Name PID

UNIVERSITY OF CALIFORNIA, SAN DIEGO

Electrical and Computer Engineering Department

ECE 65 – Spring 2021

Components and Circuits lab

Final Exam

You should submit your handwritten solutions in a PDF format to Gradescope on Thursday, 6/10, by 2:30 pm (Pacific Time).

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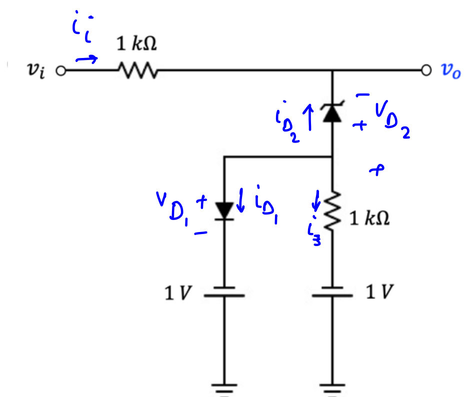
Problem 1. (10 points)

The diodes in the below circuit have $V_{D0} = 0.7\ V$, and $V_Z = 5\ V$.

a) Write the possible cases of the operation of the diodes.

b) For each case, include the calculation of finding the relationship between v_o and v_i and the range of v_i.

c) Sketch the output signal when $v_i = \sin(\omega t)$. You do not need to label the time axis.



Show your work.

Case 1: D_2 in Zener & D_1 is off

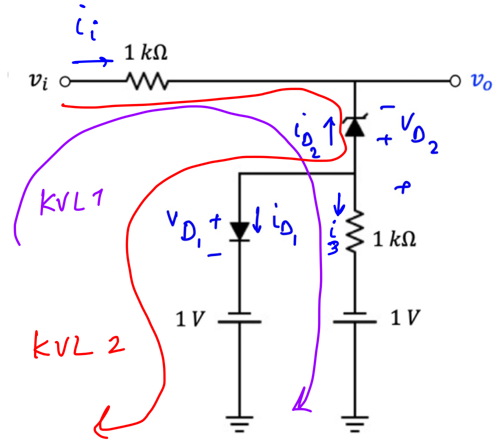
Case 2: D_2 in Zener & D_1 is ON

Case 3: D_2 is ON & D_1 is off

Case 4: D_1 & D_2 off

$i_i = -i_{D_2}$ and $i_{D_1} + i_{D_2} + i_3 = 0$

Case 1 analysis:



D_2 in Zener & D_1 is off

$V_{D_2} = -V_Z$, $i_{D_2} \leq 0$, $i_{D_1} = 0$, $V_{D_1} < V_{D_0}$

KVL 1:

$$V_i = 1k\Omega \times i_i - V_{D_2} + 1k\Omega \times i_3 - 1$$

$i_i = -i_{D_2}$ and $i_{D_1} + i_{D_2} + i_3 = 0 \rightarrow i_3 = -i_{D_2}$

$$V_i = 1k\Omega \times (-i_{D_2}) + 5 + 1k\Omega \times (-i_{D_2}) - 1$$

$$\rightarrow i_{D_2} = \frac{-V_i + 4}{2k\Omega} \qquad i_{D_2} \leq 0 \Longrightarrow V_i \geq 4V$$

V_{D_1} must also be less than 0.7 V

KVL 2:

$$V_i = 1k\Omega \times i_i - V_{D_2} + V_{D_1} - 1 = 1k\Omega \times (-i_{D_2}) + 4 + V_{D_1}$$

$$V_i = 1k\Omega \times \left(\frac{+V_i - 4}{2k\Omega}\right) + 4 + V_{D_1} \rightarrow \frac{1}{2}V_i - 2 = V_{D_1}$$

$$V_{D_1} < V_{D_0} \rightarrow \frac{1}{2}V_i - 2 < 0.7V \rightarrow V_i < 5.4V$$

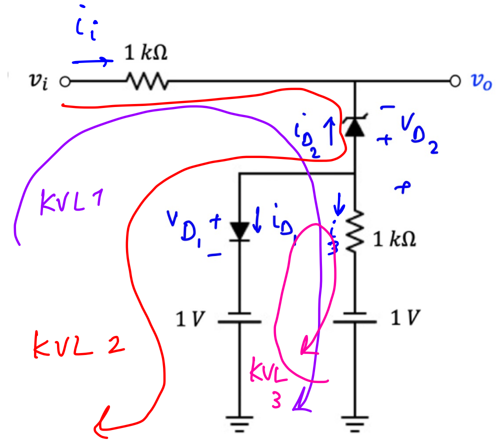
the range of V_i for this case is $4 \leq V_i < 5.4V$

$$V_0 = -V_{D_2} + 1k\Omega \times i_3 - 1 = 5V + 1k\Omega \times (-i_{D_2}) - 1 = 4V + 1k\Omega \times \frac{V_i - 4}{2k\Omega}$$

$$V_0 = \frac{1}{2}V_i + 2$$

Case 2 analysis:

D_2 in Zener & D_1 is ON



$V_{D_2} = -V_Z$, $i_{D_2} \leq 0$, $i_{D_1} \geq 0$, $V_{D_1} = V_{D_0}$

KVL 1:

$$V_i = 1k\Omega \times i_i - V_{D_2} + 1k\Omega \times i_3 - 1$$

$$i_i = -i_{D_2} = i_{D_1} + i_3$$

KVL 3:

$$1V - V_{D_1} + 1k\Omega \times i_3 - 1 = 0 \longrightarrow 1k\Omega \times i_3 = V_{D_0}$$

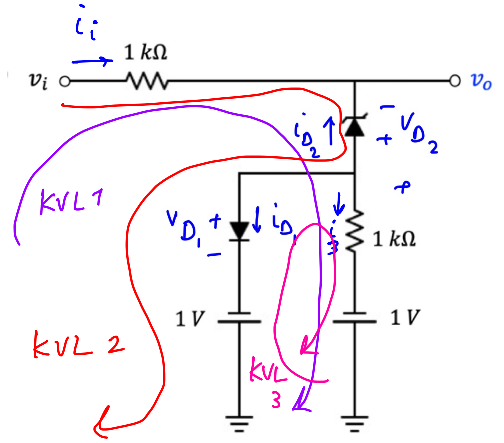
$$V_i = 1k\Omega \times (i_{D_1} + \frac{0.7}{1k\Omega}) + 5 + 0.7 - 1 \longrightarrow i_{D_1} = \frac{V_i - 5.4}{1k\Omega}$$

$$i_{D_1} \geq 0 \longrightarrow V_i \geq 5.4\,V$$

the range of V_i for this case is $V_i \geq 5.4\,V$

$$V_o = -V_{D_2} + V_{D_1} - 1 = 4.7\,V \longrightarrow V_o = 4.7\,V$$

Case 3 analysis:



D_2 is ON & D_1 is off

$V_{D_2} = V_{D_0}$, $i_{D_2} \geq 0$, $i_{D_1} = 0$, $V_{D_1} < V_{D_0}$

KVL 1:

$$V_i = 1k\Omega \times i_i - V_{D_2} + 1k\Omega \times i_3 - 1$$

$$i_i = -i_{D_2} \quad , \quad i_3 = -i_{D_2}$$

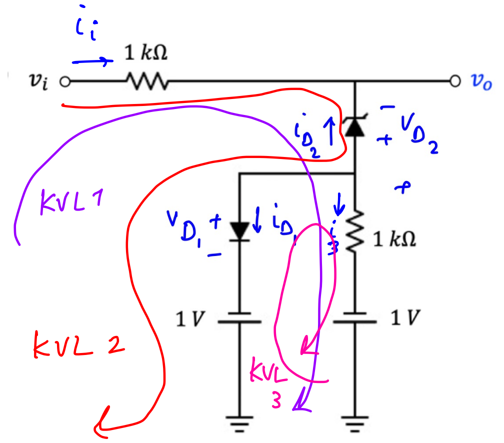
$$V_i = 1k\Omega \times (-i_{D_2}) - 0.7 + 1k\Omega \times (-i_{D_2}) - 1$$

$$\rightarrow i_{D_2} = \frac{-V_i - 1.7}{2k\Omega} \qquad i_{D_2} \geq 0 \implies V_i \leq -1.7 \text{ V}$$

$$V_o = 1k\Omega \times (-i_i) + V_i = 1k\Omega \times (i_{D_2}) + V_i = -\frac{V_i}{2} - \frac{1.7}{2} + V_i = \frac{1}{2} V_i - \frac{1.7}{2}$$

$$V_o = \frac{1}{2} V_i - \frac{1.7}{2} \text{ V}$$

Case 4 analysis :



D_2 & D_1 are off

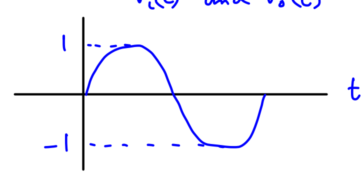
$-V_2 < V_{D_2} < V_{D_0}$, $i_{D_2} = 0$, $i_{D_1} = 0$, $V_{D_1} < V_{D_0}$

$-1.7\,V < V_i < 4\,V$

$V_0 = 1k\Omega \times (-i_i) + V_i = V_i \longrightarrow V_0 = V_i$

c) $V_i(t) = \sin\omega t \rightarrow$ The amplitude of V_i will change between -1 V and 1V. Both diodes will be off in this range. $\Rightarrow V_0(t) = \sin\omega t$

$V_i(t)$ and $V_o(t)$



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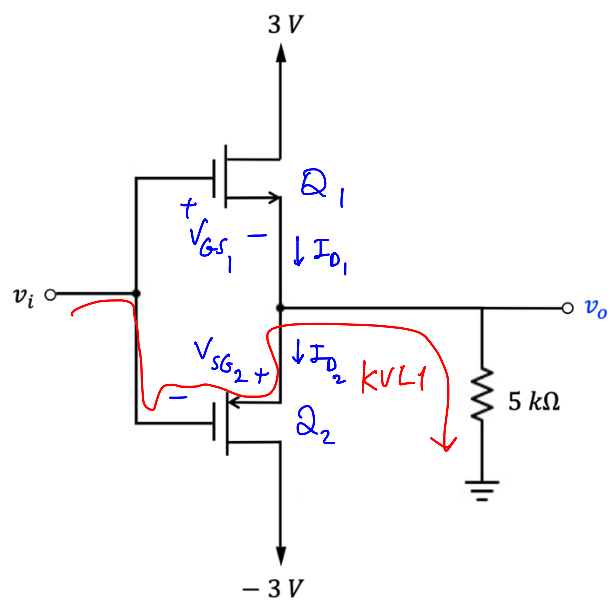
Problem 2. (10 points)

In the following circuit, $V_{tn} = -V_{tp} = 1\,V$, $k_n = k_p = 0.5\,mA/V^2$, and $\lambda = 0$ for both transistors.

If $v_i = -3\,V$, find the drain currents of <u>both</u> transistors and v_o .



Show your work.

Assume both Q_1 & Q_2 are off : $I_{D_1} = I_{D_2} = 0$ & $V_{GS_1} < V_{tn}$ & $V_{SG_2} < |V_{tp}|$

KVL1 : $V_i = -V_{SG_2} + V_o = -V_{SG_2} \rightarrow V_{SG_2} = 3\,V > 1\,V \Rightarrow$ assumption is wrong.

$V_{GD_2} = 0\,V \Rightarrow Q_2$ is in saturation. $I_{D_2} = \frac{1}{2} k_p V_{ov_2}^2 = \frac{1}{2} \times \frac{1}{2} \times V_{ov_2}^2$

From KVL 1 : $V_i = -V_{SG_2} - 5k\Omega \times I_{D_2} \rightarrow -3V = -V_{SG_2} - 5k\Omega \times I_{D_2}$

$-|V_{tp}| + V_{SG_2} = 3\,V - 5k\Omega \times I_{D_2} - |V_{tp}| \rightarrow V_{ov_2} = 2\,V - 5k\Omega \times I_{D_2} \rightarrow V_{ov} = 1.73\,V$

Assume Q_1 is ON and Q_2 is off

Q_1 is ON $\longrightarrow$ $V_{ov_1} \geq 0$ and $I_{D_1} \geq 0$ $\rightarrow$ $V_{S_1} = 5k\Omega \times I_{D_1} \geq 0$

$V_{G_1} = V_i = -3V$, $V_{GS_1} = -3 - V_{S_1} \leq -3$ $\longrightarrow$ $V_{ov_1} = V_{GS_1} - V_{tn} = -4 - V_{S_1} \leq -4V$

$V_{ov_1} < 0 \longrightarrow Q_1$ is not ON.

assumption was wrong.

Assume Q_1 is off and Q_2 is ON

$V_{GD_2} = 0V \Longrightarrow Q_2$ is in saturation. $I_{D_2} = \frac{1}{2} k_p V_{ov_2}^2 = \frac{1}{2} \times \frac{1}{2} \times V_{ov_2}^2$

From KVL 1 : $V_i = -V_{SG_2} - 5k\Omega \times I_{D_2} \rightarrow -3V = -V_{SG_2} - 5k\Omega \times I_{D_2}$

$-|V_{tp}| + V_{SG_2} = 3V - 5k\Omega \times I_{D_2} - |V_{tp}| \longrightarrow V_{ov_2} = 2V - 5k\Omega \times I_{D_2}$

$5V_{ov_2}^2 + 4V_{ov_2} - 8 = 0$

$$\begin{cases} I_{D_2} = \frac{1}{2} k_p V_{ov_2}^2 = \frac{1}{2} \times \frac{1}{2} \times V_{ov_2}^2 \\ V_{ov_2} = 2V - 5k\Omega \times I_{D_2} \end{cases} \longrightarrow V_{ov_2} = 2V - 5 \times \frac{1}{4} V_{ov_2}^2 \longrightarrow V_{ov} = 0.93 \ V$$

$I_{D_2} = \frac{1}{2} \times \frac{1}{2} \times V_{ov_2}^2 = \frac{1}{4} (mA/V^2) \times (1.73)^2 \longrightarrow \boxed{I_{D_2} = 0.216 \ mA}$

$V_o = -5k\Omega \times I_{D_2} = -1.08 \ V$ $\boxed{V_o = -1.08 \ V}$

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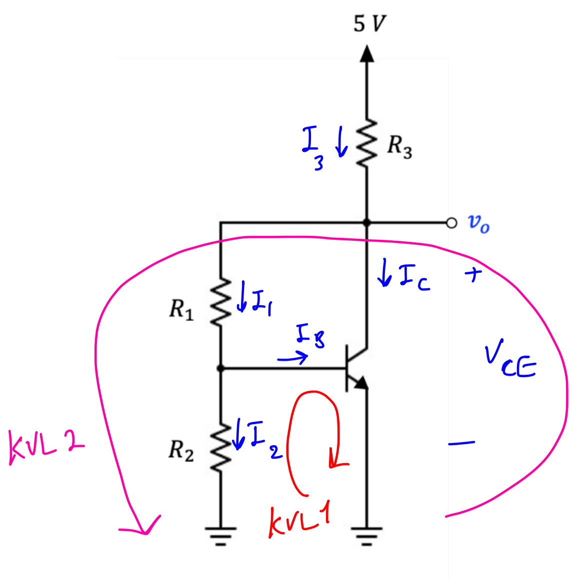
Problem 3 (5 points)

Design the following circuit (find the resistor values) to have $I_C = 1\ mA$ and $V_C = 2.5$ V. Assume $\beta = 100$, $V_D = 0.7$ V, and $V_{sat} = 0.2$V.

The resistors must have finite non-zero values.



Show your work.

$I_C = 1mA > 0 \implies$ BJT is ON $\implies V_{BE} = 0.7\ V$

$V_{CE} = V_C - V_E = V_C - 0 = V_C = 2.5V$

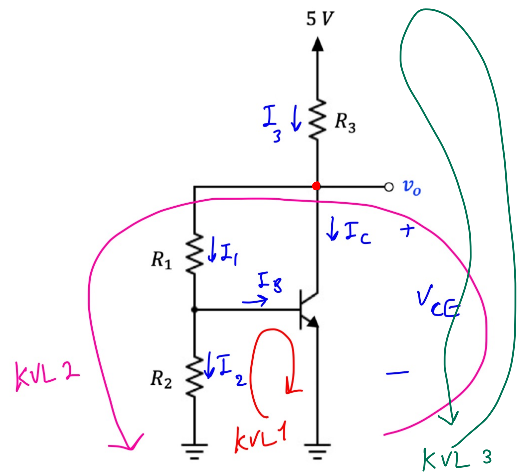
$V_{CE} > V_{D_0} \implies$ BJT is in active mode. $\implies I_C = \beta I_B$

KVL 1: $R_2 I_2 = V_{BE} = 0.7\ V$

KVL 2: $V_{CE} = R_1 I_1 + R_2 I_2 = R_1 I_1 + 0.7\ V \implies R_1 I_1 = 1.8\ V$

KVL 3:

$5\,V = R_3 I_3 + V_{CE} \Rightarrow \boxed{R_3 I_3 = 2.5\,V}$



KCL at Collector: $I_3 = I_c + I_1 = 1\,mA + I_1$

KCL at Base: $I_1 = I_B + I_2 = 0.01\,mA + I_2$

Choose $\boxed{R_2 = 10k} \rightarrow I_2 = \frac{0.7\,V}{10\,k} = 0.07\,mA$

$\boxed{R_1 I_1 = 1.8\,V}$

$\boxed{R_2 I_2 = 0.7\,V}$

$I_1 = I_B + I_2 = 0.08\,mA$

$R_1 I_1 = 1.8\,V \rightarrow R_1 = \frac{1.8\,V}{0.08\,mA} = 22.5\,k\Omega$

$\boxed{R_1 = 22.5\,k\Omega}$

$I_3 = I_c + I_1 = 1.08\,mA$

$R_3 I_3 = 2.5\,V \rightarrow R_3 = \frac{2.5\,V}{1.08\,mA} = 2.32\,k\Omega$

$\boxed{R_3 = 2.32\,k\Omega}$

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Problem 4

In the below amplifier circuit,

a) Find the DC emitter currents and the DC Base node voltages of Q_1 and Q_2

b) Find the small signal parameters (g_m and r_π) for Q_1 and Q_2.

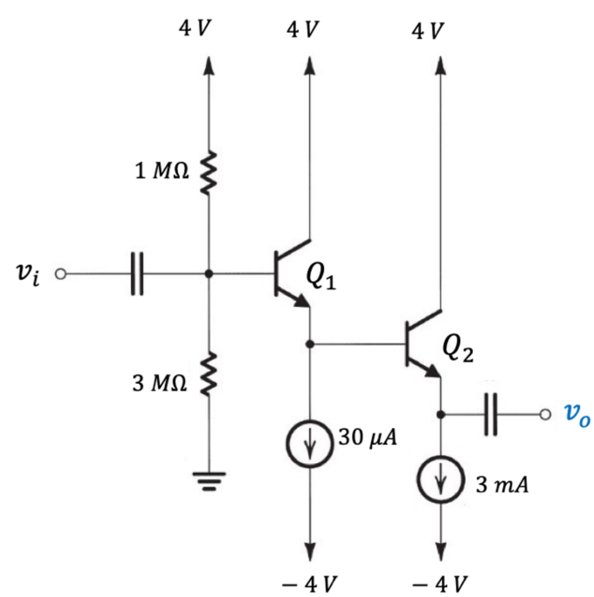
c) Draw the small signal equivalent circuit.

d) If a load resistance $R_L = 10 \;\; k\Omega$ is connected to the output terminal, and a signal source with $R_{sig} = 0$ is connected to the input terminal, find $A = \frac{v_o}{v_{sig}}$.

Assume Q_1 has $\beta_1 = 60$ and Q_2 has $\beta_2 = 100$, $V_{D0} = 0.7V$, $V_T = 25mV$. Neglect the early effect in the bias and signal circuits. The capacitors are short for the signal circuit.

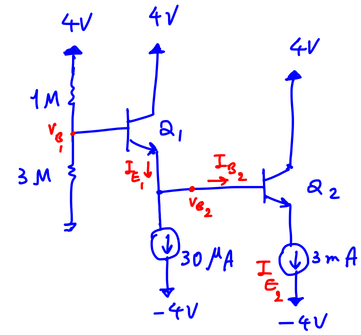
Hint: The input resistance of the second stage will act as the load resistor for the first stage.

For a common-collector amplifier use $A_{vo} = \frac{(1/g_m) \| r_\pi \| R_E \| r_o}{(1/g_m) \| r_\pi}$.



Show your work.

Bias circuit:



a) $\boxed{I_{E_2} = 3\,mA}$

KCL at Emitter of Q_1 : $I_{E_1} = I_{B_2} + 30\,\mu A$

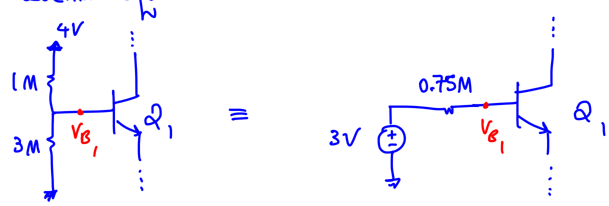
This is an amplifier circuit so the BJTs will be in active mode.

$$I_{B_2} = \frac{I_{E_2}}{1+\beta_2} = \frac{3\,mA}{101} = 29.7\,\mu A$$

$$I_{E_1} = 29.7\,\mu A + 30\,\mu A = 59.7\,\mu A$$

$$\boxed{I_{E_1} = 59.7\,\mu A}$$

The Thevenin equivalent



$$V_{B_1} = 3V - 0.75M\Omega \times I_{B_1}$$

$$I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = \frac{59.7\,\mu A}{61} \simeq 0.98\,\mu A$$

$$V_{B_1} = 3V - 0.75 \times 0.98 = 2.265\ V$$

$$\boxed{V_{B_1} = 2.265\ V}$$

$$V_{B_2} = V_{E_1} = V_{B_1} - V_{BE_1} = 2.265 - 0.7 = 1.565\ V$$

$$\boxed{V_{B_2} = 1.565\ V}$$

b)

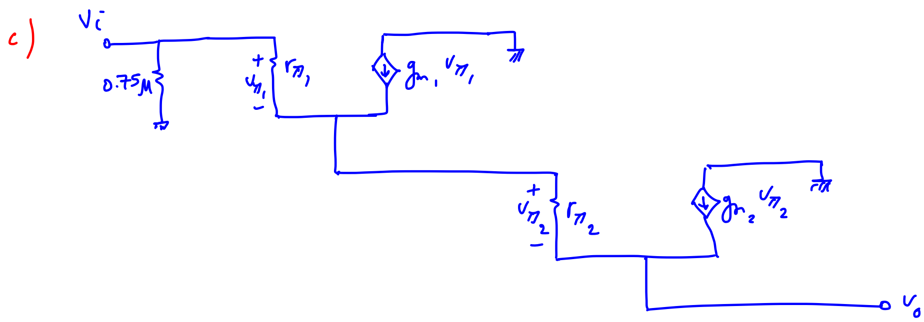
$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{3\,mA \times \frac{100}{101}}{25\,mV} \simeq 118.8\ mA/V$$

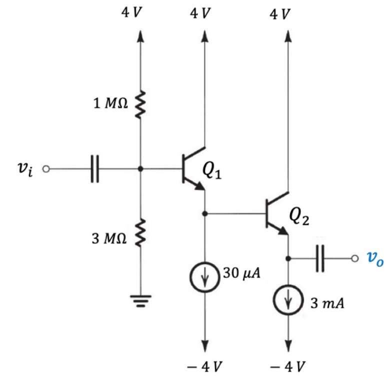
$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{59.7\,\mu A \times \frac{60}{61}}{25\,mV} = 2.35\ mA/V$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 842\ \Omega$$

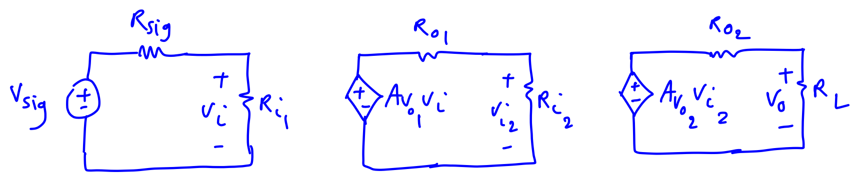
$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = 25.53\ k\Omega$$

Small signal circuit:

c) 



d) We will find R_{i_1}, R_{i_2}, R_{o_1}, R_{o_2}, $A_{v_{o_1}}$, $A_{v_{o_2}}$, and use the voltage amplifier model to answer this question.



$g_{m_2} = 118.8\ mA/V$

$r_{\pi_2} = 842\ \Omega$

$g_{m_1} = 2.35\ mA/V$

$r_{\pi_1} = 25.53\ k\Omega$

$$R_{o_2} = \frac{1}{g_{m_2}} \parallel r_{\pi_2} \simeq 8.32\ \Omega$$

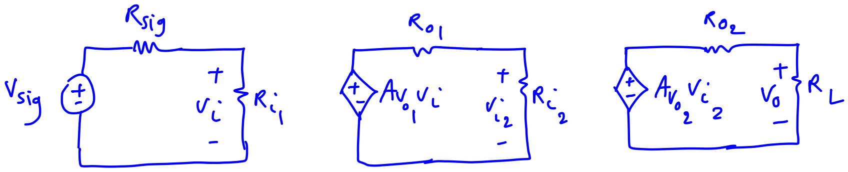
$$R_{i_2} = r_{\pi_2} + (\beta_2 + 1) R_L \simeq 1010.8\ k\Omega$$

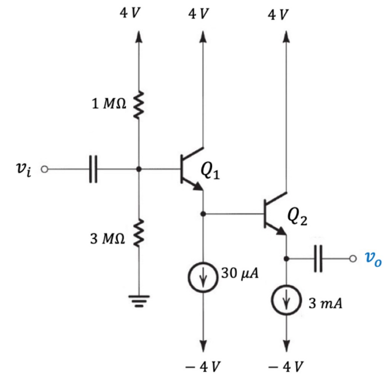
$$R_{o_1} = \frac{1}{g_{m_1}} \parallel r_{\pi_1} \simeq 418\ \Omega$$

$$R_{i_1} = R_{B_1} \parallel [r_{\pi_1} + (\beta_1 + 1) R_{i_2}] \simeq 741\ k\Omega$$

$$A_{v_{o_2}} = \frac{\frac{1}{g_{m_2}} \parallel r_{\pi_2}}{\frac{1}{g_{m_2}} \parallel r_{\pi_2}} = 1\ V/V$$

$$A_{v_{o_1}} = \frac{\frac{1}{g_{m_1}} \parallel r_{\pi_1}}{\frac{1}{g_{m_1}} \parallel r_{\pi_1}} = 1\ V/V$$





$$\frac{V_o}{V_{sig}} = \frac{R_L}{R_L + R_{o_2}} \cdot A_{v_{o_2}} \cdot \frac{R_{i_2}}{R_{i_2} + R_{o_1}} \cdot A_{v_{o_1}} \cdot \frac{R_{i_1}}{R_{i_1} + R_{sig}}$$

$$\frac{V_o}{V_{sig}} = 0.997 \ \text{V/V}$$