Lecture 2

Rectangle, step functions, signal transformations, periodic, even and odd

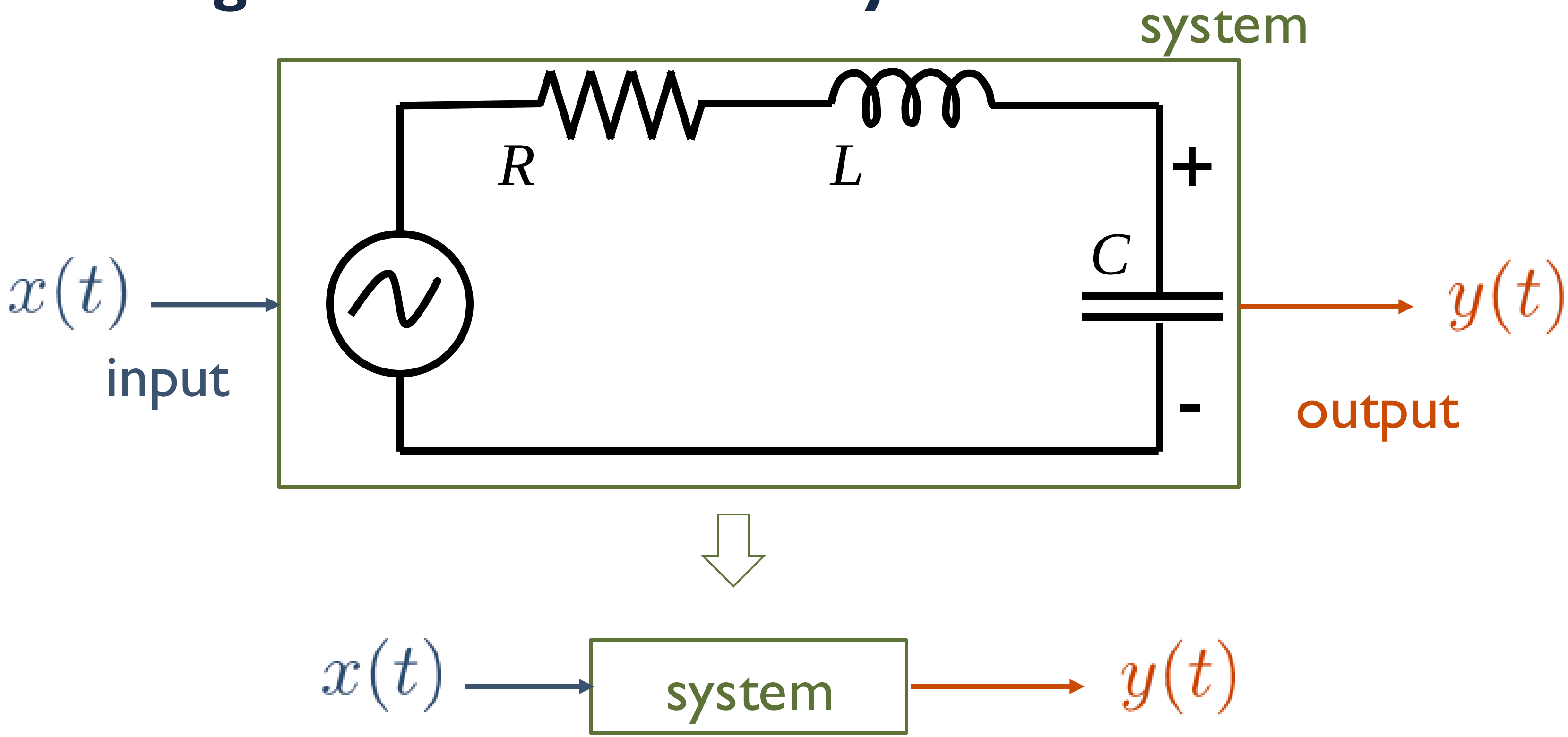
Preview of today's lecture

- Signals, unit–step and rectangle functions
  - Describe mathematically the unit step and rectangle functions

- Basic signal transformations
  - Apply different transformations on continuous-time signals
  - Create new signals from these transformations

- Signal characteristics: periodic, even, odd, and conjugate symmetric
  - Distinguish between periodic and aperiodic signals
  - Compute even and odd parts of an arbitrary signal

Thinking about circuits as a system



A mathematical description of how the input is transformed into the output

Connections back to ECE 45

Lectures 2 - 4 working with signals

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Lectures 5 - 7 LTI systems in the time domain

Lectures 11-12 LTI systems in the frequency domain

$$X(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega)$$

Lectures 8 - 10 Fourier series

Lectures 13 - 17 Fourier transform

Fourier

UC San Diego

	Date	Theme	Topic	Readings	Out	In
1	1/7	Signals	Signals, systems, circuits and phasors	1.1	HW1	
2	1/9	Signals	Rectangle, step functions, signal transformations, periodic, even and odd	1.2	HW2	HW1
3	1/14	Signals	Exponential, sinusoids, complex exponentials, phasors	1.3		
4	1/16	Signals	Dirac delta, Kronecker delta, Sha function	1.4	HW3	HW2
5	1/21	LTI in time	Linear and time-invariant systems	2.1		
6	1/23	LTI in time	Convolution, convolution with a sinusoid, connection to phasors	2.2	HW4	HW3
7	1/28	LTI in time	Convolution properties	2.3		
8	1/30	Fourier series	Fourier series	3.1 - 3.3	HW5	HW4
	2/4		**Midterm 1**			
9	2/6	Fourier series	Fourier series convergence and properties	3.4	HW6	HW5
10	2/11	Fourier series	Fourier series properties	3.5		
11	2/13	LTI in frequency	Frequency response of LTI systems	3.9	HW7	HW6
12	2/18	LTI in frequency	Filters, bode plots	3.10, 6.2.3		
13	2/20	Fourier transform	Fourier transform	4.1-4.2	HW8	HW7
	2/25		**Midterm 2**			
14	2/27	Fourier transform	Fourier transform properties	4.3	HW9	HW8
15	3/4	Fourier transform	Rectangle and sinc functions	4.3		
16	3/6	Fourier transform	Convolution property	4.4	HW10	
17	3/11	Fourier transform	Multiplication property	4.5		
18	3/13	Sampling	Sampling theorem	7.1		HW10
	3/19		**Final exam Tuesday 3-6pm**			

Signals, unit–step and rectangle functions

Learning objectives

- Describe mathematically the unit step and rectangle functions

What is a "signal"?

ECE 45 focuses on CT signals

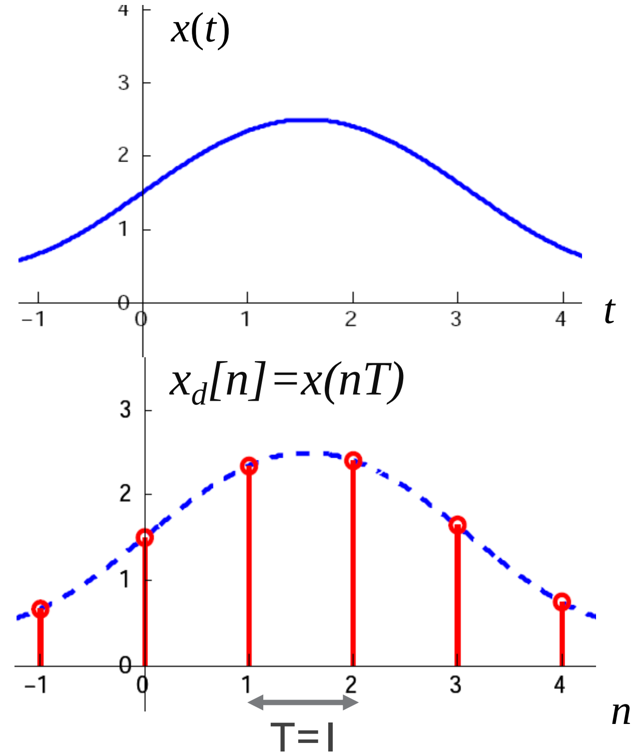
- Representation of a value/info. relative to an independent variable
  - Often a time variable but could be something else
- Continuous-time (CT) signals $x(t)$

Bracket notation used to denote a discrete-time sequence

- Discrete-time (DT) signals $x[n]$
  - Often obtained by *sampling* CT signal
  - Taken at values nT

sampling period

$\cancel{x[n/2]}$



Complex signals

- Signals may take complex values (equivalently they have an amplitude and a phase when in polar form)

$$x(t) = \mathrm{Re}\{x(t)\} + j\mathrm{Im}\{x(t)\} \qquad x[n] = \mathrm{Re}\{x[n]\} + j\mathrm{Im}\{x[n]\}$$

- Complex signals are found in many practical problems
  - Most digital signal processors support complex operations
- Example: "in phase" (real) and "quadrature" (imag) EM signals

"rides" a cosine

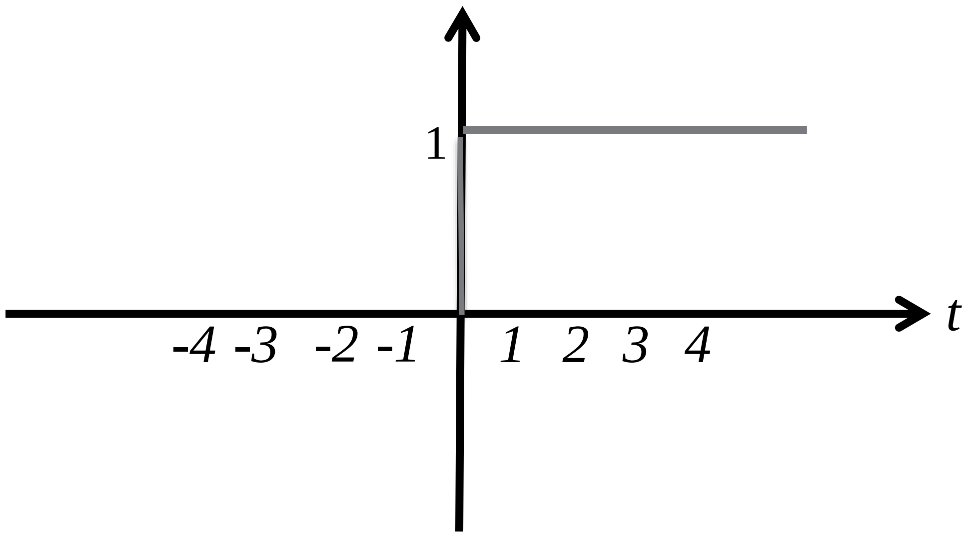
$$x(t) = \mathrm{Re}\{x(t)\} + j\mathrm{Im}\{x(t)\}$$

"rides" a sine

Many connections between complex signals, Maxwell's equations, phasors, etc.

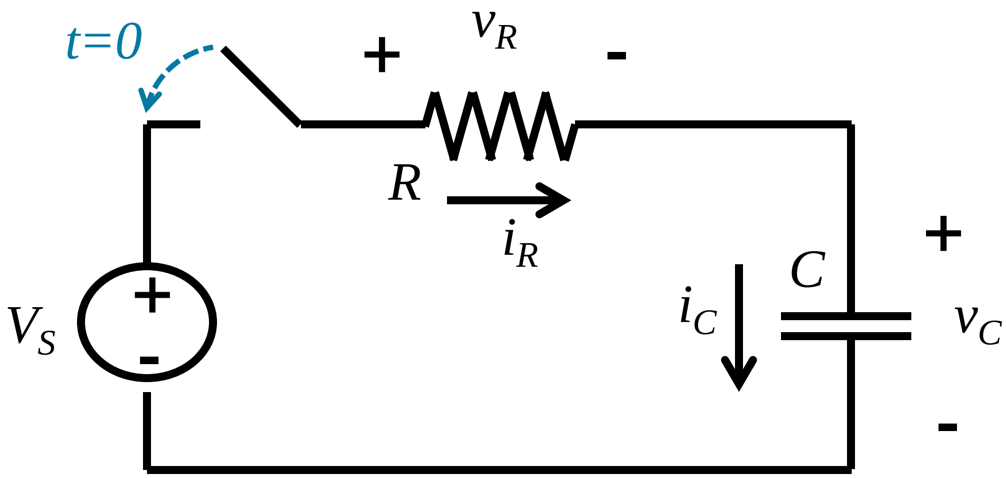
Unit step function

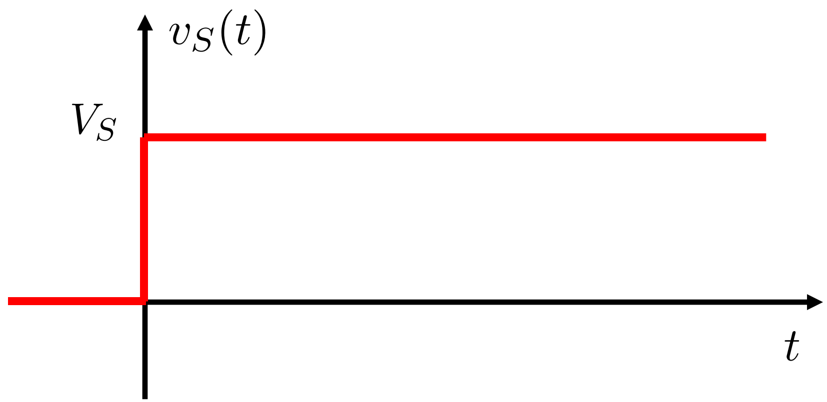
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 1 \end{cases}$$



- At t=0, u(t) may be either 0, 1, or ½ depending on the book
  - The specific choice is only important in a mathematical analysis class

Where have you seen the unit step function?



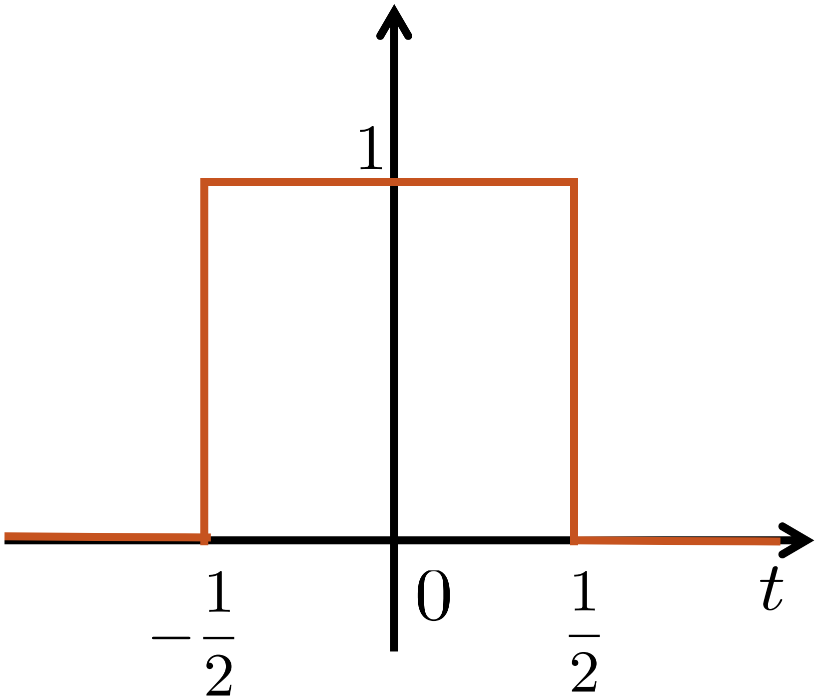


The voltage signal created from switching in a DC source is written using the unit step function

$$v_S(t) = V_S\, u(t)$$

Rectangle function

$$\mathrm{rect}(t) = \begin{cases} 0, & |t| > \frac{1}{2} \\ 1, & |t| \leqslant \frac{1}{2} \end{cases}$$

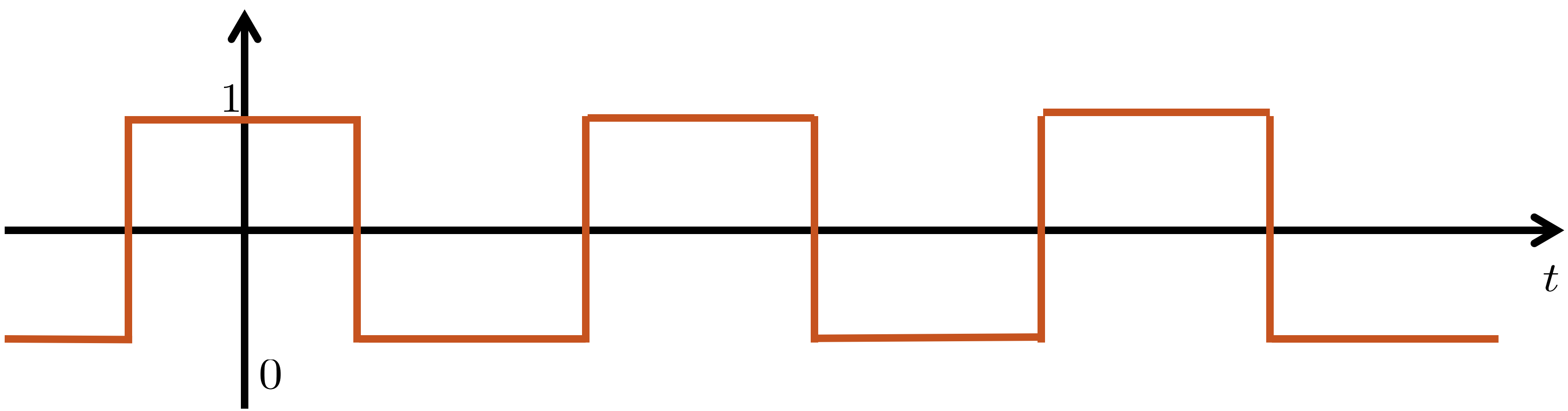


Also called the box function

May have different notation in other books like $\Pi(t)$

Where will you see the rectangle function?

- Part of many common convolution examples
- Used to build a square waves, which is analyzed via Fourier Series



- Shows up as part of "windowing," e.g. $x(t)\,\mathrm{rect}(t)$

Why spend time on these basic signals?

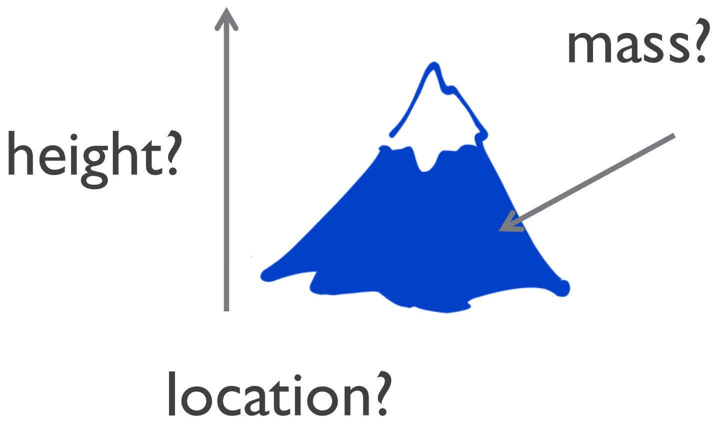
- Unit step functions
  - An important building block signals for other parts of the course
  - Shows up often when signals ”start at zero”
- Rectangle function
  - Another important building block signal
  - Used for many examples in convolution and Fourier

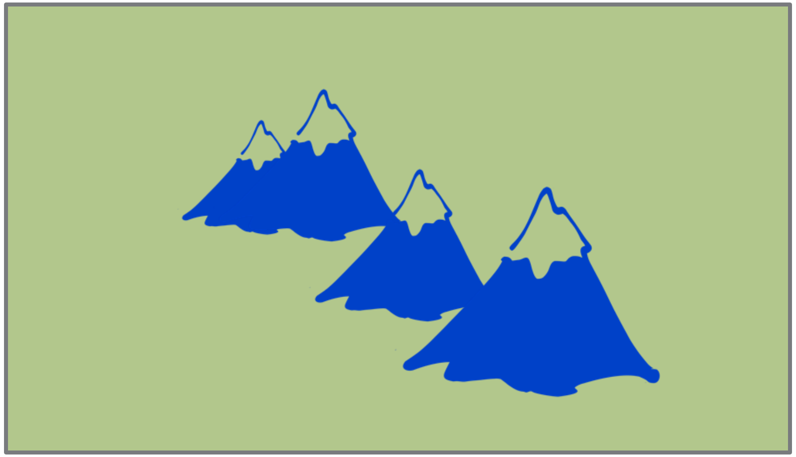
Basic signal transformations

Learning objectives

- Apply different transformations on continuous-time signals
- Create new signals from these transformations

Moving mountains (function is over space here)



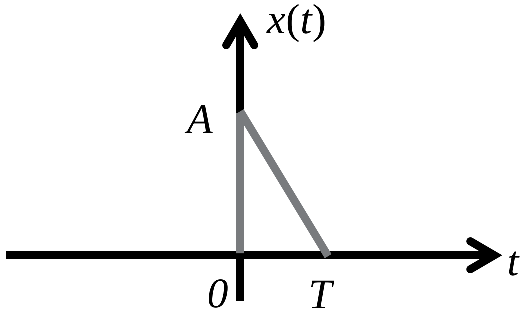


- Use mountain description to build a map
  - Where are the mountains located?
  - What are their size?

Signal transformations are ways to describe and manipulate signals

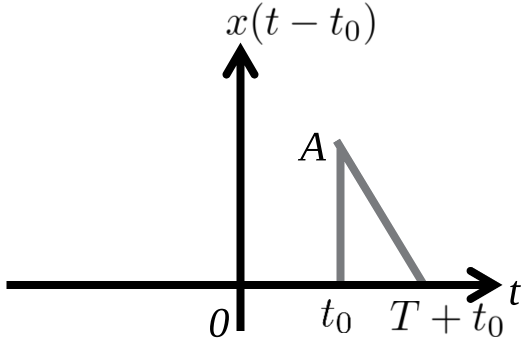
from clipartpanda.com

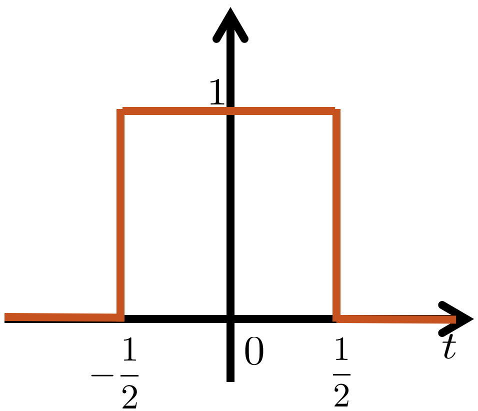
Example: time shift



$$x(t - t_0)$$

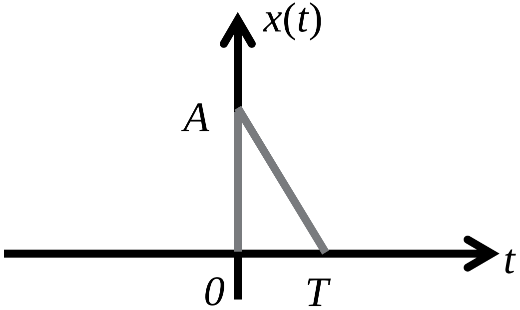
right shift



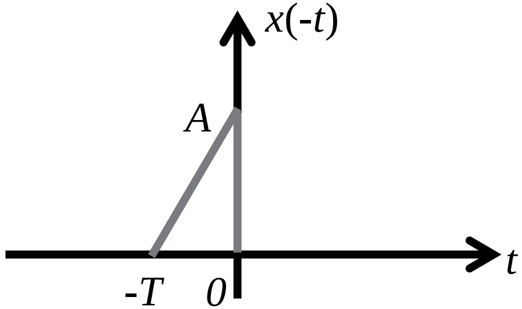


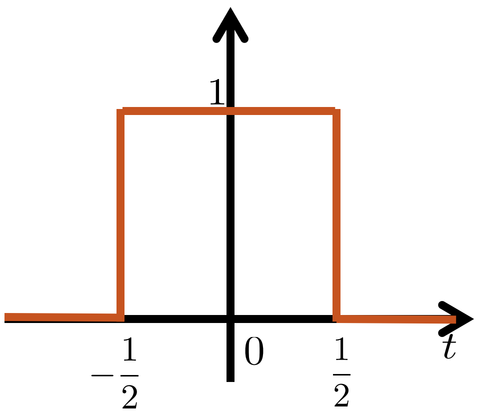
$$\mathrm{rect}(t + 3)$$

Example: reflection or time reversal



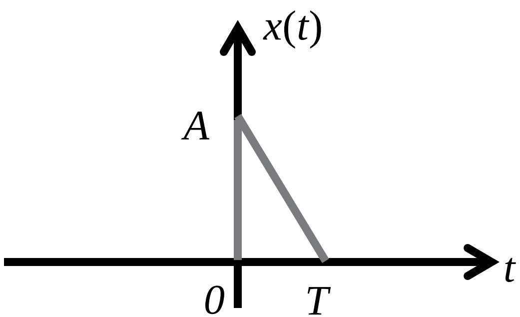
$x(-t)$



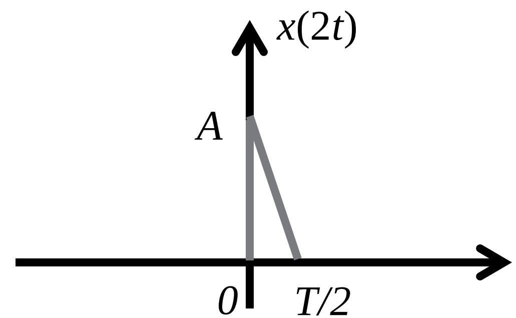


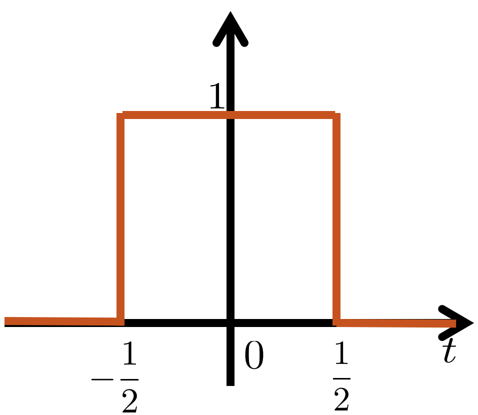
$\mathrm{rect}(-t)$

Example: time compression / downsampling



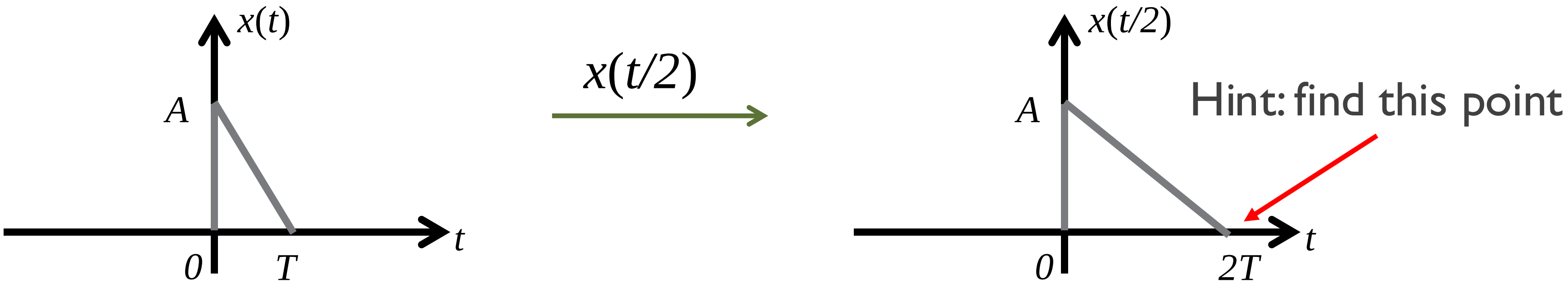
$$\xrightarrow{x(2t)}$$

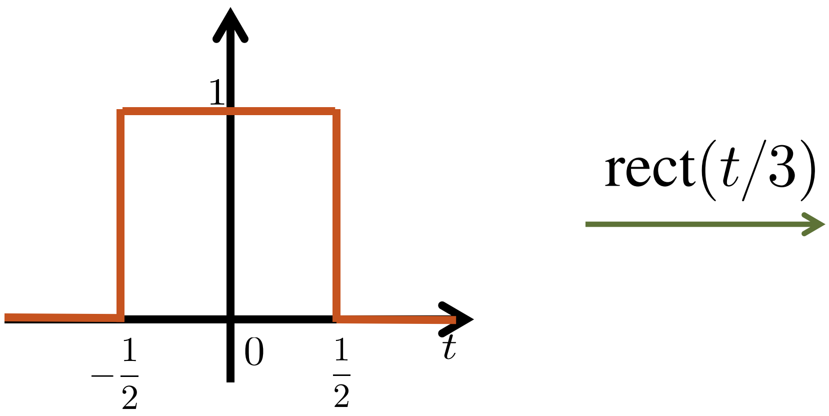




$$\xrightarrow{\mathrm{rect}(3t)}$$

Example: time expansion (upsampling)





Example

- Plot the signal $\mathrm{rect}(2t-3)$

- You may shift then scale, or scale then shift (carefully)

Example

- Plot the signal $\mathrm{rect}(2t-3)$

- You may shift then scale, or scale then shift (carefully)

Signal transformation summary

- There are different ways to transform a signal
  - This section focused on transformations of the independent variable

- Transformations
  - Time shifting changes the starting point of a signal
  - Time scaling changes how fast the signal is "played"
  - Time reversal flips a signal
  - Compression / expansion change the "speed" of a signal

Signal characteristics: periodic, even, and odd

Learning objectives

- Distinguish between periodic and aperiodic signals
- Compute the period of a periodic signal
- Compute even and odd parts of an arbitrary signal

Periodic signals

- Periodic signals satisfy for some finite non-zero T or N

$$x(t) = x(t+T)$$ periodic with period T for all t

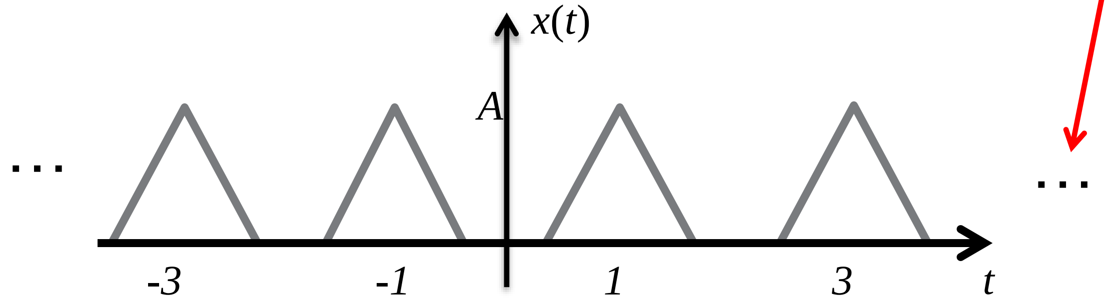
- Period is the smallest non-zero solution
  - This is called the fundamental period
  - Normally period means fundamental period

Periodic signals can be treated with special mathematical tools

Examples of periodic signals

pulse train of triangles

Dots indicate that the signal continues



period?

Establishing periodicity

- To prove a CT signal is periodic
  - Direct: find a $T>0$ such that $x(t)=x(t+T)$ for all t
  - Indirect: show that $x(t)$ is in a known class of periodic signals

- To prove a CT signal is aperiodic
  - Direct: Show that there is no $T>0$ such that $x(t)=x(t+T)$ for all t

- Remember the fundamental period is the smallest non-zero T

Example of direct method

- Determine whether or not the following signal is periodic? If it is periodic, determine its fundamental period.

$$x(t) = \sin(2t) + \cos(4t + \pi/2)$$

- Want to find T such that $x(t) = x(t+T)$

$$x(t+T) = \sin(2(t+T)) + \cos(4(t+T) + \pi/2)$$ shifted signal

$$\sin(2(t+T)) = \sin(2t) \text{ for } T = k\pi \text{ where } k \in \mathbb{Z}$$ multiple solutions

$$\cos(4(t+T) + \pi/2) = \cos(4t + \pi/2) \text{ for } T = k\pi/2 \text{ where } k \in \mathbb{Z}$$ multiple solutions

$$\boxed{T = \pi}$$ fundamental period

Example of indirect method

- Determine whether or not the following signal is periodic? If it is periodic, determine its fundamental period.

$$x(t) = [\cos(2t - \pi/3)]^2$$

- Recall that $\cos^2(x) = \frac{1}{2}(1 + \cos 2x)$

$x(t) = [1 + \cos(4t - 2\pi/3)]/2$. Periodic, period $= 2\pi/(4) = \pi/2$.

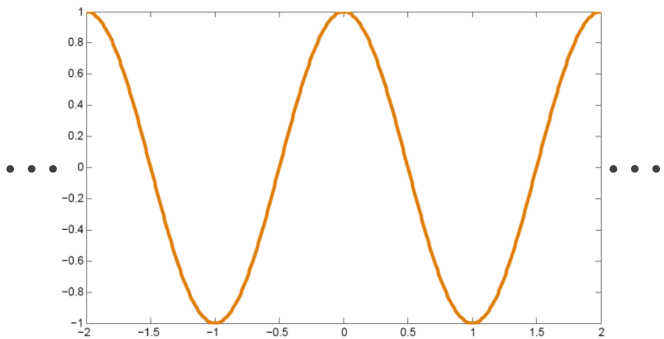
- By showing this simplifies to a known periodic function, we can also conclude it is period and find the frequency

Even signals (real)

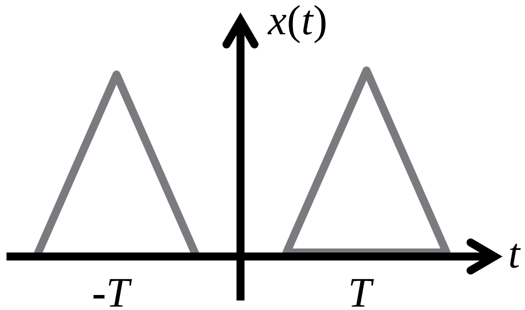
- Even signals satisfy

$$x(-t) = x(t)$$

- Examples (need not be periodic)



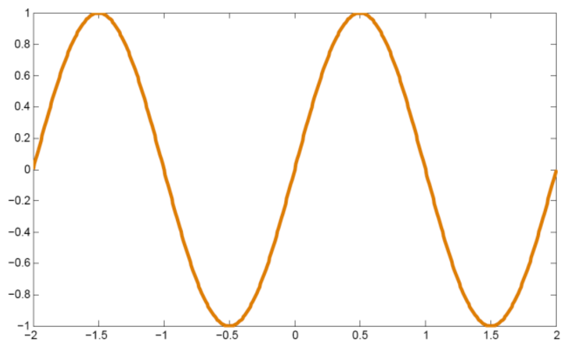
$\cos(2\pi f t)$



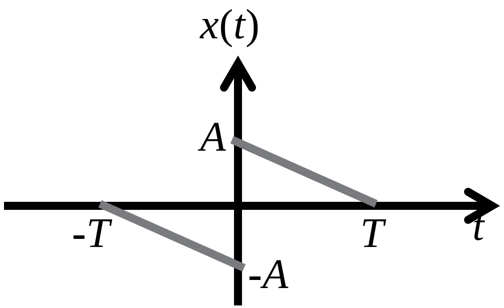
Odd signals (real)

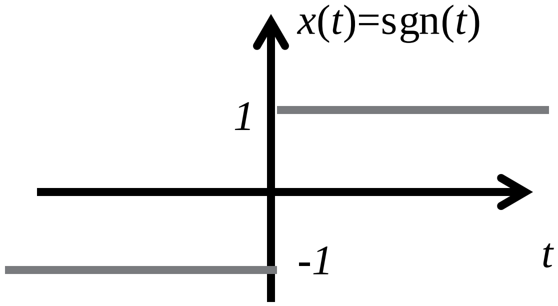
- Odd signals satisfy $$x(-t) = -x(t)$$

- Examples



$\sin(2\pi f t)$





Generalization to complex signals

- Even becomes conjugate symmetric

$$x^*(-t) = x(t)$$

- Odd becomes conjugate antisymmetric

$$x^*(-t) = -x(t)$$

Interesting facts about even and odd functions

- Any real function can be written in terms of its even and odd parts

$$e(t) = \frac{1}{2}[f(t) + f(-t)]$$

$$o(t) = \frac{1}{2}[f(t) - f(-t)]$$

$$e(-t) = \frac{1}{2}[f(-t) + f(t)] = e(t)$$

$$o(-t) = \frac{1}{2}[f(-t) - f(t)] = -o(t)$$

$$f(t) = e(t) + o(t)$$

Interesting facts about even and odd functions

- Integration properties

$$\int_{-\infty}^{\infty} e(t)dt = 2\int_{0}^{\infty} e(t)dt$$

$$\int_{-\infty}^{\infty} o(t)dt = 0 = \int_{-T_0}^{T_0} o(t)dt$$

- Multiplications of two odd or two even functions → even function

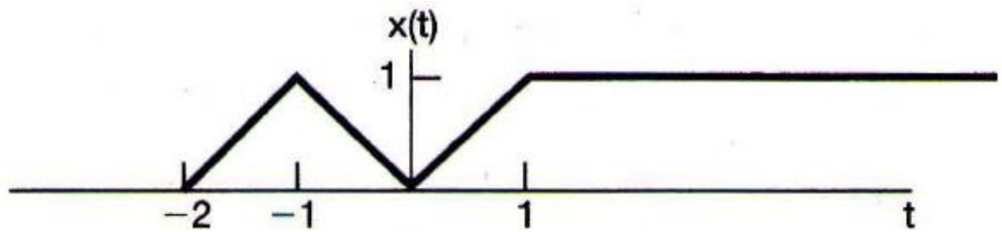
$$y(t) = o_1(t)o_2(t) = e(t)$$

$$y(t) = e_1(t)e_2(t) = \text{even function}$$

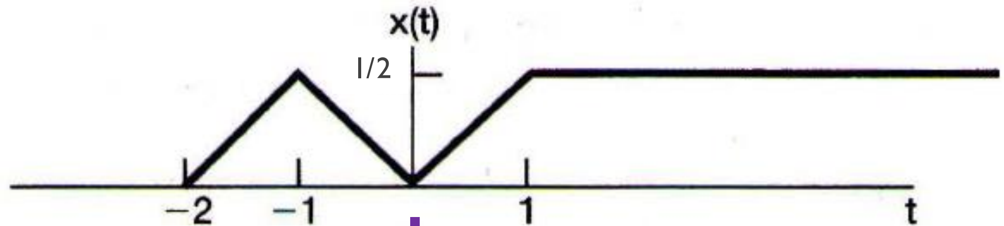
$$y(t) = o(t)e(t) = \text{odd function}$$

Example

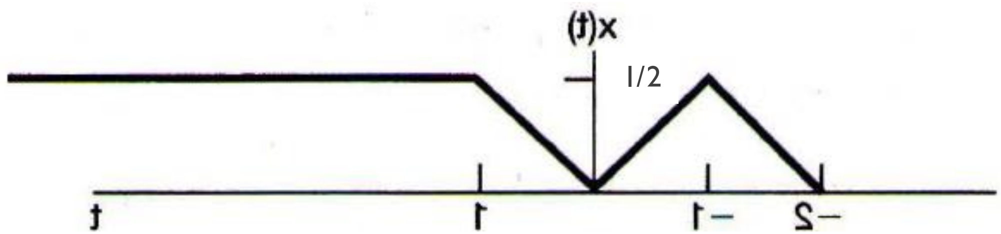
- Determine and sketch the even and odd parts of the signal



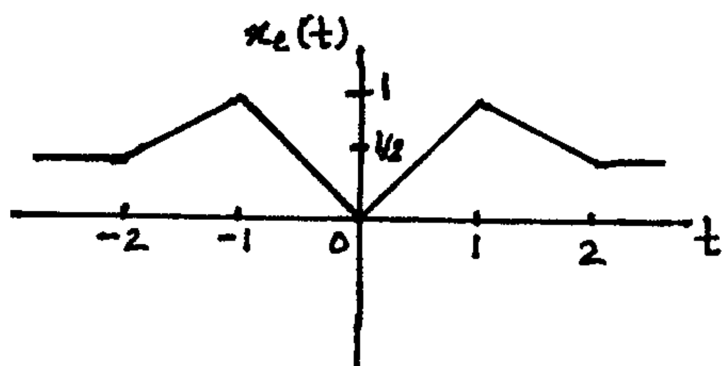
Sketching the even solution



plus

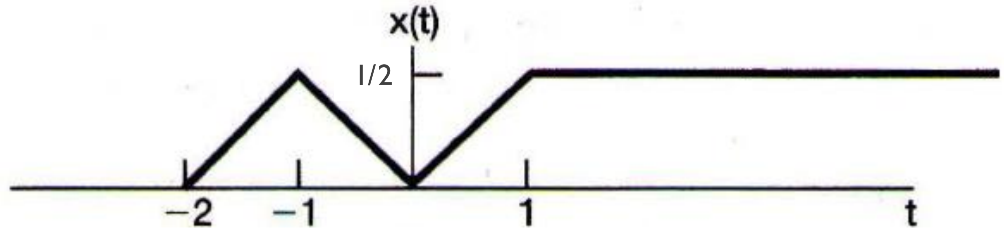


e(t) = ½ (x(t) + x(-t))

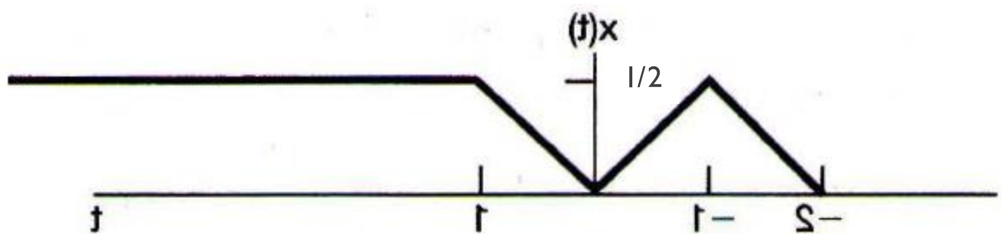


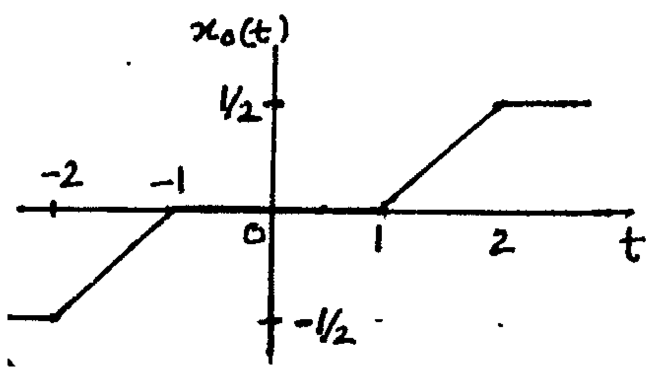
Sketching the odd solution

o(t) = ½ (x(t) - x(-t))



minus





Similar properties for complex signals

$$E(t) = \frac{1}{2}[f(t) + f^*(-t)]$$ Conjugate symmetric

$$O(t) = \frac{1}{2}[f(t) - f^*(-t)]$$ Conjugate antisymmetric

$$E^*(-t) = \frac{1}{2}[f^*(-t) + f(t)] = E(t)$$

$$O^*(-t) = \frac{1}{2}[f^*(-t) - f(t)] = -O(t)$$

$$f(t) = E(t) + O(t)$$

Note: Conjugate symmetric & real is even (why?)

Signal characteristics summary

- Periodic signals
  - Special type of signals that repeat
  - Need to determine if a signal is periodic and its period
- Even, odd, conjugate symmetric, conjugate antisymmetric
  - Signals with symmetry about the y axis
  - Fact used to simplify computations and derive intuition
- These special signal structures will be used in Fourier analysis