Problem 1:

What value of C makes the period of the signal $(1 + \cos(Ct))^2$ equal to 6π ?

(a) $1/3$

(b) 2

(c) $2/3$

(d) $1/6$

(e) 3π

(f) $\pi/6$

(g) $\pi/3$

(h) 3

(i) 6

(j) None of these

Problem 2:

What is the impedance of a parallel combination of a 5 microfarad capacitor and a 1 microhenry inductor with a 500 kilohertz sinusoidal voltage across it?

(a) $\left(\frac{\pi}{1-5\pi^2}\right) j$

(b) $\left(\frac{\pi}{5\pi^2-1}\right) j$

(c) $\left(\frac{\pi}{1+5\pi^2}\right) j$

(d) $\left(\frac{\pi}{5\pi^2+1}\right) j$

(e) $2j$

(f) $-2j$

(g) $-j/2$

(h) $j/2$

(i) 2

(j) None of these

Problem 3:

If a periodic signal $f(t)$ with period 3 has exponential Fourier Series coefficients

$$F_n = \begin{cases} 2 & \text{if } |n| \leq 5 \\ 0 & \text{else} \end{cases}$$

then what is $\int_0^3 |f(t)|^2 dt$?

(a) 132

(b) 66

(c) 120

(d) 60

(e) 44

(f) 22

(g) 40

(h) 20

(i) None of these

Problem 4:

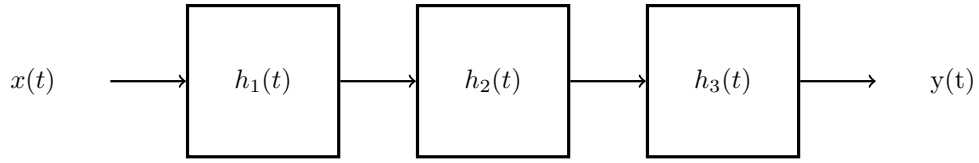
What is the value of

$$\int_{\sqrt{2}}^{\sqrt{5}} \frac{t^2 \delta(t-2) e^t}{t+1} dt?$$

1. $4e^2/3$
2. $4e^2$
3. $e^2/3$
4. e^2
5. $4/3$
6. $4e/3$
7. 0
8. $\sqrt{5} - \sqrt{2}$
9. e^{-2}
10. None of these

Problem 5:

Suppose the signal $x(t) = \cos(4t)$ is the input to the cascade of three LTI systems, with impulse responses $h_1(t) = 2 \cdot \text{sinc}(5t)$, $h_2(t) = 3 \cdot \text{sinc}(3t)$, and $h_3(t) = 4 \cdot \text{sinc}(6t)$, respectively, as shown below. What is the output signal $y(t)$?



(a) 0

(b) $\cos(4t)$

(c) $24\cos(4t)$

(d) $12\cos(4t)$

(e) $\cos(18t)$

(f) $\cos(4t)/2$

(g) $2\cos(4t)$

(h) $3\cos(4t)$

(i) $4\cos(4t)$

(j) None of these

Problem 6:

What is the inverse Fourier transform of $X(\omega) = 4\pi\delta(\omega) - 2\pi\delta(\omega - 2\pi) + 2\pi\delta(\omega + 2\pi)$?

(a) $2(1 - j\sin(2\pi t))$

(b) $2(1 - \sin(2\pi t))$

(c) $2(1 - \cos(2\pi t))$

(d) $2(1 - j\cos(2\pi t))$

(e) $2(1 + j\sin(2\pi t))$

(f) $2(1 + j\cos(2\pi t))$

(g) $1 - j\sin(2\pi t)$

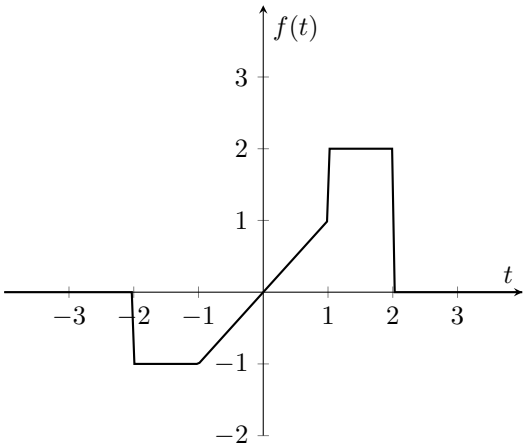
(h) $2\pi(1 - j\sin(2\pi t))$

(i) $2(1 + j\sin(2\pi t))$

(j) None of these

Problem 7:

If $F(\omega)$ is the Fourier transform of the piecewise linear function $f(t)$ shown below, then what is $F(0)$?



(a) 1

(b) 0

(c) -1

(d) 2

(e) -2

(f) 0.5

(g) 1.5

(h) -0.5

(i) -1.5

(j) None of these

Problem 8:

An LTI system with impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. What would the output be if an input $x(t-3\tau)$ were applied to a system whose impulse response is $h(t-5\tau)$?

(a) $y(t-8\tau)$

(b) $y(t+8\tau)$

(c) $y(t+2\tau)$

(d) $y(t+2\tau)$

(e) $y(2t-8\tau)$

(f) $y(8\tau-t)$

(g) $y(2\tau-t)$

(h) $y(2t-8\tau)$

(i) $y(2t-2\tau)$

(j) None of these

Problem 9:

If a linear time-invariant system has frequency response $H(\omega) = \omega e^{j(\omega+(\pi/2))}$, then what is the output at time $t = -1$ if the input is $x(t) = 3e^{4jt}$?

(a) $12j$

(b) 12

(c) $3e^{-4j}$

(d) 4

(e) $4j$

(f) 3

(g) $3j$

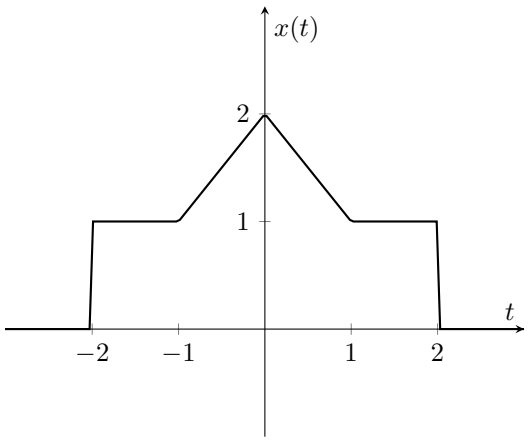
(h) $12e^{-4j}$

(i) 6

(j) None of these

Problem 10:

The piecewise linear signal $x(t)$ is shown below. What is $X(0)$, where $X(\omega)$ is the Fourier transform of $x(t)$?



(a) 5

(b) 4

(c) 6

(d) 5π

(e) $5/2\pi$

(f) 0

(g) 10π

(h) 6π

(i) $9/2$

(j) None of these

Problem 11:

The output of an LTI system is $5\cos(3t) + 8\sin(3t)$, when the input is the unit step function $u(t)$. What is the impulse response of the system?

(a) $-15\sin(3t) + 24\cos(3t)$

(b) $5\sin(3t) + 8\cos(3t)$

(c) $15\sin(3t) + 24\cos(3t)$

(d) $-5\sin(3t) + 8\cos(3t)$

(e) $8\sin(3t) + 5\cos(3t)$

(f) $8\sin(3t) - 5\cos(3t)$

(g) 0

(h) $(5/3)\sin(3t) - (8/3)\cos(3t)$

(i) $(5/3)\sin(3t) + (8/3)\cos(3t)$

(j) None of these

Problem 12:

Which one of the statements below is true? The function $x(t) = \sin(3t) + \sin(\pi t)$ has an exponential Fourier Series $\sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$ where

(a) None of these.

(b) $X_1 = -1/(2j)$

(c) $X_1 = 1/(2j)$

(d) $X_1 = -1/2$

(e) $X_1 = 1/2$

(f) $X_2 = -1/(2j)$

(g) $X_2 = 1/(2j)$

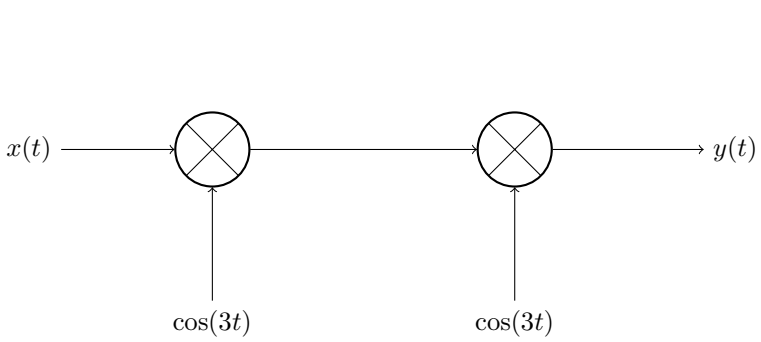
(h) $X_2 = -1/2$

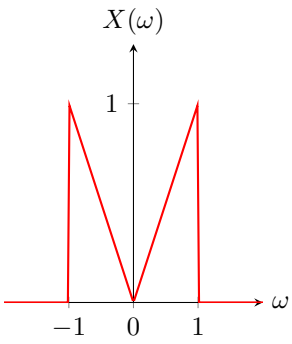
(i) $X_2 = 1/2$

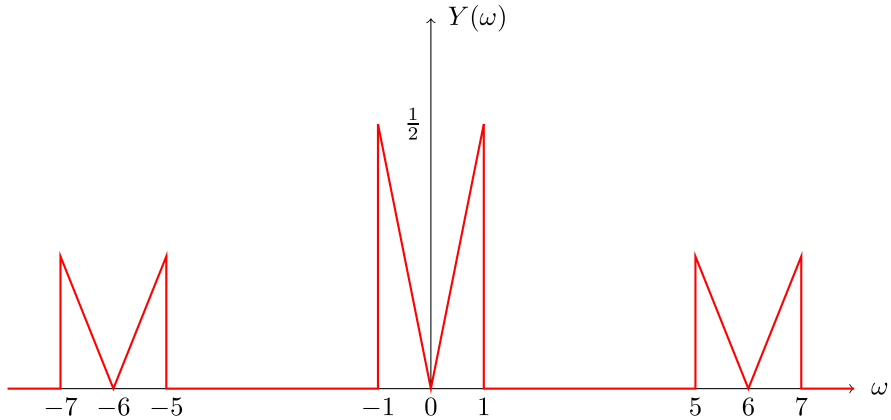
(j) $X_0 = 0$

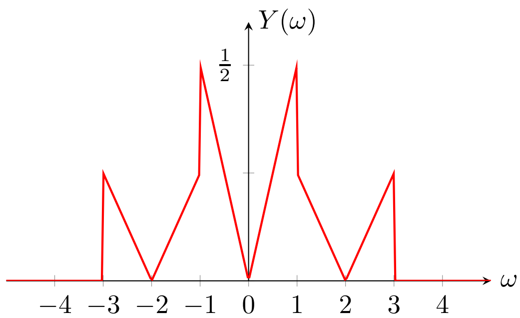
Problem 13:

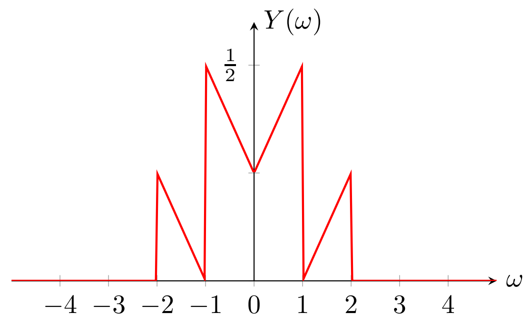
The input signal $x(t)$ in the system below with two multipliers, has Fourier transform $X(\omega)$ as shown. What is the Fourier transform $Y(\omega)$ of the output signal $y(t)$?

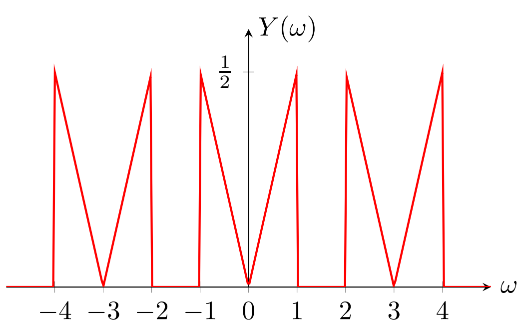


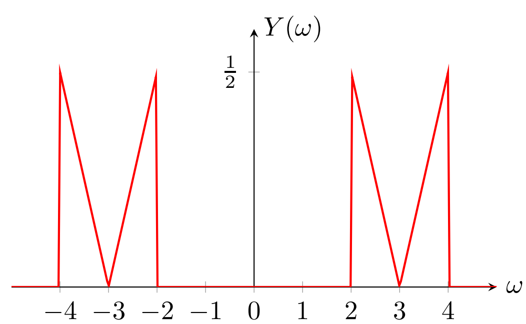


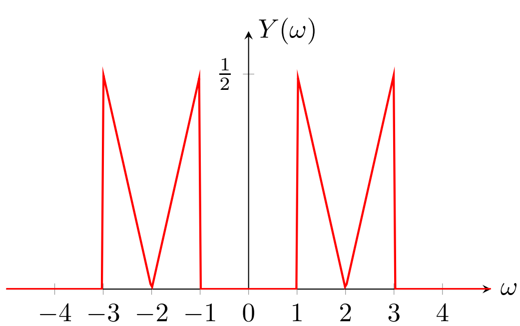

$Y(\omega)$
$\frac{1}{2}$
ω
-7 -6 -5 -1 0 1 5 6 7

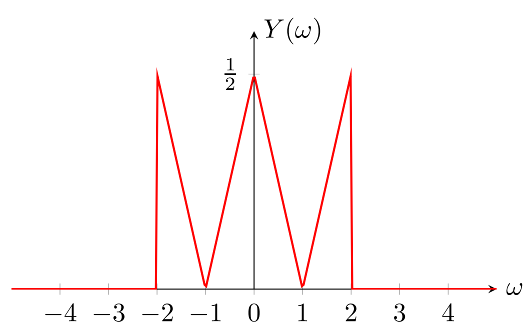

$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4


$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4


$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4


$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4


$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4


$Y(\omega)$
$\frac{1}{2}$
ω
-4 -3 -2 -1 0 1 2 3 4

(a) Figure (a)

(b) Figure (b)

(c) Figure (c)

(d) Figure (d)

(e) Figure (e)

(f) Figure (f)

(g) Figure (g)

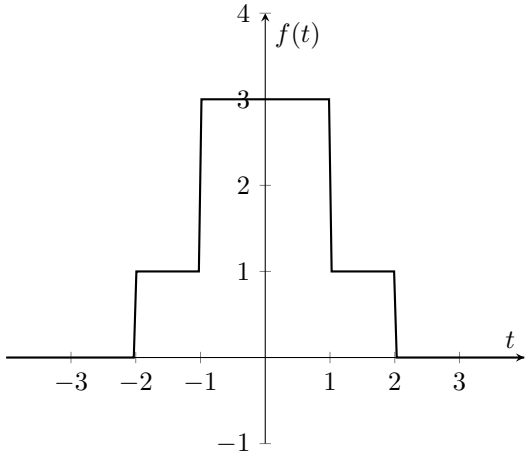
(h) $Y(\omega) = X(\omega)$.

(i) $Y(\omega) = 0$.

(j) None of these.

Problem 14:

What is the Fourier transform of the piecewise constant function $f(t)$ shown below?



(a) $4\text{sinc}(\omega) + 4\text{sinc}(2\omega)$

(b) $6\text{sinc}(\omega) + 4\text{sinc}(2\omega)$

(c) $4\text{sinc}(\omega) + 4\text{sinc}(\omega)$

(d) $(1/4)\text{sinc}(\omega) + (1/4)\text{sinc}(2\omega)$

(e) $2\text{sinc}(\omega/2) + 2\text{sinc}(\omega)$

(f) $3\text{sinc}(\omega/2) + 2\text{sinc}(\omega)$

(g) $2\text{sinc}(\omega) + 2\text{sinc}(\omega/2)$

(h) $(1/2)\text{sinc}(\omega) + (1/2)\text{sinc}(2\omega)$

(i) None of these

Problem 15:

Suppose $y(t)$ is the output of an LTI system, with impulse response $h(t) = \delta(t+1) + \delta(t-2)$, when $x(t) = \delta(t+2) + \delta(t) + \delta(t-3)$ is the input. What is $\int_{-2}^{3} y(t)dt$?

(a) 4

(b) 0

(c) 1

(d) 2

(e) 3

(f) 5

(g) 6

(h) None of these

Problem 16:

If $x(t) = f(t-4) + f(t+4)$ and the Fourier transform of $f(t)$ is $F(\omega)$, then which of the following is the Fourier transform of $x(t)$?

(a) $2\cos(4\omega)F(\omega)$

(b) $F(\omega - 4) + F(\omega + 4)$

(c) $F(\omega e^{j4\omega}) + F(\omega e^{-j4\omega})$

(d) $2\sin(4\omega)F(\omega)$

(e) $2\cos(4\omega)F(4\omega)$

(f) $2\sin(4\omega)F(4\omega)$

(g) $\cos(4\omega)F(\omega)$

(h) $\cos(\omega)F(\omega)$

(i) $2\sin(\omega)F(\omega)$

(j) None of these

Problem 17:

Suppose $x(t)$ is a periodic function with exponential Fourier Series

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\pi t/2}$$

where

$$X_n = \begin{cases} 3^{-n} & \text{if } n \geq 0 \\ 0 & \text{else.} \end{cases}$$

Determine the value of $x(t)$ when $t = 12$.

(a) 3/2

(b) 1

(c) 0

(d) 2/3

(e) 2

(f) 3

(g) 6

(h) 12

(i) 3/4

(j) None of these

Problem 18:

Let $f(t)$ be a periodic function with period 3π, and suppose the nonzero exponential Fourier series coefficients F_n are $F_{-1} = 1$, $F_1 = 2$, and $F_2 = 4$. Then what is $\int_0^2 F(\omega)d\omega$?

(a) 12π

(b) 8

(c) 6π

(d) 4

(e) 4π

(f) 3

(g) 10π

(h) 5

(i) 20π

(j) 10

(k) 0

(l) None of these

Problem 19:

Suppose $y(t)$ is the output of an LTI system, with impulse response $h(t) = u(t) - u(t-2)$, when $x(t)$ is the input. If $y(3) = 1$, then which of the following could be $x(t)$?

(a) $(1/2)(u(t-1) - u(t-3))$

(b) $u(t-1) - u(t-3)$

(c) $2(u(t-1) - u(t-3))$

(d) $(1/2)(u(t+3) - u(t+1))$

(e) $u(t+3) - u(t+1)$

(f) $2(u(t+3) - u(t+1))$

(g) None of these

Problem 20:

The input voltage $x(t)$ and output voltage $y(t)$ of a circuit in sinusoidal steady state satisfy

$$2y(t) = 4x(t) - \frac{dy(t)}{dt}.$$

What is the impulse response of the system at $t = 1$?

(a) $4/e^2$

(b) $4e^2$

(c) $2/e^4$

(d) $2e^4$

(e) $2/e^2$

(f) $2e^2$

(g) $4e^4$

(h) $4/e^4$

(i) $2/\sqrt{e}$

(j) None of these.

Problem 21:

If $x(t) = \text{sinc}(3t)$, $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - 2\pi n/3)$, and $y(t) = x(t)s(t)$, then what is the Fourier transform of $y(t)$ at $\omega = 2$, i.e., $Y(2)$?

(a) 1

(b) $\pi/3$

(c) $\pi/4$

(d) $2\pi/3$

(e) $4\pi^2/9$

(f) $3/\pi$

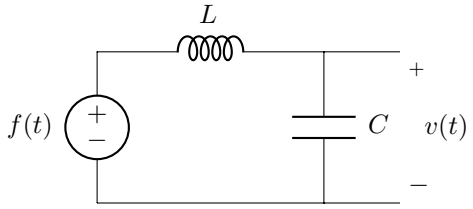
(g) $1/2$

(h) $\pi/2$

(i) None of these

Problem 22:

Consider the following sinusoidal steady-state circuit, with voltage source $f(t)$.



If $L = 2$ and $C = 4$, what is the value of the voltage $v(t)$ when $f(t) = 2\cos(t/4) + 4\cos(2t)$?

(a) $4\cos(t/4) - \frac{4}{31}\cos(2t)$

(b) $4\cos(t/4) + \frac{4}{31}\cos(2t)$

(c) $2\cos(t/4) - \frac{2}{31}\cos(2t)$

(d) $2\cos(t/4) + \frac{2}{31}\cos(2t)$

(e) $-2\cos(t/4) - \frac{4}{15}\cos(2t)$

(f) $-2\cos(t/4) + \frac{4}{15}\cos(2t)$

(g) $-\cos(t/4) - \frac{2}{15}\cos(2t)$

(h) $-\cos(t/4) + \frac{2}{15}\cos(2t)$

(i) None of these

Problem 23:

Which of the following is true about the integral:

$$\int_{-\infty}^{\infty} \cos(6(t-\tau)) \left(\frac{\sin(5\tau)}{5\tau}\right) d\tau.$$

(a) It equals 0 for all t.

(b) It never equals 0.

(c) It equals 1 whenever $0 < t < 1$.

(d) It is positive whenever $0 < t < 2$.

(e) It is positive whenever $2 < t < 3$.

(f) It is positive whenever $-1 < t < 1$.

(g) It is positive whenever $-2 < t < -1$.

(h) It is positive whenever $t = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.

(i) It equals $\cos(6t)$ for all t.

(j) None of these

Problem 24:

What is the impulse response of an LTI system whose relationship between the input $x(t)$ and output $y(t)$ is given by $y(t) = \int_{-\infty}^{t} e^{\tau - t} x(\tau) d\tau$?

(a) $e^{-t}u(t)$

(b) $e^{-t}u(-t)$

(c) $-e^{-t}u(-t)$

(d) $-e^{-t}u(t)$

(e) $e^{t}u(-t)$

(f) $e^{t}u(t)$

(g) e^{-t}

(h) e^{t}

(i) $e^{-t}u(-t+\tau)$

(j) None of these

Problem 25:

If $x(t) = \cos(2t) + \cos(3t)$ is the input to an LTI system with frequency response

$$H(\omega) = \begin{cases} 1 & 0 \leq \omega \leq 8 \\ 0 & \text{else} \end{cases}$$

then what is the output?

(a) $y(t) = e^{5jt/2}\cos(t/2)$

(b) $y(t) = \pi e^{5jt/2}\cos(t/2)$

(c) $y(t) = e^{-5jt/2}\cos(t/2)$

(d) $y(t) = \pi e^{-5jt/2}\cos(t/2)$

(e) $y(t) = e^{5jt/2}\cos(t)$

(f) $y(t) = e^{-2jt}\cos(t)$

(g) $y(t) = e^{2jt}\cos(t/2)$

(h) $y(t) = e^{5jt/2}$

(i) $y(t) = (1/\pi)e^{5jt/2}\cos(t/2)$

(j) None of these

Problem 26:

If the input $e^{-2jt} + e^{3jt}$ produces the output $4e^{-2jt+4j} + 2e^{3jt-3j}$ from an LTI system whose impulse response is real, then what is the output from the same system when the input is $e^{2jt} + 2e^{-3jt}$?

(a) $4e^{2jt-4j} + 4e^{-3jt+3j}$

(b) $4e^{2jt+4j} + 4e^{-3jt-3j}$

(c) $-4e^{2jt-4j} - 4e^{-3jt+3j}$

(d) $-4e^{2jt+4j} - 4e^{-3jt-3j}$

(e) $4e^{2jt-4j} + 2e^{-3jt+3j}$

(f) $4e^{2jt+4j} + 2e^{-3jt-3j}$

(g) $-4e^{2jt-4j} - 2e^{-3jt+3j}$

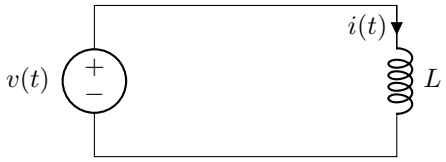
(h) $-4e^{2jt+4j} - 2e^{-3jt-3j}$

(i) $e^{2jt-4j} + e^{-3jt+3j}$

(j) None of these.

Problem 27:

Consider the following sinusoidal steady-state circuit, with voltage source $v(t)$.



What is the value of the current $i(t)$ if $v(t) = 4\cos(2t) + 2\cos(3t)$ and $L = 2$?

(a) $\sin(2t) + \frac{1}{3}\sin(3t)$

(b) $2\sin(2t) + \frac{1}{3}\sin(3t)$

(c) $2\sin(2t) + \frac{1}{3}\sin(3t)$

(d) $\sin(2t) + \frac{1}{6}\sin(3t)$

(e) $2\sin(2t) + \sin(3t)$

(f) $\cos(2t) + \frac{1}{3}\cos(3t)$

(g) $2\cos(2t) + \frac{2}{3}\cos(3t)$

(h) $2\cos(2t) + \cos(3t)$

(i) None of these

Problem 28:

Suppose $x(t) = 2e^{-2jt} + 3e^{2jt} - 4e^{4jt}$ is the input to an LTI system with frequency response

$$H(\omega) = \begin{cases} \omega & |\omega| < 10 \\ 0 & \text{else} \end{cases}.$$

If Y_n are the exponential Fourier Series coefficients of the output $y(t)$, then what is $Y_{-1} + Y_2$?

(a) -20

(b) -12

(c) 12

(d) 20

(e) 6

(f) -6

(g) 2

(h) -10

(i) 0

(j) None of these

Problem 29:

If a signal $x(t)$ is periodic with period 16 and its non-zero Fourier series coefficients are $X_2 = X_{-2} = 1$ and $X_4 = X_{-4} = j$, then which of the following is the signal?

(a) $2(\cos(\frac{\pi}{4}t) + j\cos(\frac{\pi}{2}t))$

(b) $2(\cos(\frac{\pi}{4}t) + \sin(\frac{\pi}{2}t))$

(c) $2(\cos(\frac{\pi}{2}t) - \sin(\frac{\pi}{4}t))$

(d) $2(\cos(\frac{\pi}{2}t) + \sin(\frac{\pi}{2}t))$

(e) $2(\cos(\frac{\pi}{2}t) - \sin(\frac{\pi}{4}t))$

(f) $2(\cos(\frac{\pi}{2}t) - \sin(\frac{\pi}{4}t))$

(g) $\cos(\frac{\pi}{4}t) - \sin(\frac{\pi}{2}t)$

(h) $\cos(\frac{\pi}{4}t) + \sin(\frac{\pi}{2}t)$

(i) $\cos(\frac{\pi}{4}t) - \sin(\frac{\pi}{4}t)$

(j) None of these

Problem 30:

Let $f(t) = 2\delta(t) + 5\delta(t-1)$. If g is obtained by convolving f with itself three times, i.e. $g(t) = f(t) * f(t) * f(t)$, then what is $\int_{\pi/4}^{\pi/2} g(t)e^{j\pi t}dt$?

(a) -60

(b) 60

(c) 20

(d) -20

(e) 343

(f) -343

(g) 7

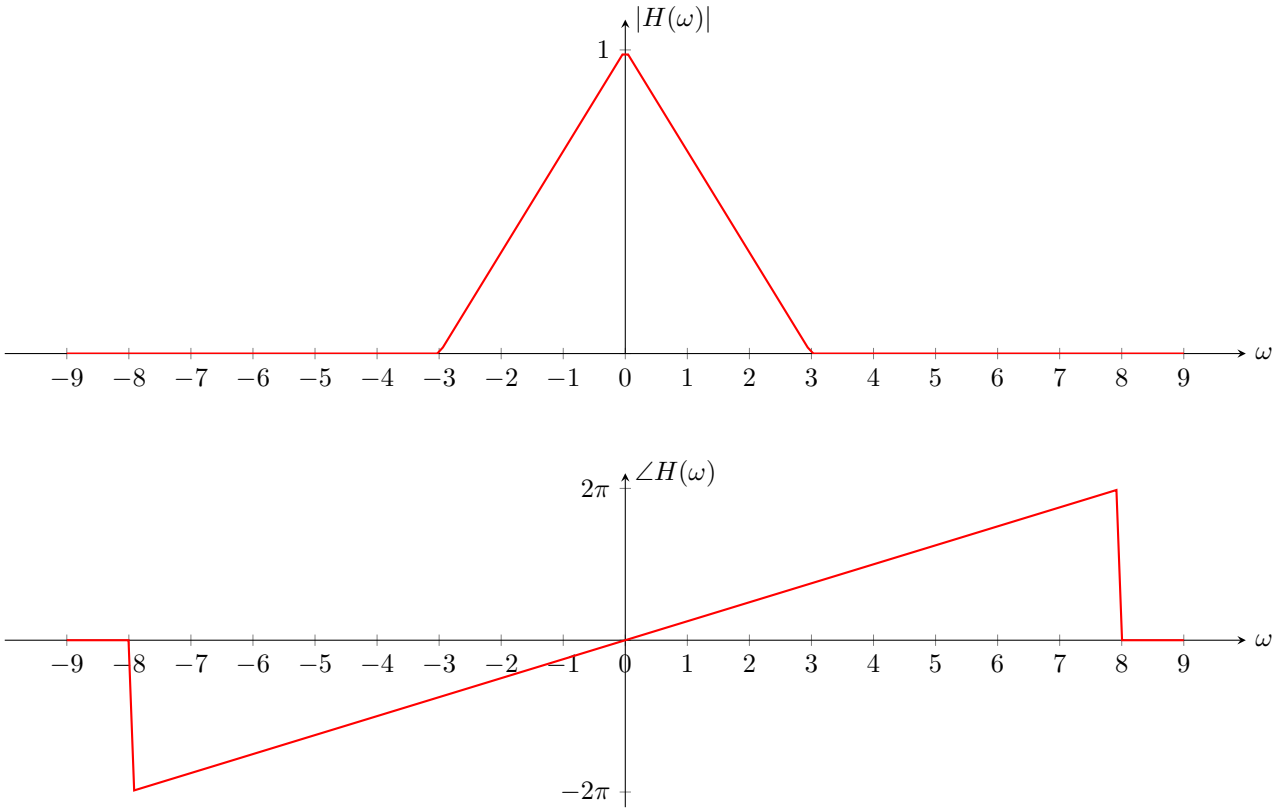
(h) -7

(i) 0

(j) None of these

Problem 31:

If $\sin(2t)$ is the input to an LTI system whose frequency response magnitude and phase are shown below, then what is the output signal?



(a) $(1/3)\cos(2t)$

(b) $(1/3)\sin(2t)$

(c) $-(1/3)\sin(2t)$

(d) $-(1/3)\cos(2t)$

(e) 0

(f) $(2/3)\cos(2t)$

(g) $-(2/3)\cos(2t)$

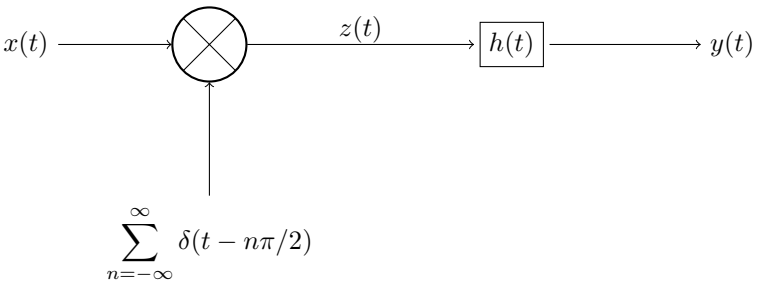
(h) $(2/3)\sin(2t)$

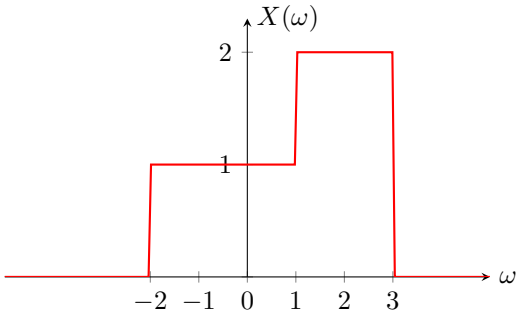
(i) $-(2/3)\sin(2t)$

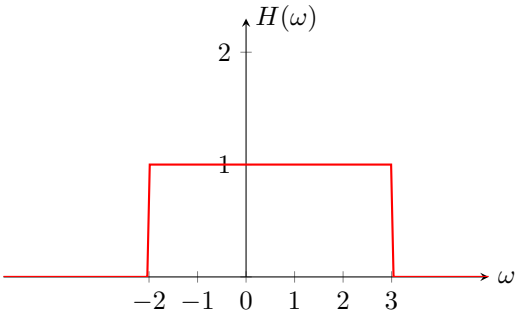
(j) None of these.

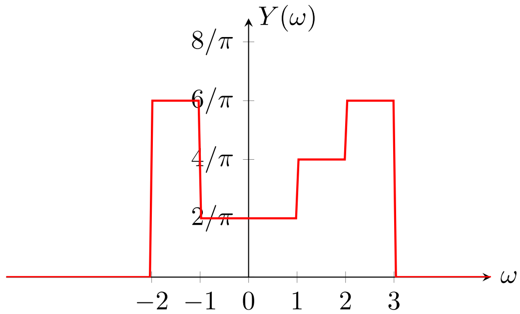
Problem 32:

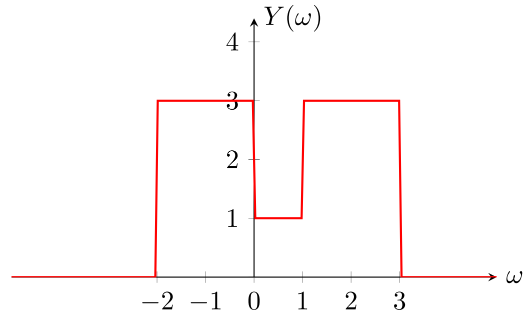
The input signal $x(t)$ in the system below has Fourier transform $X(\omega)$ as shown. Also shown is the Fourier transform of $h(t)$. What is the Fourier transform $Y(\omega)$ of the output signal $y(t)$?

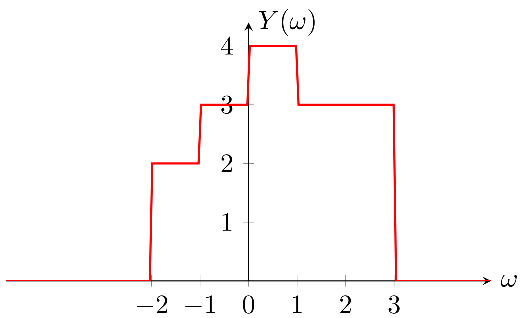


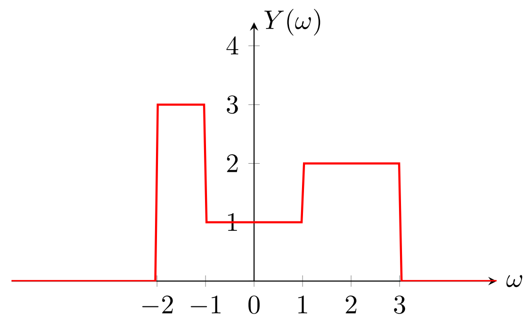


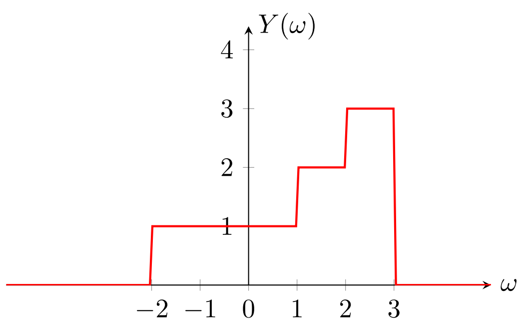


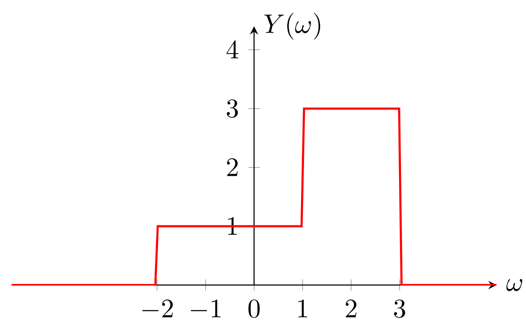

$Y(\omega)$
$8/\pi$
$6/\pi$
$4/\pi$
$2/\pi$
ω
-2 -1 0 1 2 3

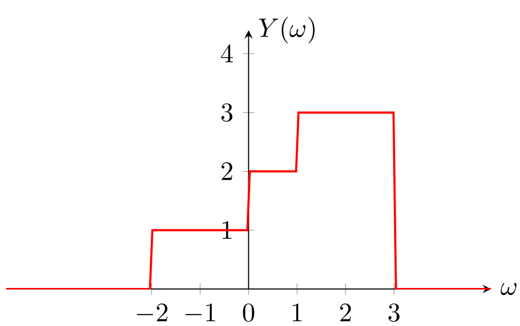

$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3

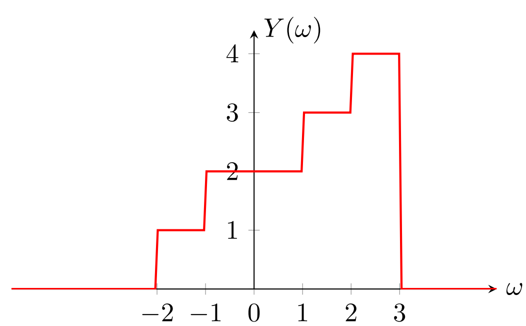

$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3

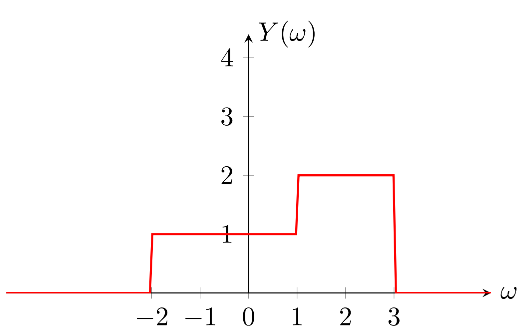

$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3


$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3


$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3


$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3


$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3


$Y(\omega)$
4
3
2
1
ω
-2 -1 0 1 2 3

(a) Figure (a)

(b) Figure (b)

(c) Figure (c)

(d) Figure (d)

(e) Figure (e)

(f) Figure (f)

(g) Figure (g)

(h) Figure (h)

(i) Figure (i)

(j) None of these.

Problem 33:

Suppose the bilateral Laplace transform of a signal $x(t)$ is $(s^2 - s)^{-1}$, which is known to converge when $s = e^{j\pi/4}$. What is the value of $x(-1) + x(1)$?

(a) $-(e+1)/e$

(b) $(e+1)/e$

(c) $(e-1)/e$

(d) $1/e$

(e) $-1/e$

(f) 1

(g) $(1-e)/e$

(h) -1

(i) e

(j) None of these

Problem 34:

What is the following function of t equal to?

$$\int_{-\infty}^{\infty} e^{j\omega t} \left(3\pi^2 + \frac{\omega^2}{3}\right)^{-1} d\omega$$

(a) $e^{-3\pi|t|}$

(b) $e^{-3t}u(t)$

(c) $\frac{1}{2\pi}e^{-3\pi|t|}$

(d) $\frac{1}{2\pi}e^{-3t}u(t)$

(e) $e^{-\sqrt{3}\pi|t|}$

(f) $e^{-\sqrt{3}\pi t}u(t)$

(g) $\frac{1}{2\pi}e^{-\sqrt{3}\pi|t|}$

(h) $\frac{1}{2\pi}e^{-\sqrt{3}t}u(t)$

(i) None of these

Problem 35:

If the output of an LTI system is $y(t) = e^{-2t}u(t)$, when the input is $x(t) = e^{-5t}u(t)$, then what is the impulse response of the system?

(a) $\delta(t) + 3e^{-2t}u(t)$

(b) $\delta(t) - 3e^{-2t}u(t)$

(c) $\delta(t) + e^{-2t}u(t)$

(d) $\delta(t) - e^{-2t}u(t)$

(e) $\delta(t) + 3e^{-5t}u(t)$

(f) $\delta(t) - 3e^{-5t}u(t)$

(g) $3e^{-2t}u(t)$

(h) $3e^{-5t}u(t)$

(i) $2e^{-3t}u(t)$

(j) None of these