ECE 101

Problem Set # 5B Solutions

(send questions/comments to psiegel@ucsd.edu)

Speed Problems

9.1

a.

$$\int_0^\infty e^{-5t}e^{-(\sigma+j\omega)t}dt \ \ converges\ if -5-\sigma<0 \ \ or\ equivalently, \boldsymbol{\sigma} > \ -\mathbf{5}$$

b.

$$\int_{-\infty}^0 e^{-5t}e^{-(\sigma+j\omega)t}dt \ \ converges\ if -5-\sigma>0\ or\ equivalently\ \boldsymbol{\sigma} < \ -\mathbf{5}$$

c.

$$\int_{-5}^5 e^{-5t}e^{-(\sigma+j\omega)t}dt \ \ converges\ for\ all -\infty < \boldsymbol{\sigma} < \infty$$

d.

$$\int_{-\infty}^\infty e^{-5t}e^{-(\sigma+j\omega)t}dt \ \ does\ not\ converge\ for\ any\ value\ of\ \sigma$$

e.

$$\int_{-\infty}^\infty e^{-5|t|}e^{-(\sigma+j\omega)t}dt = \int_{-\infty}^0 e^{-5|t|}e^{-(\sigma+j\omega)t}dt + \int_0^\infty e^{-5|t|}e^{-(\sigma+j\omega)t}dt$$

$$= \int_{-\infty}^0 e^{5t}e^{-(\sigma+j\omega)t}dt + \int_0^\infty e^{-5t}e^{-(\sigma+j\omega)t}dt$$

$$\int_{-\infty}^\infty e^{-5|t|}e^{-(\sigma+j\omega)t}dt \ \ converges\ if\ 5-\sigma>0 \ \rightarrow\ \sigma < \ 5\ \mathbf{and} -5-\sigma<0 \ \rightarrow\ \sigma>-5$$

$$In\ other\ words \int_{-\infty}^\infty e^{-5|t|}e^{-(\sigma+j\omega)t}dt\ converges\ if\ |\boldsymbol{\sigma}| < \mathbf{5}$$

f.

$$\int_{-\infty}^0 e^{-5|t|}e^{-(\sigma+j\omega)t}dt = \int_{-\infty}^0 e^{5t}e^{-(\sigma+j\omega)t}dt\ converges\ if\ 5-\sigma>0\ that\ is, \boldsymbol{\sigma} < \ \mathbf{5}$$

Note : In e. and f., we use the fact that $|t| = t\ if\ t>0\ and\ |t| = -t\ if\ t<0$

9.3

$x(t) = \ e^{-5t}u(t) + e^{-\beta t}u(t)$

The ROC is given by $\Re\mathrm{e}\{s\} > -3$.

x(t) is a sum of two signals. The ROC of the first signal $e^{-5t}u(t)$ is $R_1 = \{s \mid \Re\mathrm{e}\{s\} > -5\}$. The ROC of the second signal is $R_2 = \{s \mid \Re\mathrm{e}\{s\} > -\Re\mathrm{e}\{\beta\}\ \}$.

The ROC of the sum is atleast $R_1 \cap R_2$

Therefore for the ROC to be $\Re e\{s\} > -3$ we need to have $\Re e\{\beta\} = 3$ and there is no restriction on the imaginary part of β.

9.6

A. Since x(t) has a pole, it cannot be of finite duration.
B. Since x(t) has a pole at 2 and it is absolutely integrable (ROC includes the imaginary axis), its ROC cannot be right sided. Hence x(t) could be left-sided.
C. x(t) cannot be right-sided as reasoned in B.
D. x(t) can be two-sided, if it has atleast one more pole on to the left of the imaginary axis.

9.10

A. The poles are both on the real axis. If we move on the imaginary axis starting from the origin, the denominator of the transfer function will be smallest in magnitude at the origin and increase monotonically as we move away from the origin. Thus the magnitude of the transfer function will be maximum at the origin and will decrease monotonically as we move away from the origin, along the imaginary axis. This corresponds to a low pass filter.
B. The poles are at $s = -\frac{1}{2} \pm \frac{j\sqrt{3}}{2}$. If we move on the imaginary axis starting from the origin, the denominator of the transfer function will be small in magnitude around the points $\omega = \pm\frac{\sqrt{3}}{2}$ and increase as we move away from these points. And there is a zero at the origin which will make the magnitude of the numerator go to zero at $\omega = 0$. This corresponds to a band pass filter.
C. The poles are both on the real axis, similar to A. But now we have two zeros at the origin. This will force the magnitude of the numerator to zero at $\omega = 0$. As we move away from the origin, the numerator increases monotonically and the denominator decreases as in A. This corresponds to a high pass filter.

9.28.

A. Since the ROC is bounded by poles and cannot included any poles, the four possible ROCs for the pole-zero configuration are
   a. $R_1 = \{s \mid \Re e\{s\} < -2\}$
   b. $R_2 = \{s \mid \Re e\{s\} > -2 \ and \ \Re e\{s\} < -1\}$
   c. $R_3 = \{s \mid \Re e\{s\} > -1 \ and \ \Re e\{s\} < 1\}$
   d. $R_4 = \{s \mid \Re e\{s\} > 1\}$
B.

ROC	Causal (right-sided)	Stable (ROC includes the imaginary axis)

R1	No	No
R2	No	No
R3	No	Yes
R4	Maybe	No

9.32

From 1, we can infer that H(2) = 1/6.

From 2, by taking the Laplace transform of the differential equation, we get,

$$sH(s) + \; 2H(s) = \frac{1}{s+4} + \frac{b}{s}, \Re\{s\} > 0$$

$$H(s) = \frac{s + bs + 4b}{s(s+4)(s+2)}, \Re\{s\} > 0$$

Since H(2) = 1/6, we have

$$\frac{1}{6} = \frac{2 + 2b + 4b}{2(2+4)(2+2)}$$

Which gives us **b=1.**

$$H(s) = \frac{2s+4}{s(s+4)(s+2)}, \Re\{s\} > 0$$

$$\boldsymbol{H(s) = \frac{2}{s(s+4)}, \Re\{s\} > 0}$$

9.33

We will first find X(s).

$$x(t) = \; e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s-1} = \; -\frac{2}{(s+1)(s-1)}, \qquad -1 < \Re\{s\} < 1$$

$$H(s) = \frac{s+1}{s^2+2s+2} = \frac{s+1}{(s+1-j)(s+1+j)}, \qquad \Re\{s\} > -1$$

$$Y(s) = \; X(s).H(s)$$

$$Y(s) = \; -\frac{2}{(s-1)(s+1-j)(s+1+j)}, \qquad -1 < \Re\{s\} < 1$$

Expanding Y(s) using partial fraction expansion,

$$Y(s) = \frac{A}{(s-1)} + \frac{B}{(s+1-j)} + \frac{C}{(s+1+j)}$$

$$A = -\frac{2}{5}, B = \frac{1-2j}{5} \, and \; C = \frac{1+2j}{5}$$

$$Y(s) = \frac{-\frac{2}{5}}{(s-1)} + \frac{\frac{1-2j}{5}}{(s+1-j)} + \frac{\frac{1+2j}{5}}{(s+1+j)}, \qquad -1 < \Re\mathfrak{e}\{s\} < 1$$

$$y(t) = \frac{2}{5} e^{t} u(-t) + \frac{1-2j}{5} e^{(-1+j)t} u(t) + \frac{1+2j}{5} e^{(-1-j)t} u(t)$$

$$y(t) = \frac{2}{5} e^{t} u(-t) + \frac{1}{5} e^{-t} u(t)\left\{e^{jt} + e^{-jt}\right\} - \frac{2j}{5} e^{-t} u(t)\{e^{jt} - e^{-jt}\}$$

$$\boldsymbol{y(t) = \frac{2}{5} e^{t} u(-t) + \frac{2}{5} e^{-t} \cos(t)\, u(t) + \frac{4}{5} e^{-t} \sin(t)\, u(t)}$$

Note that another way of doing the partial fraction expansion would be as follows.

$$Y(s) = \frac{A}{(s-1)} + \frac{Bs + C}{(s^2 + 2s + 2)}$$

And then using Table 9.2, transform pair 13 and 14 to obtain the inverse transform. This method will give the same answer at the end.