ECE 45 – Circuits and Systems
Winter 2025

Homework #2

Due: January 16 at 11:59pm, submitted via GradeScope.

You can make multiple upload attempts to experiment with the system and the best way to upload. You must correctly mark the answers to the problems in GradeScope, e.g. problem 1, problem 2, problem 3, to get full credit. Note that you must tag your problems when uploading to GradeScope or they will not be graded and you will not receive credit. Any regrade requests must be placed through GradeScope within one week of the return of the homework.

Remember, discussion of homework questions is encouraged. Please be absolutely sure to submit your own independent homework solution.

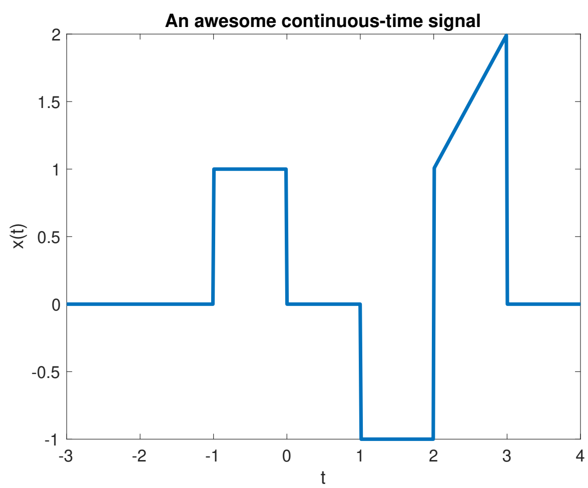


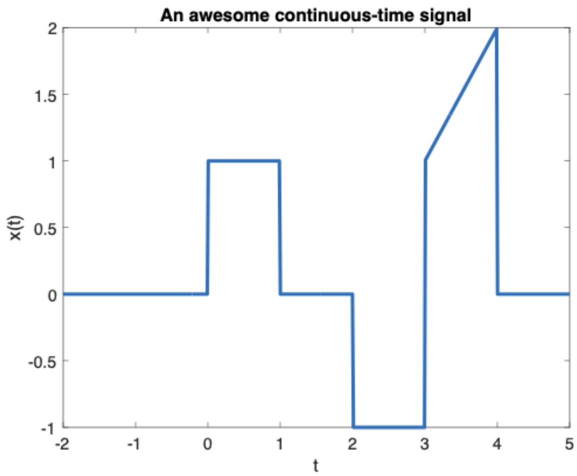
Figure 1: A continuous time function, x(t).

1. The signal $x(t)$ is shown in Fig. 1. Sketch carefully each of the following signals. The values not shown are zero.
   - (a) $x(t-1)$
   - (b) $2x(2-t)$
   - (c) $x(3t-1)+1$
   - (d) $x(t/3-1)+1$
   - (e) Even part of $x(t)$
   - (f) Odd part of $x(t)$
   - (g) $x(t)u(t)$

   Solution:

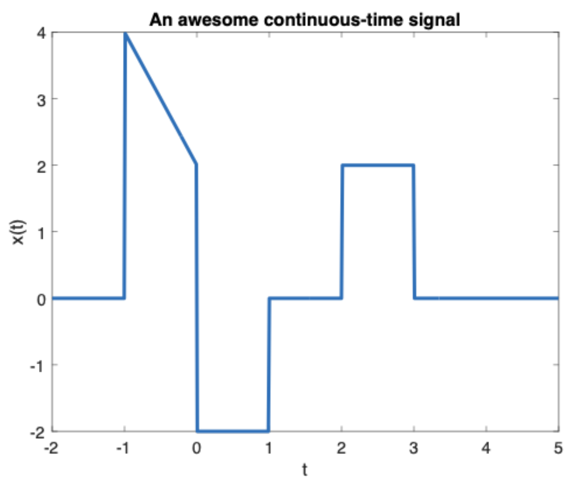
(a) $x(t-1)$

$x(t)$ is shifted 1 unit to the right:



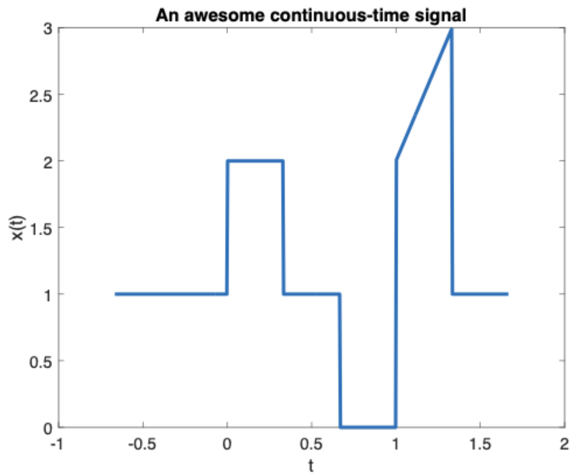
(b) $2x(2-t)$

$x(t)$ is shifted 2 to the left, then reverse, and the vertically scaled by 2:



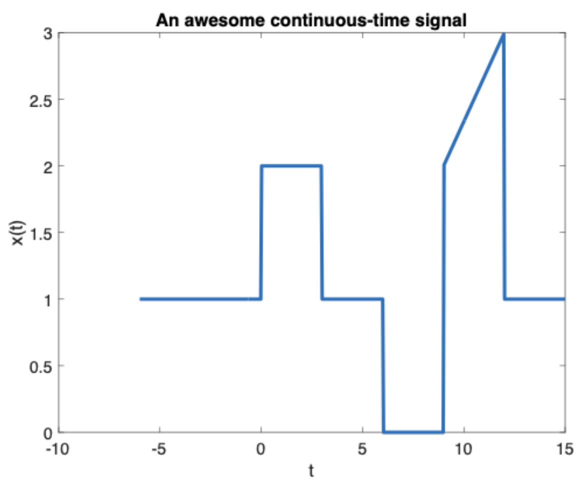
(c) $x(3t-1)+1$

$x(t)$ is shifted 1 to the right, compressed by factor 3, and vertically shifted 1 unit up:



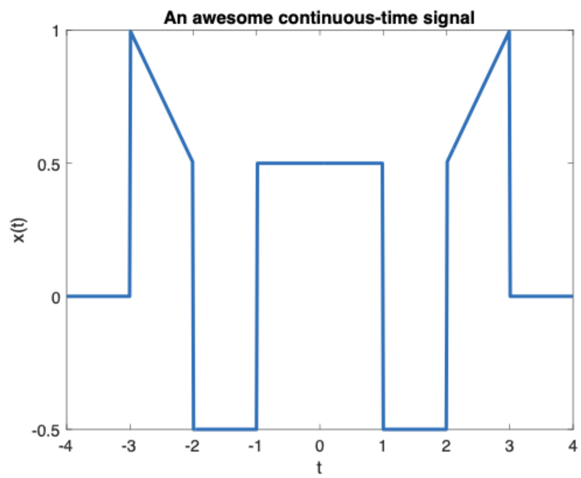
(d) $x(t/3-1)+1$

$x(t)$ is shifted 1 to the right, expanded by factor 3, and vertically shifted 1 unit up:



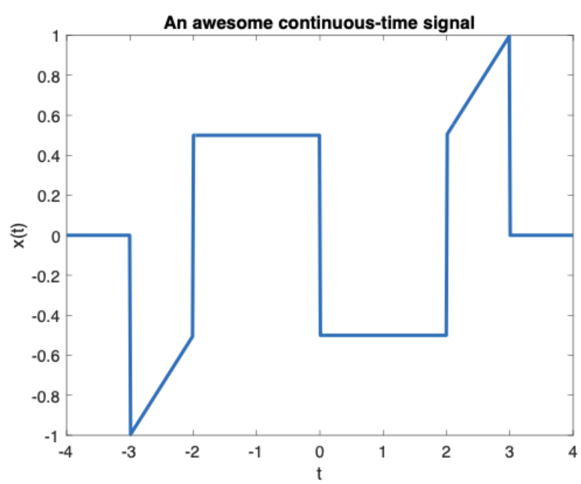
(e) Even part of $x(t)$

Even part of $x(t)$ is calculated through $\frac{1}{2}(x(t)+x(-t))$:



(f) Odd part of $x(t)$

Odd part of $x(t)$ is calculated through $\frac{1}{2}(x(t) - x(-t))$:

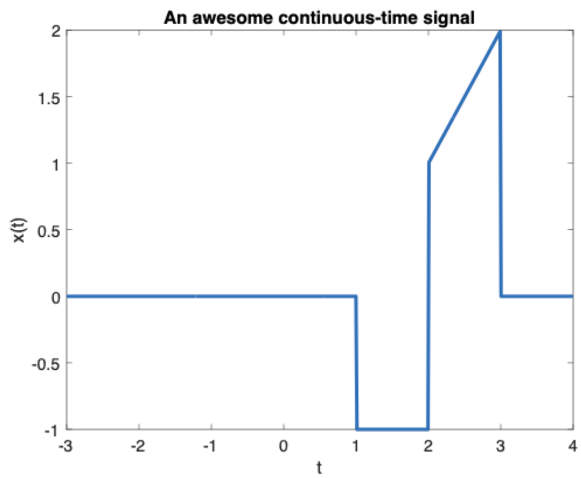


(g) $x(t)u(t)$

$u(t)$ is the unit step function:

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}.$$

$x(t)u(t)$ is:



2. Sketch carefully each of the following signals.

   (a) $2\text{rect}(3t-2)$

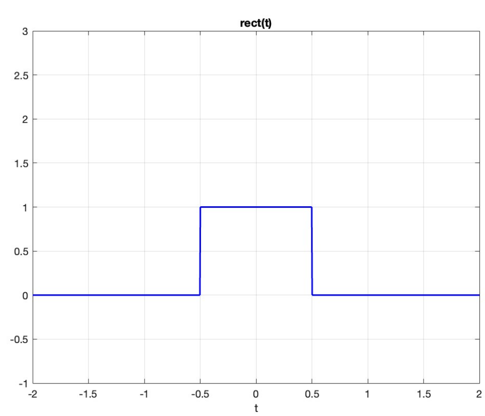
   (b) $\text{rect}(t-1/2)+\text{rect}(t+1/2)$

   (c) $\text{rect}(t)u(t)$

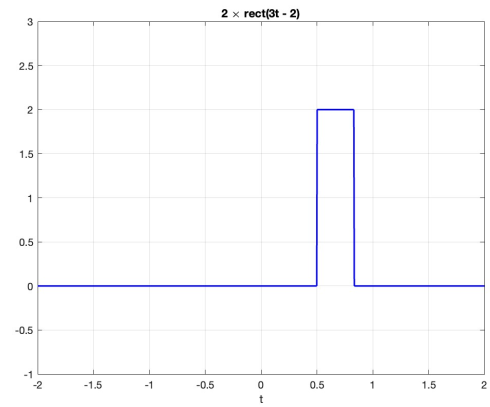
   Solution:

   (a) **Signal:** $2\textbf{rect}(3t-2)$
   The rect function is defined as:

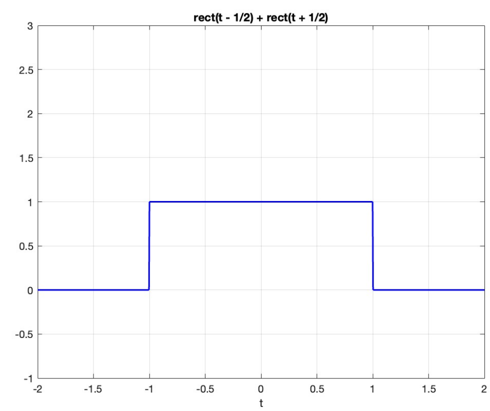
$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \leq \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$$



For $2\text{rect}(3t-2)$, the function is shifted 2 unit to the right and compressed by a factor of 3. The signal is scaled vertically by 2.



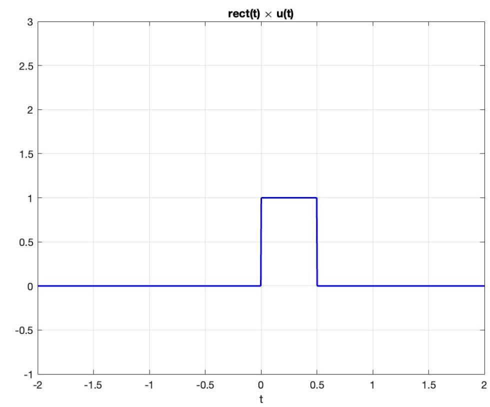
(b) **Signal: rect**$(t - \frac{1}{2}) +$ **rect**$(t + \frac{1}{2})$
This is a sum of two rect functions. The first term, $\text{rect}(t - \frac{1}{2})$, shifts the rect function to the right by $\frac{1}{2}$. The second term, $\text{rect}(t + \frac{1}{2})$, shifts it to the left by $\frac{1}{2}$.



(c) **Signal: rect**$(t)u(t)$
The $\text{rect}(t)$ function is defined as earlier, and $u(t)$ is the unit step function:

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{if } t < 0. \end{cases}$$

The product $\text{rect}(t)u(t)$ effectively halves the $\text{rect}(t)$ function, setting its left part to zero.



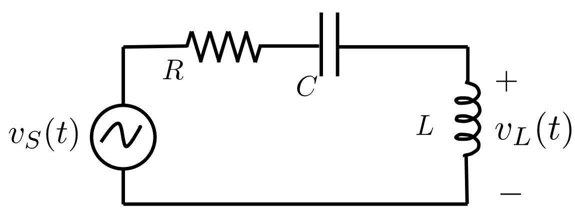


Figure 2: Revisiting the RLC circuit

3. Consider the circuit shown in Fig. 2. Let $v_S(t)$ denote the input voltage signal and let $v_L(t)$ denote the output measured voltage signal. Using the phasor method with $v_S(t) = A\cos(\omega t)$ answer the following questions.

   (a) Find $\mathbf{V}_S$

   (b) Find the effective impedance that relates $\mathbf{V}_L = \mathbf{Z}_E \mathbf{V}_S$

   (c) Find $\mathbf{V}_L$

   (d) Find $|\mathbf{Z}_E|$

   (e) Find the phase of $\mathbf{Z}_E$

   (f) Compute $v_L(t)$

   Solution:

   (a) The phasor representation of the input voltage $v_S(t) = A\cos(\omega t)$ is given by:

   $$\mathbf{V}_S = A$$

   where A is the amplitude of the input voltage.

   (b) The total impedance of the circuit is:

   $$\mathbf{Z} = R + \mathbf{Z}_C + \mathbf{Z}_L$$

   where:

   $$\mathbf{Z}_C = -j\frac{1}{\omega C}, \quad \mathbf{Z}_L = j\omega L.$$

Therefore:

$$\mathbf{Z} = R - j\frac{1}{\omega C} + j\omega L.$$

The voltage across the inductor is related to the source voltage through the effective impedance $\mathbf{Z}_E$, defined as:

$$\mathbf{Z}_E = \frac{\mathbf{Z}_L}{\mathbf{Z}} = \frac{j\omega L}{R + j(\omega L - \frac{1}{\omega C})}.$$

(c) **Find $\mathbf{V}_L$:**
Using Ohm's law, the phasor representation of the output voltage is:

$$\mathbf{V}_L = \mathbf{Z}_E \mathbf{V}_S = \frac{j\omega L}{R + j(\omega L - \frac{1}{\omega C})} \cdot A.$$

(d) **Find $|\mathbf{Z}_E|$:**
The magnitude of $\mathbf{Z}_E$ is:

$$|\mathbf{Z}_E| = \left| \frac{j\omega L}{R + j(\omega L - \frac{1}{\omega C})} \right| = \frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

(e) **Find the phase of $\mathbf{Z}_E$:**
The phase of $\mathbf{Z}_E$ is given by:

$$\angle \mathbf{Z}_E = \angle\left(j\omega L\right) - \angle\left(R + j(\omega L - \frac{1}{\omega C})\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right).$$

(f) **Compute $v_L(t)$:**
The time-domain output voltage is:

$$v_L(t) = |\mathbf{V}_L| \cos(\omega t + \angle \mathbf{V}_L),$$

where:

$$|\mathbf{V}_L| = |\mathbf{Z}_E| A = \frac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \cdot A,$$

and:

$$\angle \mathbf{V}_L = \angle \mathbf{Z}_E.$$

Therefore:

$$v_L(t) = \frac{\omega L A}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \cos\left(\omega t + \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right).$$

4. Using the provided **MATLAB script** as a template, recreate the answer to the Problem 1. Note that your script should output the figures with proper labeling and formatting. You do not need to have the corresponding text outputs. You will create a PDF of your MATLAB script, PDFs of your figures and submit them all together to GradeScope along with your homework.

5. Using the provided **MATLAB Live script** as a template to solve again the previous problem. You mainly have to copy your previous solution in the correct places, to get an understanding about how MATLAB Live works versus scripts. MATLAB Live is a computational notebook, similar to a Jypter notebook as commonly used in Python. These notebooks are a way of mixing text, equations and code. They are very popular in the machine learning research community as a convenient way of sharing code. You can find more information about MATLAB live scripts here:

   `https://www.mathworks.com/help/matlab/matlab_prog/create-live-scripts.html`

   You will create a PDF of your MATLAB live script and submit it to GradeScope along with your homework.

6. Correct your previous week's homework using a colored pen (or annotation) so it's obvious what you've corrected. If you got a problem exactly right, just use a red check mark to indicate as such.