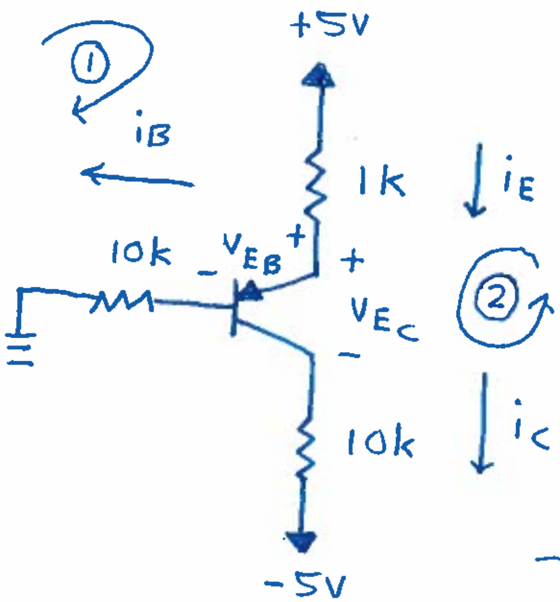
Problem 1,1



Assume cutoff:

$i_B = 0$, $i_C = 0$ $\rightarrow$ $i_E = 0$

① $-5 + 1k \cdot i_E + V_{EB} + 10k \cdot i_B = 0$

$i_E \rightarrow 0$, $i_B \rightarrow 0$

$V_{EB} = 5 > V_{Do} = 0.7$ X

$\rightarrow$ Therefore, BJT is not in cutoff

Assume Active:

$i_B \geq 0$

$V_{EB} = 0.7V$

$\beta i_B = i_C$

$V_{EC} \geq 0.7$

① $-5 + i_E \cdot 1k + V_{EB} + i_B \cdot 10k = 0$

$i_B + i_C = i_E \rightarrow i_B + \beta i_B = i_E \rightarrow i_E = (1+\beta)\, i_B$

$i_E = 101\, i_B$

$\rightarrow -5 + (101)\, i_B \cdot 1k + V_{EB} + i_B \cdot 10k = 0 \rightarrow i_B = 0.038\,mA$

$i_C = \beta i_B = 3.8\,mA$

$i_E = 101\, i_B = 3.91\,mA$

* Now, need to check if actually in active: ②

$-5 + i_E \cdot 1k + V_{EC} + i_C \cdot 10k - 5 = 0 \rightarrow V_{EC} = -32 < 0.7$ X

$\rightarrow$ Therefore, our assumption is incorrect, BJT is not in active.

$\hookrightarrow$ Not cutoff, not active, must be saturation.

Saturation:

$V_{EC} = 0.2V$

$i_C < \beta i_B$

$V_{EB} = 0.7V$

$i_B \geq 0$

2 $\cancel{1}$ $-5 + 1k \cdot i_E + V_{EC} + 10k \cdot i_C - 5 = 0$

$-9.8 + 1k \cdot i_E + 10k \cdot i_C = 0$

1 $\cancel{2}$ $-5 + 1k \cdot i_E + V_{EB} + 10k \cdot i_B = 0$

$-4.3 + 1k \cdot i_E + 10k \cdot i_B = 0$

* Next, we know $i_B + i_C = i_E$

Plugging that into the 2 equations above gives us:

$-9.8 + 1k(i_B + i_C) + 10k \cdot i_C = 0$

$-4.3 + 1k(i_B + i_C) + 10k \cdot i_B = 0$

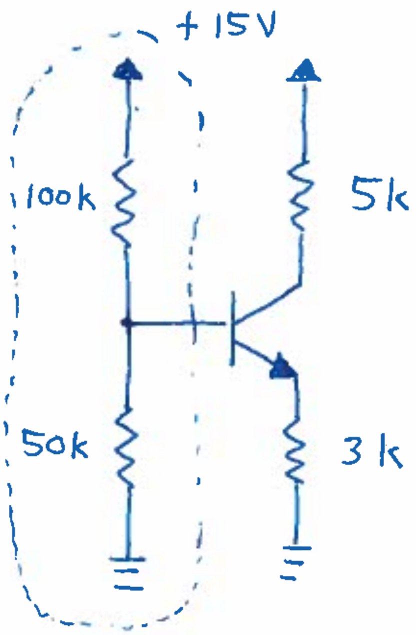
$\boxed{i_B = 0.3125\,mA, \quad i_C = 0.8625\,mA, \quad i_E = 1.175\,mA}$

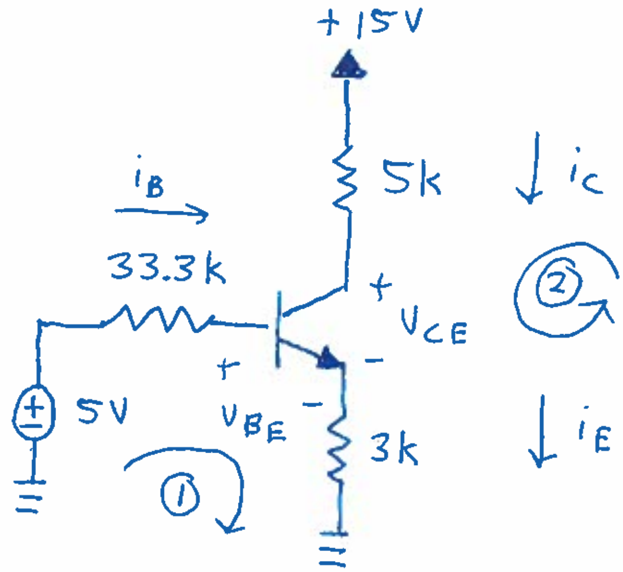
$\boxed{V_E = 5 - 1k \cdot i_E = 3.825V}$

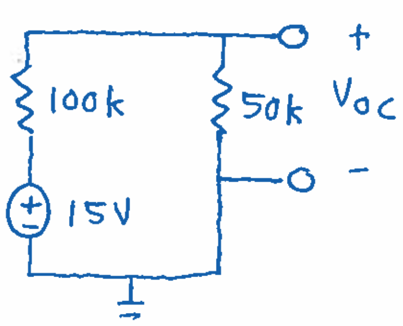
$\boxed{V_C = i_C \cdot 10k - 5 = 3.625V}$

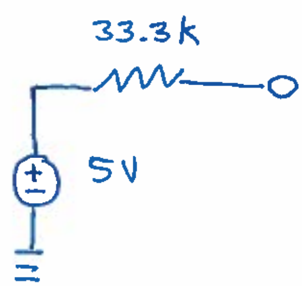
$\boxed{V_B = V_E - V_{EB} = 3.125V}$

Problem 1,2









$$V_{th} = 15V\left(\frac{50k}{100k + 50k}\right) = 5V$$

$$R_{th} = 100k \parallel 50k = 33.3k$$

Assume cutoff:

$i_B = i_C = 0 \rightarrow i_E = 0$, $V_{BE} < 0.7$

① $-5 + \underset{\downarrow \atop 0}{i_B} \cdot 33.3k + V_{BE} + \underset{\downarrow \atop 0}{i_E} \cdot 3k = 0 \rightarrow V_{BE} = 5 > 0.7$ X

→ Therefore, assumption is incorrect, BJT is not in cutoff.

Assume Active:

$i_B \geq 0$

$V_{BE} = 0.7$

$i_C = \beta i_B$

$V_{CE} \geq 0.7$

① $-5 + i_B \cdot 33.3k + V_{BE} + i_E \cdot 3k = 0$

$(\beta+1) i_B = i_E$

$-5 + i_B \cdot 33.3k + 0.7 + 101 i_B \cdot 3k = 0$

$$\boxed{\begin{aligned} i_B &= 0.0128\,mA \\ i_C &= \beta i_B = 1.28\,mA \\ i_E &= 101\, i_B = 1.29\,mA \end{aligned}}$$

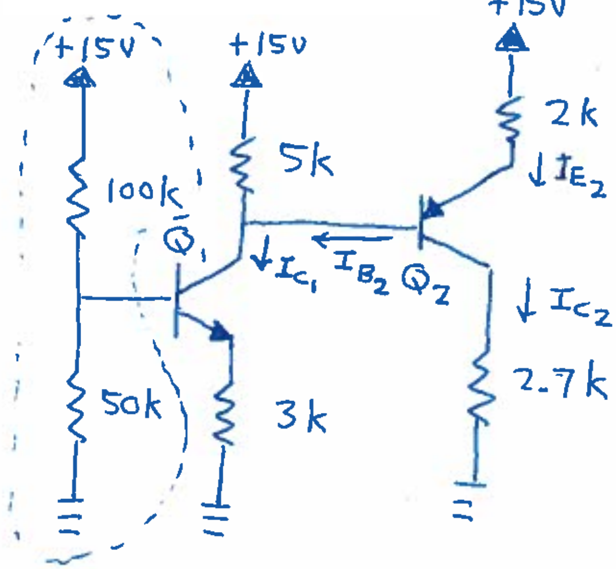
Next, need to check assumption: (Is $V_{CE} \geq 0.7$?)

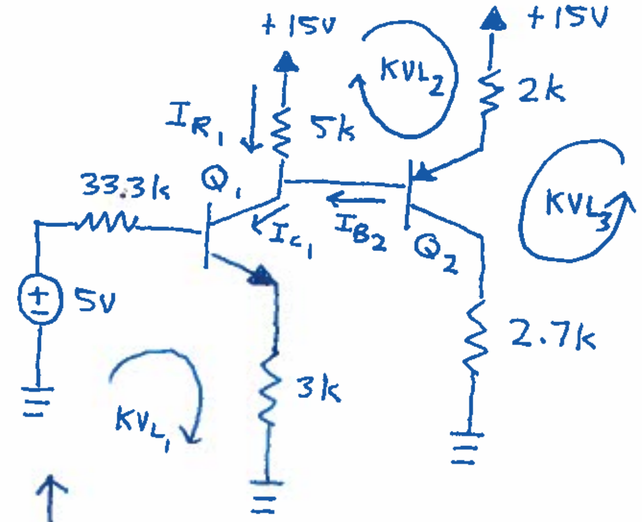
$$\boxed{\begin{aligned} V_C &= 15 - i_C \cdot 5k = 8.6V \\ V_E &= i_E \cdot 3k = 3.87V \end{aligned}} \rightarrow V_{CE} = V_C - V_E = 4.83 \geq 0.7 \checkmark$$

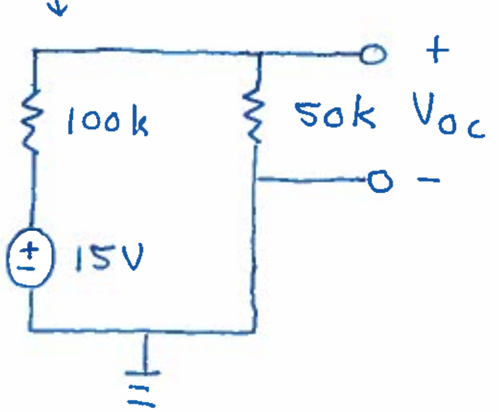
Therefore, assumption is correct, BJT is operating in active mode.

$$\boxed{V_B = V_E + V_{BE} = 4.57V}$$

Problem 2

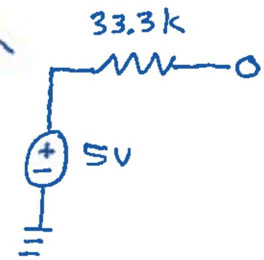






$$V_{th} = 15V\left(\frac{50k}{150k}\right) = 5V$$

$$R_{th} = 100k \,||\, 50k = 33.3k$$



First, need to see if Q_1 in cutoff:

$$KVL_1: \; -5 + 33.3k\cdot \underset{0}{\underset{\downarrow}{i_B}} + V_{BE} + 3k\cdot \underset{0}{\underset{\downarrow}{i_E}} = 0 \rightarrow V_{BE} = 5 > 0.7 \quad \times$$

- Therefore, Q_1 is not in cutoff, must be on.

Assume Q_1 in active:

$i_{B_1} \geq 0$

$V_{BE_1} = 0.7$

$i_{C_1} = \beta i_{B_1}$

$V_{CE_1} \geq 0.7$

$$-5 + 33.3k\cdot i_{B_1} + V_{BE_1} + 3k\cdot \underset{(\beta+1)\, i_{B_1} = i_{E_1}}{\underset{\uparrow}{i_{E_1}}} = 0$$

$$-5 + 33.3k\cdot i_{B_1} + 0.7 + 3k\,(101)\, i_{B_1} = 0$$

$$\boxed{\begin{aligned} i_{B_1} &= 0.0128\,mA \\ i_{C_1} &= \beta i_{B_1} = 1.28\,mA \\ i_{E} &= 101\, i_{B} = 1.29\,mA \end{aligned}}$$

Next, need to check assumption ($V_{CE} \geq 0.7$?)

- However, we don't know V_{C_1} (since I_{B_2} is unknown)

→ So, make assumption of Q_2 (we will come back later)

Assume Q_2 in cutoff: $I_{B_2} = I_{C_2} = 0 \rightarrow I_{E_2} = 0$

$$-15 + 2k \cdot i_{E_2} + V_{EB_2} = V_{C_1}$$

$$I_{B_2} = 0 \rightarrow I_{R_1} = I_{C_1} = 1.28\,mA$$

$$V_{C_1} = 15 - I_{R_1} \cdot 5k = 8.6\,V$$

$$V_{EB_2} = 15 - V_{C_1} = 6.4 > 0.7 \quad X$$

Assumption incorrect, Q_2 is not in cutoff

Assume Q_2 in active:

$i_{B_2} \geq 0$

$V_{EB_2} = 0.7$

$i_{C_2} = \beta i_{B_2}$

$V_{EC_2} \geq 0.7$

KVL$_2$:

$$-15 + 2k \cdot \underbrace{i_{E_2}}_{i_{E_2} = (\beta+1) i_{B_2}} + V_{EB_2} - \underbrace{I_{R_1}}_{I_{R_1} = i_{C_1} - i_{B_2}} \cdot 5k + 15 = 0$$

i_{C_1} = 1.28 mA

$$-15 + 2k\,(101)\, i_{B_2} + 0.7 - (1.28 - i_{B_2}) \cdot 5k + 15 = 0$$

$$\boxed{\begin{aligned} i_{B_2} &= 0.0275\,mA \\ i_{C_2} &= 100\, i_{B_2} = 2.75\,mA \\ i_{E_2} &= 101 \cdot i_{B_2} = 2.78\,mA \end{aligned}}$$

Now, we can verify assumption of Q_1:

$$V_{C_1} = 15 - I_{R_1} \cdot 5k = 8.74V$$

$$I_{R_1} = i_{C_1} - i_{B_2} = (1.28 - 0.0275)\,mA$$

$$V_{CE_1} = V_{C_1} - V_{E_1} = 8.74V - 3.87V = \underline{4.87V > 0.7} \quad \checkmark$$

$$V_{E_1} = i_{E_1} \cdot 3k = (1.29mA)(3k) = 3.87V$$

Assumption for Q_1 correct, but only if Q_2 is correct

Verify Assumption of Q_2:

$$KVL_3: \; -15 + 2k \cdot i_E + V_{EC_2} + 2.7k \cdot i_C = 0$$

$$V_{EC} = 2.015V > 0.7 \quad \checkmark$$

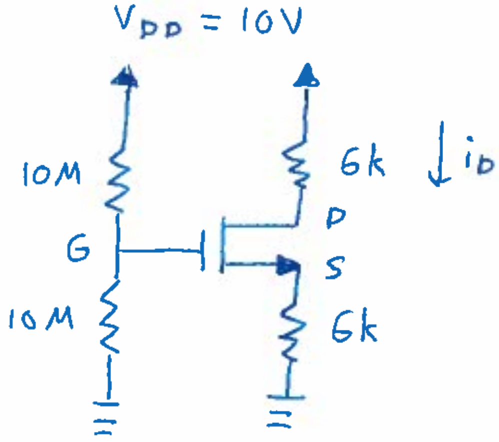
Assumption correct, Q_2 is in active as well.

– Both Q_1 and Q_2 in active.

$$\boxed{\begin{aligned} V_{B_1} &= 5 - i_{B_1}(33.3k) = 4.57V \\ V_{C_1} &= 15 - I_{R_1} \cdot 5k = 8.74V \\ V_{E_1} &= 3k \cdot i_{E_1} = 3.87V \end{aligned}}$$

$$\boxed{\begin{aligned} V_{B2} &= V_{C_1} = 8.74V \\ V_{C2} &= 2.7k \cdot i_{C2} = 7.43V \\ V_{E2} &= 15 - i_{E2} \cdot 2k = 9.44V \end{aligned}}$$

Problem 3



$$V_G = 10\text{V}\left(\frac{10M}{10M + 10M}\right) = \boxed{5\text{V}}$$

Assume Saturation:

$$i_D = \frac{1}{2} k_n (V_{OV}^2)$$

$$= \frac{1}{2} k_n (V_{gs} - V_t)^2$$

$$= \frac{1}{2} k_n (V_g - V_s - V_t)^2$$

$$= \frac{1}{2} k_n (5 - V_s - 1)^2$$

$$= \frac{1}{2} k_n (4 - V_s)^2$$

$$V_s = i_D \cdot 6k$$

$$= \frac{1}{2} k_n (4 - i_D \cdot 6k)^2$$

$$i_D = 18 i_D^2 - \overset{24 i_D}{} + 8 \rightarrow 0 = 18 i_D^2 - 25 i_D + 8$$

$i_D =$ 0.5 mA or 0.89 mA

Not valid: Would result in $V_{OV} < 0$, and transistor not in cutoff

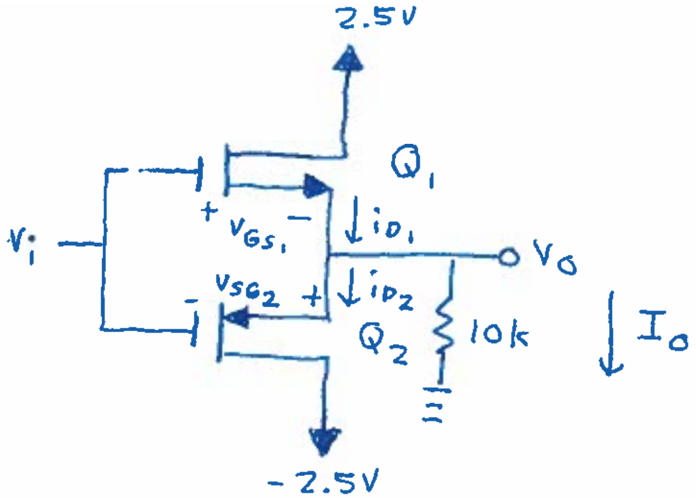
$$V_S = (0.5\text{mA})(6k) = \boxed{3\text{V}}$$

$$V_D = V_{DD} - i_D \cdot 6k = 10 - 3 = \boxed{7\text{V}}$$

$V_{OV} \leq V_{DS}$? $\rightarrow$ $V_{OV} = 4 - (0.5\text{mA})(6k) = 1\text{V} < V_{DS} = 4\text{V}$ ✓

Yes, in saturation.

Problem 4



$v_i = 0 \rightarrow$ Assume both in cutoff

$\boxed{i_{D_1} = i_{D_2} = 0} \rightarrow V_O = I_O \cdot 10k = 0V$

$V_{OV_1} < 0$? $\quad V_{OV_2} < 0$?

$V_{GS_1} = V_i - V_O = 0 \rightarrow V_{OV_1} = 0 - 1V < 0$ ✓

$V_{SG_2} = V_O - V_i = 0 \rightarrow V_{OV_2} = 0 - 1V < 0$ ✓

Assumption Correct, both are in cutoff : $\boxed{V_O = 0V}$

$V_i = 2.5V \rightarrow$ Assume Q_1 in sat., Q_2 cutoff

$i_{D_1} = \frac{1}{2} k_n V_{OV_1}^2$ $\quad$ * $I_O = i_{D_1}$ (Since if Q_2 is off, all current from Q_1 goes to I_O)

$i_{D_2} = 0$

$i_{D_1} = \frac{1}{2} k_n (V_{GS_1} - V_t)^2 = \frac{1}{2} k_n (V_i - V_O - 1)^2 = \frac{1}{2} k_n (1.5 - V_O)^2$

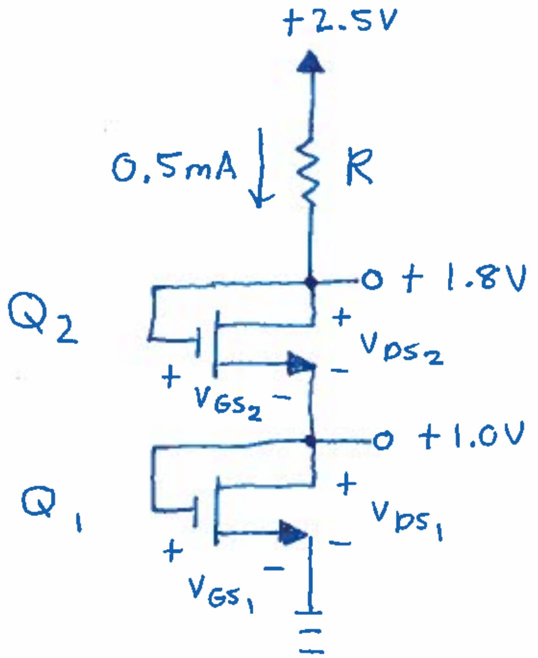
$V_O = i_{D_1} \cdot 10k \rightarrow i_{D_1} = \frac{1}{2} k_n (1.5 - i_{D_1} \cdot 10k)^2$

$0 = 50 i_{D_1}^2 - 16 i_{D_1} + 1.125 \rightarrow i_{D_1} = 0.104\,mA$ or $0.216\,mA$

Not valid, because if so, $V_{OV_1} < 0$, and Q_1 is not [illegible]

$V_O = i_{D_1} \cdot 10k = \boxed{1.04V}$

Problem 5



* We know both Q_1 and Q_2 are in saturation, because gate is connected to drain, meaning:

$$V_{GS_1} = V_{DS_1} \rightarrow V_{OV_1} < V_{DS_1}$$

$$V_{GS_2} = V_{DS_2} \rightarrow V_{OV_2} < V_{DS_2}$$

$$R = \frac{2.5V - 1.8V}{0.5mA} = \boxed{1.4\,k\Omega}$$

$$i_{D_1} = i_{D_2} = 0.5\,mA$$

$$V_{GS_2} = 1.8V - 1V = 0.8V \rightarrow V_{OV_2} = 0.8V - 0.5V = 0.3V$$

$$V_{GS_1} = 1V - 0V = 1V \rightarrow V_{OV_1} = 1V - 0.5V = 0.5V$$

$$i_{D_2} = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}(V_{OV_2})^2 \longrightarrow W_2 = \frac{2 i_{D_2} \cdot L_2}{\mu_n C_{ox}(V_{OV_2})^2} = \boxed{11.1\,\mu m}$$

$$i_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}(V_{OV_1})^2 \longrightarrow W_1 = \frac{2 i_{D_1} \cdot L_1}{\mu_n C_{ox}(V_{OV_1})^2} = \boxed{4\,\mu m}$$