UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2019
Linear Systems Fundamentals

FINAL EXAM

You are allowed four 2-sided sheets of notes.

No books, no other notes, no electronics.

PRINT YOUR NAME ______________________________

Signature ______________________________

Your signature affirms that you have completed this exam on your own and in accordance with the ECE 101 honor code.

Student ID Number ______________________________

Problem	Weight	Score
1	20 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
6	20 pts	
Total	120 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Tables 3.1, 3.2, 4.1, 4.2, 5.1, 5.2, 9.1, and 9.2 are attached.
Good luck!

Name/Student ID: ____________________

Problem 1 [CTFT] (20 points, 4pts each part)

Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ defined by

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2 - t, & 1 \leq t \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following problems about $X(j\omega)$. You can answer all of them **without** actually computing $X(j\omega)$ by using properties of the CTFT.
Justify your answers.

(a) Find $X(j0)$.

(b) Evaluate $\int_{-\infty}^{\infty} X(j\omega) d\omega$

Name/Student ID: ____________________

Problem 1 [CTFT] (cont.)

(c) Evaluate $\int_{-\infty}^{\infty} X(j\omega) 2\frac{\sin(\omega)}{\omega} d\omega$.

(d) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

Name/Student ID: ____________________

Problem 1 [CTFT] (cont.)

(e) Sketch the inverse Fourier transform of the real part of $X(j\omega)$, $\mathcal{R}e\{X(j\omega)\}$.

Name/Student ID: ______________________________

Problem 2 [DTFT] (20 points, 10pts each part)

(a) (DTFT analysis)

Let $x[n]$ be the discrete-time signal defined by:

$$x[n] = \begin{cases} 1, & 2 \leq |n| \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

(i) (2 points)

Sketch $x[n]$ precisely.

Name/Student ID: ____

Problem 2 [DTFT] (cont.)

(a) (cont.)

Let $x[n]$ be the discrete-time signal defined by:

$$x[n] = \begin{cases} 1, & 2 \leq |n| \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

(ii) (8 points)

Determine the discrete-time Fourier transform of $x[n]$. Express the answer using trigonometric functions. Write your answer in the box below. **Justify your answer.**

$X(e^{j\omega}) \; =$

Name/Student ID: ____________________

Problem 2 [DTFT] (cont.)

(b) (DTFT synthesis)

The Fourier transform of a discrete-time signal $x[n]$ is given by

$$X(e^{j\omega}) = e^{-j\omega/4} \text{ for } 0 \leq \omega \leq 2\pi.$$

Determine $x[n]$.

Express your answer in rectangular form, $x[n] = R(n) + jI(n)$, where $R(n)$ and $I(n)$ are real-valued functions of n.

Write your answer in the box below. **Justify your answer.**

$x[n] \;=$

Name/Student ID: ______________________________

Problem 3 [Filtering] (20 points, 4pts each part)

Consider a causal and stable LTI system whose input $x[n]$ and output $y[n]$ are related by the first-order difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

(a) Determine the frequency response $H(e^{j\omega})$ of the system.
Justify your answer.

Name/Student ID: ______________________________

Problem 3 [DT Filtering] (cont.)

(b) Evaluate $H(e^{j\omega})$ at $\omega = 0$ and $\omega = \pi$.

Does the system represent a low-pass or a high-pass filter?

$H(e^{j0}) =$		Low-pass	High-Pass
$H(e^{j\pi}) =$		☐	☐

(c) Determine the impulse response $h[n]$ of the system.
Justify your answer.

Name/Student ID: ______________________________

Problem 3 [DT Filtering] (cont.)

(d) Determine the step response $s[n]$ of the system.

(e) Let $x[n] = (-1)^n$ be the system input. Determine the output $y[n]$.

Name/Student ID: ____________________

Problem 4 [Amplitude Modulation] (20 pts, 4pts each part)

Consider the signals $x_1(t) = \frac{\sin(4t)}{\pi t}$ and $x_2(t) = \frac{\sin(2t)}{\pi t}$.

(a) Determine their Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$.
Sketch them precisely.

Name/Student ID: ____________________

Problem 5 [Amplitude Modulation] (cont.)

(b) Let $y_1(t) = x_1(t)\cos(7t)$. Express $Y_1(j\omega)$ in terms of $X_1(j\omega)$.
Sketch $Y_1(j\omega)$ precisely.

(c) Let $y_2(t) = x_2(t)\cos(14t)$. Express $Y_2(j\omega)$ in terms of $X_2(j\omega)$.
Sketch $Y_2(j\omega)$ precisely.

Name/Student ID: ______________________________

Problem 5 [Amplitude Modulation] (cont.)

(d) The signal $y(t) = y_1(t) + y_2(t)$ is transmitted. It is demodulated using multiplication by $\cos(7t)$ to form the signal $z(t) = y(t)\cos(7t)$.

Express the Fourier Transform $Z(j\omega)$ in terms of $Y_1(j\omega)$ and $Y_2(j\omega)$.

Express $Z(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$.

Sketch precisely the portion of $Z(j\omega)$ in the frequency range $[-18, 18]$.

(e) Can $x_1(t)$ be recovered from $z(t)$? If not, why not.
If so, describe precisely how it can be recovered.

Name/Student ID: ____

Problem 5 [Sampling Theory] (20 pts)

(a) (10 points, 5pts each part)

Let $x(t)$ be a continuous-time band-limited signal with

$$X(j\omega) = 0 \text{ for } |\omega| > W$$

For each of the signals below, indicate the conditions that, according to the sampling theorem, the sampling frequency ω_s and sampling period T should satisfy so that the signal can be reconstructed from its samples.

Justify your answers by explicit reference to the Fourier Transforms $Y(j\omega)$ of the signals.

(i) $y(t) = x(t) * x(t-1)$.

Condition on ω_s :	Condition on T :

(ii) $y(t) = x(t)x(-4t)$.

Condition on ω_s :	Condition on T :

Name/Student ID: ______________________________

Problem 5 [Sampling Theory] (cont.)

(b) (10 points, 5pts each part)

Consider the signal $x(t) = \sin(2\pi t)$. Let $x_p(t)$ be obtained by impulse-train sampling $x(t)$ with sampling frequency $\omega_s = 3\pi$.

(i) Let $X_p(j\omega)$ be the Fourier transform of the signal $x_p(t)$.
Sketch $X_p(j\omega)$ precisely in the frequency range $[-6\pi, 6\pi]$.

(ii) Suppose $x_p(t)$ is passed through a low-pass filter with gain $T = 2\pi/\omega_s = 2/3$ and cutoff frequency $\omega_c = \omega_s/2 = 3\pi/2$.
Determine the output $y(t)$ of the low-pass filter and determine whether aliasing occurred.

Name/Student ID: ______________________________

Problem 6 [Laplace Transform](20 points)

(a) (8 points)

Let S be a **causal** LTI system whose input-output relationship is described by the differential equation:

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+5y(t) = \frac{d^2x(t)}{dt^2} - x(t).$$

(i) Determine the system function $H(s)$.

$H(s) =$

Name/Student ID: ______________________________

Problem 6 [Laplace Transform](cont.)

(a) (cont.)

(ii) Sketch precisely the pole-zero plot associated with the system function $H(s)$. Indicate the order of all poles and zeros.

Name/Student ID: ______________________________

Problem 6 [Laplace Transform] (cont.)

(a) (cont.)

(iii) Determine the region of convergence (ROC) associated with $H(s)$.
Justify your answer.

(iv) Is the systems stable? Check the box. **Justify your answer.**

Yes	No
☐	☐

Name/Student ID: ____________________

Problem 6 [Laplace Transform] (cont.)

(b) (6 points)

Let S be a **causal, stable** LTI system with system function

$$H(s) = \frac{s(s-2)}{(s+1)(s+2)}.$$

Let $H(j\omega)$ be the frequency response of the system.

(i) Sketch the magnitude of the frequency response $\left|H(j\omega)\right|$.

(ii) Is the frequency response lowpass, highpass, bandpass, or all-pass? Indicate your answer below by checking the appropriate box.

Low-pass	High-pass	Band-pass	All-pass
☐	☐	☐	☐

Name/Student ID: ______________________________

Problem 6 [Laplace Transform] (cont.)

(c) (6 points)

Let S be a **stable** LTI system with system function

$$H(s) = \frac{1}{s^2 - 2s - 3} = \frac{s}{(s-1)(s+3)}.$$

Determine the impulse response $h(t)$.

$h(t) =$

Scratch page **Name/Student ID:** ______________________

Scratch page **Name/Student ID:** ______________________

Scratch page **Name/Student ID:** ______________________

Scratch page **Name/Student ID:** ______________________