ECE 65: Components & Circuits Lab

Lecture 19

BJT Amplifier small signal model

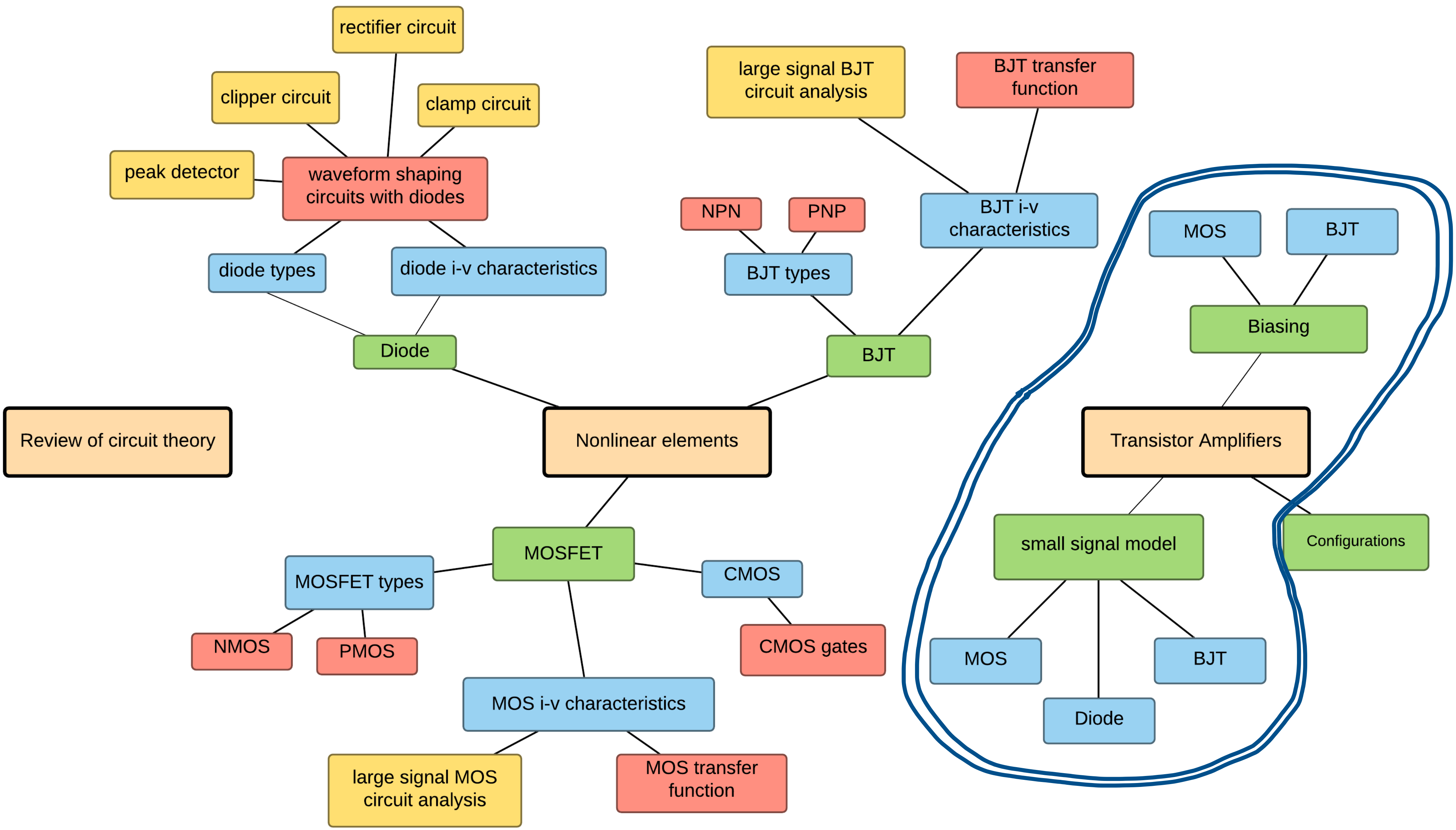
Reference notes: sections 5.1, 5.2

Sedra & Smith (7th Ed): sections 7.1

Saharnaz Baghdadchi

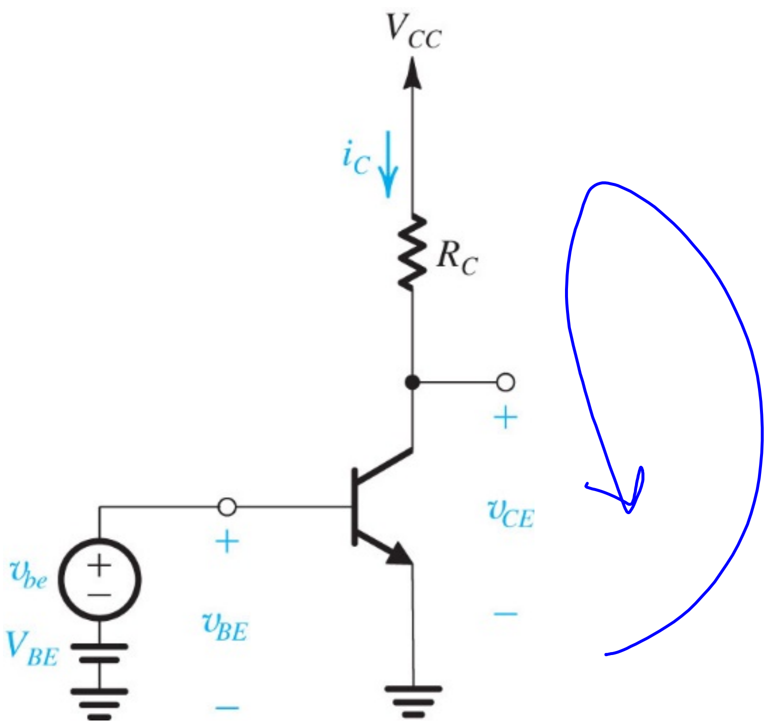
Course map

5. Transistor Amplifiers – Bias and small signal



Derivation of BJT small signal model

The DC Bias point: $v_{be} = 0$



$$I_C = I_s\, e^{V_{BE}/V_T}$$

[The early effect is neglected here ($\lambda = 0$)]

$$I_B = I_C/\beta$$

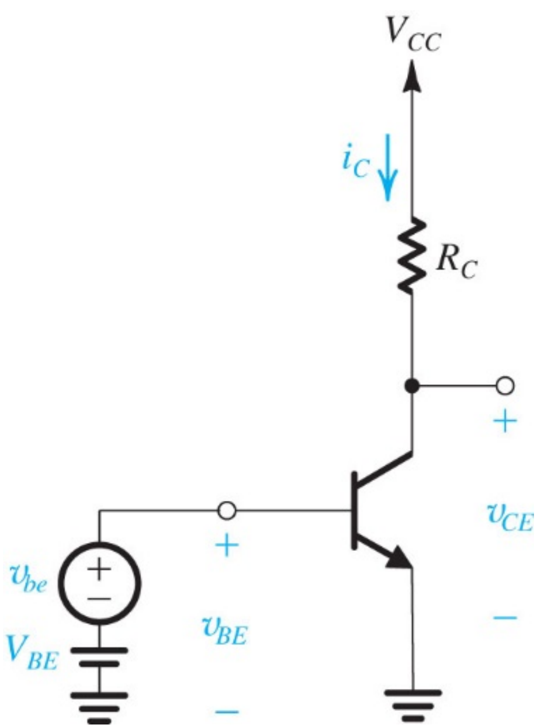
$$I_E = \frac{\beta + 1}{\beta}\, I_C$$

$$V_{CE} = V_{CC} - R_C I_C$$

Derivation of BJT small signal model

When v_{be} is applied:

The total instantaneous base-emitter voltage is: $v_{BE} = V_{BE} + v_{be}$



$$i_C = I_s\, e^{v_{BE}/V_T} = I_s\, e^{(V_{BE}+v_{be})/V_T}$$

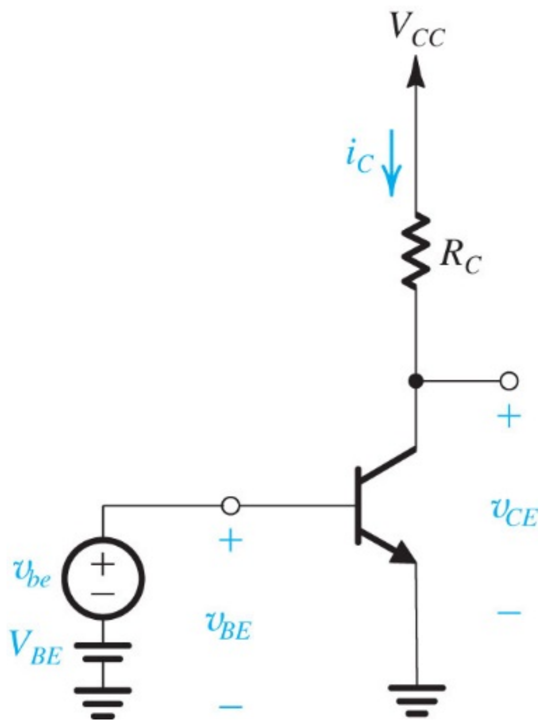
$$i_C = I_s\, e^{V_{BE}/V_T}\, e^{v_{be}/V_T}$$

Since $I_C = I_s\, e^{V_{BE}/V_T}$,

$$i_C = I_C\, e^{v_{be}/V_T}$$

Derivation of BJT small signal model

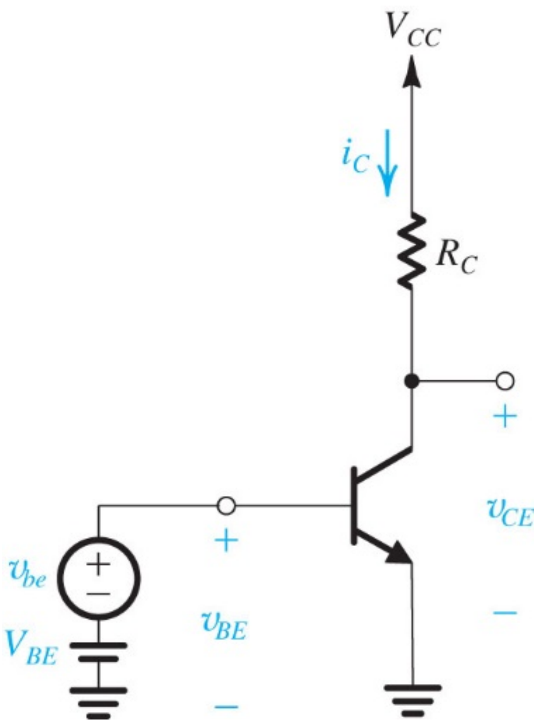
$$i_C = I_C\, e^{v_{be}/V_T}$$



If $v_{be} \ll V_T$, using Taylor series expansion and neglecting the higher order terms in the exponential series expansion,

$$i_C \simeq I_C\left(1 + \frac{v_{be}}{V_T}\right)$$

Derivation of BJT small signal model



$$i_C \simeq I_C \left(1 + \frac{v_{be}}{V_T}\right)$$

Under small signal approximation for BJT ($v_{be} \ll V_T$),

$$i_C = I_C + \frac{I_C}{V_T} v_{be}$$

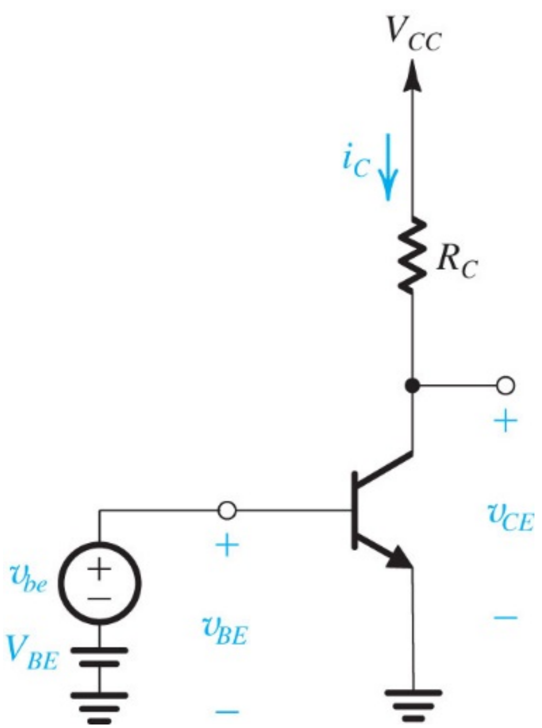
$$i_c = \frac{I_C}{V_T} v_{be}$$

The BJT transconductance g_m is defined as $g_m \equiv \frac{i_c}{v_{be}}$

$$\boxed{g_m = \frac{I_C}{V_T}}$$

$$i_c = g_m v_{be}$$

Derivation of BJT small signal model



$$i_C = I_C + \frac{I_C}{V_T} v_{be}$$

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} \frac{I_C}{V_T} v_{be}$$

$$i_B = I_B + i_b$$

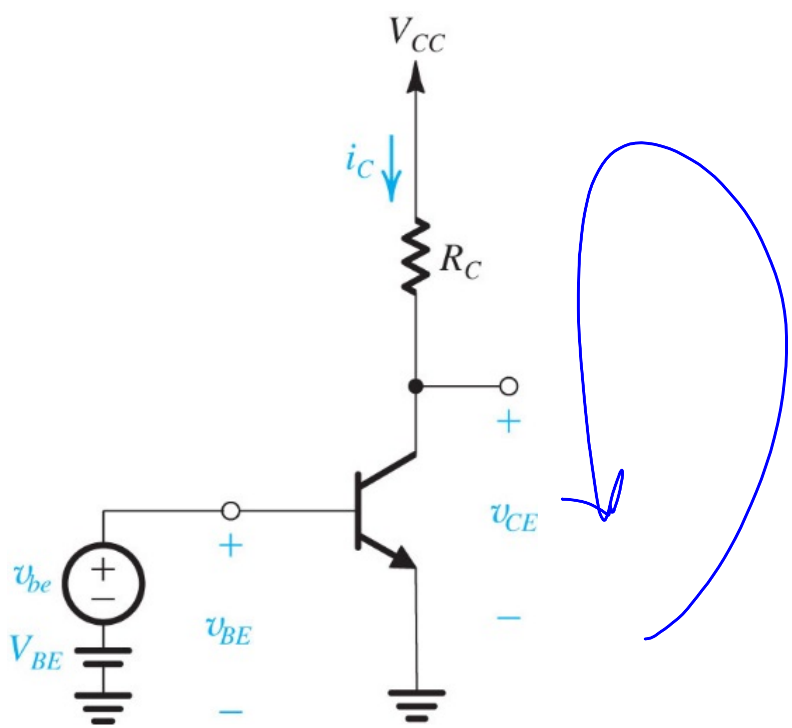
$$i_b = \frac{1}{\beta} \frac{I_C}{V_T} v_{be} = \frac{g_m}{\beta} v_{be}$$

The small signal input resistance between base and emitter is denoted by r_π

$$r_\pi \equiv \frac{v_{be}}{i_b} \rightarrow \quad r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

Derivation of BJT small signal model

The voltage gain in this amplifier configuration:



$$v_{CE} = V_{CC} - i_C R_C$$

$$= V_{CC} - (i_c + I_C)\, R_C$$

$$= (V_{CC} - I_C R_C) \; - \; i_c R_C$$

$$= V_{CE} - i_c R_C$$

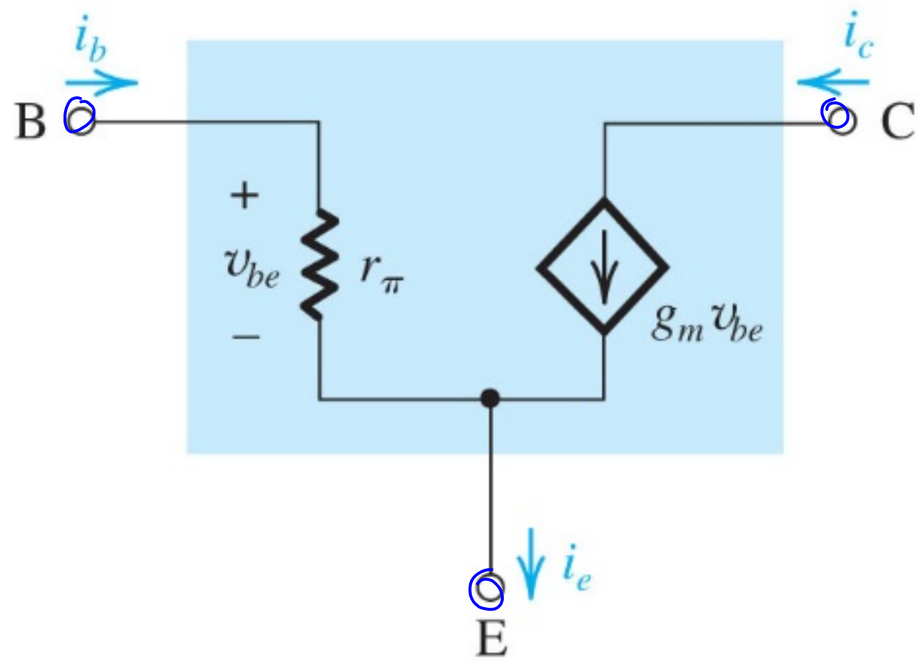
$$v_{ce} = -\, i_c \; R_C = -g_m v_{be} R_C$$

$$A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C$$

BJT small signal model

$$i_b = \frac{v_{be}}{r_\pi}$$

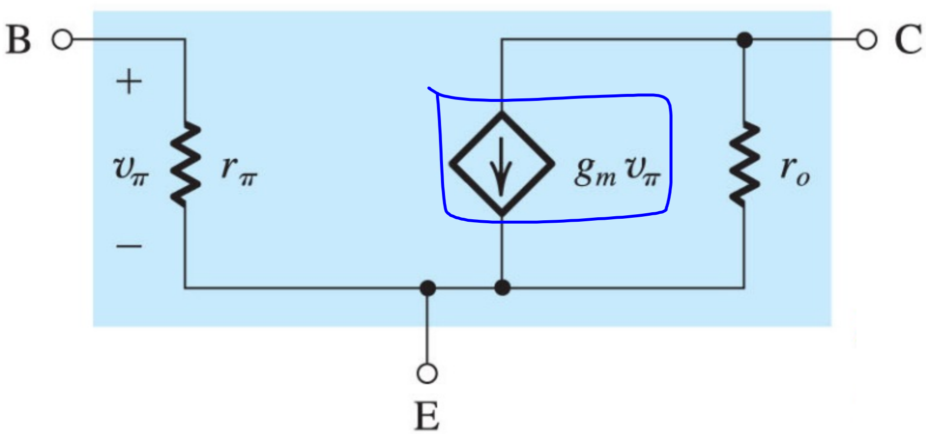
$$i_c = g_m v_{be}$$

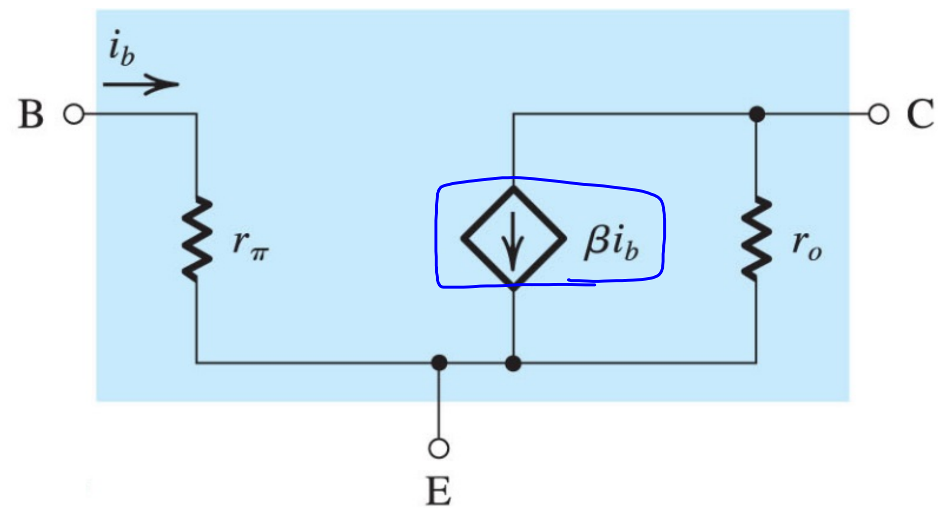


BJT small signal model

$v_\pi = v_{be}$

$$i_b = \frac{v_{be}}{r_\pi} \qquad i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$





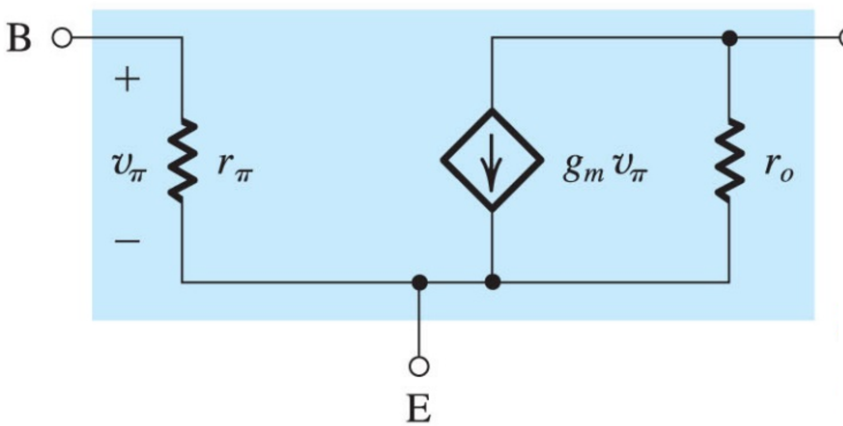
$V_A = \frac{1}{\lambda}$

$$g_m = \frac{I_C}{V_T} \qquad r_o \approx \frac{V_A}{I_C} \qquad r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m v_{be} = g_m(i_b r_\pi) = (g_m r_\pi) i_b = \beta i_b$$

PNP small signal model is identical to NPN

PNP



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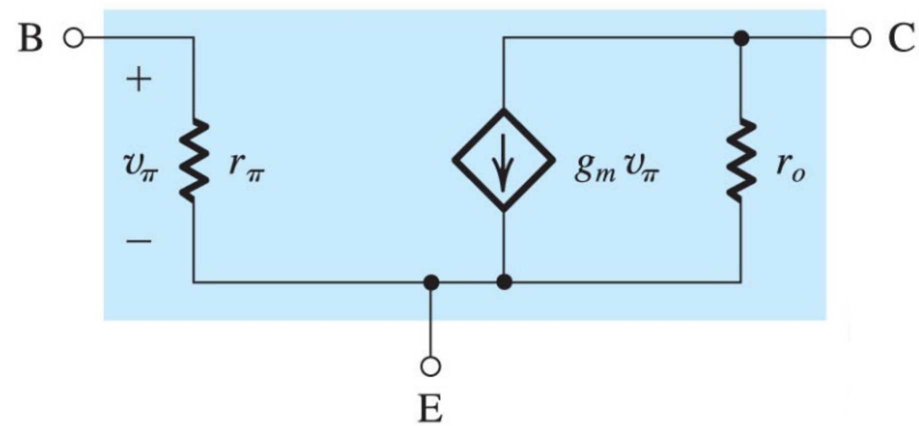
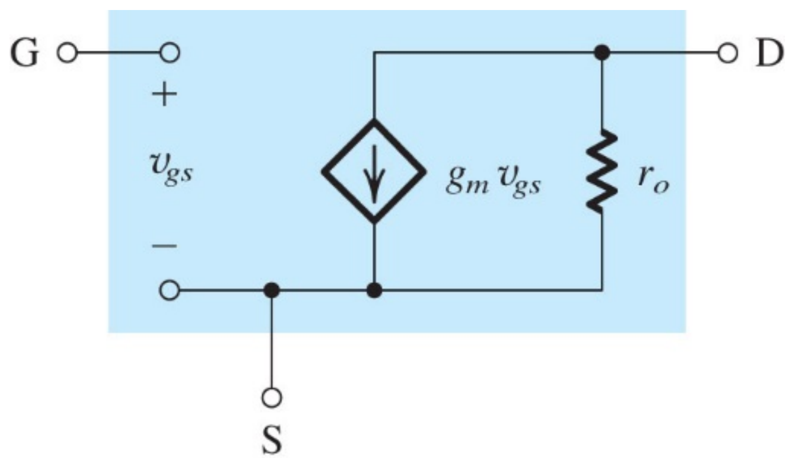
NPN



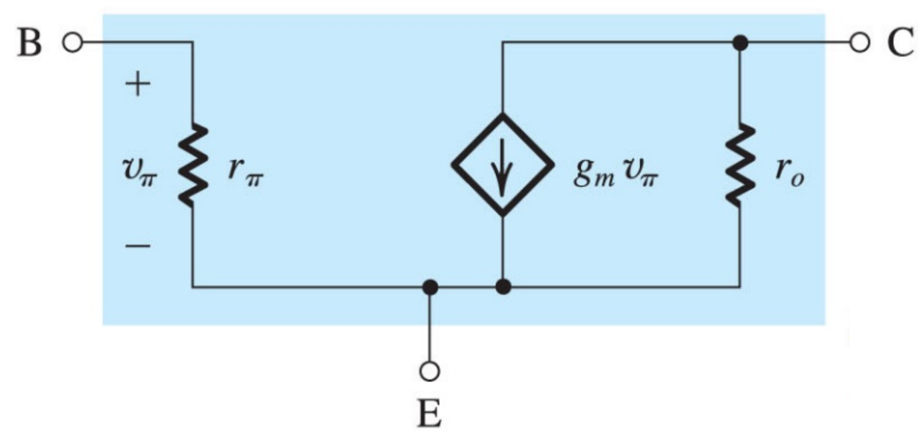
PNP small-signal circuit model is identical to NPN

Summary of transistor small signal models

NMOS/PMOS



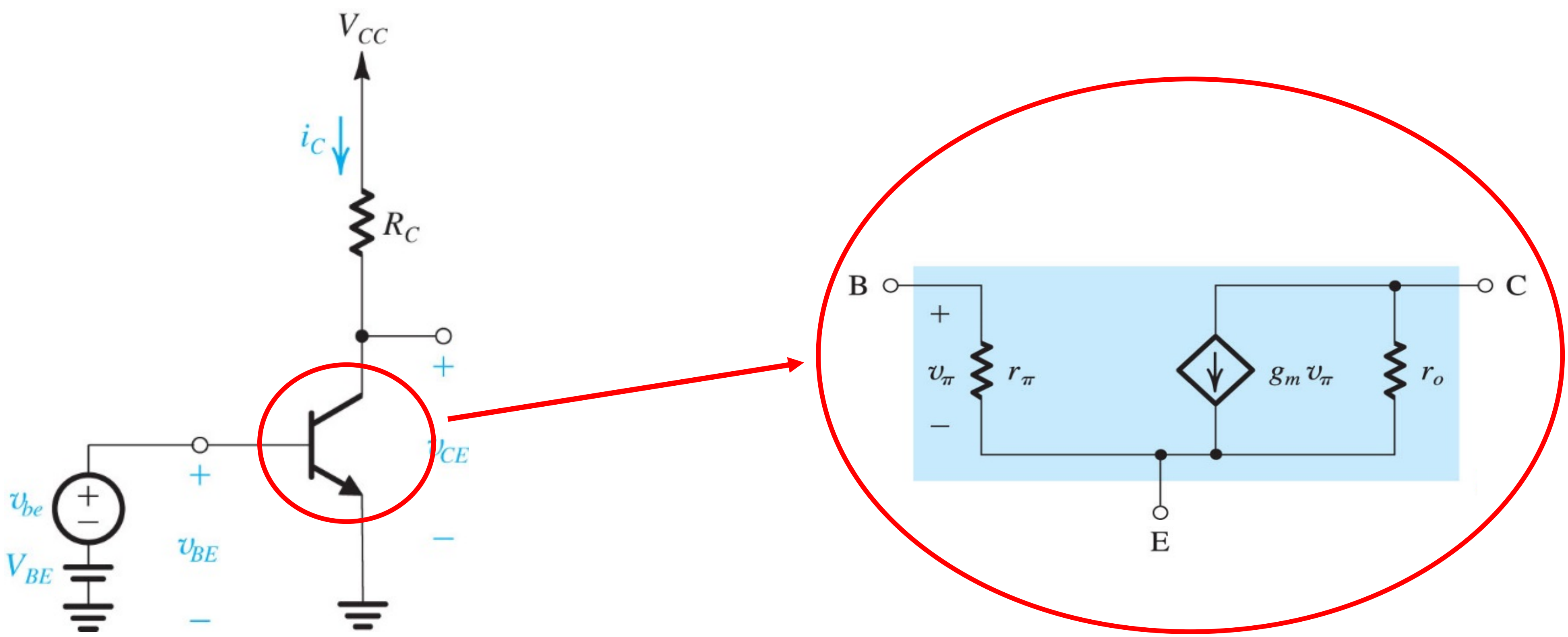
NPN/PNP BJT



Comparison of MOS and BJT small-signal circuit models:

1. MOS has an infinite resistor in the input (v_{gs}) while BJT has a finite resistor, r_π (typically several kΩ).
2. BJT g_m is substantially larger than that of a MOS (BJT has a much higher gain).
3. r_o values are typically similar (10s of kΩ).

BJT Small Signal Model



Review of amplifier circuit analysis

- Under small signal approximation, we can analyze the **Signal** and **Bias** circuits separately for a given amplifier circuit.

- The **Signal** and **Bias** circuits are different.

- **Bias** is the state of the system when there is no signal. In drawing and analyzing the **Bias** circuit, the capacitors are open and signal sources are set to zero.

- **Signal** circuit $\equiv$ signal equivalent circuit

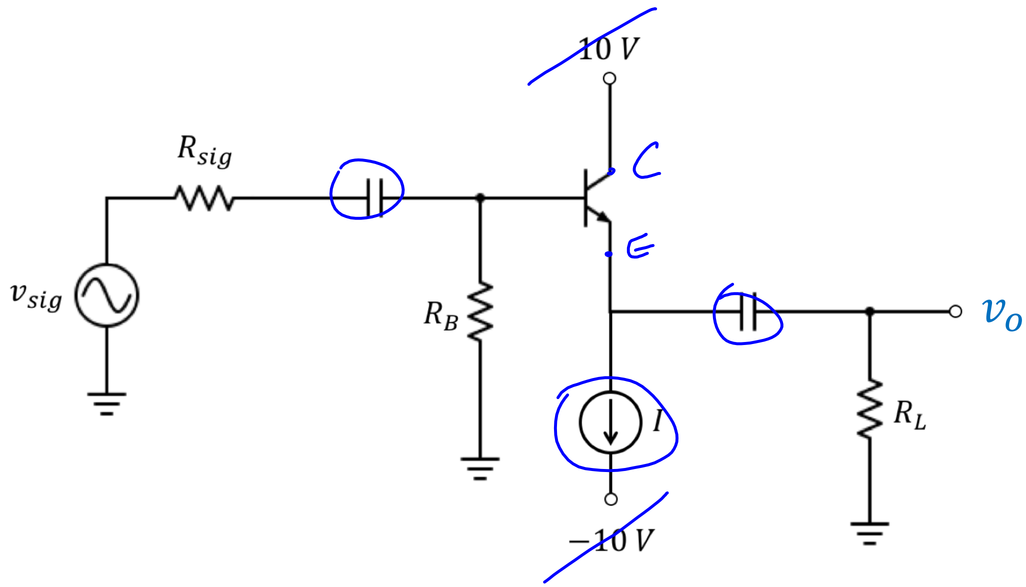
  $\equiv$ small signal equivalent circuit

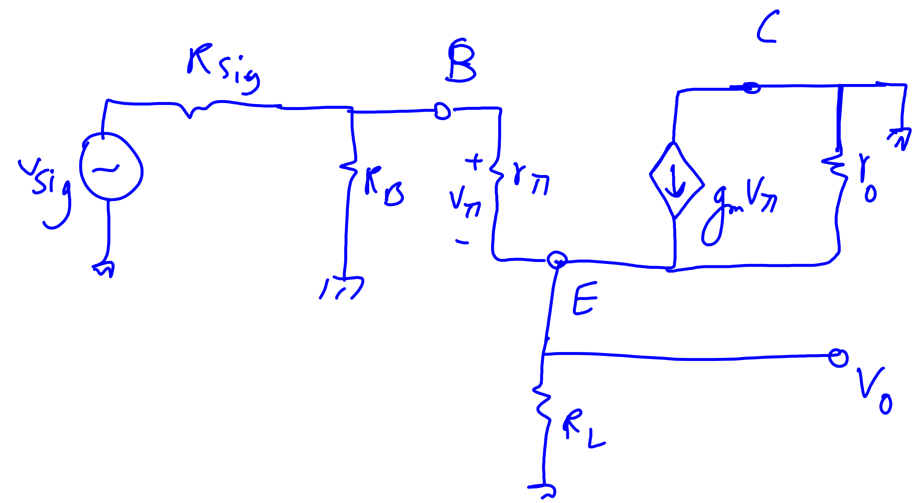
Review of amplifier analysis

- In drawing the **Signal** circuit you should
  - replace the transistors with their small signal models without changing anything in the model
  - keep the resistors – find the node they are connected to in the original circuit and connect them to the right node in the signal circuit.
  - short the capacitors
  - set the independent DC current and voltage sources to zero.

Example:

Draw the signal circuit (assume capacitors are short for signal).

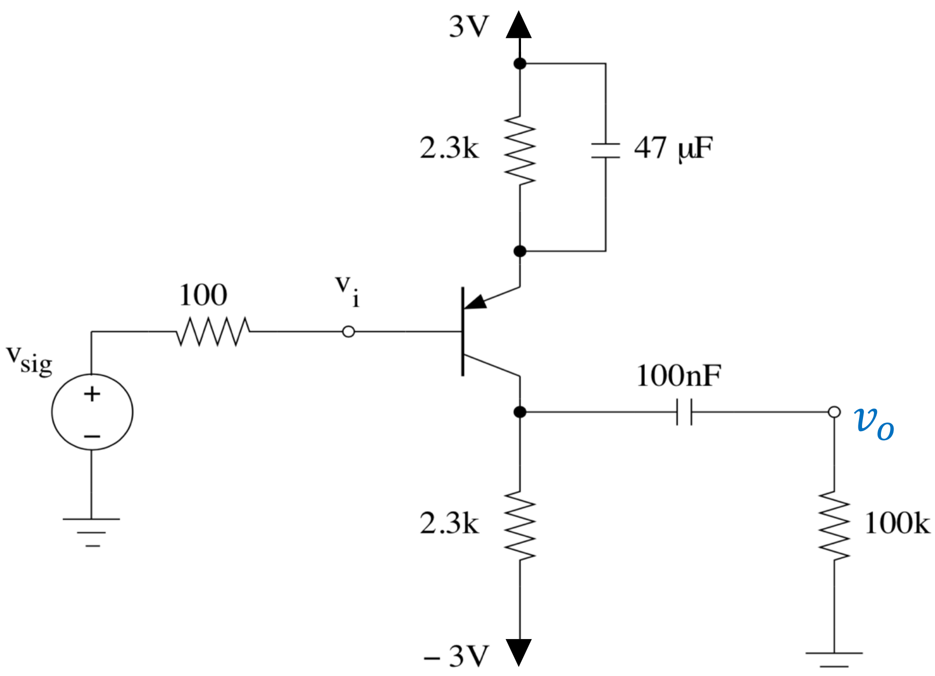




Lecture 19 reading quiz:

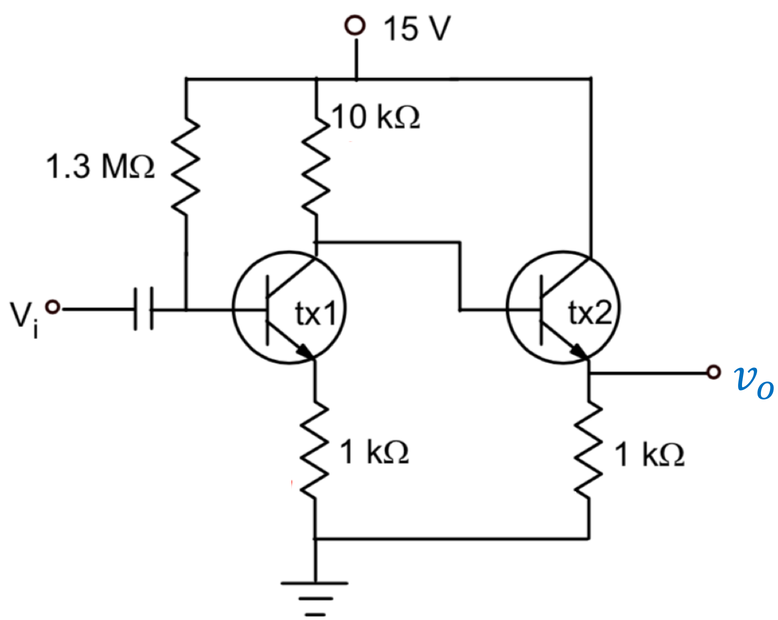
Find the transconductance, g_m , in this circuit ($V_{D0} = 0.7\ V,\ V_T = 26\ \mathrm{mV}$

$V_A = 150\ V$.



Discussion question 1.

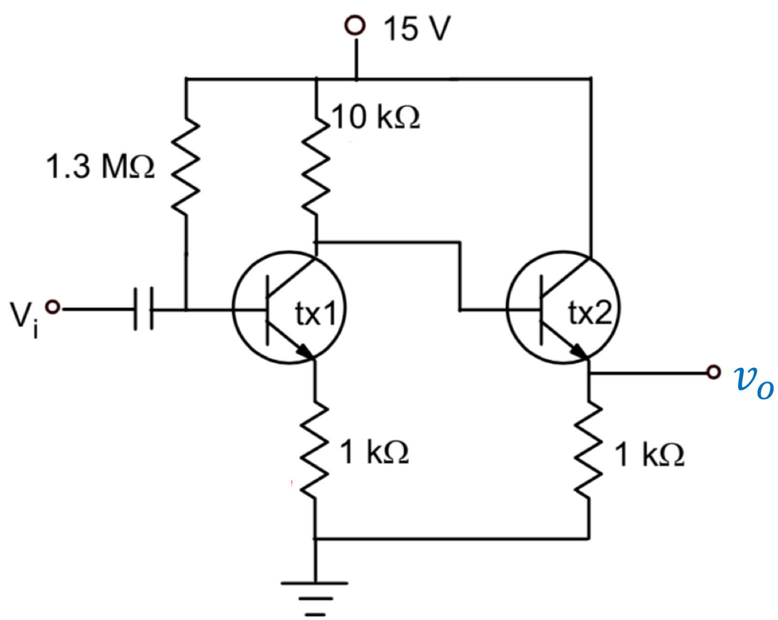
Draw the signal circuit (assume capacitors are short for signal).



Hints:

Discussion question 1.

Draw the signal circuit (assume capacitors are short for signal).



- Label all the node voltages and currents on the given circuit.

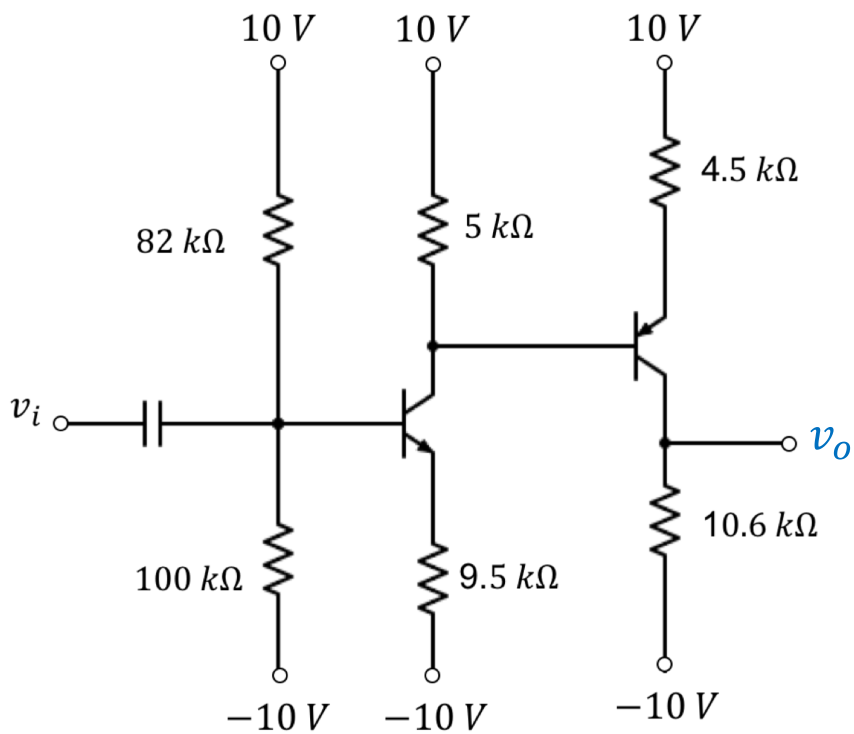
- Draw the small signal model for two BJTs, and label all the node voltages.

- Add the other circuit elements to the small signal circuits according to how they are connected to the base, collector, and emitter terminals of the BJTs in the given circuit.

- The capacitors will be short and the DC voltage sources will be set to zero in the signal circuit.

Discussion question 2.

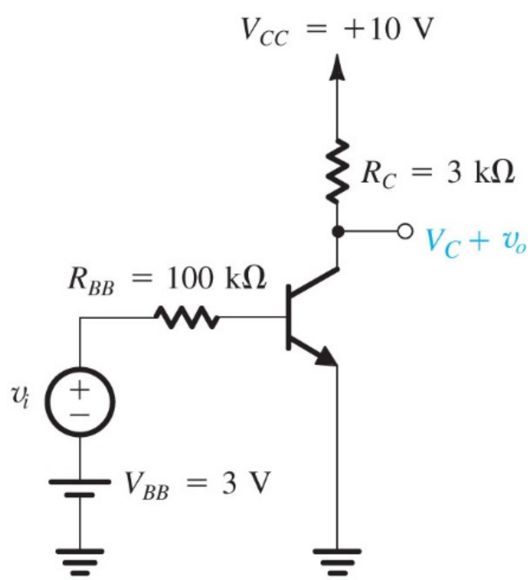
Draw the signal circuit (assume capacitors are short for signal).



Discussion question 3.

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

Assume $\beta = 100$, $V_T = 25\ mV$ and neglect the early effect.



Hints:

Discussion question 3.

For this amplifier circuit, find the small signal parameters, draw the signal circuit (assume capacitors are short for signal) and find v_o/v_i .

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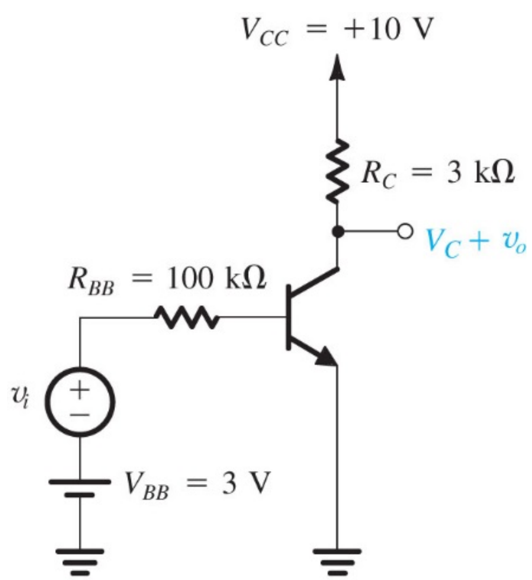

- Draw and solve the DC circuit to get all the DC node voltages and currents.

- Calculate the small signal parameters (r_pi, r_o, g_m)

- Draw the small signal model for and label all the node voltages.

- Add the other circuit elements to the small signal circuit according to how they are connected to the base, collector, and emitter terminals of the BJT in the given circuit.

- To find v_o/v_i, write a KVL(or KCL) to relate v_o to v_{pi} and another one to relate v_{pi} to v_i. Use those two equations to relate v_o to v_i.