ECE 45 – Circuits and Systems
Winter 2025

Homework #1

Due: January 10 at 5:00pm, submitted via GradeScope.

You can make multiple upload attempts to experiment with the system and the best way to upload. You must correctly mark the answers to the problems in GradeScope, e.g. problem 1, problem 2, problem 3, to get full credit. Note that you must tag your problems when uploading to GradeScope or they will not be graded and you will not receive credit. Any regrade requests must be placed through GradeScope within one week of the return of the homework.

Remember, discussion of homework questions is encouraged. Please be absolutely sure to submit your own independent homework solution.

1. (40 points) Calculus review. Compute the following sums or integrals. We will use these calculations later in the course. You may find the reference equations in the Chapter 1 lecture notes useful. Use results on geometric sums to help in computing the summations.

   (a) $\int_0^\infty e^{-t}dt$

   Solution:

$$\begin{aligned}\int_0^\infty e^{-t}dt &= \int_0^\infty e^{-t}dt \\ &= \lim_{a\to\infty} -e^{-t}\Big|_0^a \\ &= \lim_{a\to\infty} (-e^{-a} + e^0) \\ &= 1\end{aligned}$$

   (b) $\int_1^\infty e^{-t}dt$

   Solution:

$$\begin{aligned}\int_1^\infty e^{-t}dt &= \lim_{a\to\infty} -e^{-t}\Big|_1^a \\ &= \lim_{a\to\infty} (-e^{-a} + e^{-1}) \\ &= e^{-1}\end{aligned}$$

   (c) $\int_{-1}^\infty e^{-t}dt$

Solution:

$$\begin{aligned}\int_{-1}^{\infty} e^{-t}dt &= \lim_{a\to\infty} -e^{-t}\Big|_{-1}^{a} \\ &= \lim_{a\to\infty}(-e^{-a}+e^{1}) \\ &= e\end{aligned}$$

(d) $\int_{-\infty}^{\infty} e^{-2|t|}dt$

Solution:

$$\begin{aligned}\int_{-\infty}^{\infty} e^{-2|t|}dt &= \int_{-\infty}^{0} e^{2t}dt + \int_{0}^{\infty} e^{-2t}dt \\ &= \frac{1}{2}\left(\lim_{a\to-\infty} e^{2t}\Big|_{a}^{0} - \lim_{b\to\infty} e^{-2t}\Big|_{0}^{b}\right) \\ &= \frac{1}{2}\left(\lim_{a\to-\infty}(e^{0}-e^{2a}) + \lim_{b\to\infty}(-e^{-2b}+e^{0})\right) \\ &= \frac{1}{2}(2) = 1\end{aligned}$$

(e) $\int_{0}^{\infty} te^{-t}dt$

Solution:

$$\begin{aligned}\int_{0}^{\infty} te^{-t}dt &= (-te^{-t})\Big|_{0}^{\infty} - \int_{0}^{\infty} -e^{-t}dt \quad (Using\ IBP) \\ &= (0) + \int_{0}^{\infty} e^{-t}dt \\ &= (-e^{-t})\Big|_{0}^{\infty} = 1\end{aligned}$$

(f) $\int_{0}^{\infty} e^{-\beta t}dt$ where $\beta \in \mathbb{R}$. Hint: Be sure to carefully consider the values of the real variable α as the integral does not always exist.

Solution: For this question, β is a variable, that can assume any value in the domain of real numbers.
In the case of $\beta \geq 0$, $e^{\beta t}$ remains greater than or equal to 1 throughout the range of the integration, which is from 0 to ∞.
For $\beta < 0$:

$$\begin{aligned}\int_{0}^{\infty} e^{-\beta t}dt &= \frac{-1}{\beta}e^{-\beta t}\Big|_{0}^{\infty} \\ &= \lim_{t\to\infty}\left(\frac{-1}{\beta}e^{-\beta t}\right) - \lim_{t\to 0}\left(\frac{-1}{\beta}e^{-\beta t}\right)\end{aligned}$$

Since $\beta < 0$, the first limit approaches infinity as t approaches infinity.

$$\int_{0}^{\infty} e^{-\beta t}dt \to \infty$$

For $\beta = 0$:

$$\int_0^\infty e^{-\beta t} dt = \int_0^\infty 1 dt \to \infty$$

For $\beta > 0$:

$$\int_0^\infty e^{-\beta t} dt = \frac{-1}{\beta} e^{-\beta t}\Big|_0^\infty$$
$$\text{Since } \beta \text{ is positive, } e^{-\beta\infty} \to 0$$
$$\int_0^\infty e^{-\beta t} dt = \frac{-1}{\beta}(0-1) = \frac{1}{\beta}$$

Therefore, the integration converges to $\frac{1}{\beta}$ when $\beta < 0$, and diverges for other values of $\beta \leq 0$.

(g) Rewrite $4 + 4j$ in polar coordinates.

Solution:

$$\begin{aligned}
&z = 4 + 4j \\
&r = \sqrt{\mathrm{Re}(z)^2 + \mathrm{Im}(z)^2} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}, \\
&\theta = \arctan\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right) = \arctan\left(\frac{4}{4}\right) = \arctan(1) = \frac{\pi}{4}.
\end{aligned}$$

Therefore, the polar form of z is:

$$z = r \cdot e^{j\theta} = 4\sqrt{2} \cdot e^{j\frac{\pi}{4}}.$$

(h) Simplify the following and put in polar form $(4 + 3j) - (2 - 6j)$.

Solution:

$$\begin{aligned}
z &= (4 + 3j) - (2 - 6j) \\
&= 4 + 3j - 2 + 6j \\
&= (4 - 2) + (3 + 6)j \\
&= 2 + 9j.
\end{aligned}$$

Now, we rewrite $z = 2 + 9j$ in polar form.

$$\begin{aligned}
&r = \sqrt{\mathrm{Re}(z)^2 + \mathrm{Im}(z)^2} = \sqrt{2^2 + 9^2} = \sqrt{4 + 81} = \sqrt{85}, \\
&\theta = \arctan\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right) = \arctan\left(\frac{9}{2}\right).
\end{aligned}$$

Therefore, the polar form of z is:

$$z = r \cdot e^{j\theta} = \sqrt{85} \cdot e^{j \arctan\left(\frac{9}{2}\right)}.$$

(i) Use Euler's formula to write $f(t) = e^{jt} - e^{-jt} + 2e^{3jt}$ as a finite sum of sinusoidal functions.

Solution:

$$f(t) = e^{jt} - e^{-jt} + 2e^{3jt}.$$

Using Euler's formula $e^{jt} = \cos(t) + j\sin(t)$ and $e^{-jt} = \cos(t) - j\sin(t)$, we get:

$$f(t) = (\cos(t) + j\sin(t)) - (\cos(t) - j\sin(t)) + 2(\cos(3t) + j\sin(3t)).$$

Simplifying:

$$f(t) = 2\cos(3t) + j(2\sin(t) + 2\sin(3t)).$$

(j) Use Euler's formula to write $g(t) = 1+\cos(t)+\sin(2t)$ as a finite sum of complex exponentials.

Solution:

$$\begin{aligned} g(t) &= 1 + \cos(t) + \sin(2t) \\ &= e^{j\cdot 0} + \frac{e^{jt} + e^{-jt}}{2} + \frac{e^{2jt} - e^{-2jt}}{2j} \\ &= e^{j\cdot 0} + \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt} + \frac{1}{2j}e^{2jt} - \frac{1}{2j}e^{-2jt} \end{aligned}$$

(k) Determine the phasor associated with the voltage signal $v(t) = 4\cos(4t) - 4\sin(4t)$.

Solution:

$$\begin{aligned} v(t) &= 4\cos(4t) - 4\sin(4t) \\ &= \mathrm{Re}[4e^{4jt}] - \mathrm{Re}[4e^{j(4t-\frac{\pi}{2})}] \\ &= \mathrm{Re}[4e^{4jt} - 4e^{-\frac{\pi}{2}j}e^{4jt}] \\ &= \mathrm{Re}\left[(4 - 4(-j))e^{4jt}\right] \\ &= \mathrm{Re}\left[4\sqrt{2}e^{j\frac{\pi}{4}}e^{4jt}\right]. \end{aligned}$$

So, the phasor is $V = 4\sqrt{2}e^{j\frac{\pi}{4}}$.

2. (50 pts) Complete the MATLAB Onramp course.

   `https://matlabacademy.mathworks.com`

   Turn in your course completion certificate. This will be included as part of the PDF that you submit to GradeScope.

3. (10 pts) Write an acknowledgement that you have reviewed all the course policies from the syllabus including but not limited to (a) the requirement to mark problems when you submit to GradeScope to receive any credit for the problem, and (b) policies related to no excused homeworks and the two homework drop policy.