UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2021
Linear Systems Fundamentals

MIDTERM EXAM

You are allowed one 2-sided sheet of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME ____

Signature ____

Your signature confirms that you have completed this exam on your own and in accordance with the ECE 101 Academic Integrity Agreement.

Student ID Number ____

Problem	Weight	Score
1	36 pts	
2	32 pts	
3	32 pts	
Total	100 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Useful facts and Tables 3.1 and 3.2 from the textbook are attached to the exam.

Good luck!

Name/Student ID: ____

Problem 1 (36 points)

A discrete-time (DT) system S is described by

$$y[n] = \sum_{k=n-1}^{n+1} (x[k])^2.$$

(a) (18 points)

Check the appropriate box indicating whether or not the system S satisfies the specified property. **Justify your answers.**

True	**False**	
☐	☐	Invertible
☐	☐	Causal
☐	☐	Stable

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Problem 1 (cont.)

(b) (18 points)

Consider the same system S as in part (a). Check the appropriate box indicating whether or not the system S satisfies the specified property. **Justify your answers.**

True	**False**	
☐	☐	Time-invariant
☐	☐	Scalable
☐	☐	LTI

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Problem 2 (32 points)[8 points each part]

Let S be the DT LTI system with impulse response

$$h[n] = \delta[n] + \frac{1}{4}\delta[n-2].$$

(a) Determine a difference equation that describes S of the form

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

.

(b) Determine the transfer function $H(z)$ of S.

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Problem 2 (cont.)

(c) Determine the output $y[n]$ of the system S for input signal $x[n] = (\frac{1}{2})^n$.

(d) Determine the output $y[n]$ of the system S for the input signal

$$x[n] = \left(\frac{1}{2}\right)^n u[n].$$

Express your answer in the simplest form you can.

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Problem 3 (32 points)[16 points each part]

(a) Let $x[n]$ be a DT signal with fundamental period 4 and Fourier series coefficients $a_0 = 1, a_1 = -j, a_2 = 1, a_3 = j$. Determine and sketch precisely one period of $x[n]$, for $n = 0, 1, 2, 3$.

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Problem 3 (cont.)

(b) Let $x[n]$ be a DT signal with fundamental period 6 and Fourier series coefficients $a_0 = 1, a_1 = \frac{1}{2j}, a_2 = 0, a_3 = 1, a_4 = 0, a_5 = -\frac{1}{2j}$. Consider the DT LTI system whose frequency response in the interval $[-\pi, \pi]$ is given by

$$H(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{\pi}{2} \leq |\omega| \leq \pi \\ 0 & \text{otherwise.} \end{cases}$$

Suppose the signal $x[n]$ is the input to this system.

Determine and sketch precisely the output $y[n]$ in the interval $[0, \ldots, 5]$.

Scratch page **Name/Student ID:** ______________________

Scratch page **Name/Student ID:** ____________________

Chapter 2: LTI systems

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \qquad\qquad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

If $h(t)$ is the impulse response to the CT system S, and $g(t)$ is the impulse response to the CT system S', then it follows that if $g(t) * h(t) = \delta(t)$, then S' is the inverse of S. The same property holds true for DT systems.

An LTI system is causal iff $h[n] = 0$ for $n < 0$ ($h(t) = 0$ for $t < 0$).

If an impulse response is absolutely summable ($\sum_{k=-\infty}^{\infty} |h[k]| < \infty$), then the LTI system is stable.

If an impulse response is absolutely integrable ($\int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty$), then the LTI system is stable.

Chapter 3: Fourier series and Fourier coefficients

synthesis equation — analysis equation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt \qquad \omega_0 = 2\pi/T$$

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} \qquad a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\omega_0 n} \qquad \omega_0 = 2\pi/N$$

Response of LTI system to complex exponential

e^{st} (z^n) is called the eigenfunction and $H(s)$ ($H(z)$) the eigenvalue of the LTI system.

$$e^{st} \to H(s)e^{st} \qquad H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$$

$$z^n \to H(z)z^n \qquad H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

Let $x(t) \to y(t)$ ($x[n] \to y[n]$) in an LTI system. Then

$$\text{if} \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{then} \quad y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)e^{jk\omega_0 t} \qquad \omega_0 = 2\pi/T$$

$$\text{if} \quad x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} \quad \text{then} \quad y[n] = \sum_{k=<N>} a_k H(e^{jk\omega_0})e^{jk\omega_0 n} \qquad \omega_0 = 2\pi/N$$

Chapters 4 and 5

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega \qquad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Triangle Inequality and summation formula:

$$|X+Y| \le |X| + |Y| \qquad \sum_{k=0}^{\infty} z^k = \frac{1}{1-z} \quad \text{if} \quad |z| < 1 \qquad \sum_{k=0}^{N-1} z^k = \frac{1-z^N}{1-z}$$

Table 3.1 PROPERTIES OF THE CONTINUOUS-TIME FOURIER SERIES
$x(t)$ and $y(t)$ are periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$

Section	Property	Periodic Signal	Fourier Series Coefficients
	both $x(t)$ and $y(t)$ periodic with period T and frequency $\omega_0 = 2\pi/T$	$x(t)$ $y(t)$	a_k b_k
3.5.1	Linearity	$A\,x(t) + B\,y(t)$	$A\,a_k + B\,b_k$
3.5.2	Time Shifting	$x(t - t_0)$	$a_k\, e^{-jk\omega_0 t_0}$
	Frequency Shifting	$\exp[jM\omega_0 t]\, x(t)$	a_{k-M}
3.5.6	Conjugation	$x^*(t)$	a^*_{-k}
3.5.3	Time Reversal	$x(-t)$	a_{-k}
3.5.4	Time Scaling	$x(\alpha t)$, $\alpha > 0$ (periodic with period T/α)	a_k
	Periodic Convolution	$\int_T x(\tau) y(t-\tau) d\tau$	$T\,a_k\,b_k$
3.5.5	Multiplication	$x(t)\,y(t)$	$\sum_{\ell=-\infty}^{\infty} a_\ell\, b_{k-\ell}$
	Differentiation	$dx(t)/dt$	$j\,k\,\omega_0\,a_k$
	Integration (requires $a_0 = 0$)	$\int_{-\infty}^{t} x(t')\,dt'$	$a_k/(j\,k\,\omega_0)$
3.5.6	Conjugate Symmetry	$x(t)$ is real	$\begin{cases} \|a_k\| = \|a_{-k}\| \\ a_k = a^*_{-k} \\ \mathfrak{Re}\{a_k\} = \mathfrak{Re}\{a_{-k}\} \\ \mathfrak{Im}\{a_k\} = -\mathfrak{Im}\{a_{-k}\} \\ \measuredangle a_k = -\measuredangle a_{-k} \end{cases}$
3.5.6	real and even signals	$x(t)$ real and even	a_k purely real and even
3.5.6	real and odd signals	$x(t)$ real and odd	a_k purely imaginary and odd
3.5.6	Even Decomposition ($x(t)$ is real)	$x_e(t) = \mathrm{Ev}\,\{x(t)\}$	$\mathfrak{Re}\,\{a_k\}$
3.5.6	Odd Decomposition ($x(t)$ is real)	$x_o(t) = \mathrm{Od}\,\{x(t)\}$	$j\mathfrak{Im}\,\{a_k\}$

Parseval's Relation for Periodic Signals $\quad (1/T) \int_T |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

Table 3.2 PROPERTIES OF THE DISCRETE-TIME FOURIER SERIES

$x[n]$ and $y[n]$ are periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$

Property	Periodic Signal	Fourier Series Coefficients
both $x[n]$ and $y[n]$ periodic with period N and frequency $\omega_0 = 2\pi/N$	$x[n]$ $y[n]$	a_k (periodic, period N) b_k (periodic, period N)
Linearity	$A\,x[n] + B\,y[n]$	$A\,a_k + B\,b_k$
Time Shifting	$x[n-n_0]$	$a_k\,e^{-jk\omega_0 n_0}$
Frequency Shifting	$\exp[jM\omega_0 n]\,x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ Mod } m = 0 \\ 0, & n \text{ Mod } m \neq 0 \end{cases}$	$\frac{a_k}{m}$ (periodic, period mN)
Periodic Convolution	$\sum_{r=<N>} x[r]y[n-r]$	$N\,a_k\,b_k$
Multiplication	$x[n]\,y[n]$	$\sum_{\ell=<N>} a_\ell\,b_{k-\ell}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-j\,k\,\omega_0})a_k$
Running Sum (requires $a_0 = 0$)	$\sum_{k=-\infty}^{n} x[k]$	$\left(\frac{1}{1 - e^{-j\,k\,\omega_0}}\right) a_k$
Conjugate Symmetry	$x[n]$ is real	$\begin{cases} \lvert a_k \rvert = \lvert a_{-k} \rvert \\ a_k = a^*_{-k} \\ \mathfrak{Re}\{a_k\} = \mathfrak{Re}\{a_{-k}\} \\ \mathfrak{Im}\{a_k\} = -\mathfrak{Im}\{a_{-k}\} \\ \angle a_k = -\angle a_{-k} \end{cases}$
real and even signals	$x[n]$ real and even	a_k purely real and even
real and odd signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even Decomposition ($x[n]$ is real)	$x_e[n] = \text{Ev}\{x[n]\}$	$\mathfrak{Re}\{a_k\}$
Odd Decomposition ($x[n]$ is real)	$x_o[n] = \text{Od}\{x[n]\}$	$j\mathfrak{Im}\{a_k\}$

Parseval's Relation for Periodic Signals $\quad \frac{1}{N} \sum_{n=<N>} \lvert x[n] \rvert^2 = \sum_{k=<N>} \lvert a_k \rvert^2$