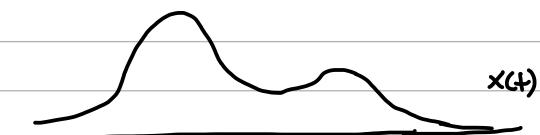


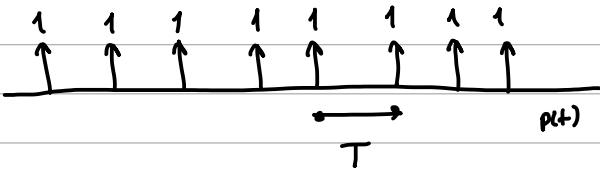
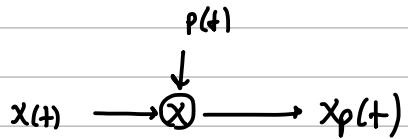
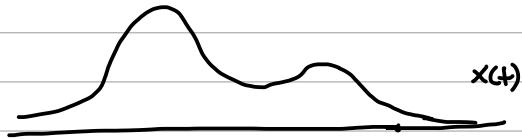
Discussion 10.

Sampling and reconstruction

Imagine you have a signal $x(t)$



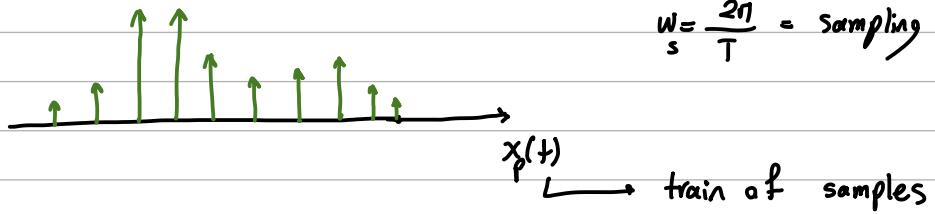
We want to sample $x(t)$, with sampling period T .



Sampling function (periodic impulse train)

T = sampling period

$$w_s = \frac{2\pi}{T} = \text{sampling frequency}$$



$$p(t) = \sum_{-\infty}^{\infty} \delta(t - nT)$$

$$x_p(t) = x(t) p(t) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$

What happen in frequency domain?

Time \longleftrightarrow Freq

$$x(t) p(t) \longleftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

What is $P(j\omega) = F(p(t))$

$p(t)$ is a periodic function with period T . So its Fourier transform is

$$P(j\omega) = 2n \sum_{k=-\infty}^{\infty} a_k \underbrace{\delta(\omega - k \frac{2\pi}{T})}_{w_s}$$

Where a_k is Fourier series coefficient of train of delta functions, which is $\frac{1}{T}$

$$\rightarrow p(j\omega) = \frac{2\pi}{T} \sum \delta(\omega - kw_s), \quad w_s = \frac{2\pi}{T}$$

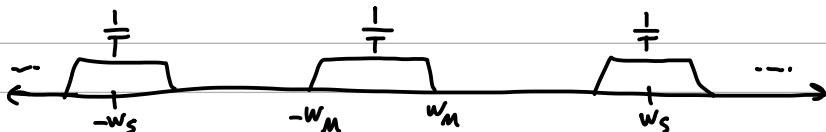
$$\Rightarrow X_p(j\omega) = \frac{1}{2n} X(j\omega) * P(j\omega) = \frac{1}{2n} \left[X(j\omega) * \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - kw_s) \right]$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - kw_s))$$

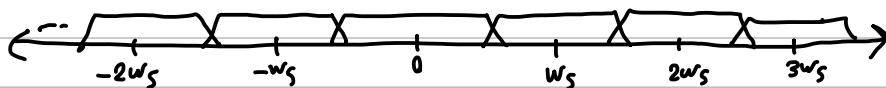
So if $X(j\omega)$ is



$X_p(j\omega)$ will be



OR



Depending on w_M and w_s , we can have overlap (⊗) or no overlap (⊕)

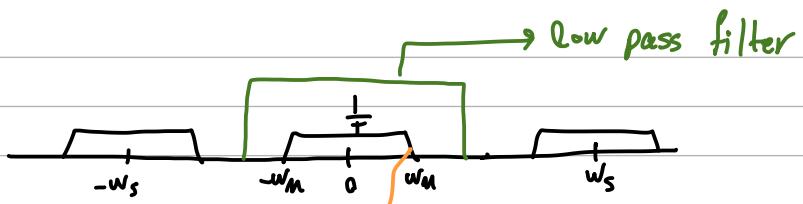
When we have no overlap? $w_M < w_s - w_M \rightarrow w_s > 2w_M$

w_M = Nyquist frequency

$2w_M$ = Nyquist rate

So if the sampling frequency (w_s) is larger than Nyquist rate ($2w_M$), we will have no overlap, no distortion, no aliasing,

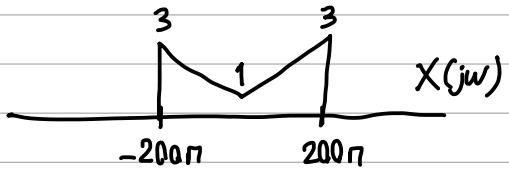
Why we are interested in no overlap? We want to be able to reconstruct the original message $x(t)$ from the train of samples $X_p(t)$.



→ (scaled) Fourier transform of original signal

Using a low pass filter, you can recover $X(j\omega)$ from $X_p(j\omega)$ if there is no aliasing.

Example: For $x(t)$ and its FT, $X(j\omega)$,



a) Find Nyquist frequency and rate.

b) What is the biggest sampling period, T_s , that leads to no distortion.

c) Plot $X_p(j\omega)$ for $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$, for $T_s = \frac{T}{3}$ (T calculated in part b)

Bode Plot Review

Step ①: Write $H(j\omega)$ in this format: $H(j\omega) = A \frac{(1 + \frac{j\omega}{z_1}) \dots (1 + \frac{j\omega}{z_n})}{(1 + \frac{j\omega}{p_1}) \dots (1 + \frac{j\omega}{p_m})}$

Step ② Identify all breaking points.

- For magnitude: $\{z_k\}_{k=1}^n$ and $\{p_k\}_{k=1}^m$.

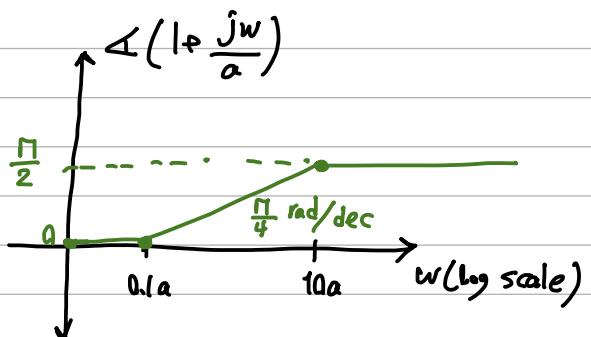
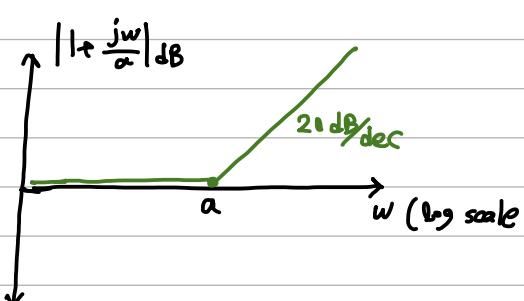
- For phase: $\left\{\frac{z_k}{10}, 10z_k\right\}_{k=1}^n$ and $\left\{\frac{p_k}{10}, 10p_k\right\}_{k=1}^m$.

Step ③: Plot the individual Bode plot for each term

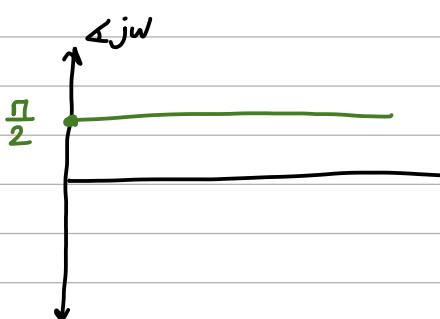
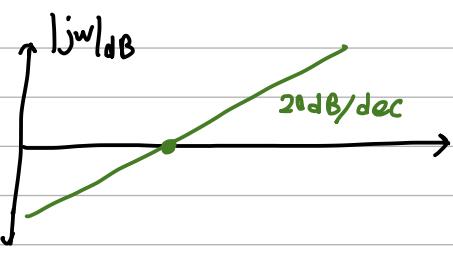
$$\begin{aligned}|H(j\omega)|_{dB} &= 20 \log |H(j\omega)| = 20 \log |A| + 20 \log \left|1 + \frac{j\omega}{z_1}\right| + \dots + 20 \log \left|1 + \frac{j\omega}{z_n}\right| \\ &\quad - 20 \log \left|1 + \frac{j\omega}{p_1}\right| - \dots - 20 \log \left|1 + \frac{j\omega}{p_m}\right|\end{aligned}$$

$$\begin{aligned}\angle H(j\omega) &= \angle A + \angle \left(1 + \frac{j\omega}{z_1}\right) + \dots + \angle \left(1 + \frac{j\omega}{z_n}\right) \\ &\quad - \angle \left(1 + \frac{j\omega}{p_1}\right) - \dots - \angle \left(1 + \frac{j\omega}{p_m}\right)\end{aligned}$$

for a general case: $\left|1 + \frac{j\omega}{a}\right|_{dB}$ and $\angle \left(1 + \frac{j\omega}{a}\right)$



For $|j\omega|_{dB}$ and $\angle j\omega$



Step ④: Sum the result.

Step ⑤: Label all breaking points, slopes, and transition points

Example: $H(jw) = \frac{3(jw+2)}{(1+\frac{jw}{5})^2}$

$$H(jw) = \frac{3(jw+2)}{(1+\frac{jw}{5})^2} = \frac{3 \times 2 \times (1+\frac{jw}{2})}{(1+\frac{jw}{5})^2} = 6 \frac{1+\frac{jw}{2}}{(1+\frac{jw}{5})^2}$$

$$\begin{aligned}\text{Magnitude: } |H(jw)|_{dB} &= 20 \log 6 + 20 \log \left|1 + \frac{jw}{2}\right| - 20 \log \left|\left(1 + \frac{jw}{5}\right)^2\right| \\ &= 20 \log 6 + 20 \log \left|1 + \frac{jw}{2}\right| - 40 \log \left|1 + \frac{jw}{5}\right|\end{aligned}$$

$$\text{Example: } \frac{(1 + \frac{jw}{2})}{(jw)^2 (1 + \frac{jw}{10})}$$