ECE 65: Components & Circuits Lab

Lecture 19

BJT Amplifier small signal model

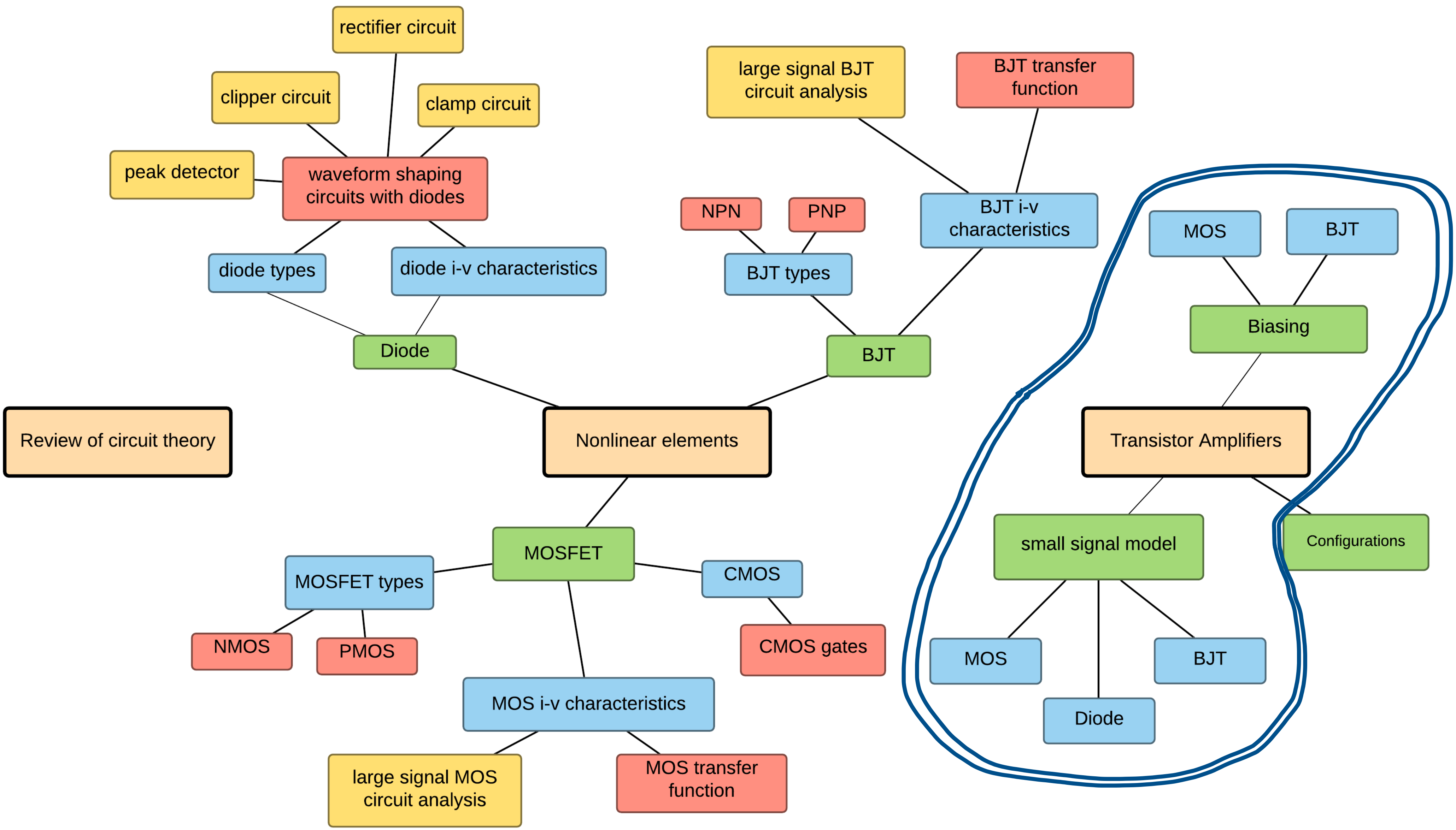
Reference notes: sections 5.1, 5.2

Sedra & Smith (7th Ed): sections 7.1

Saharnaz Baghdadchi

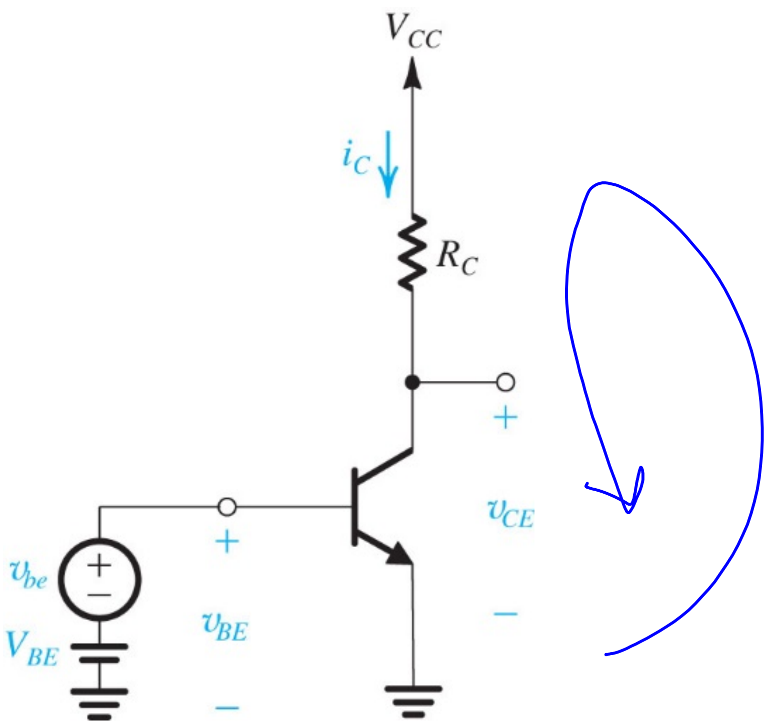
Course map

5. Transistor Amplifiers – Bias and small signal



Derivation of BJT small signal model

The DC Bias point: $v_{be} = 0$



$$I_C = I_s\, e^{V_{BE}/V_T}$$

[The early effect is neglected here ($\lambda = 0$)]

$$I_B = I_C/\beta$$

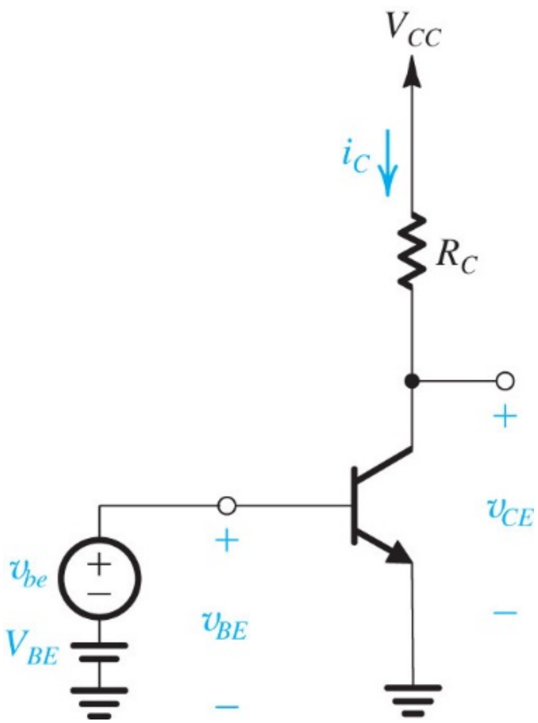
$$I_E = \frac{\beta + 1}{\beta}\, I_C$$

$$V_{CE} = V_{CC} - R_C I_C$$

Derivation of BJT small signal model

When v_{be} is applied:

The total instantaneous base-emitter voltage is: $v_{BE} = V_{BE} + v_{be}$



$$i_C = I_s\, e^{v_{BE}/V_T} = I_s\, e^{(V_{BE}+v_{be})/V_T}$$

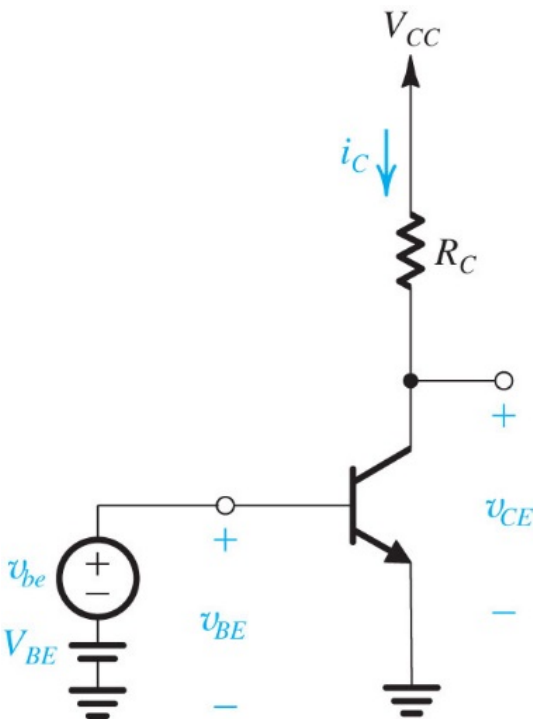
$$i_C = I_s\, e^{V_{BE}/V_T}\, e^{v_{be}/V_T}$$

Since $I_C = I_s\, e^{V_{BE}/V_T}$,

$$i_C = I_C\, e^{v_{be}/V_T}$$

Derivation of BJT small signal model

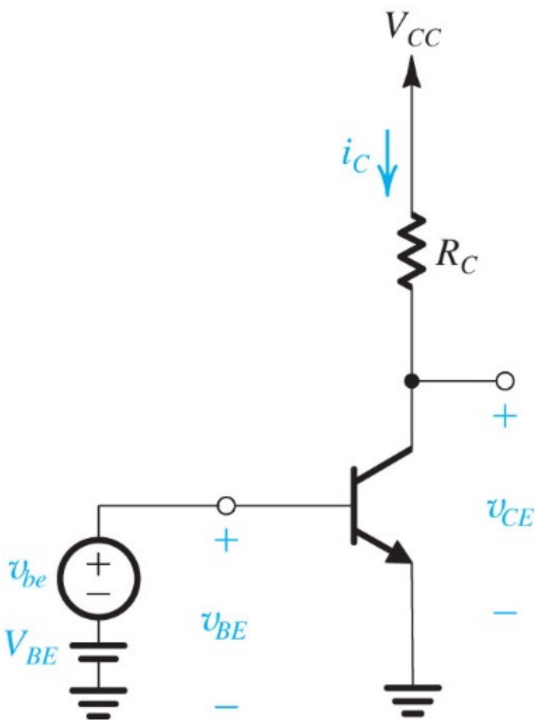
$$i_C = I_C\, e^{v_{be}/V_T}$$



If $v_{be} \ll V_T$, using Taylor series expansion and neglecting the higher order terms in the exponential series expansion,

$$i_C \simeq I_C\left(1 + \frac{v_{be}}{V_T}\right)$$

Derivation of BJT small signal model



$$i_C \simeq I_C\left(1+\frac{v_{be}}{V_T}\right)$$

Under small signal approximation for BJT ($v_{be} \ll V_T$),

$$i_C = I_C + \frac{I_C}{V_T}\, v_{be}$$

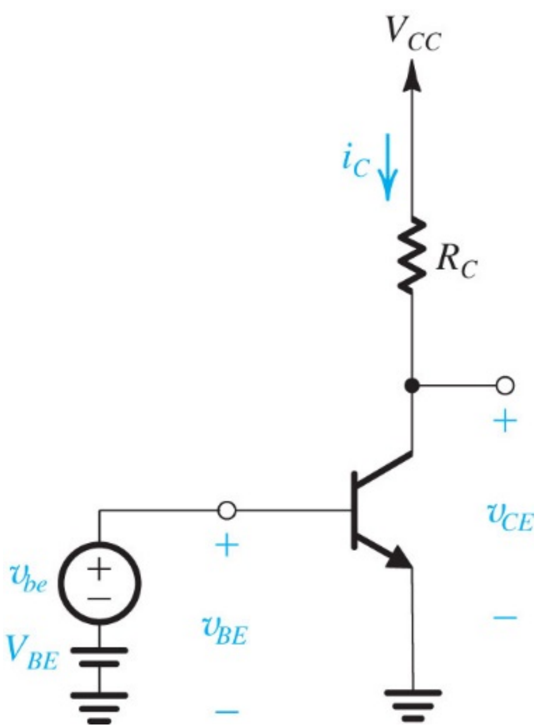
$$i_c = \frac{I_C}{V_T}\, v_{be}$$

The BJT transconductance g_m is defined as $g_m \equiv \dfrac{i_c}{v_{be}}$

$$\boxed{g_m = \frac{I_C}{V_T}}$$

$$i_c = g_m v_{be}$$

Derivation of BJT small signal model



$$i_C = I_C + \frac{I_C}{V_T} v_{be}$$

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T} v_{be}$$

$$i_B = I_B + i_b$$

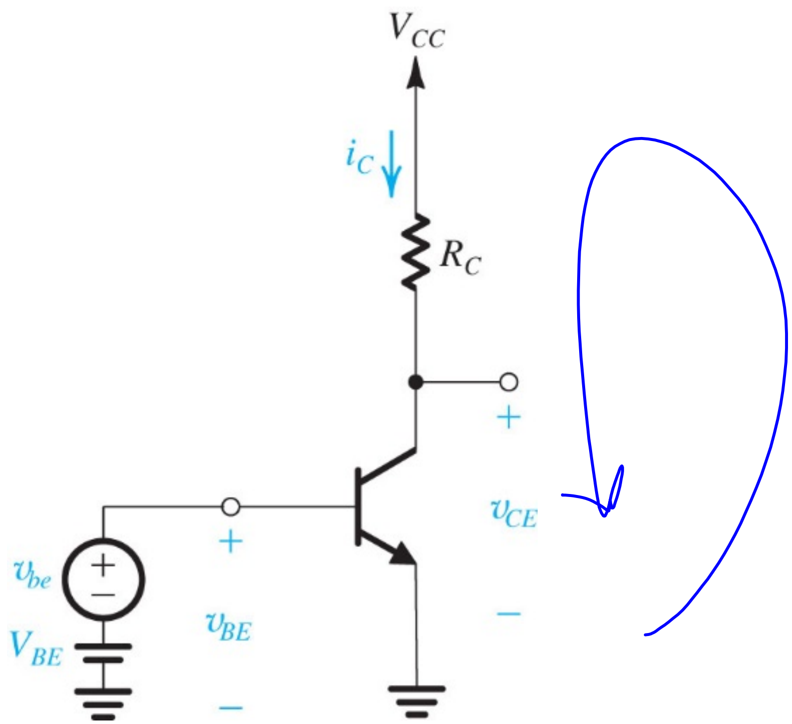
$$i_b = \frac{1}{\beta}\frac{I_C}{V_T} v_{be} = \frac{g_m}{\beta} v_{be}$$

The small signal input resistance between base and emitter is denoted by r_π

$$r_\pi \equiv \frac{v_{be}}{i_b} \rightarrow \quad r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

Derivation of BJT small signal model

The voltage gain in this amplifier configuration:



$$v_{CE} = V_{CC} - i_C R_C$$

$$= V_{CC} - (i_c + I_C)\, R_C$$

$$= (V_{CC} - I_C R_C) \ - \ i_c R_C$$

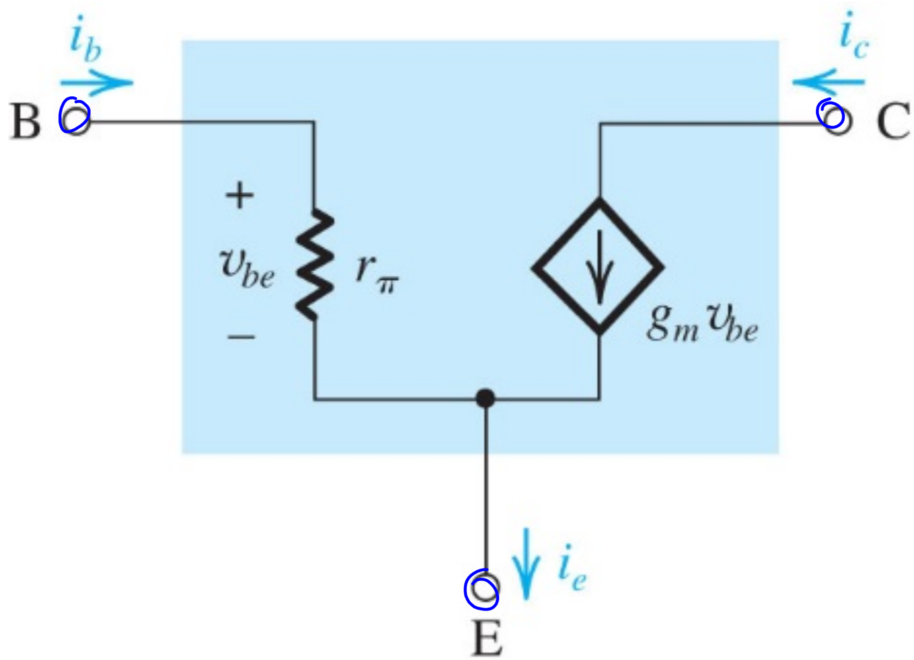
$$= V_{CE} - i_c R_C$$

$$v_{ce} = -\, i_c \ R_C = -g_m v_{be} R_C$$

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

BJT small signal model

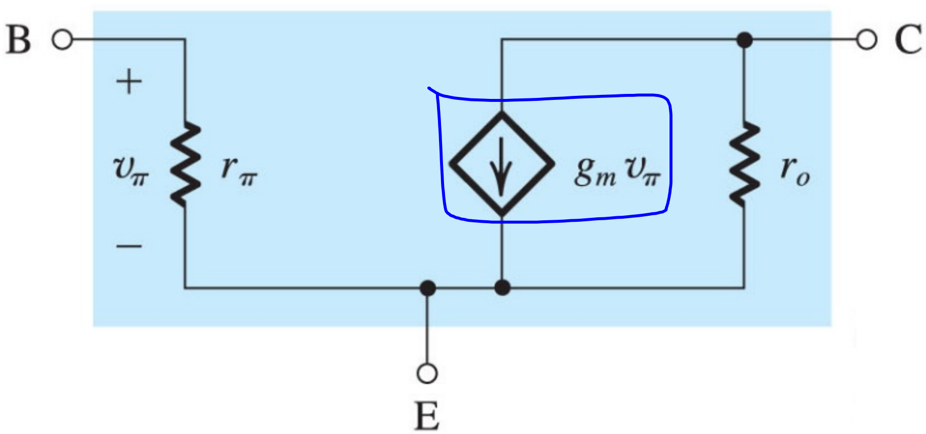
$$i_b = \frac{v_{be}}{r_\pi} \qquad\qquad i_c = g_m v_{be}$$

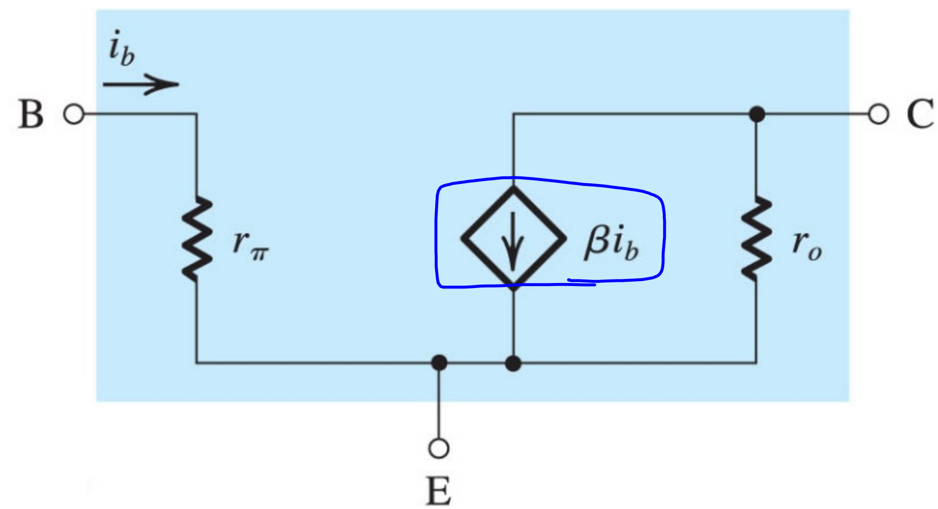


BJT small signal model

$v_\pi = v_{be}$

$$i_b = \frac{v_{be}}{r_\pi} \qquad i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$





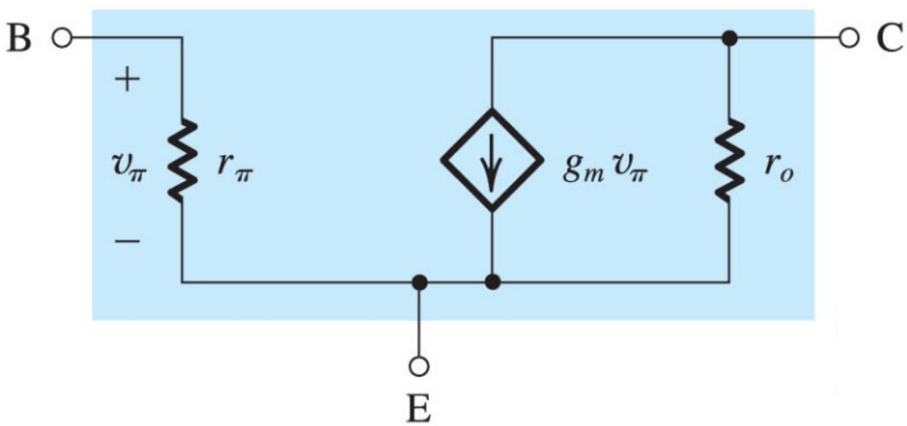
$V_A = \frac{1}{\lambda}$

$$g_m = \frac{I_C}{V_T} \qquad r_o \approx \frac{V_A}{I_C} \qquad r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m v_{be} = g_m(i_b\, r_\pi) = (g_m r_\pi) i_b = \beta i_b$$

PNP small signal model is identical to NPN

PNP



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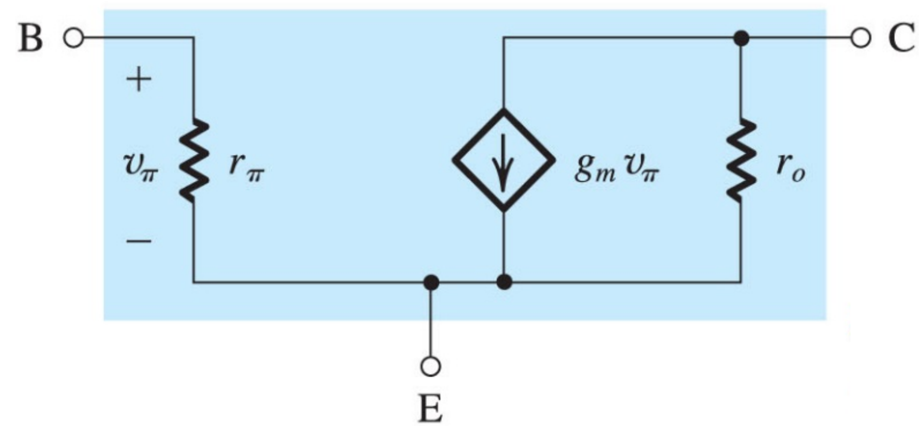
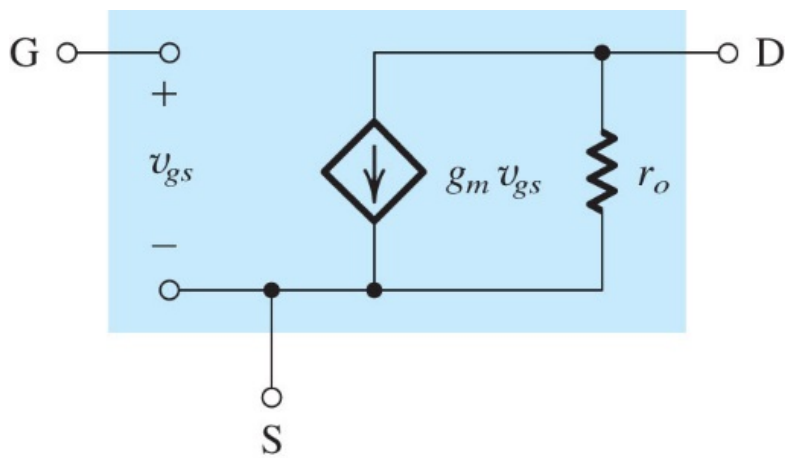
NPN



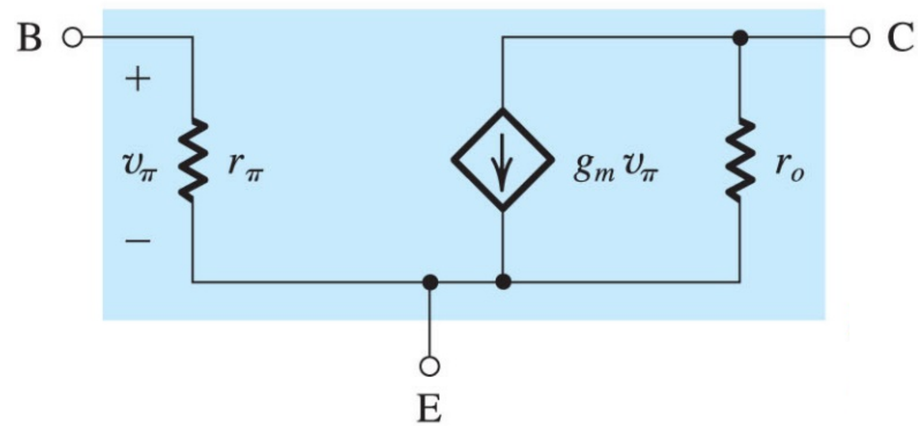
PNP small-signal circuit model is identical to NPN

Summary of transistor small signal models

NMOS/PMOS



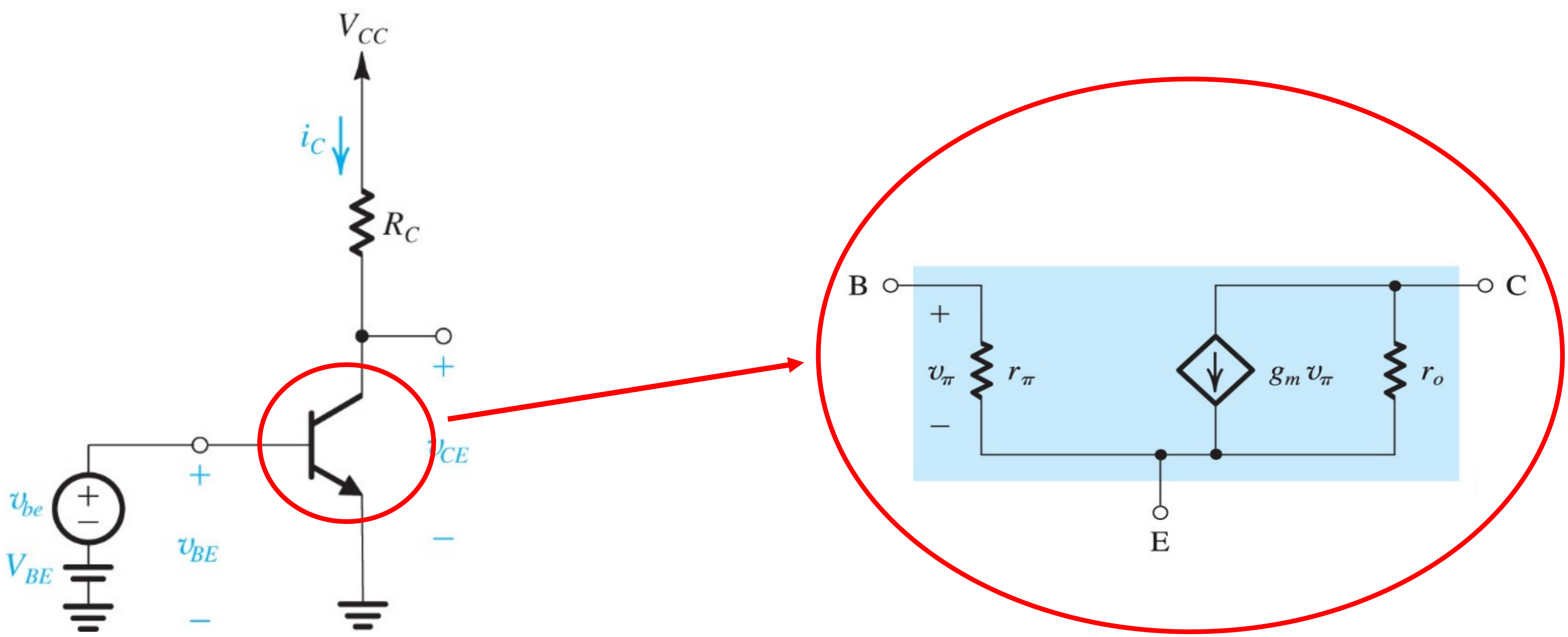
NPN/PNP BJT



Comparison of MOS and BJT small-signal circuit models:

1. MOS has an infinite resistor in the input (v_{gs}) while BJT has a finite resistor, r_π (typically several kΩ).
2. BJT g_m is substantially larger than that of a MOS (BJT has a much higher gain).
3. r_o values are typically similar (10s of kΩ).

BJT Small Signal Model



Review of amplifier circuit analysis

- Under small signal approximation, we can analyze the **Signal** and **Bias** circuits separately for a given amplifier circuit.

- The **Signal** and **Bias** circuits are different.

- **Bias** is the state of the system when there is no signal. In drawing and analyzing the **Bias** circuit, the capacitors are open and signal sources are set to zero.

- **Signal** circuit ≡ signal equivalent circuit

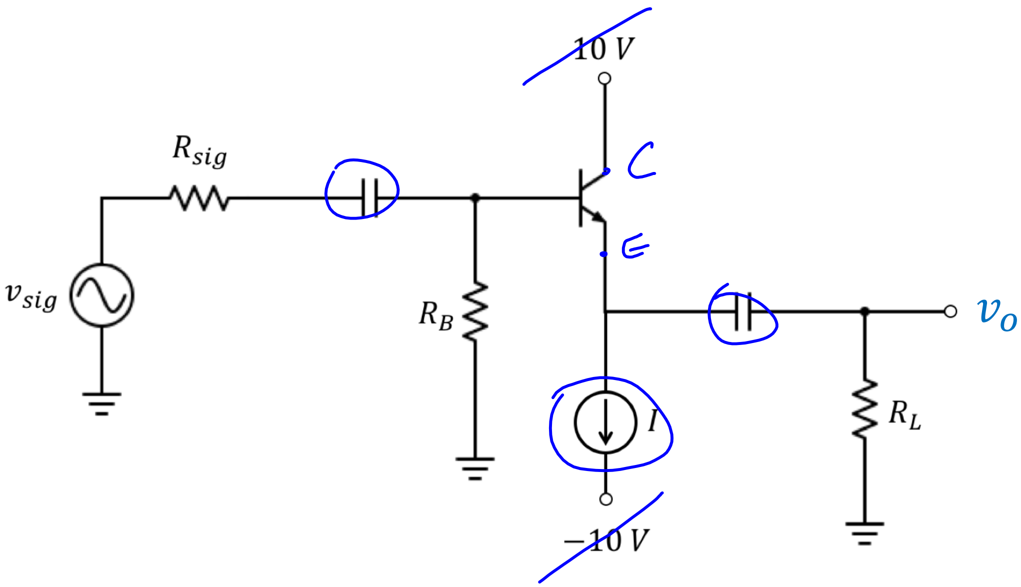
  ≡ small signal equivalent circuit

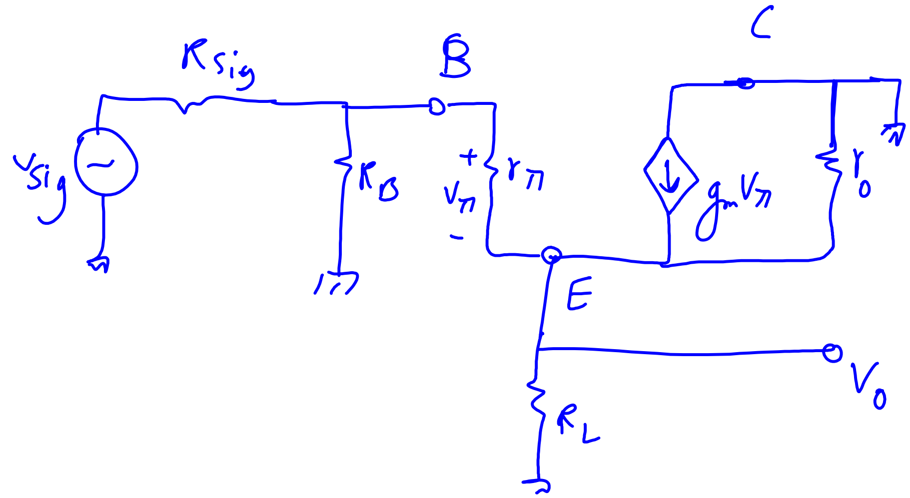
Review of amplifier analysis

- In drawing the **Signal** circuit you should
  - replace the transistors with their small signal models without changing anything in the model
  - keep the resistors – find the node they are connected to in the original circuit and connect them to the right node in the signal circuit.
  - short the capacitors
  - set the independent DC current and voltage sources to zero.

Example:

Draw the signal circuit (assume capacitors are short for signal).

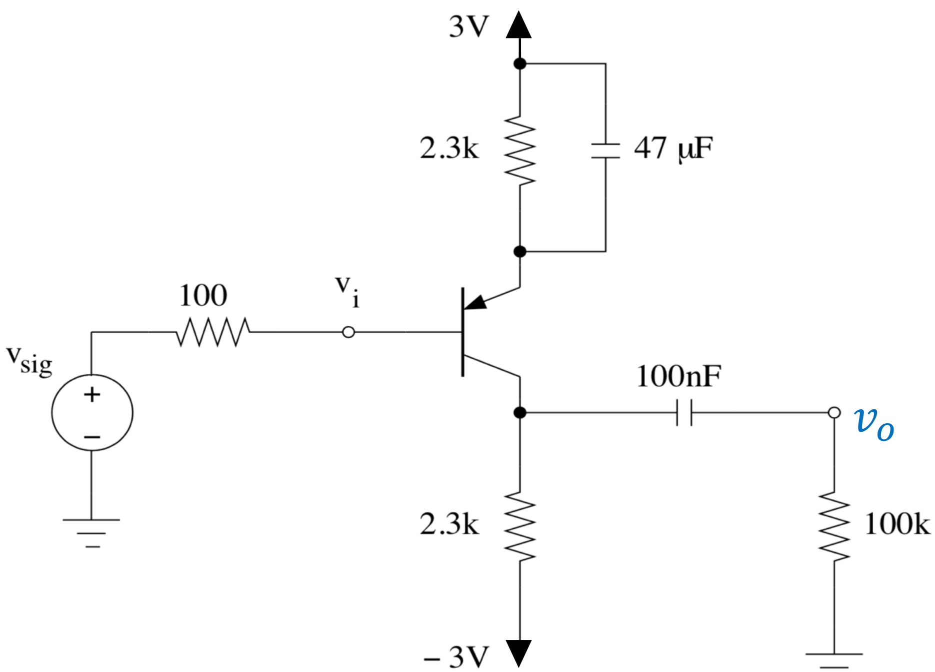


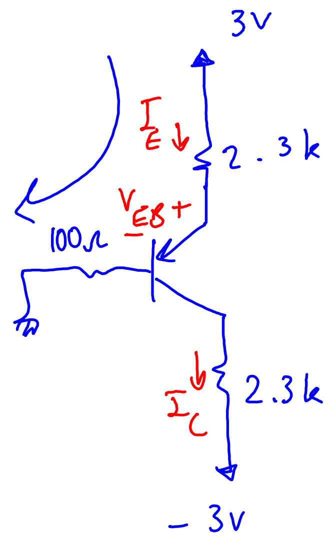


Lecture 19 reading quiz:

Find the transconductance, g_m , in this circuit ($V_{D0} = 0.7\ V,\ V_T = 26\ \text{mV}$

$V_A = 150\ V$.





EB KVL: $3V = 2.3k \times I_E + V_{EB} + 0.1k\, I_B$

BJT is in active: $I_B = \dfrac{I_E}{1+\beta} = \dfrac{I_E}{201}$

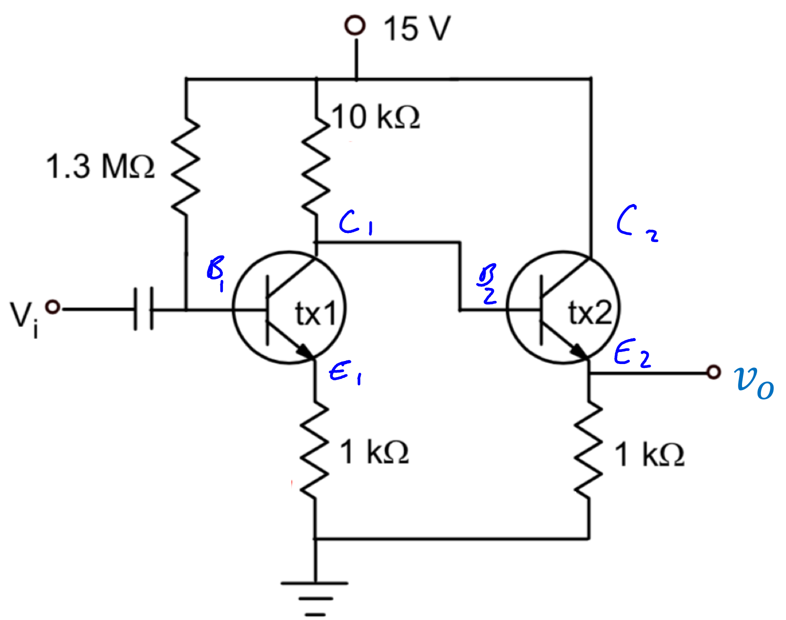
EB KVL: $3V \simeq 2.3k\Omega \times I_E + 0.7V \rightarrow I_E \simeq 1mA$ and $I_c \simeq I_E$

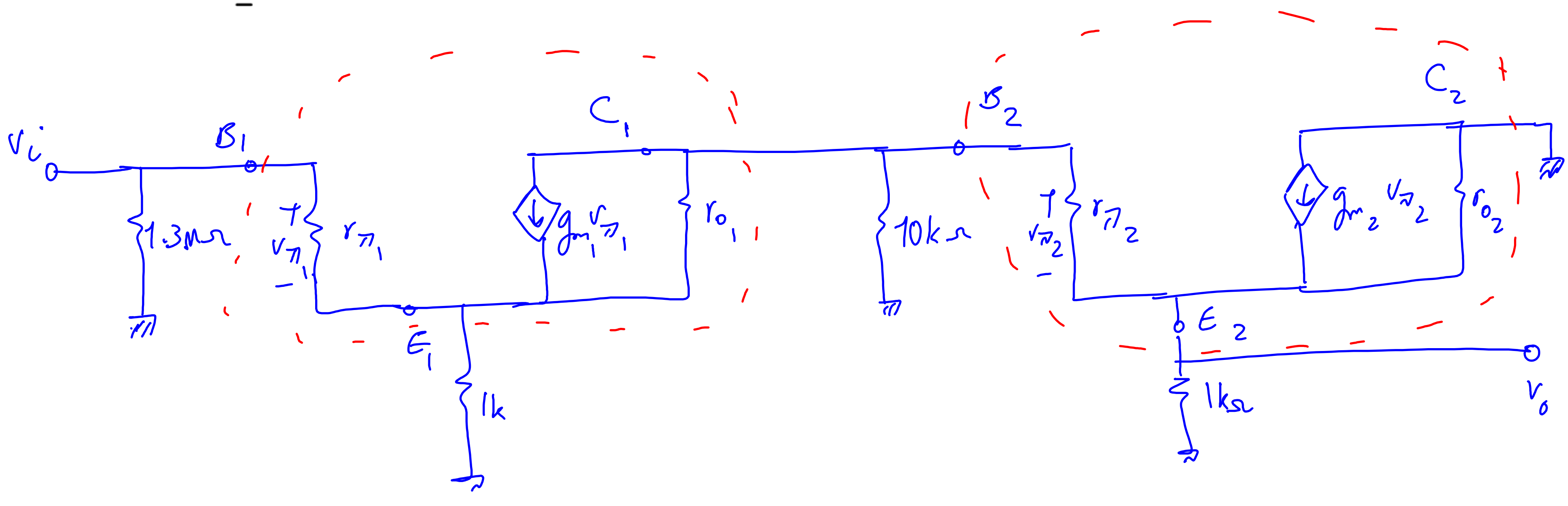
EC KVL: $3V = 2.3k \times I_E + V_{EC} + 2.3k \times I_c - 3V \longrightarrow V_{EC} = 1.3V > 0.7V$

$g_m = \dfrac{I_c}{V_T} = \dfrac{1\times10^{-3}\,A}{26\times10^{-3}\,V} = 38.5\ mA/V$

Discussion question 1.

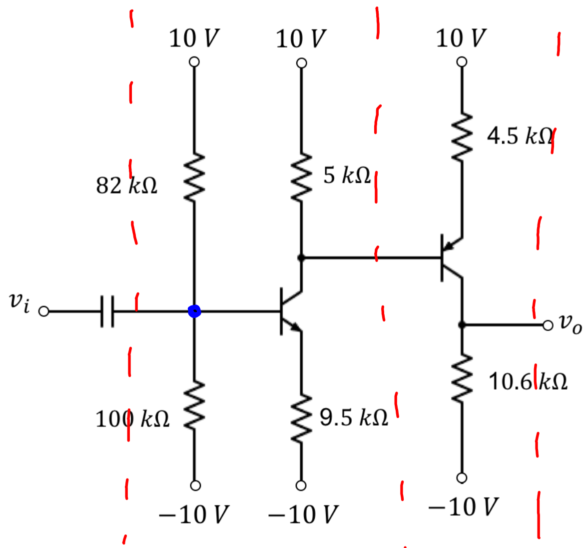
Draw the signal circuit (assume capacitors are short for signal).

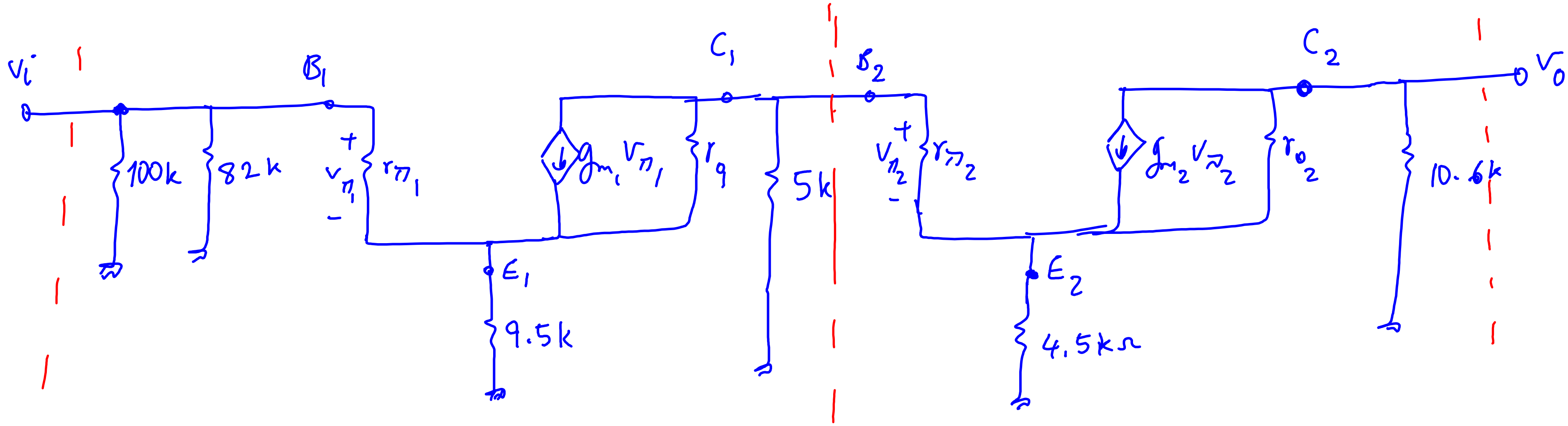




Discussion question 2.

Draw the signal circuit (assume capacitors are short for signal).

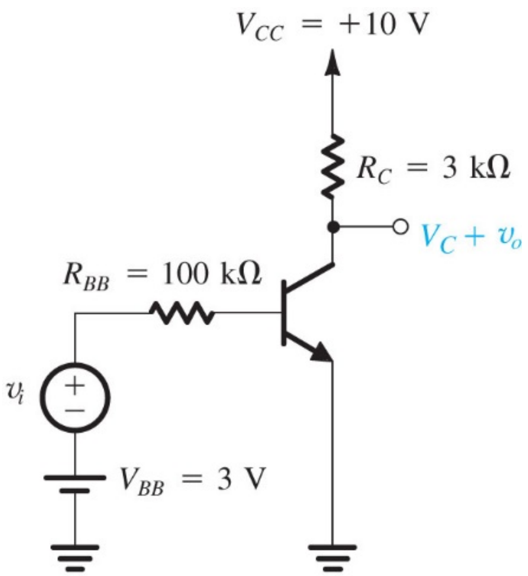




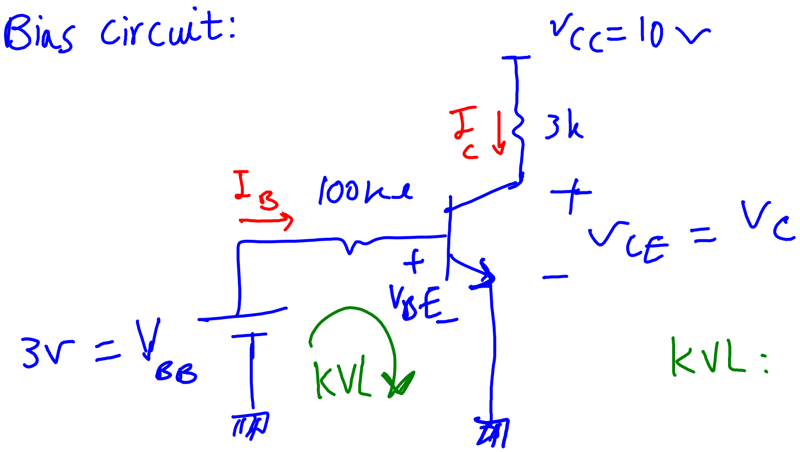
Discussion question 3

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

Assume $\beta = 100$, $V_T = 25\ mV$ and neglect the early effect.



Bias circuit:



KVL: $-V_{BB} + 100k \times I_B + V_{BE} = 0$

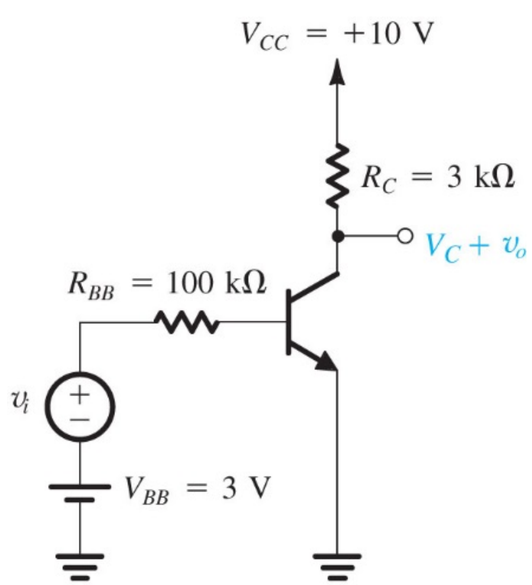
$$I_B = \frac{3 - 0.7}{100\ k\Omega} = 0.023\,mA$$

$$I_C = \beta I_B = 100 \times 0.023\,mA = 2.3\,mA$$

Discussion question 3

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

Assume $\beta = 100, V_T = 25\ mV$ and neglect the early effect.



small signal parameters:

$$g_m = \frac{I_C}{V_T} = \frac{2.3\,mA}{25\,mV} = 0.092\ A/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.092} \simeq 1.09\ k\Omega$$

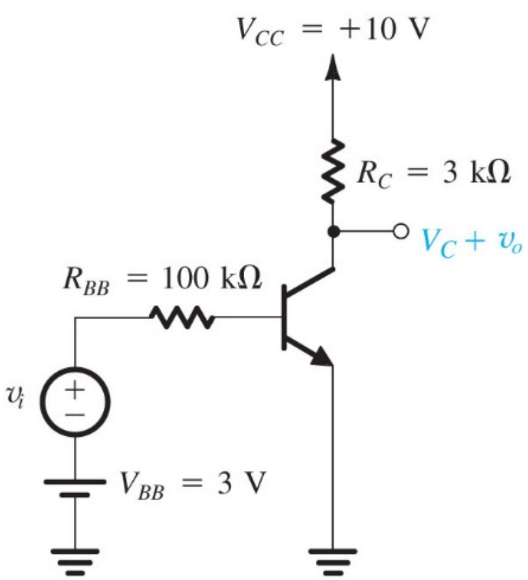
neglect the early effect: $\lambda = 0$ or $V_A = \infty$

$$r_o = \infty$$

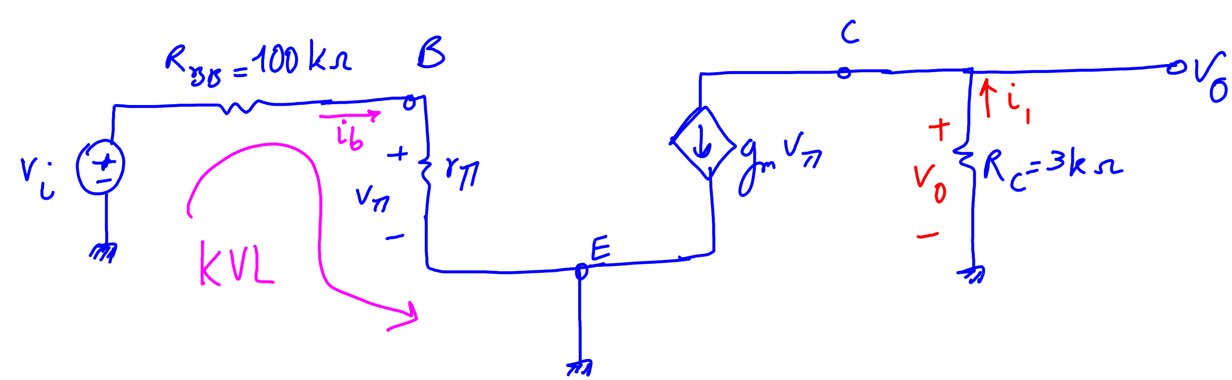
Discussion question 3

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

Assume $\beta = 100$, $V_T = 25\ mV$ and neglect the early effect. $v_\pi = v_{be}$



Signal circuit:



KCL @ node (C) : $i_1 = g_m v_\pi \longrightarrow v_o = -3k\Omega \times i_1$

$$\boxed{v_o = -3k\Omega \times g_m v_\pi}$$

KVL : $v_i = R_{BB}\, i_b + r_\pi\, i_b \rightarrow i_b = \dfrac{v_i}{R_{BB} + r_\pi}$

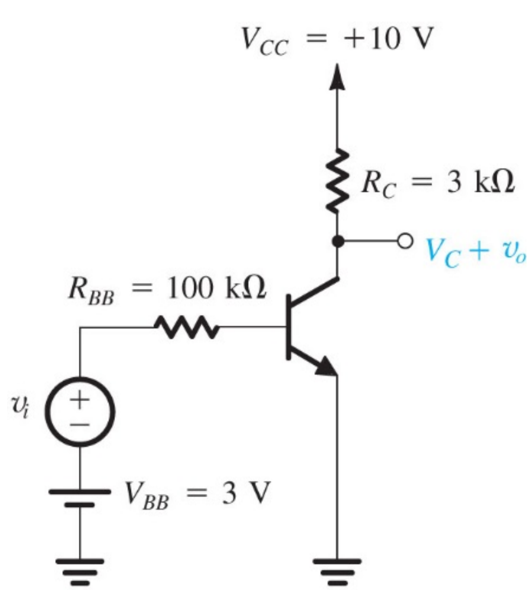
and $v_\pi = r_\pi \times i_b$

$$\longrightarrow \boxed{v_\pi = \frac{r_\pi}{r_\pi + R_{BB}} \times v_i}$$

Discussion question 3

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

Assume $\beta = 100, V_T = 25\ mV$ and neglect the early effect.



$$V_o = -3k\, g_m\, V_\pi = -276\, V_\pi \Rightarrow \boxed{V_o = -276\, V_\pi}$$

$$V_\pi = \frac{1.09\,k\Omega}{1.09\,k\Omega + 100\,k\Omega}\, V_i \Rightarrow \boxed{V_\pi = 0.011\, V_i}$$

$$\Rightarrow \boxed{A_v = \frac{V_o}{V_i} = -3.04\ V/V}$$