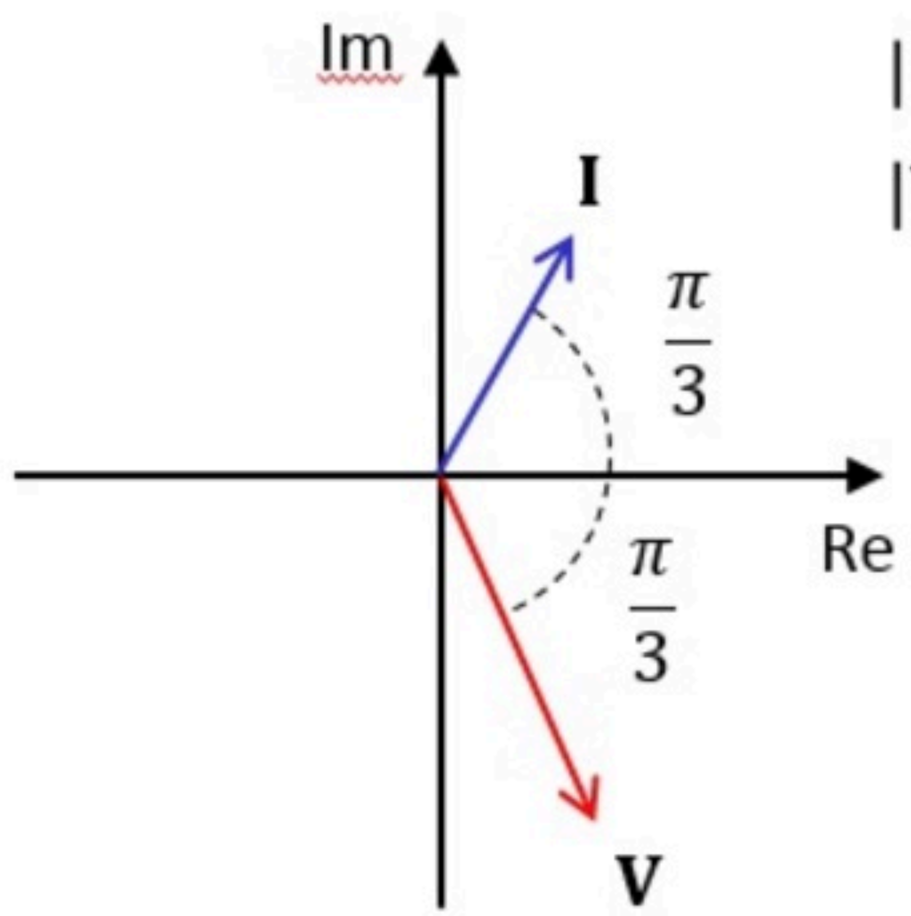
PP AC power 001

Unlimited Attempts.

In the diagram are the phasors of the voltage across an element and the current through that element (according to the passive sign convention).

What is the average power P received by the element?



$|\mathbf{I}| = 3$ A

$|\mathbf{V}| = 4$ V

Given Variables:

. : . .

Calculate the following:

P (W) :

-3

Hint: Write average power as a function of the angle between the voltage and current.

$$I = 3 e^{j\frac{\pi}{3}}$$

$$V = 4 e^{-j\frac{\pi}{3}}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \cdot 4 \cdot 3 \cdot \cos\left(-\frac{\pi}{3} - \frac{\pi}{3}\right)$$

$$= \frac{1}{2} \cdot 4 \cdot 3 \cdot \cos\left(-\frac{2\pi}{3}\right)$$

$$= \frac{1}{2} \cdot 4 \cdot 3 \left(-\frac{1}{2}\right)$$

$$= -3$$

$$\boxed{P = -3W}$$

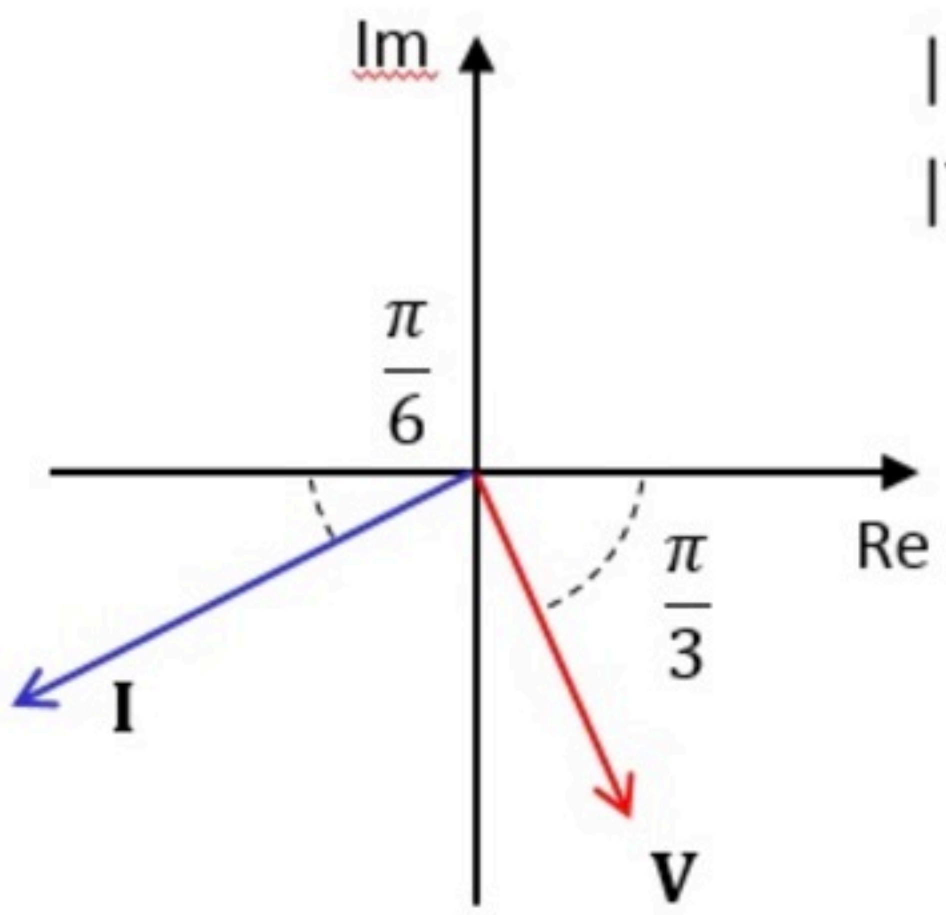
↳ SO THIS ELEMENT SUPPLIES POWER

PP AC power 002

Unlimited Attempts.

In the diagram are the phasors of the voltage across an element and the current through that element (according to the passive sign convention).

What is the average power *P* received by the element?



$|\mathbf{I}| = 8$ A

$|\mathbf{V}| = 5$ V

Given Variables:

. : . .

Calculate the following:

P (W) :

0

Hint: Write average power as a function of the angle between the voltage and current.

$$I = 8 e^{-j\frac{5\pi}{6}}$$

$$V = 5 e^{-j\frac{\pi}{3}}$$

$$P = \frac{1}{2} \cdot I_m \cdot V_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \cdot 8 \cdot 5 \cdot \cos\left(-\frac{\pi}{3} + \frac{5\pi}{6}\right)$$

$$= \frac{1}{2} \cdot 8 \cdot 5 \cdot \cos\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{2} \cdot 8 \cdot 5 \cdot 0$$

$\boxed{P = 0 \text{ W}}$ ⇝ AS EXPECTED SINCE V & I ARE ORTHOGONAL

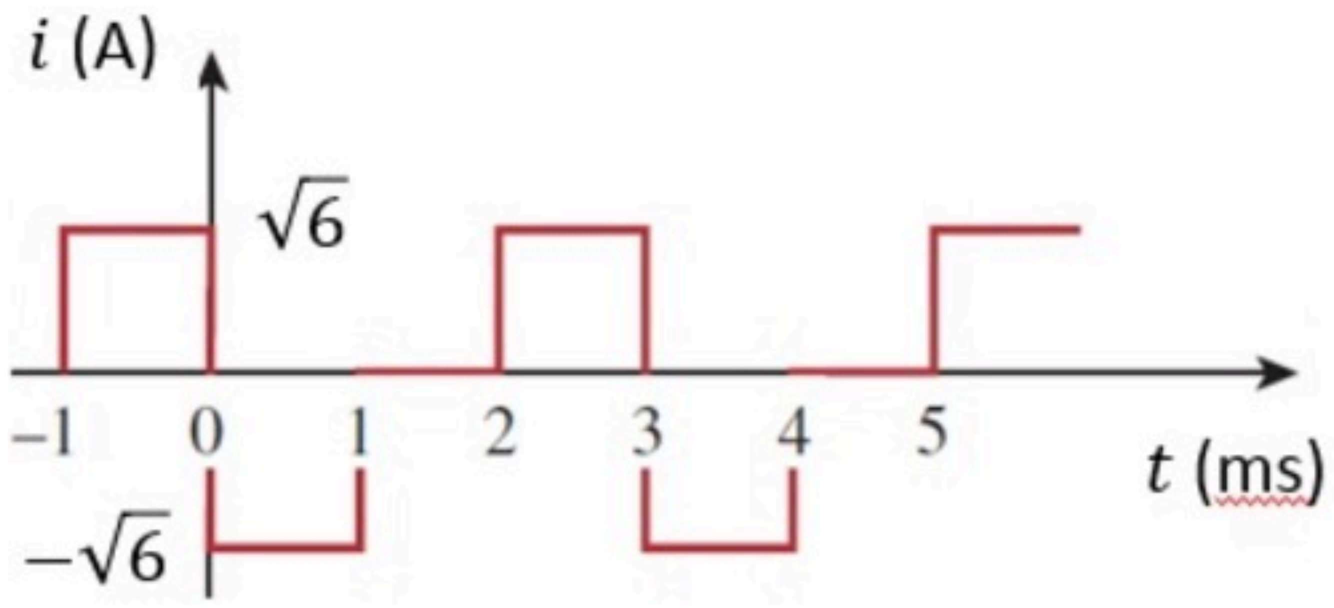
IN THIS CASE, "I LAGS V" BY $90°$ ⇒ THE ELEMENT IS AN INDUCTOR

$$I = \frac{V}{j\omega L} = \frac{V}{\omega L} e^{-j\frac{\pi}{2}}$$

PP AC power 003

Unlimited Attempts.

Find I_{rms} for this waveform.



Given Variables:

. : . .

Calculate the following:

Irms (A) :

2

Hint: Square the waveform first.

$$A = \frac{1}{T} \int_0^T i^2 \, dt$$

$$= \frac{1}{3 \cdot 10^{-3}} \cdot \left[(-\sqrt{6})^2 \cdot 10^{-3} + 0 + (\sqrt{6})^2 \cdot 10^{-3} \right]$$

$$= \frac{1}{3} \left[6 + 6 \right]$$

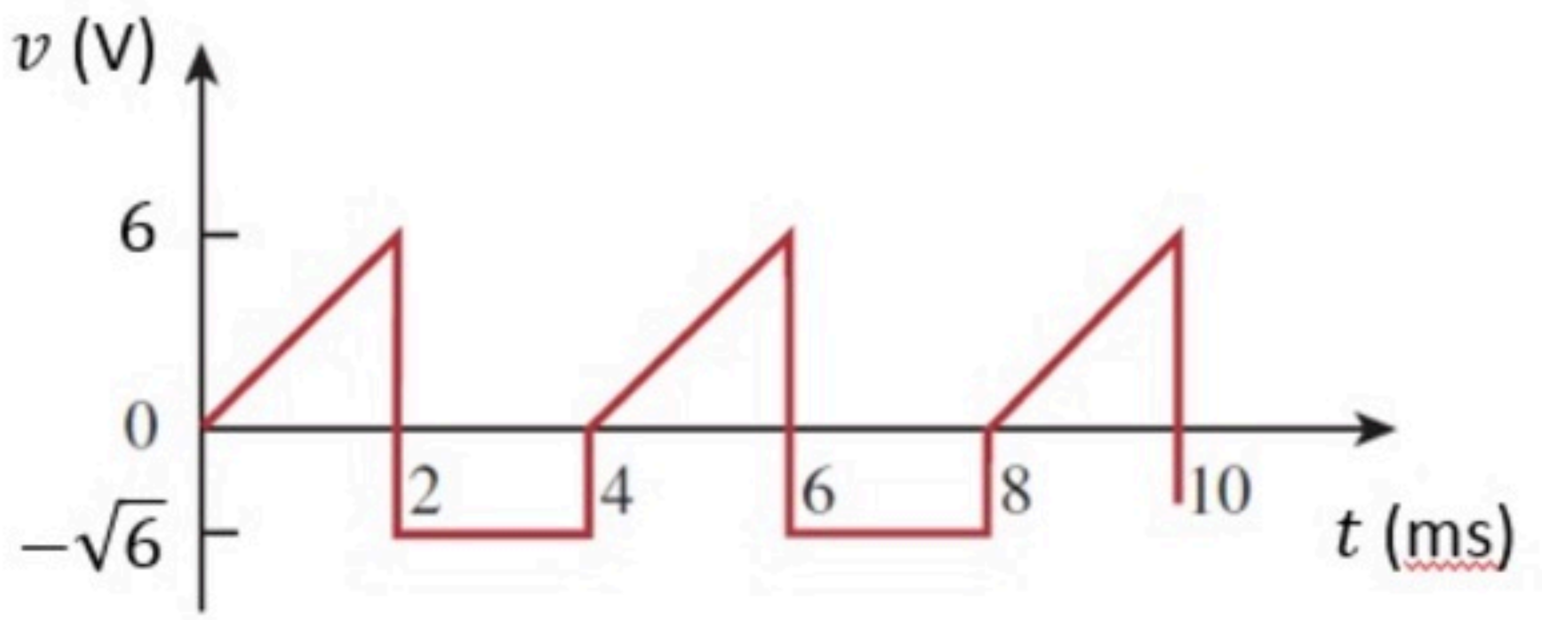
$$= 4$$

$$I_{RMS} = \sqrt{A} \implies \boxed{I_{RMS} = 2}$$

PP AC power 004

Unlimited Attempts.

Find V_{rms} for this waveform.



Given Variables:

. : . .

Calculate the following:

Vrms (V) :

3

Hint: Write the equation of the waveform in parts.

$\boxed{\text{OPTION 1}}$ t EXPRESSED IN SECONDS

$$f(t) = 3\cdot 10^{3}\cdot t \quad , \quad 0 \le t < 2\cdot 10^{-3}$$
$$= -\sqrt{6} \quad , \quad 2\cdot 10^{-3} \le t < 4\cdot 10^{-3}$$

$$\frac{1}{T}\int_0^T f^2(t)\,dt = \frac{1}{4\cdot 10^{-3}}\left[\int_0^{2\cdot 10^{-3}} (3\cdot 10^3\, t)^2 dt + \int_{2\cdot 10^{-3}}^{4\cdot 10^{-3}} (-\sqrt{6})^2 dt\right]$$

$$= \frac{1}{4\cdot 10^{-3}}\left[9\cdot 10^{6}\cdot \frac{t^3}{3}\Big|_0^{2\cdot 10^{-3}} + 6\cdot 2\cdot 10^{-3}\right]$$

$$= \frac{1}{4\cdot 10^{-3}}\left[\frac{9\cdot 8}{3}\cdot 10^{-3} + 12\cdot 10^{-3}\right]$$

$$= \frac{24+12}{4} = \frac{36}{4} = 9$$

$$\sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \boxed{V_{RMS} = 3V}$$

$\boxed{\text{OPTION 2}}$ t EXPRESSED IN ms

$$f(t) = 3\cdot t \quad , \quad 0 \le t < 2$$
$$= -\sqrt{6} \quad , \quad 2 \le t < 4$$

$$\frac{1}{T}\int_0^T f^2(t)\,dt = \frac{1}{4}\left[\int_0^2 (3t)^2 dt + \int_2^4 (-\sqrt{6})^2 dt\right] = \frac{1}{4}\left[9\,\frac{t^3}{3}\Big|_0^2 + 6\cdot 2\right]$$

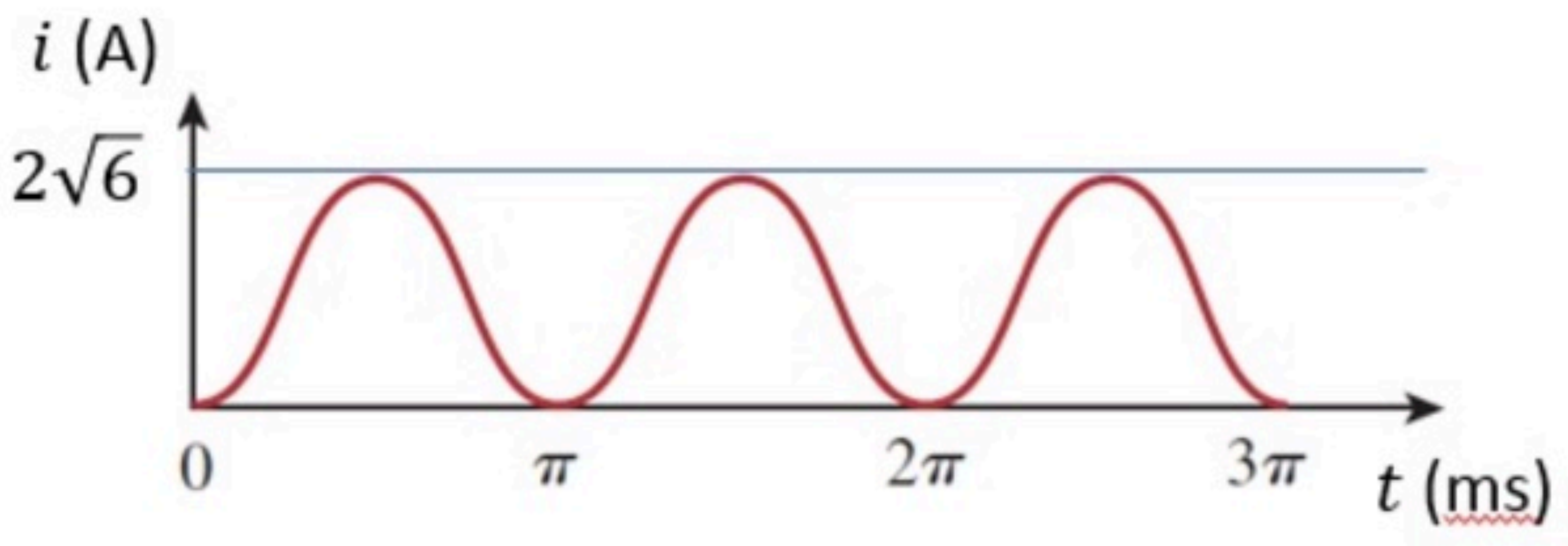
$$= \frac{1}{4}\left[\frac{9\cdot 8}{3} + 12\right] = \frac{36}{4} = 9$$

$$\boxed{V_{RMS} = 3\,V}$$

PP AC power 005

Unlimited Attempts.

Find I_{rms} for this waveform (it is a sine wave that has an offset, i.e., it is shifted up).



Given Variables:

. : . .

Calculate the following:

Irms (A) :

3

Hint: Utilize the fact that you know the Irms for a sine wave.

$$i = \sqrt{6} - \sqrt{6}\cos(\omega t) = \sqrt{6}\left(1 - \cos(\omega t)\right)$$

$$i^2 = 6\left(1 - \cos(\omega t)\right)^2$$

$$A = \frac{1}{T}\int_0^T i^2 dt = \frac{6}{T}\int_0^T \left[1 - 2\cos(\omega t) + \cos^2(\omega t)\right] dt$$

$$= \frac{6}{T}\int_0^T dt - \frac{12}{T}\int_0^T \cos(\omega t)\, dt + \frac{6}{T}\int_0^T \cos^2(\omega t)\, dt$$

$$= \frac{6}{T}\cdot T - \frac{12}{T}\cdot 0 + 6\cdot\frac{1}{2}$$

$\hookrightarrow$ WE KNOW FOR A COS

$$I_{RMS} = \frac{I_M}{\sqrt{2}} \Rightarrow I_{RMS}^2 = \frac{I_M^2}{2}$$

$$I_{RMS}^2 = \frac{I_M^2}{T}\int_0^T \cos^2(\omega t)\, dt$$

$$\Rightarrow \frac{1}{T}\int_0^T \cos^2(\omega t) dt = \frac{1}{2}$$

$$A = 6 + \frac{6}{2} = 9$$

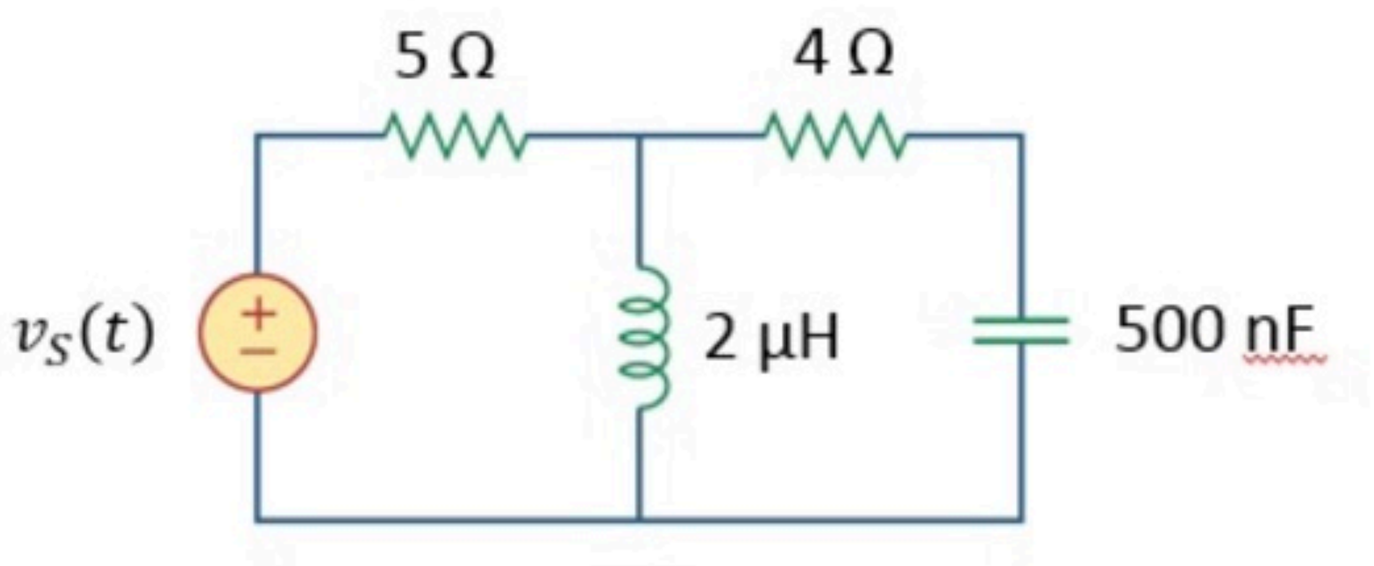
$$I_{RMS} = \sqrt{A} \Rightarrow \boxed{I_{RMS} = 3\,A}$$

PP AC power 006

Unlimited Attempts.

$v_S(t) = 10\sqrt{2} \cdot \cos\left(10^6 t + \frac{\pi}{6}\right)$ V

Find the average power P_1 supplied by the source v_S.
Find the average power P_2 received by the 5 Ω resistor.
Find the average power P_3 received by the 4 Ω resistor.
Find the average power P_4 received by the capacitor.
Find the average power P_5 received by the inductor.



Given Variables:

. : . .

Calculate the following:

P1 (W) :

15

P2 (W) :

12.5

P3 (W) :

2.5

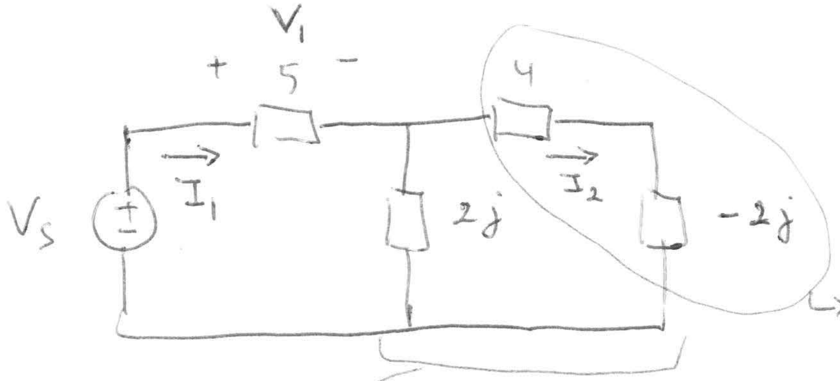
P4 (W) :

0

P5 (W) :

0

Hint: Find S first for each element. Keep Vs algebraic. Can you predict P for the cap and inductor? Check.



$\hookrightarrow Z_1 = 4 - 2j$

$$\hookrightarrow Z_2 = \frac{1}{\frac{1}{2j} + \frac{1}{4-2j}} = \frac{2j(4-2j)}{4-\cancel{2j}+\cancel{2j}} = j(2-j) = 1+2j$$

$$I_1 = \frac{V_s}{5+Z_2} = \frac{V_s}{6+2j}$$

① $S_1 = \frac{1}{2} V_s \cdot I_1^*$ (SUPPLIED !) $V_s \pm \uparrow I_1$

$$= \frac{1}{2} V_s \frac{V_s^*}{6-2j} = \frac{|V_s|^2}{2} \cdot \frac{6+2j}{40} = \frac{100 \cdot 2}{2}\left(\frac{6+2j}{40}\right) \qquad P_1 = Re[S_1]$$

$$\boxed{P_1 = 15W}$$

② $S_2 = \frac{1}{2} V_1 I_1^* = \frac{1}{2} \cdot Z_{5\Omega} \cdot I_1 \cdot I_1^* = \frac{1}{2} Z_{5\Omega} |I_1|^2$

$$S_2 = \frac{1}{2} \cdot 5 \cdot \frac{|V_s|^2}{|6+2j|^2} = \frac{5}{2} \cdot \frac{100 \cdot 2}{40} = 12.5 \qquad \boxed{P_2 = 12.5\,W}$$

③ $I_2 = I_1 \cdot \frac{2j}{2j + Z_1} = I_1 \cdot \frac{2j}{2j+4-2j} = I_1 \frac{\cancel{2}j}{\cancel{4}} = I_1 \frac{j}{2}$

$$S_3 = \frac{1}{2} Z_{4\Omega} |I_2|^2 = \frac{1}{2} \cdot 4 \cdot |I_1|^2 \cdot \frac{|j|^2}{4} = \frac{|I_1|^2}{2} = \frac{1}{2} \cdot \frac{|V_s|^2}{|6+2j|^2}$$

$$= \frac{1}{2} \cdot \frac{100 \cdot 2}{40} = 2.5 \qquad \boxed{P_3 = 2.5\,W}$$

WE KNOW A CAPACITOR AND INDUCTOR ONLY HAVE REACTIVE POWER $\Rightarrow \boxed{P_4 = P_5 = 0}$

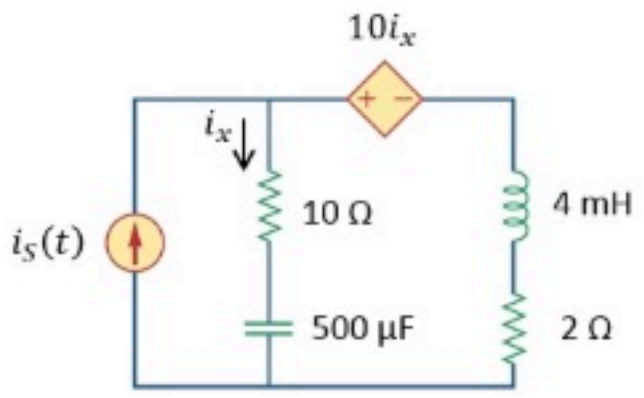
CHECK : $P_{REC.} = P_{SUPPLIED} \Rightarrow 12.5 + 2.5 = 15$ OK

PP AC power 007

Unlimited Attempts.

$$i_S(t) = 2 \cdot \cos\left(10^3 t + \frac{\pi}{4}\right) \text{ A}$$

Find the complex power $\mathbf{S_1} = a_1 + b_1 j$ supplied by the source i_S.
Find the complex power $\mathbf{S_2} = a_2 + b_2 j$ received by the 10 Ω resistor.
Find the complex power $\mathbf{S_3} = a_3 + b_3 j$ received by the 2 Ω resistor.
Find the complex power $\mathbf{S_4} = a_4 + b_4 j$ received by the CCVS.
Find the complex power $\mathbf{S_5} = a_5 + b_5 j$ received by the inductor.
Find the complex power $\mathbf{S_6} = a_6 + b_6 j$ received by the capacitor.



Given Variables:

. : . .

Calculate the following:

a1 (W) :

32

b1 (VAR) :

4

a2 (W) :

50

b2 (VAR) :

0

a3 (W) :

2

b3 (VAR) :

0

a4 (W) :

-20

b4 (VAR) :

10

a5 (W) :

0

b5 (VAR) :

4

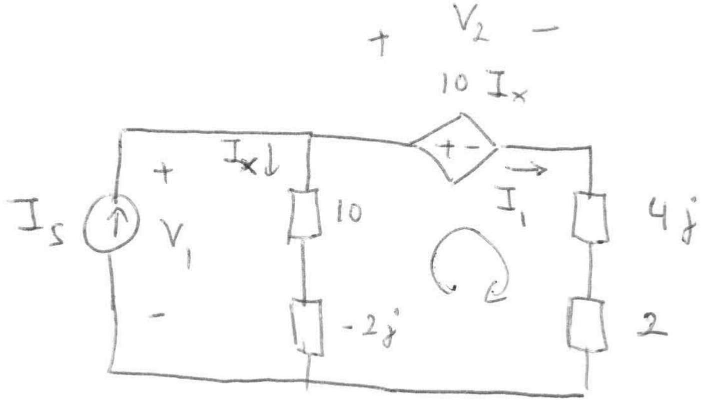
a6 (W) :

0

b6 (VAR) :

-10

Hint: Keep Is algebraic.



$$I_1 = I_S - I_x$$

$$|I_S|^2 = 4$$

<u>KVL</u>: $I_x\,(10 - 2j) = 10 I_x + (2+4j)\,I_1$

$$= 10\,I_x + (2+4j)(I_S - I_x)$$

$$I_x\,(\cancel{10} - 2j - \cancel{10} + 2 + 4j) = (2+4j)\,I_S$$

$$I_x\,(2+2j) = I_S\,(2+4j) \implies I_x = I_S\,\frac{(1+2j)}{(1+j)} = I_S\,\frac{(3+j)}{2}$$

① $V_1 = I_x\,(10-2j) = I_S\,\dfrac{(3+j)(10-2j)}{2} = I_S\,(16+2j)$

NOTE DIRECTION OPPOSITE PASSIVE SIGN CONVENTION

$$S_1 = \frac{1}{2} V_1 \cdot I_S^* = \frac{1}{2} I_S\,(16+2j)\,I_S^* = |I_S|^2\,(8+j) \implies \boxed{S_1 = 32 + 4j}$$

② $S_2 = \dfrac{1}{2} \cdot Z_{10\Omega} \cdot |I_x|^2 = \dfrac{1}{2} \cdot 10 \cdot |I_S|^2 \cdot \dfrac{(10)}{4} \implies \boxed{S_2 = 50 + 0j}$ AS EXPECTED

③ $I_1 = I_S - I_x = I_S\left(1 - \dfrac{(3+j)}{2}\right) = I_S\,\dfrac{(-1-j)}{2}$

$$S_3 = \frac{1}{2} Z_{2\Omega}\,|I_1|^2 = \frac{1}{2} \cdot 2\,|I_S|^2 \cdot \frac{|-1-j|^2}{4} = \frac{|I_S|^2}{4} \cdot 2 \implies \boxed{S_3 = 2 + 0j}$$

④ $S_4 = \dfrac{1}{2} V_2\,I_1^* = \dfrac{1}{2} \cdot 10\,I_x \cdot I_1^* = 5 \cdot I_S\,\dfrac{(3+j)}{2} \cdot I_S^*\,\dfrac{(-1+j)}{2} = |I_S|^2 \cdot \dfrac{5}{4}\,(-4+2j)$

$\implies \boxed{S_4 = -20 + 10j}$ RECEIVED SINCE WE USED PASSIVE SIGN CONVENTION

⑤ $S_5 = \dfrac{1}{2} Z_L \cdot |I_1|^2 = \dfrac{1}{2} \cdot 4j\,|I_S|^2 \cdot \dfrac{|-1-j|^2}{4} = 2j \cdot 4 \cdot \dfrac{2}{4} \implies \boxed{S_5 = 0 + 4j}$

AS EXPECTED

⑥ $S_6 = \dfrac{1}{2} Z_C\,|I_x|^2 = \dfrac{1}{2}(-2j)\,|I_S|^2\,\dfrac{|3+j|^2}{4} = -j \cdot 4 \cdot \dfrac{10}{4} \implies \boxed{S_6 = 0 - 10j}$

CHECK: $\sum P_{REC} = \sum P_{SUPPL} \implies 50 + 2 - 20 = 32$ <u>OK</u>

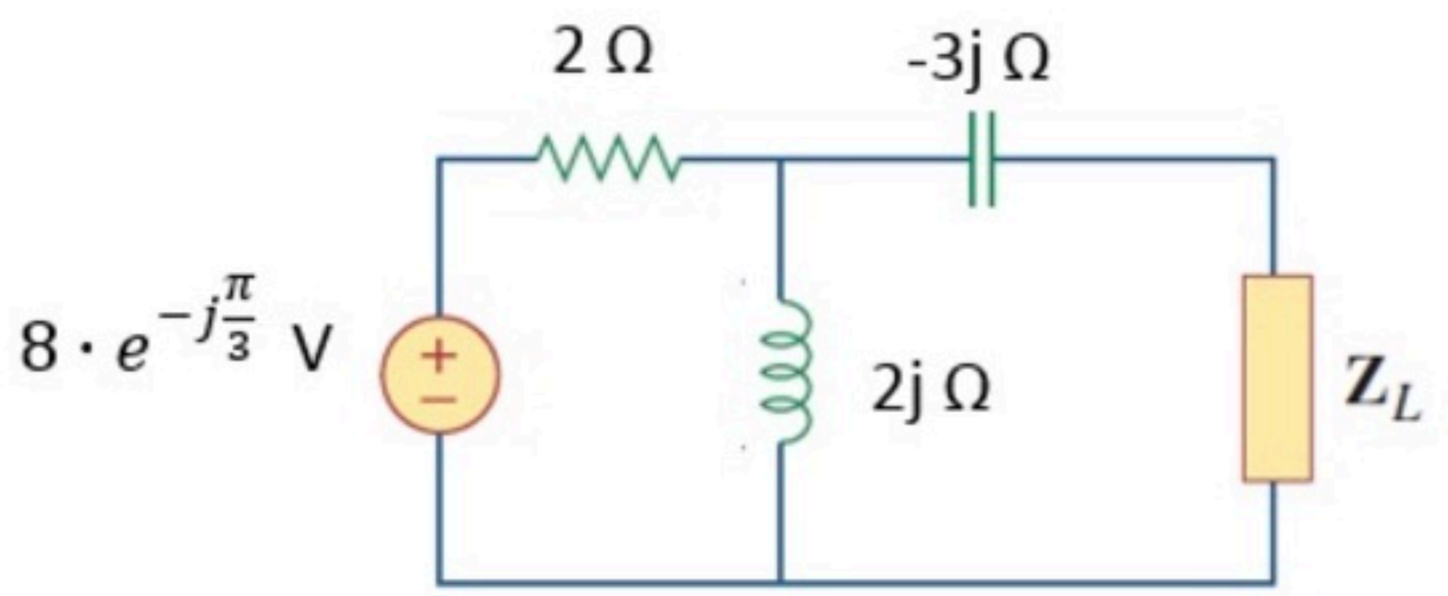
$\sum Q_{REC} = \sum Q_{SUPPL} \implies 10 + 4 - 10 = 4$ <u>OK</u>

PP AC power 008

Unlimited Attempts.

Find the value of $\mathbf{Z_L} = a + jb$ that will receive the maximum amount of power

Find the resulting complex power $\mathbf{S} = c + jd$ received by the load.



Given Variables:

. : . .

Calculate the following:

a (ohm) :

1

b (ohm) :

2

c (W) :

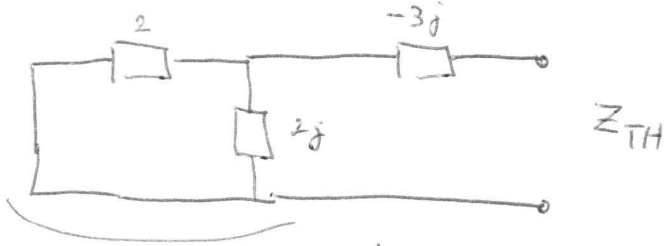
4

d (VAR) :

8

Hint: Set the independent source to zero. Find Zth.

(1)

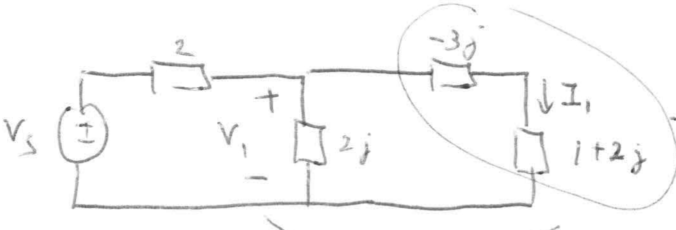


$$\hookrightarrow Z_1 = \frac{1}{\frac{1}{2}+\frac{1}{2j}} = \frac{4j}{2+2j} = \frac{2j}{1+j} = \frac{2j(1-j)}{2} = 1+j$$

$$\Rightarrow Z_{TH} = Z_1 - 3j = 1-2j$$

MAX POWER: $\boxed{Z_L = Z_{TH}^* = 1+2j}$

(2) OPTION 1:



$\rightarrow Z_2 = 1-j$

$$\hookrightarrow Z_3 = \frac{1}{\frac{1}{2j}+\frac{1}{1-j}} = \frac{2j(1-j)}{1-j+2j} = \frac{2j+2}{1+j} = 2$$

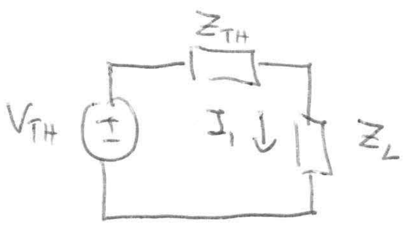
$$V_1 = V_S \frac{Z_3}{Z_3+2} = \frac{V_S}{2}$$

$$I_1 = \frac{V_1}{Z_2} \quad \Rightarrow S_L = \frac{1}{2} Z_L \cdot |I_1|^2 = \frac{1}{2}(1+2j)\frac{|V_S|^2}{4}\cdot\frac{1}{|Z_2|^2}$$

$$= \frac{1}{2}\cdot\frac{64}{4}\cdot\frac{1}{2}(1+2j)$$

$$\boxed{S_L = 4+8j}$$

OPTION 2: FIND V_{TH} $\Rightarrow$ $V_{OC} = V_S \cdot \frac{2j}{2+j} = V_S \frac{j}{1+j} = V_{TH}$



$$I_1 = \frac{V_{TH}}{Z_{TH}+Z_L} = \frac{V_{TH}}{(1-2j)+(1+2j)} = \frac{V_{TH}}{2}$$

AS EXPECTED

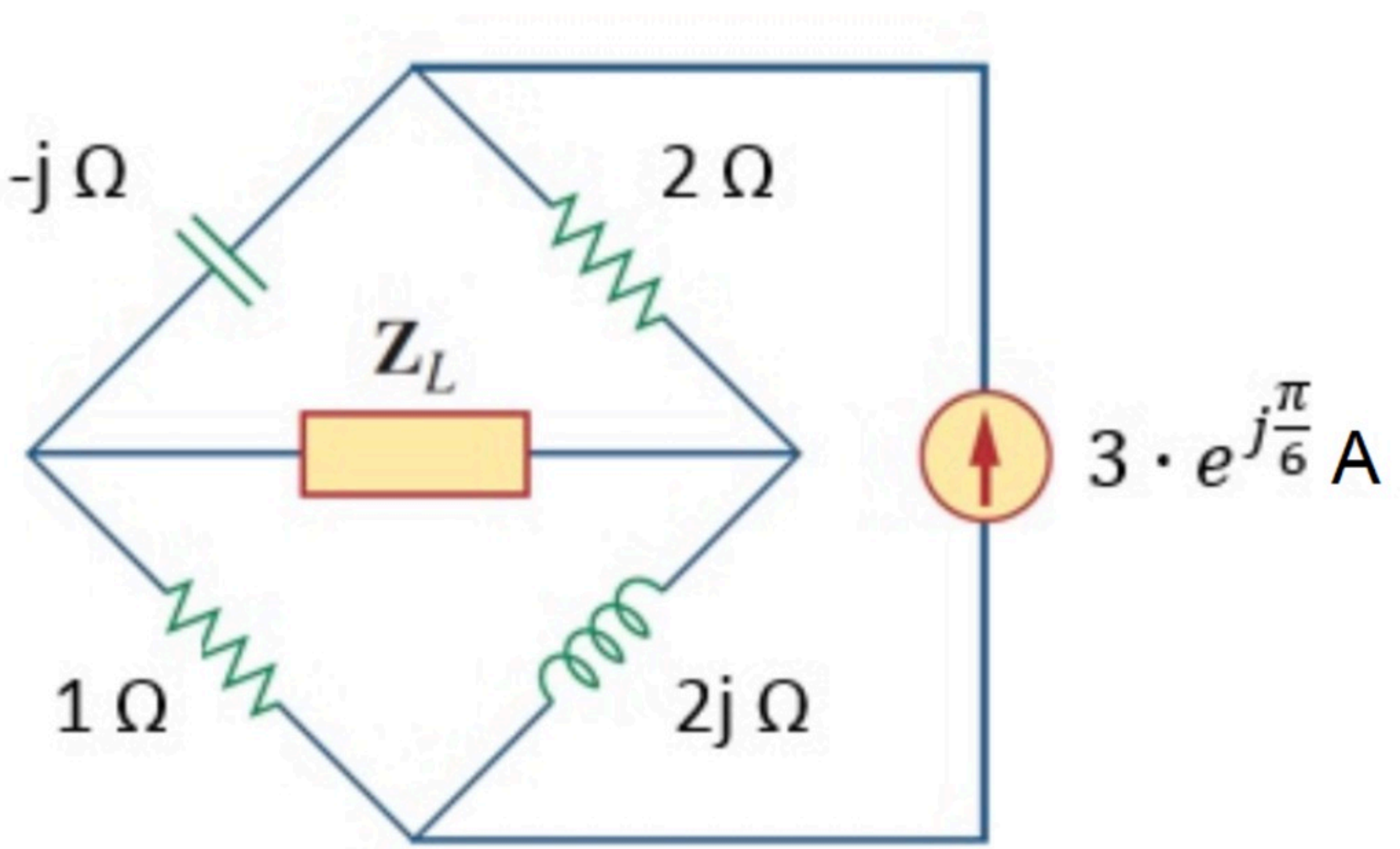
$$S_L = \frac{1}{2} Z_L |I_1|^2 = \frac{1}{2}(1+2j)\frac{|V_{TH}|^2}{4} = \frac{(1+2j)}{8}|V_S|^2 \frac{|j|^2}{|1+j|^2}$$

$$S_L = (1+2j)\cdot\frac{64}{8}\cdot\frac{1}{2} \Rightarrow \boxed{S_L = 4+8j}$$

PP AC power 009

Unlimited Attempts.

Find the value of $\mathbf{Z_L} = a + jb$ that will receive the maximum amount of power



Given Variables:

. : . .

Calculate the following:

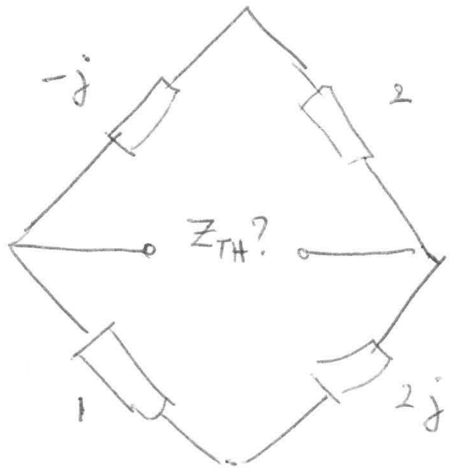
a (ohm) :

1.5

b (ohm) :

-0.5

Hint: Find ZTh by setting independent sources to zero.



$$Z_{TH} = (2-j) \,//\, (1+2j) = \frac{1}{\frac{1}{2-j} + \frac{1}{1+2j}}$$

$$= \frac{(2-j)(1+2j)}{2-j+1+2j} = \frac{2-j+4j+2}{3+j} \cdot \frac{3-j}{3-j}$$

$$= \frac{1}{10} \cdot (4+3j)(3-j)$$

$$= \frac{1}{10}(12+9j-4j+3)$$

$$= 1.5 + 0.5j$$

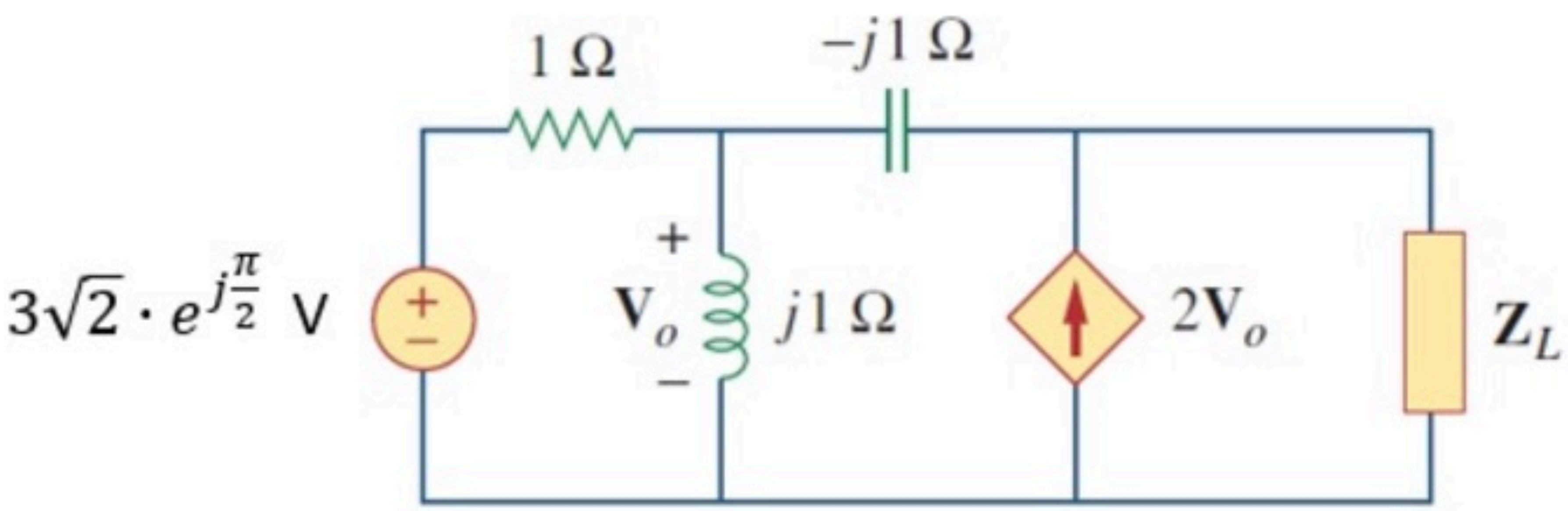
$Z_L^* = Z_{TH}$ FOR MAX POWER

$$\boxed{Z_L = 1.5 - 0.5j}$$

PP AC power 010

Unlimited Attempts.

Find the value of $\mathbf{Z_L} = a + jb$ that will receive the maximum amount of power



Given Variables:

. : . .

Calculate the following:

a (ohm) :

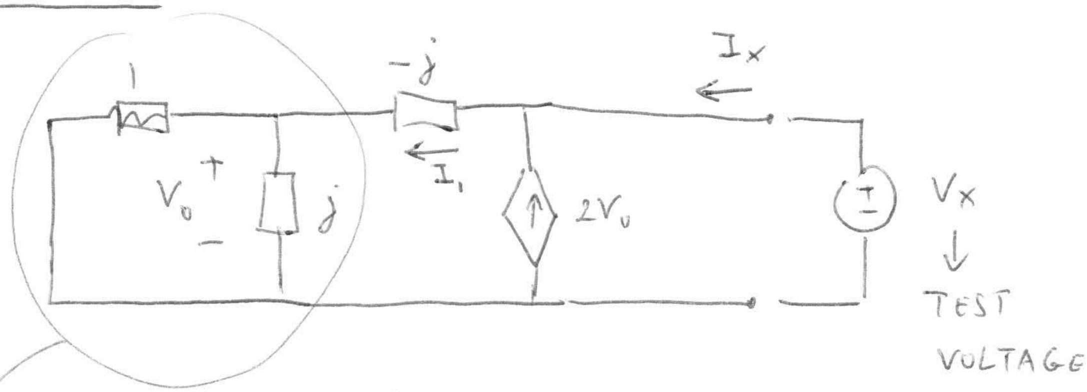
0.5

b (ohm) :

-0.5

Hint: Use a test source to find the ZTh.

FIND Z_{TH}:



$$Z_1 = \frac{1}{\frac{1}{1} + \frac{1}{j}} = \frac{j}{1+j} = \frac{j(1-j)}{2} = \frac{1+j}{2}$$

$$I_1 = \frac{V_x}{Z_1 - j} = \frac{V_x}{\frac{1-j}{2}}$$

$$V_0 = V_x \cdot \frac{Z_1}{Z_1 - j} = V_x \frac{(1+j)}{(1-j)}$$

$$I_x = I_1 - 2V_0 = \frac{2V_x}{1-j} - 2V_x \frac{(1+j)}{1-j} = V_x \frac{(-2j)}{1-j}$$

$$Z_{TH} = \frac{V_x}{I_x} = \frac{V_x(1-j)}{V_x(-2j)} = \frac{1-j}{-2j} = \frac{(1-j)\cdot j}{2} = \frac{1+j}{2}$$

MAX POWER: $Z_L = Z_{TH}^*$

$$\boxed{Z_L = 0.5 - 0.5j}$$