

AC power 002

Unlimited Attempts.

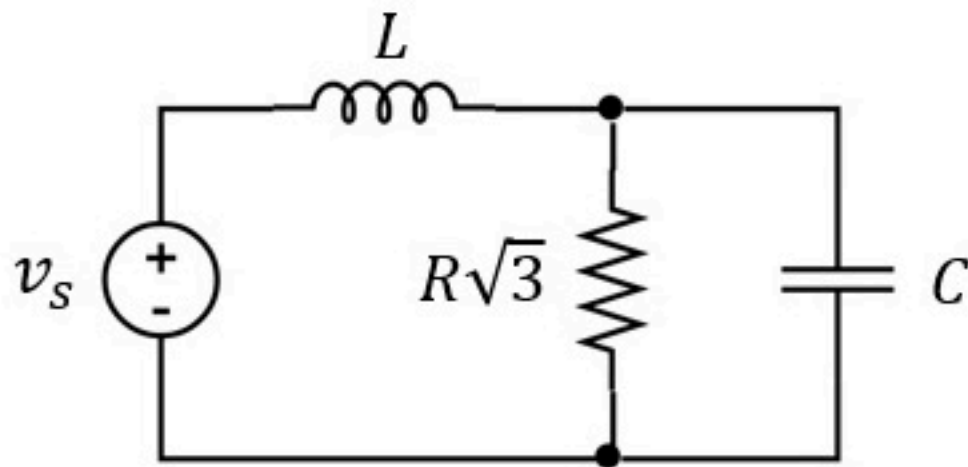
$$v_s(t) = A_1 \cos(1000t + B_1)$$

(a) Find the instantaneous power supplied by the power supply

$$p = A_2\sqrt{3} + A_3 \cos(2000t + B_3) \quad \text{with } -180^\circ < B_3 \leq 180$$

(b) Find the instantaneous power received by the inductor

$$p = A_4\sqrt{3} + A_5 \cos(2000t + B_5) \quad \text{with } -180^\circ < B_5 \leq 180$$



Given Variables:

A1 : 6 V

B1 : 45 degrees

R : 2 ohm

C : 500 uF

L : 1 mH

Calculate the following:

A2 (W) :

9

✓

A3 (W) :

18

✓

B3 (degrees) :

120

✓

A4 (W) :

0

✓

A5 (W) :

18

✓

B5 (degrees) :

-120

✓

Hint: Find the $v(t)$ and $i(t)$, or use the equations (video lecture)

$$v_s(t) = A_1 \cos(1000t + B_1)$$

$$A_1 : 8 \text{ V}$$

(a) Find the instantaneous power supplied by the power supply

$$B_1 : 135 \text{ degrees}$$

$$p = A_2 \sqrt{3} + A_3 \cos(2000t + B_3)$$

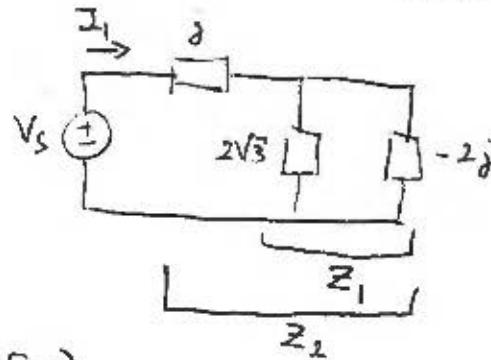
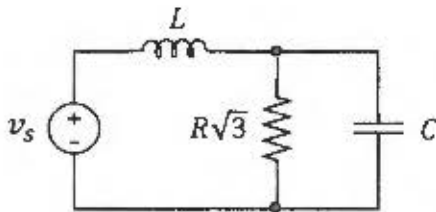
$$R : 2 \text{ ohm}$$

(b) Find the instantaneous power received by the inductor

$$C : 500 \text{ uF}$$

$$p = A_4 \sqrt{3} + A_5 \cos(2000t + B_5)$$

$$L : 1 \text{ mH}$$



$$V_s = 8 e^{j135^\circ}$$

$$Z_1 = \frac{1}{\frac{1}{2\sqrt{3}} + \frac{j}{2}} = \frac{2\sqrt{3}}{1 + \sqrt{3}j} = \frac{\sqrt{3}}{2} (1 - \sqrt{3}j)$$

$$Z_2 = Z_1 + j = \frac{\sqrt{3}}{2} - \frac{j}{2} = 1 \cdot e^{-30^\circ}$$

$$I_1 = \frac{V_s}{Z_2} = 8 e^{j165^\circ}$$

INSTANTANEOUS POWER

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$\textcircled{a} \quad V = V_s \Rightarrow V_m = 8 \text{ V} \quad \theta_v = 135^\circ$$

$$I = I_1 \Rightarrow I_m = 8 \text{ A} \quad \theta_i = 165^\circ$$

$$A_2 \sqrt{3} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \cdot 8 \cdot 8 \cdot \cos(-30^\circ) = 16\sqrt{3} \Rightarrow \boxed{A_2 = 16 \text{ W}}$$

$$A_3 = \frac{1}{2} V_m I_m = \frac{1}{2} \cdot 8 \cdot 8 \Rightarrow \boxed{A_3 = 32 \text{ W}} \quad B_3 = \theta_v + \theta_i = 300^\circ \quad \boxed{B_3 = -60^\circ}$$

$$\textcircled{b} \quad I = I_1 \Rightarrow I_m = 8 \text{ A} \quad \theta_i = 165^\circ$$

$$V = j I_1 \Rightarrow V_m = 8 \text{ V} \quad \theta_v = 165^\circ + 90^\circ = 255^\circ$$

$$A_4 \sqrt{3} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \cdot 8 \cdot 8 \cos(90^\circ) = 0 \Rightarrow \boxed{A_4 = 0 \text{ W}}$$

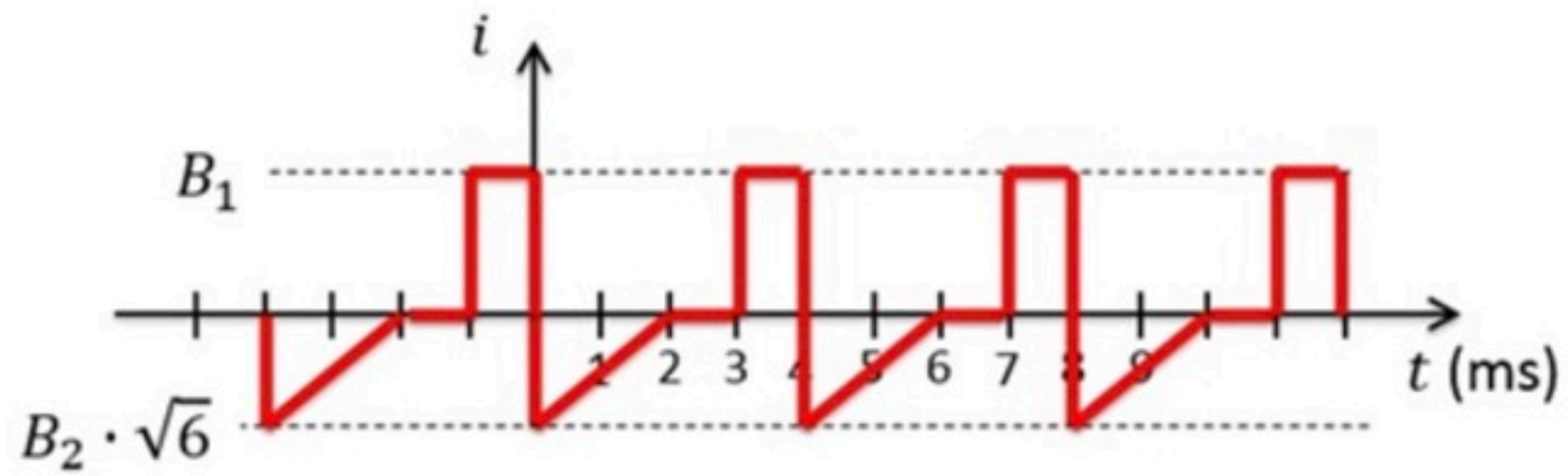
$$A_5 = \frac{1}{2} V_m I_m = \frac{1}{2} \cdot 8 \cdot 8 \Rightarrow \boxed{A_5 = 32 \text{ W}} \quad B_5 = \theta_v + \theta_i = 420^\circ$$

$$\boxed{B_5 = 60^\circ}$$

AC power 003

Problem has been graded.

Find I_{rms} for this waveform.



Given Variables:

B_1 : 6 A

B_2 : -4 A

Calculate the following:

I_{rms} (A) :

5

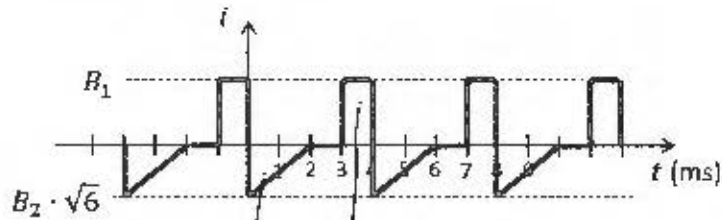


Hint: Square the time waveform, integrate, average and take sqrt()

Find I_{rms} for this waveform.

B1: 6 A

B2: -4 A



$$i(t) = 6 \Rightarrow i(t)^2 = 36$$

$$i(t) = 2\sqrt{6}(t-2) \Rightarrow i(t)^2 = 24(t-2)^2$$

$$\int_0^4 i^2 dt = \int_0^2 24(t-2)^2 dt + \int_2^4 36 dt$$

$$= 24 \left[\frac{(t-2)^3}{3} \right]_0^2 + 36$$

$$= 8 \cdot (0 - (-2)^3) + 36$$

$$= 64 + 36$$

$$= 100$$

$$\sqrt{\frac{1}{4} \int_0^4 i^2 dt} = \sqrt{\frac{1}{4} \cdot 100} = 5 \Rightarrow \boxed{I_{rms} = 5A}$$

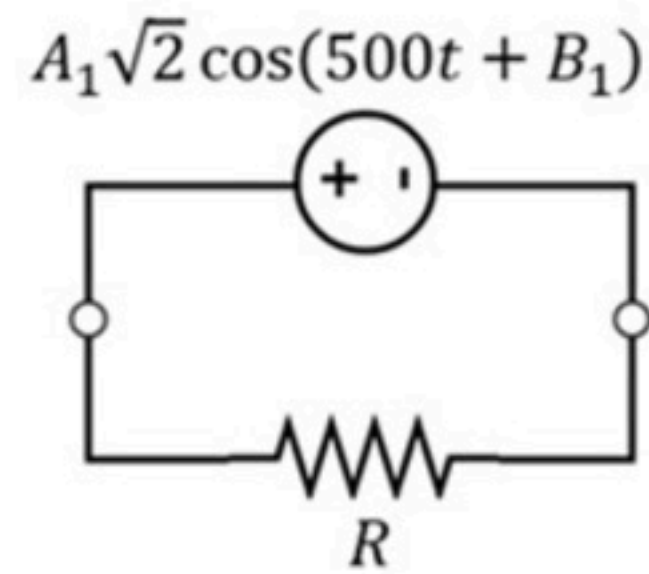
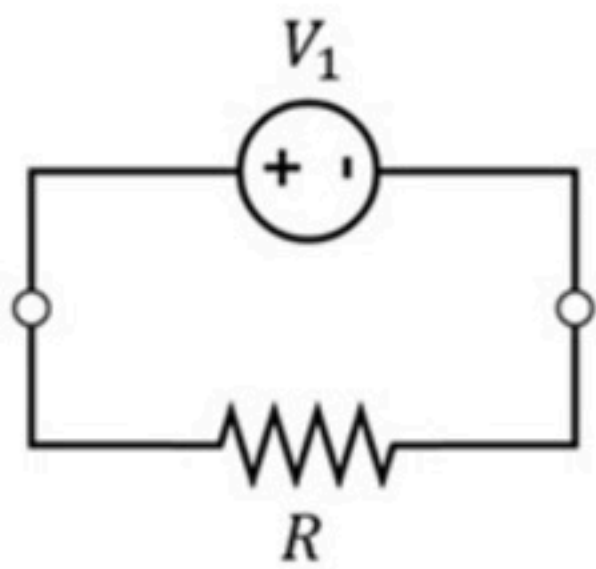
AC power 004

Problem has been graded.

You are told that the sources in the two circuits below supply the same average power.

(a) Find the average power P supplied.

(b) Find A_1 .



Given Variables:

$V_1 : 10 \text{ V}$

$R : 5 \text{ ohm}$

$B_1 : 135 \text{ degrees}$

Calculate the following:

$P \text{ (W)} :$

20



$A_1 \text{ (V)} :$

10



Hint: What is the definition of V_{rms} ?

You are told that the sources in the two circuits below supply the same average power.

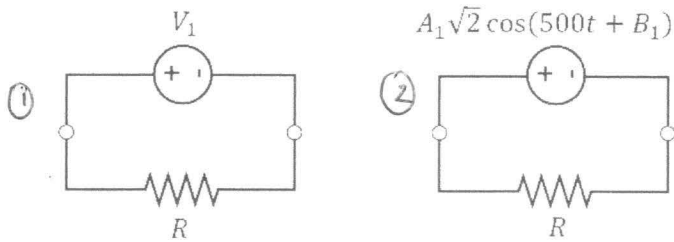
(a) Find the average power P supplied.

(b) Find A_1 .

$$V_1 : 6 \text{ V}$$

$$R : 2 \text{ ohm}$$

$$B_1 : 60 \text{ degrees}$$



(a) CIRCUIT ① : $P = \frac{V_1^2}{R} = \frac{36}{2} \Rightarrow \boxed{P = 18 \text{ W}}$

(b) FOR THE SAME AVERAGE POWER

$$V_{\text{RMS}} = V_1 = 6 \text{ V}$$

$$V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} = \frac{A_1 \sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \boxed{A_1 = 6 \text{ V}}$$

AC power 005

0 of 5 attempts made

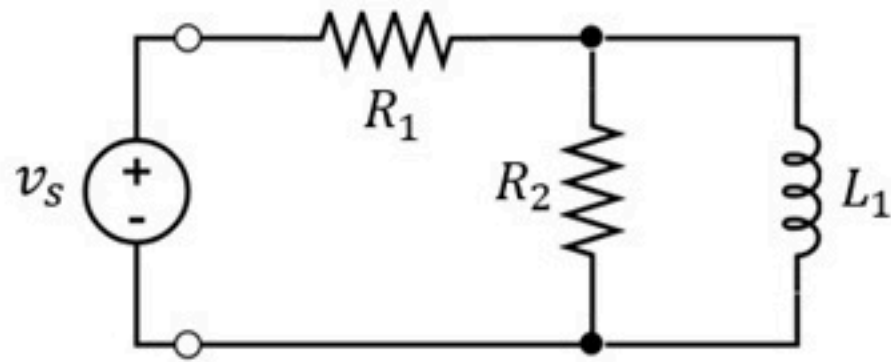
$$v_s(t) = A_1 \cos(500t + B_1)$$

Find the complex power $\mathbf{S}_1 = a_1 + b_1j$ received by the source v_s .

Find the complex power $\mathbf{S}_2 = a_2 + b_2j$ received by the resistor R_1 .

Find the complex power $\mathbf{S}_3 = a_3 + b_3j$ received by the resistor R_2 .

Find the complex power $\mathbf{S}_4 = a_4 + b_4j$ received by the inductor L_1 .



Given Variables:

A1 : 6 V

B1 : 45 degrees

R1 : 4 ohm

R2 : 4 ohm

L1 : 8 mH

Calculate the following:

a1 (W) :

-2.7



b1 (VAR) :

-0.9



a2 (W) :

1.8



b2 (VAR) :

0



a3 (W) :

0.9



b3 (VAR) :

0



a4 (W) :

0



b4 (VAR) :

0.9



$$v_s(t) = A_1 \cos(500t + B_1)$$

Find the complex power $S_1 = a_1 + b_1j$ received by the source v_s .

Find the complex power $S_2 = a_2 + b_2j$ received by the resistor R_1 .

Find the complex power $S_3 = a_3 + b_3j$ received by the resistor R_2 .

Find the complex power $S_4 = a_4 + b_4j$ received by the inductor L_1 .

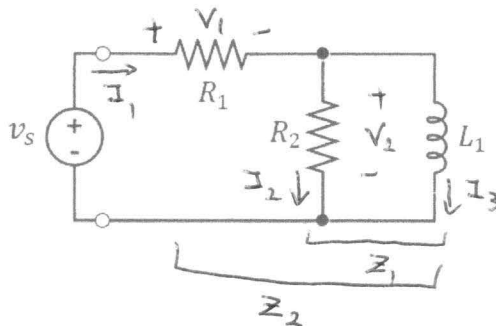
$$A_1 : 3 \text{ V}$$

$$B_1 : 30 \text{ degrees}$$

$$R_1 : 1 \text{ ohm}$$

$$R_2 : 2 \text{ ohm}$$

$$L_1 : 4 \text{ mH}$$



$$Z_L = 2j$$

$$Z_1 = \frac{1}{\frac{1}{2} - \frac{j}{2}} = \frac{2}{1-j} = 1+j$$

$$Z_2 = Z_1 + 1 = 2+j$$

$$V_s = 3e^{j30^\circ}$$

$$I_1 = \frac{V_s}{Z_2}$$

$$V_1 = I_1$$

$$V_2 = Z_1 I_1$$

$$I_2 = \frac{V_1}{2}$$

$$I_3 = \frac{V_1}{2j}$$

$$\textcircled{a} S_1 = \frac{1}{2} V_s (-I_1^*) = -\frac{1}{2} \frac{V_s \cdot V_s^*}{Z_2^*} = -\frac{|V_s|^2}{2} \frac{Z_2}{|Z_2|^2} = -\frac{9}{2} \frac{(2+j)}{5}$$

$$a_1 = -1.8 \text{ W}$$

$$b_1 = -0.9 \text{ VAR}$$

$$\textcircled{b} S_2 = \frac{1}{2} V_1 I_1^* = \frac{I_1 I_1^*}{2} = \frac{|I_1|^2}{2} = \frac{|V_s|^2}{2|Z_2|^2} = \frac{9}{2 \cdot 5}$$

$$a_2 = 0.9 \text{ W}$$

$$b_2 = 0 \text{ VAR}$$

$$\textcircled{c} S_3 = \frac{1}{2} V_2 I_2^* = \frac{1}{2} V_2 \frac{V_1^*}{2} = \frac{|V_2|^2}{4} = \frac{|Z_1|^2 |I_1|^2}{4} = \frac{|Z_1|^2}{4} \frac{|V_s|^2}{|Z_2|^2} = \frac{2 \cdot 9}{4 \cdot 5}$$

$$a_3 = 0.9 \text{ W}$$

$$b_3 = 0 \text{ VAR}$$

$$\textcircled{d} S_4 = \frac{1}{2} V_2 I_3^* = \frac{1}{2} V_2 \frac{V_1^*}{(2j)^*} = j \frac{|V_2|^2}{4} = j \frac{|Z_1|^2}{4} \frac{|V_s|^2}{|Z_2|^2} = j \frac{2 \cdot 9}{4 \cdot 5}$$

$$a_4 = 0 \text{ W}$$

$$b_4 = 0.9 \text{ VAR}$$

$$\text{CHECK: } S_1 + S_2 + S_3 + S_4 = -1.8 + 0.9 + 0.9 + 0 = 0$$

$$-0.9j + 0j + 0j + 0.9j = 0$$

$$(\sum \text{REC.} = \sum \text{SUPPL.})$$

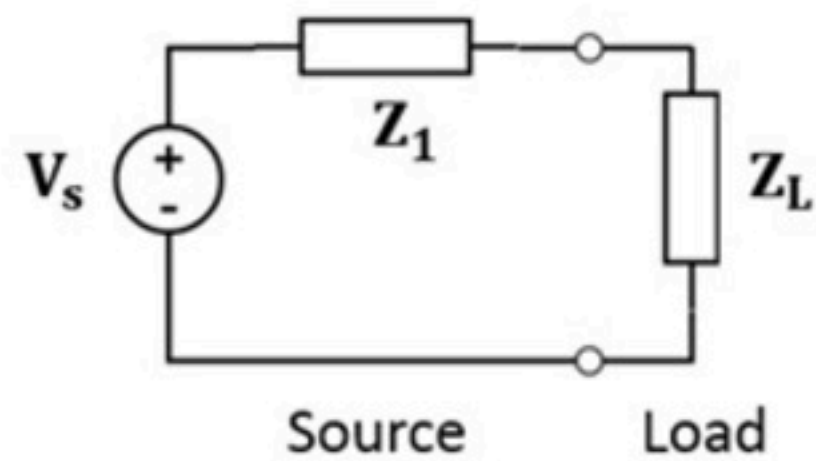
AC power 006

No more attempts left.

For the system below, the source is represented in phasor-notation as:

$$\mathbf{Z}_1 = A_1 + jB_1 \quad \mathbf{V}_S = A_2 \cdot e^{jB_2}$$

- (a) Find the load impedance $\mathbf{Z}_L = A_3 + jB_3$ that results in the maximum power being received by this load.
- (b) Find the maximum average power P received by the load.



Given Variables:

A_1 : 5 ohm

B_1 : 4 ohm

A_2 : 2 V

B_2 : 75 degrees

Calculate the following:

A_3 (ohm) :

5



B_3 (ohm) :

-4



P (W) :

0.1



Hint: Keep your calculations algebraic as long as you can.

For the system below, the source is represented in phasor-notation as:

$$\mathbf{Z}_1 = A_1 + jB_1 \quad \mathbf{V}_S = A_2 \cdot e^{jB_2}$$

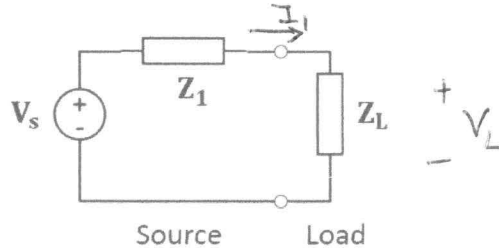
A1 : 1 ohm

B1 : -8 ohm

A2 : 4 V

B2 : 65 degrees

- (a) Find the load impedance $\mathbf{Z}_L = A_3 + jB_3$ that results in the maximum power being received by this load.
 (b) Find the maximum average power P received by the load.



(a) $\mathbf{Z}_L = \mathbf{Z}_1^* \Rightarrow \mathbf{Z}_L = 1 + j8$

$A_3 = 1 \Omega$
$B_3 = 8 \Omega$

(b) $\mathbf{I}_1 = \frac{\mathbf{V}_S}{\mathbf{Z}_1 + \mathbf{Z}_L} \quad \mathbf{Z}_1 + \mathbf{Z}_L = 2$

$\mathbf{V}_S = 4 e^{j65^\circ} \quad \mathbf{V}_L = \mathbf{Z}_L \cdot \mathbf{I}_1$

$$S_L = \frac{1}{2} \mathbf{V}_L \cdot \mathbf{I}_1^* = \frac{1}{2} \mathbf{Z}_L \cdot \mathbf{I}_1 \cdot \mathbf{I}_1^* = \frac{\mathbf{Z}_L}{2} |\mathbf{I}_1|^2 = \frac{\mathbf{Z}_L}{2} \frac{|\mathbf{V}_S|^2}{|\mathbf{Z}_1 + \mathbf{Z}_L|^2}$$

$$= \frac{(1+j8)}{2} \cdot \frac{4^2}{2^2} = (1+j8) \cdot 2 = \underbrace{2}_P + \underbrace{16j}_Q$$

$P = 2 \text{ W}$

AC power 008

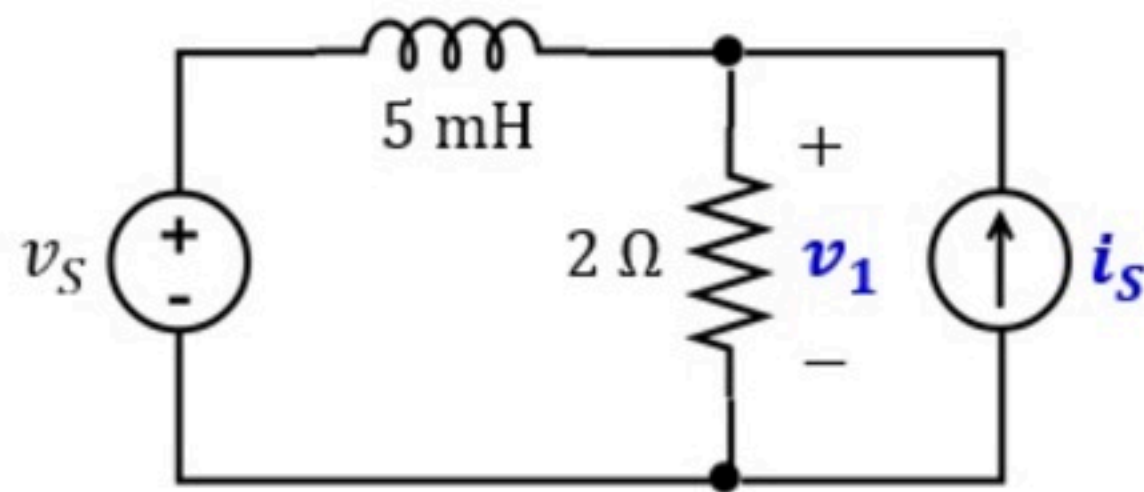
Problem has been graded.

Both v_S and i_S are AC sources with $\omega = 400$ rad/s. Someone did two measurements on the circuit and found:

$$v_1(t) = A_1 \cos(400t + B_1) \quad (\text{this is } v_L, \text{ not } v_S !)$$

$$i_S(t) = 2 \cos\left(400t + \frac{\pi}{6}\right)$$

- (a) Find the average power P_1 received by the resistor
- (b) Find the average power P_2 supplied by current source i_S
- (c) Find the average power P_3 received by the inductor



Given Variables:

A1 : 8 V

B1 : 90 degrees

Calculate the following:

P1 (W) :

16

✓

P2 (W) :

4

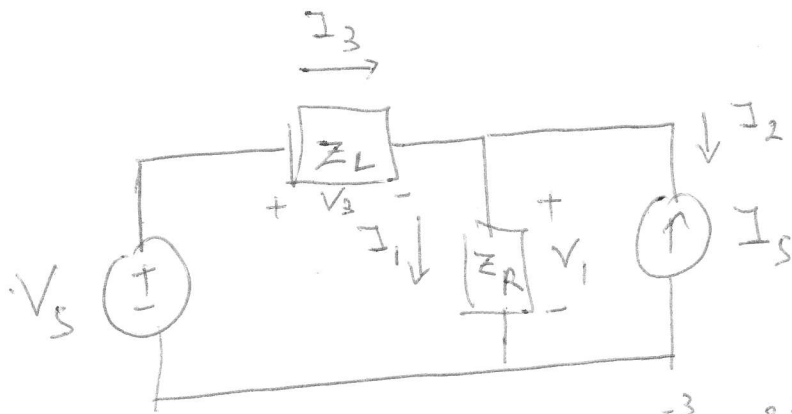
✓

P3 (W) :

0

✓

Hint: Find S symbolically first. Find phasors and plug in. For c, what do you know about the element?



$$A_1: 10 \text{ V}$$

$$B_1: -90^\circ$$

$$Z_L = j\omega L = j \cdot 400 \cdot 5 \cdot 10^{-3} = 2j$$

$$Z_R = 2$$

(a) $S_1 = \frac{1}{2} V_1 \cdot I_1^*$ $I_1 = \frac{V_1}{Z_R}$

$$= \frac{1}{2} V_1 \left(\frac{V_1}{Z_R} \right)^* = \frac{1}{2} \frac{V_1 V_1^*}{Z_R} = \frac{|V_1|^2}{2} \cdot \frac{1}{Z_R} = \frac{A_1^2}{2} \cdot \frac{1}{Z_R} = \frac{100}{2 \cdot 2} = 25$$

$P_1 = \text{Re}[S_1]$ $P_1 = 25 \text{ W}$ received

(b) $S_2 = \frac{1}{2} V_1 I_2^*$ \leadsto PASSIVE SIGN CONVENTION (POWER RECEIVED)

$$= \frac{1}{2} V_1 (-I_S)^*$$

$$= -\frac{1}{2} V_1 I_S^*$$

$$= -\frac{1}{2} \cdot 10 e^{-j\frac{\pi}{2}} \cdot 2 e^{-j\frac{\pi}{6}} = -10 e^{-j\frac{4\pi}{6}} = -10 e^{-j\frac{2\pi}{3}}$$

$P_2 = \text{Re}[S_2] = -10 \cos(-\frac{2\pi}{3}) = -10 \left(-\frac{1}{2}\right) = +5 \leadsto$ received

$P_2 = -5 \text{ W}$ supplied

(c) $P_3 = 0 \text{ W}$ inductor always has $P=0$

we can check: $S_3 = \frac{1}{2} V_3 I_3^*$ $V_3 = Z_L \cdot I_3$

$$= \frac{1}{2} Z_L I_3 \cdot I_3^* = \frac{Z_L}{2} |I_3|^2 = \frac{|I_3|^2}{2} \cdot (2j)$$

no real part $\Rightarrow P = \text{Re}[S] = 0$