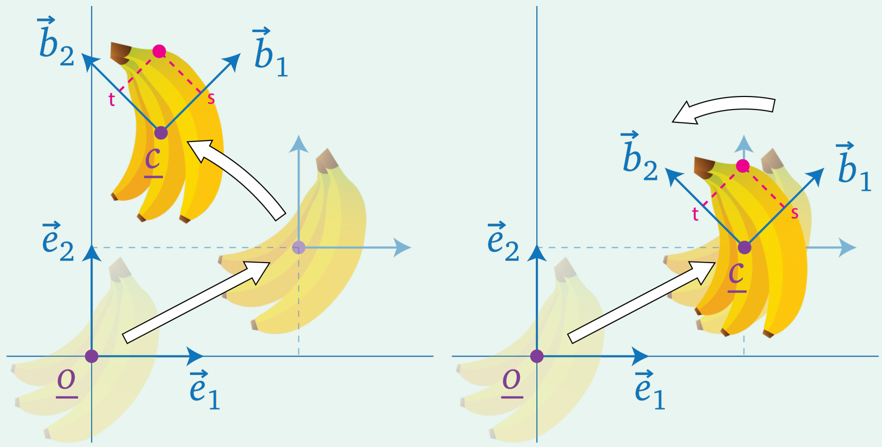
CSE 167 (WI 2025) Exercise 4 — Due 2/7/2025

Exercise 4.1 — 2 pts. In 2D, let us fix a world frame $(\vec{e}_1, \vec{e}_2, \underline{o})$

(a) What is the affine transformation matrix $\mathbf{M}_{(a)}$ that first translates by $(2, 1)$ followed by a counterclockwise rotation about the (world) origin by angle θ? (Left figure)

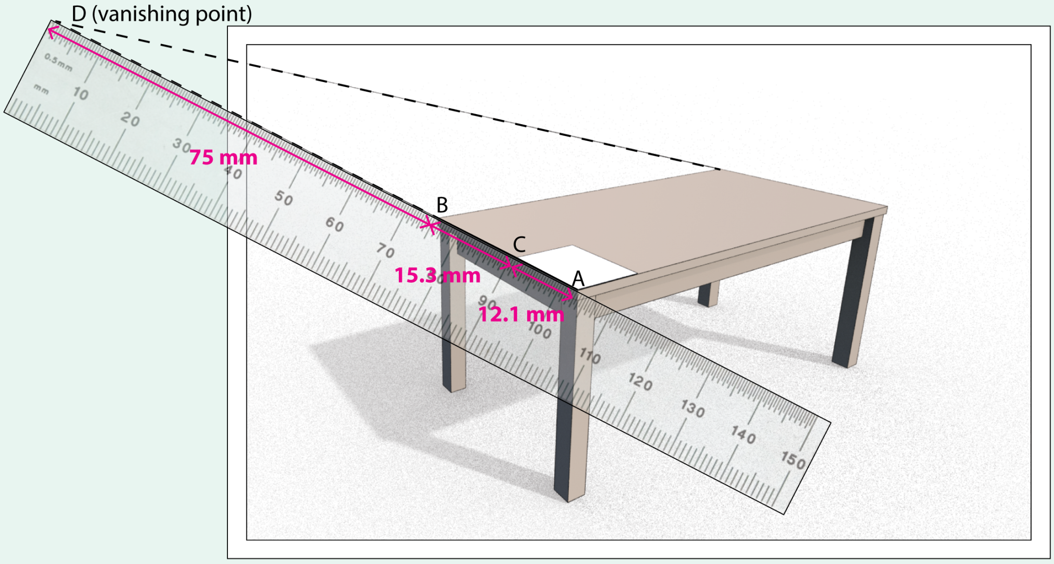
(b) What is the affine transformation matrix $\mathbf{M}_{(b)}$ that first performs a counterclockwise rotation about the origin by angle θ followed by a translation by $(2, 1)$ (in world frame)? (Right figure)



(Geometrically, starting by putting an object upright centered at the origin in the world frame $(\vec{e}_1, \vec{e}_2, \underline{o})$, in **(a)** we translate the object by $2\vec{e}_1 + \vec{e}_2$ and then rotate it about $\underline{o}$ by angle θ, resulting in a new object frame $(\vec{b}_1, \vec{b}_2, \underline{c})$ with respect to which the transformed object is upright and centered. In fact, Senario **(b)** is also first translating the object by $2\vec{e}_1 + \vec{e}_2$, but then we rotate *about the center of the object* by angle θ, resulting in a new object frame $(\vec{b}_1, \vec{b}_2, \underline{c})$. In either scenario, after the transformation, a point with a given coordinate (s, t) in the object frame will have coordinate (x, y) in the world frame, given by $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{M}_{(a)\text{ or }(b)} \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$.) ▪

Exercise 4.2 — 2 pt. Where will the 3D point with coordinates $\begin{bmatrix} 3 \\ 0.5 \\ 5 \end{bmatrix}$ be mapped to by being first lifted to the homogeneous coordinate, applied by projective transformation matrix $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, and finally dehomogenized back as a 3D point? ▪

Exercise 4.3 — 1 pt. Suppose we have a perspective photograph of a rectangular table, and on top of the table there is a lettersize (11×8.5 inches) paper placed at a corner. The 11-inch side of the paper runs along the width (shorter side) of the table. Along the extension line of that side of the table, we mark four points: the two corners of the table A, B, the other corner of the lettersize paper C, and the vanishing point D which is the intersection with the extension line of the opposite side of the table. We measure the lengths between the four points on the photograph as shown below.



What is the width of the table (in the unit of inches)? (That is, what is the distance of between A, B in real world?)

Hint *Use the projective invariance of the cross ratio. You may use a calculator. You can also use the fact that in reallife the ratio of any two infinite distances is one:* $\frac{\overline{AD}_{\text{reallife}}}{\overline{BD}_{\text{reallife}}} = 1, \frac{\overline{BD}_{\text{reallife}}}{\overline{CD}_{\text{reallife}}} = 1, \frac{\overline{AD}_{\text{reallife}}}{\overline{CD}_{\text{reallife}}} = 1.$ ■