# ECE 101: Linear Systems Fundamentals

Spring 2025 - Lecture 3

Paul Siegel

University of California, San Diego

# Today's topics

- Discrete-time unit impulse signal
- Discrete-time unit step signal

Signals and Systems (2th Edition): section 1.4.1

# Unit Step and Unit Impulse Signals

# Discrete-Time: Unit Step

- We define the discrete-time unit step function as:

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

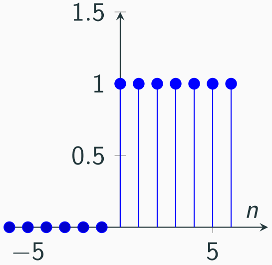



**Figure 1:** Plot of step signal $u[n]$

# Discrete-Time: Unit Impulse

- We define the discrete-time unit impulse as:

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases} = u[n] - u[n-1]$$

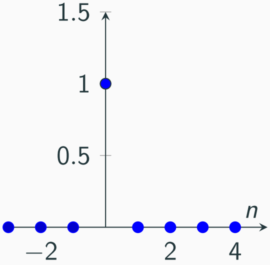



**Figure 2:** Plot of unit impulse signal $\delta[n]$

# Discrete-Time: Unit Impulse

- Relations between $u[n]$ and $\delta[n]$

  - $\delta[n] = u[n] - u[n-1]$
  - $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$
  - $u[n] = \sum_{k=-\infty}^{\infty} u[k]\delta[n-k].$

# Properties of $\delta[n]$

## Sampling Property

$$x[n]\delta[n-k] = x[k]\delta[n-k].$$

- By the definition of $\delta[n]$, $\delta[n-k] = 1$ if $n = k$, and 0 otherwise. Therefore,

$$\begin{aligned} x[n]\delta[n-k] &= \begin{cases} x[n], & n = k \\ 0, & n \neq k \end{cases} \\ &= x[k]\delta[n-k]. \end{aligned}$$

# Properties of $\delta[n]$

- Example of sampling property: When $k = 0$, we have $x[n]\delta[n] = x[0]\delta[n]$!

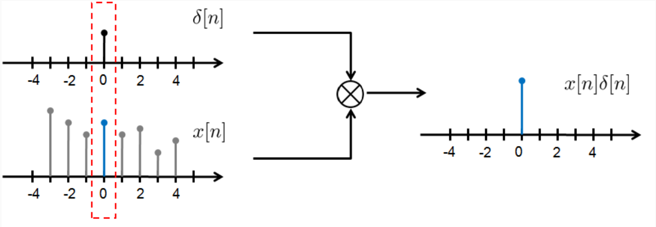



**Figure 3:** Example of sampling property of $\delta[n]$

# Properties of $\delta[n]$

## Sifting Property

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0].$$

**Proof.**

- The sifting property follows from the sampling property:

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] &= \sum_{n=-\infty}^{\infty} x[n_0]\delta[n-n_0] \\
&\overset{(a)}{=} x[n_0]\left(\sum_{n=-\infty}^{\infty} \delta[n-n_0]\right) \\
&\overset{(b)}{=} x[n_0].
\end{aligned}$$

□

# Properties of $\delta[n]$

**Representation Property**

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

**Proof.**

- Consider the sampling property:

$$x[n]\delta[n-k] = x[k]\delta[n-k].$$

- Instead of summing over $n$ (sifting property), sum over $k$:

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = \sum_{k=-\infty}^{\infty} x[n]\delta[n-k] = x[n].$$

$\square$

# Representation property of $\delta[n]$

- The representation property shows that every discrete-time signal x[n] can be represented as a linear combination of time-shifted unit impulses.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

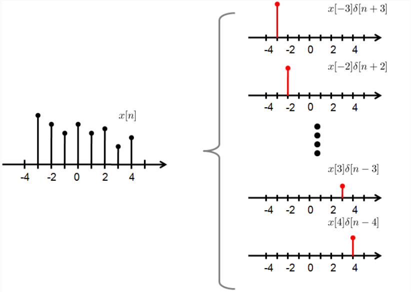
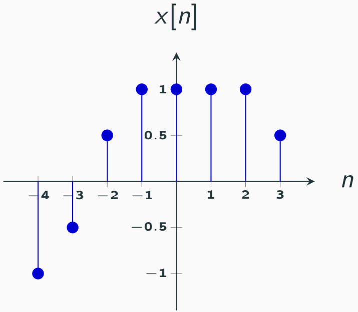


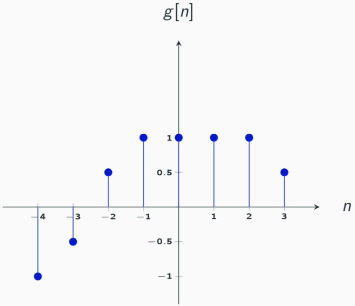

**Figure 4:** Representing a signal, $x[n]$, using a train of impulses, $\delta[n-k]$

# Lecture 3 reading quiz

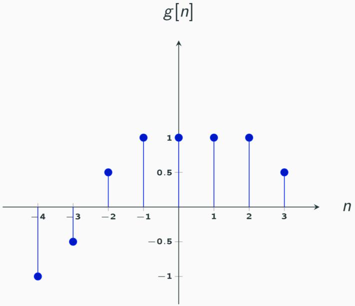
- Signal $x[n]$ is as shown below. Sketch and label $g[n] = x[n]\,(u[3-n] + u[n-2])$.



# Lecture 3 reading quiz

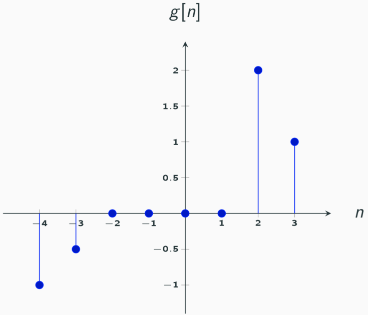


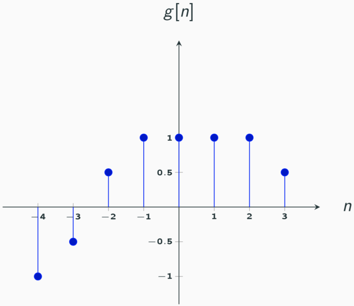

$g[n] = x[n]\,(u[3-n] + u[n-2])$?




$x[n]$

# Lecture 3 reading quiz

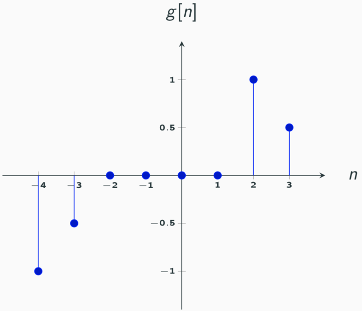


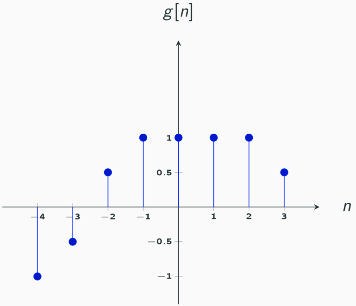

$g[n] = x[n]\,(u[3-n] + u[n-2])$?

$x[n]$

# Lecture 3 reading quiz

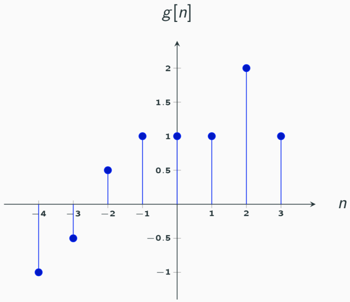



$$g[n] = x[n]\,(u[3-n] + u[n-2])?$$

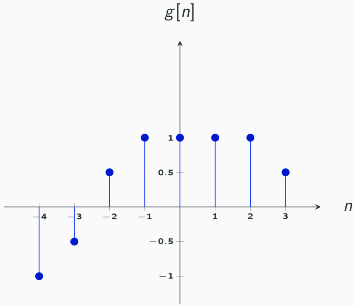



$x[n]$

# Lecture 3 reading quiz




$$g[n] = x[n]\left(u[3-n] + u[n-2]\right)?$$




$$x[n]$$

# Clicker question 1

- Let $x[n] = 1 - \sum_{k=2}^{\infty} \delta[n-k]$. Which of the following statements is correct?

  A. $x[n] = u[-n+1]$

  B. $x[n] = u[n-1]$

  C. $x[n] = u[-n-1]$

  D. $x[n] = -u[n+1]$

# Clicker question 1

- Let $x[n] = 1 - \sum_{k=2}^{\infty} \delta[n-k]$. Which of the following statements is correct?

# Clicker question 2

- A discrete-time signal is defined as $x[n] = \delta[n+1] + \delta[n] + \delta[n-4]$.

  Which answer is correct for $y[n] = x[2n-1] + x[2n+2]$?

  A. $y[n] = \delta[n+1] + \delta[n-1]$

  B. $y[n] = \delta[n] + \delta[n+1] + \delta[n-1]$

  C. $y[n] = \delta[2n-1] + \delta[2n-5] + \delta[2n+3]$

  D. $y[n] = \delta[n-1] + \delta[2n-5] + \delta[2n+3] + \delta[2n-2]$

# Clicker question 2

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  Which answer is correct for $y[n] = x[2n-1] + x[2n+2]$?

# Clicker question 3

- If $x[n] = 1 + \delta[n-1] + u[n-2]$, what is the value of $\sum_{k=-\infty}^{\infty} x[k]\delta[k-2]$?

  A. 1

  B. 2

  C. $2\delta[n-1]$

  D. $u[n-2]$

# Clicker question 3

- If $x[n] = 1 + \delta[n-1] + u[n-2]$, what is the value of $\sum_{k=-\infty}^{\infty} x[k]\delta[k-2]$?

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# Class activity 1

- For the signal given below, determine all the values of the independent variable at which the even part of the signal is guaranteed to be zero.

$$x[n] = u[n] - u[n-4]$$

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