

**UNIVERSITY OF CALIFORNIA, SAN DIEGO**  
**Electrical & Computer Engineering Department**  
**ECE 101 - Fall 2019**  
*Linear Systems Fundamentals*

**MIDTERM EXAM**

**You are allowed two 2-sided sheet of notes.**

**No books, no other notes, no electronics.**

PRINT YOUR NAME \_\_\_\_\_

Signature \_\_\_\_\_

**Your signature confirms that you have completed this exam on your own and in accordance with the ECE 101 honor code.**

Student ID Number \_\_\_\_\_

Problem	Weight	Score
1	20 pts	
2	30 pts	
3	20 pts	
4	30 pts	
Total	100 pts	

**Please do not begin until told.**

**Show your work.**

**Use back of previous page and attached scratch sheets as needed.  
Tables 3.1 and 3.2 from the textbook are attached to the exam.**

**Good luck!**

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**Problem 1 (20 pts)**

(a) (8 pts)

Let  $x(t) = 2^{-t} (u(t+1) - u(t-1))$ . Define  $y(t) = x((-2t) + 1)$ .

To obtain  $y(t)$  from  $x(t)$  you can

First shift  $x(t)$  by \_\_\_\_\_

Then scale the result by \_\_\_\_\_

or

First scale  $x(t)$  by \_\_\_\_\_

Then shift the result by \_\_\_\_\_

(b) (6 pts)

Let  $x(t)$  and  $y(t)$  be as in part (a) above.

Sketch precisely  $y(t)$ .

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**Problem 1 (cont.)**

(c) (6 pts)

Let  $x(t)$  and  $y(t)$  be as in part (a) above.

The signal  $x(t)$  can be written as  $x(t) = y(ct - d)$ . Find  $c$  and  $d$ .

Confirm mathematically that your answer is correct.

$$c = \underline{\hspace{2cm}}$$

$$d = \underline{\hspace{2cm}}$$

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**Problem 2 (30 pts)**

(a) (5 pts)

Consider the discrete-time system  $S_1$  defined by

$$y[n] = \begin{cases} x[n-1], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1. \end{cases}$$

Determine and precisely sketch the impulse response  $h[n]$ .

(b) (5 pts)

Give the difference equation for the linear, time-invariant (LTI) system  $S_2$  with the impulse response  $h[n]$  you found in part (a).

Describe the action of the system  $S_2$  in words.

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**Problem 2 (cont.)**

(c) (8 pts)

Is the system  $S_1$  of part (a) an LTI system?

More specifically, is  $S_1$  linear? Is  $S_1$  time-invariant?

Indicate below whether the system satisfies each listed property.

**Justify your answers with specific reference to the system  $S_1$ .**

**True    False**

       LTI

       Linear

       Time-invariant

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**Problem 2 (cont.)**

(d) (12 pts)

Indicate below whether the system  $S_1$  of part (a) satisfies each listed property.

**Justify your answers with specific reference to the system  $S_1$ .**

**True    False**

       Memoryless

       Invertible (if so, give the inverse system)

       Causal

       Stable

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**Problem 3 (20 pts)**

(a) (6 pts)

Consider the continuous-time LTI system  $S$  defined by

$$y(t) = x(t) + \frac{1}{2}x(t - 1).$$

Determine the impulse response  $h(t)$  of the system.

(b) (6 pts)

Determine the system function and frequency response of the system in part (a). **Justify your answers.**

$$H(s) = \text{_____}$$

$$H(j\omega) = \text{_____}$$

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**Problem 3 (cont.)**

(c) (8 points)

Let  $x(t) = \cos(\pi t)$  be the input to the system in part (a).

Determine the output  $y(t)$ . Express it in the form  $y(t) = A \cos(\omega_0 t + \theta)$ .

**Justify your answer.**

$$A = \underline{\hspace{2cm}}$$

$$\omega_0 = \underline{\hspace{2cm}}$$

$$\theta = \underline{\hspace{2cm}}$$

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**Problem 4 (30 pts)**

(a) (16 points)

Let  $x[n] = \cos(\frac{\pi}{2}n) \sin(\frac{\pi}{4}n)$ . The signal has fundamental period  $N = 8$ .

Is  $x[n]$  even, odd, or neither?

Determine the Fourier series of  $x[n]$ .

$x[n]$  is even \_\_\_\_\_      odd \_\_\_\_\_      neither \_\_\_\_\_

$a_0 =$  \_\_\_\_\_       $a_4 =$  \_\_\_\_\_

$a_1 =$  \_\_\_\_\_       $a_5 =$  \_\_\_\_\_

$a_2 =$  \_\_\_\_\_       $a_6 =$  \_\_\_\_\_

$a_3 =$  \_\_\_\_\_       $a_7 =$  \_\_\_\_\_

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**Problem 4 (cont.)**

(b) (7 points)

Let  $x[n]$  be the signal of part (a). Determine  $\sum_{n=0}^7 x[n]$ .

How does this sum relate to the Fourier series coefficient  $a_0$ ?

(c) (7 points)

Let  $y[n] = x[n - 4]$ , where  $x[n]$  is the signal of part (a).

Is  $y[n]$  even, odd, or neither?

Determine the relationship between the Fourier series  $b_k$  of  $y[n]$  and the Fourier series  $a_k$  of  $x[n]$ .

$y[n]$  is even \_\_\_\_\_      odd \_\_\_\_\_      neither \_\_\_\_\_

$b_k =$  \_\_\_\_\_