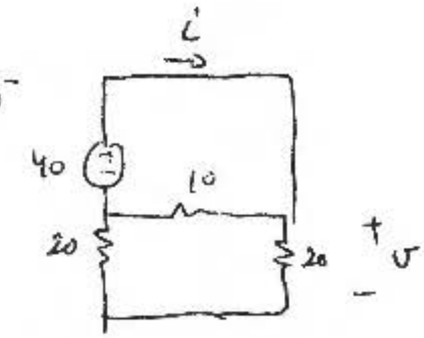
When $t > 0$, $v_s = V_1$ — V1 : 64 V

Find $i(t) = A_1 + B_1 \cdot e^{-t/\tau_1}$ for $t > 0$ — L1 : 2 mH

and $v(t) = A_2 + B_2 \cdot e^{-t/\tau_2}$ for $t > 0$



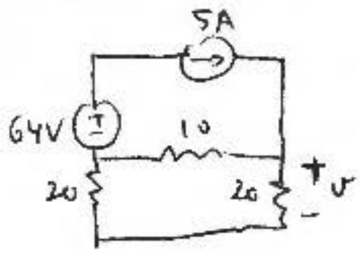
(a) $t = 0^-$



$$i(0^-) = \frac{40}{10 /\!/ 40} = \frac{40}{8} = 5\text{ A}$$

$$v(0^-) = 40 \cdot \frac{20}{20+20} = 20\text{ V}$$

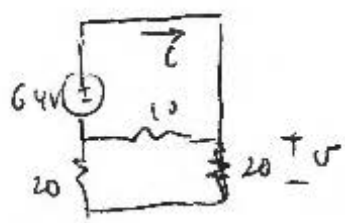
(b) $t = 0^+$



$$i(0^+) = 5\text{ A}$$

$$v(0^+) = (5\text{ A}) \cdot (10 /\!/ 40) \frac{20}{20+20} = 20\text{ V}$$

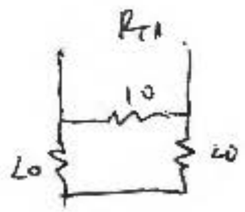
(c) $t = \infty$



$$i(\infty) = \frac{64}{10 /\!/ 40} = 8\text{ A}$$

$$v(\infty) = 64 \cdot \frac{20}{20+20} = 32\text{ V}$$

(d)



$$R_{TH} = 10 /\!/ 40 = 8\,\Omega$$

$$\tau = \frac{L_1}{R_{TH}} = \frac{2 \cdot 10^{-3}}{8} = 0.25 \cdot 10^{-3}$$

$A_1 = i(\infty) \Rightarrow \boxed{A_1 = 8\text{ A}}$ $\qquad$ $A_2 = v(\infty) \Rightarrow \boxed{A_2 = 32\text{ V}}$

$B_1 + A_1 = i(0^+) \Rightarrow \boxed{B_1 = -3\text{ A}}$ $\qquad$ $B_2 + A_2 = v(0^+) \Rightarrow \boxed{B_2 = -12\text{ V}}$

$$\boxed{\tau_1 = \tau_2 = 0.25\text{ ms}}$$

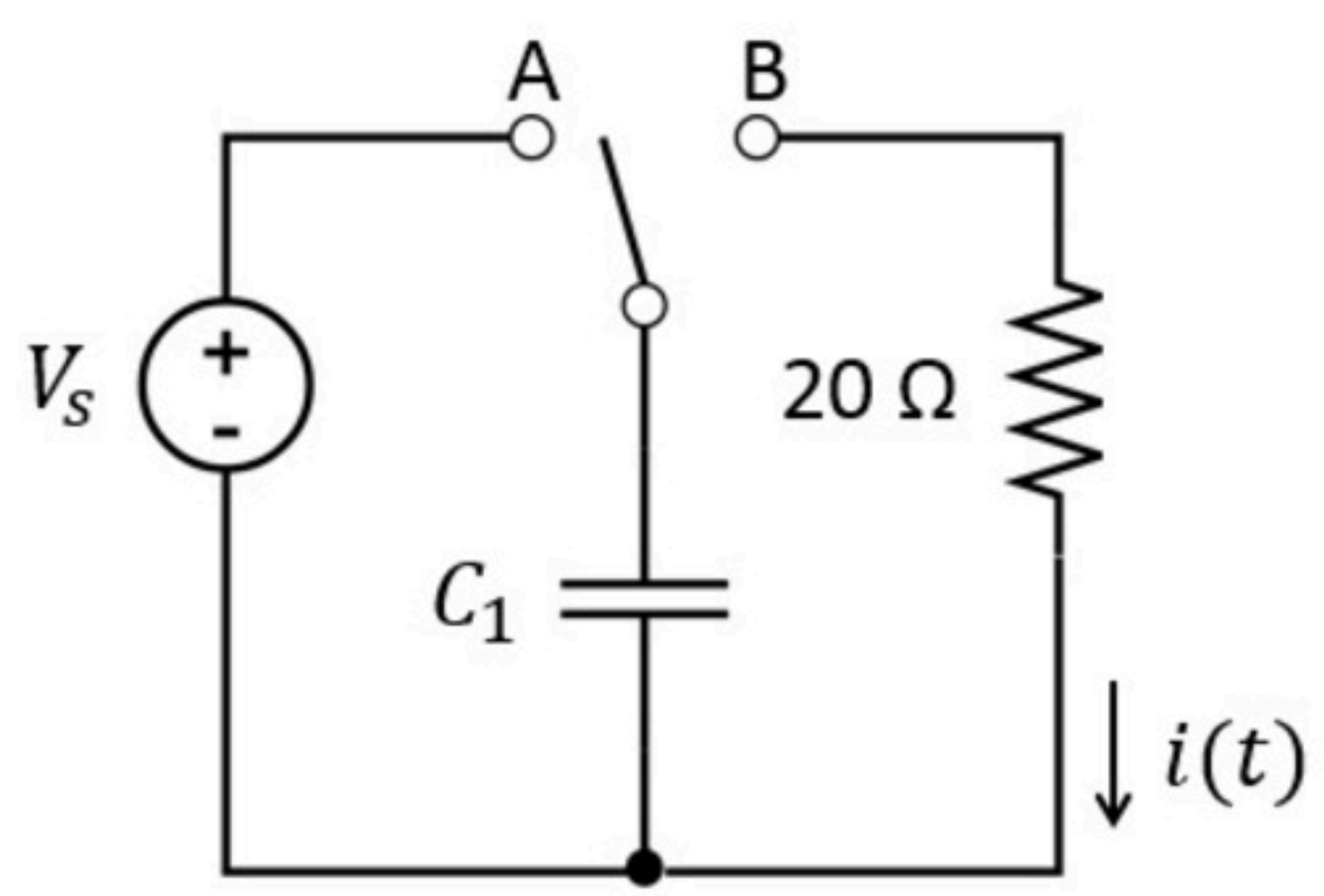
First order circuits 009

Problem has been graded.

For $t < 0$, the switch has been in position *A* for a long time. At time $t = 0$, it moves from *A* to *B*.

Find the time t_1 it takes to reduce the capacitor voltage to 37% of its initial voltage ($\frac{1}{e} \approx .37$).

What is the total energy *E* received by the resistor from the moment the switch is flipped until the capacitor is completely discharged?



Given Variables:

Vs : 12 V

C1 : 2 uF

Calculate the following:

t1 (ms) :

0.04

E (mJ) :

0.144

Hint: To calculate E, consider where the energy is coming from. Verify by integrating the P(t).

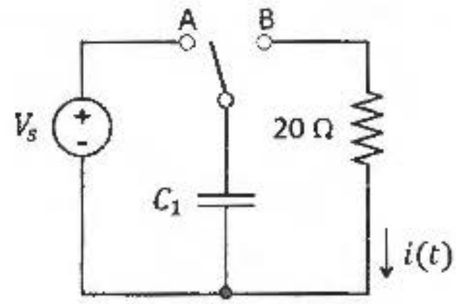
For $t < 0$, the switch has been in position A for a long time. At time $t = 0$, it moves from A to B.

Find the time t_1 it takes to reduce the capacitor voltage to 37% of its initial voltage ($\frac{1}{e} \approx .37$).

What is the total energy E received by the resistor from the moment the switch is flipped until the capacitor is completely discharged?

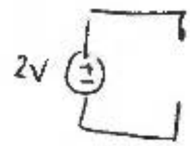
Vs : 2 V

C1 : 2 uF



1

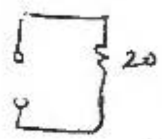
ⓐ $t = 0^-$:



$v_c(0^-) = 2V$

ⓑ $t = 0^+$: $v_c(0^+) = 2V$

ⓒ $t = \infty$



$v_c(\infty) = 0V$

ⓓ $R_{TH} = 20\,\Omega$

$\tau = R_{TH} \cdot C = 40\,\mu s$

$\Rightarrow v_c(t) = 2e^{-t/\tau}$ $\qquad v_c(t_1) = \frac{1}{e} \cdot v_c(0^+) = \frac{2}{e}$

$\Rightarrow 2e^{-t_1/\tau} = 2e^{-1} \Rightarrow \frac{t_1}{\tau} = 1 \Rightarrow t_1 = \tau = 40\,\mu s$ $\qquad \boxed{t_1 = 0.04\ ms}$

2 $E_{rec.} = E_{supplied} \Rightarrow$ Coming from Capacitor

$E = \frac{1}{2}CV^2 = \frac{1}{2} 2 \cdot 10^{-6}\, 2^2 = 4\,\mu J$ $\qquad \boxed{E = 0.004\ mJ}$

OR $\quad i(t) = Ae^{-t/\tau} + B = 0{,}1\, e^{-t/\tau}$

$i(0^+) = \frac{2V}{20\,\Omega} = 0.1\ A$ $\qquad E = \int P = \int_0^\infty i^2 R = 20 \cdot (0.1)^2 \int_0^\infty e^{-\frac{2t}{\tau}} dt$

$i(\infty) = 0$ $\qquad = -0.2 \frac{\tau}{2} e^{-2t/\tau} \Big|_0^\infty$

$= 0.1\,\tau \Rightarrow E = 4\,\mu J$

PP First order circuits 011

Unlimited Attempts.

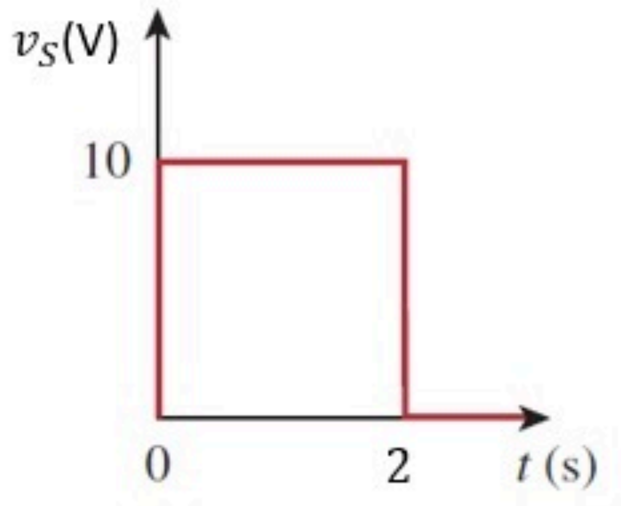
The voltage source behaves as shown (and assume it has been zero for a long time for $t < 0$).

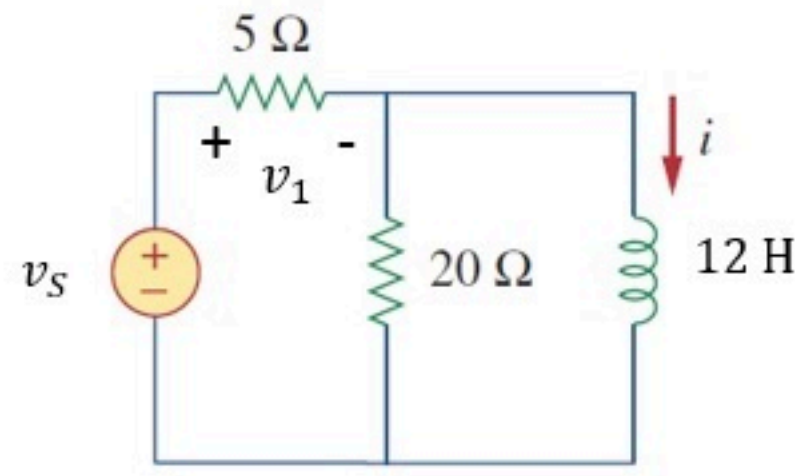
Find $v_1(t) = D$ for $t = 0^-$

$= A_1 \cdot e^{-t/\tau_1} + B_1$ for $0 < t < 2$ s

$= A_2 \cdot e^{-(t-2\,s)/\tau_2} + B_2$ for 2 s $< t$

Note, for your calculations, use: $e^{-1/1.5} \approx 0.5$





Given Variables:

. : . .

Calculate the following:

D (V) :

0

A1 (V) :

-8

B1 (V) :

10

tau1 (s) :

3

A2 (V) :

4

B2 (V) :

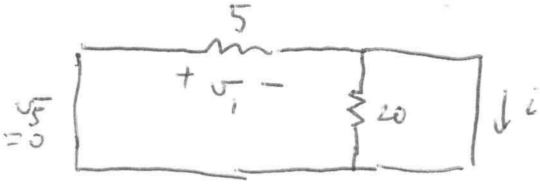
0

tau2 (s) :

3

Hint: Find the result of the first transition first.

(a) $\boxed{v_s = 0}$ $\quad t = 0^-$:

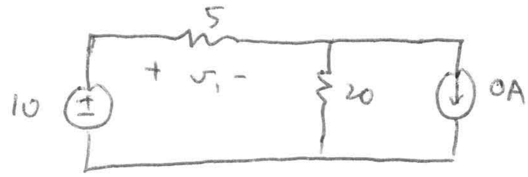


$v_1(0^-) = 0\,V$

$\boxed{D = 0V}$

$i(0^-) = 0A$

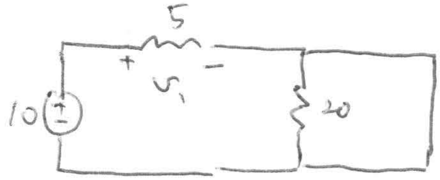
(b) $\boxed{v_s = 10}$ $\quad t = 0^+$:



$i(0^+) = 0A \rightsquigarrow$ open

$v_1(0^+) = 10 \cdot \frac{5}{5+20} = 2V$

$t = \infty$:



$i(\infty) = \frac{10}{5} = 2A$

$v_1(\infty) = 10V$

R_{TH}: $\quad R_{TH} = 5 /\!/ 20 = 4\,\Omega \Rightarrow \tau = \frac{L}{R} = \frac{12}{4} = 3s$

$\boxed{A_1 = -8V}$

$\boxed{B_1 = 10V}$

$\boxed{\tau_1 = 3s}$

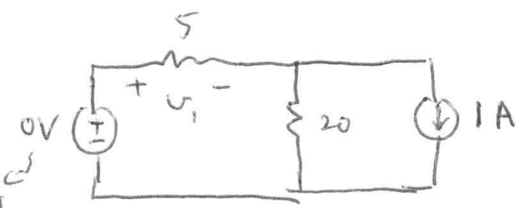
$v_1(t) = -8e^{-t/3} + 10, \quad 0 < t < 2$

$i(t) = -2e^{-t/3} + 2, \quad 0 < t < 2$

$t = 2^-$: $\quad i(2^-) = -2e^{-2/3} + 2 = -2e^{-\frac{1}{1.5}} + 2 = -2 \cdot \frac{1}{2} + 2 = 1A$

(c) $\boxed{v_s = 0}$

$t = 2^+$:

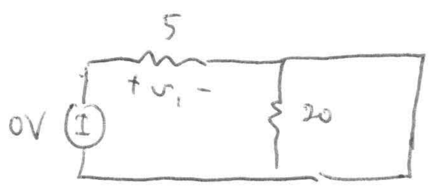


SHORT

$i(2^+) = i(2^-) = 1A$

$v_1 = 1 \cdot (5 /\!/ 20) = 1 \cdot 4 = 4V$

$t = \infty$:



$i(\infty) = 0$

$v_1(\infty) = 0V$

R_{TH}: SAME AS BEFORE $\Rightarrow \tau = 3s$

$v_1(t) = 4e^{-t/3} + 0$

$\boxed{A_2 = 4V}$

$\boxed{B_2 = 0V}$

$\boxed{\tau_2 = 3s}$

Second order circuits 001

Problem has been graded.

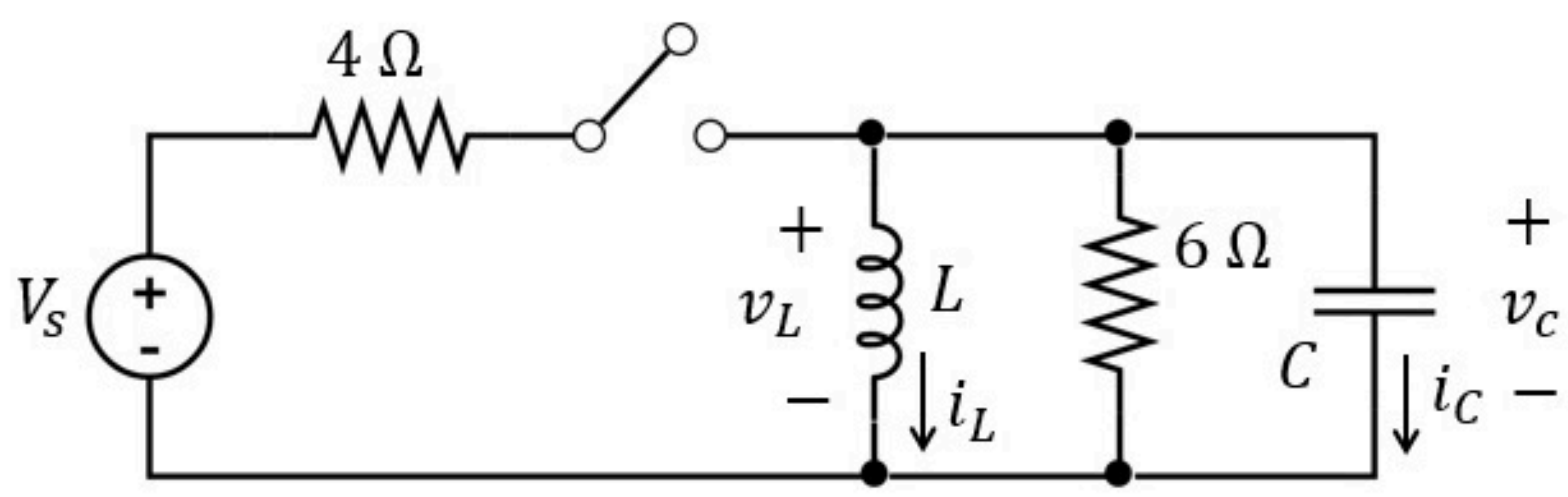
The switch opens at time t = 2 s. Before the switch opens, the system has reached steady state.

Find these voltages and currents (i.e., just before the switch opens):

$$i_{L1} = i_L(2^-) \quad v_{L1} = v_L(2^-) \quad i_{C1} = i_C(2^-) \quad v_{C1} = v_C(2^-)$$

Find these voltages and currents (i.e., just after the switch opens):

$$i_{L2} = i_L(2^+) \quad v_{L2} = v_L(2^+) \quad i_{C2} = i_C(2^+) \quad v_{C2} = v_C(2^+)$$



Given Variables:

Vs : 8 V
C : 2 nF
L : 2 mH

Calculate the following:

iL1 (A) :

2

vL1 (V) :

0

iC1 (A) :

0

vC1 (V) :

0

iL2 (A) :

2

vL2 (V) :

0

iC2 (A) :

-2

vC2 (V) :

0

Hint: In steady state, the capacitor and inductor behave as an open and short respectively.

The switch opens at time $t = 2$ s. Before the switch opens, the system has reached steady state.

Vs : 20 V

C : 2 nF

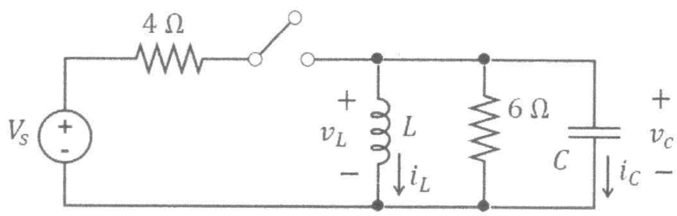
L : 1 mH

Find these voltages and currents (i.e., just before the switch opens):

$i_{L1} = i_L(2^-) \quad v_{L1} = v_L(2^-) \quad i_{C1} = i_C(2^-) \quad v_{C1} = v_C(2^-)$

Find these voltages and currents (i.e., just after the switch opens):

$i_{L2} = i_L(2^+) \quad v_{L2} = v_L(2^+) \quad i_{C2} = i_C(2^+) \quad v_{C2} = v_C(2^+)$



(a) $t = 2^-$: SWITCH IS CLOSED

$i_L = \frac{20}{4} = 5$

$\boxed{v_L(2^-) = 0\,V}$ $\boxed{i_c(2^-) = 0\,A}$ $\boxed{v_c(2^-) = 0\,V}$ $\boxed{i_L(0^-) = 5\,A}$

(b) $t = 2^+$

$\boxed{i_L(2^+) = 5\,A}$ $\boxed{v_c(2^+) = 0\,V}$ $\boxed{v_L(2^+) = 0\,V}$ $\boxed{i_c(2^+) = -5\,A}$