ECE 101: Linear Systems Fundamentals

Spring 2020 - Lecture 1

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Today's topics

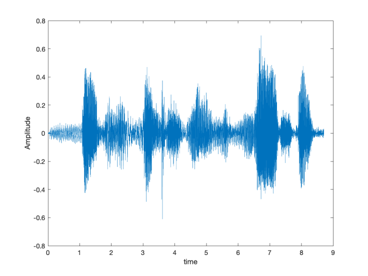
- Introduction to the course
- Signals
- Basic operations on the signals

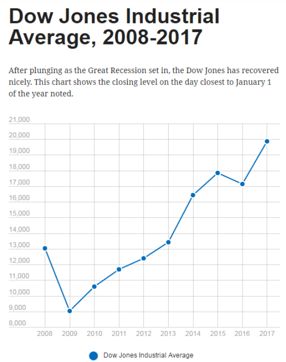
Signals and Systems (2th Edition): sections 1.1.1 and 1.2.1

Signals

What is a signal?

- A signal is a quantitative description of a physical phenomenon, event or process that often has *time-dependency*.
- Examples:





What is a signal?

- Signals:
  - Continuous-time (CT)
  - Discrete-time (DT)

Continuous-time Signals

- **Continuous-time Signals**: Signals that are functions of real-valued independent variables (time variable t).

- We use notation: $x(t), y(t), z(t), \ldots$:
    - reserve time variable t for continuous-time signals' independent variable
    - use parenthesis $(\cdot)$

Discrete-time Signals

- **Discrete-time Signals**: Signals that are defined only at discrete times (time variable n).
- For these signals, the independent variable takes on only a discrete set of values.
- We use notation: $x[n], y[n], z[n], \ldots$:
  - reserve time variable n for discrete-time signals' independent variable
  - use square bracket $[\cdot]$

Complex signals

Complex Signals - Notation

- A complex CT signal $x(t)$ is formed by the signal pair $\{x_r(t), x_j(t)\}$, where both $x_r(t)$ and $x_j(t)$ are real-valued signals.

- The relationship between these signals is given by:

  $x(t) = x_r(t) + j\, x_j(t)$

  where $j = \sqrt{-1}$

Complex Signals - Notation

- Similarly, a complex DT signal $x[n]$ can be represented as

  $$x[n] = x_r[n] + j\, x_j[n]$$

  where both $x_r[n]$ and $x_j[n]$ are real-valued DT signals.

Basic Notations

- $\mathbb{Z}$: the set of integers $\ldots, -2, -1, 0, 1, 2, \ldots$
- $\mathbb{R}$: the set of real numbers
- $\mathbb{C} := \{(a + bj) \mid a, b \in \mathbb{R}\}$ the set of complex numbers
  - $x = a + bj$ is Cartesian coordinate representation
  - $x = re^{j\theta}$, where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(\frac{b}{a})$ is the polar coordinate representation
  - Euler's Formula:

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Basic Operations on Signals

Time Shift

- For any $t_0 \in \mathbb{R}$ and $n_0 \in \mathbb{Z}$, **time shift** is defined as

$$\begin{aligned} x(t) &\longrightarrow x(t - t_0) \qquad \text{CT} \\ x[n] &\longrightarrow x[n - n_0] \qquad \text{DT} \end{aligned}$$

- For $t_0 > 0$ ($n_0 > 0$), it is called **delay** and for $t_0 < 0$ ($n_0 < 0$) it is called **advance**.

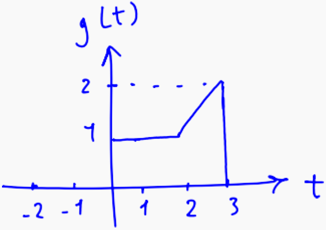
Time Shift: example

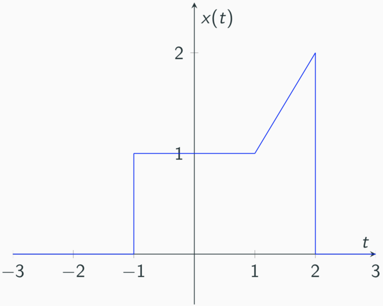
$g(t) = x(t-1)$

$x(t-t_0)$

- Example: Suppose that $x(t)$ is as shown below. Sketch $x(t-1)$.

here $t_0 = 1$





$g(-2) = x(-3)$

$g(-1) = x(-2) = 0$

$g(t)\big|_{t=0} = x(t)\big|_{t=-1}$

$g(1) = x(0)$

Time Reversal

- Time reversal is defined as

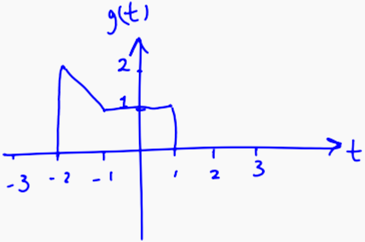
$$\begin{aligned} x(t) &\longrightarrow x(-t) \\ x[n] &\longrightarrow x[-n], \end{aligned}$$

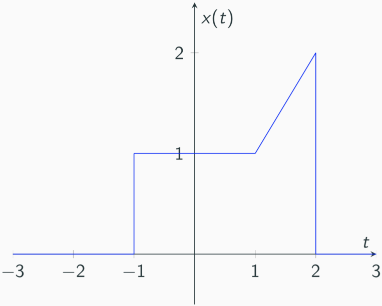
- Interpretation: Flipping over y-axis

Time Reversal: example

$g(t) = x(-t)$

- Example: Suppose that $x(t)$ is as shown below. Sketch $x(-t)$.





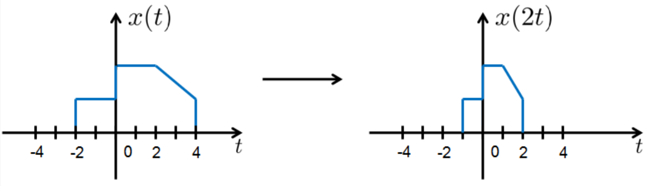
Time Scaling: Continuous Time

- For CT Signals, time-scaling by a factor $a > 0$ is defined as

$$x(t) \longrightarrow x(at), \quad a > 0.$$

- If $a > 1$, it is called **Decimation** (squeezing)
- If $0 < a < 1$, it is called **Expansion** (enlarging)

$g(t) = x(2t)$



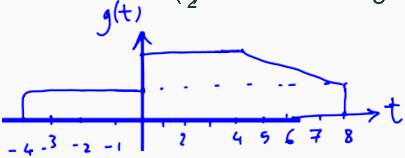
$g(t)\big|_{t=0^-} = g(0^-) = x(t)\big|_{t=0^-} = 1$, $g(t)\big|_{t=0^+} = x(t)\big|_{t=0^+} = 2$

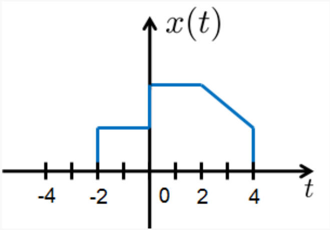
$g(t)\big|_{t=-0.5} = x(-2\times 0.5) = x(-1) = 1$

Class activity:

- Sketch $x\left(\frac{t}{2}\right)$ for the below signal.

$g(t) = x\left(\frac{t}{2}\right)$



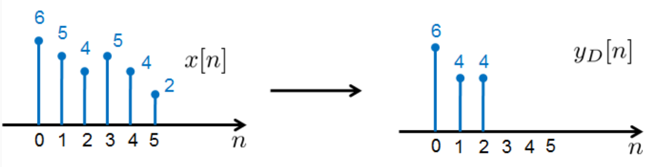


Time Scaling: Discrete Time

- **Decimation**: For a DT Signal, and an **integer** factor $M \geq 1$, the decimated signal is defined by:

$$y_D[n] = x[Mn].$$

- An example of decimation for M = 2:

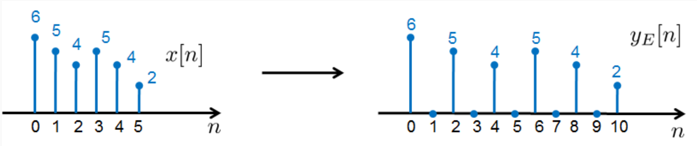


Time Scaling: Discrete Time

- **Expansion**: For an integer $L \geq 1$, the discrete-time expanded signal (by a factor L) is:

$$y_E[n] = \begin{cases} x\left[\frac{n}{L}\right], & n = \text{integer multiple of } L \\ 0, & \text{otherwise.} \end{cases}$$

- An example of expansion for L = 2:



Linear-Time Transformation

- Often we are interested in the transformation $x(t) \to x(at - b)$
- A combination of time shift and time-scaling is needed (order matters):
    1. Define $\boxed{v(t) = x(t - b)}$,
    2. Define $\boxed{y(t) = v(at)} = x(at - b)$.

$v(t) = x(t - b)$

$v(at) = x(at - b)$

Combination of Operations

- Often we are interested in the transformation $x(t) \to x(at - b)$
- A combination of time shift and time scaling is needed (order matters):
- The recommended method is
    1. Define $v(t) = x(t - b)$,
    2. Define $y(t) = v(at) = x(at - b)$.
- So $x(t) \to x(at - b)$ is equivalent to time delay then scale!

Combination of Operations

- The second method is

  1. Define $v(t) = x(at)$,

  2. Define $y(t) = v(t - b/a) = x(at - b)$.

- This method will not always result in a correct answer for the discrete-time signals.

Linear Time Transformation: example

- Example: Plot the signal $x(-t + 2)$ for the signal $x(t)$ as bellow,

$v(t) = x(t+2)$, $y(t) = v(-t) = x(-t+2)$