AC power 008

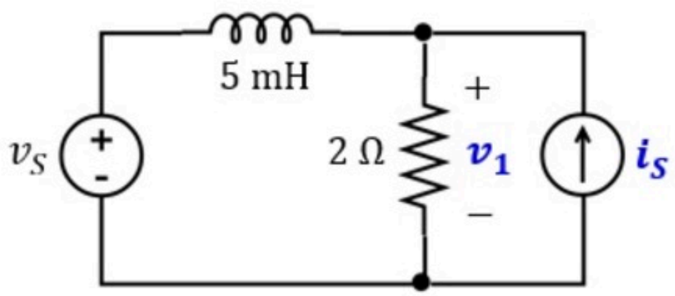
Problem has been graded.

Both v_S and i_S are AC sources with $\omega = 400$ rad/s. Someone did two measurements on the circuit and found:

$$v_1(t) = A_1 \cos(400t + B_1) \qquad \textit{(this is } v_1\textit{, not } v_S \textit{ !)}$$

$$i_S(t) = 2\cos\left(400t + \frac{\pi}{6}\right)$$

(a) Find the average power P_1 received by the resistor
(b) Find the average power P_2 supplied by current source i_S
(c) Find the average power P_3 received by the inductor



Given Variables:

A1 : 8 V

B1 : 90 degrees

Calculate the following:

P1 (W) :

16 ✓

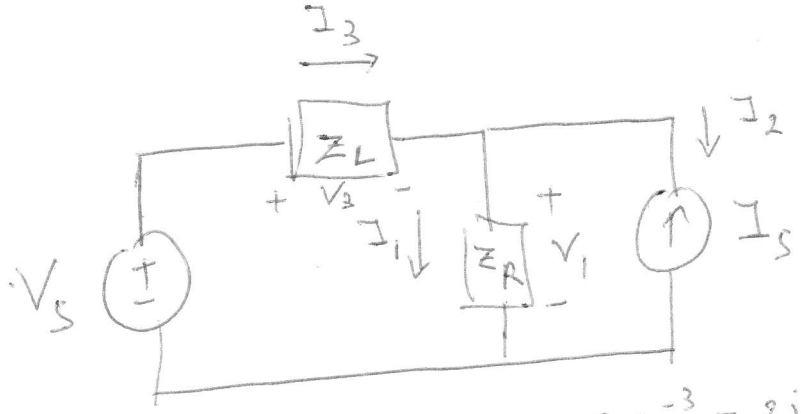
P2 (W) :

4 ✓

P3 (W) :

0 ✓

Hint: Find S symbolically first. Find phasors and plug in. For c, what do you know about the element?



$A_1 : 10\,V$

$B_1 : -90^\circ$

$Z_L = j\omega L = j \cdot 400 \cdot 5 \cdot 10^{-3} = 2j$ $\qquad Z_R = 2$

$I_1 = \dfrac{V_1}{Z_R}$

(a) $S_1 = \frac{1}{2} V_1 \cdot I_1^*$

$= \frac{1}{2} V_1 \left(\frac{V_1}{Z_R}\right)^* = \frac{1}{2} \frac{V_1 V_1^*}{Z_R} = \frac{|V_1|^2}{2} \cdot \frac{1}{Z_R} = \frac{A_1^2}{2} \cdot \frac{1}{Z_R} = \frac{100}{2 \cdot 2} = 25$

$P_1 = Re[S_1]$ $\qquad$ **$P_1 = 25\,W$** received

(b) $S_2 = \frac{1}{2} V_1 I_2^*$ ⇝ PASSIVE SIGN CONVENTION (POWER RECEIVED)

$= \frac{1}{2} V_1 (-I_S)^*$

$= -\frac{1}{2} V_1 I_S^*$

$= -\frac{1}{2} \cdot 10 e^{-j\frac{\pi}{2}} \cdot 2 e^{-j\frac{\pi}{6}} = -10 e^{-j\frac{4\pi}{6}} = -10 e^{-j\frac{2\pi}{3}}$

$\left(V_1 = A_1 e^{jB_1} = 10 e^{-j\frac{\pi}{2}} \right.$

$\left. I_S = 2 e^{j\frac{\pi}{6}} \right.$

$P_2 = Re[S_2] = -10 \cos\left(-\frac{2\pi}{3}\right) = -10\left(-\frac{1}{2}\right) = +5$ ⇝ received

$P_2 = -5\,W$ supplied

(c) **$P_3 = 0\,W$** inductor always has $P = 0$

we can check: $S_3 = \frac{1}{2} V_3 I_3^*$ $\qquad V_3 = Z_L \cdot I_3$

$= \frac{1}{2} Z_L I_3 I_3^* = \frac{Z_L}{2} |I_3|^2 = \frac{|I_3|^2}{2} \cdot (2j)$

no real part ⇒ $P = Re[S] = 0$