ECE 101: Linear Systems Fundamentals

Summer Session II 2020 - Lecture 7

Saharnaz Baghdadchi

University of California, San Diego

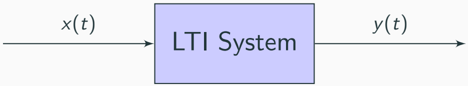
Today's topics

- CT LTI systems and convolution integral
- Properties of LTI systems using impulse response

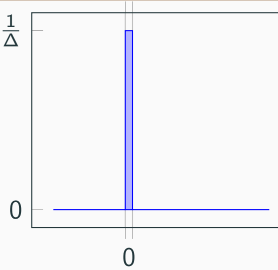
Signals and Systems (2th Edition): sections 2.2 and 2.3.4-7

CT LTI Systems and Convolution Integral

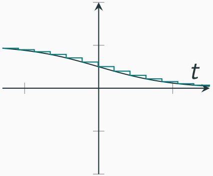
Output of Discrete-time LTI Systems for an Arbitrary Input



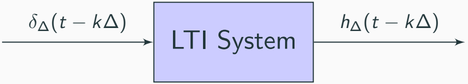
Continuous-time LTI Systems



- Reminder: Define $\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & \text{if } t \in [0, \Delta] \\ 0 & \text{else.} \end{cases}$
- $\Delta\delta_\Delta(t)$ is a pulse of height 1 and width Δ
- We can approximate a nice-enough signal $x(t) \approx \hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\Delta\delta_\Delta(t - k\Delta)$

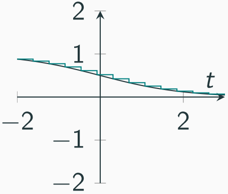


- Let $h_\Delta(t)$ be the response to the input $\delta_\Delta(t)$
- By Time-Invariant property:



Continuous-time LTI Systems

- $x(t) \approx \hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\Delta\delta_\Delta(t - k\Delta)$



- By LTI property:

- Letting $\Delta \to 0$:
  - $\delta_\Delta(t) \to \delta(t)$
  - $\hat{x}(t) \to x(t)$ (piece-wise approximation becomes exact)
  - $\hat{y}(t) \to y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ (by the Riemann approximation of the integral),

  where $h(t)$ is the response of the system to $\delta(t)$.

Continuous-time LTI Systems

- Define the impulse response of a CT system to be the response of the system to $x(t) = \delta(t)$ and denote it by $h(t)$.

Response of CT LTI System to an Arbitrary Input

Let $h(t)$ be the impulse response of an LTI system. Then for any input $x(t)$, the output is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau.$$

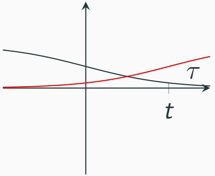
- This integral is called **the convolution (integral)** of $x(t)$ and $h(t)$. Denote it by $y(t) \stackrel{def}{=} x(t) * h(t)$.

Convolutions integral

For signals $x(t)$ and $h(t)$, convolution integral $y(t) = h(t) * x(t)$ is given by:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau.$$

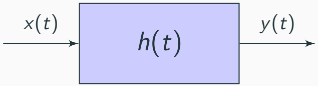
How to compute convolution in CT?



- Given input $x(t)$ and impulse response $h(t)$
- The main idea: **fix time** t
- $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$
- As a function of τ, $x(\tau)$ remains the same
- As a function of τ, $h(t-\tau)$: $h(\tau)$ flipped and shifted to t
- Output at time t: sample by sample multiply $x(\tau)$ by $h(t-\tau)$ and then ~~add~~ integrate

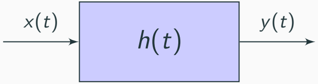
Properties of LTI Systems Using Impulse Response

Impulse Response



- We know that impulse response tells us everything about a system.
- We can determine the properties (memorlyess, causality, invertibility, BIBO stability) of an **LTI** system by investigating its impulse response.

Memoryless



Memoryless Property for LTI Systems

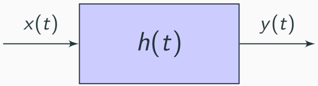
A discrete-time LTI system is memoryless if and only if

$$h[n] = a\delta[n], \text{for some } a \in \mathbb{C}. \tag{1}$$

A continuous-time LTI system is memoryless if and only if

$$h(t) = a\delta(t), \text{for some } a \in \mathbb{C}.$$

Causal



Causal Property for LTI Systems

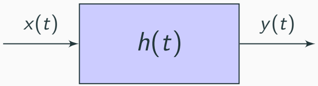
A discrete-time LTI system is causal if and only if

$$h[n] = 0, \text{for } n < 0. \tag{2}$$

A continuous-time LTI system is causal if and only if

$$h(t) = 0, \text{for } t < 0.$$

Invertibility



Invertible Property for LTI Systems

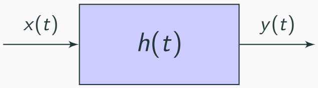
A discrete-time LTI system is invertible if and only if there exists a $g[n]$ such that:

$$g[n] * h[n] = \delta[n]. \tag{3}$$

A continuous-time LTI system is invertible if and only if there exists a $g(t)$ such that:

$$g(t) * h(t) = \delta(t).$$

BIBO Stability



Invertible Property for LTI Systems

A discrete-time LTI system is BIBO stable if and only if:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty. \tag{4}$$

A continuous-time LTI system is BIBO stable if and only if:

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty.$$