ECE 45 WI25 Professor Heath
January 23, 2025 11:59PM
Andrew Onozuka A16760043
Homework #3

1) Simplify:
    a) $(1 + v + v^2)\delta(t)$

       $(1 + v + v^2)\delta(t) = (1 + 0 + 0^2)\delta(t) = \delta(t)$

    b) $\int_{-\infty}^{\infty} \delta(t - 1)x(1 - t)dt$

       $\int_{-\infty}^{\infty} \delta(t - a)f(t)dt = f(a)$

       $\int_{-\infty}^{\infty} \delta(t - 1)x(1 - t)dt = x(1 - 1) = x(0)$

    c) $\int_{-\infty}^{\infty} \delta(t - 4)\frac{sin(\pi t^2)}{\pi t^2}dt = \frac{sin(\pi 4^2)}{\pi 4^2} = \frac{sin(16\pi)}{16\pi} = 0$

    d) $\sum_{n=0}^{\infty} (t + 1)^n\delta(t)$

       $(t + 1)^n = (0 + 1)^n = 1$ for all n, when substituting $t = 0$.

       $\sum_{n=0}^{\infty} (t + 1)^n\delta(t) = \sum_{n=0}^{\infty} \delta(t)$

    e) $f(t) = \int_{-\infty}^{t} \delta(\tau - 3)d\tau$

       $\int_{-\infty}^{t} \delta(\tau - 3)d\tau = \{0\ if\ t < 3,\ 1\ if\ t \geq 3\}$

       $f(t) = u(t - 3)$

    f) $sin(2\pi t)\delta(\frac{1}{2} - 2t)$

       $\frac{1}{2} - 2t = 0 \qquad t = \frac{1}{4}$

       $\delta(at - b) = \frac{1}{|a|}\delta(t - \frac{b}{a}) \qquad a =- 2, b = \frac{1}{2}$

       $sin(2\pi t) = sin(2\pi * \frac{1}{4}) = sin(\frac{\pi}{2}) = 1$

       $\frac{1}{2}\delta(t - \frac{1}{4})$

    g) $\int_{-\infty}^{\infty} (\frac{du(t)}{dt} - rect(t))dt$

       $\frac{du(t)}{dt} = \delta(t)$

       $\int_{-\infty}^{\infty} rect(t)dt = 1$

       $\int_{-\infty}^{\infty} (\frac{du(t)}{dt} - rect(t))dt = 1 - 1 = 0$

ECE 45 WI25 Professor Heath
January 23, 2025 11:59PM
Andrew Onozuka A16760043
Homework #3

2) Determine Time Invariance/Linearity:
   a) $y(t) = 2x(t - 3)$
      i) When we shift the input, the output also shifts:
      $x(t) \rightarrow x(t - t_0)$ $\quad$ $y(t) \rightarrow 2x((t - t_0) - 3) = 2x(t - t_0 - 3)$
      If we shift the output $y(t) \rightarrow y(t - t_0)$, we get the same answer, so the system **is time-invariant.**
      ii) The system satisfies additivity $y(t) = x(t - 3) + x(t - 3)$ and homogeneity (scaling both by constant k), therefore, **is linear.**
   b) $y(t) = \int_{-\infty}^{t} x(\gamma)d\gamma$
      i) When we shift the input, the output also shifts:
      $x(t) \rightarrow x(t - t_0)$ $\quad$ $y(t) = \int_{-\infty}^{t} x(\gamma - t_0)d\gamma$
      When we shift the output, however, we get $\int_{-\infty}^{t-t_0} x(\gamma)d\gamma$.
      The limits of integration are different and therefore do not match, so the system **is not time-invariant.**
      ii) The system satisfies additivity $\int_{-\infty}^{t} [x_1(\gamma) + x_2(\gamma)]d\gamma$ and homogeneity (scaling both by constant k), therefore, **is linear.**
   c) $y(t) = Re\{x(t)\}$
      i) $Re\{x(t - t_0)\}$ matches $y(t) = Re\{x(t - t_0)\}$, therefore the system **is time-invariant.**
      ii) The system satisfies additivity $Re\{x_1(t) + x_2(t)\}$ and homogeneity (scaling both by constant k), therefore, **is linear.**
   d) $y(t) = x(t - 2) + x(2 - t)$
      i) Both shifted match at $y(t) = x((t - t_0) - 2) + x(2 - (t - t_0))$, system **is time-invariant.**
      ii) Additivity is already included in the $x(t - 2) + x(2 - t)$ and homogeneity holds, system **is linear.**
   e) $y(t) = log_2(1 + |x(t)|^2)$
      i) Both match at $y(t) = log_2(1 + |x(t - t_0)|^2)$, system **is time-invariant.**
      ii) Logarithmic and magnitude-squared operations are not linear, therefore the system **is not linear.**
   f) $y(t) = cos(x(t))$
      i) Both shifts match at $y(t) = cos(x(t - t_0))$. System **is time-invariant.**

ECE 45 WI25 Professor Heath
January 23, 2025 11:59PM
Andrew Onozuka A16760043
Homework #3

ii) Cosine operation is not additive (same wave) and it is not homogeneous as scaling x(t) does not scare the system linearly. Therefore this system **is not linear.**

g) $y(t) = \{0\ if\ x(t) < 1, \int_0^1 x(t - \tau)d\tau\ if\ x(t) \geq 0\}$

i) The thresholds mean that the behavior is dependent on input, therefore the system **is not time-invariant.**

ii) The piecewise definition and integration means this system **is not linear.**

3) Homework 2 Corrections

c) $V_L = Z_E V_S = \frac{j\omega L}{R+j(\omega L - \frac{1}{\omega C})} A\angle 0$

didn't put it under $j\omega L$

d) $|Z_E| = \frac{\omega L}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}}$

didn't put it under ωL

e) $\angle Z_E = \frac{\pi}{2} - tan^{-1}(\frac{\omega L - \frac{1}{\omega C}}{R})$

didn't put $\frac{\pi}{2} -$