UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2018
Linear Systems Fundamentals

MIDTERM EXAM

You are allowed one 2-sided sheet of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME _____

Signature _____

Your signature confirms that you have completed this exam on your own and in accordance with the ECE 101 honor code.

Student ID Number _____

Problem	Weight	Score
1	30 pts	
2	34 pts	
3	36 pts	
Total	100 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Table 3.1 from the textbook is attached to the exam.

Good luck!

Name/Student ID: _____

Problem 1 (30 pts)

(a) (10 pts)

Determine three distinct complex numbers that satisfy the equation

$$z^3 = j.$$

Write your answers in polar form and rectangular form.

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Problem 1 (cont.)

(b) (10 pts)

Determine the fundamental period of the discrete-time signal

$$x[n] = e^{j\frac{3\pi}{2}n} + \cos\left(\frac{\pi}{3}n\right).$$

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Problem 1 (cont.)

(c) (10 pts)

Let $x(t)$ be the continuous-time signal defined by

$$x(t) = \begin{cases} 0 \ , & t < 0 \\ t \ , & 0 \leq t < 1 \\ 0 \ , & t \geq 1. \end{cases}$$

Precisely sketch the signal $y(t) = x(-2t + 3)$.

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Problem 2 (34 pts)

(a) (12 pts)

Let $x(t) = u(t) - u(t-2)$ and $y(t) = u(t-1) - u(t-4)$.

Determine and precisely sketch the convolution $z(t) = x(t) * y(t)$.

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Problem 2 (cont.)

(b) (12 pts; 4pts each part)

Consider the discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n-1] + \left(-\frac{1}{4}\right)^{n+1} u[n]$$

For each property below, indicate whether or not the system satisfies the specified property.

Justify your answers citing properties of this particular $h[n]$.

True	False	
☐	☐	Memoryless
☐	☐	Causal
☐	☐	Stable

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Problem 2 (cont.)

(c) (10 points)

Consider the system S defined by:

$$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ n, & n \text{ odd.} \end{cases}$$

Determine if the system is invertible. (Justify your answer.)

If it is invertible, write an expression for the inverse system.

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Problem 3 (36 points)

(a) (12 points)

Consider three discrete-time systems S_1, S_2, and S_3. For each one, an input-output pair is shown below.

$$\begin{aligned} S_1: &\ \ j^n \rightarrow j^{n+1} \\ S_2: &\ \ e^{j5n} \rightarrow \cos(5n) \\ S_3: &\ \ e^{j\pi n/3} \rightarrow 2e^{j7\pi n/3} \end{aligned}$$

For each system, determine whether the given information is sufficient to conclude that the system is definitely **not** LTI.

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Problem 3 (cont.)

(b) (12 points)

Let $x(t)$ be a **real-valued** continuous-time signal with fundamental period $T = 4$. Denote the Fourier series coefficients of $x(t)$ by a_k. Suppose $x(t)$ satisfies the following properties:

(i) $\int_1^5 x(t)dt = 4$

(ii) $a_1 = 2$

(iii) $a_2 = e^{j\frac{\pi}{3}}$

(iv) $a_k = 0$, $k > 2$.

Express $x(t)$ in the form $x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$

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Problem 3 (cont.)

(c) (12 points)

The signal $x(t) = 1 + \cos(3\pi t)\sin(\pi t)$ is passed through the continuous-time filter whose frequency response is given by:

$$H(j\omega) = \begin{cases} 1 - \frac{|\omega|}{3\pi}, & |\omega| \leq 3\pi \\ 0, & \text{otherwise.} \end{cases}$$

Determine the output signal $y(t)$.