CSE 167 (WI 2025) Exercise 2 — Due 1/24/2025

You've probably seen it when you first learned matrix algebra. The inversion of 2×2 matrices has a simple formula: swap the diagonals, add a minus sign on the off diagonals, and divide the result by the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \tag{1}$$

Exercise 2.1 — 3 pts. Let V be a 2-dimensional vector space. Let $\vec{a}_1, \vec{a}_2 \in V$ be a pair of linearly independent vectors (a basis). Let $\vec{b}_1, \vec{b}_2 \in V$ be another basis satisfying the relation:

$$\vec{b}_1 = -\vec{a}_1 + 2\vec{a}_2, \quad \vec{b}_2 = 3\vec{a}_1 - 5\vec{a}_2. \tag{2}$$

Now, let $\vec{v} \in V$ be a vector with coefficients $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ under the basis $(\vec{a}_1, \vec{a}_2)$; that is,

$$\vec{v} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \vec{a}_1 - \vec{a}_2. \tag{3}$$

What is the coefficients of $\vec{v}$ under the basis $(\vec{b}_1, \vec{b}_2)$? That is, what is x, y in

$$\vec{v} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x\vec{b}_1 + y\vec{b}_2? \tag{4}$$

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Exercise 2.2 — 1 pt. In the lecture we mentioned that matrix multiplication is not commutative. Given an example of $\mathbf{A}, \mathbf{B}$, both square matrices of the same size, so that $\mathbf{AB} \neq \mathbf{BA}$.

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