Discussion Session 7

Review on Fourier Series

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jw_n t} = \sum_{n=-\infty}^{\infty} F_n e^{jnw_0 t} \qquad w_0 = \frac{2\pi}{T} : \text{fundamental frequency}$$

$$F_n = \frac{1}{T} \int_T f(t) e^{-jw_n t} dt \qquad F_n : \text{Fourier coefficient}$$

Example : $T = 2\pi$ $\quad f(x) = e^{ax}$ for $x \in (-\pi, \pi)$

Example : $T = 2\pi$, $f(t) = 1 + \sin^2(t)$ for $t \in (-\pi, \pi)$

Fourier Coefficient and signal transformation

$$f(t) \xleftrightarrow{FS} F_n \qquad \text{period} = T$$

$$f(t-t_0) \xleftrightarrow{FS} F_n\, e^{-jnw_0 t_0}$$

$$f(-t) \xleftrightarrow{FS} F_{-n}$$

$$f(at) \longleftrightarrow F_n \qquad \text{period} = \frac{T}{a}$$

$$\frac{d}{dt} f(t) \longleftrightarrow (jkw_0)\, F_n$$

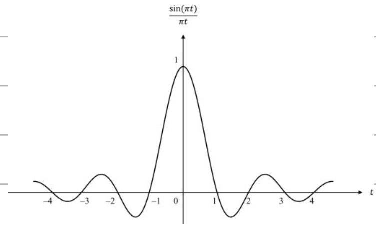
$$f^*(t) \longleftrightarrow F_{-n}^*$$

$$f_1(t) \xleftrightarrow{FS} a_n \qquad f_2(t) \xleftrightarrow{FS} b_k$$

$$A f_1(t) + B f_2(t) \xleftrightarrow{FS} A a_k + B b_k$$

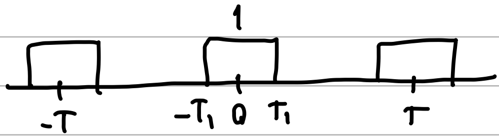
$$f_1(t)\, f_2(t) \xleftrightarrow{FS} a_k * b_k$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

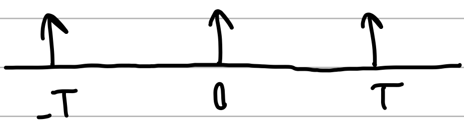


Famous fourier series pairs:

$$\text{pulse train} = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-kT}{2T_1}\right) \xleftrightarrow{FS} a_k = \frac{2T_1}{T}\, \text{sinc}\left(\frac{2T_1}{T} k\right)$$



$$\text{Delta train} = \sum_{k=-\infty}^{\infty} \delta(t-kT) \xleftrightarrow{FS} a_k = \frac{1}{T}$$



$$\cos(\ell w_0 t) = \cos\left(\ell \frac{2\pi}{T} t\right) \xleftrightarrow{FS} a_k = \begin{cases} 0 & k \neq \ell \\ \frac{1}{2} & k = \pm \ell \end{cases}$$

$$\sin(\ell w_0 t) = \sin\left(\ell \frac{2\pi}{T} t\right) \xleftrightarrow{FS} \begin{cases} 0 & k \neq \ell \\ \frac{1}{2j} & k = \ell \\ \frac{-1}{2j} & k = -\ell \end{cases}$$

Example :

What is $x(t)$ if its coefficients are $a_k = e^{-jk\frac{\pi}{4}} \frac{\sin(k\frac{\pi}{3})}{2k}$ with $T=12$.