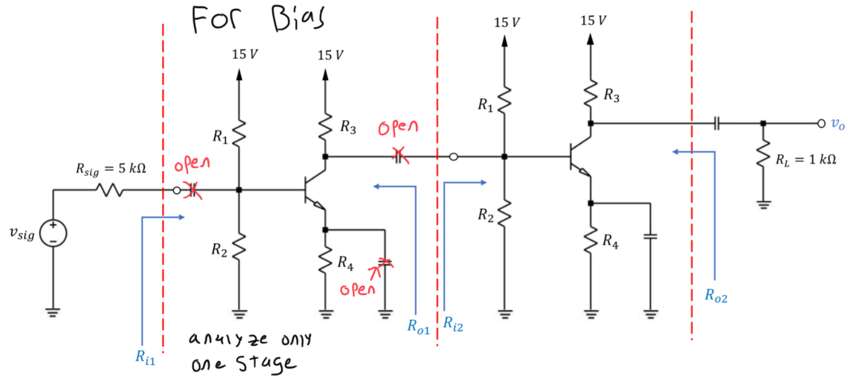
UC San Diego, ECE 65

Spring 2021 Midterm 3 Solutions

We first note we are working with two cascaded amplifiers with identical components. Therefore, for bias analysis we may analyze only one stage and apply conclusions to both.



Moreover, being that the BJTs function as amplifiers, both will be operating in the active region, so we will follow the active region definitions: $V_{BE} = V_{D0}$; $V_{CE} \geq V_{D0}$; $I_C = \beta I_B$; $I_B \geq 0$.

We can start by calculating the currents in the base and emitter:

$$I_C = \beta I_B \implies I_B = \frac{I_C}{\beta} \implies I_B = 10\mu A$$

$$I_E = I_C + I_B \implies I_E = 1.01mA$$

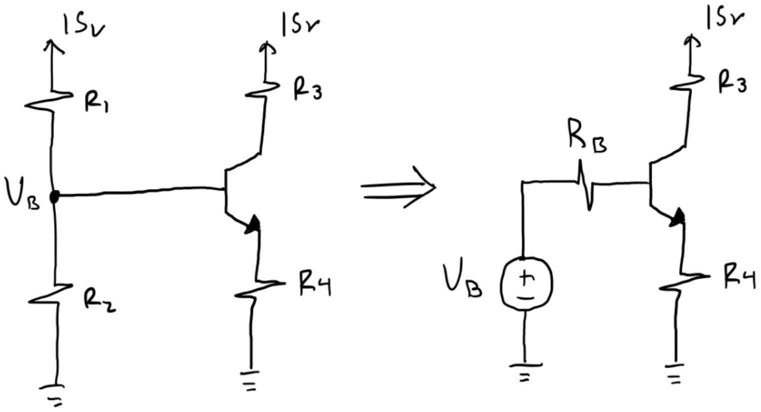
With these currents, we can easily calculate our small signal parameters:

$$r_\pi = \frac{V_t}{I_B} = \frac{25mV}{10\mu A} \implies r_\pi = 2.5k\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100V}{1mA} \implies r_o = 100k\Omega$$

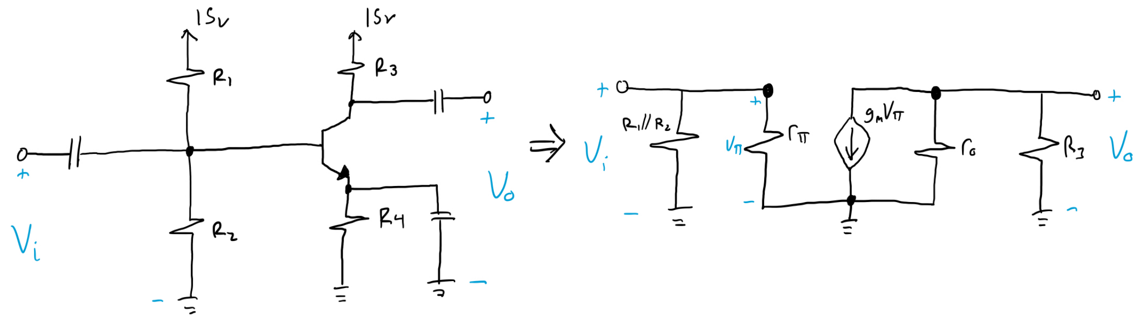
$$g_m = \frac{I_C}{V_t} = \frac{1mA}{25mV} \implies g_m = 0.04\frac{A}{V}$$

To simplify our analysis, we will find the Thevenin equivalent model for the base.



We note that the voltage value at the base is our Thevenin source value($V_{th} = V_B$), and our Thevenin resistor will be R_1 and R_2 in parallel ($R_B = R_1 /\!/ R_2$).

We note our restriction on input resistance for a single stage, $R_i = 2k\Omega$. We see that we have a common emitter amplifier since the signal input goes in to the base and the output comes out of the collector. Moreover, we take this to not have a resistor along the emitter path in the small signal model, as there is a capacitor parallel to the resistor at the emitter.



We conclude we have a common emitter WITHOUT emitter resistor, so we may conclude our input resistance,R_i, is as follows:

$$R_i = R_B /\!/ r_\pi$$

From this, we can extract a value for R_B.

$$R_i = \frac{R_B \times r_\pi}{R_B + r_\pi} \implies R_i R_B + R_i r_\pi = R_B r_\pi \implies R_i r_\pi = R_B(r_\pi - R_i) \implies R_B = \frac{R_i r_\pi}{r_\pi - R_i}$$

$$R_B = \frac{2k\Omega \times 2.5k\Omega}{2.5k\Omega - 2k\Omega} \implies R_B = 10k\Omega$$

As noted before, this base resistor value is simply R1 and R2 in parallel, so we may apply some values to R1 and R2. For simplicity, let us say they are equal.

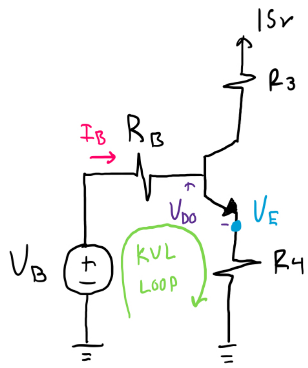
$$R_1 = R_2 = R \implies R_B = \frac{R^2}{2R} \implies R = 2R_B$$

$$R_1 = R_2 = 20k\Omega$$

We can now easily find the voltage at the base, which is equal to the Thevenin voltage source at the base.

$$V_B = 15V(\frac{R_2}{R_1 + R_2}) = 7.5V$$

We then analyze the KVL loop from the base to the emitter to find the voltage at the emitter node, V_E.



$$7.5V = I_B \times R_B + V_{D0} + V_E \implies V_E = 6.7V$$

With the emitter voltage, R_4 is a simple Ohm's law calculation.

$$V_E = I_E \times R_4 \implies R_4 = \frac{V_E}{I_E} = \frac{6.7V}{1.01mA} \implies R_4 = 6.63k\Omega$$

In this iteration, since $V_E \geq 5V$, then automatically $V_C \geq 5V$. Moreover, we must also take in to account for these transistors to operate as amplifiers they must operate in the active region. Therefore,

$$V_{CE} \geq V_{D0} \implies V_C - V_E \geq V_{D0} \implies V_C \geq V_{D0} + V_E = 7.4V$$

We can use an Ohm's law calculation to find the range of values the collector resistor may take:

$$15V - V_C \geq I_C R_3 \implies R_3 \leq \frac{7.6V}{1mA} \implies R_3 \leq 7.6k\Omega$$

Let's keep these transistors operating well within the active region, so let's give it a value of $3k\Omega$.

$$R_3 = 3k\Omega \implies V_C = 15V - 1mA \times 3k\Omega \implies V_C = 12V$$

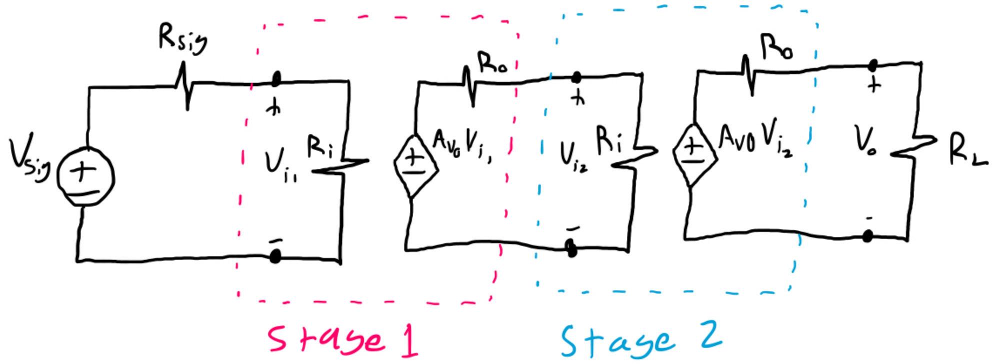
Having found R_3, we can easily calculate output resistance, R_o, and open loop gain, A_{vo}:

$$R_o = R_C /\!/ r_o = R_3 /\!/ r_o \implies R_o = 2.91k\Omega$$

$$A_{vo} = -g_m(R_C /\!/ r_o) = -g_m(R_3 /\!/ r_o) \implies A_{vo} = -116.5\frac{V}{V}$$

To find the overall gain of our cascaded amplifiers, we have two options.

METHOD 1: draw and analyze the amplifier voltage models.



Working from right to left, we first make a relationship between v_o and v_{i2} using the voltage divider technique.

$$v_o = A_{vo} v_{i2} (\frac{R_L}{R_o + R_L})$$

We repeat this for the relationship between V_{i2} and V_{i1}, as well as the relationship between V_{i1} and V_{sig}.

$$v_{i2} = A_{vo} v_{i1} (\frac{R_{i2}}{R_o + R_{i2}})$$

$$v_{i1} = v_{sig}(\frac{R_{i1}}{R_{sig} + R_{i1}})$$

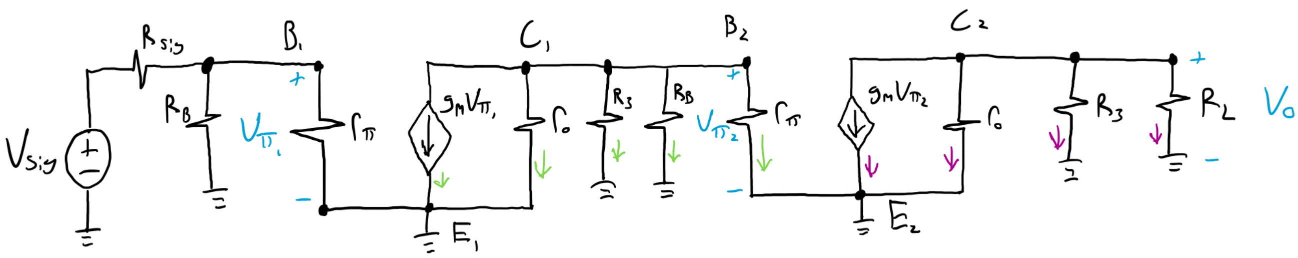
Putting this all together, we get the following expression:

$$v_o = A_{vo}^2 v_{sig}(\frac{R_{i1}}{R_{sig} + R_{i1}})(\frac{R_{i2}}{R_o + R_{i2}})(\frac{R_L}{R_o + R_L})$$

$$\implies A = \frac{v_o}{v_{sig}} = A_{vo}^2(\frac{R_{i1}}{R_{sig} + R_{i1}})(\frac{R_{i2}}{R_o + R_{i2}})(\frac{R_L}{R_o + R_L})$$

$$A = 403.5\frac{V}{V}$$

Method 2: Draw and analyze small signal model.



The goal is to find an expression for $\frac{v_o}{v_{sig}}$. We can break this up by finding expressions for $\frac{v_{c1}}{v_{sig}}$ and $\frac{v_{c2}}{v_{c1}}$ and taking their product, noting specifically $v_{c2} = v_o$.

Starting with $\frac{v_{c1}}{v_{sig}}$, we note we can express $v_{\pi 1}$ as a function of v_{sig} as follows:

$$v_{\pi 1} = v_{sig}(\frac{R_B /\!/ r_\pi}{R_{sig} + R_B /\!/ r_\pi}) \implies v_\pi = \frac{2}{7}v_{sig}$$

We can then perform node analysis on C_1 (green arrows) to find $\frac{v_{c1}}{v_{sig}}$.

$$\frac{v_{c1}}{r_o} + \frac{v_{c1}}{R_3} + \frac{v_{c1}}{R_B} + \frac{v_{c1}}{r_\pi} + g_m v_\pi = 0$$

$$\implies v_{c1}(\frac{1}{r_o} + \frac{1}{R_3} + \frac{1}{R_B} + \frac{1}{r_\pi}) = -g_m(\frac{2}{7}v_{sig})$$

$$\implies \frac{v_{c1}}{v_{sig}} = -\frac{2}{7}g_m(r_o /\!/ R_3 /\!/ R_B /\!/ r_\pi) = -13.55\frac{V}{V}$$

Next, to find $\frac{v_{c2}}{v_{c1}}$, we first note $v_{c1} = v_{\pi 2}$, and then perform node analysis at C_2 (magenta arrows).

$$\frac{v_{c2}}{r_o} + \frac{v_{c2}}{R_3} + \frac{v_{c2}}{R_L} + g_m v_{\pi 2} = 0$$

$$\implies v_{c2}(\frac{1}{r_o} + \frac{1}{R_3} + \frac{1}{R_L}) = -g_m v_{c1}$$

$$\implies \frac{v_{c2}}{v_{c1}} = -g_m(r_o /\!/ R_3 /\!/ R_L) = -29.78\frac{V}{V}$$

We then take the product to find the gain of the system.

$$A = \frac{v_o}{v_{sig}} = \frac{vc2}{v_{c1}}\frac{v_{c1}}{v_{sig}} = -13.55\frac{V}{V} \times -29.78\frac{V}{V} = 403.5\frac{V}{V}$$