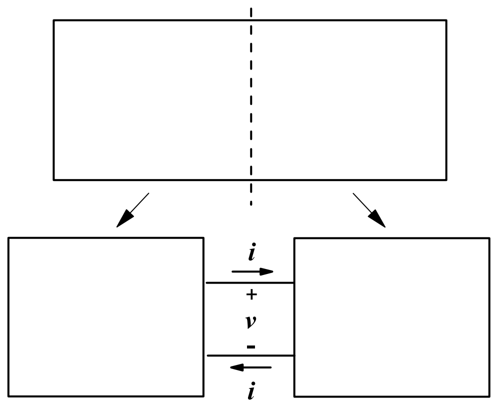
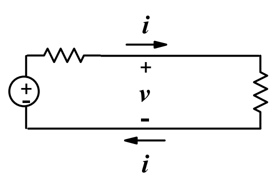
VI. Transistor Amplifiers

6.1. Amplifier System Concepts

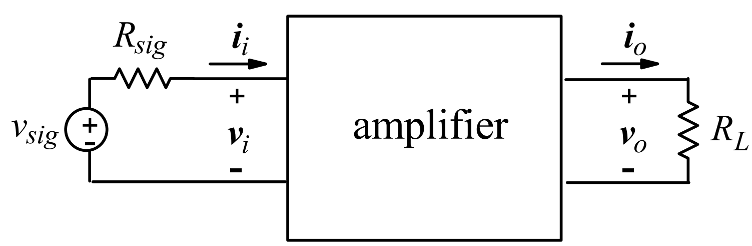
6.1.1. Basic Treatment of Linear Circuits

When a linear circuit is partitioned into two sub-circuits, or two one-port networks, each one-port network is represented by its Thevenin's equivalent circuit. When combined, the two unknowns at the terminals, *i* and *v*, are solved from the two coupled linear equations.





The above concept is generalized to a two-port linear network, e.g., an amplifier, with a signal circuit to the left and a load to the right.



In this case, there are four unknowns, two voltages and two currents. The amplifier, as a linear two-port network can be represented by a 2×2 matrix with four elements.

$$\begin{bmatrix} v_o \\ i_o \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_i \\ i_i \end{bmatrix}$$

It represents two linear relations among the four variables, v_i, i_i, v_o, i_o at the input and output terminals of the amplifier. When coupled to the equation describing the signal circuit,

$$v_{sig} = R_{sig} i_i + v_i \ ,$$

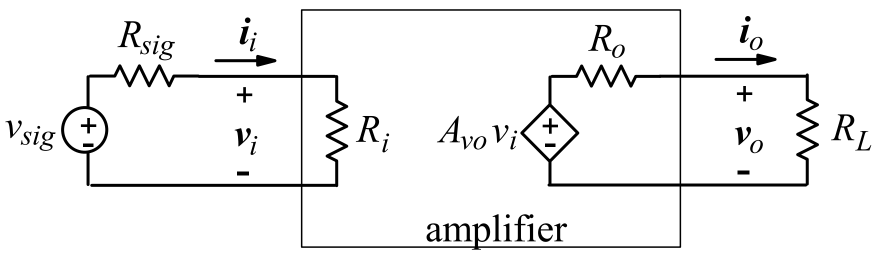
and the load resistor,

$$v_o = R_L i_o \ ,$$

all four variables, v_i, i_i, v_o, i_o, are solved.

Equivalent Circuit of an Amplifier

The following equivalent circuit is used to represent a unilateral amplifier.



The voltage gain of the amplifier is

$$\frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_o + R_L}$$

where A_{vo} is the open loop voltage gain,

$$A_{vo} \equiv \left. \frac{v_o}{v_i} \right|_{R_L \to \infty} .$$

R_o is the output resistance defined as,

$$R_o \equiv - \left. \frac{v_o}{i_o} \right|_{v_i = 0}$$

R_i is the input resistance defined as the Thevenin resistance seen by the signal circuit,

$$R_i \equiv \frac{v_i}{i_i}$$

R_i depends on the load R_L.

The overall gain from v_{sig} to v_o is

$$\frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i} = \frac{R_i}{R_i + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

Amplifier Parameters in terms of the Matrix Elements:

By eliminating i_i from the two linear equations given by the 2×2 matrix,

$$(a_{11} - a_{12}a_{21} / a_{22})v_i = -(a_{12} / a_{22})i_o + v_o .$$

Hence,

$$A_{vo} = a_{11} - a_{12}a_{21} / a_{22}$$

and

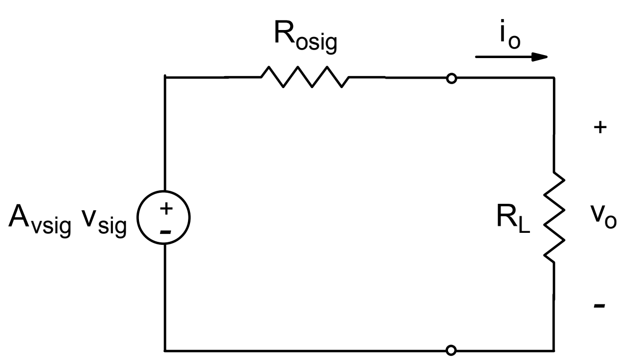
$$R_o = -a_{12} / a_{22} \qquad (> 0)$$

To find R_i, we let $v_o = R_L i_o$ in the two matrix equations to obtain the ratio of v_i/i_i,

$$R_i = \frac{v_i}{i_i} = \frac{a_{22}R_L - a_{12}}{a_{11} - a_{21}R_L} .$$

Thus in contrast to R_o which only depends on the amplifier parameters, R_i depends on the load resistance R_L as well.

The Thevenin's equivalent circuit that includes both the amplifier and the signal circuit, as seen by the load, takes the following form:



A_{vsig} and R_{osig} can be found by eliminating v_i and i_i from the two matrix equations plus the signal circuit relation, $v_{sig} = R_{sig}i_i + v_i$,

$$(a_{11}a_{22} - a_{12}a_{21})v_{sig} = (a_{22} - a_{21}R_{sig})v_o - (a_{12} - a_{11}R_{sig})i_o \,.$$

Therefore,

$$A_{vsig} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22} - a_{21}R_{sig}}$$

and

$$R_{osig} = \frac{-a_{12} + a_{11}R_{sig}}{a_{22} - a_{21}R_{sig}}$$

Note that $R_{osig} = R_o$ only when $R_{sig} = 0$.

Consistency check:

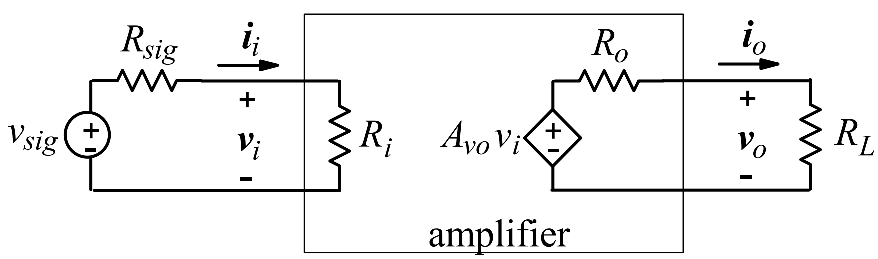
From the last circuit,

$$v_o = A_{vsig} v_{sig} \frac{R_L}{R_L + R_{osig}}$$

So

$$\frac{v_o}{v_{sig}} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22} - a_{21}R_{sig}} \frac{R_L}{R_L + \dfrac{-a_{12} + a_{11}R_{sig}}{a_{22} - a_{21}R_{sig}}} = \frac{(a_{11}a_{22} - a_{12}a_{21})R_L}{(a_{22} - a_{21}R_{sig})R_L - a_{12} + a_{11}R_{sig}}$$

On the other hand, from the amplifier equivalent circuit on the page before the last,



$$\frac{v_o}{v_{sig}} = \frac{v_o}{A_{vo}v_i} A_{vo} \frac{v_i}{v_{sig}} = \frac{R_L}{R_L + R_o} A_{vo} \frac{R_i}{R_i + R_{sig}}$$

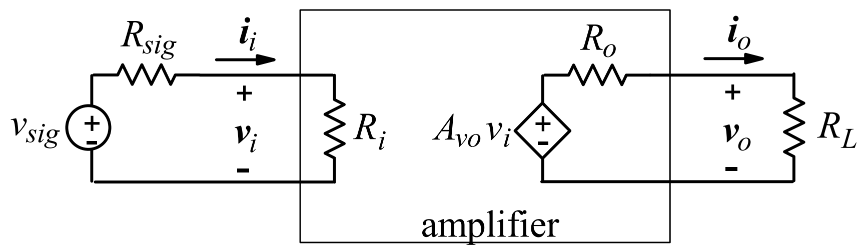
$$= \frac{R_L}{R_L - a_{12}/a_{22}} (a_{11} - a_{12}a_{21}/a_{22}) \frac{\dfrac{a_{22}R_L - a_{12}}{a_{11} - a_{21}R_L}}{\dfrac{a_{22}R_L - a_{12}}{a_{11} - a_{21}R_L} + R_{sig}}$$

$$= \frac{R_L(a_{11}a_{22} - a_{12}a_{21})}{a_{22}R_L - a_{12}} \frac{a_{22}R_L - a_{12}}{a_{22}R_L - a_{12} + (a_{11} - a_{21}R_L)R_{sig}}$$

$$= \frac{R_L(a_{11}a_{22} - a_{12}a_{21})}{a_{22}R_L - a_{12} + (a_{11} - a_{21}R_L)R_{sig}}$$

The two v_o/v_{sig} expressions are mathematically identical.

Current gain and power gain



In addition to the voltage gain v_o/v_i, there is also the current gain i_o/i_i:

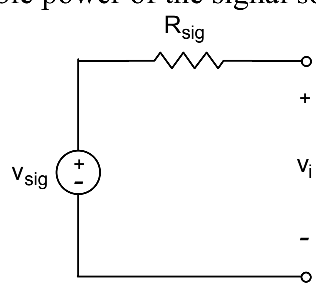
$$\frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / R_i} = \frac{v_o}{v_i}\frac{R_i}{R_L}$$

and the power gain,

$$\frac{v_o i_o}{v_i i_i} = \left(\frac{v_o}{v_i}\right)^2 \frac{R_i}{R_L}.$$

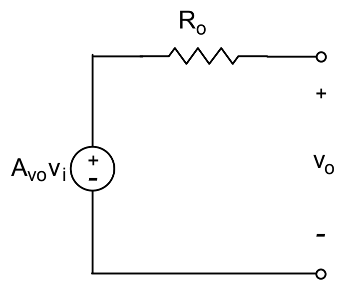
For the emitter follower and the source follower amplifiers, voltage gain ≈ 1, but both the current gain and the power gain can be >> 1.

The maximum available power of the signal source without the amplifier is



$$P_{sig}(\max) = \frac{v_{sig}^2}{8R_{sig}}$$

The maximum available power at the output of the amplifier is

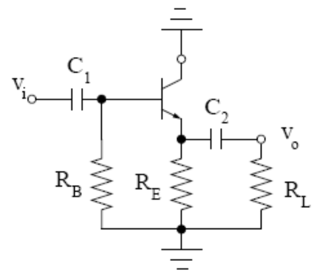


$$P_{out}(\max) = \frac{(A_{vo}v_i)^2}{8R_o}$$

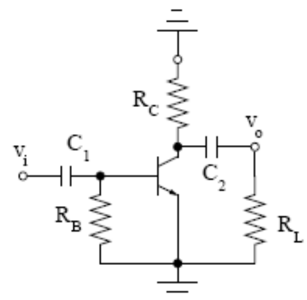
The available power gain of the amplifier is $P_{out}(\max)/P_{sig}(\max)$.

Types of transistor amplifiers covered in this section:

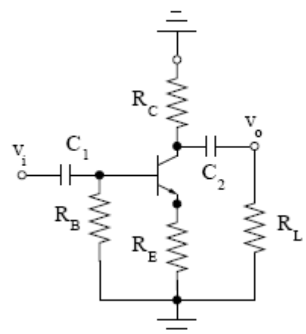
- Common collector BJT amplifier, a.k.a., emitter follower.



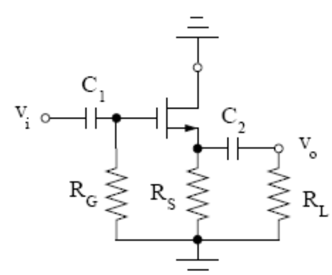
- Common emitter BJT amplifier.



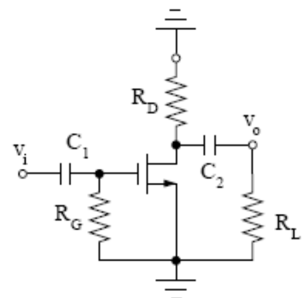
- Common emitter BJT amplifier with emitter resistance.



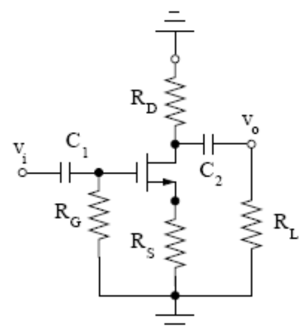
- Common drain MOS amplifier, a.k.a., source follower.



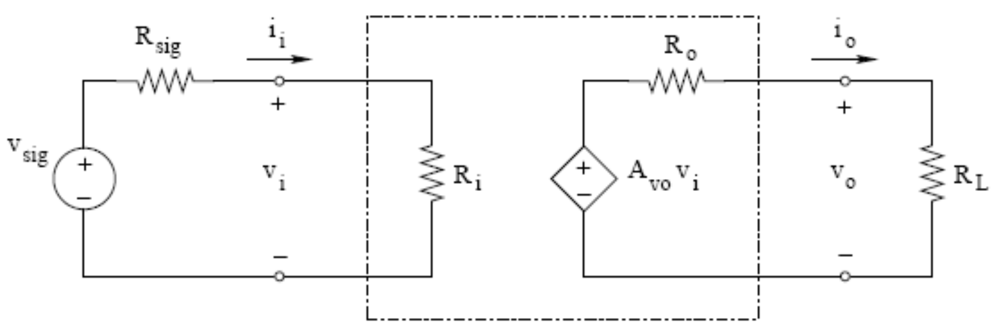
- Common source MOS amplifier.



- Common source MOS amplifier with source resistance.



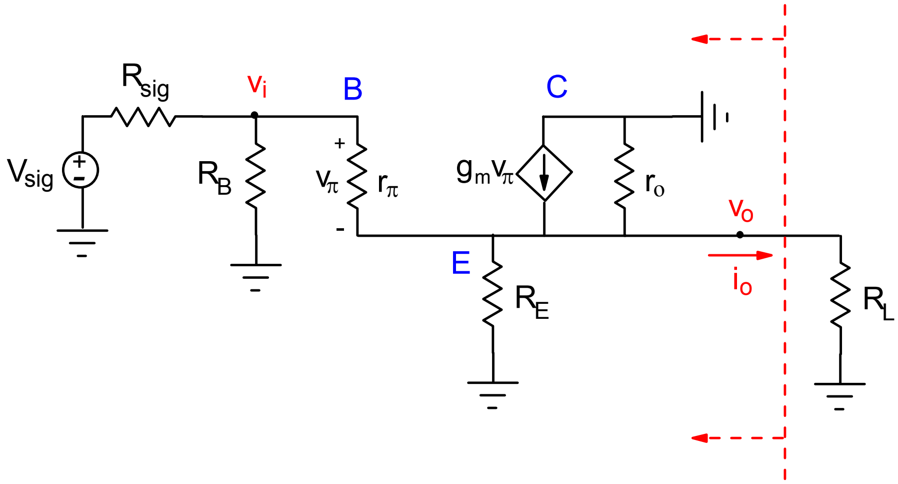
General equivalent circuit for any type of amplifier:



A_{vo}, R_o, R_i of the common collector BJT amplifier and common emitter BJT amplifier with emitter resistance are derived in detail below. The parameters of the other types of amplifiers can be obtained by simple substitution afterwards.

6.2.1. Derivation of A_{vo}, R_o, R_i for the common collector BJT amplifier.

- A_{vo} and R_o:



Assume v_o, i_o at the output node and looking back toward the left. KCL at node E,

$$g_m v_\pi + \frac{v_\pi}{r_\pi} = \frac{v_o}{R_E} + \frac{v_o}{r_o} + i_o$$

$v_\pi = v_i - v_o$,

$$\left(g_m + \frac{1}{r_\pi}\right)(v_i - v_o) = \frac{v_o}{R_E} + \frac{v_o}{r_o} + i_o$$

$$\left(g_m + \frac{1}{r_\pi}\right)v_i = \left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}\right)v_o + i_o$$

The above can be abbreviated as

$$\frac{v_i}{(1/g_m)//r_\pi} = \frac{v_o}{(1/g_m)//r_\pi//R_E//r_o} + i_o$$

By definition of the amplifier equivalent circuit, $A_{vo}\, v_i = v_o + R_o\, i_o$. Therefore,

$$R_o = (1/g_m)//r_\pi//R_E//r_o$$

and

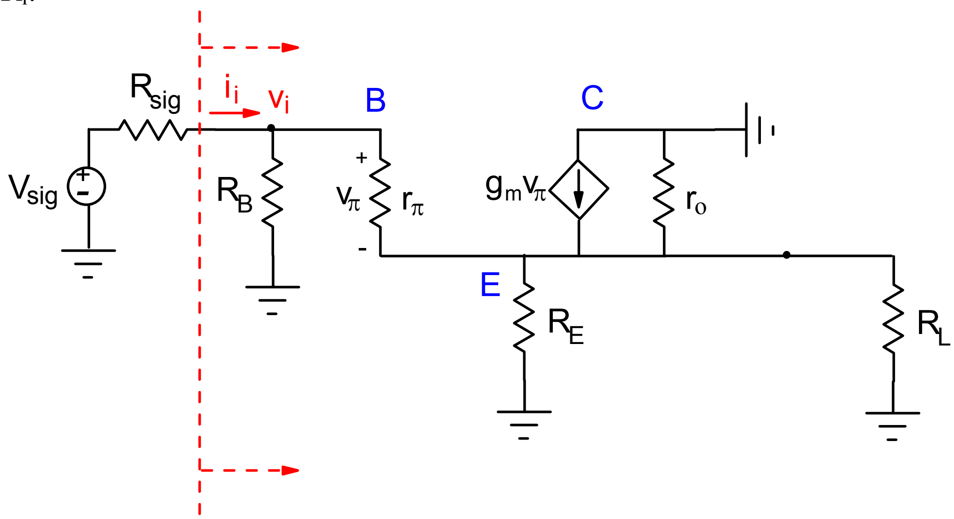
$$A_{vo} = \frac{(1/g_m)//r_\pi//R_E//r_o}{(1/g_m)//r_\pi}$$

Alternatively, since $g_m = \beta/r_\pi$,

$$R_o = \frac{r_\pi}{\beta+1}//R_E//r_o$$

$$A_{vo} = \frac{\frac{r_\pi}{\beta+1} // R_E // r_o}{r_\pi/(\beta+1)} = \frac{R_E // r_o}{r_\pi/(\beta+1) + R_E // r_o} \approx \frac{R_E // r_o}{1/g_m + (R_E // r_o)} = \frac{g_m(R_E // r_o)}{1 + g_m(R_E // r_o)}$$

- R_i:



$$i_i = \frac{v_i}{R_B} + \frac{v_\pi}{r_\pi}$$

The voltage at node E is $v_i - v_\pi$. KCL node E:

$$g_m v_\pi + \frac{v_\pi}{r_\pi} = \frac{v_i - v_\pi}{R_E} + \frac{v_i - v_\pi}{r_o} + \frac{v_i - v_\pi}{R_L}$$

or

$$(\beta+1)\frac{v_\pi}{r_\pi} = \frac{v_i - v_\pi}{R_E // r_o // R_L}$$

and

$$\left[1 + \frac{\beta+1}{r_\pi}(R_E // r_o // R_L)\right] v_\pi = v_i$$

Then

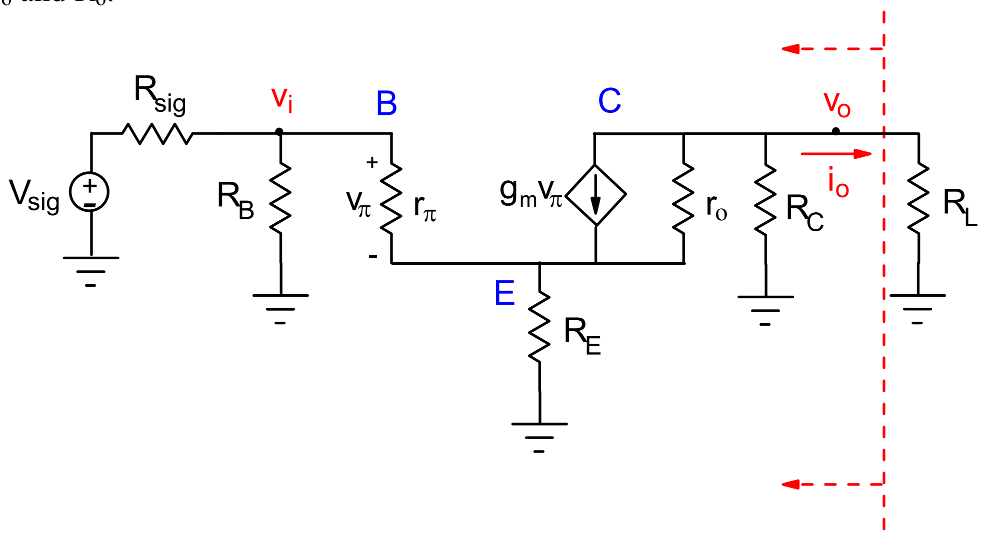
$$i_i = \frac{v_i}{R_B} + \frac{v_i}{r_\pi + (\beta+1)(R_E // r_o // R_L)}$$

From the amplifier equivalent circuit, $i_i = v_i/R_i$. Therefore,

$$R_i = R_B // [r_\pi + (\beta+1)(R_E // r_o // R_L)]$$

6.2.2. Derivation of A_{vo}, R_o, R_i for common emitter BJT amplifier with emitter resistance.

- A_{vo} and R_o:



Assume v_o, i_o at the output node and looking back toward the left. The voltage at node E is $v_i - v_\pi$. KCL supernode E + C,

$$\frac{v_\pi}{r_\pi} = \frac{v_i - v_\pi}{R_E} + \frac{v_o}{R_C} + i_o$$

Solve for v_π,

$$\frac{v_\pi}{r_\pi // R_E} = \frac{v_i}{R_E} + \frac{v_o}{R_C} + i_o$$

KCL at node C,

$$g_m v_\pi + \frac{v_o}{R_C} + i_o = \frac{v_i - v_\pi - v_o}{r_o}$$

Solve for v_π,

$$\frac{v_\pi}{(1/g_m)//r_o} = \frac{v_i - v_o}{r_o} - \frac{v_o}{R_C} - i_o$$

Eliminate v_π by taking ratio of 2nd and 4th eqs,

$$[(1/g_m)//r_o]\left[\frac{v_i - v_o}{r_o} - \frac{v_o}{R_C} - i_o\right] = (r_\pi // R_E)\left[\frac{v_i}{R_E} + \frac{v_o}{R_C} + i_o\right]$$

Group v_i terms on the left and v_o, i_o terms on the right:

$$\left[\frac{1}{1+g_m r_o}-\frac{r_\pi}{r_\pi+R_E}\right]v_i=\left[\frac{(1/g_m)//r_o}{r_o//R_C}+\frac{r_\pi//R_E}{R_C}\right]v_o+\left[(r_\pi//R_E)+(1/g_m)//r_o\right]i_o$$

Identify the above eq. with $A_{vo}v_i = v_o + R_o i_o$, we have

$$A_{vo}=\left[\frac{1}{1+g_m r_o}-\frac{r_\pi}{r_\pi+R_E}\right]\left[\frac{(1/g_m)//r_o}{r_o//R_C}+\frac{r_\pi//R_E}{R_C}\right]^{-1}$$

and

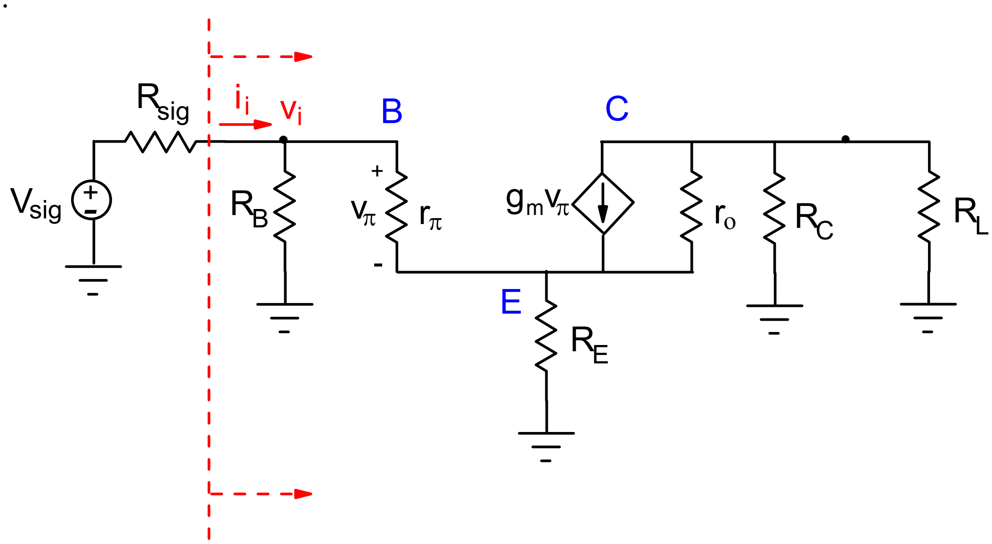
$$R_o=\left[\frac{(1/g_m)//r_o}{r_o//R_C}+\frac{r_\pi//R_E}{R_C}\right]^{-1}\left[(r_\pi//R_E)+(1/g_m)//r_o\right]$$

Take approximations that $g_m r_o >> 1$, and use $g_m r_\pi = \beta$,

$$A_{vo}=\left[-\frac{r_\pi}{r_\pi+R_E}\right]\left[\frac{1/g_m}{r_o//R_C}+\frac{r_\pi//R_E}{R_C}\right]^{-1}=\frac{-R_C}{R_E+(1+R_C/r_o)\dfrac{R_E+r_\pi}{\beta}}$$

$$R_o=\left[\frac{1/g_m}{r_o//R_C}+\frac{r_\pi//R_E}{R_C}\right]^{-1}\left[(r_\pi//R_E)+(1/g_m)\right]=R_C//\left[r_o+r_o g_m(r_\pi//R_E)\right]$$

- R_i:



$$i_i=\frac{v_i}{R_B}+\frac{v_\pi}{r_\pi}$$

The voltage at node E is $v_i - v_\pi$. Assume the voltage at node C is v_c, KCL node C:

$$g_m v_\pi + \frac{v_c}{R_C} + \frac{v_c}{R_L} + \frac{v_c - (v_i - v_\pi)}{r_o} = 0$$

Solve for v_c,

$$\frac{v_c}{R_C // R_L // r_o} = \frac{v_i - v_\pi}{r_o} - g_m v_\pi$$

KCL supernode E + C,

$$\frac{v_\pi}{r_\pi} = \frac{v_c}{R_C} + \frac{v_c}{R_L} + \frac{v_i - v_\pi}{R_E}$$

Eliminate v_c,

$$\frac{v_\pi}{r_\pi} = \frac{R_C // R_L // r_o}{R_C // R_L}\left[\frac{v_i - v_\pi}{r_o} - g_m v_\pi\right] + \frac{v_i - v_\pi}{R_E}$$

Approximate $g_m + 1/r_o$ as g_m,

$$\frac{v_\pi}{r_\pi} = \frac{r_o}{r_o + R_C // R_L}\left[\frac{v_i}{r_o} - g_m v_\pi\right] + \frac{v_i - v_\pi}{R_E}$$

Solve for v_π/r_π:

$$\frac{v_\pi}{r_\pi}\left[1 + \frac{r_\pi}{R_E} + \frac{g_m r_\pi r_o}{r_o + R_C // R_L}\right] = \frac{v_i}{r_o + R_C // R_L} + \frac{v_i}{R_E} = \frac{v_i}{R_E //[r_o + R_C // R_L]}$$

Plug v_π/r_π back to the 1st eq.,

$$i_i = \frac{v_i}{R_B} + \frac{v_\pi}{r_\pi} = \frac{v_i}{R_B} + \frac{v_i}{\left[1 + \frac{r_\pi}{R_E} + \frac{g_m r_\pi r_o}{r_o + R_C // R_L}\right]\left\{R_E //[r_o + R_C // R_L]\right\}}$$

Multiply out the last denominator:

$$i_i = \frac{v_i}{R_B} + \frac{v_i}{\frac{(R_E + r_\pi)[r_o + R_C // R_L] + g_m r_\pi r_o R_E}{R_E + r_o + R_C // R_L}} = \frac{v_i}{R_B} + \frac{v_i}{R_E + r_\pi + \frac{[g_m r_\pi r_o - (R_E + r_\pi)]R_E}{R_E + r_o + R_C // R_L}}$$

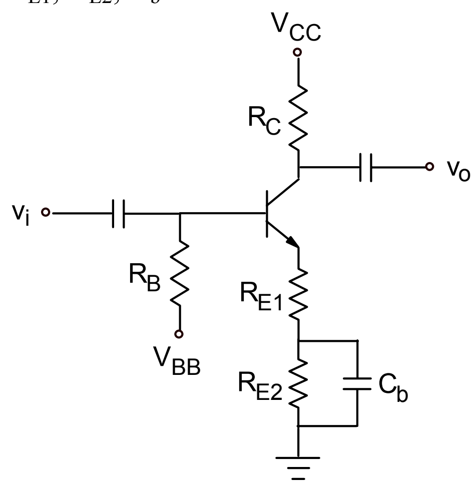
Approximation that $g_m r_\pi r_o = \beta r_o >> R_E + r_\pi$,

$$i_i \approx \frac{v_i}{R_B} + \frac{v_i}{R_E + r_\pi + \frac{\beta r_o R_E}{R_E + r_o + R_C // R_L}} = \frac{v_i}{R_B} + \frac{v_i}{R_E + r_\pi + \frac{\beta R_E}{1 + [R_E + R_C // R_L]/r_o}}$$

With $i_i = v_i/R_i$,

$$R_i = R_B // \left\{ R_E + r_\pi + \frac{\beta R_E}{1+[R_E + R_C // R_L]/r_o} \right\}$$

- Cut-off frequency with R_{E1}, R_{E2}, C_b:



$$A_{vo} = \frac{-R_C}{R_E + (1 + R_C / r_o)\dfrac{R_E + r_\pi}{\beta}} \approx \frac{-R_C}{R_E + r_\pi / \beta}$$

Replace R_E with $R_{E1} + R_{E2}//(1/j\,\omega\,C_b) = R_{E1} + R_{E2}/(1 + j\,\omega\,C_b\,R_{E2})$,

$$A_{vo} = \frac{-R_C}{R_{E1} + \dfrac{R_{E2}}{1 + j\omega C_b R_{E2}} + r_\pi / \beta} = \frac{-R_C(1 + j\omega C_b R_{E2})}{R_{E1} + R_{E2} + r_\pi / \beta + j\omega C_b R_{E2}(R_{E1} + r_\pi / \beta)}$$

or

$$A_{vo} = \frac{-R_C(1 + j\omega C_b R_{E2})}{(R_{E1} + R_{E2} + r_\pi / \beta)\{1 + j\omega C_b [R_{E2} //(R_{E1} + r_\pi / \beta)]\}}$$

and

$$|A_{vo}| = \frac{R_C \sqrt{1 + (\omega C_b R_{E2})^2}}{(R_{E1} + R_{E2} + r_\pi / \beta)\sqrt{1 + \{\omega C_b [R_{E2} //(R_{E1} + r_\pi / \beta)]\}^2}}$$

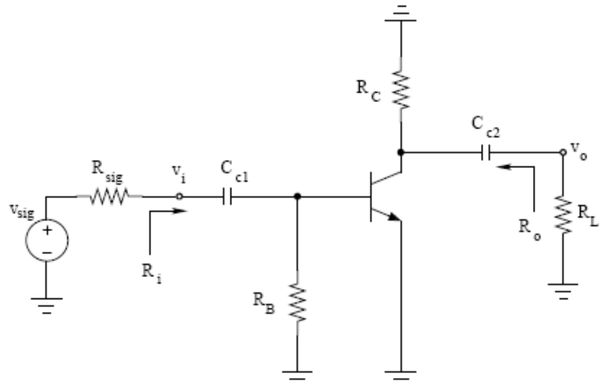
$|A_{vo}|$ is a monotonically increasing function of ω. Its high frequency limit is

$$|A_{vo}|(\omega \to \infty) = \frac{R_C}{R_{E1} + r_\pi / \beta}.$$

For $|A_{vo}|$ to be close to this limit, the condition is $\omega C_b [R_{E2} //(R_{E1} + r_\pi / \beta)] >> 1$. (Note that this assures $\omega C_b R_{E2} >> 1$ also.) Therefore, the cut-off frequency with C_b is:

$$f_3 = \frac{1}{2\pi C_b[R_{E2} // (R_{E1} + r_\pi / \beta)]}$$

6.2.3. Common emitter BJT amplifier:



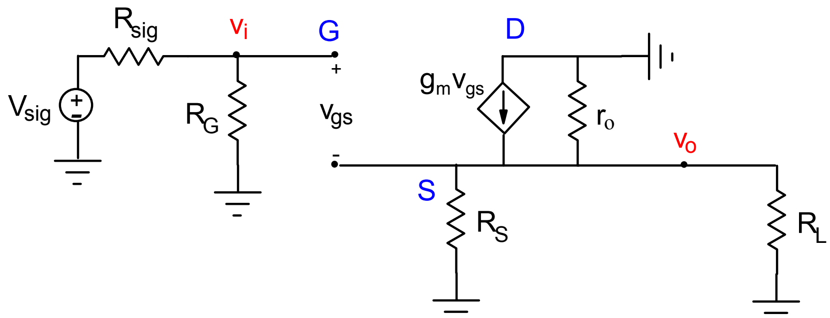
Simply take the common emitter BJT amplifier with emitter resistance derived in 6.2.2 and set $R_E = 0$.

$$A_{vo} = -g_m (R_C // r_o)$$

$$R_o = R_C // r_o$$

$$R_i = R_B // r_\pi$$

6.2.4. Common drain MOS amplifier:



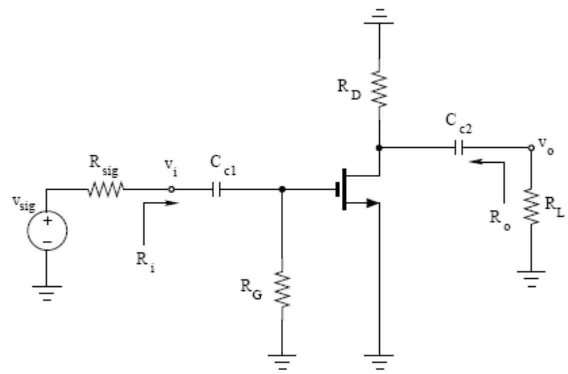
Simply take the common collector BJT amplifier parameters derived in 6.2.1 and replace R_B with R_G, R_E with R_S, β/r_π with g_m, and let $r_\pi \rightarrow \infty$:

$$A_{vo} = \frac{g_m (R_S // r_o)}{1 + g_m (R_S // r_o)}$$

$$R_o = (1/g_m) // R_S // r_o$$

$$R_i = R_G$$

6.2.5. Common source MOS amplifier:



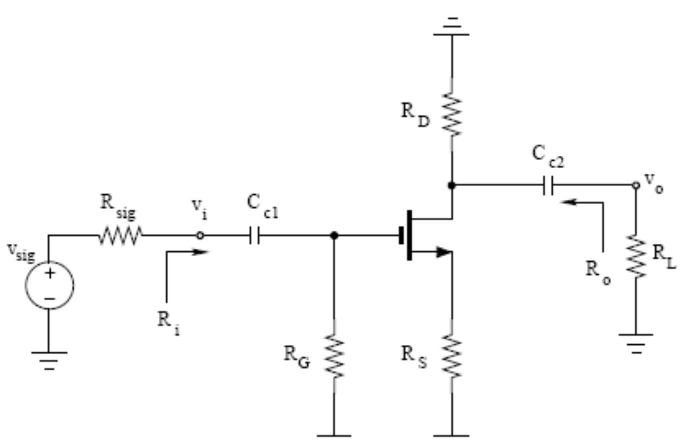
Simply take the common emitter BJT amplifier parameters in 6.2.3 and replace R_B with R_G, R_C with R_D, and let $r_\pi \to \infty$:

$$A_{vo} = -g_m (R_D // r_o)$$

$$R_o = R_D // r_o$$

$$R_i = R_G$$

6.2.6. Common source MOS amplifier with source resistance:



Simply take the common emitter BJT amplifier with emitter resistance derived in 6.2.2 and replace R_B with R_G, R_E with R_S, R_C with R_D, β/r_π with g_m, and let $r_\pi \to \infty$:

$$A_{vo} = -\left[\frac{1/g_m}{r_o // R_D} + \frac{R_S}{R_D}\right]^{-1} = \frac{-g_m R_D}{1 + g_m R_S + R_D / r_o}$$

$$R_o = R_D // \left[r_o(1 + g_m R_S)\right]$$

$$R_i = R_G$$

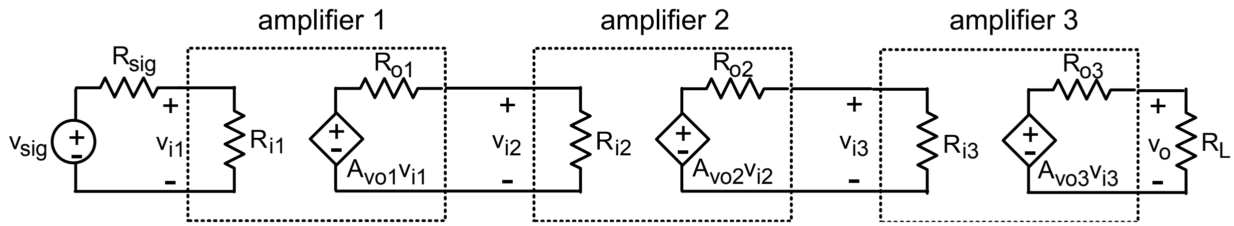
Cut-off frequency if R_S is replaced by $R_{S1} + R_{S2}//(1/j\omega C_b)$ where C_b is the bypass capacitor:

$$f_3 = \frac{1}{2\pi C_b[R_{S2} //(R_{S1} + 1/g_m)]}$$

6.2.7. Summary of Amplifier Configurations

- The common-source (CS) and common-emitter (CE) amplifiers have a high gain and are the main configuration in a practical amplifier. Ignoring bias resistors R_G or R_B, the CS configuration has an infinite input resistance while the CE amplifier has a modest input resistance. Both CS and CE amplifier have a rather high output resistance r_o and a limited high-frequency response (you will see this in 102).
- Addition of source or emitter resistor (degenerated CS or CE) leads to several benefits: a gain which is less sensitive to temperature, a much larger input resistance for CE configuration, a better control of amplifier saturation, and a much improved high-frequency response. However, these are realized at the expense of a lower gain.
- The common-gate (CG) and commons-base (CB) amplifiers have a high gain (similar to CS and CE) but a low input resistance. As such, they are only used for specialized applications. CG and CB amplifiers have an excellent high-frequency response. They are typically used in combination with a CS or CE stage (such as cascode amplifiers)
- The source-follower and emitter-follower configurations have a high input resistance, a gain close to unity, and a low output resistance. They are employed as a voltage buffer and/or as the output stage to increase the current and power to the load.

Multi-stage amplifiers

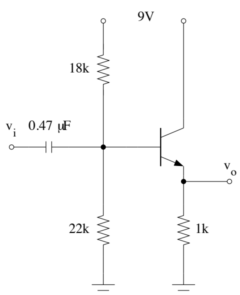


For a chain of amplifiers in cascade, first work out the parameters that depend on the amplifier only: A_{vo1} and R_{o1} for amplifier 1, A_{vo2} and R_{o2} for amplifier 2, A_{vo3} and R_{o3} for amplifier 3. Those parameters can be determined without knowing the signal circuit and the load resistance. Then we calculate the input resistances in reverse order, i.e., from the last stage working toward the first stage. In the case shown, R_{i3} depends on the load R_L. Once R_{i3} is calculated, it acts as the load for amplifier 2. In other words, R_{i3} is used to calculate R_{i2} for amplifier 2. Likewise, R_{i2} is used to calculate R_{i1} for amplifier 1. After all the R_i's are calculated, the overall gain of the 3-stage amplifier is

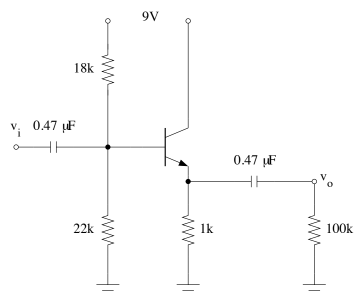
$$\frac{v_o}{v_{sig}} = \frac{R_{i1}}{R_{i1}+R_{sig}} A_{vo1} \frac{R_{i2}}{R_{i2}+R_{o1}} A_{vo2} \frac{R_{i3}}{R_{i3}+R_{o2}} A_{vo3} \frac{R_L}{R_L+R_{o3}}$$

6.3. Exercise Problems

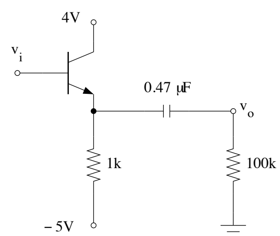
Problem 1 to 12: Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



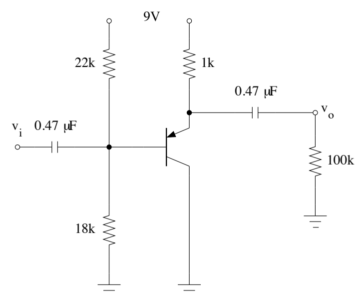
Problem 1



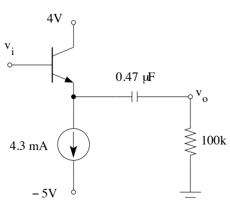
Problem 2



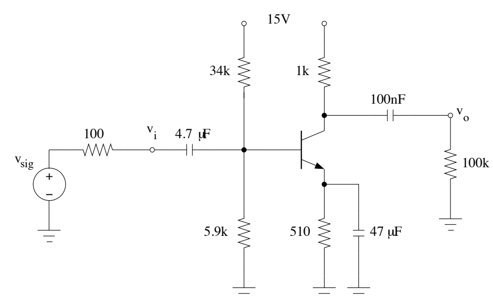
Problem 3



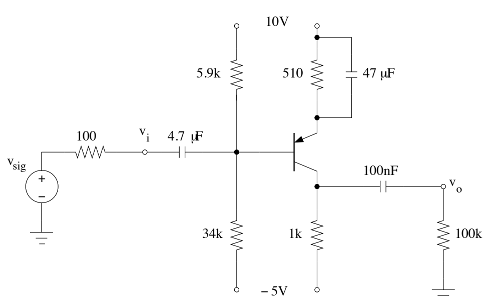
Problem 4



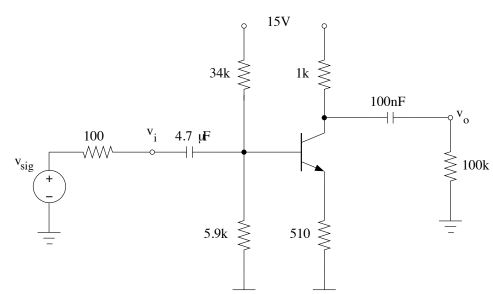
Problem 5



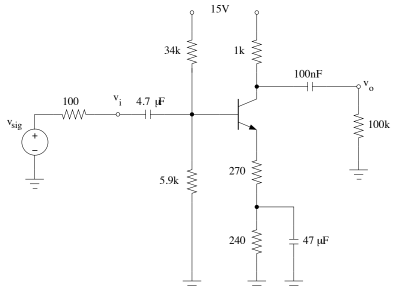
Problem 6



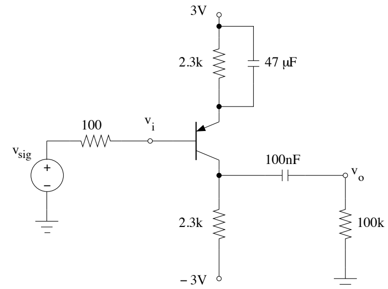
Problem 7



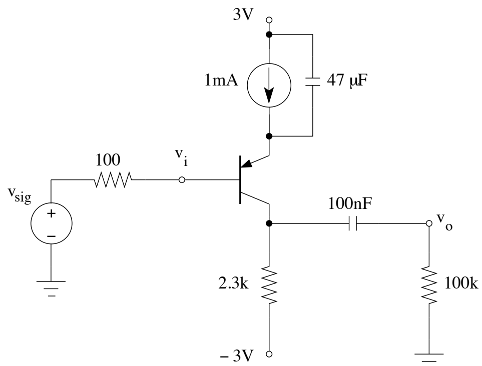
Problem 8



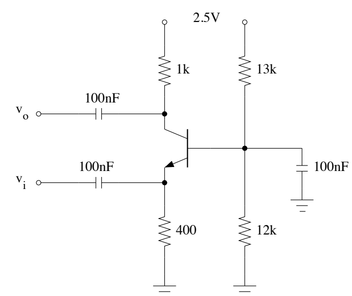
Problem 9



Problem 10

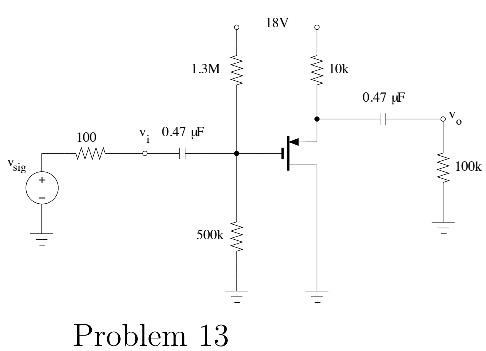


Problem 11

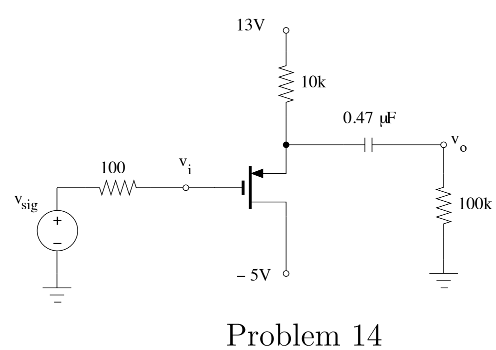


Problem 10 Problem 12

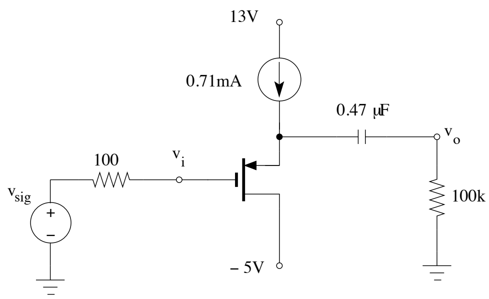
Problem 13-16. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 4$ V, $V_{tp} = -4$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.4$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.)



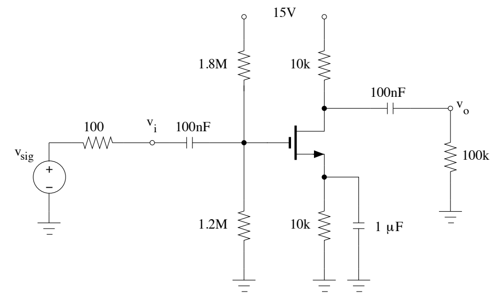
Problem 13



Problem 14

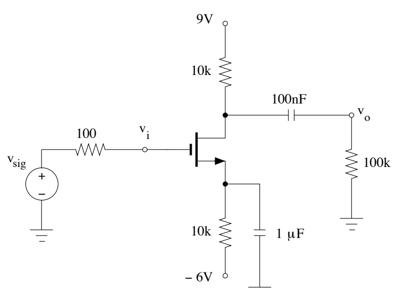


Problem 15

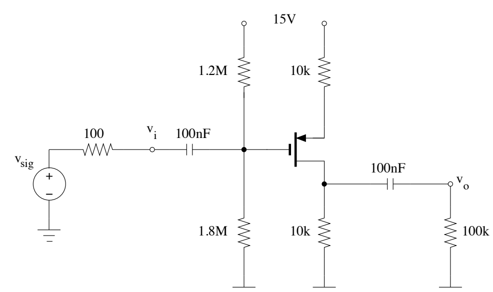


Problem 16

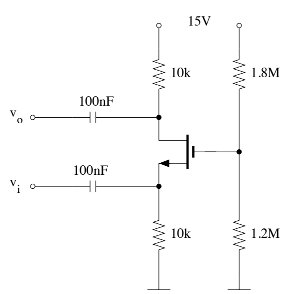
Problem 18-24. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 1$ V, $V_{tp} = -1$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.8$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.



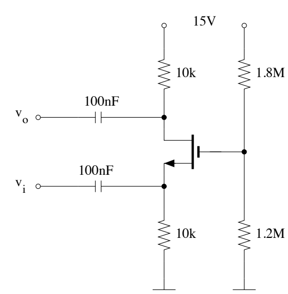
Problem 17



Problem 18



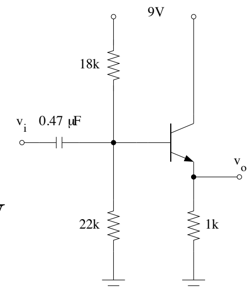
Problem 19



Problem 19

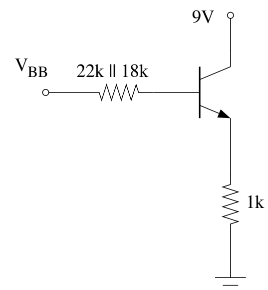
6.4. Solution to Selected Exercise Problems

Problem 1. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



Bias: Set $v_i = 0$ and capacitors open. Set $v_i = 0$ and capacitors open. Replace R_{B1}/R_{B2} voltage divider with its Thevenin equivalent. Assuming BJT in active,

$$R_B = 18 \text{ k} \parallel 22 \text{ k} = 9.9 \text{ k}\Omega \qquad V_{BB} = \frac{22}{18+22} \times 9 = 4.95 \text{ V}$$



KVL: $\quad V_{BB} = R_B I_B + V_{BE} + 10^3 I_E$

$$4.95 = 9.9 \times 10^3 I_E/(\beta+1) + 0.7 + 10^3 I_E$$

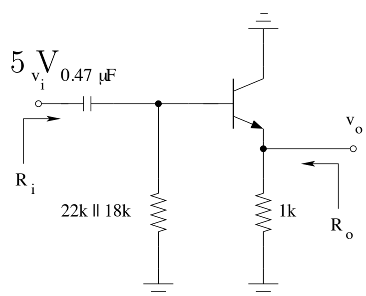
$$I_E = 4.05 \text{ mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 20.3\ \mu\text{A}$$

KVL: $\quad 9 = V_{CE} + 10^3 I_E$

$$V_{CE} = 9 - 10^3 \times 4 \times 10^{-3} = 5 \text{ V}$$

Since $V_{CE} > V_{D0} = 0.7$, assumption of BJT in active is correct.

Bias summary: $\quad I_E \approx I_C = 4.05$ mA, $\quad I_B = 20.3\ \mu$A, $\quad V_{CE} = 5$ V



Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{4 \times 10^{-3}}{26 \times 10^{-3}} = 156 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 1.28 \text{ k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{4 \times 10^{-3}} = 37.0 \text{ k}$$

Note that we could have ignored V_{CE} compared to V_A in the above expression for r_o. Proceeding with the small signal analysis, we zero bias sources (see circuit). As the input is at the base and output is at the emitter, this is a common-collector amplifier (emitter follower). Using formulas of page 6-21 and noting $R_L \to \infty$, $R_E \ll r_o$, and $R_E \gg r_e$:

$$\frac{v_o}{v_i} = \frac{g_m(r_o \parallel R_E \parallel R_L)}{1 + g_m(r_o \parallel R_E \parallel R_L)} = \frac{152}{153} \approx 1$$

$$R_i \approx R_B \parallel [r_\pi + \beta(r_o \parallel R_E \parallel R_L)] = (9.9 \text{ k}) \parallel (1.28 \text{ k} + 195 \text{ k}) = 9.42 \text{ k} \quad (\approx R_B)$$

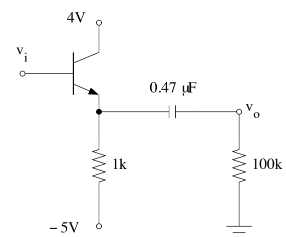
$$R_o = R_E \parallel r_o \parallel \frac{r_\pi + R_B \parallel R_{Sig}}{1+\beta} = 6.4\ \Omega \quad (\approx \frac{r_\pi}{\beta} = \frac{1}{g_m})$$

$$f_l = f_{p1} = \frac{1}{2\pi C_{c1}(R_i + R_{sig})} = \frac{1}{2\pi \times 0.47 \times 10^{-6} \times (9.9 \times 10^3 + 0)} = 34.2 \text{ Hz}$$

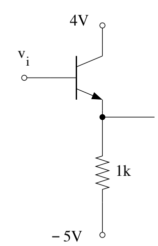
Problem 2. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).

This is the same circuit as Problem 1 with exception of C_{c2} and R_L. The bias point is exactly the same. As $R_E \ll R_L$, the amplifier parameters would be the same except $f_l = f_{p1} + f_{p2} = 34.3 + 3.39 = 37.6$ Hz.

Problem 3. Find the bias point and amplifier parameters of this circuit (Si BJT with $n = 2$, $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



This circuit is similar to Problem 1 expect that the transistor is biased with two voltage sources (values are chosen to give approximately the same bias point).



Bias: Set $v_i = 0$ and capacitors open:

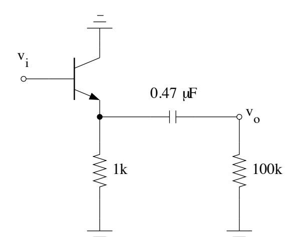
$$\text{BE-KVL:} \quad 0 = V_{BE} + 10^3 I_E - 5$$

$$I_E = 4.3\text{mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 21.5\ \mu\text{A}$$

$$\text{CE-KVL:} \quad 4 = V_{CE} + 10^3 I_E - 5$$

$$V_{CE} = 9 - 10^3 \times 4.3 \times 10^{-3} = 4.7\ \text{V}$$

Bias summary: $I_E \approx I_C = 4.3$ mA, $I_B = 21.5\ \mu$A, $V_{CE} = 4.7$ V



Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{4.3 \times 10^{-3}}{26 \times 10^{-3}} = 165.4\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 1.21\ \text{k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{4.3 \times 10^{-3}} = 34.9\ \text{k}$$

Proceeding with the signal analysis, we zero bias sources (see circuit). As the input is at the base and output is at the emitter, this is a common-collector amplifier (emitter follower). The difference with Problem 1 is that there is no R_B ($R_B = \infty$) which affects R_i only.

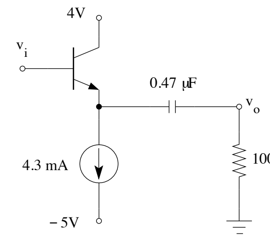
$$A_v = \frac{g_m(r_o \parallel R_E \parallel R_L)}{1 + g_m(r_o \parallel R_E \parallel R_L)} = \frac{161}{162} \approx 1$$

$$R_i \approx R_B \parallel [r_\pi + (1+\beta)(r_o \parallel R_E \parallel R_L)] = \infty \parallel (1.21\ \text{k} + 194\ \text{k}) = 195\ \text{k}$$

$$R_o = R_E \parallel r_o \parallel \frac{r_\pi + R_B \parallel R_{Sig}}{1+\beta} = 6.0\ \Omega \quad (\approx \frac{r_\pi}{\beta} = \frac{1}{g_m})$$

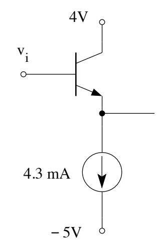
$$f_l = f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} = \frac{1}{2\pi \times 0.47 \times 10^{-6} \times (100 \times 10^3 + 6)} = 3.39\ \text{Hz}$$

Problem 5. Find the bias point and amplifier parameters of this circuit (Si BJT with$\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



This circuit is similar to the circuit of Problem 3 except that the transistor is biased with a current source.

Bias: Set $v_i = 0$ and capacitors open.



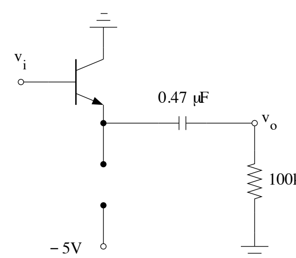
$$I_E = 4.3 \text{ mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 21.5\ \mu\text{A}$$

$$\text{BE-KVL:} \quad 0 = V_{BE} + V_E \quad \rightarrow \quad V_E = -0.7 \text{ V}$$

$$\text{CE-KVL:} \quad 4 = V_{CE} + V_E \quad \rightarrow \quad V_{CE} = 4.7 \text{ V}$$

Bias summary: $I_E \approx I_C = 4.3$ mA, $I_B = 21.5\ \mu$A, $V_{CE} = 4.7$ V

Small-Signal: First we calculate the small-signal parameters:



$$g_m = \frac{I_C}{V_T} = \frac{4.3 \times 10^{-3}}{26 \times 10^{-3}} = 165.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 1.21 \text{ k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{4.3 \times 10^{-3}} = 34.9 \text{ k}$$

Amplifier Response: we zero bias sources (the current source becomes an open circuit. As the input is at the base and output is at the emitter, this is a common-collector amplifier (emitter follower). The difference with problem 3 is that here $R_E \rightarrow \infty$. Using formulas of page 6-21 and noting $R_E \parallel R_L = R_L \gg r_e$

$$A_v = \frac{g_m(r_o \parallel R_E \parallel R_L)}{1 + g_m(r_o \parallel R_E \parallel R_L)} = \frac{4,279}{4,280} \approx 1$$

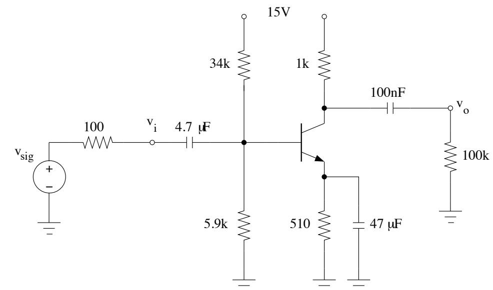
$$R_i \approx R_B \parallel [r_\pi + \beta(r_o \parallel R_E \parallel R_L)] = \infty \parallel (1.21 \text{ k} + 5.17 \text{ M}) = 5.17 \text{ M}$$

$$R_o = R_E \parallel r_o \parallel \frac{r_\pi + R_B \parallel R_{Sig}}{1 + \beta} = 6.0\ \Omega \quad (\approx \frac{r_\pi}{\beta} = \frac{1}{g_m})$$

$$f_l = f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} = 3.39 \text{ Hz}$$

Comparing results from Problems 1 through 5 highlights the impact of each element on the amplifier performance as in successive problems, R_L and C_{C2} were added, and then R_B, C_{C1} and R_E were eliminated.

Problem 6. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



Bias: Set $v_i = 0$ and capacitors open. Replace R_{B1}/R_{B2} voltage divider with its Thevenin equivalent:

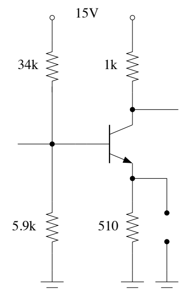
$$R_B = 5.9 \text{ k} \parallel 34 \text{ k} = 5.0 \text{ k},$$

$$V_{BB} = \frac{5.9}{5.9 + 34} \times 15 = 2.22 \text{ V}$$

BE-KVL: $\quad V_{BB} = R_B I_B + V_{BE} + 510 I_E$

$$2.22 = 5.0 \times 10^3 I_E / * \beta + 1) + 0.7 + 510 I_E$$

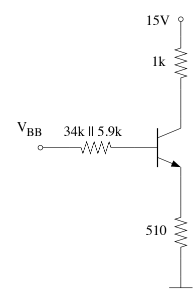
$$I_E = 2.84 \text{ mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 14.2\ \mu\text{A}$$



CE-KVL: $\quad 15 = 1000 I_C + V_{CE} + 510 I_E$

$$V_{CE} = 10.5 \text{ V} \qquad > V_{D0}$$

Bias summary: $\quad I_C \approx I_E = 2.84$ mA, $\quad I_B = 14.2\ \mu$A, $\quad V_{CE} = 10.5$ V



Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{2.84 \times 10^{-3}}{26 \times 10^{-3}} = 109 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 1.83 \text{ k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{2.84 \times 10^{-3}} = 52.8 \text{ k}$$

Proceeding with the small signal analysis, we zero bias sources (see circuit). As the input is at the base and output is at the collector, this is a common-emitter amplifier with NO emitter resistor as there is bypass capacitor.

$$\frac{v_o}{v_i} = -g_m (r_o \parallel R_C \parallel R_L) = -106$$

$$R_i = R_B \parallel r_\pi = 5.0 \parallel 1.83 = 1.34 \text{ k}$$

$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = -0.93 \times 106 = -99$$

$$R_o = R_C \parallel r_o = 0.98 \text{ k} \quad (\approx R_C)$$

$$f_{p1} = \frac{1}{2\pi C_{c1}(R_i + R_{sig})} = \frac{1}{2\pi \times 4.7 \times 10^{-6} \times (1{,}340 + 0)} = 25.3 \text{ Hz}$$

$$f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} = \frac{1}{2\pi 100 \times 10^{-9}(100 \times 10^3 + 980)} = 15.9 \text{ Hz}$$

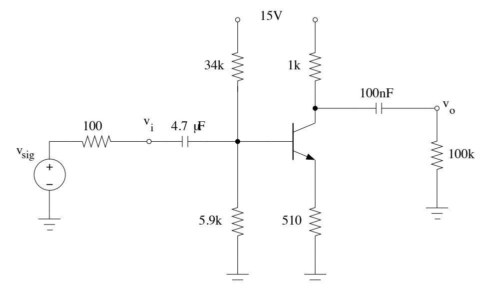
$$f_{p3} = \frac{1}{2\pi C_e[R_E \parallel (1/g_m + (R_B \parallel R_{sig})/\beta)]}$$

$$f_{p3} = \frac{1}{2\pi C_e[R_E \parallel 9.67]} = 356 \text{ Hz}$$

$$f_l = f_{p1} + f_{p2} + f_{pb} = 25.3 + 15.9 + 356 = 397 \text{ Hz}$$

Note that although C_b is the largest capacitor in the circuit (*e.g.,* 10 times larger than C_{c1}, f_{p3} is 10 times larger than the other poles.

Problem 8. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



Bias: Set $v_i = 0$ and capacitors open. The bias circuit is exactly that of Problem 7 with $R_B = 5.0$ k.

Bias summary:
$I_C \approx I_E = 2.84$ mA, $I_B = 14.2\ \mu$A, $V_{CE} = 10.5$ V

Small-Signal: The small-signal parameters are also the same as those of Problem 7: $g_m = 109$ mA/V, $r_\pi = 1.83$k, and $r_o = 52.8$ k.

Proceeding with the small signal analysis, we zero bias sources (see circuit). As the input is at the base and output is at the collector, this is a degenerated common-emitter amplifier (*i.e,* with a emitter resistor):

$$\frac{v_o}{v_i} \approx -\frac{g_m(R_C \parallel R_L)}{1 + g_m R_E} = -1.91$$

$$R_i = R_B \parallel [r_\pi + (1+\beta)R_E] = 4.8 \text{ k} \quad (\approx R_B)$$

$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = -0.98 \times 1.91 = -1.87$$

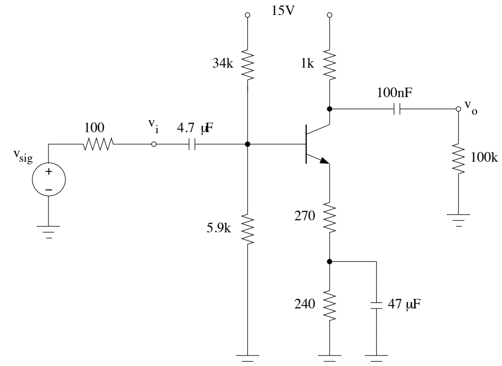
$$R_o = R_C \parallel \left[r_o \left(1 + \frac{\beta R_E}{r_\pi + R_E + R_B \parallel R_{sig}} \right) \right] \approx R_C = 1 \text{ k}$$

$$f_{p1} = \frac{1}{2\pi C_{c1}(R_i + R_{sig})} = \frac{1}{2\pi \times 4.7 \times 10^{-6} \times (4,800 + 100)} = 6.91 \text{ Hz}$$

$$f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} \approx \frac{1}{2\pi C_{c2}(R_L + R_c)} \approx \frac{1}{2\pi 100 \times 10^{-9}(100 \times 10^3 + 10^3)} = 15.8 \text{ Hz}$$

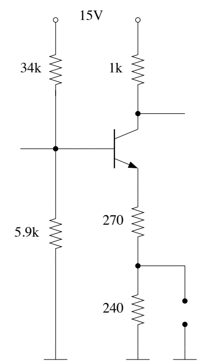
$$f_l = f_{p1} + f_{p2} = 6.9 + 15.8 = 22.7 \text{ Hz}$$

Problem 9. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



Bias:

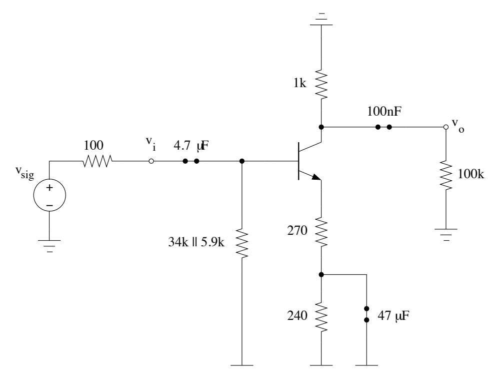
Set $v_i = 0$ and capacitors open. Because the 47 μF capacitor across the 240 Ω resistor becomes an open circuit, the total R_E for bias is $270 + 240 = 510$ Ω and the bias circuit is exactly that of Problem 7 (or Problem 9) with $R_B = 5.0$ k.



Bias summary:
$I_C \approx I_E = 2.84$ mA, $\quad I_B = 14.2$ μA, $\quad V_{CE} = 10.5$ V

Small-Signal: The small-signal parameters are also the same as those of Problem 7: $\quad g_m = 109$ mA/V, $r_\pi = 1.83$k, and $r_o = 52.8$ k.

Proceeding with the small signal analysis, we zero bias sources (see circuit). As the input is at the base and output is at the collector, this is a degenerated common-emitter amplifier (*i.e,* with a emitter resistor). For midband amplifier parameters calculations, the 47 μF capacitor across the 240 Ω resistor becomes a short circuit and the total R_E for small-signal is 270 Ω.



$$\frac{v_o}{v_i} \approx -\frac{g_m(R_C \parallel R_L)}{1 + g_m R_E} = -3.55$$

$$R_i = R_B \parallel [r_\pi + (1+\beta) R_E] = 4.6 \text{ k} \quad (\approx R_B)$$

$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = -0.98 \times 1.91 = -3.47$$

$$R_o = R_C \parallel \left[r_o \left(1 + \frac{\beta R_E}{r_\pi + R_E + R_B \parallel R_{sig}} \right) \right] \approx R_C = 1 \text{ k}$$

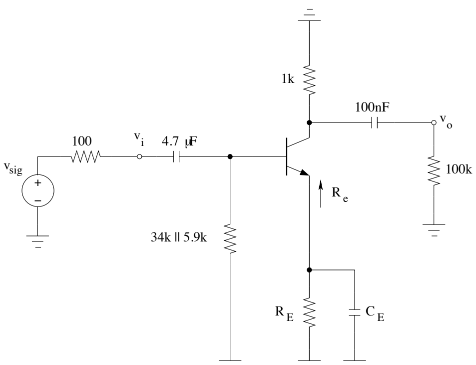
$$f_{p1} = \frac{1}{2\pi C_{c1}(R_i + R_{sig})} = \frac{1}{2\pi \times 4.7 \times 10^{-6} \times (4,600 + 100)} = 7.20 \text{ Hz}$$

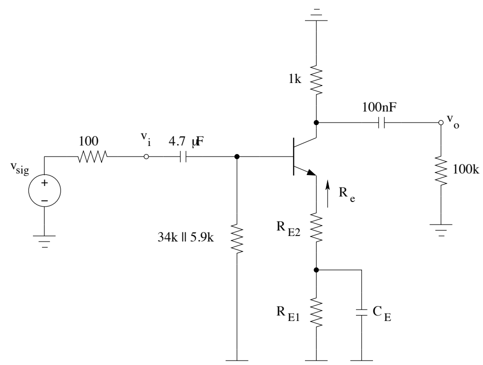
$$f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} \approx \frac{1}{2\pi C_{c2}(R_L + R_c)} \approx \frac{1}{2\pi 100 \times 10^{-9}(100 \times 10^3 + 10^3)} = 15.8 \text{ Hz}$$

We need to find the pole introduced by the 47 μF by-pass capacitor, f_{pb}. Although this configuration was not included in the formulas for BJT elementary configuration of page 6-21, we can extend those formulas to cover this case.

The pole introduced by the by-pass capacitor in the common emitter case is (see figure below left)

$$f_{p3} = \frac{1}{2\pi C_e[R_E \parallel (1/g_m + (R_B \parallel R_{sig})/\beta)]}$$





Per our discussion of Section 6.7 on how to find poles introduced by each capacitor, $R_E \parallel [1/g_m + (R_B \parallel R_{sig})/\beta)]$ is the total resistance seen across the terminal of C_e. As can be seen from the circuit (above right), the resistance across C_c terminals consists of two resistors in parallel, R_E and R_e. R_e is the resistance seen between the emitter of the BJT and the ground and is: $R_e \equiv 1/g_m + (R_B \parallel R_{sig})/\beta)$ from the above formula.

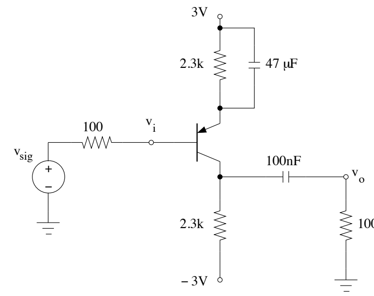
For the circuit here (defined $R_{E1} = 240\ \Omega$ and $R_{E2} = 270\ \Omega$), the resistance across C_e is made of two resistances in parallel: R_{E1} and the combination of R_{E2} and R_e, the resistance seen through the emitter of BJT in series. Thus:

$$f_{p3} = \frac{1}{2\pi C_b[R_{E1} \parallel (R_{E2} + 1/g_m + (R_B \parallel R_{sig})/\beta)]}$$

$$f_{pb} = \frac{1}{2\pi \times C_e[240 \parallel (270 + 9.17 + 0.49]} = \frac{1}{2\pi \times 47 \times 10^{-6} \times 129} = 26.2 \text{ Hz}$$

$$f_l = f_{p1} + f_{p2} = 7.20 + 15.8 + 26.2 = 49.2 \text{ Hz}$$

Problem 10. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



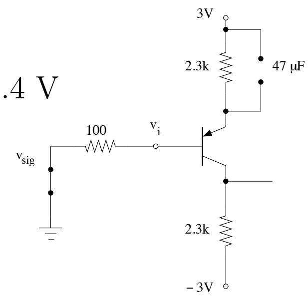
Bias: Set $v_i = 0$ and capacitors open.

$$\text{BE-KVL:} \qquad 3 = 2.3 I_E + V_{EB}$$

$$I_E = 1 \text{ mA} \approx I_C, \qquad I_B = I_E/(1+\beta) = 5\ \mu\text{A}$$

$$\text{CE-KVL:} \qquad 3 = 2.3 \times 10^3 I_E + V_{EC} + 2.3 \times 10^3 I_C - 3$$

$$V_{EC} = 6 - 4.6 \times 10^3 \times 1 \times 10^{-3} = 1.4 \text{ V}$$



Bias summary: $\quad I_C \approx I_E = 1$ mA, $\quad I_B = 5.0\ \mu$A, $\quad V_{CE} = 1.4$ V

Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{1 \times 10^{-3}}{26 \times 10^{-3}} = 38.5 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 5.26 \text{ k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{1 \times 10^{-3}} = 150 \text{ k}$$

Proceeding with the small signal analysis, we zero bias sources. As the input is at the base and output is at the collector, this is a common-emitter amplifier. It does not have an emitter resistor as 47 μF capacitor shorts out R_E for signals.

$$A_v = -g_m(r_o \parallel R_C \parallel R_L) = -38.5 \times 10^{-3}(150 \text{ k} \parallel 2.3 \text{ k} \parallel 100 \text{ k}) = -85.3$$

$$R_i = R_B \parallel r_\pi = 10.4 \text{ k}$$

$$R_o = R_C \parallel r_o \approx 2.3 \text{ k}$$

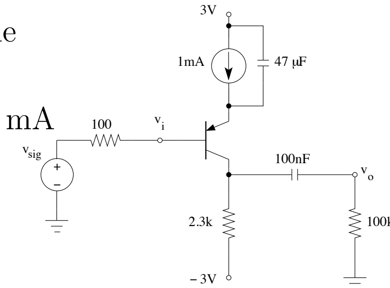
$$f_{p1} = 0$$

$$f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} = \frac{1}{2\pi 100 \times 10^{-9}(10^5 + 10^3)} = 15.8 \text{ Hz}$$

$$f_{p3} = \frac{1}{2\pi C_e[R_E \parallel (1/g_m + (R_B \parallel R_{sig})/\beta)]} = \frac{1}{2\pi C_e[2,300 \parallel 26]} = 132 \text{ Hz}$$

$$f_l = f_{p1} + f_{p2} + f_{pb} = 0 + 15.8 + 132 = 148 \text{ Hz}$$

Problem 11. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



This is the same circuit as that of Problem 11 expect that the transistor is biased with a current source

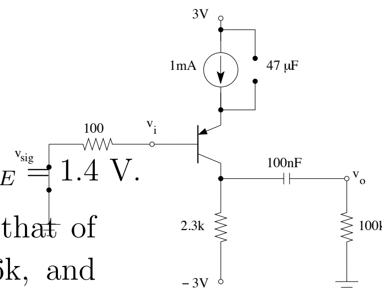
Bias: Set $v_i = 0$ and capacitors open. From the circuit $I_E = 1$ mA

$$I_E = 1 \text{ mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 5\ \mu\text{A}$$

BE-KVL: $\quad V_E = V_{EB} = 0.7$ V

CE-KVL: $\quad V_E = V_{CE} + 2.3 \times 10^3 I_C - 3 = V_{CE} - 0.7$

$$V_{CE} = 1.4 \text{ V}$$

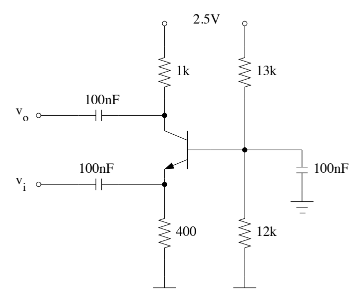


Bias summary: $\quad I_C \approx I_E = 1$ mA, $\quad I_B = 50\ \mu$A, $\quad V_{CE} = 1.4$ V.

Small-Signal: As the bias point is exactly the same as that of problem 11, we have: $g_m = 38.5$ mA/V, $r_\pi = 5.26$k, and $r_o = 150$ k.

Amplifier response: The only difference with problem 11 is that $R_E \to \infty$ in this circuit. R_E only appears in f_{p3} but $R_E = \infty$ does not change results:
$A_v = -37.9$, $R_i = 10.4$ k, $R_o = 0.99$ k, and $f_l = 0 + 15.8 + 132 = 148$ Hz.

Problem 12. Find the bias point and amplifier parameters of this circuit (Si BJT with $\beta = 200$ and $V_A = 150$ V. Ignore the Early effect in biasing calculations).



Bias: Set $v_i = 0$ and capacitors open.

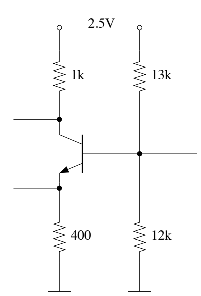
$$R_B = 12 \text{ k} \parallel 13 \text{ k} = 6.24 \text{ k},$$

$$V_{BB} = \frac{12}{12 + 13} \times 2.5 = 1.2 \text{ V}$$

BE-KVL: $\quad V_{BB} = R_B I_B + V_{BE} + 510 I_E$

$$1.2 = 6.24 \times 10^3 I_E/(1 + \beta) + 0.7 + 400 I_E$$

$$I_E = 1.16 \text{ mA} \approx I_C, \qquad I_B = \frac{I_C}{\beta} = 5.8\ \mu\text{A}$$



CE-KVL: $\quad 2.5 = 1000 I_C + V_{CE} + 400 I_E$

$$V_{CE} = 2.5 - 1{,}400 \times 1.16 \times 10^{-3} = 0.88 \text{ V}$$

Bias summary: $\quad I_C \approx I_E = 1.16$ mA, $\quad I_B = 5.8\ \mu$A, $\quad V_{CE} = 0.88$ V

Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{1.16 \times 10^{-3}}{26 \times 10^{-3}} = 44.6 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 4.48 \text{ k} \qquad r_o \approx \frac{V_A}{I_C} = \frac{150}{1.16 \times 10^{-3}} = 129 \text{ k}$$

Proceeding with the small signal analysis, we zero bias sources. As the input is at the emitter and output is at the collector, this is a common-base amplifier (note $R_L = \infty$).

$$\frac{v_o}{v_i} = g_m(r_o \parallel R_C \parallel R_L) = 44.6 \times 10^{-3}(129 \text{ k} \parallel 1 \text{ k} \parallel \infty) = 44.26$$

$$R_i = R_E \parallel r_\pi \parallel \frac{1 + (R_C \parallel R_L)/r_o}{g_m} = 1 \text{ 1k} \parallel 4.48 \text{ 1k} \parallel 22.4 = 21.8\ \Omega \quad (\approx 1/g_m)$$

$$R_o = R_C \parallel [r_o(1 + g_m(R_E \parallel r_\pi \parallel R_{sig}))] \approx R_C = 1 \text{ k}$$

$$f_{p1} = \frac{1}{2\pi C_{c1}(R_i + R_{sig})} = \frac{1}{2\pi 1000 \times 10^{-9}(21.8)} = 7.30 \text{ kHz}$$

$$f_{p2} = \frac{1}{2\pi C_{c2}(R_L + R_o)} = 0$$

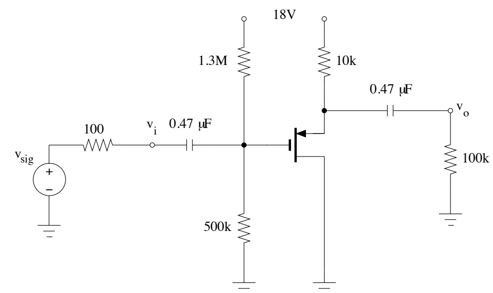
$$R_{CB} \equiv R_B \parallel [r_\pi + (1+\beta)(R_{sig} \parallel R_E)] = 6.24 \text{ k} \parallel 84.9 \text{ k} \approx 5.81 \text{ k}$$

$$f_{p3} = \frac{1}{2\pi C_b R_{CB}} == \frac{1}{2\pi \times 100 \times 10^{-9} \times 5.81 \times 10^3} = 274\ Hz$$

$$f_l = f_{p1} + f_{p2} + f_{pb} = 7,300 + 0 + 274 = 7.57 \text{ kHz}$$

Note the small input resistance of this amplifier and corresponding large f_{p1}.

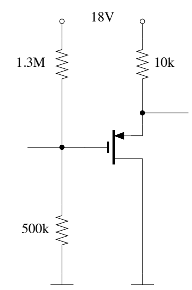
Problem 13. Find the bias point and amplifier parameters of this circuit ($V_{tn} =$ 4 V, $V_{tp} = -4$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.4$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.)



Bias: Since $I_G = 0$:

$$V_G = \frac{0.5 \text{ M}}{1.3 \text{ M} + 0.5 \text{ M}} \times 18 = 5 \text{ V}$$
$$R_G = 1.3 \text{ M} \parallel 500 \text{ k} = 361 \text{ k}$$

Assume PMOS is in the active state,



$$I_D = 0.5\mu_p C_{ox}(W/L)V_{OV}^2$$
$$\text{SG-KVL:} \quad 18 = 10^4 I_D + V_{SG} + V_G = 10^4 I_D + V_{OV} + |V_{tp}| + V_G$$
$$10^4 \times 0.5 \times 0.4 \times 10^{-3} V_{OV}^2 + V_{OV} - 18 + 4 + 5 = 0$$
$$2V_{OV}^2 + V_{OV} - 9 = 0$$

Negative root is unphysical, $V_{OV} = 1.89$ V and $V_{SG} = V_{OV} + |V_{tp}| = 5.89$ V.

$$I_D = 0.5 \times 0.4 \times 10^{-3} V_{OV}^2 = 0.71 \text{ mA}$$
$$\text{SD-KVL:} \quad 18 = V_{SD} + 10^4 I_D \quad \rightarrow \quad V_{SD} = 18 - 10^4 \times 0.71 \times 10^{-3} = 10.9 \text{ V}$$

Since $V_{SD} = 10.9 \geq V_{OV} = 1.89$ V, our assumption of PMOS in active is justified.

Bias summary: $I_D = 0.71$ mA, $V_{OV} = 1.89$ V, $V_{SG} = 5.89$ V, $V_{SD} = 10.9$ V.

Small-Signal: First we calculate the small-signal parameters:

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.71 \times 10^{-3}}{1.89} = 0.751 \text{ mA/V}$$
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 0.71 \times 10^{-3}} = 141 \text{ k}$$

Proceeding with the small signal analysis, we zero bias sources. As the input is at the gate and output is at the source, this is a common-drain amplifier (source follower).

$$\frac{v_o}{v_i} = \frac{g_m(r_o \parallel R_S \parallel R_L)}{1 + g_m(r_o \parallel R_S \parallel R_L)} = \frac{6.41}{7.41} = 0.866$$
$$R_i = R_G = 361 \text{ k}$$

$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = 0.866$$

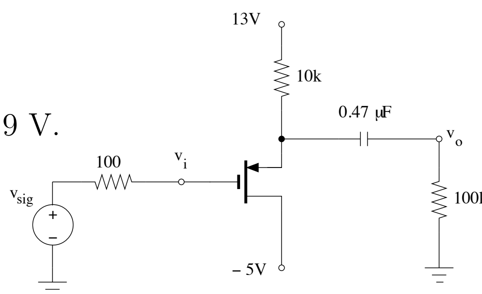
$$R_o = \frac{1}{g_m} \parallel R_S = 1.33 \text{ k} \parallel 10 \text{ k} = 1.17 \text{ k}$$

$$f_{p1} = 1/[2\pi C_{c1}(R_i + R_{sig})] = 0.94 \text{ Hz}$$

$$f_{p2} = 1/[2\pi C_{c2}(R_L + R_o)] = 3.39 \text{ Hz}$$

$$f_l = f_{p1} + f_{p2} = 4.33 \text{ Hz}$$

Problem 14. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 4$ V, $V_{tp} = -4$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.4$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.



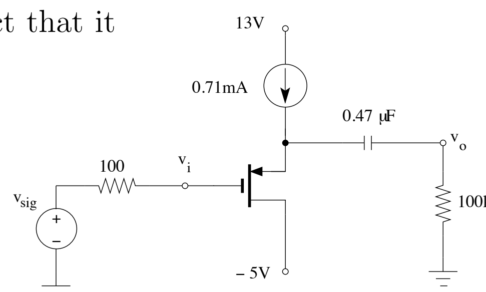
Answer:

$I_D = 0.71$ mA, $V_{OV} = 1.89$ V, $V_{SG} = 5.89$ V, $V_{SD} = 10.9$ V.

$v_o/v_{sig} = 0.866$, $R_i = \infty$, $R_o = 1.17$ k,
$f_{p1} = 0$, $f_{p2} = 3.35$ Hz, $f_l = 4.3$ Hz.

Problem 15. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 4$ V, $V_{tp} = -4$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.4$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.)

This circuit is also similar to that of problem 14 expect that it is biased with a current source:



$$I_D = 0.71 \text{ mA}$$

$$I_D = 0.5\mu_p C_{ox}(W/L)V_{OV}^2$$

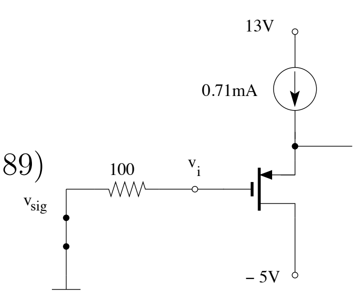
$$0.71 \times 10^{-3} = 0.5 \times 0.4 \times 10^{-3} V_{OV}^2$$

$$V_{OV} = 1.88 \text{ V}$$

$$V_{SG} = V_{OV} + |V_{tp}| = 5.88 \text{ V}$$

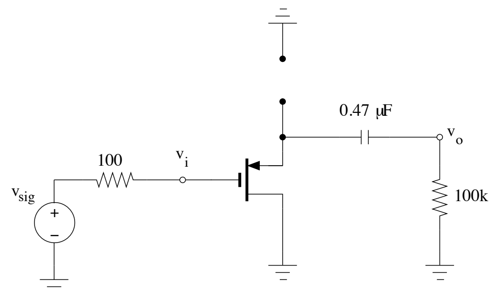
$$V_{SG} = V_S - V_G = 5.88 \quad \rightarrow \quad V_S = 5.88 \text{ V}$$

$$V_{SD} = V_S - V_D = 5.88 - (-5) = 10.9 \text{ V} \qquad (> V_{OV} = 1.89)$$



Bias summary:
$I_D = 0.71$ mA, $V_{OV} = 1.88$ V, $V_{SG} = 5.88$ V, $V_{SD} = 10.9$ V.

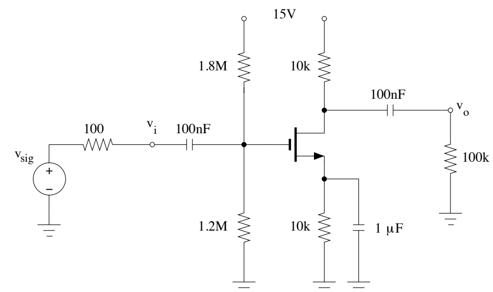
$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.71 \times 10^{-3}}{1.89} = 0.751 \text{ mA/V}$$
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 0.71 \times 10^{-3}} = 141 \text{ k}$$



This is a common-drain amplifier (source follower). Note $R_G = \infty$, $R_s = \infty$.

$$\frac{v_o}{v_i} = \frac{g_m(r_o \parallel R_S \parallel R_L)}{1 + g_m(r_o \parallel R_S \parallel R_L)} = \frac{43.9}{44.9} = 0.98$$
$$R_i = R_G = \infty$$
$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = 0.98$$
$$R_o = \frac{1}{g_m} \parallel R_S = 1.33 \text{ k} \parallel \infty = 1.33 \text{ k}$$
$$f_{p1} = 1/[2\pi C_{c1}(R_i + R_{sig})] = 0$$
$$f_{p2} = 1/[2\pi C_{c2}(R_L + R_o)] = 3.39 \text{ Hz}$$
$$f_l = f_{p1} + f_{p2} = 3.39 \text{ Hz}$$

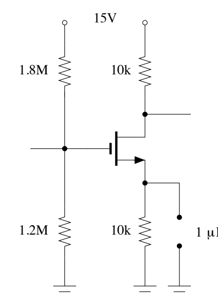
Problem 16. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 1$ V, $V_{tp} = -1$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.8$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.



Bias: Since $I_G = 0$:

$$V_G = \frac{1.2 \text{ M}}{1.2 \text{ M} + 1.8 \text{ M}} \times 15 = 6 \text{ V}$$
$$R_G = 1.2 \text{ M} \parallel 1.8 \text{ M} = 720 \text{ k}$$

Assume NMOS is in the active state,



$$I_D = 0.5 \mu_p C_{ox}(W/L) V_{OV}^2$$

GS-KVL: $$V_G = V_{GS} + 10^4 I_D = V_{OV} + V_t + 10^4 I_D$$
$$10^4 \times 0.5 \times 0.8 \times 10^{-3} V_{OV}^2 + V_{OV} - 6 + 1 = 0$$
$$4V_{OV}^2 + V_{OV} - 5 = 0$$

Negative root is unphysical, $V_{OV} = 1.0$ V and $V_{GS} = V_{OV} + V_t = 2.0$ V.

$$I_D = 0.5 \times 0.8 \times 10^{-3} V_{OV}^2 = 0.40 \text{ mA}$$

DS-KVL: $$15 = 10^4 I_D + V_{DS} + 10^4 I_D \quad \rightarrow \quad V_{DS} = 7 \text{ V} \quad (> V_{OV} = 1.0)$$

Bias summary: $I_D = 0.40$ mA, $V_{OV} = 1.0$ V, $V_{GS} = 2.0$ V, $V_{DS} = 7.0$ V.

Small-Signal:

$$g_m = \frac{2I_D}{V_{OV}} = 2 \times 0.4 \times 10^{-3} = 0.8 \text{ mA/V} \qquad r_o = \frac{1}{\lambda I_D} = \frac{1}{0.4 \times 10^{-3}} = 250 \text{ k}$$

As the input is at the gate and output is at the collector, this is a common-source amplifier. There is no R_S because of the by-pass capacitor.

$$\frac{v_o}{v_i} = -g_m (r_o \parallel R_D \parallel R_L) = -0.8 \times 10^{-3} (250 \text{ k} \parallel 10 \text{ k} \parallel 100 \text{ k}) = -7.02$$
$$R_i = R_G = 720 \text{ k}$$
$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = -7.02$$
$$R_o = R_D \parallel r_o = 10 \text{ k} \parallel 100 \text{ k} = 9.09 \text{ k}$$

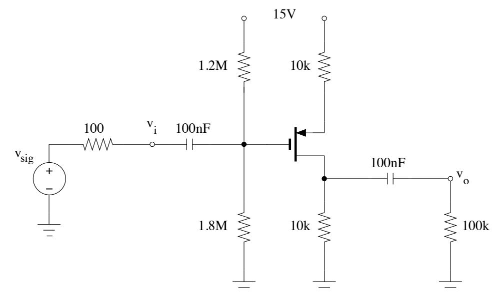
$f_{p1} = 1/[2\pi C_{c1}(R_i + R_{sig})] = 2.21$ Hz

$f_{p2} = 1/[2\pi C_{c2}(R_L + R_o)] = 14.6$ Hz

$$f_{p3} = \frac{1}{2\pi C_s[R_S \parallel (1/g_m)]} = \frac{1}{2\pi C_s[10 \text{ k} \parallel 1.25 \text{ k}]} = \frac{1}{2\pi \times 10^{-6} \times 1.11 \times 10^3} = 143 \text{ Hz}$$

$f_l = f_{p1} + f_{p2} + f_{pb} = 2.21 + 14.6 + 143 = 160$ Hz

Problem 18. Find the bias point and amplifier parameters of this circuit ($V_{tn} =$ 1 V, $V_{tp} = -1$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.8$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.

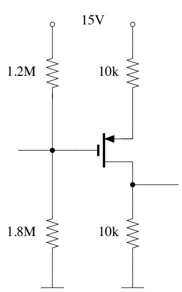


Bias: Since $I_G = 0$:

$$V_G = \frac{1.8 \text{ M}}{1.2 \text{ M} + 1.8 \text{ M}} \times 15 = 9 \text{ V}$$

$$R_G = 1.2 \text{ M} \parallel 1.8 \text{ M} = 720 \text{ k}$$

Assume PMOS is in the active state,



$$I_D = 0.5\mu_p C_{ox}(W/L)V_{OV}^2$$

SG-KVL: $15 = 10^4 I_D + V_{SG} + V_G = 10^4 I_D + V_{OV} + |V_{tp}| + V_G$

$$10^4 \times 0.5 \times 0.8 \times 10^{-3} V_{OV}^2 + V_{OV} - 15 + 9 + 1 = 0$$

$$4V_{OV}^2 + V_{OV} - 5 = 0$$

Negative root is unphysical, $V_{OV} = 1.0$ V and $V_{SG} = V_{OV} + |V_{tp}| = 2.0$ V.

$$I_D = 0.5 \times 0.8 \times 10^{-3} V_{OV}^2 = 0.40 \text{ mA}$$

SD-KVL: $15 = 10^4 I_D + V_{SD} + 10^4 I_D \quad \rightarrow \quad V_{SD} = 7 \text{ V} \quad (> V_{OV} = 1.0)$

Bias summary: $I_D = 0.40$ mA, $V_{OV} = 1.0$ V, $V_{SG} = 2.0$ V, $V_{SD} = 7.0$ V. Small-Signal:

$$g_m = \frac{2I_D}{V_{OV}} = 2 \times 0.4 \times 10^{-3} = 0.8 \text{ mA/V} \qquad r_o = \frac{1}{\lambda I_D} = \frac{1}{0.4 \times 10^{-3}} = 250 \text{ k}$$

As the input is at the gate and output is at the collector, this is a common-source amplifier with R_S.

$$\frac{v_o}{v_i} = -\frac{g_m(R_D \parallel R_L)}{1 + g_m R_S + (R_D \parallel R_L)/r_o} = -\frac{0.8 \times 10^{-3}(10 \text{ k} \parallel 100 \text{ k})}{1 + 0.8 \times 10^{-3} \times 10^4 + (10 \text{ k} \parallel 100 \text{ k})/(250 \text{ k})}$$

$$\frac{v_o}{v_i} = -\frac{7.27}{9.04} = -0.805$$

$$R_i = R_G = 720 \text{ k}$$

$$A_v = \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = -0.805$$

$$R_o = R_D \parallel [r_o(1 + g_m R_S)] = 10 \text{ k} \quad (\approx R_D)$$

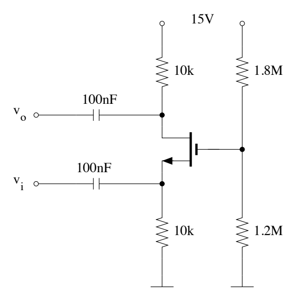
$$f_{p1} = 1/[2\pi C_{c1}(R_i + R_{sig})] = 2.21 \text{ Hz}$$

$$f_{p2} = 1/[2\pi C_{c2}(R_L + R_o)] = 14.5 \text{ Hz}$$

$$f_l = f_{p1} + f_{p2} = 16.7 \text{ Hz}$$

Note one needs to choose R_D to be several times R_S for this amplifier to have a gain larger than unity.

Problem 19. Find the bias point and amplifier parameters of this circuit ($V_{tn} = 1$ V, $V_{tp} = -1$ V, $\mu_p C_{ox}(W/L) = \mu_n C_{ox}(W/L) = 0.8$ mA/V^2, and $\lambda = 0.01$ V^{-1}. Ignore the channel-width modulation effect in biasing calculations.



Answer:

$I_D = 0.40$ mA, $V_{OV} = 1.0$ V, $V_{GS} = 2.0$ V, $V_{SD} = 7.0$ V.

$v_o/v_{sig} = 0.866$, $R_i = \infty$, $R_o = 1.17$ k,
$f_{p1} = 0$, $f_{p2} = 3.35$ Hz, $f_l = 4.3$ Hz.

Amp Response (common-gate amp):
$A_v = 7.7$, $R_i = 1.1$ k, $R_o = 10$ k, and $f_l = 1.45 \times 10^3 + 22 = 1.47$ kHz,