Discussion Session 9:

Convolution property and Fourier Transform:

$$x(t) \longrightarrow \boxed{\;h(t)\;} \longrightarrow y(t)$$

$h(t)$ = Impulse response of a system is its output when the input is a unit impulse function $\delta(t)$.

$H(jw)$ = The frequency response of a system describe how the system react to different frequency components.

$$H(jw) = F\big(h(t)\big) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

If we have an input signal $x(t)$ and we want to find the output $y(t)$ of a LTI system, we can compute it in two different ways.

1) Time-domain approach (Convolution)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

2) Frequency-domain approach (Multiplication)

$$\left.\begin{array}{l} X(jw) = F\big(x(t)\big) \\ H(jw) = F\big(h(t)\big) \end{array}\right\} \Rightarrow Y(jw) = X(jw)\, H(jw) \Rightarrow y(t) = F^{-1}\big(Y(jw)\big)$$

Example: $x(t) = e^{-t}\, u(t) \qquad h(t) = u(t)$

Modulation:

Modulation is the process of modifying a carrier signal to transmit information.

AM modulation: $s(t) = \underbrace{m(t)}_{\text{message}} \cos(\underbrace{w_c}_{\text{carrier frequency}} t)$

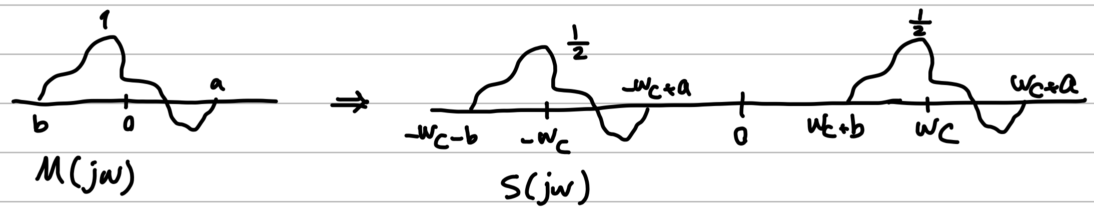
It is called amplitude modulation (AM) as the message signal is embedded in the amplitude variation of the career signal.

To transmite the signal, we multiply the message with a high frequency cosine wave.

$s(t) = m(t)\cos(w_c t) \longrightarrow$

This is corresponds to shifting the fourier transform of signal to $\pm w_c$.

$$S(jw) = \frac{1}{2} M(j(w - w_c)) + \frac{1}{2} M(j(w + w_c))$$

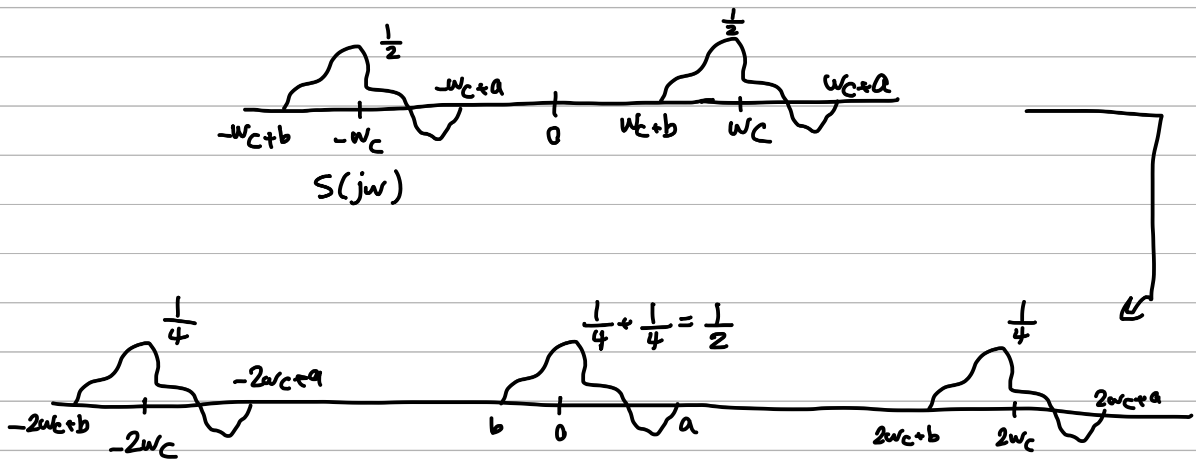


Demodulation:

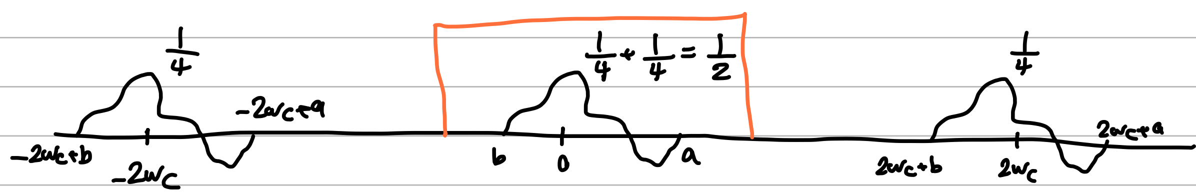
At receiver, the original signal is recovered by multiplying to $\cos(w_c t)$

$$R(t) = s(t)\cos(w_c t) = m(t)\cos(w_c t)\cos(w_c t)$$

$$= m(t)\,\frac{1}{2}\left[1 + \cos(2w_c t)\right]$$



A low pass filter can recover the signal

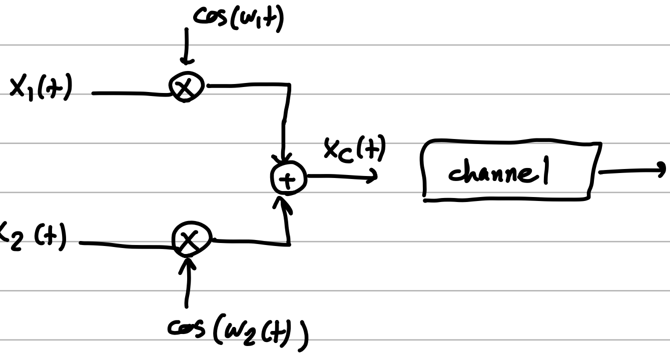


Handling multiple signal.

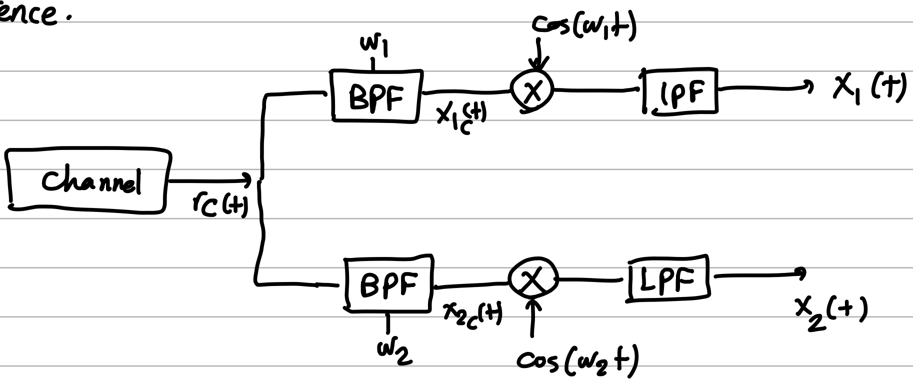
In practical, multiple signals are transmitted simultaneously over different carrier frequency.

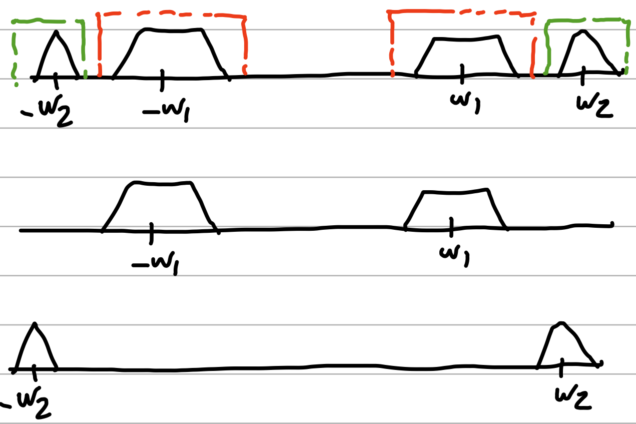
user 1:  , w_{c1}

user 2:  , w_2



At receiver, a band pass filter is used to seperate desired signals, and avoid interference.





Bode plot:

It is a graphical representation of a system's frequency response, showing how system's gain (magnitude) and phase shift vary with frequency.

Constructing Bode plots by hand involves approximating these responses using straight line segments, which simplifies the analysis.

Steps to plot Bode Plot:

① Express the transfer function in standard form.

$$H(j\omega) = k \frac{(j\omega/z_1 + 1)(j\omega/z_2 + 1)\cdots}{(j\omega/p_1 + 1)(j\omega/p_2 + 1)\cdots}$$

Here, k is the gain, z_i are zeros, and p_i are the poles.

② Identify the corner (break) frequencies.

poles and zeros of transfer function.

③ Magnitude plot:

$$\log|H(j\omega)| = \log|k| + \log\left|\frac{j\omega}{z_1}+1\right| + \log\left|\frac{j\omega}{z_2}+1\right| + \cdots - \log\left|\frac{j\omega}{p_1}+1\right| - \log\left|\frac{j\omega}{p_2}+1\right| \cdots$$

* Initial gain: start by plotting the initial gain of $20\log|k|$ in descibels (dB)

* slop changes: at each zero frequency (ω_z), increase the slop by +20 dB/decade. At each pole frequency, decrease the slop by -20 dB/decade.

$$H(j\omega) = \frac{10(j\omega+1)}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)}$$

④ Construct the phase plot.

* Initial phase: start with the phase angle contributed by the gain k. If k is positive, the initial phase is 0, if negative, it is π.

* Phase contribution of poles and zeros.

zero at w_z. Contribute $+\frac{\pi}{4}$ per decade, starting from one decade before w_z and leveling off at $\frac{\pi}{2}$ one decade after.

pole at w_p: Contribute $\frac{-\pi}{4}$ per decade, starting from one decade w_p and leveling off at $\frac{-\pi}{2}$ one decade after.

Example: $$H(jw) = \frac{10\,(jw+1)}{\left(\frac{jw}{10}+1\right)\left(\frac{jw}{100}+1\right)}$$

Notes :

1) Handling $(\frac{jw}{z}+1)^n$ or $(\frac{jw}{p}+1)^n$

* If zero or pole is repeated n times, it amplifies the effect.

Magnitude : instead of 20 $dB/_{decade}$, change slop by $20n$ $dB/_{decade}$.

phase : Each zero contributes +90 per factor, and each pole contribute -90 per factor.

$$H(jw) = \frac{1}{(jw+1)^2} \left\{ \begin{array}{l} \text{Magnitude : start at 0 dB, drops by } -40\ dB/_{decade} \text{ after } w=1 \\ \text{phase : start at 0, then drops to } \frac{-\pi}{2} \text{ at } w=1 \text{ and settles at } -\pi. \end{array} \right.$$

2) Handling jw and $\frac{1}{jw}$.

jw : contributes $+20$ $dB/_{decade}$ indefinil and a constant $+\frac{\pi}{2}$ phase shift.

$\frac{1}{jw}$: Constributes -20 $dB/_{decade}$ and a constant $\frac{-\pi}{2}$ phase shift.

Magnitude

Phase

(jw)