ECE 65: Components & Circuits Lab

Lecture 18

MOSFET Amplifier small signal model

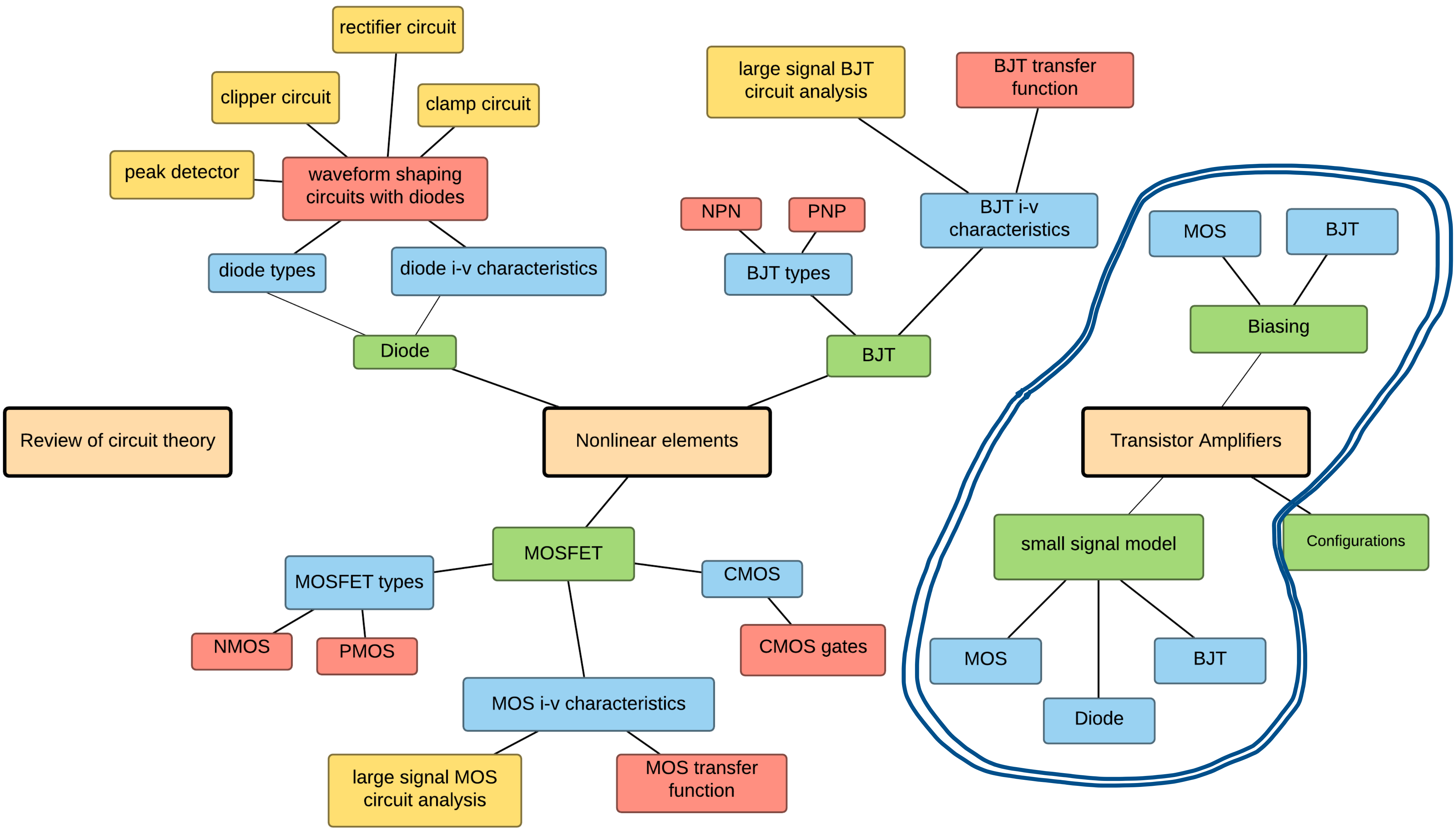
Reference notes: sections 5.1, 5.2

Sedra & Smith (7th Ed): sections 7.1

Saharnaz Baghdadchi

Course map

5. Transistor Amplifiers – Bias and small signal



Review of definitions

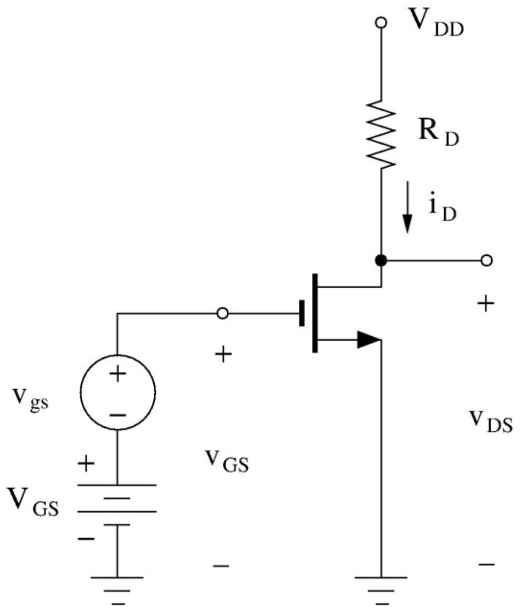
Signal: We want the response of the circuit to this input.

Bias: State of the system when there is no signal.

- Bias is constant in time (may vary extremely slowly compared to signal).
- Purpose of the bias is to ensure that MOS is in saturation at all times.

Derivation of MOS small signal model

$\lambda = 0$



The DC Bias point: $v_{gs} = 0$

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2 = \frac{1}{2}k_n {V_{OV}}^2$$

$$k_n = \mu_n C_{ox}\left(\frac{W}{L}\right)_n$$

$$V_{DS} = V_{DD} - R_D I_D$$

In the presence of the input signal, v_{gs}:

The total instantaneous gate-to-source voltage is: $v_{GS} = V_{GS} + v_{gs}$

$$i_D = \frac{1}{2}k_n\left(V_{GS} + v_{gs} - V_t\right)^2$$

$$i_D = \frac{1}{2} k_n \left(V_{GS} + v_{gs} - V_t\right)^2$$

$$= \frac{1}{2} k_n (V_{GS} - V_t)^2 + k_n (V_{GS} - V_t)\, v_{gs} + \frac{1}{2} k_n \left(v_{gs}\right)^2$$

Consider the case:

$$\frac{1}{2} k_n \left(v_{gs}\right)^2 \ll k_n (V_{GS} - V_t)\, v_{gs}$$

or

$$\underline{v_{gs}} \ll \underline{2\,(V_{GS} - V_t)}$$

In this case, v_{gs} is small enough and the **Small Signal Condition** is satisfied.

we can neglect the $\frac{1}{2} k_n \left(v_{gs}\right)^2$ term in the i_D equation and

$$i_D \simeq \underline{I_D} + \underline{i_d}$$

$$\begin{cases} i_D \simeq \frac{1}{2}k_n(V_{GS}-V_t)^2 + k_n\,(V_{GS}-V_t)\,v_{gs} \\ \\ i_D \simeq I_D + i_d \end{cases}$$

$$\boxed{I_D = \frac{1}{2}k_n(V_{GS}-V_t)^2}$$

$$\boxed{i_d = k_n\,(V_{GS}-V_t)\,v_{gs}}$$

The MOSFET transconductance g_m is defined as

$$g_m \equiv \frac{i_d}{v_{gs}} = k_n\,(V_{GS}-V_t)$$

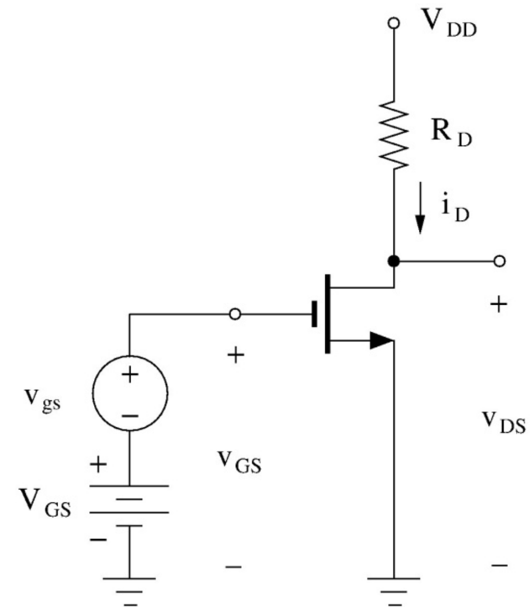
$$\boxed{g_m = k_n\,V_{OV} = \frac{2\,I_D}{V_{OV}}}$$

$$\boxed{i_d = g_m\,v_{gs}}$$

$$v_{DS} = V_{DD} - R_D i_D$$

$$= V_{DD} - R_D(I_D + i_d)$$

$$\left\{ \begin{aligned} v_{DS} &= V_{DD} - R_D I_D - R_D i_d \\ v_{DS} &= V_{DS} + v_{ds} \end{aligned} \right.$$



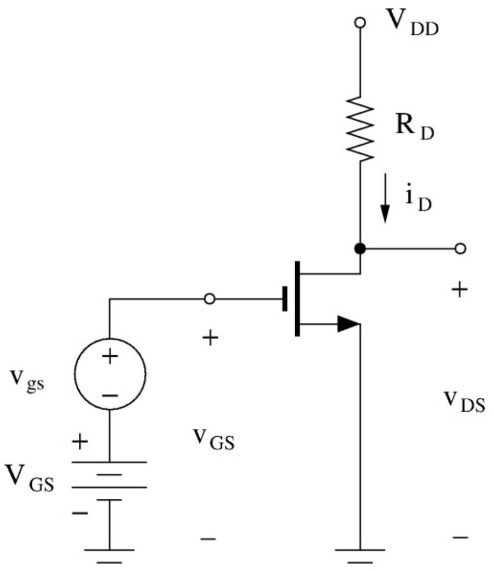
$$\boxed{V_{DS} = V_{DD} - R_D I_D}$$

$$v_{ds} = -i_d R_D = -g_m v_{gs} R_D$$

Amplifier voltage gain is $$\boxed{A_v = \frac{v_{ds}}{v_{gs}} = -g_m R_D}$$

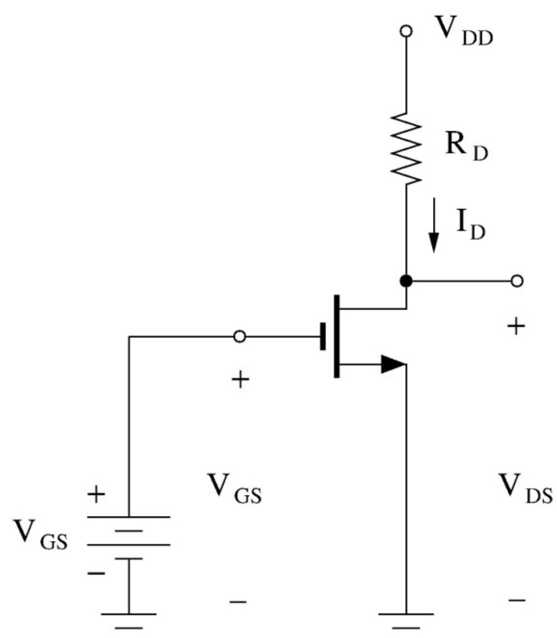
Bias and Signal circuits under small signal approximation

Bias & Signal



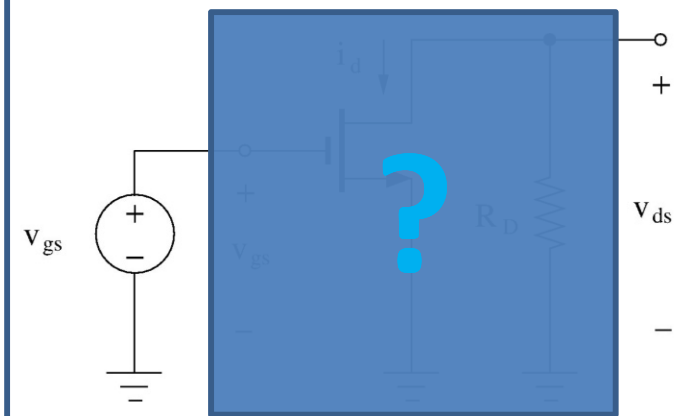
$v_{GS}, i_D, v_{DS}, \ldots$

Bias



$V_{GS}, I_D, V_{DS}, \ldots$

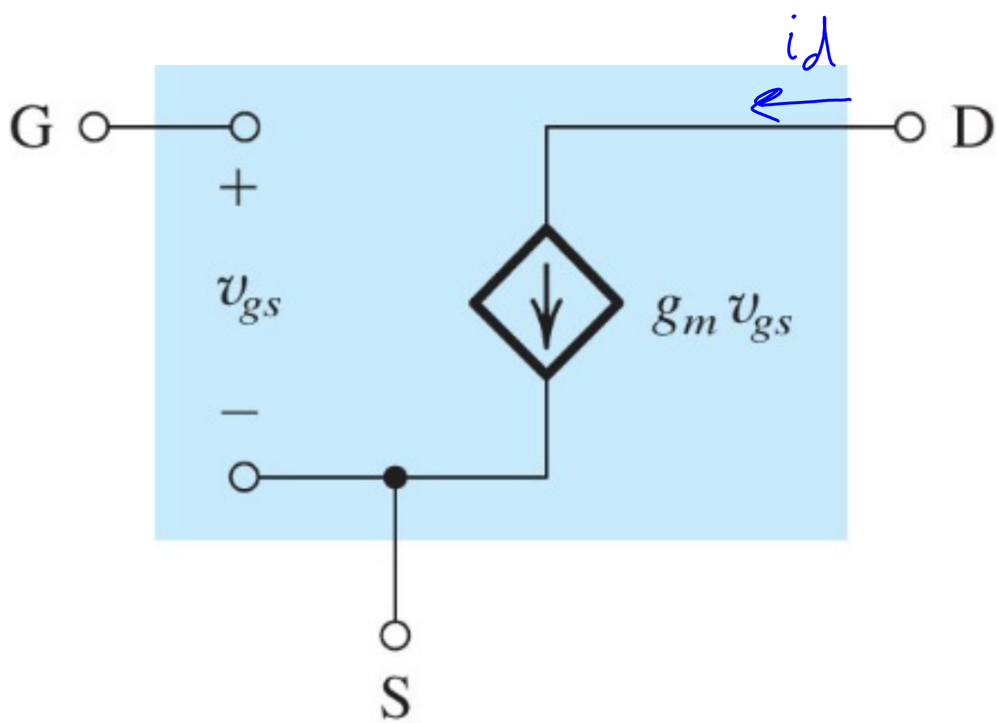
Signal only
= (Bias + Signal) - Bias



$v_{gs}, i_d, v_{ds}, \ldots$

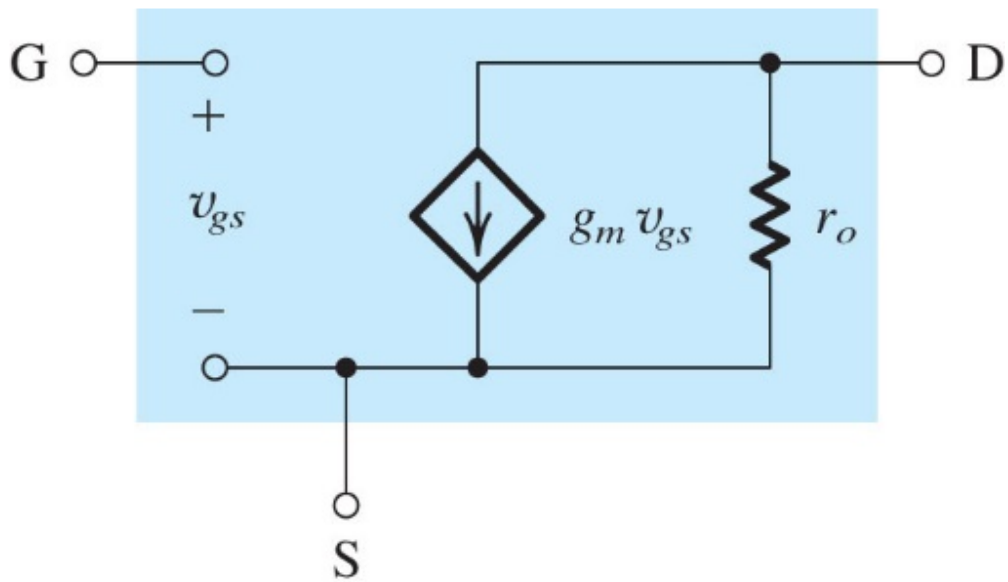
MOS small signal model

$i_g = 0$ $\qquad$ $i_d = g_m v_{gs}$



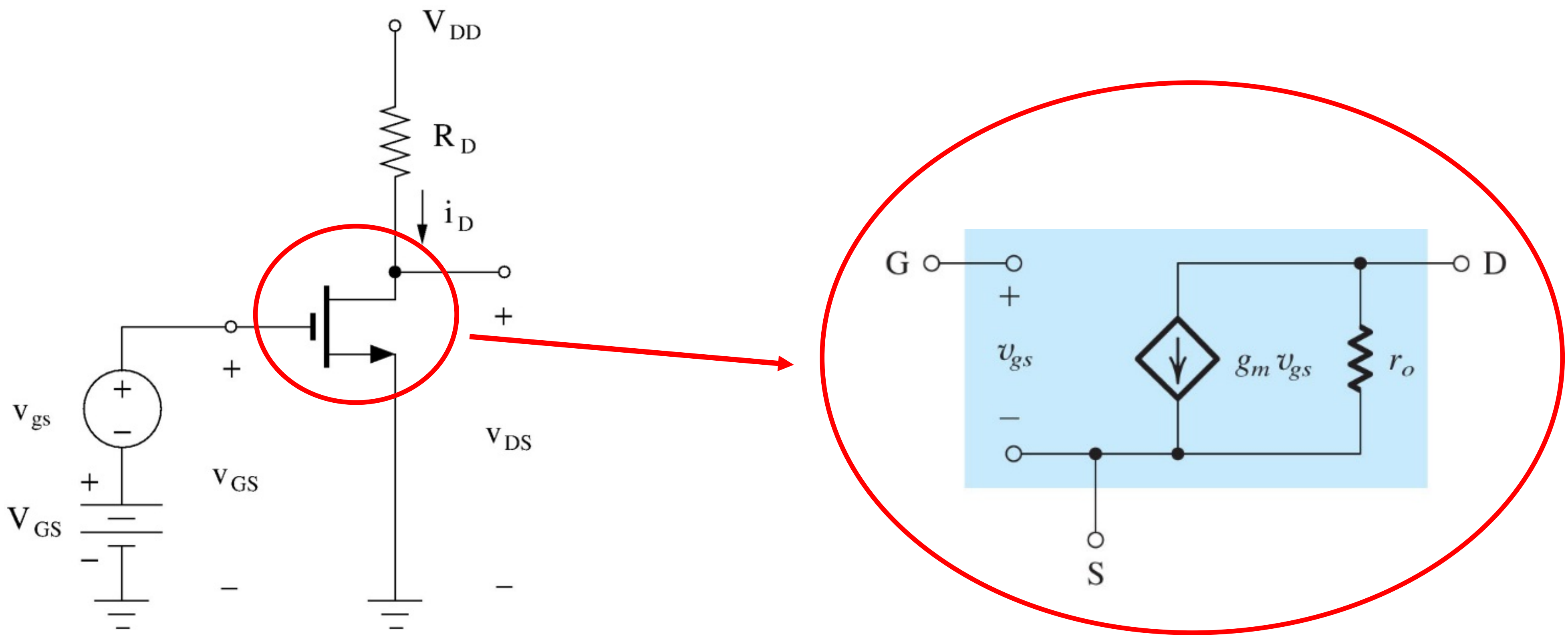
MOS small signal model

$$i_g = 0 \qquad\qquad i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$



$$\boxed{g_m = \frac{2\, I_D}{V_{OV}}} \qquad\qquad \underline{r_o \approx \frac{1}{\lambda . I_D}}$$

MOS Small Signal Model



What about the rest of the circuit element?

Other circuit elements in the small signal equivalent circuit

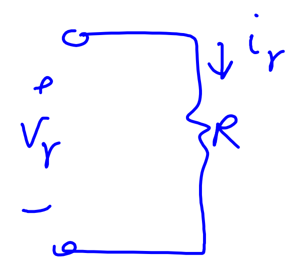
A resistor remains as a resistor in the signal circuit.

$$v_R = V_R + v_r$$

Bias + Signal: $v_R = Ri_R$

Bias: $V_R = RI_R$

Signal: $v_r = v_R - V_R = Ri_r \Longrightarrow \boxed{v_r = Ri_r}$



Other circuit elements in the small signal equivalent circuit

A capacitor remains as a capacitor in the signal circuit.

Unless we perform the frequency analysis of amplifier circuits, <u>the capacitors will be short in the signal equivalent circuit</u>.

Reminder:

A capacitor acts as an open circuit in the bias circuit.

Other circuit elements in the small signal equivalent circuit

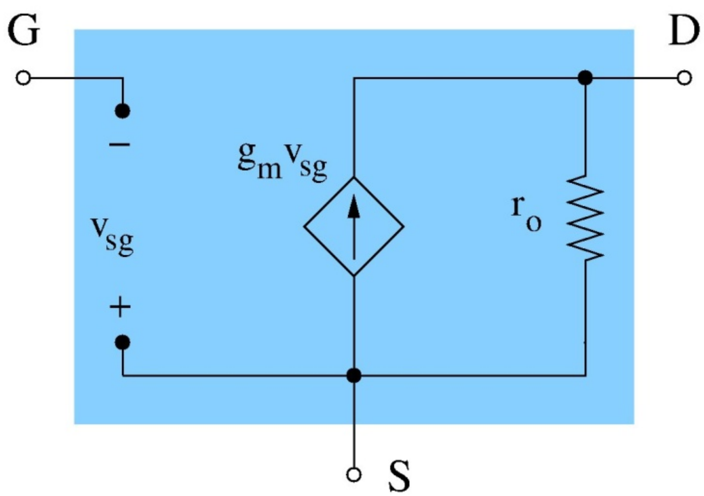
A DC voltage source becomes a <u>short</u> circuit.

For example, V_{DD} , V_{SS} will be effectively grounded.

A DC current source becomes an <u>open</u> circuit.

PMOS small signal model is identical to NMOS

PMOS



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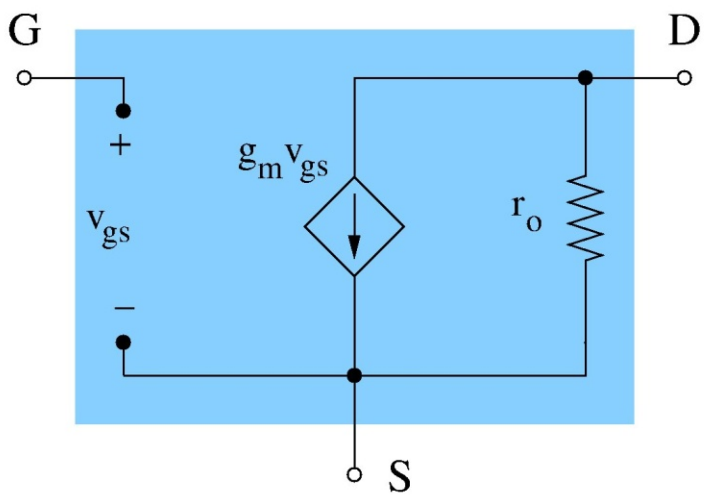
NMOS or PMOS ✓



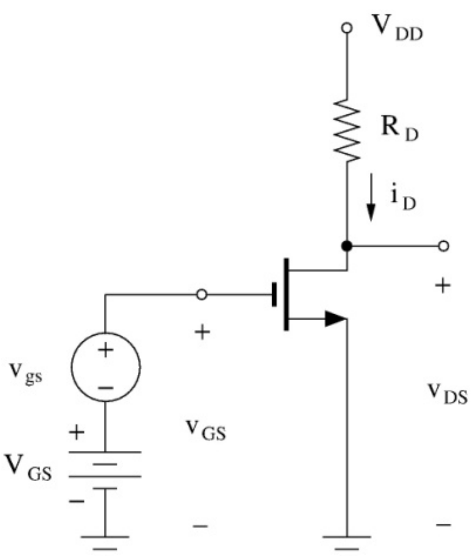
PMOS small-signal circuit model is identical to NMOS

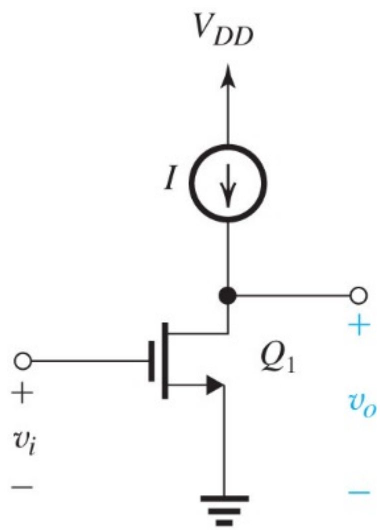
We will use NMOS circuit model for both!

How to add signal to the bias

1. Direct Coupling

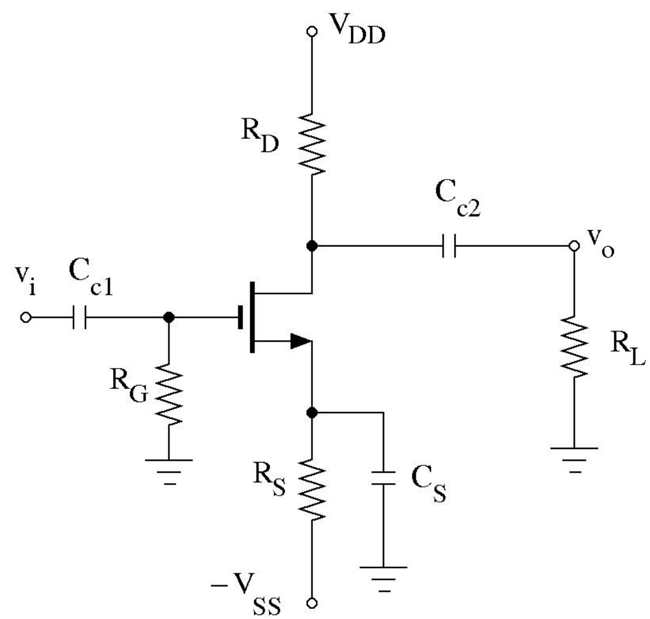
The signal is directly applied to the transistor.





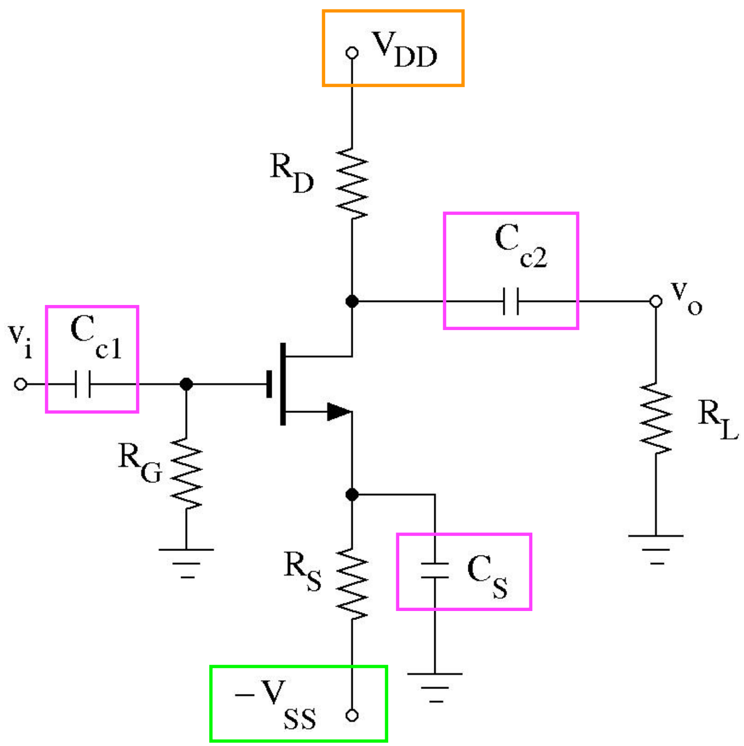
2. Capacitive Coupling

A capacitor is used to couple the signal to the transistor.

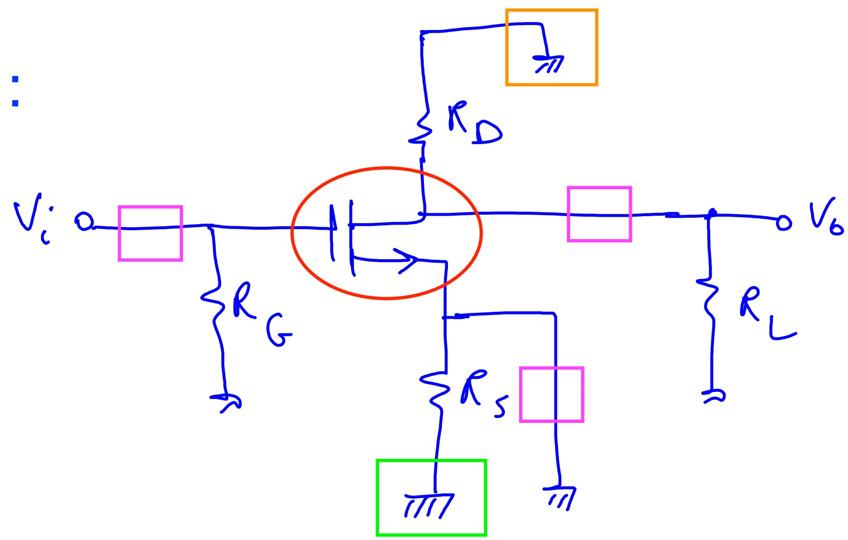


Example 1:

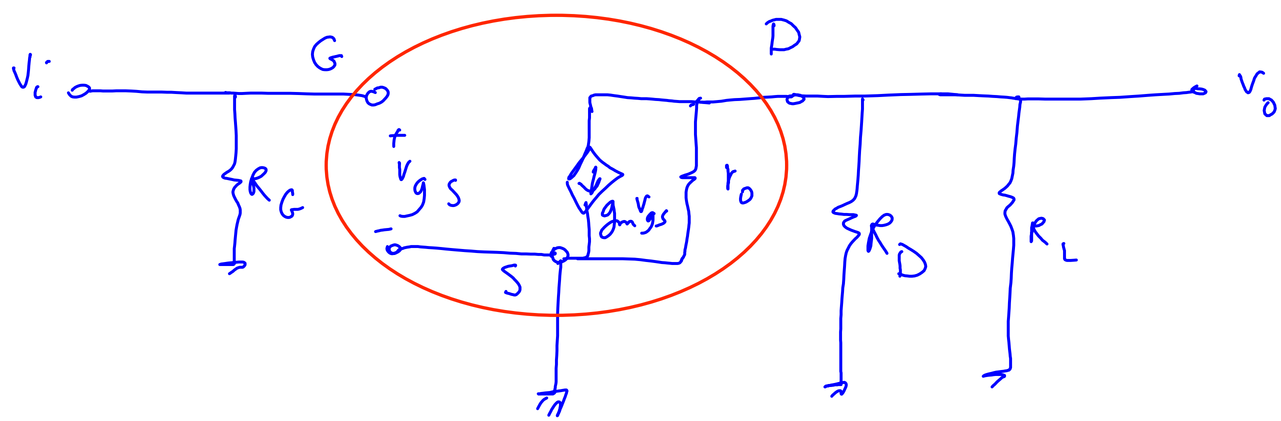
Draw the signal circuit (assume capacitors are short for signal).



step 1:

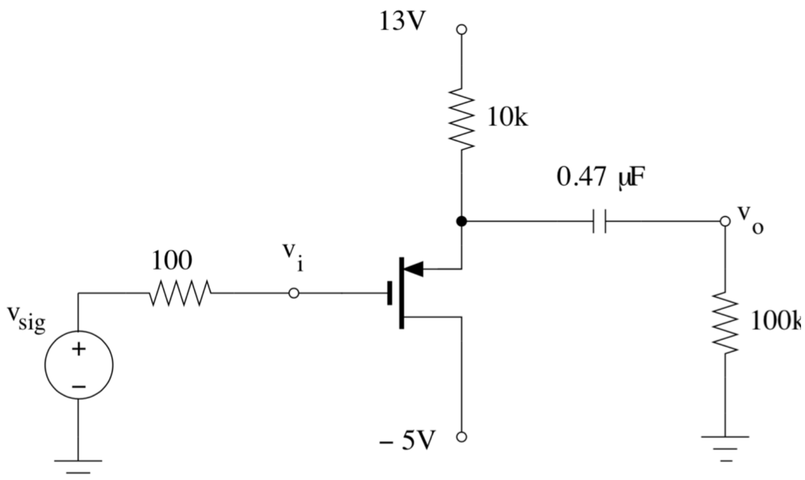


step 2:



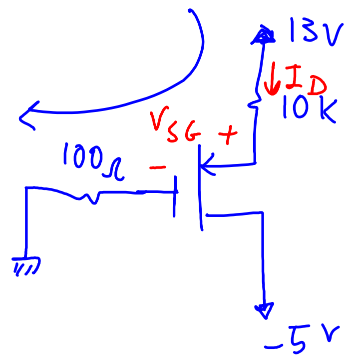
Lecture 18 reading quiz.

Find the transconductance, g_m , in this circuit ($V_{tn} = |V_{tp}| = 4\ V$, $k_n = k_p = 0.4\ mA/V^2, \lambda = 0.01\ V^{-1}$. Assume $\lambda = 0$ for the bias circuit.



$$g_m = \frac{2 I_D}{V_{ov}} = \frac{2 I_D}{(V_{SG} - |V_{tp}|)}$$

Bias circuit to find I_D and V_{GS}.



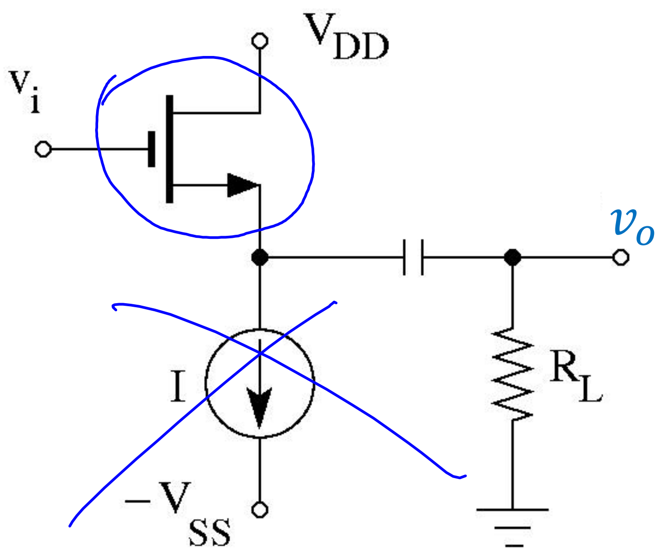
$$I_D = \frac{1}{2} \times k_p \times V_{ov}^2 \Rightarrow I_D = 0.2 (V_{SG} - 4)^2$$

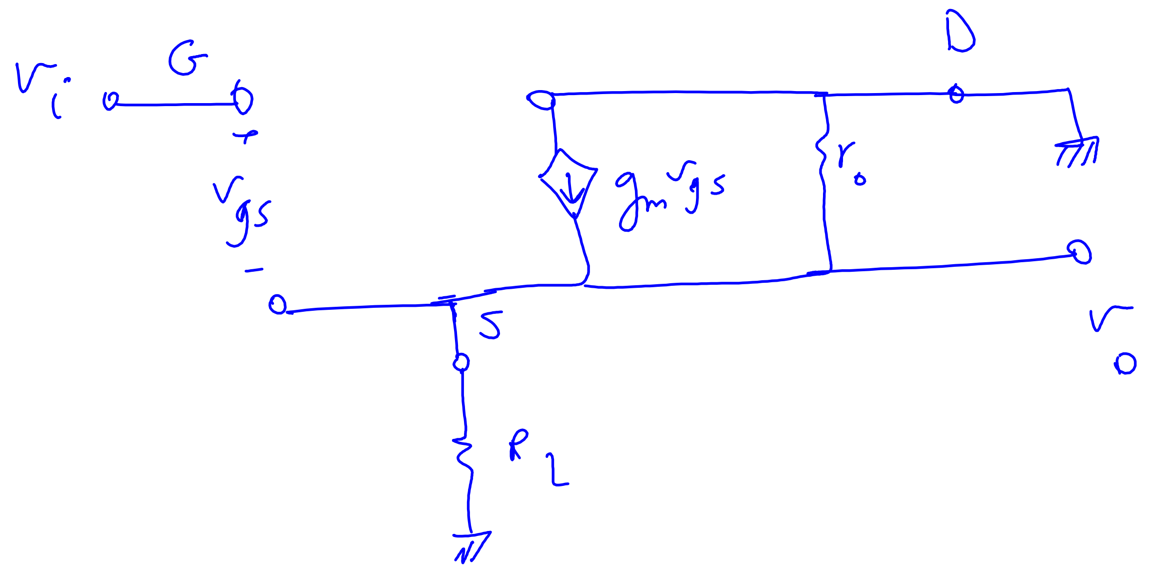
SG KVL: $13 = 10k \times I_D + V_{SG} \longrightarrow 13 = 10 \times 0.2 (V_{SG} - 4)^2 + V_{SG} \longrightarrow V_{SG} = \begin{cases} 1.61 \ \times \\ 5.89 \ \checkmark \end{cases}$

$V_{SG} = 5.89\ V$ and $I_D = 0.71\ mA \longrightarrow g_m = \frac{2 \times 0.71 mA}{5.89 - 4} = 0.751\ mA/V$

Discussion question 1.

Draw the signal circuit (assume capacitors are short for signal).





Discussion question 2.

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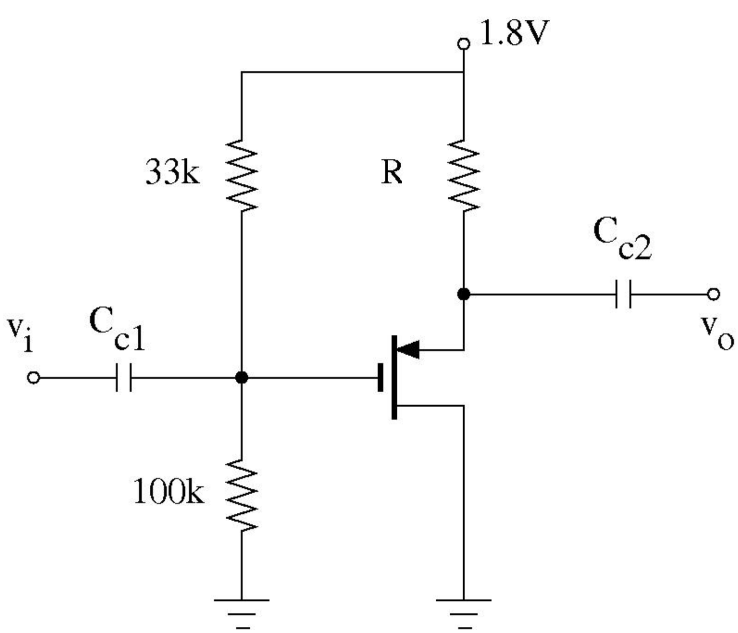
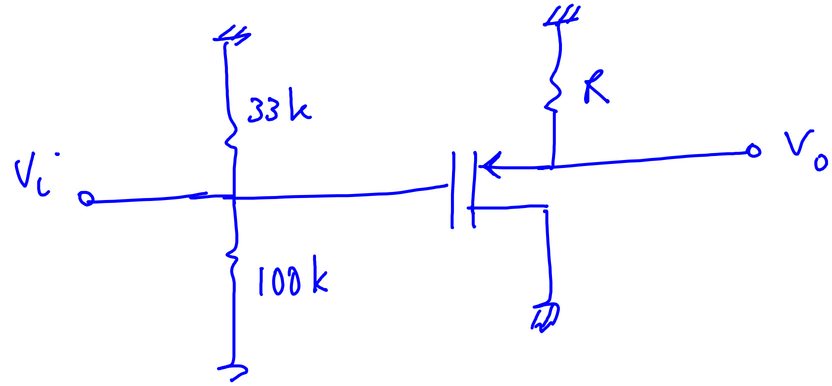
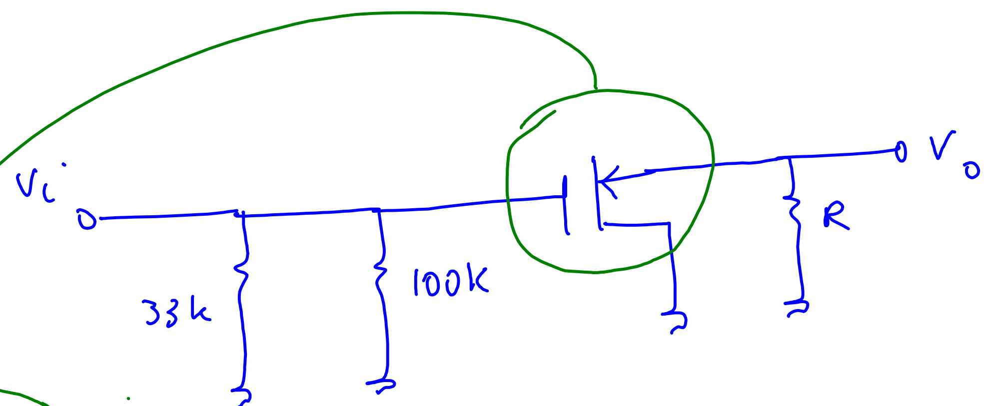
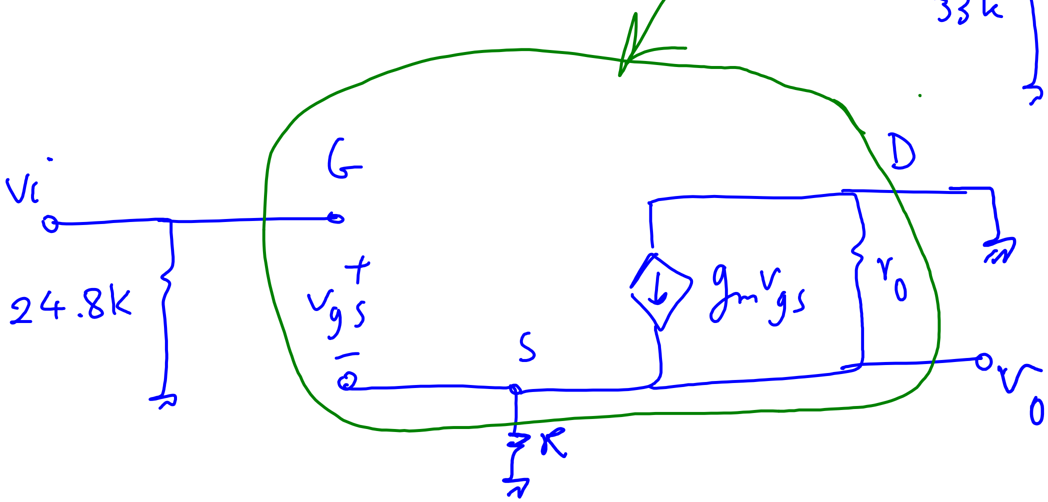






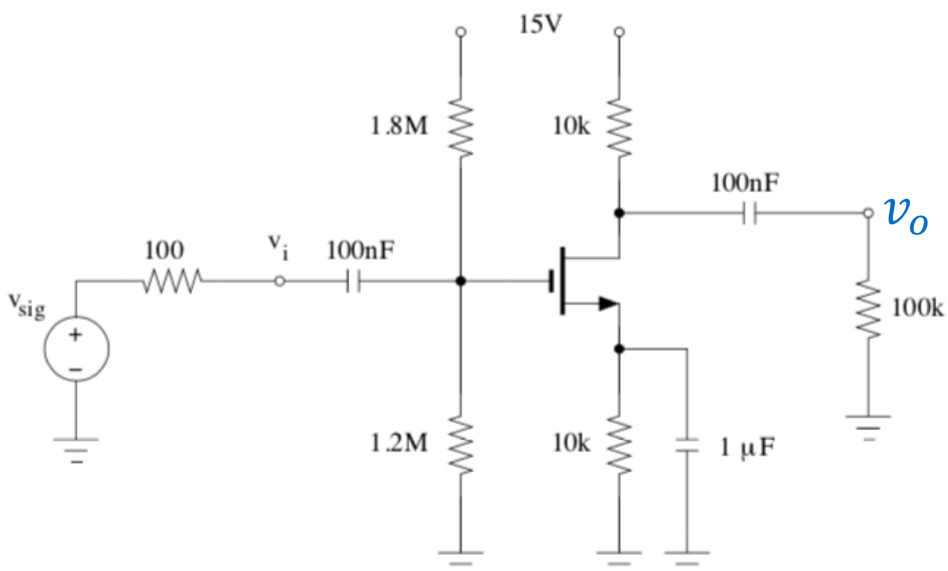


Discussion question 3.

Find the small signal parameters and draw the signal circuit (assume capacitors are short for signal).

$V_{tn} = 1\,V$, $k_n = 0.8\,mA/V^2$. $\lambda = 0.01\,V^{-1}$.

Ignore the channel-width modulation effect in biasing calculations.



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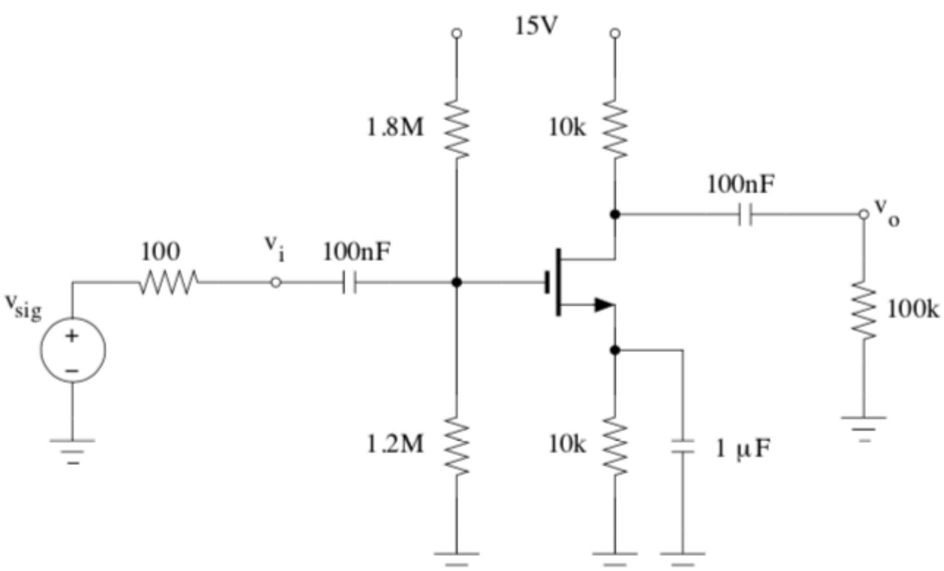
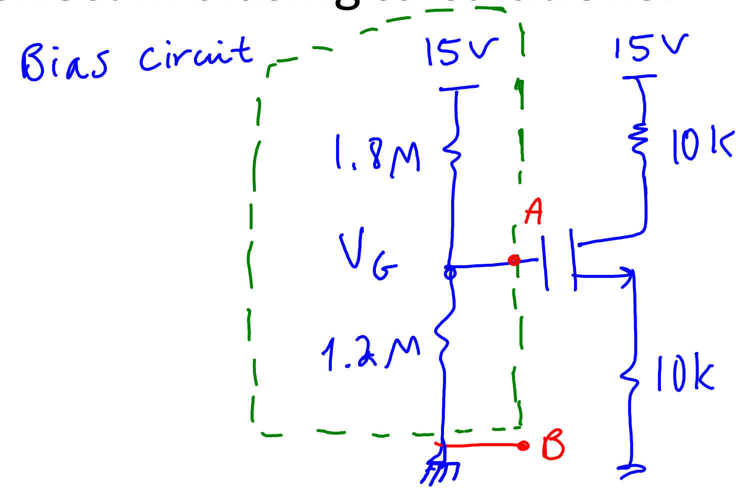
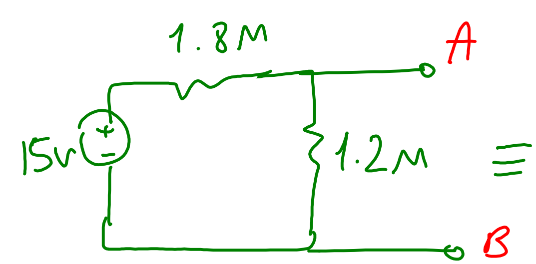
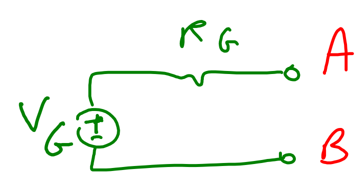




$$V_G = \frac{1.2\ M}{1.2\ M + 1.8\ M} \times 15V = 6V$$



$$R_G = 1.2\ M \parallel 1.8\ M = 720k$$



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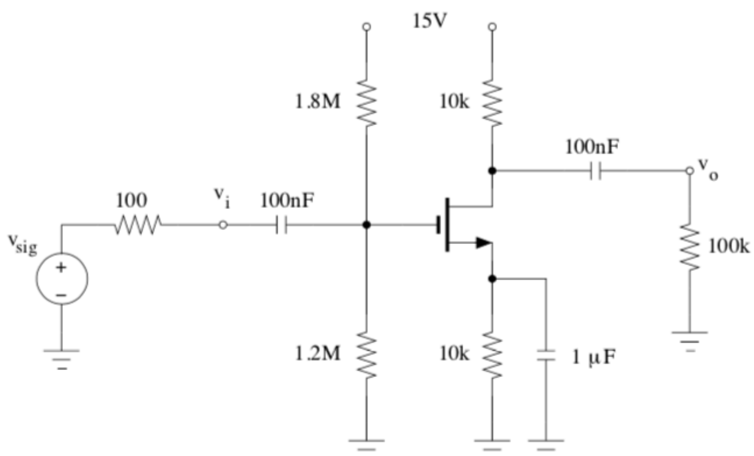


$$I_D = \frac{1}{2} k_n V_{ov}^2$$

KVL: $V_G = V_{GS} + 10k\, I_D$

$$6 = V_{ov} + V_t + 10k\, I_D$$

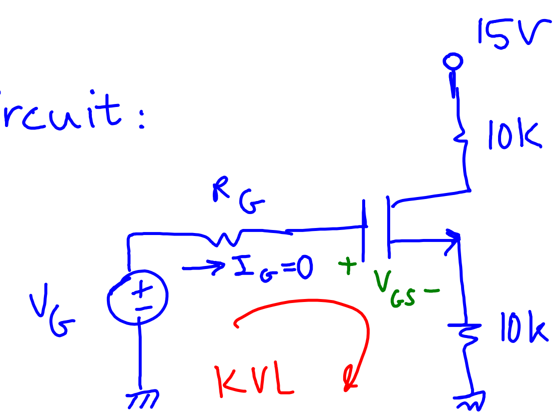
$$\rightarrow 10k\, I_D + V_{ov} - 6 + 1 = 0$$

$$10 \times 10^3 \times \frac{1}{2} \times k_n V_{ov}^2 + V_{ov} - 5 = 0$$

$$\rightarrow 4V_{ov}^2 + V_{ov} - 5 = 0$$

$$\rightarrow V_{ov} = 1\,V \quad \text{and} \quad V_{GS} = V_{ov} + V_t = 2\,V$$

Bias circuit:



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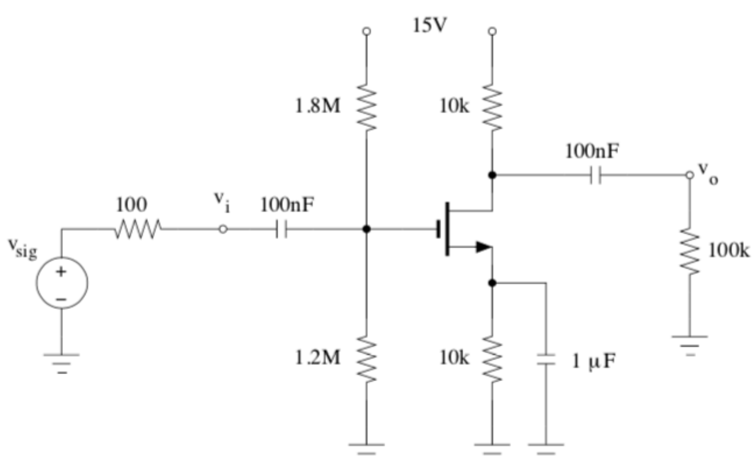

$I_D = 0.5 \times 0.8 \times 10^{-3} \times V_{ov}^2 = 0.4\,mA$

DS KVL: $15 = 10\,k\,I_D + V_{DS} + 10^4 I_D$

$\longrightarrow V_{DS} = 7V > V_{ov}$ MOS is in saturation

Small signal parameters:

$$\begin{cases} g_m = \dfrac{2 I_D}{V_{ov}} = \dfrac{2 \times 0.4 \times 10^{-3}}{1} = 0.8\,mA/V \\ r_0 = \dfrac{1}{\lambda I_D} = \dfrac{1}{10^{-2} \times 0.4 \times 10^{-3}} = 250K \end{cases}$$

Discussion question 3.

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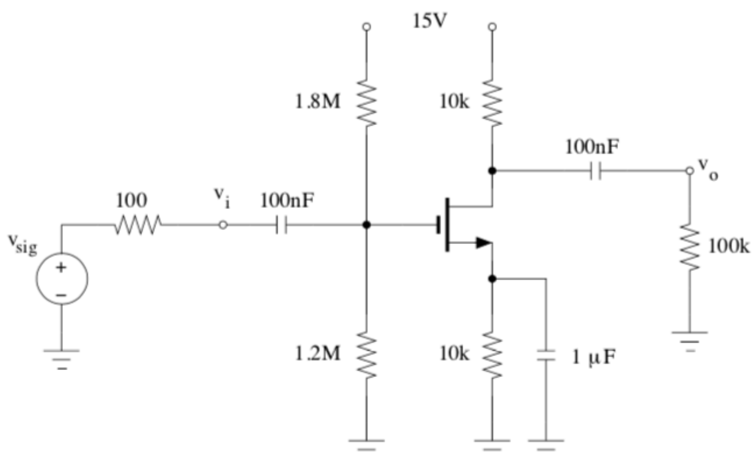

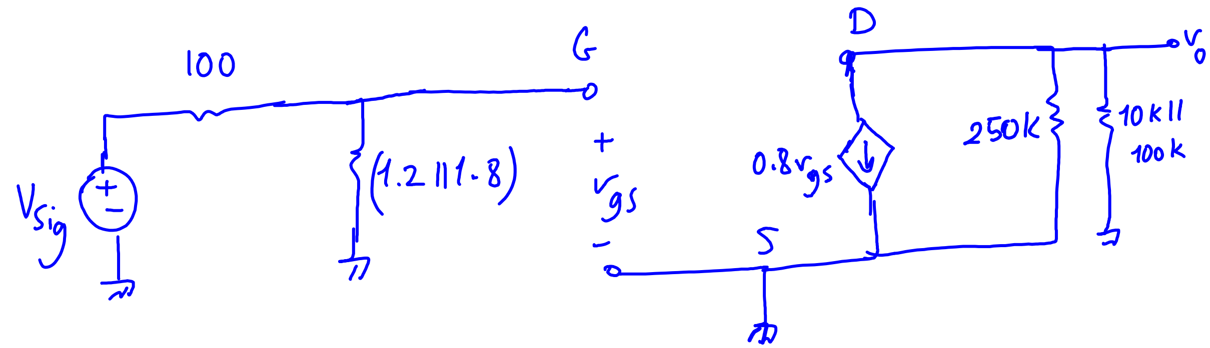

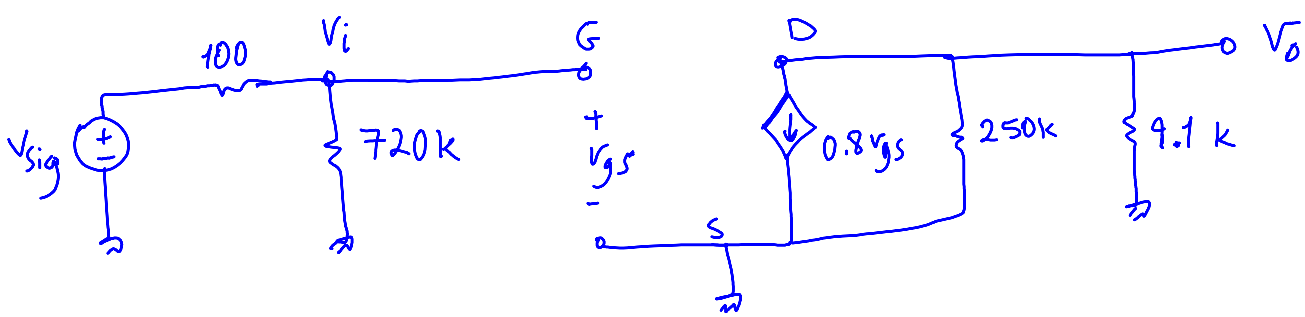

v_{gs} is the same as v_i