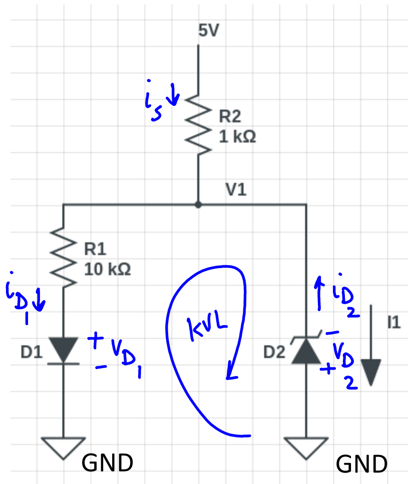
ECE 65 - HW#1 solutions

Problem 1.

In the following circuit find V1 and I1. ($V_Z = 3\ V$ and $V_{D0} = 0.7\ V$)



Assume D_1 is ON and D_2 is in Zener region:

$V_{D_1} = V_{D_0} = 0.7V$ & $V_{D_2} = -V_Z = -3V$ & $i_{D_2} \leq 0$ & $i_{D_1} \geq 0$

KVL: $-V_{D_1} + 10k\Omega \times (-i_{D_1}) - V_{D_2} = 0 \rightarrow -0.7V - 10k\Omega \times i_{D_1} + 3V = 0$

$\rightarrow i_{D_1} = \dfrac{2.3V}{10k\Omega} = 0.23\,mA \geq 0$ ✓

$V1 = -V_{D_2} = +V_Z = 3V$

Ohm's law: $i_s = \dfrac{5V - V1}{1k\Omega} = \dfrac{5V - 3V}{1k\Omega} = 2\,mA$

KCL: $i_s = i_{D_1} - i_{D_2} \rightarrow i_{D_2} = i_{D_1} - i_s = 0.23\,mA - 2\,mA = -1.77\,mA \leq 0$ ✓

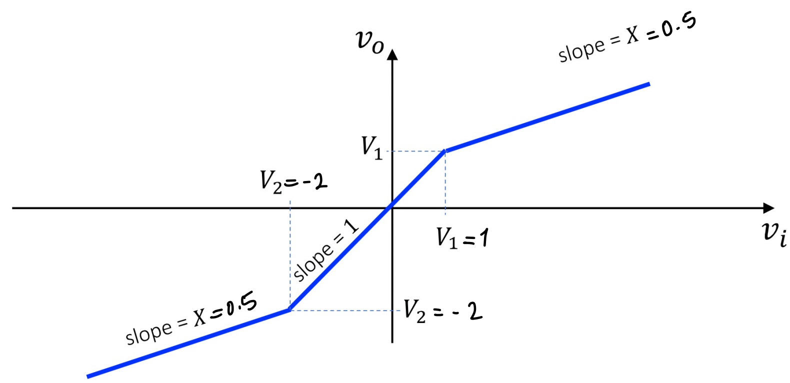
$i_{D_1} \geq 0$ & $i_{D_2} \leq 0 \rightarrow$ assumption was correct.

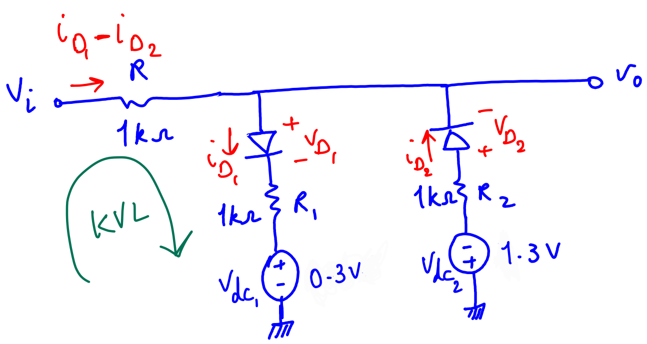
$\boxed{I1 = -i_{D_2} = 1.77\,mA}$, $\boxed{V1 = 3V}$

Problem 2.

Design a diode waveform shaping circuit that would have the below transfer function.

$V_{D_0} = 0.7V$





From the graph:

For $v_i \geq 1V$, $V_o = \frac{1}{2} v_i + \frac{1}{2}$

For $v_i \leq -2V$, $V_o = \frac{1}{2} v_i - 1$

For $-2 \leq v_i \leq 1$, $V_o = v_i$

Case 1: D_1 ON and D_2 off

$\boxed{i_{D_1} \geq 0}$, $V_{D_1} = V_{D_0}$, $i_{D_2} = 0$, $V_{D_2} < V_{D_0}$ Assumed $V_{D_0} = 0.7V$

KVL: $V_i = R\,(i_{D_1} - i_{D_2}) + V_{D_1} + R_1 i_{D_1} + V_{dc_1}$

$V_i = 1k\Omega \times i_{D_1} + 0.7V + 1k\Omega \times i_{D_1} + V_{dc_1}$

$\rightarrow i_{D_1} = \frac{V_i - 0.7V - V_{dc_1}}{2k\Omega} \geq 0 \longrightarrow V_i \geq 0.7V + V_{dc_1}$

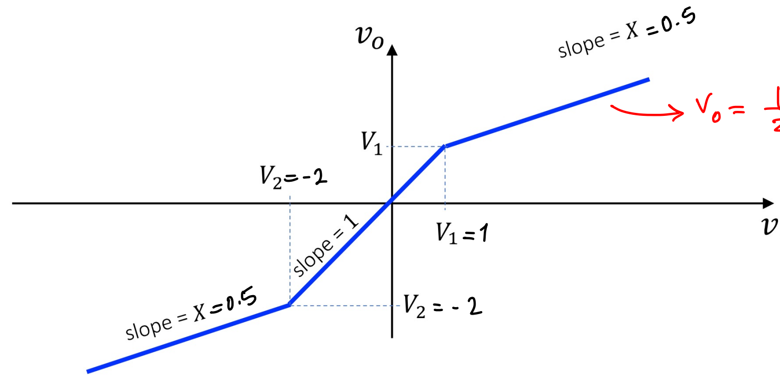
$V_o = V_{D_1} + R_1 i_{D_1} + V_{dc_1} = V_{D_0} + 1k\Omega \times \left(\frac{V_i - 0.7V - V_{dc_1}}{2k\Omega} \right) + V_{dc_1}$

$V_o = \frac{1}{2} V_i + 0.35V + \frac{1}{2} V_{dc_1}$

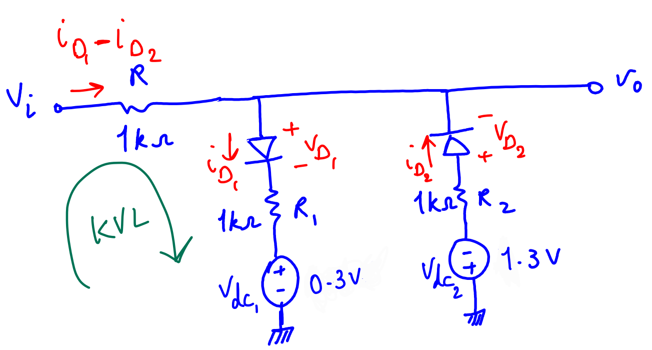
Problem 2.

Design a diode waveform shaping circuit that would have the below transfer function.

$V_{D_0} = 0.7V$



$\rightarrow V_o = \frac{1}{2} V_i + \frac{1}{2}$



From the graph:

$$\begin{cases} \text{For } V_i \geq 1V, & V_o = \frac{1}{2} V_i + \frac{1}{2} \\ \text{For } V_i \leq -2V, & V_o = \frac{1}{2} V_i - 1 \\ \text{For } -2 \leq V_i \leq 1, & V_o = V_i \end{cases}$$

Case 1: D_1 ON and D_2 off

For $V_i \geq 0.7V + V_{dc_1}$, $V_o = \frac{1}{2} V_i + 0.35V + \frac{1}{2} V_{dc_1}$

From the graph:

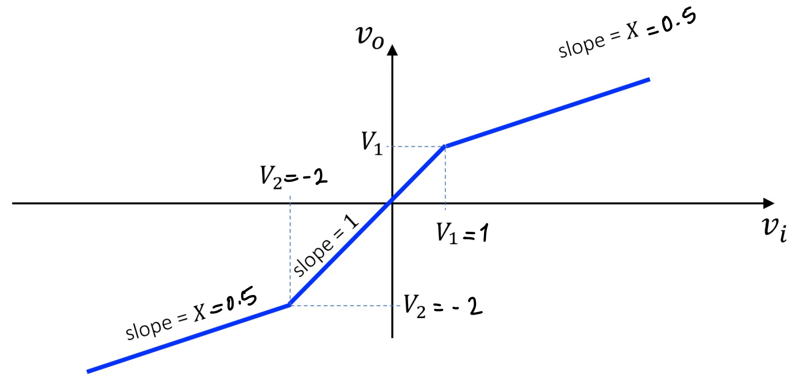
For $V_i \geq 1V$, $V_o = \frac{1}{2} V_i + \frac{1}{2}$

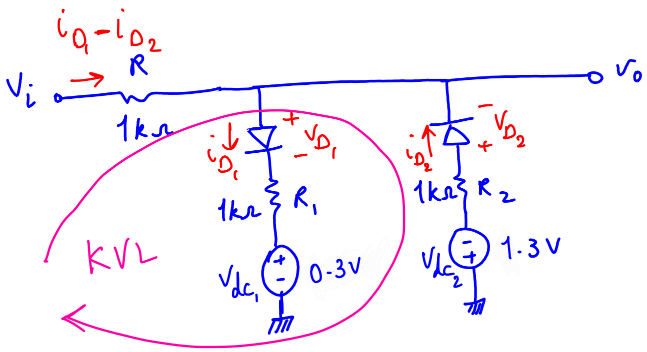
if we choose $V_{dc_1} = 0.3V$ $\longrightarrow$ the response of the circuit will match the transfer function

Problem 2.

Design a diode waveform shaping circuit that would have the below transfer function.

$V_{D_0} = 0.7V$





From the graph:

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Case 2: D_1 off and D_2 ON

$i_{D_2} \geq 0$, $V_{D_2} = V_{D_0}$, $i_{D_1} = 0$, $V_{D_1} < V_{D_0}$ Assumed $V_{D_0} = 0.7V$

KVL: $V_i = R\,(i_{D_1} - i_{D_2}) - V_{D_2} - R_2 i_{D_2} - V_{dc_2}$

$$V_i = -1k\Omega \times i_{D_2} - 0.7V - 1k\Omega \times i_{D_2} - V_{dc_2}$$

$$\rightarrow i_{D_2} = \frac{-V_i - 0.7V - V_{dc_2}}{2k\Omega} \geq 0 \longrightarrow V_i \leq -0.7V - V_{dc_2}$$

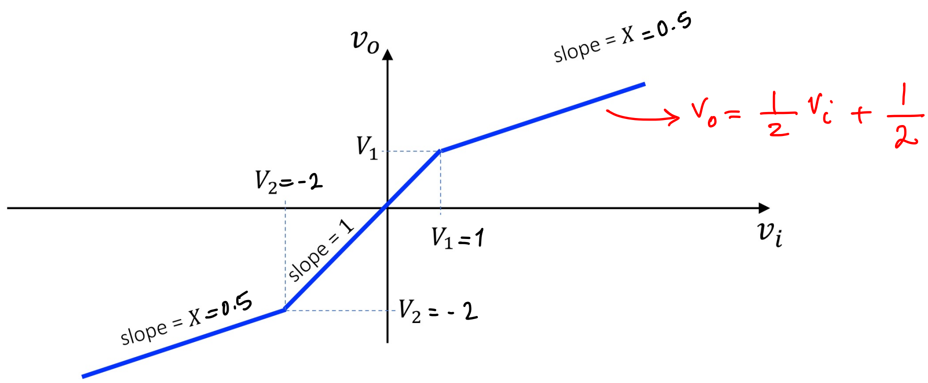
$$V_o = -V_{D_2} - R_2 i_{D_2} - V_{dc_2} = -V_{D_0} - 1k\Omega \times \left(\frac{-V_i - 0.7V - V_{dc_2}}{2k\Omega}\right) - V_{dc_2}$$

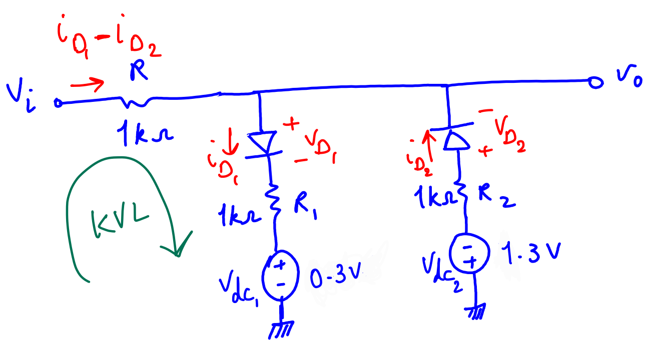
$$V_o = \frac{1}{2}V_i - 0.35V - \frac{1}{2}V_{dc_2}$$

Problem 2.

Design a diode waveform shaping circuit that would have the below transfer function.

$V_{D_0} = 0.7\,V$





From the graph:

$$\begin{cases} \text{For } V_i \geq 1V, & V_o = \frac{1}{2} V_i + \frac{1}{2} \\ \text{For } V_i \leq -2V, & V_o = \frac{1}{2} V_i - 1 \\ \text{For } -2 \leq V_i \leq 1, & V_o = V_i \end{cases}$$

Case 2: D_1 off and D_2 ON

For $V_i \leq -0.7V - V_{dc_2}$, $V_o = \frac{1}{2} V_i - 0.35V - \frac{1}{2} V_{dc_2}$

From the graph:

For $V_i \leq -2V$, $V_o = \frac{1}{2} V_i - 1$

if we choose $V_{dc_2} = 1.3V$ $\longrightarrow$ the response of the circuit will match the transfer function

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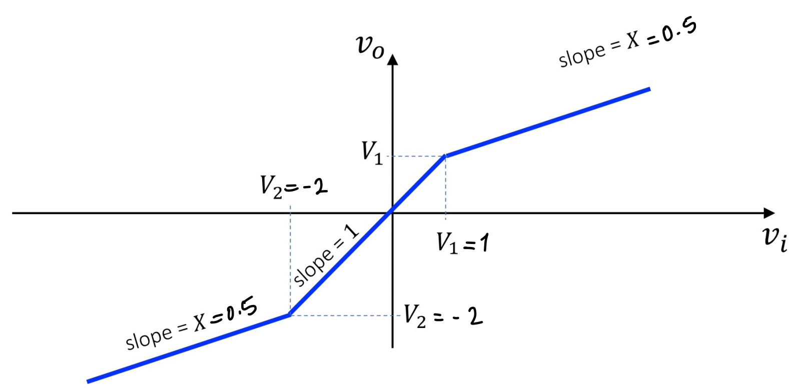
Design a diode waveform shaping circuit that would have the below transfer function.

$V_{D_0} = 0.7\,V$



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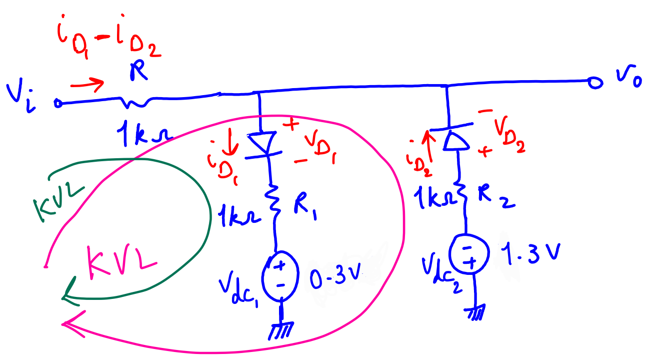


Case 3 : D_1 off and D_2 off

$i_{D_2} = 0$, $V_{D_2} < V_{D_0}$, $i_{D_1} = 0$, $V_{D_1} < V_{D_0}$ Assumed $V_{D_0} = 0.7\,V$

KVL: $V_i = R\,(i_{D_1} - i_{D_2}) - V_{D_2} - R_2 i_{D_2} - V_{dc_2}$

$V_i = -V_{D_2} - 1.3\,V \longrightarrow V_{D_2} = -V_i - 1.3V$, $V_{D_2} < 0.7 \longrightarrow V_i > -2\,V$

KVL: $V_i = R\,(i_{D_1} - i_{D_2}) + V_{D_1} + R_1 i_{D_1} + V_{dc_1}$

$V_i = V_{D_1} + 0.3V \longrightarrow V_{D_1} = V_i - 0.3\,V$, $V_{D_1} < 0.7\,V \longrightarrow V_i < 1\,V$

$-2V < V_i < 1V$

$V_0 = -R\,(i_{D_1} - i_{D_2}) + V_i \longrightarrow V_o = V_i$

Problem 3.

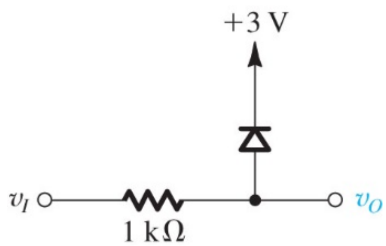
The circuits (a) and (b) shown below are connected as follows:

The two input terminals are tied together, and the two output terminals are tied together.

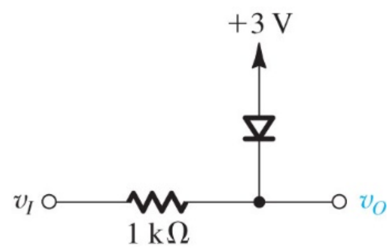
a) Find, sketch and label the transfer function. (Assume $V_{D0} = 0.7\ V$).

b) Draw the output voltage waveform if $v_i = 5\sin(\omega t)$. No need for labeling the time axis.

Show your work.

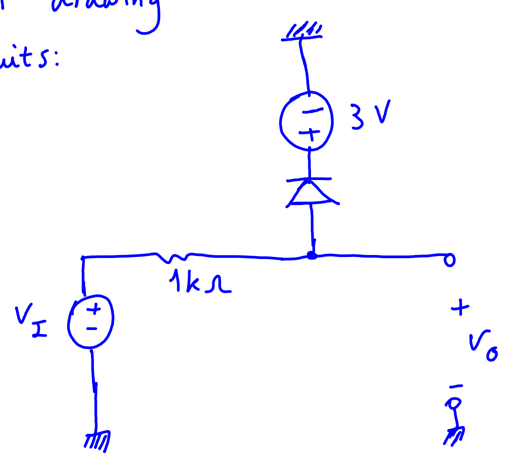


(a)



(b)

other ways of drawing the same circuits:



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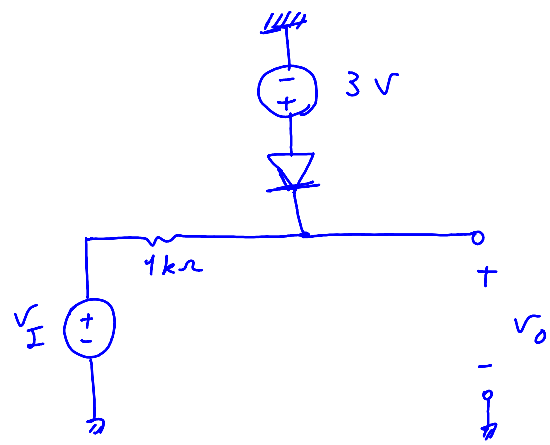
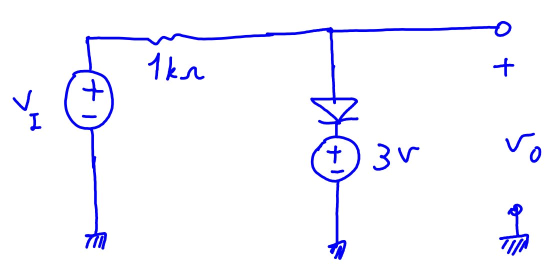


(b)



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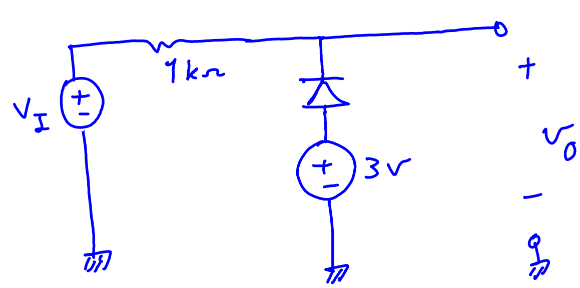
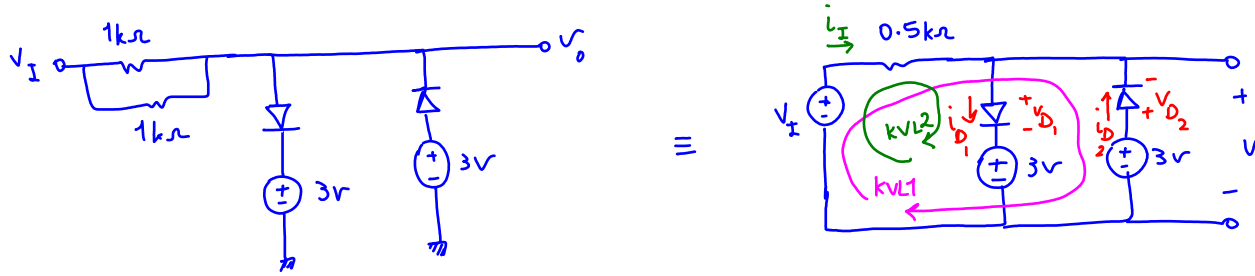


(b)

part a)



Case 1: D_1 is OFF and D_2 is ON

$i_{D_1} = 0$ & $v_{D_1} < 0.7V$, $i_{D_2} \geq 0$ & $v_{D_2} = 0.7V$

KCL : $i_I = i_{D_1} - i_{D_2}$ $\longrightarrow$ for this case : $i_I = -i_{D_2}$

KVL1 : $-V_I + 0.5\,k\Omega \times (-i_{D_2}) - V_{D_2} + 3V = 0 \longrightarrow i_{D_2} = \dfrac{-V_I - 0.7V + 3V}{0.5k\Omega} = \dfrac{-V_I + 2.3V}{0.5k\Omega}$

$i_{D_2} \geq 0 \longrightarrow \boxed{V_I \leq 2.3V}$

$V_0 = -V_{D_2} + 3V = -0.7V + 3V \longrightarrow \boxed{V_0 = 2.3V}$

Case 2: D_1 is ON and D_2 is OFF

$i_{D_1} \geq 0$ & $v_{D_1} = 0.7V$, $i_{D_2} = 0$ & $v_{D_2} < 0.7V$

KVL 2 : $-V_I + 0.5\,k\Omega \times i_{D_1} + V_{D_1} + 3V = 0 \longrightarrow i_{D_1} = \dfrac{V_I - 0.7V - 3V}{0.5k\Omega}$

$i_{D_1} \geq 0 \longrightarrow \boxed{V_I \geq 3.7V}$

$V_0 = +V_{D_1} + 3V = 0.7V + 3V \longrightarrow \boxed{V_0 = 3.7V}$

Case 3: D_1 and D_2 are both OFF:

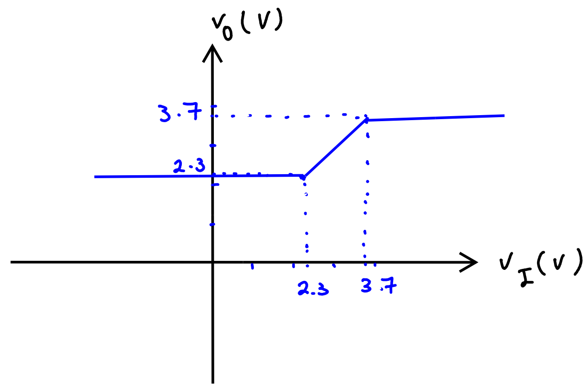
$i_{D_1} = i_{D_2} = 0$, $V_{D_1} < 0.7V$ & $V_{D_2} < 0.7V$

KVL1: $-V_I + 0.5\,k\Omega \times 0 - V_{D_2} + 3V = 0 \rightarrow V_{D_2} = -V_I + 3V < 0.7V \rightarrow$ $\boxed{V_I > 2.3V \ \& \ V_I < 3.7V}$

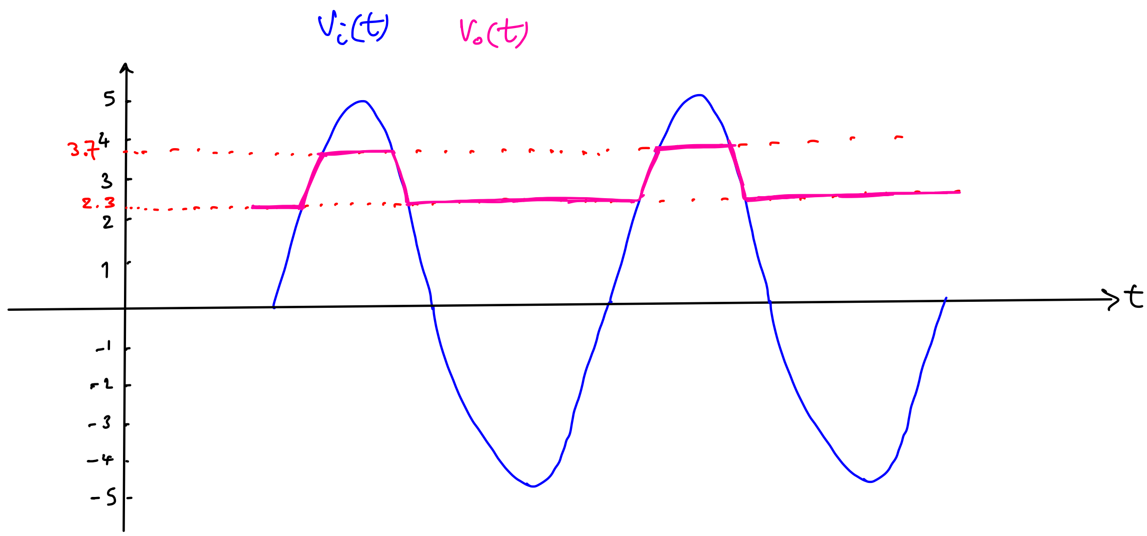
KVL2: $-V_I + 0.5\,k\Omega \times 0 + V_{D_1} + 3V = 0 \rightarrow V_{D_1} = V_I - 3V < 0.7V \rightarrow$

$\Rightarrow$ $\boxed{2.3V < V_I < 3.7V}$

$V_0 = 0.5\,k\Omega \times 0 + V_I \Rightarrow \boxed{V_0 = V_I}$

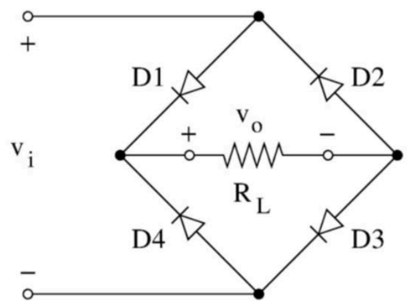


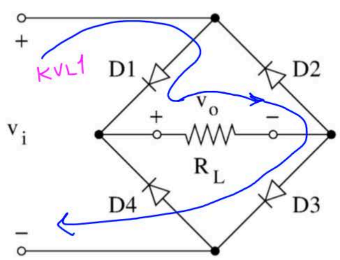
b) $V_i(t) = 5 \sin \omega t$



Problem 4.

Plot the transfer function of the following full–wave rectifier. Find vo for different ranges of vi and plot a graph that shows the relationship between vo and vi.





Case 1: D_1 and D_3 ON, D_2 and D_4 off

$i_{D_1} \geq 0$ & $V_{D_1} = V_{D_0}$ & $i_{D_3} \geq 0$ & $V_{D_3} = V_{D_0}$

$i_{D_2} = 0$ & $V_{D_2} < V_{D_0}$ & $i_{D_4} = 0$ & $V_{D_4} < V_{D_0}$

KVL1: $-V_i + V_{D_1} + R\, i_{D_1} + V_{D_3} = 0$

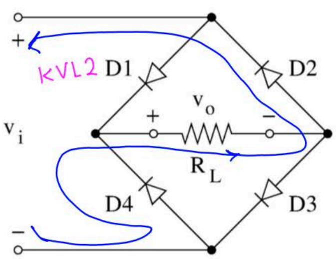
$\rightarrow -V_i + V_{D_0} + R\, i_{D_1} + V_{D_0} = 0 \rightarrow i_{D_1} = \dfrac{V_i - 2V_{D_0}}{R} \geq 0$

$\rightarrow \boxed{V_i \geq 2V_{D_0}}$

$V_0 = R \times i_{D_1} = R \times \dfrac{V_i - 2V_{D_0}}{R} \Rightarrow \boxed{V_0 = V_i - 2V_{D_0}}$

For $V_i \geq 2V_{D_0}$

$V_0 = V_i - 2V_{D_0}$



Case 2: D_2 and D_4 ON, D_1 and D_3 off

$i_{D_1}=0$ & $i_{D_3}=0$ & $v_{D_1} < V_{D_0}$ and $v_{D_3} < V_{D_0}$

$v_{D_2} = V_{D_0}$ & $v_{D_4} = V_{D_0}$ & $i_{D_2} \geq 0$ & $i_{D_4} \geq 0$

KVL 2: $v_{D_4} + R \times i_{D_4} + v_{D_2} + v_i = 0$

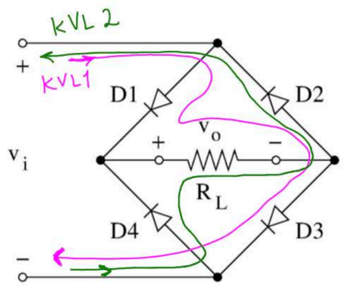
$\rightarrow v_i + 2V_{D_0} + R\, i_{D_4} = 0 \rightarrow i_{D_4} = \dfrac{-v_i - 2V_{D_0}}{R} \geq 0$

$\rightarrow \boxed{v_i \leq -2V_{D_0}}$

$v_0 = R \times i_{D_4} = R \times \dfrac{-v_i - 2V_{D_0}}{R} \Rightarrow \boxed{v_0 = -v_i - 2V_{D_0}}$

For $v_i \leq -2V_{D_0}$

$v_0 = -v_i - 2V_{D_0}$



Case 3: all diodes OFF

$i_{D_1} = i_{D_2} = i_{D_3} = i_{D_4} = 0$, $v_{D_1} < V_{D_0}$ & $v_2 < V_{D_0}$ &

$v_{D_3} < V_{D_0}$ & $v_{D_4} < V_{D_0}$

KVL1: $-v_i + v_{D_1} + R\, i_{D_1} + v_{D_3} = 0 \rightarrow v_i = v_{D_3} + v_{D_4}$

$v_{D_3} < V_{D_0}$

$v_{D_1} < V_{D_0}$

$v_{D_3} + v_{D_1} < 2V_{D_0} \rightarrow \boxed{v_i < 2V_{D_0}}$

KVL 2: $v_{D_4} + i_{D_4} \times R + v_{D_2} + v_i = 0 \rightarrow -v_i = v_{D_2} + v_{D_4}$

$v_0 = R\, i_{D_1} = 0 \rightarrow \boxed{v_0 = 0}$

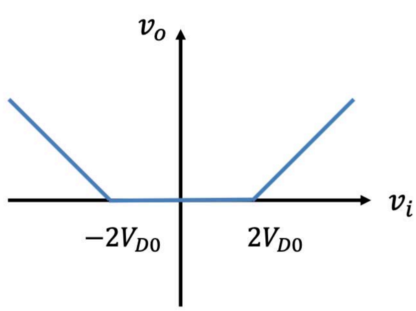
$v_{D_2} < V_{D_0}$

$v_{D_4} < V_{D_0}$

$v_{D_2} + v_{D_4} < 2V_{D_0} \rightarrow -v_i < 2V_{D_0} \rightarrow \boxed{v_i > -2V_{D_0}}$

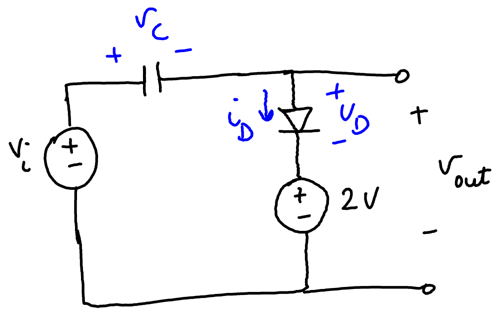
For $-2V_{D_0} < v_i < 2V_{D_0}$

$v_0 = 0$



Problem 5.

In the circuit below, $v_i(t) = 10\sin(\omega t)$ where $\omega = 1000\, rad/s$, Assume $V_{D0} = 0.7\,V$ and $v_o(0) = 0\,V$. Calculate and plot $v_o(t)$ for $0 \le t \le 5\,ms$.



$V_o(0) = V_i(0) - V_C(0) \longrightarrow V_C(0) = 0$

when $0 < V_i < 2.7V$, the diode will be off. $i_D = 0$, $V_C = 0$, $V_o = V_i$

- when $2.7\,V \leqslant V_i \leqslant 10\,V$

diode is ON. $V_D = V_{D_0} = 0.7V \longrightarrow V_o = V_D + 2V = 2.7\,V$

- after v_i passed it's first positive peak, the diode will turn off.

the voltage across the capacitor will stay at a constant value.

$V_C = V_{i_{Peak}} - 2 - 0.7 = 10 - 2.7 = 7.3\,V$

Since the diode is off, $V_D \neq V_{D_0}$ and V_o can be calculated using

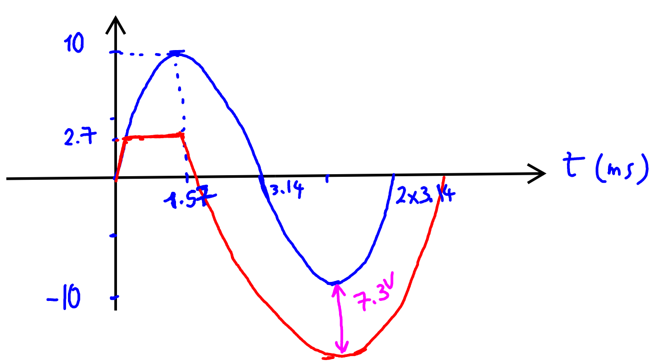
$V_o = -V_C + V_i = V_i - 7.3V$

$V_i(t) = 10\sin(1000t)$

$\omega = 1000$ rad/s

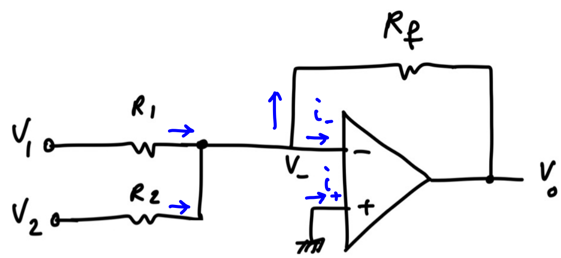
$T = \frac{2\pi}{\omega} = 2 \times 3.14$ ms

$v_i(t)$, $V_o(t)$



Problem 6.

Design the following op-amp circuit to achieve the weighted sum of v_1 (input 1), and v_1 (input 2) at the output. It is required that $v_o = -(v_1 + 5v_2)$. Assume an ideal op-amp.



ideal op-amp : $i_+ = i_- = 0$

negative feedback: $V_+ = V_-$

KCL at the inverting input terminal:

$$\frac{V_1 - V_-}{R_1} + \frac{V_2 - V_-}{R_2} = \frac{V_- - V_0}{R_f} + i_-$$

$$V_- = V_+ = 0$$

$$\rightarrow \frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} = \frac{0 - V_0}{R_f} \Rightarrow V_0 = \left(\frac{-R_f}{R_1}\right) V_1 + \left(\frac{-R_f}{R_2}\right) V_2$$

$$V_0 = -V_1 - 5V_2 \Rightarrow \frac{R_f}{R_1} = 1 \quad \text{and} \quad \frac{R_f}{R_2} = 5$$

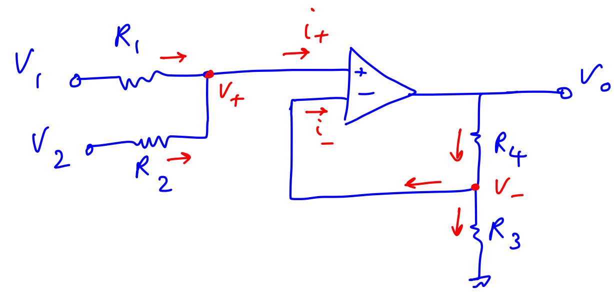
We can choose $R_f = 5\,k\Omega$ $R_1 = 5\,k\Omega$ $R_2 = 1\,k\Omega$

Problem 7.

Using the structure of a non-inverting amplifier, design an op-amp circuit to achieve

$v_o = 6v_1 + 4v_2$

Assume an ideal op-amp.



$$\begin{cases} \text{ideal op-amp} : i_+ = i_- = 0 \\ \text{negative feedback}: V_+ = V_- \end{cases}$$

KCL at the inverting input terminal: $\frac{V_o - V_-}{R_4} - \frac{V_-}{R_3} = 0$

$$\frac{V_o}{R_4} = \left(\frac{1}{R_4} + \frac{1}{R_3}\right) V_- \Longrightarrow V_o = R_4\left(\frac{R_3 + R_4}{R_3 R_4}\right) V_- \Longrightarrow V_o = \left(\frac{R_3 + R_4}{R_3}\right) V_-$$

or $\quad V_- = \left(\frac{R_3}{R_3 + R_4}\right) V_o$

KCL at the non-inverting input terminal: $\frac{V_1 - V_+}{R_1} + \frac{V_2 - V_+}{R_2} = 0$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_+ = \left(\frac{R_1 + R_2}{R_1 R_2}\right) V_+ \Rightarrow V_+ = \left(\frac{R_2}{R_1 + R_2}\right) V_1 + \left(\frac{R_1}{R_1 + R_2}\right) V_2$$

negative feedback: $V_+ = V_-$

$$\begin{cases} V_- = \left(\frac{R_3}{R_3 + R_4}\right) V_o \\ V_+ = \left(\frac{R_2}{R_1 + R_2}\right) V_1 + \left(\frac{R_1}{R_1 + R_2}\right) V_2 \end{cases}$$

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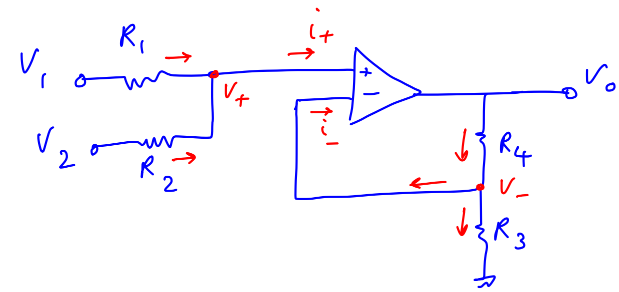

$$\begin{cases} V_- = \left(\dfrac{R_3}{R_3 + R_4}\right) V_o \\ V_+ = \left(\dfrac{R_2}{R_1 + R_2}\right) V_1 + \left(\dfrac{R_1}{R_1 + R_2}\right) V_2 \end{cases}$$

negative feedback: $V_+ = V_-$

$$\Longrightarrow \left(\frac{R_3}{R_3 + R_4}\right) V_o = \left(\frac{R_2}{R_1 + R_2}\right) V_1 + \left(\frac{R_1}{R_1 + R_2}\right) V_2$$

$$\Longrightarrow V_o = \left(\frac{R_3 + R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) V_1 + \left(\frac{R_3 + R_4}{R_3}\right)\left(\frac{R_1}{R_1 + R_2}\right) V_2$$