ECE 45 – Circuits and Systems
Winter 2025

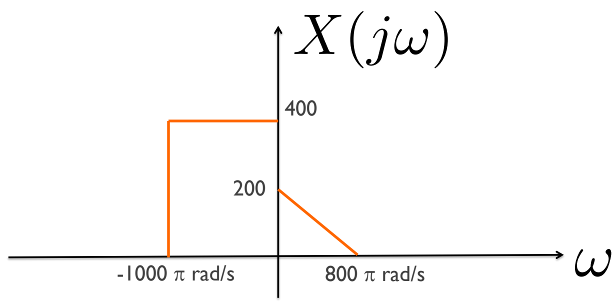
Homework #10

Due: March 14 at 11:59pm, submitted via GradeScope.

You can make multiple upload attempts to experiment with the system and the best way to upload. You must correctly mark the answers to the problems in GradeScope, e.g. problem 1, problem 2, problem 3, to get full credit. Note that you must tag your problems when uploading to GradeScope or they will not be graded and you will not receive credit. Any regrade requests must be placed through GradeScope within one week of the return of the homework.

Remember, discussion of homework questions is encouraged. Please be absolutely sure to submit your own independent homework solution.

1. (20 %) Consider a signal $x(t)$ with the following Fourier transform $X(j\omega)$ as illustrated. The spectrum is real to make sketching easier. In this problem, make the plots by hand.



The signal $x(t)$ is sampled to produce $x[n]$. The signal $y(t)$ is reconstructed from the samples $x[n]$.

(a) Determine the Nyquist frequency ω_M.

(b) Determine the Nyquist rate $2\omega_M$. Let $T_{\text{NQ}} = 2\pi/(2\omega_M) = \pi/\omega_M$.

(c) Suppose that $T = T_{\text{NQ}}/2$.

(d) Sketch $X_p(j\omega)$, the spectrum of the impulse train of samples.

(e) Sketch $Y(j\omega)$, the spectrum of the reconstructed signal.

(f) Is there any aliasing in the reconstructed signal? Does $X(j\omega) = Y(j\omega)$?

(g) Suppose that $T = 3\ T_{\text{NQ}}/2$.

(h) Sketch $X_p(j\omega)$, the spectrum of the impulse train of samples.

(i) Sketch $Y(j\omega)$, the spectrum of the reconstructed signal.

(j) Is there any aliasing in the reconstructed signal? Does $X(j\omega) = Y(j\omega)$?

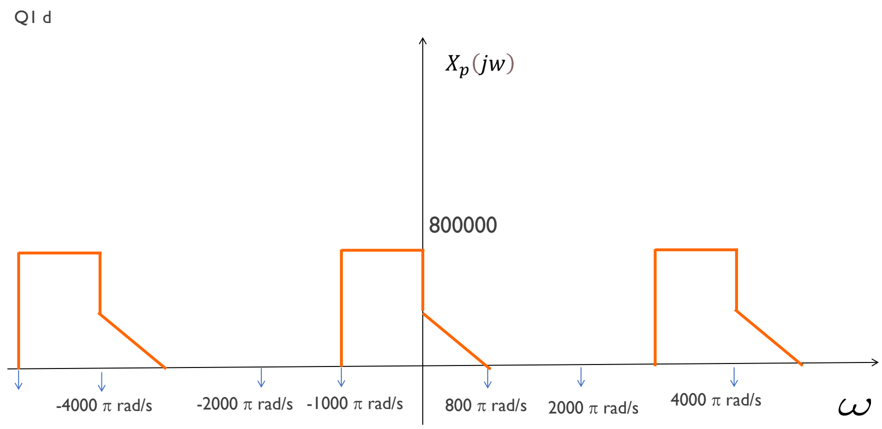
<u>Solution:</u>

(a) $\omega_M = 1000\pi$ rad/s.

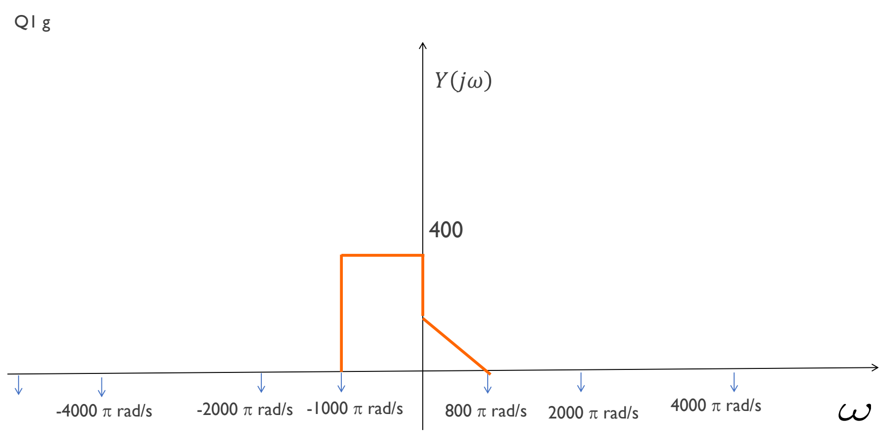
(b) Nyquist rate $= 2\omega_M = 2000\pi$ rad/s.

(c) $T = T_{NQ}/2 = \pi/2\omega_M = \pi/(2 \times 1000\pi) = 1/2000s$

(d) Here, $\omega_s = \frac{2\pi}{T} = 4000\pi$
Now, $\frac{1}{T} = \frac{400}{0.005} = 600000 sec^{-1}$
$X_p(j\omega)$ is plotted below



(e) $Y(j\omega)$ is plotted below



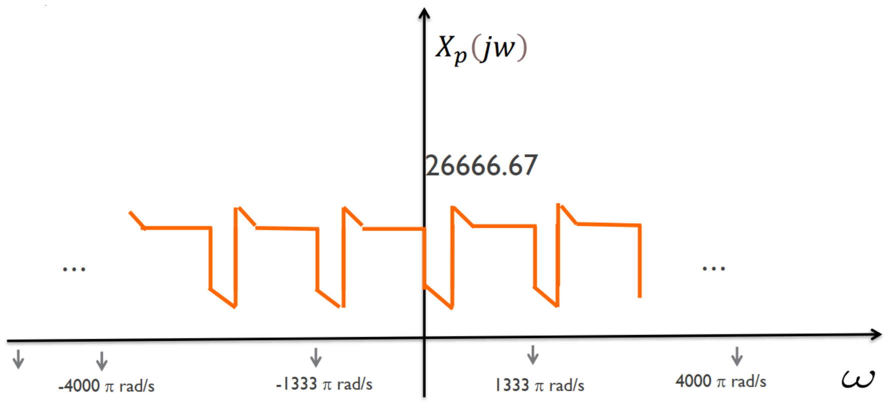
(f) No aliasing. Yes, $X(j\omega) = Y(j\omega)$

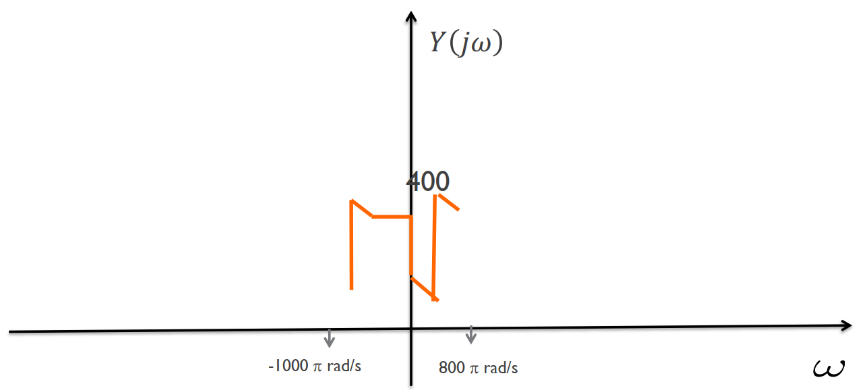
(g) $T = 3T_{NQ}/2 = 3\pi/(2 \times \omega_M) = 0.0015s$

(h) Here,
$\omega_s = \frac{2\pi}{0.0015} = 1333 \ \ rad/sec$
$X_p(j\omega)$ is plotted below

(i) $Y(j\omega)$ is plotted below

(j) Yes there is an aliasing in the constructed signal





2. (20 %) Look up the following information and reference as appropriate. Provide numbers in both Hz and rad/s. Given brand, model and other details if appropriate.

   (a) Lower range of typical human hearing.

   (b) Upper range of typical human hearing.

   (c) Nyquist rate based on the upper limit of human hearing range.

   (d) Sampling rate of the microphone that you normally use for zoom calls.

   (e) Sampling rate of a typical audio CD.

   (f) Sampling rate of a vinyl record.

   (g) Some people pay a lot of money for a DAC to listen to their digital music. The DAC connects digitally and produces an audio output. Find an example of such a DAC that costs more than $200. What is the sampling rate? How does it compare with your answers above? Why?

   <u>Solution:</u>

   (a) Lower range of typical human hearing is 20 Hz.

   (b) Upper range of typical human hearing is 20 kHz.

   (c) Nyquist rate based on the upper limit of human hearing range is 44.1 kHz

   (d) Sampling rate of a typical audio CD range from 16 to 48 kHz.

   (e) Sampling rate of a typical audio CD is 44.1 kHz.

(f) Sampling rate of a vinyl record is 44.1kHz.

(g) AudioQuest - Dragonfly Cobalt at 300 dollars. Sampling rates 0f 44.1kHz, 48kHz, 88.2kHz, or 96kHz. Compared to the sampling rate above, it's about twice as much as a vinyl.

3. (40 %) Determine the Bode plot of the following system in magnitude and phase. This is done by hand not using MATLAB.

$$H(j\omega) = (1 + 5j\omega)(1 + 0.25j\omega) \tag{1}$$

$$H(j\omega) = \frac{1}{(1 + 5j\omega)(1 + 0.25j\omega)} \tag{2}$$

$$H(j\omega) = (j\omega)^2 \tag{3}$$

$$H(j\omega) = (j\omega)^{-3} \tag{4}$$

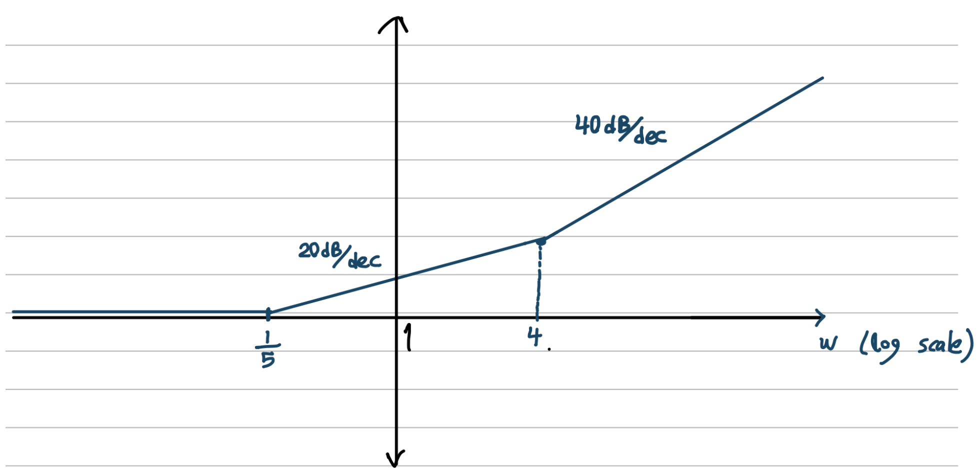
$$H(j\omega) = 3 + 4j \tag{5}$$

$$H(j\omega) = \frac{1 + 20.1j\omega + 2(j\omega)^2}{j\omega + 0.0005(j\omega)^2} \tag{6}$$

<u>Solution:</u>

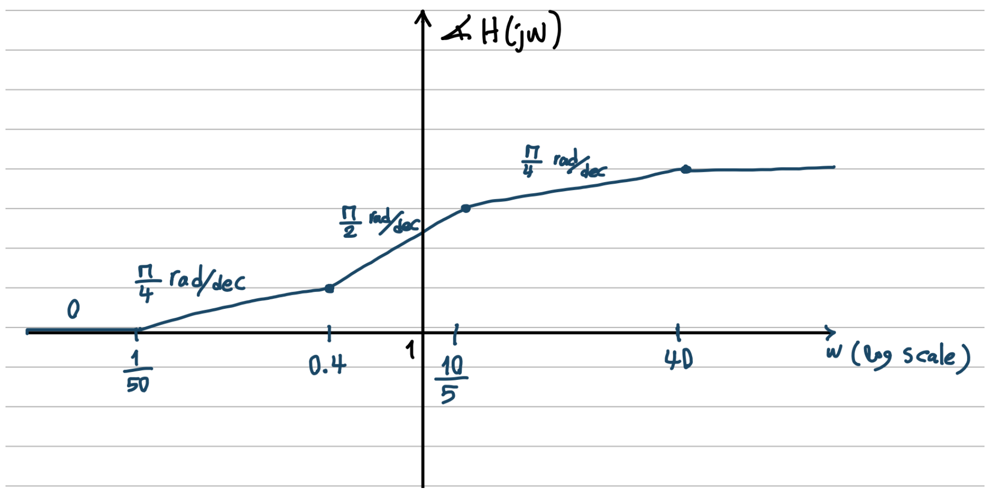
- $H(j\omega) = (1 + 5j\omega)(1 + 0.25j\omega)$. The magnitude in dB is:

$$\begin{aligned} |H(j\omega)|_{\text{dB}} &= 20\log_{10}|1 + 5j\omega| + 20\log_{10}|1 + 0.25j\omega| \\ &= 20\log_{10}|1 + \frac{j\omega}{1/5}| + 20\log_{10}|1 + \frac{j\omega}{4}| \end{aligned}$$



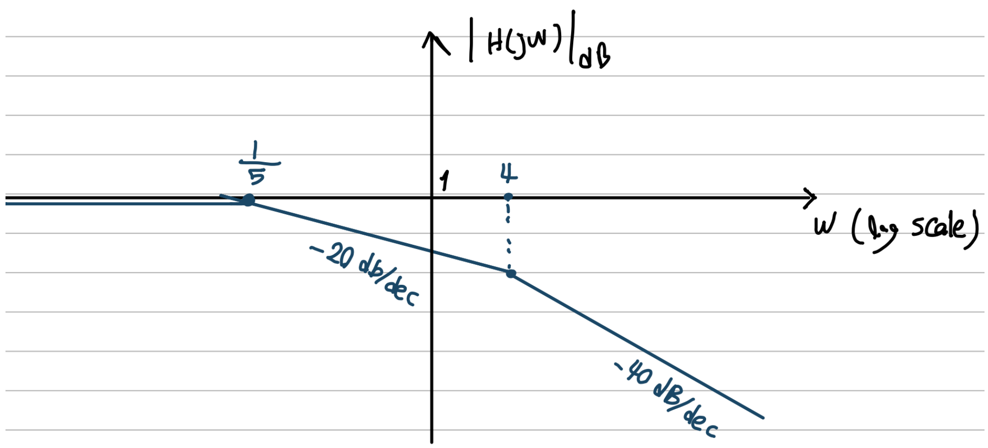
The phase is:

$$\begin{aligned} \angle H(j\omega) &= \angle(1 + 5j\omega) + \angle(1 + 0.25j\omega) \\ &= \angle(1 + \frac{j\omega}{1/5}) + \angle(1 + \frac{j\omega}{4}) \end{aligned}$$



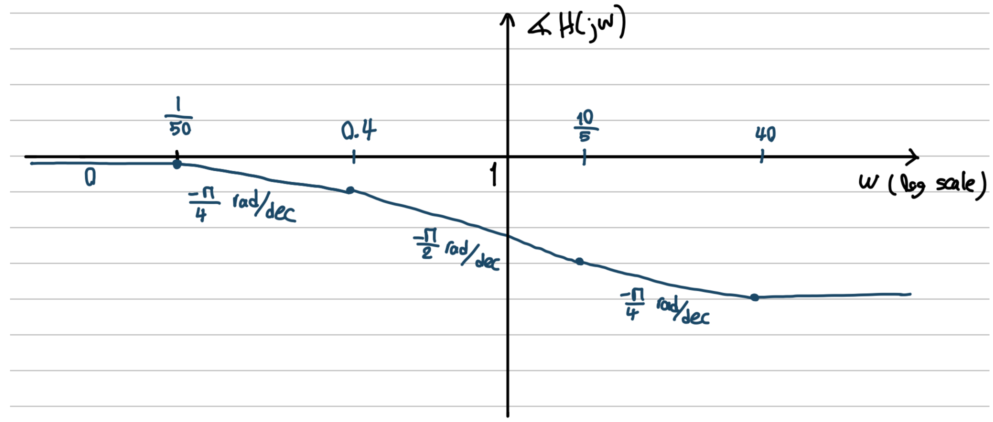
- $H(j\omega) = \frac{1}{(1+5j\omega)(1+0.25j\omega)}$.
  The magnitude in dB is:

$$\begin{aligned} |H(j\omega)|_{\text{dB}} &= -20\log_{10}|1+5j\omega| - 20\log_{10}|1+0.25j\omega| \\ &= -20\log_{10}|1+\frac{j\omega}{1/5}| - 20\log_{10}|1+\frac{j\omega}{4}| \end{aligned}$$



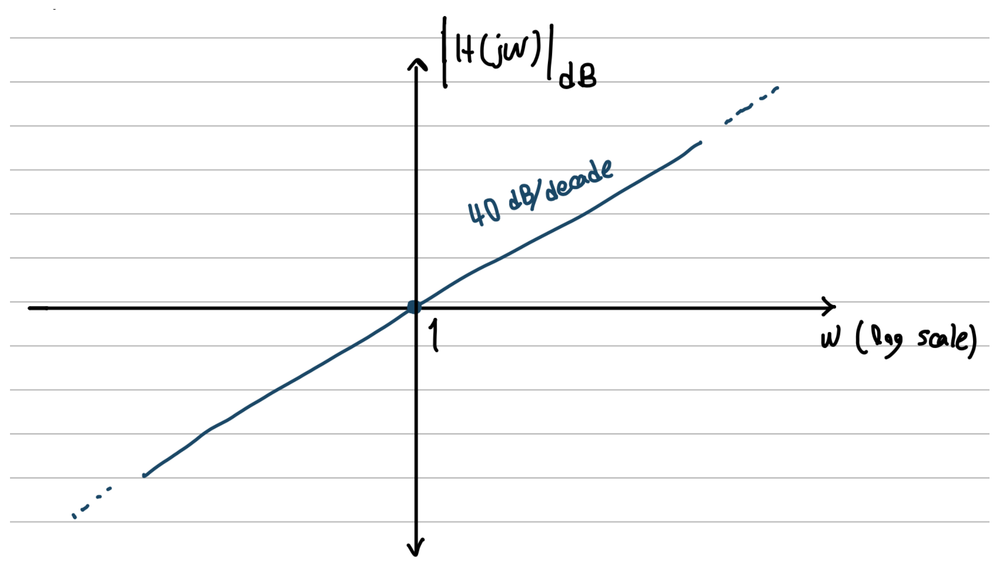
The phase is:

$$\begin{aligned} \angle H(j\omega) &= -\angle(1+5j\omega) - \angle(1+0.25j\omega) \\ &= -\angle(1+\frac{j\omega}{1/5}) - \angle(1+\frac{j\omega}{4}) \end{aligned}$$



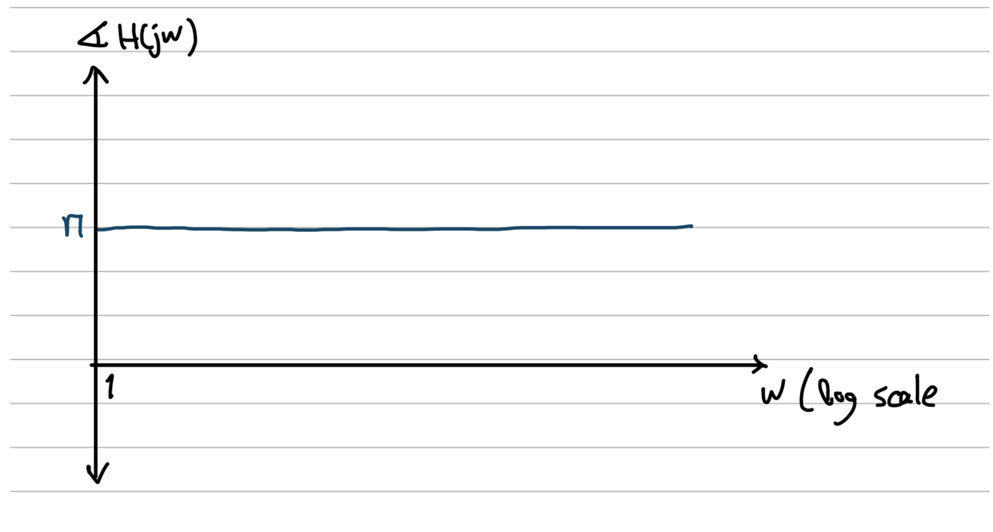
- $H(j\omega) = (j\omega)^2$. The magnitude in dB is:

$$\begin{aligned} |H(j\omega)|_{\text{dB}} &= 20\log_{10}|j\omega|^2 \\ &= 40\log_{10}|\omega| \end{aligned}$$



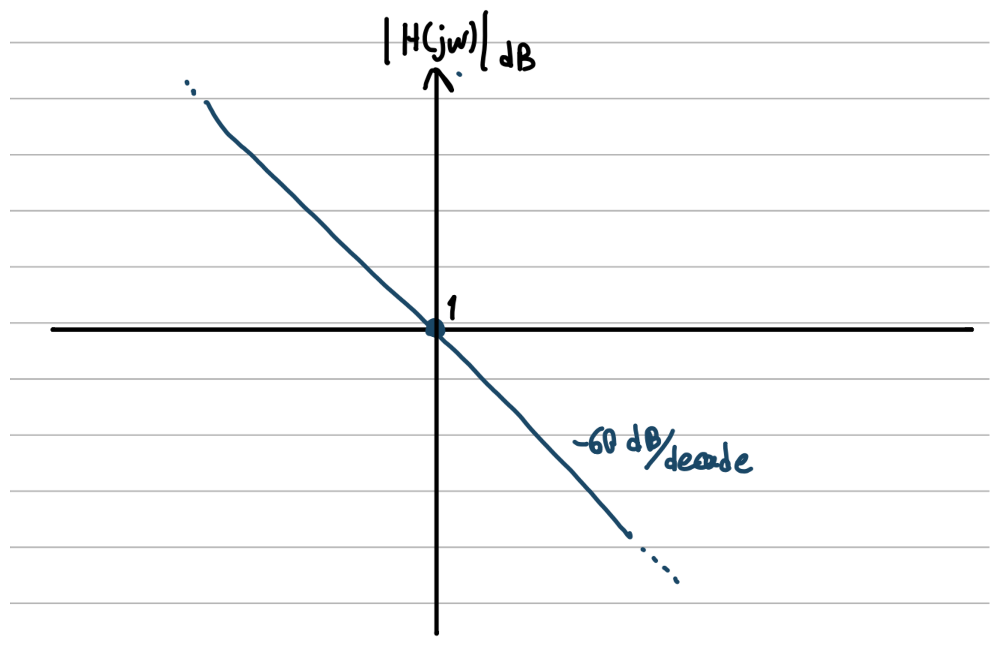
The phase is:

$$\begin{aligned} \angle H(j\omega) &= \angle(j\omega)^2 \\ &= 2\angle(j\omega) \\ &= 2 \times \frac{\pi}{2} = \pi \end{aligned}$$



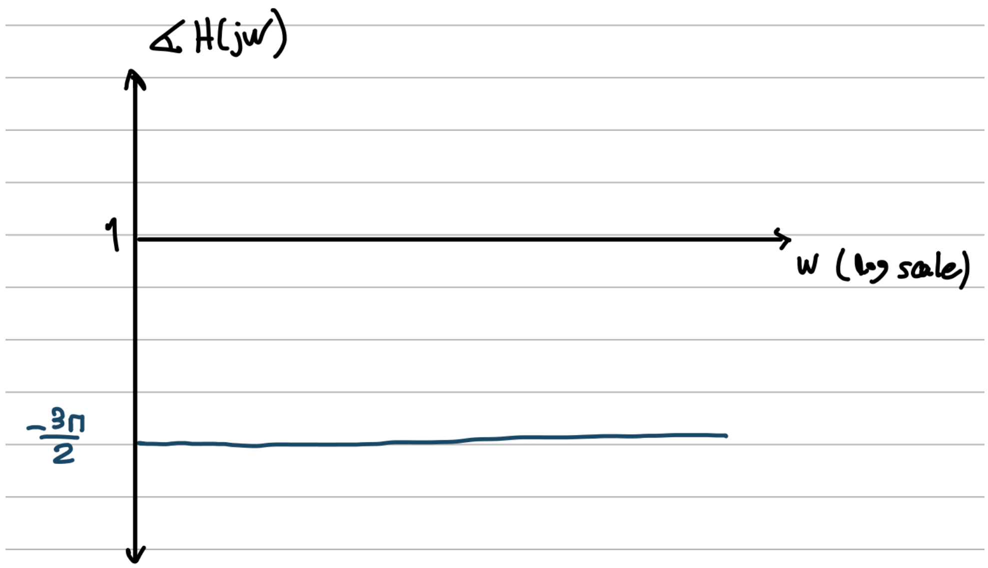
- $H(j\omega) = (j\omega)^{-3}$.
  The magnitude in dB is:

$$\begin{aligned} |H(j\omega)|_{\text{dB}} &= 20\log_{10}|j\omega|^{-3} \\ &= -60\log_{10}|\omega| \end{aligned}$$



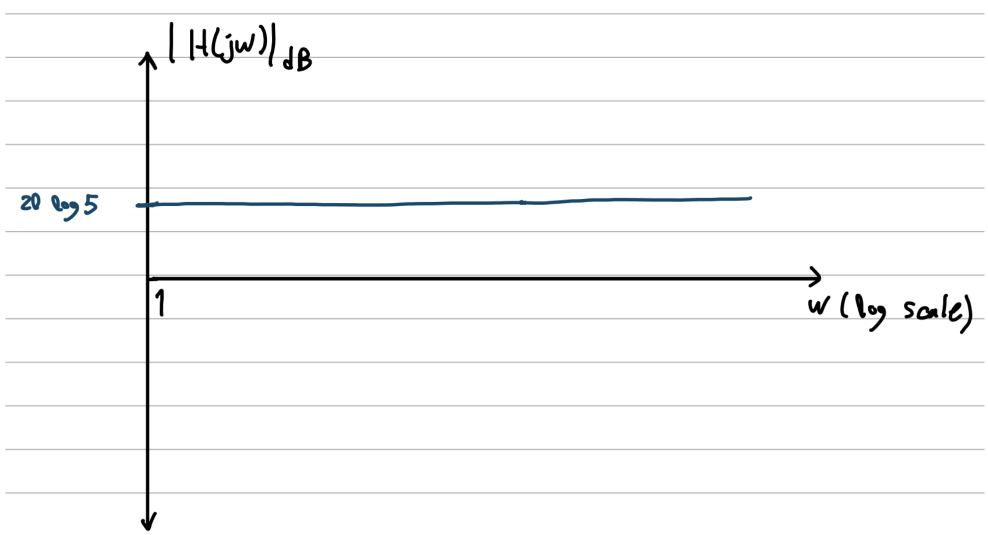
The phase is:

$$\begin{aligned} \angle H(j\omega) &= \angle(j\omega)^{-3} \\ &= -3\angle(j\omega) \\ &= -3 \times \frac{\pi}{2} = -\frac{3\pi}{2} \end{aligned}$$



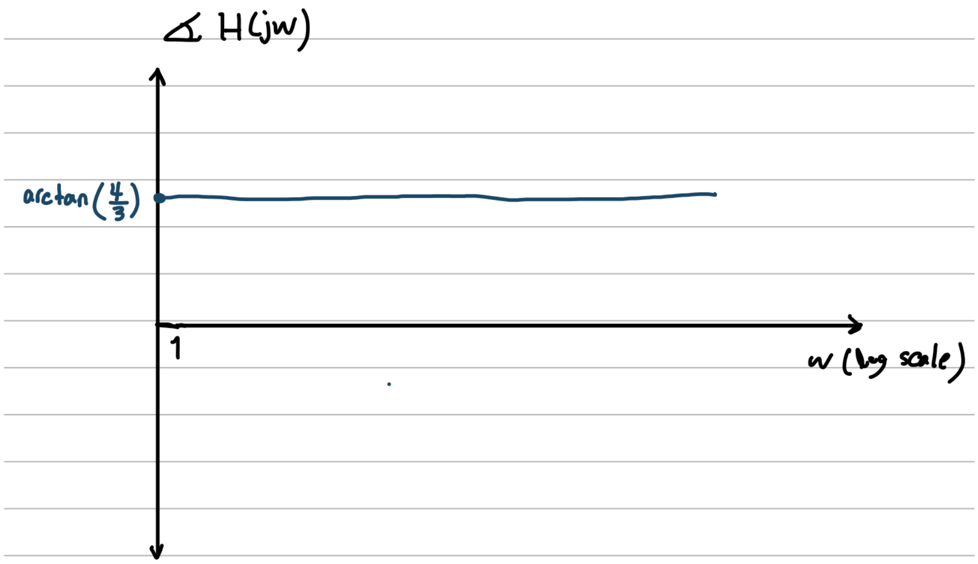
- $H(j\omega) = 3 + 4j$.
  The magnitude in dB is:

$$\begin{aligned}
|H(j\omega)|_{\text{dB}} &= 20\log_{10}|3+4j| \\
&= 20\log_{10}\sqrt{3^2+4^2} \\
&= 20\log_{10}\sqrt{9+16} \\
&= 20\log_{10}\sqrt{25} \\
&= 20\log_{10}5 \\
&= 20 \times 0.699 = 13.98 \text{ dB}
\end{aligned}$$



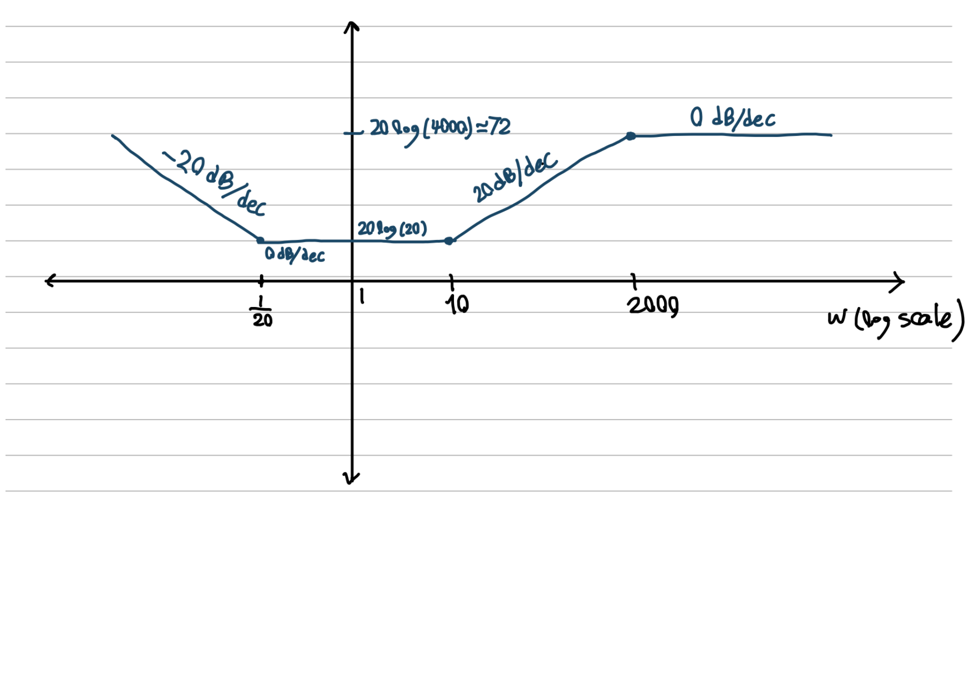
The phase is:

$$\begin{aligned}\angle H(j\omega) &= \tan^{-1}\frac{4}{3}\\ &\approx 53^\circ\end{aligned}$$



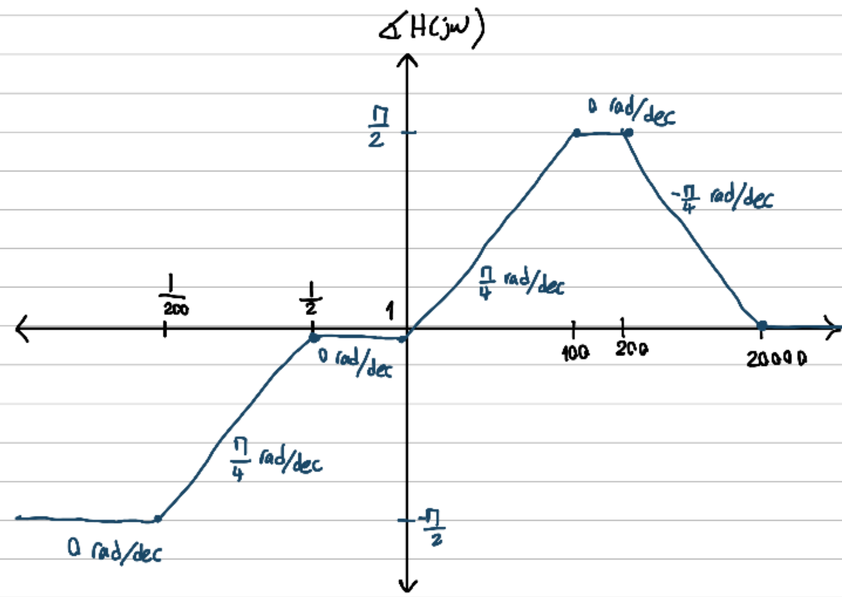
- $H(j\omega) = \frac{1+20.1j\omega+2(j\omega)^2}{j\omega+0.0005(j\omega)^2}$. The magnitude in dB is:

$$\begin{aligned}|H(j\omega)|_{\text{dB}} &= 20\log_{10}\left|1+20.1j\omega+2(j\omega)^2\right| - 20\log_{10}\left|j\omega+0.0005(j\omega)^2\right|\\ &= 20\log_{10}\left|(1+\frac{jw}{10})(1+20jw)\right| - 20\log_{10}\left|(j\omega)(1+0.0005j\omega)\right|\\ &= 20\log_{10}\left|1+\frac{jw}{10}\right| + 20\log_{10}\left|1+\frac{jw}{1/20}\right| - 20\log_{10}|jw| - 20\log_{10}\left|1+\frac{j\omega}{2000}\right|\end{aligned}$$



The phase is:

$$\angle H(j\omega) = \angle\left(1+\frac{j\omega}{10}\right) + \angle\left(1+\frac{j\omega}{1/20}\right) - \angle(j\omega) - \angle\left(1+\frac{j\omega}{2000}\right)$$



4. (20 %) Based on the solutions, correct your previous week's homework using a colored pen (or annotation) so it's obvious what you've corrected. If you got a problem exactly right, just use a red check mark to indicate as such.

Supplemental Problems

These problems will not be graded nor counted for credit, but solutions will be provided. They have been selected for their relevance. Note that the book also has additional problems with answers (but not detailed solutions).

5. Redo the windowing example from Lecture 15 where instead of using $\text{rect}(t/(2T))$ you use $\text{tria}(t/T)$ where the triangle function is defined as $\text{tria}(t) = 1 - |t|$ for $t \in [-1, 1]$ and is zero otherwise. Why would someone use the triangle instead of the rectangular window? Some hints for doing this problem:

   - You need to do this both on paper and on the computer.
   - You need to find the Fourier transform of the triangle. You can find it in various places but you can also remember how you can get the triangle by convolving two rectangle functions together.

   Windowing with different windows (e.g. Bartlett, Hamming, Tukey, etc) is a common part of spectral estimation.

   <u>Solution:</u>

   The windowed cosine becomes $\hat{x}(t) = \cos(\omega_c t)\text{tria}(t/T)$. Let $\hat{x}(t) \overset{\mathcal{F}}{\longleftrightarrow} F(w)$, we have

$$\begin{aligned}
F(w) &= \mathcal{F}\{\cos(\omega_c t)(1/T \cdot \text{rect}(t/T) * \text{rect}(t/T))\} \\
&= (\pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)) * \left(\text{sinc}\left(\frac{T\omega}{2\pi}\right) \cdot T\text{sinc}\left(\frac{T\omega}{2\pi}\right)\right) \\
&= \pi T \text{sinc}^2(T(\omega - \omega_c)/(2\pi)) + \pi T \text{sinc}^2(T(\omega + \omega_c)/(2\pi))
\end{aligned}$$

   Below shows an example with $\omega_c = 8\pi$ and $T = 1$.

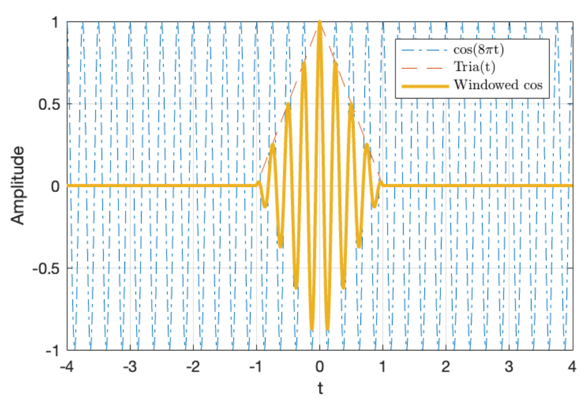


Figure 1: Windowed $\cos(8\pi t)$

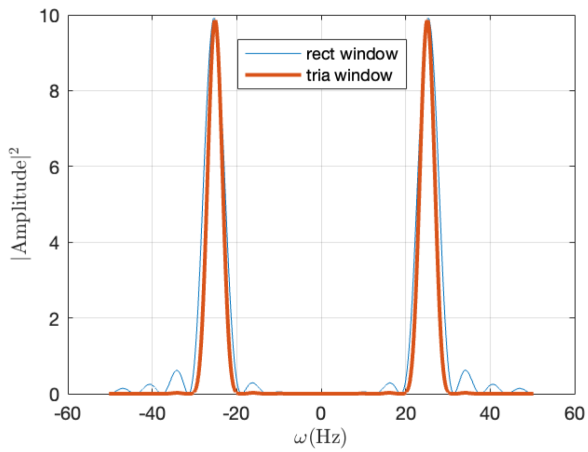
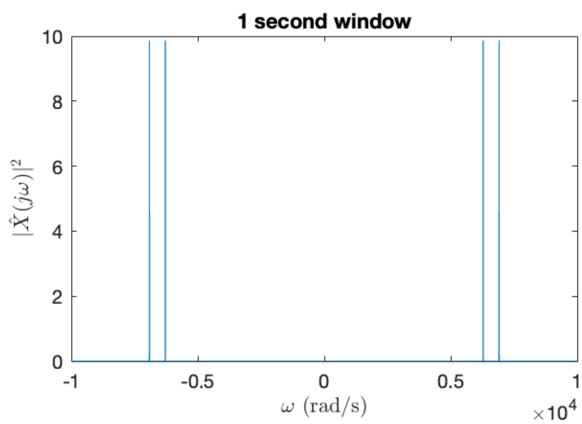
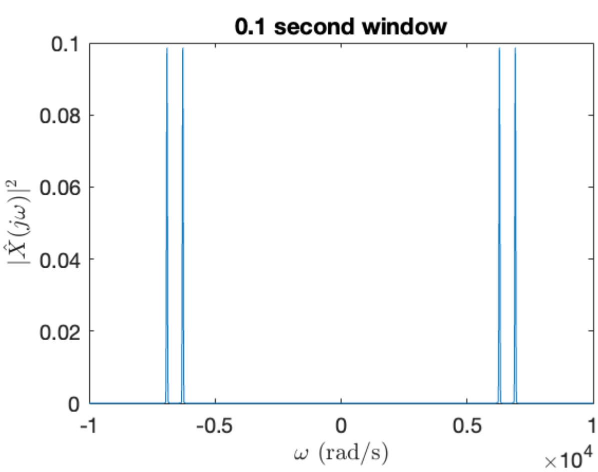
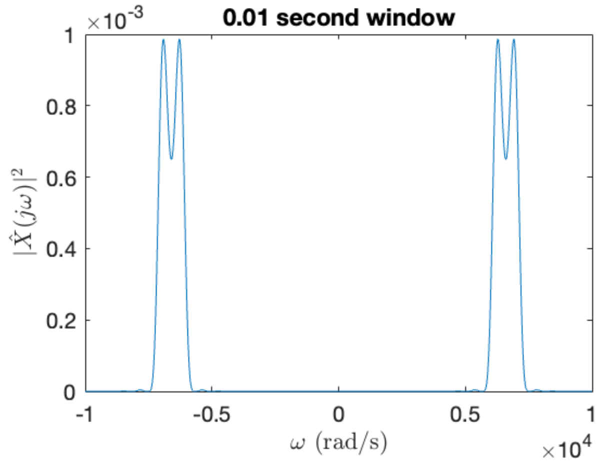


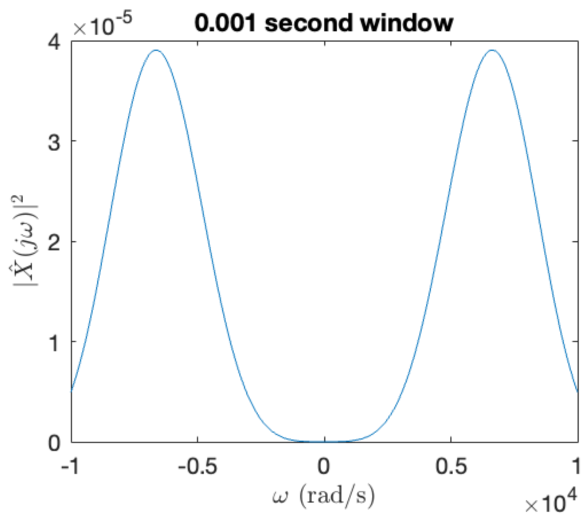
Figure 2: $F(\omega)$

Using the triangle window with different durations for the signal $x(t) = \cos(2\pi 1000t) + \cos(2\pi 1100t)$, we have









One of the reasons the triangle window is used instead of the rectangular one is that it reduces the impact of sidelobes and avoids spectral leakage.