# Discussion Session 5:

## Fourier Series Tutorial:

Fourier Series is a mathematical tool used to represent periodic function as a sum of sines and cosines. A periodic signal $f(t)$ with period $T_0$ can be expressed as:

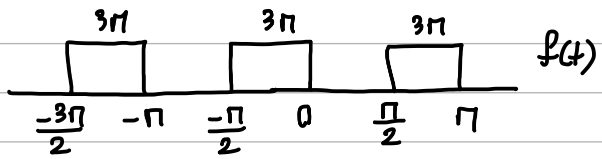
$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jw_n t} = \sum_{n=-\infty}^{\infty} \underbrace{F_n}_{\text{Fourier Coefficient}} e^{jnw_0 t}$$

$$w_n = n \frac{2\pi}{T_0} = n w_0$$

$F_n$ are given by :

$$F_n = \frac{1}{T_0} \int_{T_0} F(t)\, e^{-jw_n t}\, dt$$

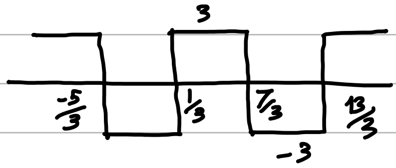
Example # 1:




a) What is the input's fundamental frequency $(w_0)$ ?

b) What is the Fourier Series coefficient $(F_n)$ ?

Example #2:




a) What is the input's fundamental frequency ($\omega_0$)?

b) What is this input's Fourier Series coefficient ($F_n$)?

If $f(x)$ is a periodic function with period $T_0$, it can be expressed as:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{2\pi}{T_0} n x\right) + b_n \sin\left(\frac{2\pi}{T_0} n x\right) \right]$$

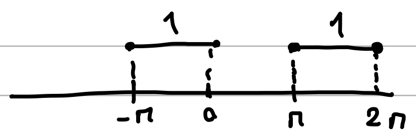
with $a_0 = \frac{1}{T_0} \int_{T_0} f(x)\, dx$

for $n \neq 0$

$$a_n = \frac{2}{T_0} \int_{T_0} f(x) \cos\left(\frac{2\pi}{T_0} n x\right) dx$$

$$b_n = \frac{2}{T_0} \int_{T_0} f(x) \sin\left(\frac{2\pi}{T_0} n x\right) dx$$

Example #3:




Find $a_n$ and $b_n$.

Pick an appropiate value of $x$, to show that: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$