ECE 35, Fall 2017
Final - Section A

Last name

First + middle name(s)

PID

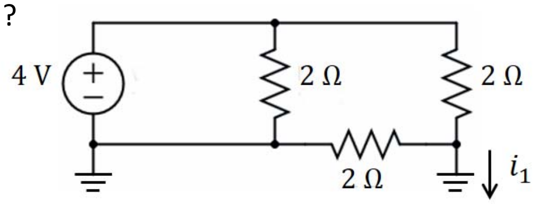
Last name	
First + middle name(s)	
PID	

Instructions:

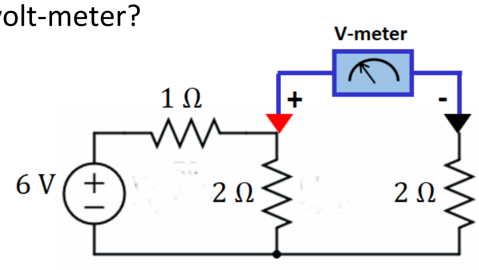
- Read each problem completely and thoroughly before beginning.
- All calculations should to be done in your blue book. It should be clear what question they belong to. Answers without supporting calculations will receive zero credit.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes on these question pages. Make sure you list units!

(1) Answer the questions below.

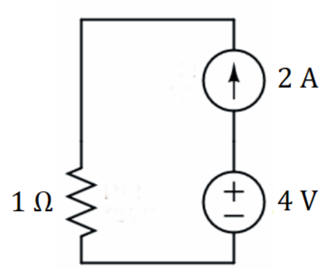
(a) What is the current i_1?



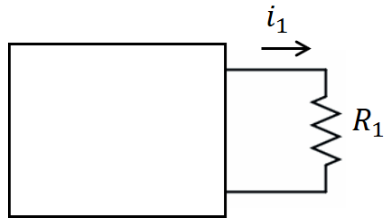
(b) What is the reading from the volt-meter?



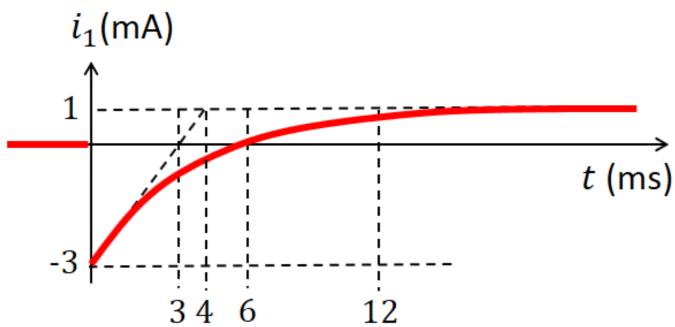
(c) What is the power supplied by the current source?



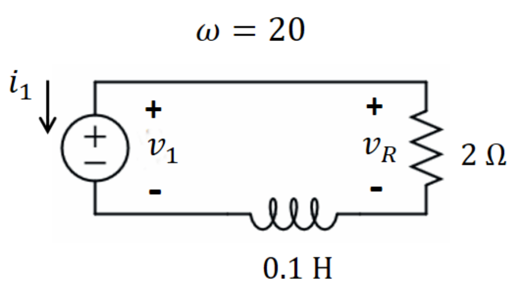
(d) When R_1 = 1 Ω, the current i_1 = 4 A. When R_1 = 2 Ω, the current i_1 = 3 A. When is the current i_1 when R_1 = 3 Ω?

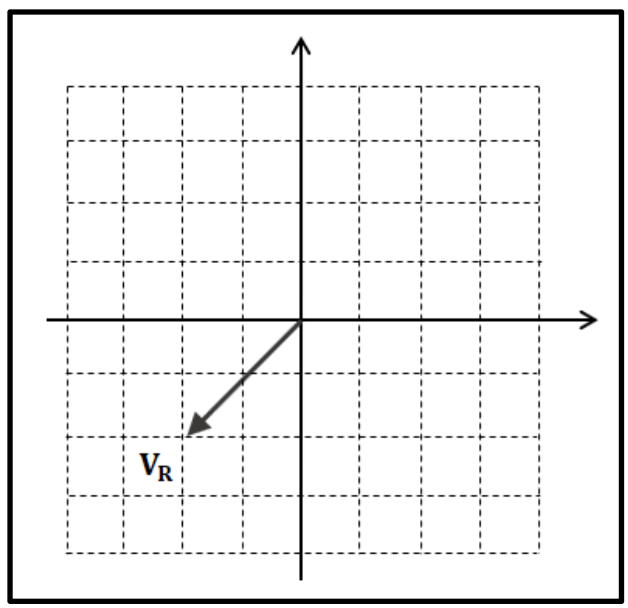


(e) The graph below shows the current in a first order RL circuit. What is the equation of $i_1(t)$ for $t > 0$?

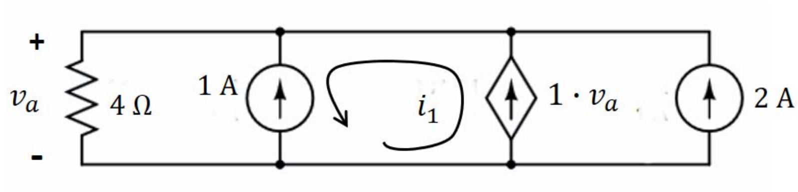


(f) With the phasor $\mathbf{V_R}$ given on right, draw the phasors $\mathbf{V_1}$ and $\mathbf{I_1}$ in the same diagram. You can draw the same scale for A and V (i.e., a 1 A current phasor has the same length as a 1 V voltage phasor).



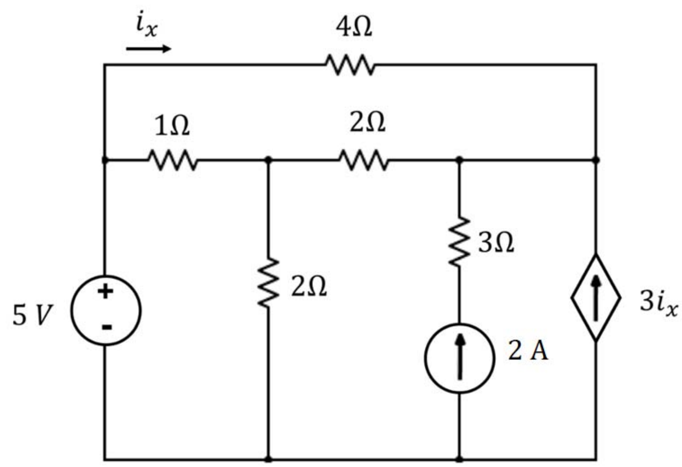


(2) What is the value of mesh current i_1?

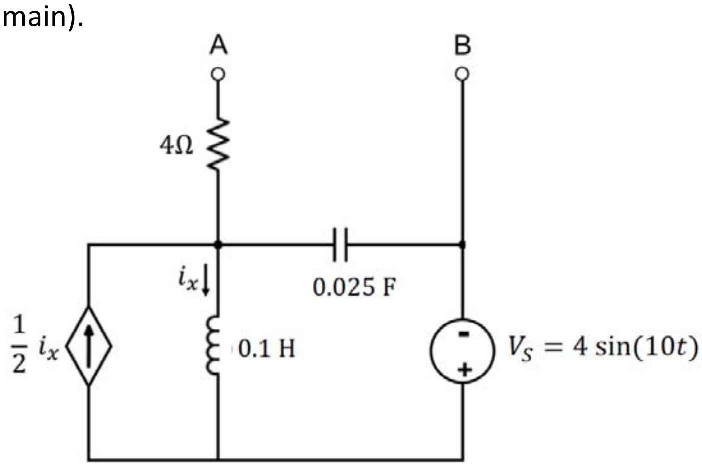


(3) (a) For the circuit below, what is the value of i_x?

(b) What is the power supplied by the dependent source?



(4) What is the Norton equivalent current source I_N (for the Norton equivalent model between A and B). Write your final answer in the time domain (don't leave it in the phasor domain).

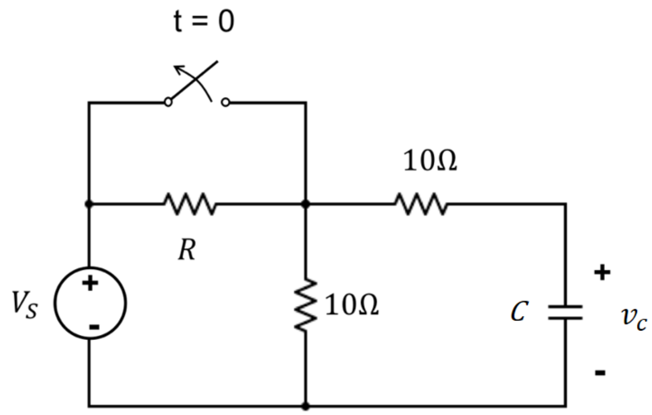


(5) Consider the circuit below. For *t* < 0, the switch is closed and the system has reached steady state. At *t* = 0, the switch opens and it remains open.

We measure that for $t > 0$, $v_C(t) = 2 + 8\, e^{-\frac{t}{12}}$.

(a) What was v_C at time *t* = 0^- (just before the switch was opened)? $v_C(0^-)$ ☐

(b) Find V_S , *R* and *C*. V_S ☐ *R* ☐ *C* ☐



(6) Consider the circuit below. At *t* < 0, the switch is open.
At *t* = 0, the switch closes and it remains closed.
We are given the capacitor voltage just before the switch closes: $v_a(0^-) = V_1$ and $v_b(0^-) = V_2$.

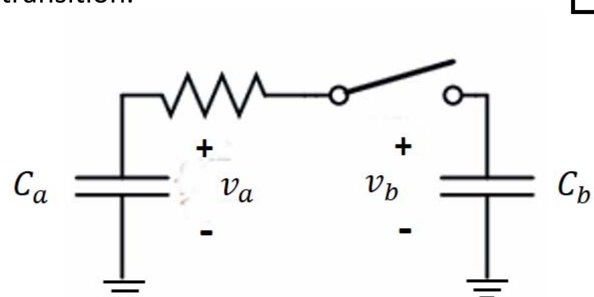
(a) What are the capacitor voltages v_a and v_b at time *t* = ∞?

v_a ☐

v_b ☐

(b) Find a concise expression of the total energy converted to heat during the transition.

☐



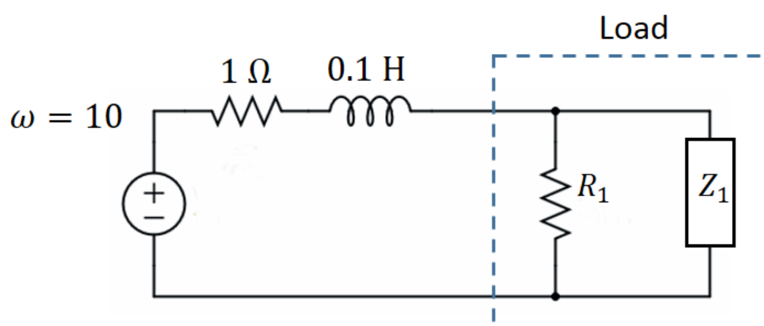
(7) Consider the circuit below, where a load is connected to a power distribution network (consisting of a resistor, an inductor and an AC voltage source). The load itself consists of a resistor R_1 in parallel with a mystery element Z_1, where Z_1 is either a capacitor or an inductor. The goal is to maximize the average power received by the load.

(a) In the mystery element a capacitor or an inductor?

(b) What is the value of the resistor R_1?

What is the value of the capacitor or inductor that makes up Z_1?

(c) If Z_1 is removed from the circuit and R_1 is chosen to be 1 Ω. If the voltage source is $v_S(t) = 2\cos\left(10t + \frac{\pi}{2}\right)$, what is the complex power of the inductor?



ECE 35, Fall 2017

Final - Section B

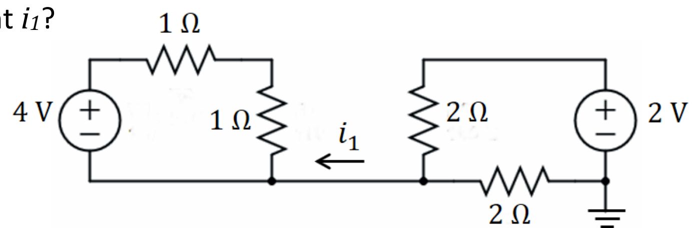
Last name	
First + middle name(s)	
PID	

Instructions:

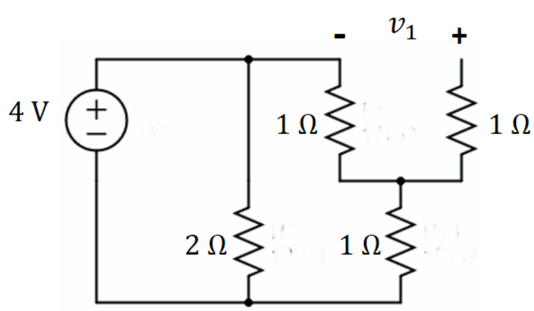
- Read each problem completely and thoroughly before beginning.
- All calculations should to be done in your blue book. It should be clear what question they belong to. Answers without supporting calculations will receive zero credit.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes on these question pages. Make sure you list units!

(1) Answer the questions below.

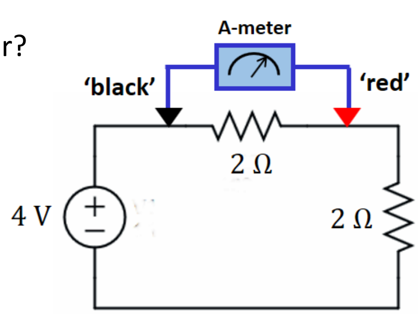
(a) What is the current i_1?



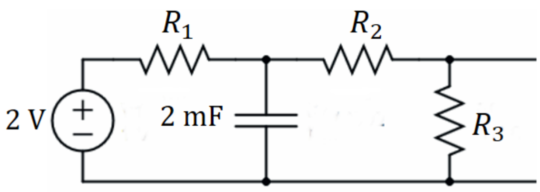
(b) What is the voltage v_1?



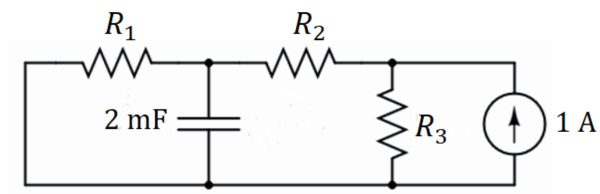
(c) What is the reading from the ammeter?



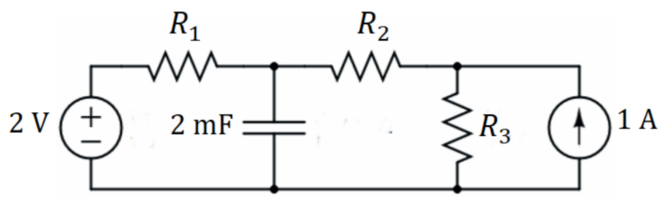
(d) The energy stored in the capacitor in scenario (1) is 1 mJ and in scenario (2) it is 4 mJ. What is the energy stored in scenario (3)?



(1)

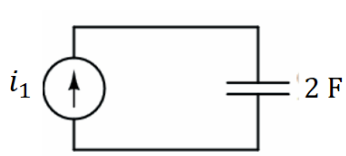


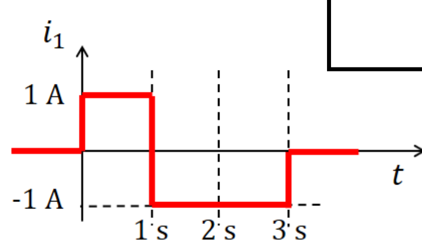
(2)



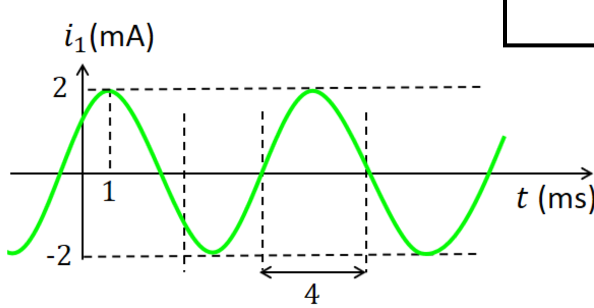
(3)

(e) Assume the capacitor is discharged at time $t = 0$. During what time period(s) does the capacitor receive power? For example, write your answer as ($t < 1$ s) and (1 s $< t < 2$ s).



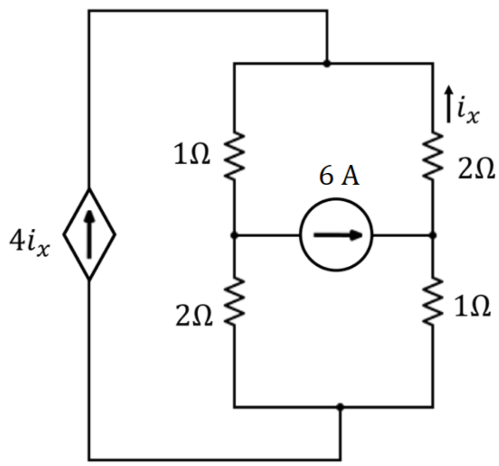


(f) What is the phasor $\mathbf{I}_1$ of this current waveform? Write the phasor in polar coordinates.



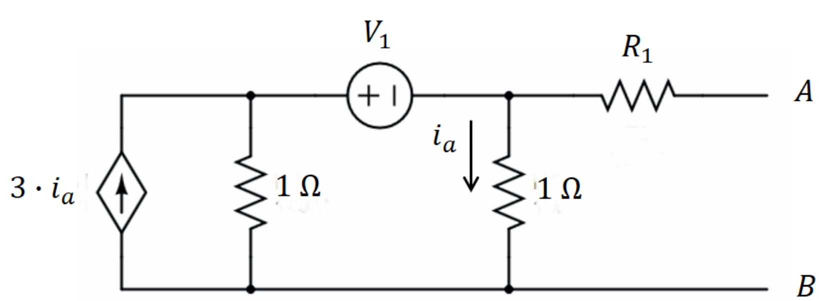
(2) (a) For the circuit below, what is the value of i_x?

(b) What is the power supplied by the dependent source?



(3) (a) For the circuit below, find the Thevenin equivalent model as seen between A and B, when V_1 = 4 V and R_1 = 0 Ω. Draw the model in the answer box with the appropriate values.

(b) What is the Thevenin equivalent resistance when I make the following changes to circuit: V_1 = 8 V and R_1 = 17 Ω?



(4) Consider the circuit below. For $t < 0$, the switch is open and the system has reached steady state. At $t = 0$, the switch closes and it remains closed.

Find $v_C(0^-)$ and $v_C(0^+)$.

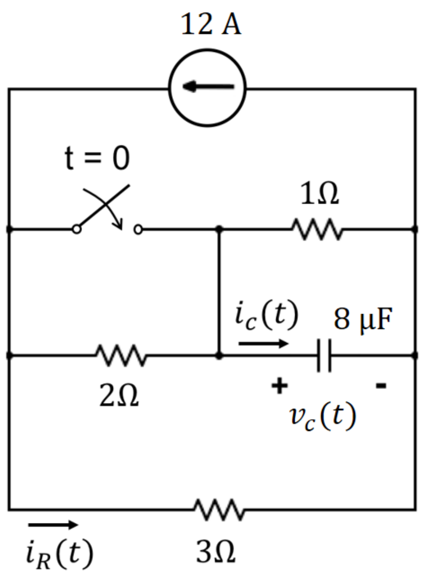
$v_C(0^-)$ ☐ $v_C(0^+)$ ☐

Find $i_C(0^-)$ and $i_C(0^+)$.

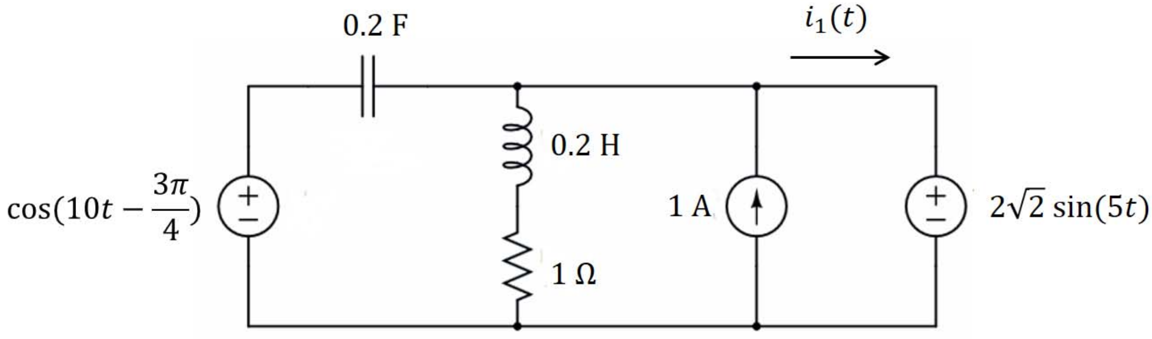
$i_C(0^-)$ ☐ $i_C(0^+)$ ☐

Find the expression for the current $i_R(t)$ for $t > 0$.

$i_R(t)$ =



(5) For the circuit below, what is $i_1(t)$? Express your answer in the time domain.



(6) (a) What is the average power received by the inductor?

(b) What is the complex power received by the combination of the resistor and the capacitor?

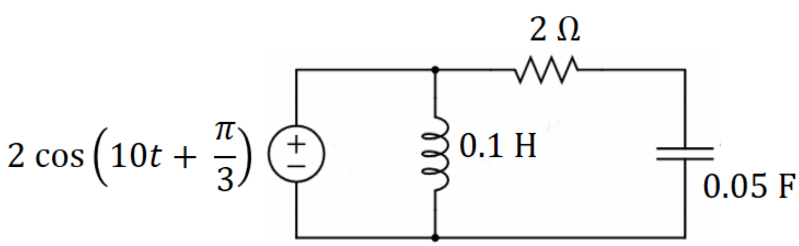
(c) Someone modified the values of the resistor, capacitor and inductor. With these new elements in the circuit (and with the same source), the total complex power received by all elements combined is $S = 1 + j$

What element (resistor, capacitor or inductor) do you need to place in parallel with the source to achieve a total complex power of $S = 1$?

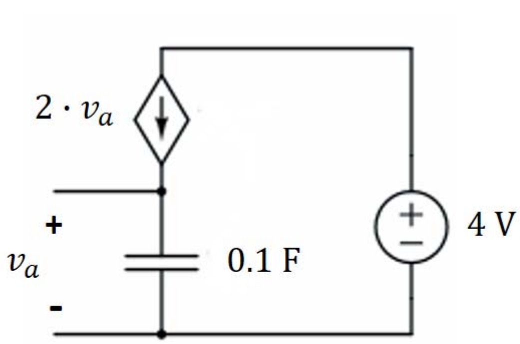
Type of element:

What is the value of that element?

Value of the element:



(7) For the circuit below, what is the expression for $v_a(t)$, for $t > 0$? At $t = 0$, the capacitor voltage is 1 V.



ECE 35, Fall 2018
Final - Section B

Sequence number	

Grade

/ 50	

Last name	
First + middle name(s)	
PID	

Instructions:

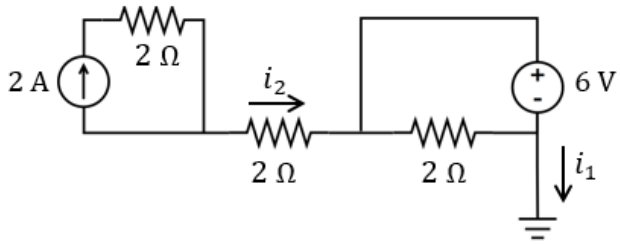
- Do not look at the questions or start writing until it is announced you can do so.
- Read each problem completely and thoroughly before beginning.
- <u>All calculations must be done in your blue book</u>. It should be clear which question they belong to. Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- <u>Write your final answers in the answer boxes on these question pages</u>. Make sure you list units!
- You must follow the Final Exam Procedures that were posted on TritonEd. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag in the front of the room (or on the table in the front). If you are found to have a phone (or other communication device) on you during the exam, your exam will not be graded.
  - Calculators are not allowed.
  - This is a closed book exam.

▲

(1) (a) Find the currents i_1 and i_2. *(2 points)*

i_1

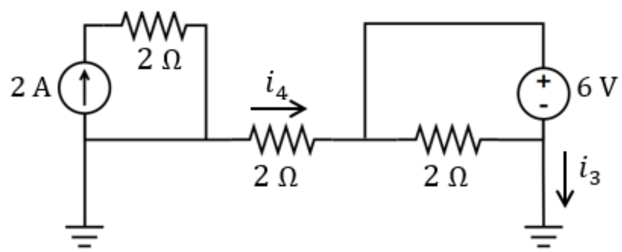
i_2



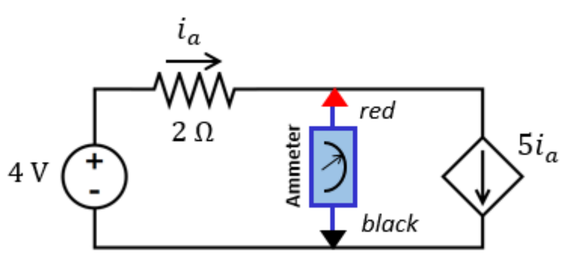
(b) Find the currents i_3 and i_4. *(2 points)*

i_3

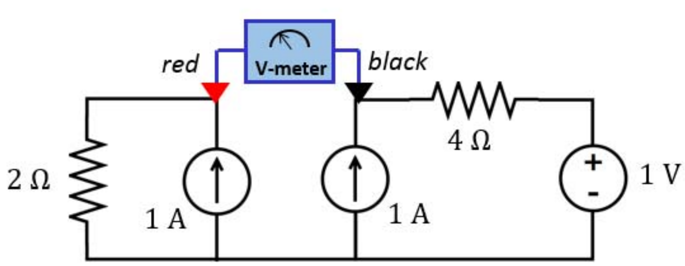
i_4



(2) (a) What is the ammeter reading? *(2 points)*



(b) What is the volt-meter reading? *(2 points)*



(c) Find the Thevenin equivalent model between a and b for the circuit shown in Fig. A. *(3 points)*

To help you find this model, someone did an experiment for you (shown in Fig. B): they attached a 1 Ω resistor and measured a current of 1 A with the ammeter.

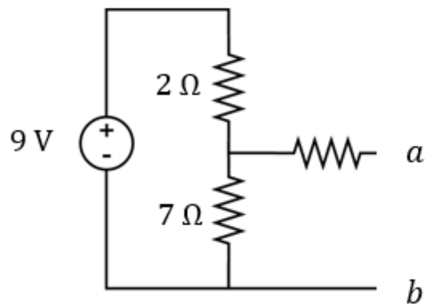


Fig. A

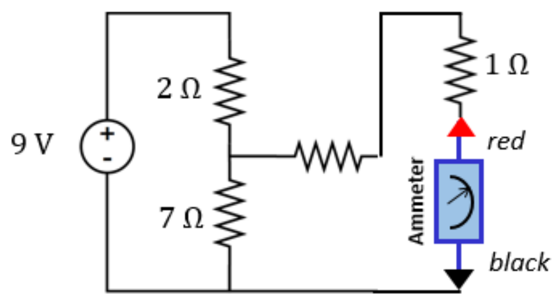
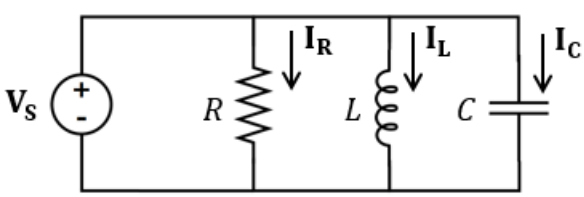


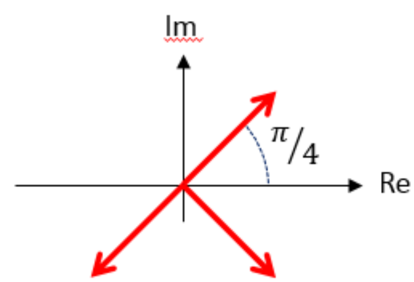
Fig. B

(3) (a) The phasor diagram shows the currents $\mathbf{I_R}$, $\mathbf{I_L}$ and $\mathbf{I_C}$. Each phasor has a length of 4 A. Find i_R and i_C at time t = 2 s. *(3 points)*

$i_R(2)$ $i_C(2)$

$\omega = 0.25\pi \quad s^{-1}$



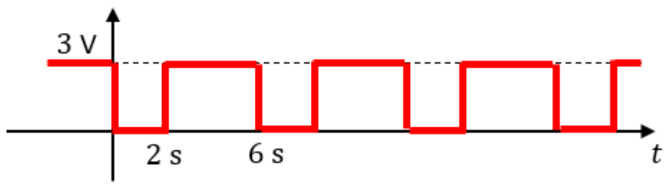


(b) Find I_{RMS} of $i_1(t)$. *(2 points)*

Find V_m (i.e., the magnitude of v_2) such that the average power received by the 2 Ω resistor in Fig. B is the same as the average power received by the 2 Ω resistor in Fig. A. *(2 points)*

I_{RMS} 

V_m 



(Note: this waveform is periodic)

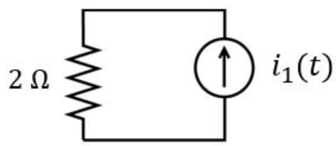


Fig. A

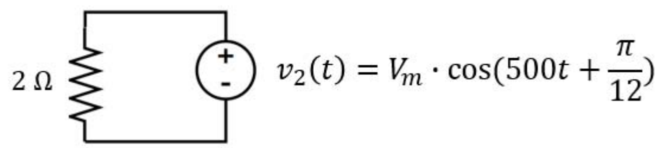
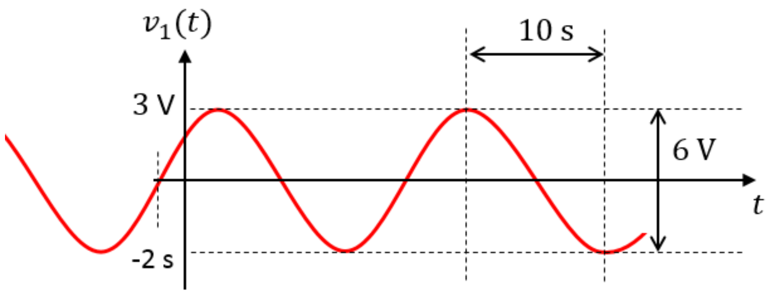


Fig. B

(c) Find the equation of the time waveform $v_1(t)$. *(3 points)*

$v_1(t)$ 

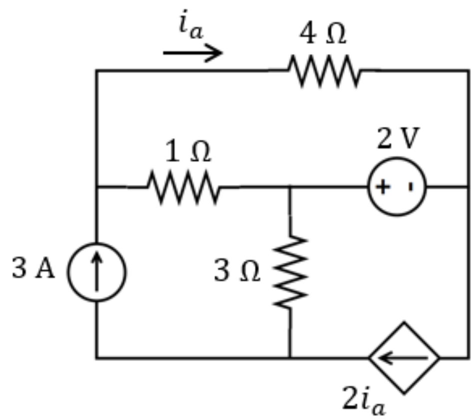


(4) (a) For the circuit below, find i_a. *(3 points)*

i_a [ ]

(b) What is the power P supplied by the dependent source? *(3 points)*

P [ ]



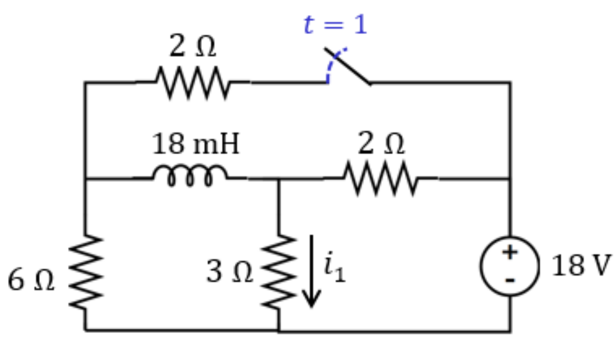
(5) For $t < 1$ s, the switch is closed and you may assume the system has reached steady state.
<u>The switch opens at time $t = 1$ s</u>.

(a) Find $i_1(1^-)$. *(1 point)*

$i_1(1^-)$ [ ]

(b) Find $i_1(t)$ for $t > 1$ s. *(6 points)*

$i_1(t)$ [ ]

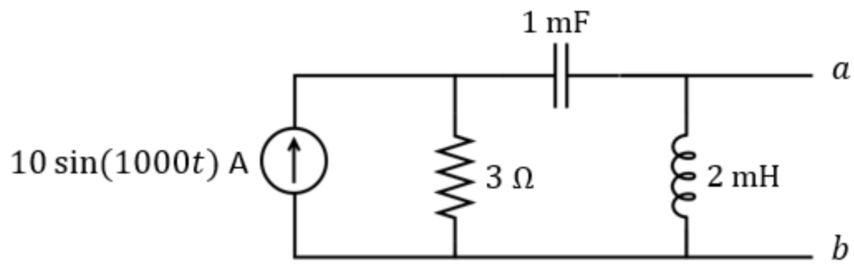


(6) (a) Find the complex power **S** received by the inductor. *(4 points)*

(b) What load impedance $\mathbf{Z_L}$ should be attached between a and b to get maximum average power received by $\mathbf{Z_L}$? You can choose to give your answer in cartesian or polar form. *(4 points)*

$\mathbf{S}$

$\mathbf{Z_L}$



(7) For $t < t_0$, the switch is in position *A* and you may assume the system is in steady state. The switch moves to position *B* at time $t = t_0 = \frac{\pi}{3000}$.

(a) Find the capacitor current i_C at time $t = t_0^-$ (i.e. with the switch in position A, just before it moves to position *B*). *(1 points)*

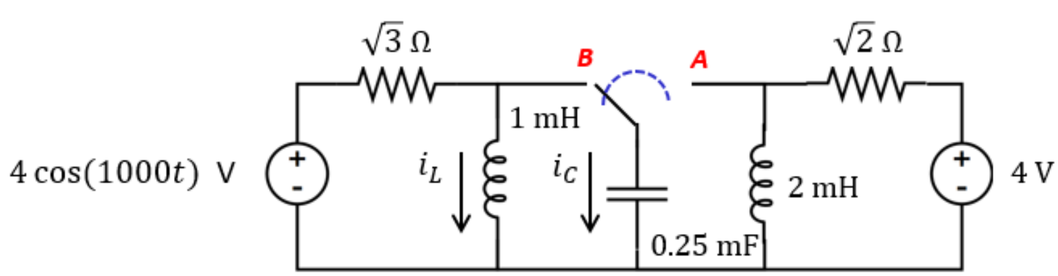
$i_C(t_0^-)$

(b) Find the inductor current i_L at time $t = t_0^+$ (i.e. immediately after the switch moves to position *B*). *(3 points)*

$i_L(t_0^+)$

(c) Find the capacitor current i_C at time $t = t_0^+$ (i.e. immediately after the switch moves to position *B*). *(4 points)*

$i_C(t_0^+)$



ECE 35, Fall 2019
Final – Section A

Sequence number

Last name

First + middle name(s)

PID

Grade

/ 50

Instructions:

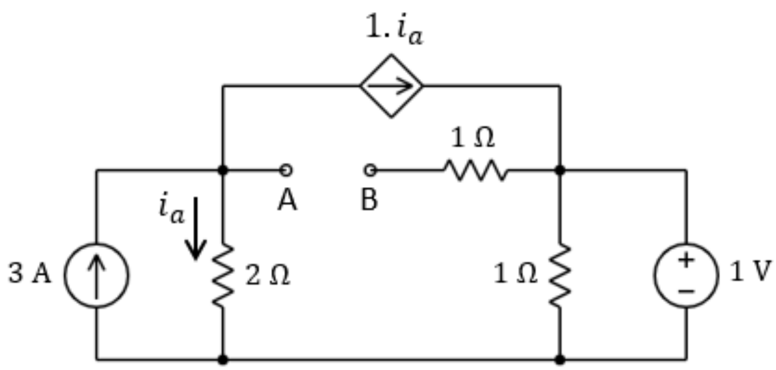
- Do not look at the questions or start writing until it is announced you can do so.
- Read each problem completely and thoroughly before beginning.
- All calculations must be done in your blue book. It should be clear which question they belong to. Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes on these question pages. Make sure you list units!
- You must follow the Final Exam Procedures that were posted on TritonEd. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag in the front of the room (or on the table in the front). If you are found to have a phone (or other communication device) on you during the exam, your exam will not be graded.
  - Calculators are not allowed.
  - This is a closed book exam.

(1) Consider the circuit below.

(a) Find the Thevenin equivalent model between A and B. *(8 points)*

(b) If we were to place a 2/3 Ω resistor between A and B in the circuit below, what is the current i_R through that resistor (measured from A to B)? *(1 point)*

i_R

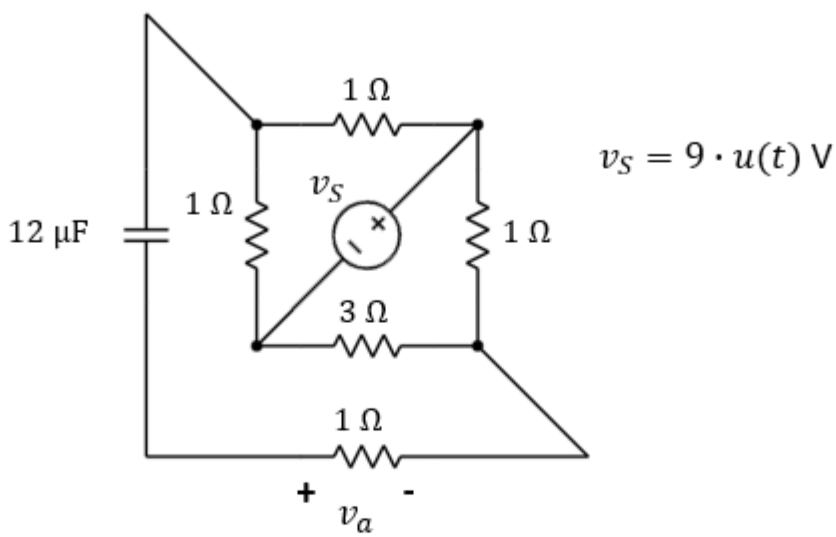


(2) Consider the circuit below. For $t < 0$ s, you may assume the system has reached steady state.

(a) Find $v_a(t)$ for $t > 0$ s.
(8 points)

$v_a(t)$

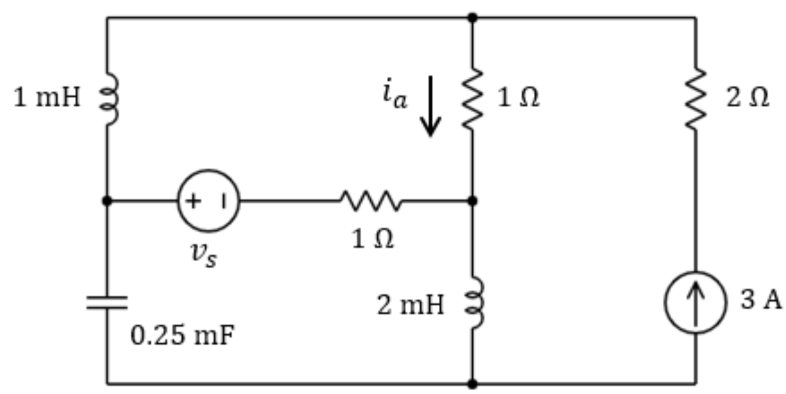
(b) Sketch the waveform $v_a(t)$ for $t > 0$ s. Include where you can observe the time constant.
(2 points)



(3) Consider the circuit below. The system is in steady state. Find $i_a(t)$. *(9 points)*

$i_a(t)$

$$v_s = 18\cos\left(1000t - \frac{\pi}{2}\right)\ \text{V}$$

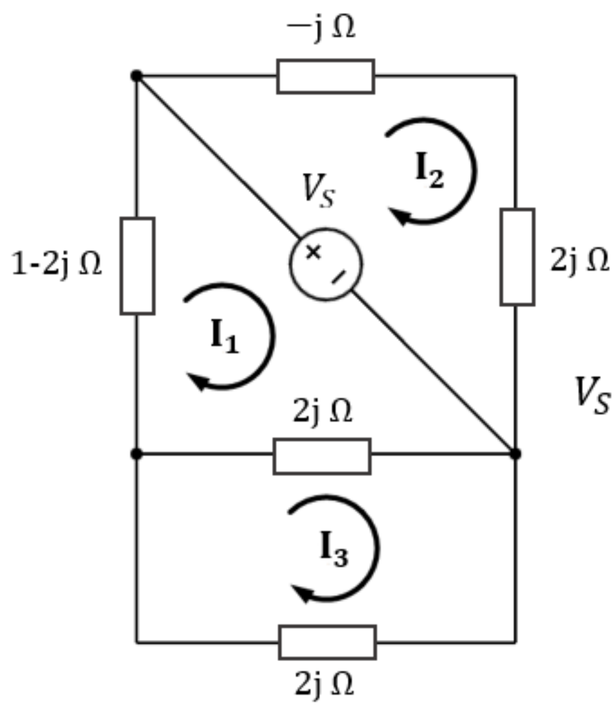


(4) Consider the circuit below. The system is in steady state and represented in phasor form.
(a) Find the mesh currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$ (expressed in polar form). *(8 points)*

$\mathbf{I_1}$ $\boxed{}$ $\mathbf{I_2}$ $\boxed{}$ $\mathbf{I_3}$ $\boxed{}$

(b) Find the complex power <u>supplied</u> by $\mathbf{V_S}$. *(2 points)*

$\mathbf{S}$ $\boxed{}$



$V_S = 10\sqrt{2}e^{j\frac{\pi}{5}}$ V

(5) Consider the circuit below. The system is in steady state and represented in phasor form. The ammeter and volt-meter are both ideal.

(a) Assume the load $\mathbf{Z_L} = 2j\ \Omega$. What is the reading X of the ammeter and the reading Y of the volt-meter? *(8 points)*

X (ammeter reading) ☐

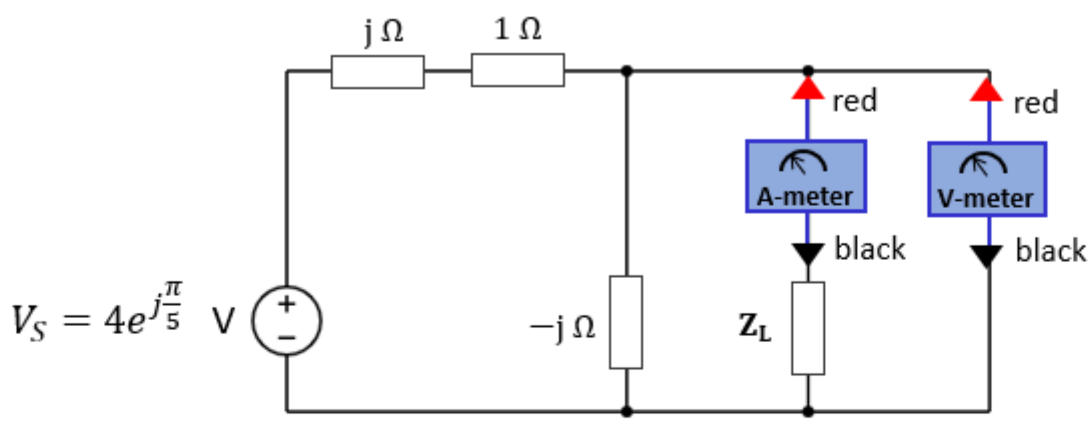
Y (voltmeter reading) ☐

(b) We now replace the load $\mathbf{Z_L}$ with a new value such that the average power received by this load is maximized. What is this new value of $\mathbf{Z_L}$? *(2 points)*

$\mathbf{Z_L}$ ☐

(c) Find the average power <u>received</u> by the load calculated in part (b). *(2 points)*

P ☐



ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho \frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2} C v^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2} L i^2$

AC power: $p(t) = \frac{1}{2} V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \qquad Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) \qquad X_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\qquad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\qquad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\qquad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\qquad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

ECE 35, Fall 2019
Final – Section B

Sequence number	
Last name	
First + middle name(s)	
PID	

Grade

/ 50

Instructions:

- Do not look at the questions or start writing until it is announced you can do so.
- Read each problem completely and thoroughly before beginning.
- <u>All calculations must be done in your blue book</u>. It should be clear which question they belong to. Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- <u>Write your final answers in the answer boxes on these question pages</u>. Make sure you list units!
- You must follow the Final Exam Procedures that were posted on TritonEd. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag in the front of the room (or on the table in the front). If you are found to have a phone (or other communication device) on you during the exam, your exam will not be graded.
  - Calculators are not allowed.
  - This is a closed book exam.

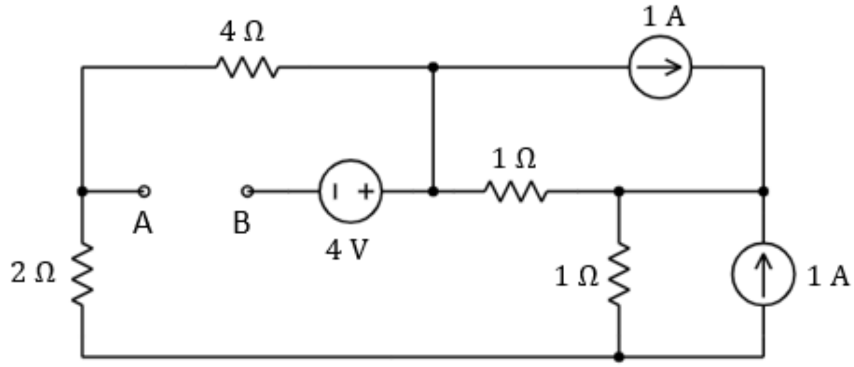
▲

(1) Consider the circuit below.

(a) Find the Thevenin equivalent model between A and B. *(8 points)*

(b) If we were to place a $2/3\ \Omega$ resistor between A and B in the circuit below, what is the current i_R through that resistor (measured from A to B)? *(1 point)*

i_R



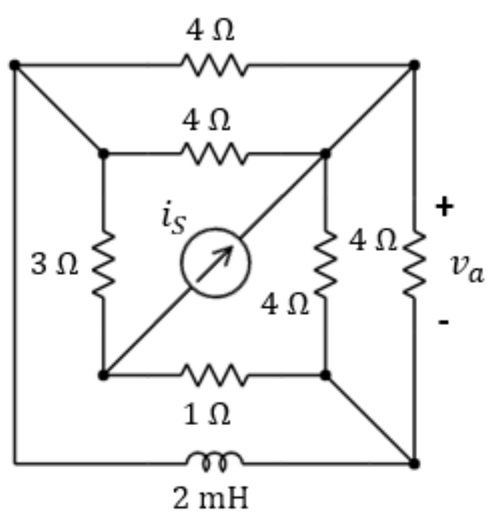
(2) Consider the circuit below. For $t < 0$ s, you may assume the system has reached steady state.

(a) Find $v_a(t)$ for $t > 0$ s.
(8 points)

$v_a(t)$

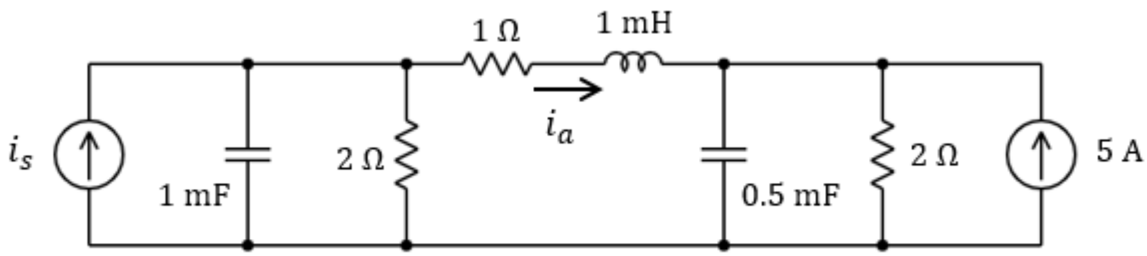
(b) Sketch the waveform $v_a(t)$ for $t > 0$ s. Include where you can observe the time constant.
(2 points)

$i_S = 4 \cdot u(t)$ A



(3) Consider the circuit below. The system is in steady state. Find $i_a(t)$. *(9 points)*

$i_a(t)$	



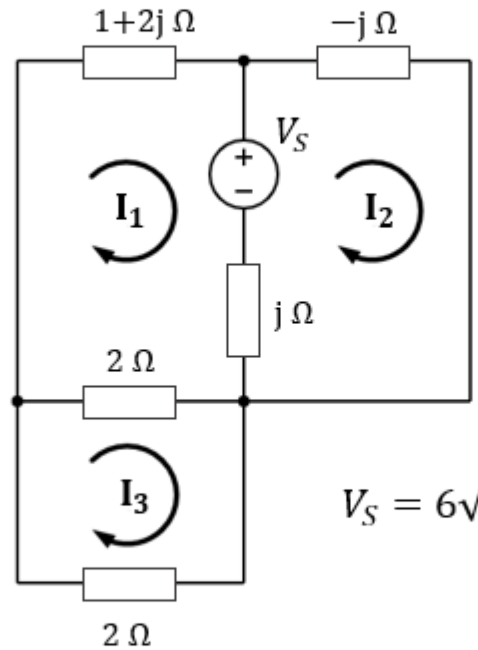
$$i_s = 20\cos\left(1000t - \frac{\pi}{2}\right)\ \text{A}$$

(4) Consider the circuit below. The system is in steady state and represented in phasor form.
(a) Find the mesh currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$ (expressed in polar form). *(8 points)*

$\mathbf{I_1}$ $\boxed{}$ $\mathbf{I_2}$ $\boxed{}$ $\mathbf{I_3}$ $\boxed{}$

(b) Find the complex power <u>supplied</u> by $\mathbf{V_S}$. *(2 points)*

$\mathbf{S}$ $\boxed{}$



$$V_S = 6\sqrt{2}e^{j\frac{2\pi}{5}} \text{ V}$$

(5) Consider the circuit below. The system is in steady state and represented in phasor form. The ammeter and volt-meter are both ideal.

(a) Assume the load $\mathbf{Z_L} = -j\ \Omega$. What is the reading X of the ammeter and the reading Y of the volt-meter? *(8 points)*

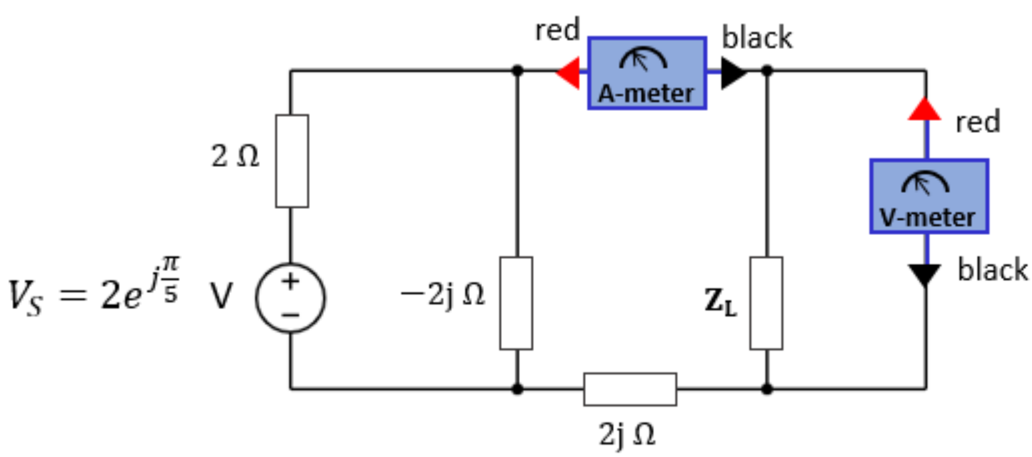
X (ammeter reading) ☐   Y (voltmeter reading) ☐

(b) We now replace the load $\mathbf{Z_L}$ with a new value such that the average power received by this load is maximized. What is this new value of $\mathbf{Z_L}$? *(2 points)*

$\mathbf{Z_L}$ ☐

(c) Find the average power <u>received</u> by the load calculated in part (b). *(2 points)*

P ☐



ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho \frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2} C v^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2} L i^2$

AC power: $p(t) = \frac{1}{2} V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ $\quad Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$ $\quad X_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt}$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\quad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\quad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\quad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\quad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

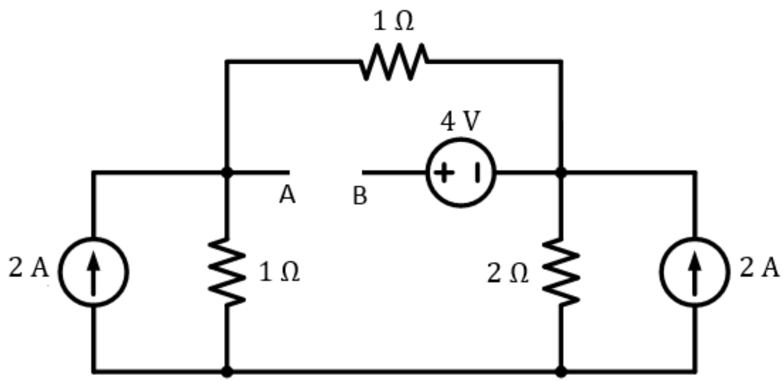
$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

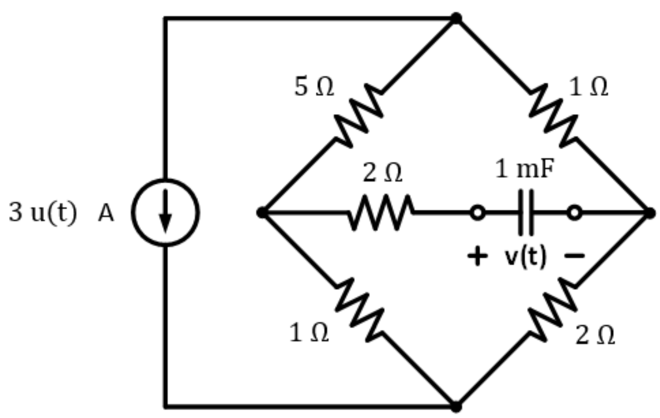
Practice Final

Problem 1



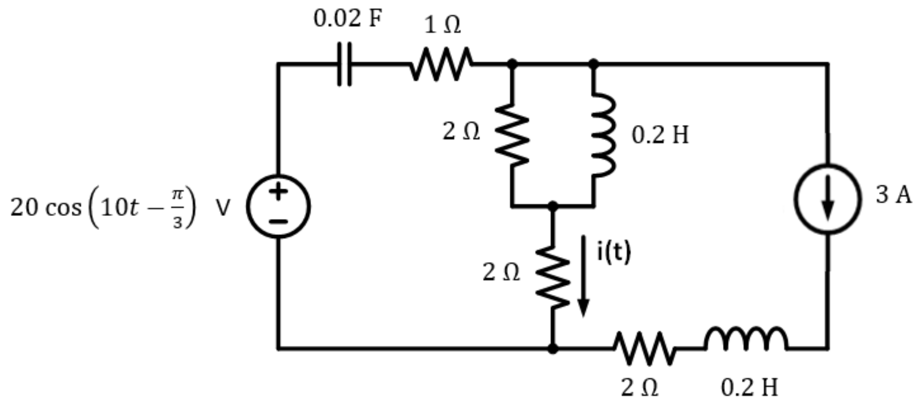
(a) Find the Thevenin equivalent model between A and B.
(b) If we were to place a 1.5 Ω resistor between A and B in the circuit above, what is the current through that resistor (measured from A to B)?

Problem 2



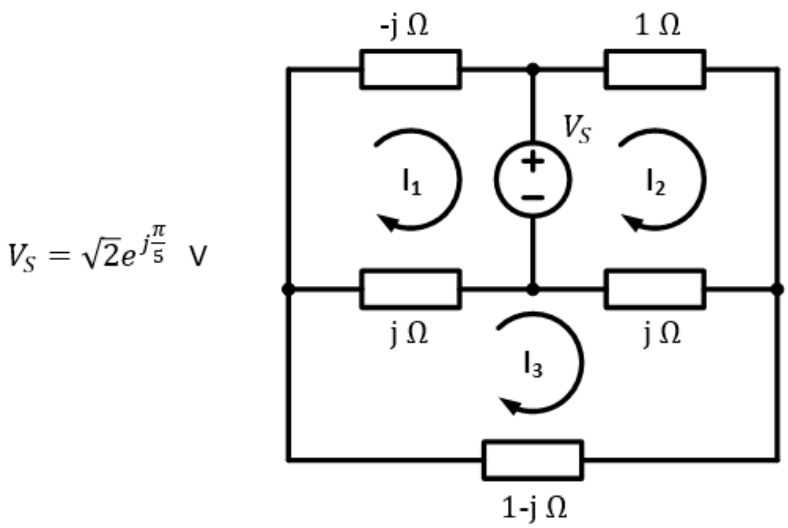
(a) Find v(t) for t ≥ 0.
(b) Sketch the waveform of v(t) for t ≥ 0.

Problem 3



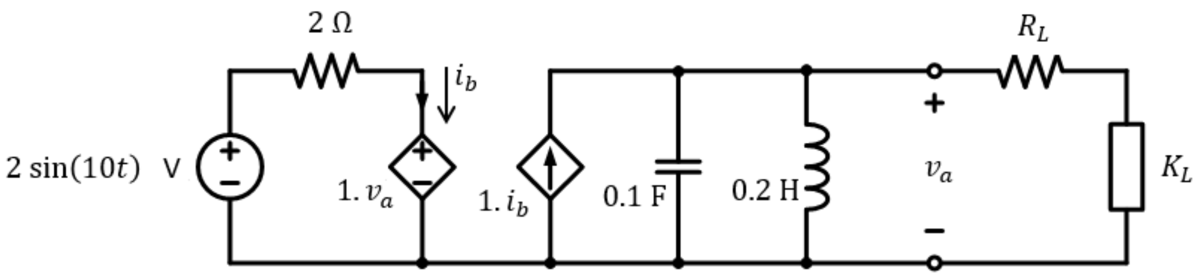
Find the steady-state response of i(t).

Problem 4



(a) Find the values of the mesh current phasors, I_1, I_2, and I_3 (in polar form).
(b) Find the complex power supplied by V_S.

Problem 5

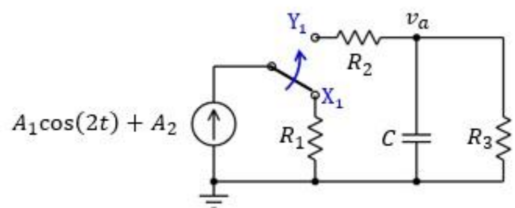


A load ($R_L + K_L$) is connected to a power distribution network consisting of a resistor, inductor, capacitor and AC voltage source. The load itself includes a resistor R_L in series with a mystery element K_L, where K_L is either a capacitor or an inductor. The goal is to maximize the average power delivered to the load.

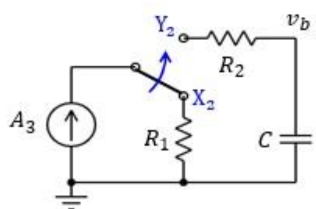
(a) Determine the values of R_L and K_L for the circuit that cause maximum power transfer to the load.
(b) Determine the mystery element (a capacitor or an inductor; and its value).
(c) Calculate the maximum power transferred to the load.

Q1

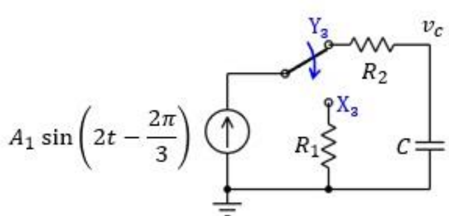
(a) For $t < \frac{\pi}{6}$ s, the switch is in position X_1, and at time $t = 0$ s the capacitor is fully discharged. At time $t = \frac{\pi}{6}$ s, the switch moves from position X_1 to position Y_1. Find the steady state node voltage waveform $v_a(t)$.



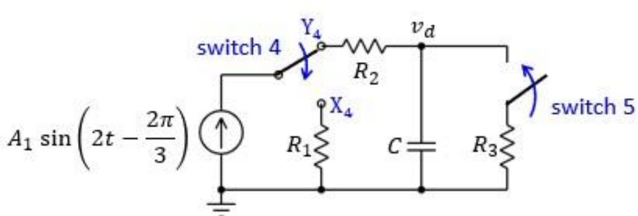
(b) For $t < \frac{\pi}{6}$ s, the switch is in position X_2, and at time $t = 0$ s the capacitor is fully discharged. At time $t = \frac{\pi}{6}$ s, the switch moves from position X_2 to position Y_2. Find the node voltage $v_b(\pi \text{ s})$, i.e., at time $t = \pi$ s.



(c) For $t < \frac{\pi}{6}$ s, the switch is in position Y_3, and you may assume the system has reached steady state. At time $t = \frac{\pi}{6}$ s, the switch moves from position Y_3 to position X_3. Find the node voltage $v_c(\pi \text{ s})$.



(d) For $t < \frac{\pi}{6}$ s, switch 4 is in position Y_4 and switch 5 is open, and you may assume the system has reached steady state. At time $t = \frac{\pi}{6}$ s, switch 4 moves from position Y_4 to position X_4 and switch 5 closes. Find the node voltage $v_d(\pi \text{ s})$.



R1:	1 Ω
R2:	1 Ω
R3:	1 Ω
C:	500 mF
A1:	3 A
A2:	2 A
A3:	3 A

Q2

The AC circuit below has ω = 10 rad/s and is in steady state. The phasor diagram shows the phasors of i_S and v_L. You are given the angles α (between $\mathbf{I_S}$ and the x-axis), β (between $\mathbf{V_L}$ and the x-axis), $|\mathbf{I_S}|$ and $|\mathbf{V_L}|$. The diagram is not necessarily drawn to scale.
The element represented by the rectangular box is either an inductor or a capacitor but you are not told which.

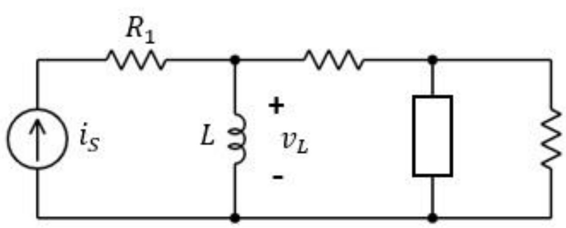
(a) Find the complex power $\mathbf{S_L}$ received by the inductor.

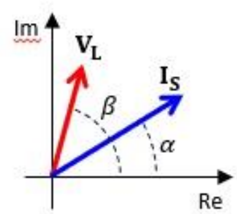
(b) Find the complex power $\mathbf{S_S}$ supplied by the current source.

(c) Find the complex power $\mathbf{S_M}$ received by the mystery element.

(d) What is the mystery element (capacitor or inductor)? (You do not need to find its value, but your answer needs to be motivated)

|Is|:	3 A
alpha:	20 degrees
|VL|:	4 V
Beta:	80 degrees
R1:	2 Ω
L:	200 mH



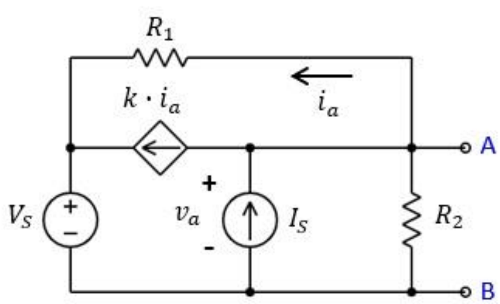


Q3

Consider the circuit below.

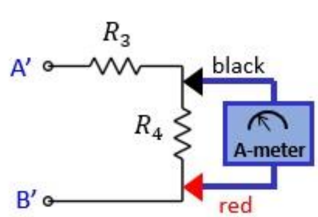
(a) Find the voltage v_a.

(b) Draw the Thevenin model for the circuit between A and B (make sure you label A and B in your drawing).



R1:	18 Ω
R2:	3 Ω
k:	2 A/A
Vs:	3 V
Is:	2 A
R3:	2 Ω
R4:	2 Ω

(c) Connect the circuit on the right to the one on top (A′ connected to A and B′ connected to B). What is the reading X of the ideal **ammeter**?

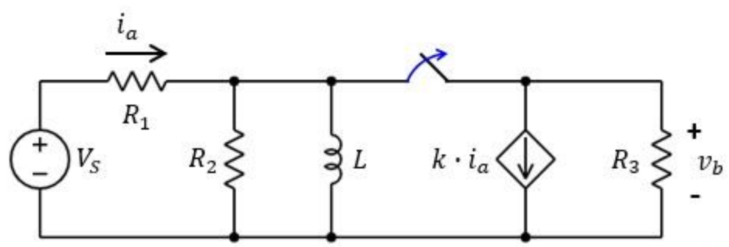


Q4

For $t < 2$ s, the switch is closed, and you may assume the system has reached steady state. The switch opens at time $t = 2$ s.

(a) Find $v_b(t)$ for $t > 2$ s.

(b) What is the instantaneous power received by the inductor at time $t = 2^+$ s.



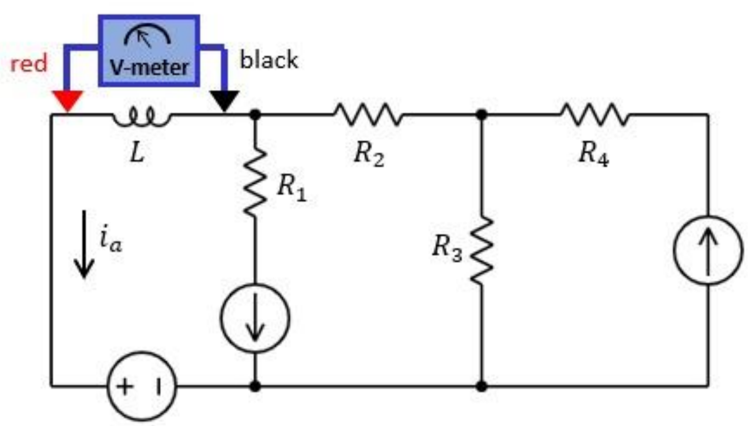
Vs:	4 V
R1:	2 Ω
R2:	2 Ω
R3:	3 Ω
L:	200 mH
k:	3 A/A

Q5

The circuit below is an AC circuit and is in steady state. The three independent sources all have ω = 10 rad/s.

You measure the waveform $i_a(t) = A_1 \cos(10t + B_1)$.

Now the circuit is changed: the inductor **L** is replaced by capacitor **C**. In this changed circuit, what will be the reading of the ideal volt-meter?



R1:	2 Ω
R2:	2 Ω
R3:	3 Ω
R4:	2 Ω
A1:	10 A
B1:	60 degrees
L:	300 mH
C:	40 mF

ECE 35, Fall 2021
Final – Section A

Your sequence number

Grade

/ 45

Last name

First + middle name(s)

PID

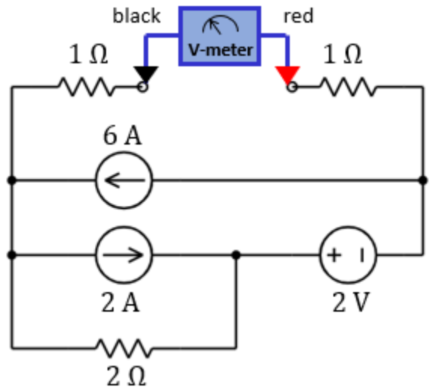
Instructions:

- Do not look at the questions or start writing until it is announced you can do so.
- Make sure you write your PID on EACH page.
- Read each problem completely and thoroughly before beginning.
- Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- All calculations must be done on these pages. It should be clear which question they belong to. Use the front for your actual work and the back as scratch paper.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes. Make sure you list units.
- You must follow the Final Exam Procedures that were posted on Canvas. If you are unsure of anything, ask. As a reminder:
    - Your phone should be turned off and put inside your bag
    - Calculators are not allowed.
    - This is a closed book exam.
    - Follow the Academic Integrity standards.

PID

(1) *(4 points)* Consider the circuit below.
What is the reading ***X*** of the voltmeter?

X



PID

(2) *(7 points)* Consider the circuit below.

(a) Find the current i_b.

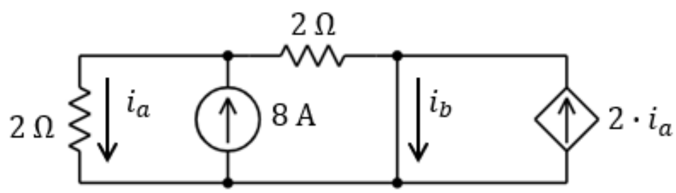
(b) Find the power P_1 <u>received</u> by the <u>independent</u> source.

(c) Find the power P_2 <u>supplied</u> by the <u>dependent</u> source.

i_b

P_1

P_2

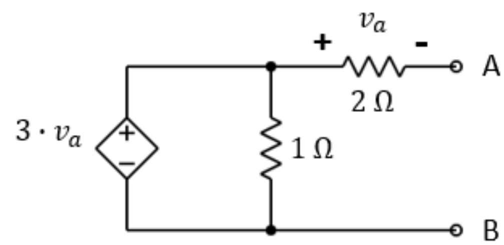


PID

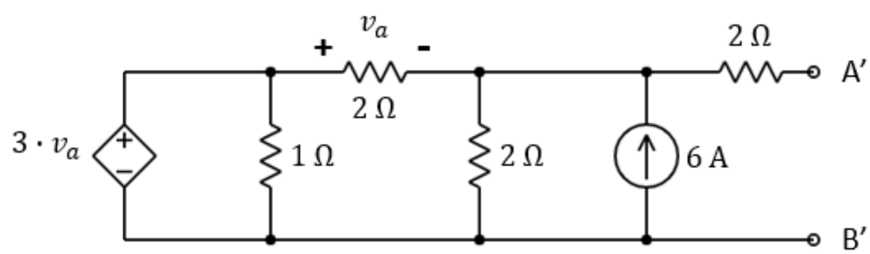
(3) *(7 points)*

(a) Consider the circuit below. What is the Thevenin equivalent resistance between A and B?

R_{TH}



(b) Consider the circuit below (which contains the circuit above). Draw the Norton equivalent model between A' and B' (make sure you label A' and B').



PID

(4) *(10 points)* Consider the circuit below. For $t < 2$ s, the switch is in position 1 and you may assume the system has reached steady state. The switch moves from position 1 to position 2 at $t = 2$ s and remains in position 2.

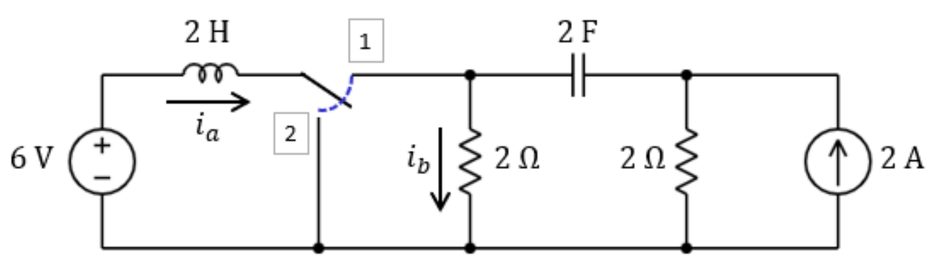
Note: make sure you don't mix up i_a and i_b in the questions below.

(a) Find the current $i_a(2^- \text{ s})$. (i.e., just before the switch moves). $i_a(2^- \text{ s})$

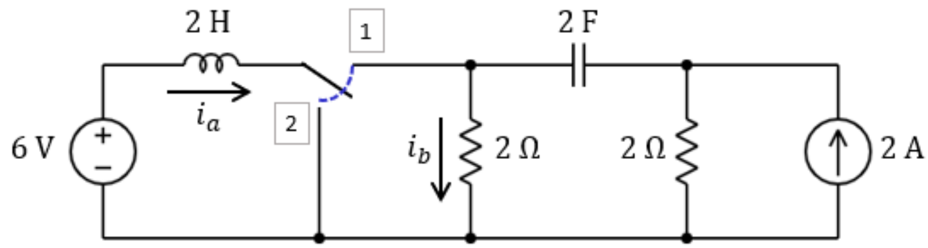
(b) Find the current $i_b(2^+ \text{ s})$. (i.e., just after the switch moves). $i_b(2^+ \text{ s})$

(c) Find the current $i_a(7 \text{ s})$. $i_a(7 \text{ s})$

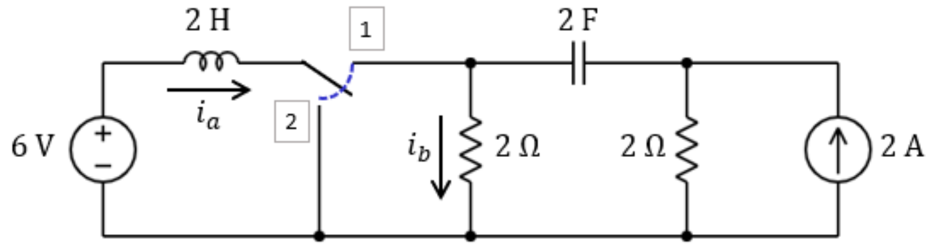
(d) Find the current $i_b(\infty)$. $i_b(\infty)$



PID



PID



PID

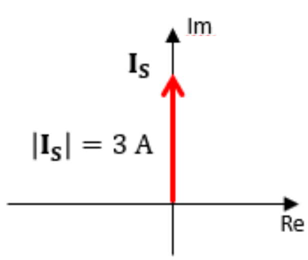
(5) *(9 points)* Consider the circuit below. For $t < \frac{\pi}{4}$ ms, the switch is closed and you may assume the system has reached steady state. The source i_S is an AC source with $\omega = 4000$ rad/s. Its phasor diagram is shown on the left. The switch opens at $t = \frac{\pi}{4}$ ms and remains open.

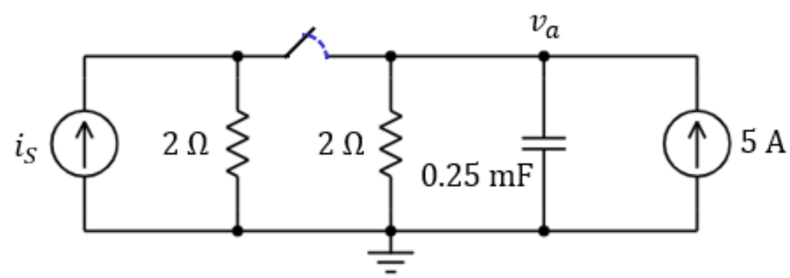
(a) Find the node voltage $v_a\left(\frac{\pi^-}{4}\ ms\right)$. (i.e., just before the switch opens).

$v_a\left(\frac{\pi^-}{4}\ ms\right)$

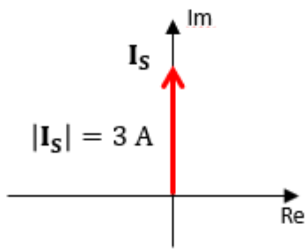
(b) Find the node voltage $v_a(\pi\ ms)$.

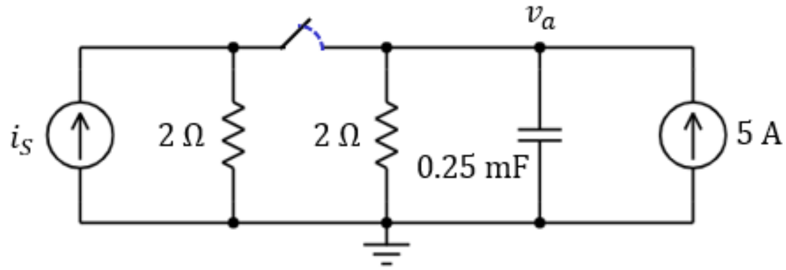
$v_a(\pi\ ms)$





PID





PID

(6) *(8 points)* The circuit below is in steady state.

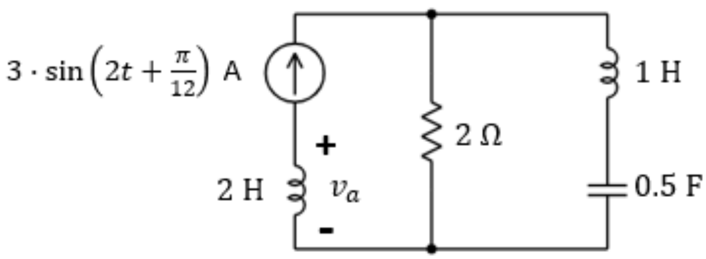
(a) Sketch the phasor of v_a (make sure the magnitude and phase are labeled).

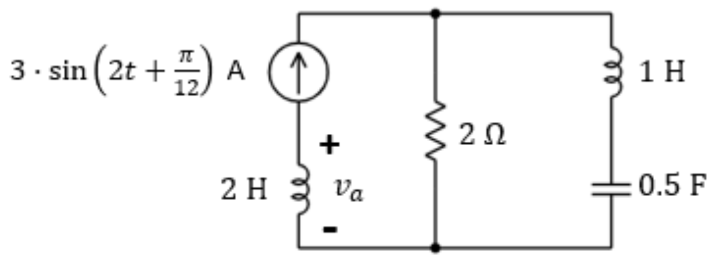
(b) Find average power P_R received by the resistor.

P_R

(c) If the reactive power received by all capacitors and inductors combined is **21.6 VAR**, what is the complex power S supplied by the independent source?

S




3 · sin(2t + π/12) A
1 H
+
2 Ω
2 H
v_a
0.5 F
-

PID

ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $v_{ab} \triangleq \frac{dw}{dq}$ $R = \rho \frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $Q = C \cdot v$ $w_C = \frac{1}{2} C v^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $B \sim i$ $w_L = \frac{1}{2} L i^2$

AC power: $p(t) = \frac{1}{2} V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ $Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$ $X_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt}$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\alpha)$:	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan(\alpha)$:	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

ECE 35, Fall 2021
Final – Section B

Your sequence number

Last name	
First + middle name(s)	
PID	

Grade

/ 45

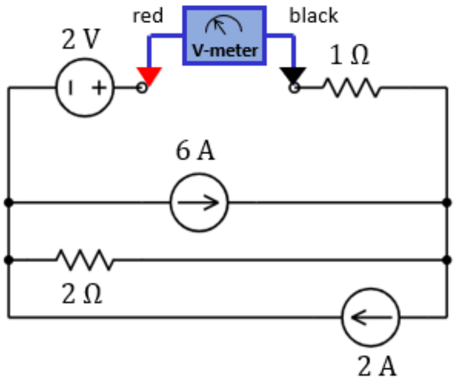
Instructions:

- Do not look at the questions or start writing until it is announced you can do so.
- Make sure you write your PID on EACH page.
- Read each problem completely and thoroughly before beginning.
- Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- All calculations must be done on these pages. It should be clear which question they belong to. Use the front for your actual work and the back as scratch paper.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes. Make sure you list units.
- You must follow the Final Exam Procedures that were posted on Canvas. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag
  - Calculators are not allowed.
  - This is a closed book exam.
  - Follow the Academic Integrity standards.

PID

(1) *(4 points)* Consider the circuit below.
What is the reading ***X*** of the voltmeter?

X



PID

(2) *(7 points)* Consider the circuit below.

(a) Find the current i_b.

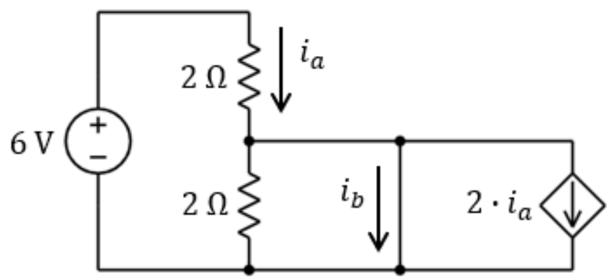
(b) Find the power P_1 received by the independent source.

(c) Find the power P_2 supplied by the dependent source.

i_b

P_1

P_2

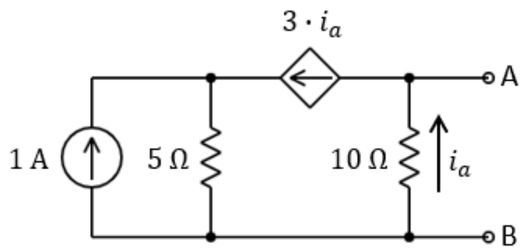


PID

(3) *(7 points)*

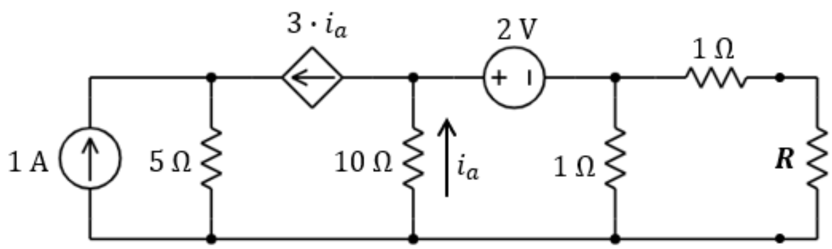
(a) Consider the circuit below. What is the Thevenin equivalent resistance between A and B?

R_{TH}



(b) Consider the circuit below (which contains the circuit above). Find the value of resistor ***R*** such that the power received by ***R*** is maximized.

R



PID

(4) *(10 points)* Consider the circuit below. For $t < 2$ s, the switch is closed and you may assume the system has reached steady state. The switch opens at $t = 2$ s and remains open.

Note: make sure you don't mix up v_a and v_b in the questions below.

(a) Find the node voltage $v_a(2^- \text{ s})$. (i.e., just before the switch opens).

$v_a(2^- \text{ s})$

(b) Find the node voltage $v_b(2^+ \text{ s})$. (i.e., just after the switch opens).

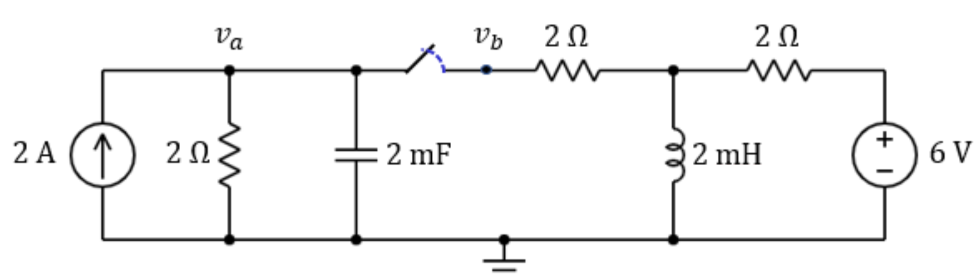
$v_b(2^+ \text{ s})$

(c) Find the node voltage $v_a(10 \text{ s})$.

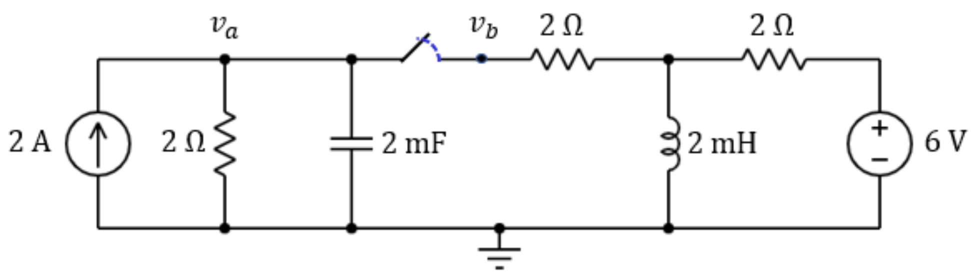
$v_a(10 \text{ s})$

(d) Find the node voltage $v_b(\infty)$.

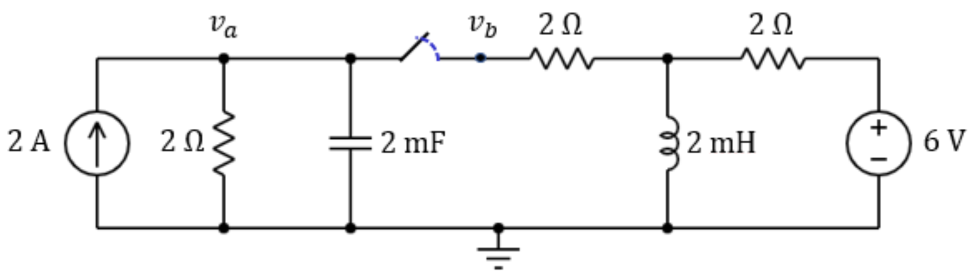
$v_b(\infty)$



PID



PID



PID

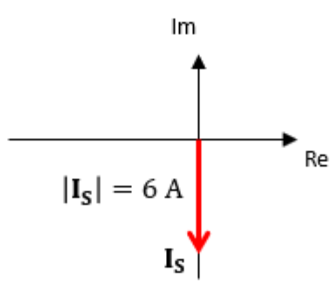
(5) *(9 points)* Consider the circuit below. For $t < \frac{\pi}{2}$ s, the switch is in position 1 and you may assume the system has reached steady state. The source i_S is an AC source with $\omega = 2$ rad/s. Its phasor diagram is shown on the left. The switch moves <u>from position 1 to position 2</u> at $t = \frac{\pi}{2}$ s and remains in position 2.

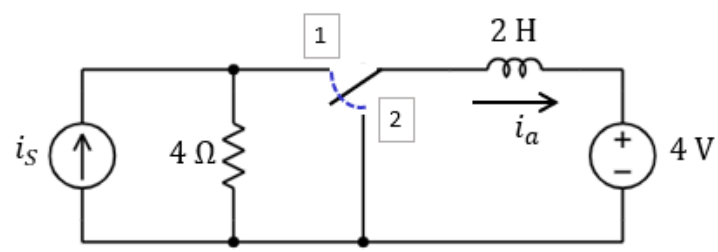
(a) Find the current $i_a\left(\frac{\pi^-}{2}\text{ s}\right)$. (i.e., just before the switch moves).

$i_a\left(\frac{\pi^-}{2}\text{ s}\right)$

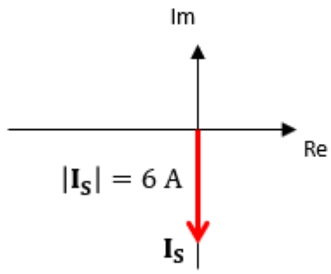
(b) Find the current $i_a(2\pi\text{ s})$.

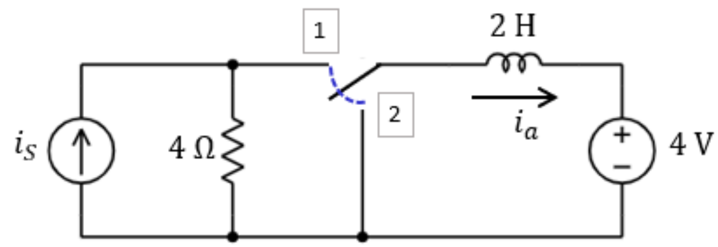
$i_a(2\pi\text{ s})$





PID





PID

(6) *(8 points)* The circuit below is in steady state.

(a) Sketch the phasor of i_a (make sure the magnitude and phase are labeled).

(b) Find complex power S_L received by the inductor.

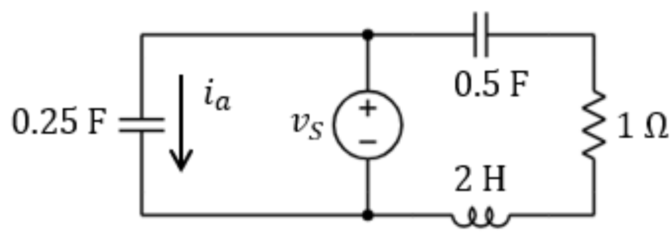
S_L

(c) If the complex power supplied by the independent source is **0.2 - 0.4j VA**, what is the total complex power S received by both capacitors ($S_{C1} + S_{C2}$) and the average power P_R received by the resistor?

$S_{C1} + S_{C2}$

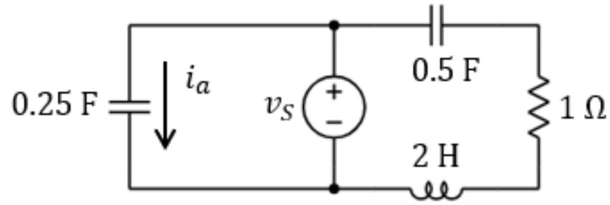
P_R

$$v_S = 2 \cdot \sin\left(2t + \frac{\pi}{12}\right) \text{ V}$$



PID

$v_S = 2 \cdot \sin\left(2t + \frac{\pi}{12}\right)$ V



PID

ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho\frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2}Cv^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2}Li^2$

AC power: $p(t) = \frac{1}{2}V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$ $\quad Q = \frac{1}{2}V_m I_m \sin(\theta_v - \theta_i)$ $\quad X_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$

Trigonometry: $\sin(-\alpha) = -\sin(\alpha)$ $\quad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\quad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\quad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\quad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

ECE 35, Fall 2022 – Section A Final

Your sequence number

Grade

/ 31

Last name

First + middle name(s)

PID

Instructions:

- Do not look at the questions or start writing until it is announced that you can do so.
- You must follow the Final Exam Procedures that were posted on Canvas. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag
  - Calculators are not allowed.
  - This is a closed book exam.
  - Follow the Academic Integrity standards

- These pages contain the exam questions.
  - Do not write any of your work here except for scratch work. It will not be graded.
  - You need to write your final answer in the answer boxes here. Make sure you list units.
  - Keep these question pages stapled together. The last page is the equation sheet; you may detach this if you want.
  - Make sure you write your PID on EACH page.
  - Read each problem completely and thoroughly before beginning.

▼

PID

(1) *(7 points)*

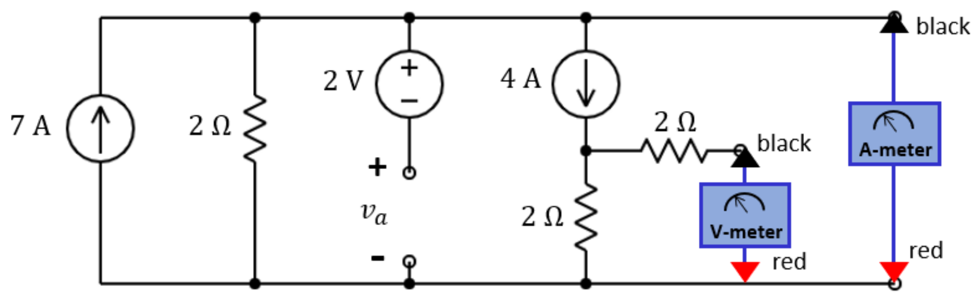
(a) In the circuit below, the ammeter and voltmeter are ideal.

What is the reading X of the voltmeter? X

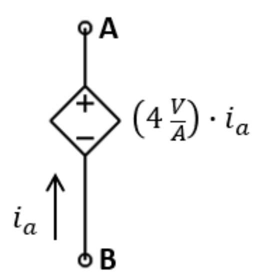
What is the reading Y of the ammeter? Y

What is the voltage v_a? v_a

What is the power P received by the 4 A current source ? P



(b) What is the Thevenin resistance R_{th} between A and B ? R_{th}



(2) *(7 points)* For this question, you should make reasonable numerical approximations if needed.

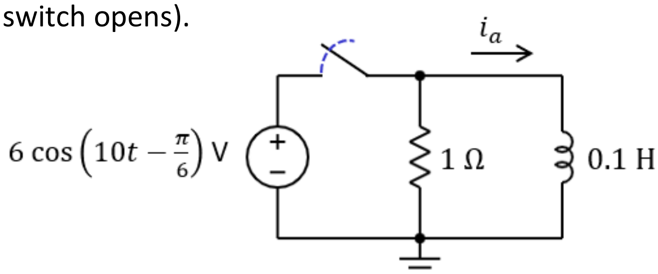
(a) Consider the circuit below. For $t < 0$ s, the switch is closed, and the system has reached steady state. At time $t = 0$ s, the switch opens.

Find the current i_a at time $t = 0^-$s (i.e., immediately before the switch opens).

Find the current i_a at time $t = 0^+$s (i.e., immediately after the switch opens).

$i_a(0^- \text{s})$

$i_a(0^+ \text{s})$



(b) Consider the circuit below. For $t < 0$ s, the switch is in position 1, and the system has reached steady state. At time $t = 0$ s, the switch moves to position 2 (and stays there). Just before the switch moves, i_x is 4 A.

Find the current i_b at time $t = 0^+$s (i.e., immediately after the switch moves to position 2).

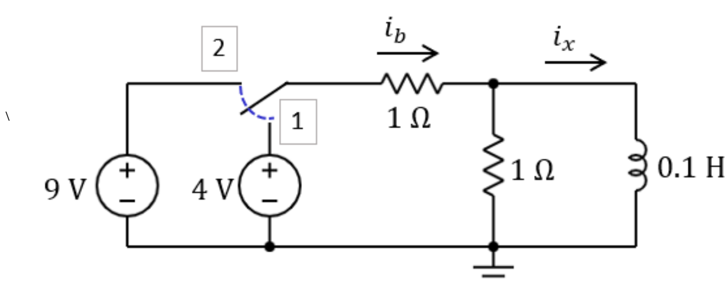
Find the current i_b at time $t = 2$ s.

Find the current i_b at time $t = \infty$.

$i_b(0^+ \text{s})$

$i_b(2 \text{ s})$

$i_b(\infty)$



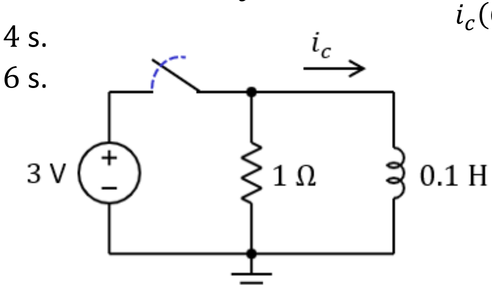
(c) Consider the circuit below. For $t < 2$ s, the switch is open, and the system may not have reached steady state. At time $t = 2$ s, the switch closes. Just after the switch closes, i_c is 5 A.

Find the current i_c at time $t = 4$ s.

Find the current i_c at time $t = 6$ s.

$i_c(4 \text{ s})$

$i_c(6 \text{ s})$



PID

(3) *(6 points)* Consider the circuit below. You may assume it is in steady-state.

(a) Find the current of current source i_{S1} at time $t = \frac{\pi}{20}$ s.

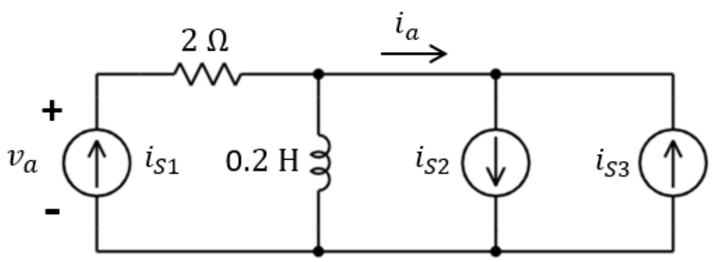
$i_{S1}\left(\frac{\pi}{20}\text{ s}\right)$

(b) Find the voltage v_a at time $t = \frac{\pi}{20}$ s.

$v_a\left(\frac{\pi}{20}\text{ s}\right)$

(c) Find the maximum value of the waveform $i_a(t)$.

$i_{a\,max}$



$i_{S1} = 3 + 5\cos(10t)$ A

$i_{S2} = 6\sin\left(20t + \frac{\pi}{4}\right)$ A

$i_{S3} = 3\sqrt{2}\cos(20t)$ A

PID

(4) *(6 points)* The circuit below represents an AC circuit in steady-state in the phasor domain (for the complex numbers, you may assume units are V, A, Ω, etc. as appropriate). The voltage source v_S is an AC source with $\omega = 5$ rad/s. Each box represents the impedance of a single circuit element (a resistor, capacitor or inductor).

(a) Find the average power P_s supplied by the voltage source.

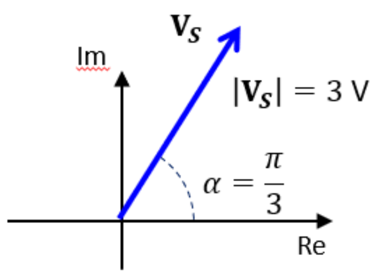
P_s

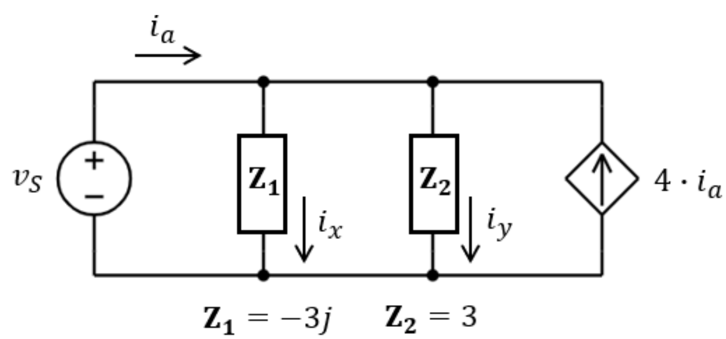
(b) Find the average power P_1 received by the element with impedance $\mathbf{Z_1}$.

P_1

(c) Find the RMS value of i_x.

i_{xRMS}





PID

(5) *(5 points)*

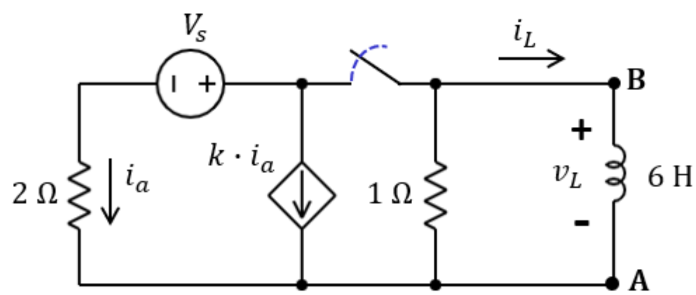
(a) Consider the circuit below. You are not given the values of k and V_s (but you are told that all sources are DC sources). We are considering the circuit after the switch closes.

- The time constant τ associated with $i_L(t)$ during this transition is τ = 2 s.
- You also measure that when current i_L reaches 4 A, the voltage v_L drops to 0 V.

What is the current i_L when the system reaches steady state? i_L

Find the Norton equivalent resistance R_N between A and B if we were to remove the inductor from the circuit. R_N

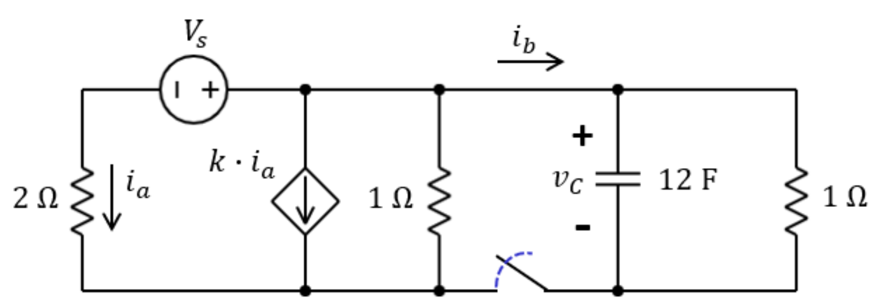
During the transition (in our original circuit with the inductor in there), when the current i_L has a value of 1 A, what is the value of the voltage v_L? v_L



(b) Consider the circuit below. The unknown values of k and V_s are the same as in the circuit above. The switch closes at time t = 400 s (you do not know if the system has reached steady state before the switch closes).

Find the time constant τ_1 associated with $v_c(t)$ for t > 400 s. τ_1

Find the time constant τ_2 associated with $i_a(t)$ for t > 400 s. τ_2



PID

ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho\frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2}Cv^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2}Li^2$

AC power: $p(t) = \frac{1}{2}V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$ $\quad Q = \frac{1}{2}V_m I_m \sin(\theta_v - \theta_i)$ $\quad X_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\quad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\quad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\quad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\quad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

ECE 35, Fall 2022 – Section B
Final

Your sequence number

Grade
/ 31

Last name	
First + middle name(s)	
PID	

Instructions:

- Do not look at the questions or start writing until it is announced that you can do so.
- You must follow the Final Exam Procedures that were posted on Canvas. If you are unsure of anything, ask. As a reminder:
    - Your phone should be turned off and put inside your bag
    - Calculators are not allowed.
    - This is a closed book exam.
    - Follow the Academic Integrity standards
- These pages contain the exam questions.
    - Do not write any of your work here except for scratch work. It will not be graded.
    - You need to write your final answer in the answer boxes here. Make sure you list units.
    - Keep these question pages stapled together. The last page is the equation sheet; you may detach this if you want.
    - Make sure you write your PID on EACH page.
    - Read each problem completely and thoroughly before beginning.

▼

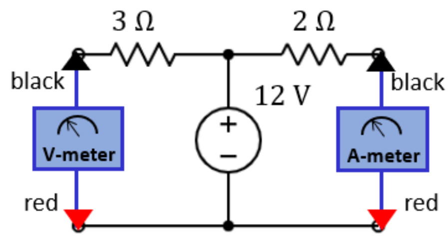
PID

(1) *(7 points)*

(a) The ammeter and voltmeter are ideal.

What is the reading X of the voltmeter?

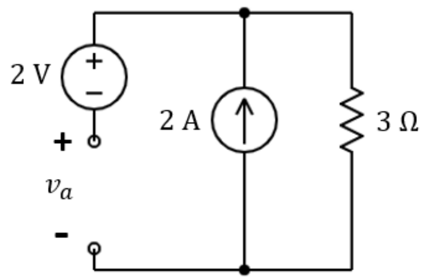
What is the reading Y of the ammeter?



X

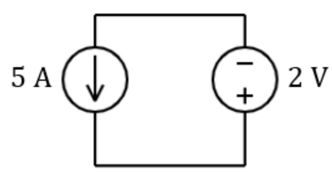
Y

(b) What is the voltage v_a?



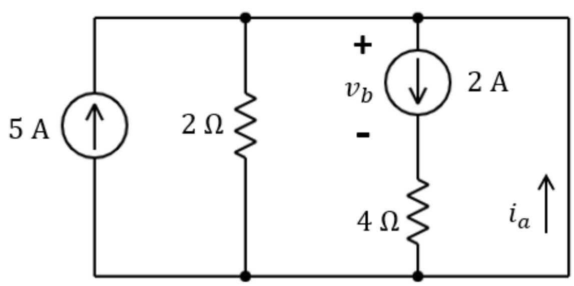
v_a

(c) What is the power P <u>received</u> by the current source ?



P

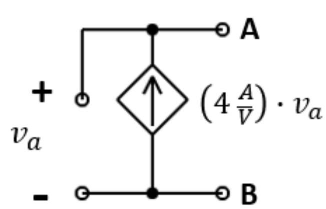
(d) What is the voltage v_b?
What is the current i_a?



v_b

i_a

(e) What is the Thevenin resistance R_{th} between A and B ?

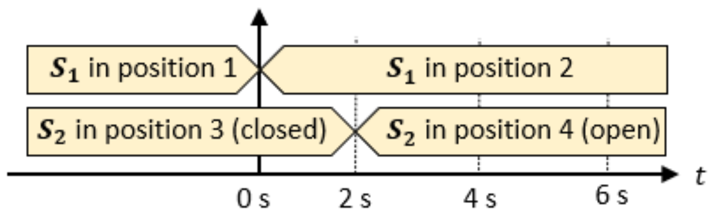


R_{th}

PID

(2) *(7 points)* For this question, you should make reasonable numerical approximations if needed. Consider the circuit below.

- For $t < 0$ s, switch S_1 is in position 1, switch S_2 is in position 3, and the system has reached steady state.
- At time $t = 0$ s, switch S_1 moves from position 1 to position 2 (you may assume that i_{S1} turns off at that point). Switch S_2 stays in position 3 (closed).
- At time $t = 2$ s, switch S_2 moves from position 3 to position 4 (open). Switch S_1 stays in position 2.



(a) Find the capacitor voltage v_c at time $t = 0^{-}$s (i.e., immediately before switch S_1 moves).

$v_c(0^{-}\text{s})$

Find the node voltage v_a at time $t = 0^{+}$s (i.e., immediately after switch S_1 moves).

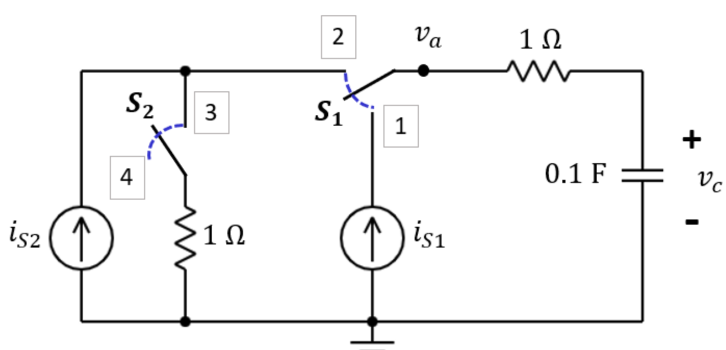
$v_a(0^{+}\text{s})$

(b) Find the node voltage v_a at time $t = 2^{-}$s. (i.e., immediately before switch S_2 moves).

$v_a(2^{-}\text{s})$

(c) Find the node voltage v_a at time $t = 6$ s.

$v_a(6\text{ s})$



$i_{S1} = 8\cos\left(10t - \frac{\pi}{2}\right)$ A

$i_{S2} = 5$ A

PID

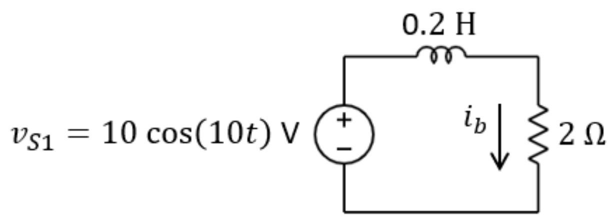
(3) *(6 points)* For all the circuits in this question, you may assume that they are in steady-state.

(a) Find the phasor of $v_x(t) = 8\sin\left(20t + \frac{\pi}{4}\right)$ V. You can write your answer in cartesian or polar coordinates.

$\mathbf{V_x}$

(b) Find the current i_b at time $t = \frac{\pi}{10}$ s.

$i_b\left(\frac{\pi}{10}\text{s}\right)$



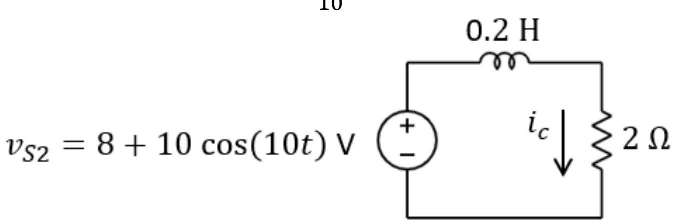
(c) Note the similarity to circuit in part (b).

Find the voltage v_{S2} of the voltage source at time $t = \frac{\pi}{10}$ s.

Find the current i_c at time $t = \frac{\pi}{10}$ s.

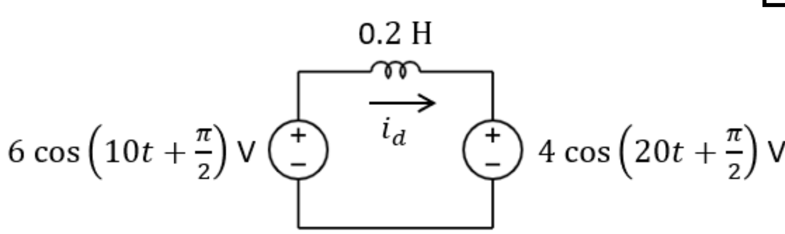
$v_{S2}\left(\frac{\pi}{10}\text{s}\right)$

$i_c\left(\frac{\pi}{10}\text{s}\right)$



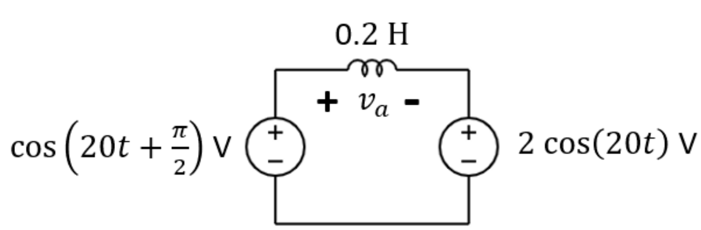
(d) Find the current i_d at time $t = \frac{\pi}{10}$ s.

$i_d\left(\frac{\pi}{10}\text{s}\right)$



(e) Find the maximum value of the waveform $v_a(t)$.

$v_{a\,max}$

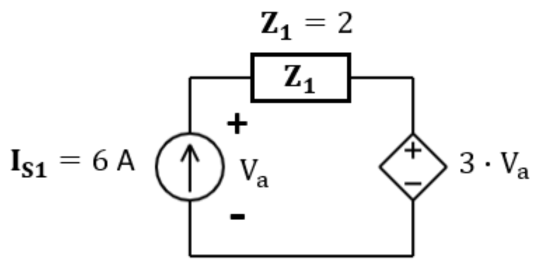


PID

(4) *(6 points)* The circuits below represent AC circuits in steady-state in the phasor domain (for the complex numbers, you may assume units are V, A, Ω, etc. as appropriate). The independent current sources are AC sources with $\omega = 5$ rad/s. Each box represents the impedance of a single circuit element (a resistor, capacitor or inductor).

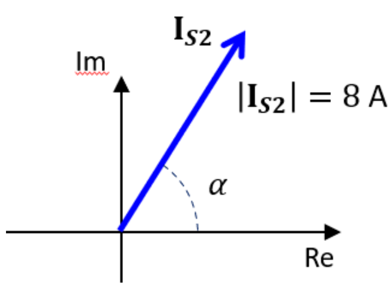
(a) Find the phasor V_a. You can write your answer in cartesian or polar coordinates.

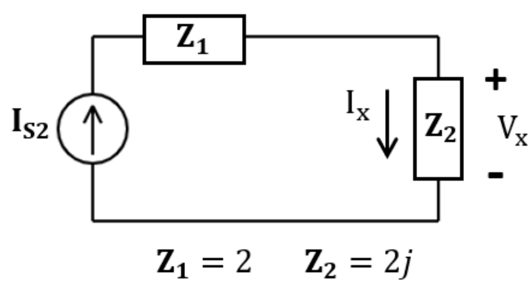
V_a



(b) In the circuit below, we set $\alpha = \frac{\pi}{2}$ (the figure is clearly not drawn to scale). Find the average power P_s supplied by the current source I_{s2}.

P_s





(c) For the same circuit as in part (b), we change α to $\alpha = \frac{\pi}{6}$.

Find the average power P_s supplied by the current source I_{s2}. P_s

Find the average power P_1 received by the element with impedance $\mathbf{Z_2}$. P_2

Find the magnitude of the phasor V_x. $|V_x|$

Find the RMS value of i_x. i_{xRMS}

PID

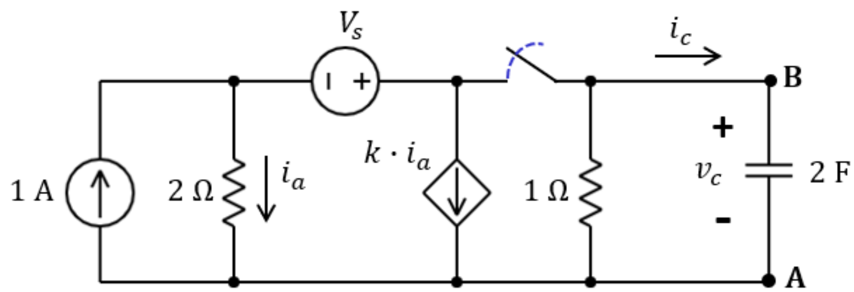
(5) *(5 points)*

(a) Consider the circuit below. You are not given the values of k and V_s (but you are told that all sources are DC sources). We are considering the circuit after the switch closes.

We measure that when the voltage v_c reaches 6 V, the current i_c is 2 A. Similarly, when the voltage v_c becomes 10 V, the current i_c drops to 0 A.

Find the Norton equivalent resistance R_N between A and B when we remove the capacitor from the circuit.

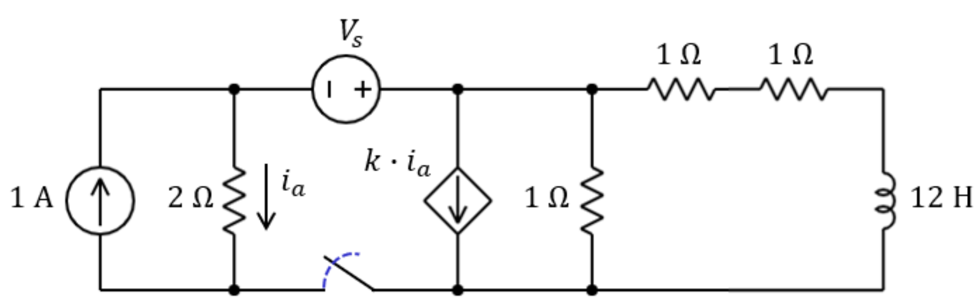
R_N



(b) Consider the circuit below. The unknown values of k and V_s are the same as in the circuit above. The switch closes at time t = 400 s (you do not know if the system has reached steady state before the switch closes).

Find the time constant τ associated with $i_a(t)$ for $t > 400$ s.

τ



PID

ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho \frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2} C v^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2} L i^2$

AC power: $p(t) = \frac{1}{2} V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ $\quad Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$ $\quad X_{rms} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\quad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\quad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\quad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ $\quad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\sin(\alpha)\sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos(\alpha)\cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha)\cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

ECE 35, Spring 2020

Quiz 3

/ 10

Last name

First + middle name(s)

PID

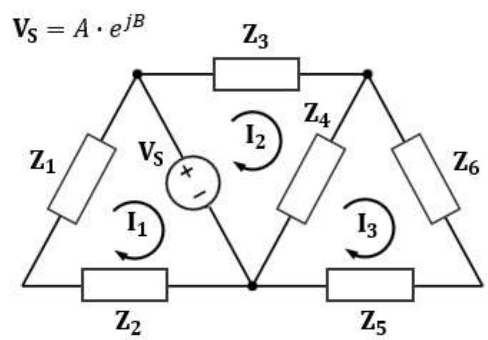
(1) *(5 points)*

Consider the circuit below. The system is in steady state and represented in phasor form.

(a) Find the mesh currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$ (expressed in polar form).

(b) Find the complex power supplied by $\mathbf{V_S}$.

A : 6 V
B : 90 degrees
Z1 : 1
Z2 : 2
Z3 : 1
Z4 : 2
Z5 : -2j
Z6 : 2+2j



(2) *(5 points)*

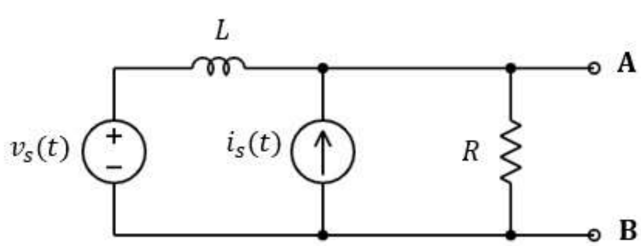
Consider the circuit below. The system is in steady state.

(a) Find the average power received by resistor *R*.

(b) What load can we place between A and B that would result in maximum average power received by that load? Give your result as two elements in series (with their values).

$v_S(t) = A_1 \cos(10t)$ $i_S(t) = A_2 \cos(10t)$

A1 : 4 V
A2 : 3 A
L : 0.2 H
R : 2 ohm

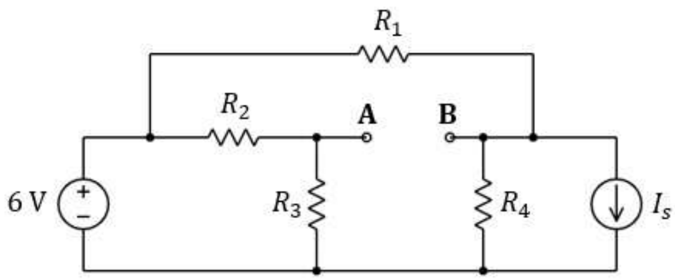


(3) *(5 points)*

Consider the circuit below.

(a) Find the Thevenin equivalent model between A and B. Make sure to label A and B in your model as well.

(b) If we were to place a resistor $\boldsymbol{R_5}$ between A and B, what is the current through that resistor (measured from A to B)?



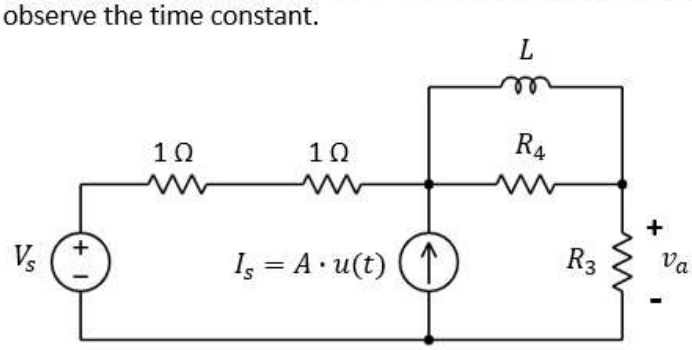
R1 : 4 ohm
R2 : 3 ohm
R3 : 6 ohm
R4 : 4 ohm
Is : 3 A
R5 : 3 ohm

(4) *(5 points)*

Consider the circuit below. For $t < 0$ s, you may assume the system has reached steady state.

(a) Find $v_a(t)$ for $t > 0$ s.

(b) Sketch the waveform $v_a(t)$ for $t > 0$ s. Include where you can observe the time constant.

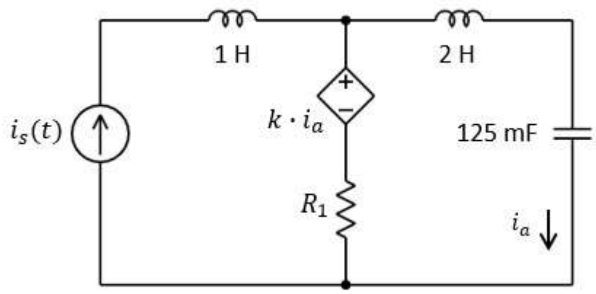


R3 : 2 ohm
R4 : 2 ohm
Vs : 2 V
A : 3 A
L : 8 mH

(5) *(5 points)*

Consider the circuit below. The system is in steady state.
Find $i_a(t)$.

$$i_S(t) = A_1\cos(2t + B_1) + A_2\cos(4t + B_2)$$



A1 : 2 A
B1 : 60 degrees
A2 : 4 A
B2 : 0 degrees
R1 : 9 ohm
k : 3 V/A

ECE 35, Fall 2019
Final

Sequence number

Last name

First + middle name(s)

PID

Grade

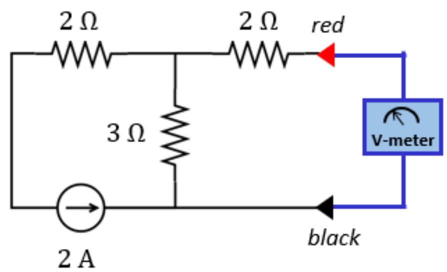
/ 45

Instructions:

- Do not look at the questions or start writing until it is announced you can do so.
- Read each problem completely and thoroughly before beginning.
- All calculations must be done in your blue book. It should be clear which question they belong to. Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation.
- Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes on these question pages. Make sure you list units!
- You must follow the Final Exam Procedures that were posted on TritonEd. If you are unsure of anything, ask. As a reminder:
    - Your phone should be turned off and put inside your bag in the front of the room (or on the table in the front). If you are found to have a phone (or other communication device) on you during the exam, your exam will not be graded.
    - Calculators are not allowed.
    - This is a closed book exam.

(1) (a) What is the volt-meter reading X? *(2 points)*

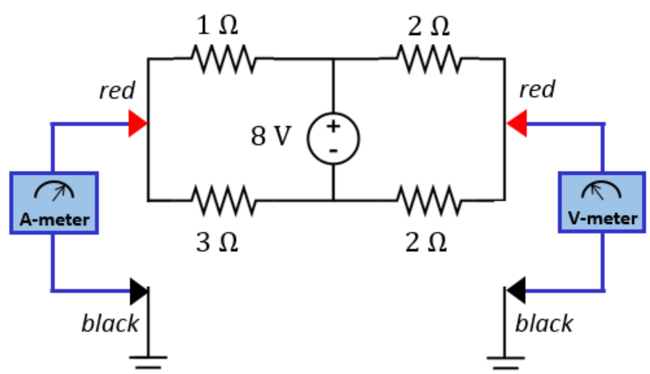
X



(b) Find **volt-meter** reading X and the **ammeter** reading Y.
(2 points)

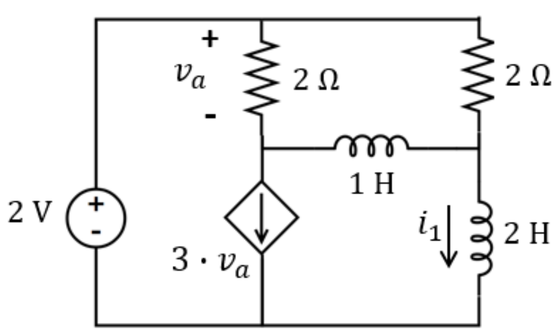
X

Y



(c) The system is in steady state. Find the current $\boldsymbol{i_1}$. *(3 points)*

i_1



(2) For $t < 0$ s, both switches are closed and you may assume the system has reached steady state. <u>Both switches open at time $t = 0$ s (at exactly the same time)</u>.

(a) Find $v_a(0^-), v_b(0^-)$ and $v_c(0^-)$. *(2 points)*

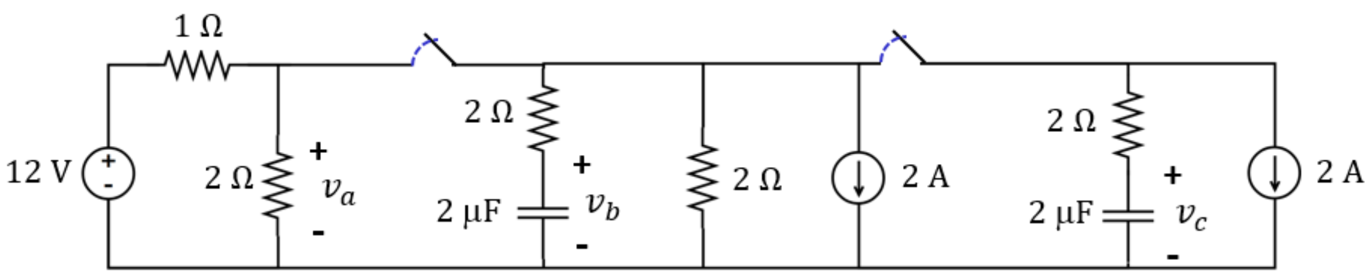
$v_a(0^-)$ $\boxed{}$ $v_b(0^-)$ $\boxed{}$ $v_c(0^-)$ $\boxed{}$

(b) Find $v_a(t), v_b(t)$ and $v_c(t)$ for $t > 0$ s. Also sketch all three waveforms in your blue book (a rough sketch is fine). *(9 points)*

$v_a(t)$ $\boxed{}$

$v_b(t)$ $\boxed{}$

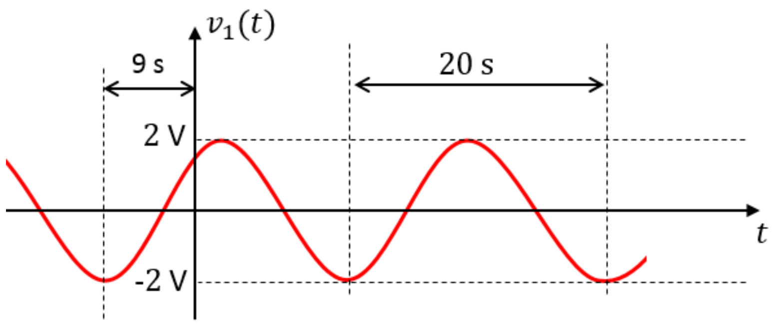
$v_c(t)$ $\boxed{}$

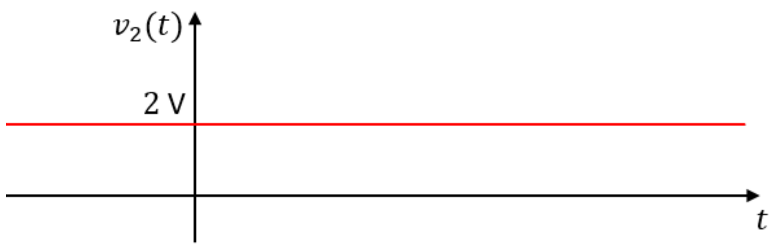


(3) Find the phasors $\mathbf{V_1}$ and $\mathbf{V_2}$ for the waveforms below. You can use polar or cartesian notation. *(5 points)*

$\mathbf{V_1}$

$\mathbf{V_2}$



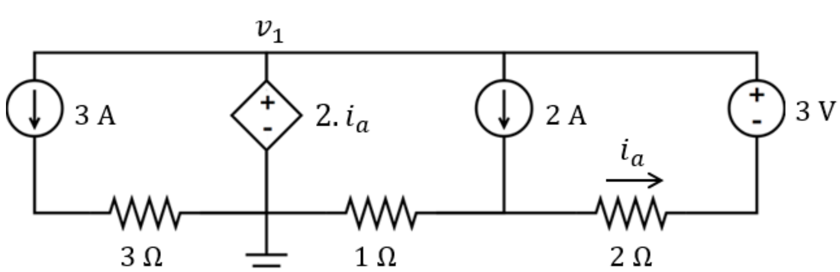


(4) (a) For the circuit below, find node voltage v_1. *(4 points)*

v_1

(b) Find the power P **supplied** by the 3 A current source (i.e., the left current source). *(2 points)*

P

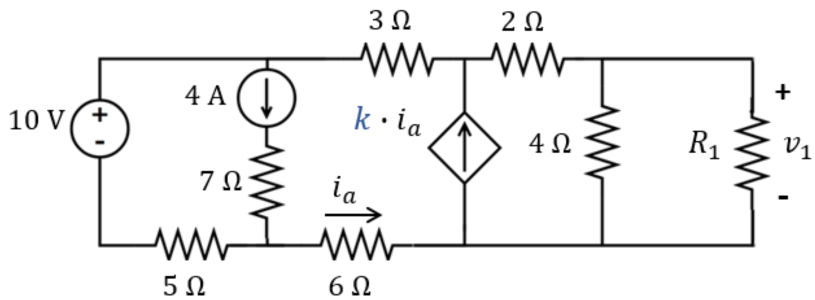


(5) (a) Consider the figure below. Note that *k* of the dependent source is unknown. We make the following measurements:

v_1 [ ]

When $R_1 = 1\ \Omega$, we find that $v_1 = 2\ \text{V}$
When $R_1 = 4\ \Omega$, we find that $v_1 = 4\ \text{V}$

Find v_1 when $R_1 = 3\ \Omega$. *(4 points)*

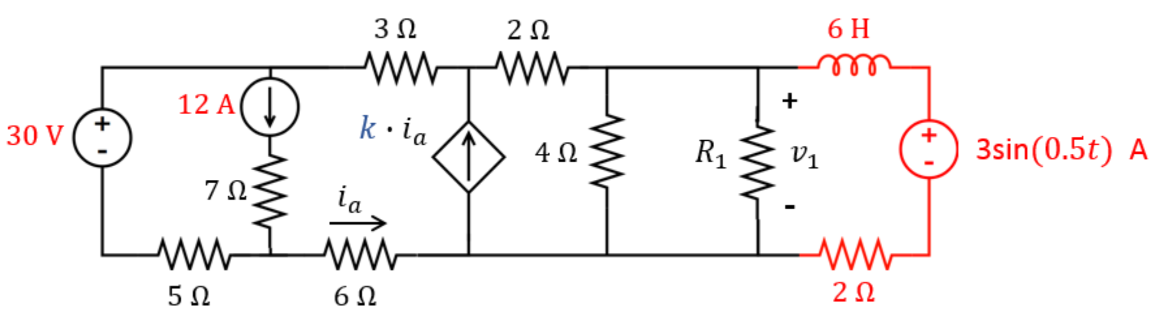


(b) The circuit below is that of part (a), with a few modifications (marked in red): the current and voltage source have a different value, and an extra inductor, resistor and voltage source are attached (on the right). The value of *k* is the same. Furthermore, we set $R_1 = 2\ \Omega$.

$v_1\left(\frac{11\pi}{6}\text{s}\right)$ [ ]

Assume the system is in steady state.

Find $v_1\left(\frac{11\pi}{6}\text{s}\right)$, i.e., v_1 at time $t = \frac{11\pi}{6}$ seconds. *(5 points)*



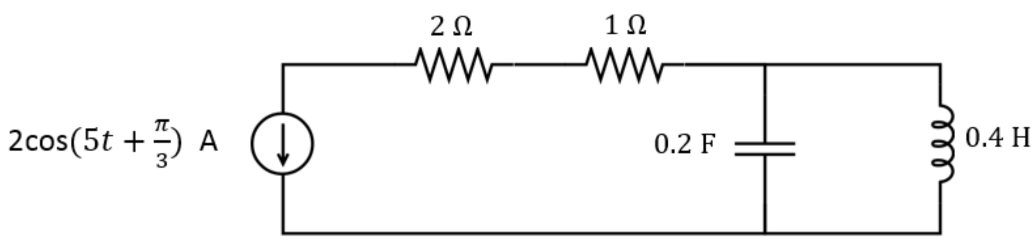
(6) (a) Find the average power P and reactive power Q **received** by the current source. *(5 points)*

P

Q

(b) We replace the 1 Ω resistor with another element (either a resistor, a capacitor or an inductor). The result is that the reactive power Q of the current source is now equal to zero. What is the value X of this new element (so we are looking for the resistance, capacitance or inductance value)? Use the correct units, so we know whether you think it is a resistor, capacitor or inductor (For example, X = 9 Ω). *(2 points)*

X



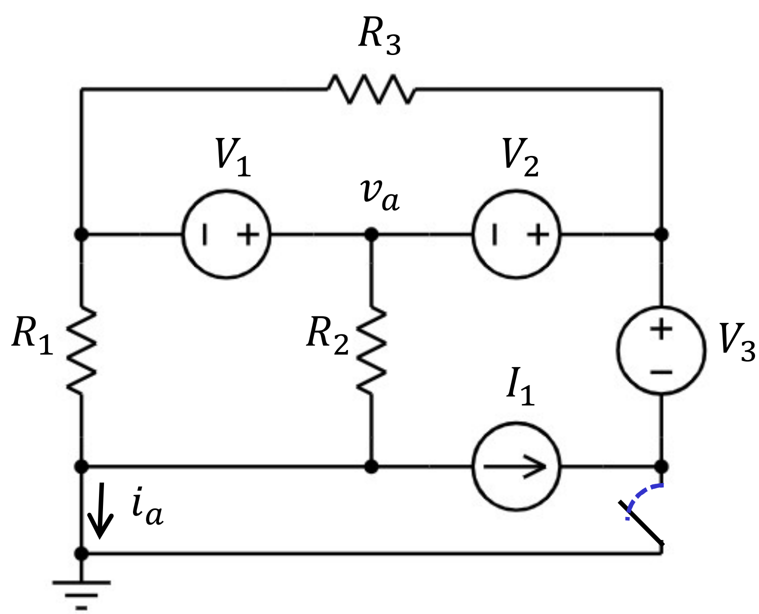
For t < 1 s, the switch is open. The switch closes at t = 1 s and remains closed.

(a) Find $v_a(0\ \mathrm{s})$

(b) Find $i_a(0\ \mathrm{s})$

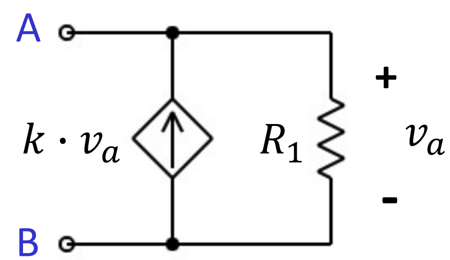
(c) Find $v_a(2\ \mathrm{s})$

(d) Find $i_a(2\ \mathrm{s})$



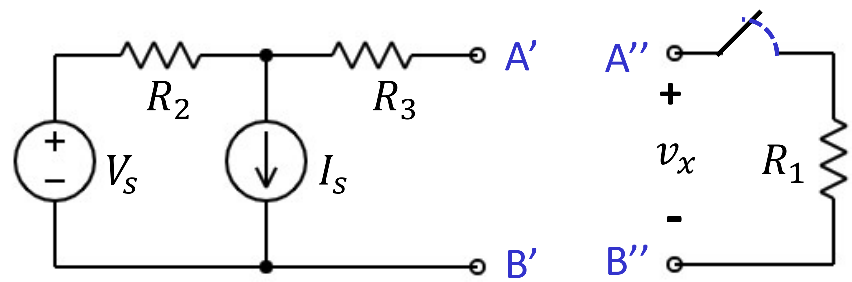
R1:	1 Ω
R2:	1 Ω
R3:	2 Ω
V1:	1 V
V2:	2 V
V3:	0 V
I1:	2 A

(a) For the circuit on the right, find the Thevenin equivalent model between A and B. Draw the model and don't forget to label A and B.

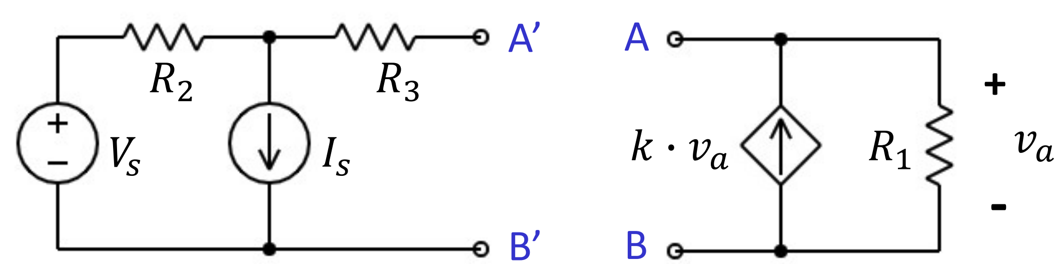


R1:	2 Ω
k:	1 A/V
X:	4 V
Y:	1 V

(b) Consider the two circuits on the right (you are not given the values of R_2, R_3, V_s or I_s). We connect the two circuits together, A' to A'' and B' to B'' and do two measurements. When the switch is open, we measure $v_x = X$. When the switch is closed, we measure $v_x = Y$.



On the right, the circuit between A' and B' is the same as the one above and the circuit between A and B is the same as the one in part (a).
If we connect them together, A' to A and B' to B, what is value of v_a?



For $t < 0$ s, the switch is closed and you may assume the system has reached steady state. The switch opens at $t = 0$ s and remains open.

(a) Find $i_a(0^- \text{ s})$ (i.e., just before the switch opens)

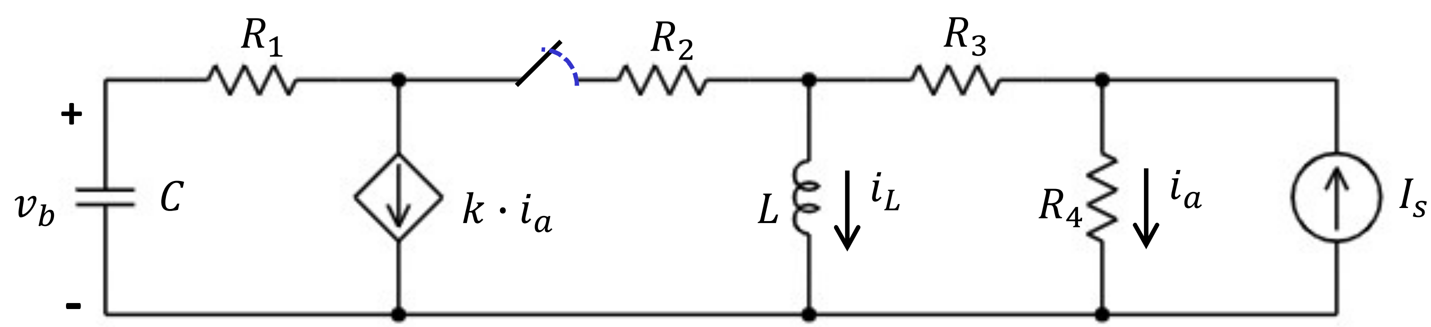
(b) Find $v_b(0^+ \text{ s})$ (i.e., right after the switch opens)

(c) Find $i_a(4 \text{ s})$

(d) Find $v_b(4 \text{ s})$

Note: For any of the parts, you can leave your answers as a function of e.

R1:	4 Ω
R2:	1 Ω
R3:	1 Ω
R4:	3 Ω
Is:	12 A
k:	2 A/A
C:	4 F
L:	8 H



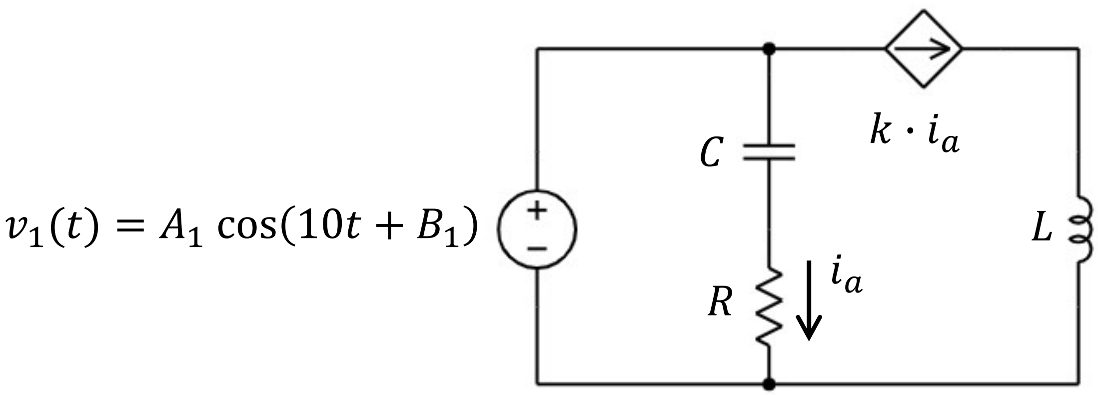
The AC circuit below is in steady state.

(a) What is the maximum value of the waveform $i_a(t)$?

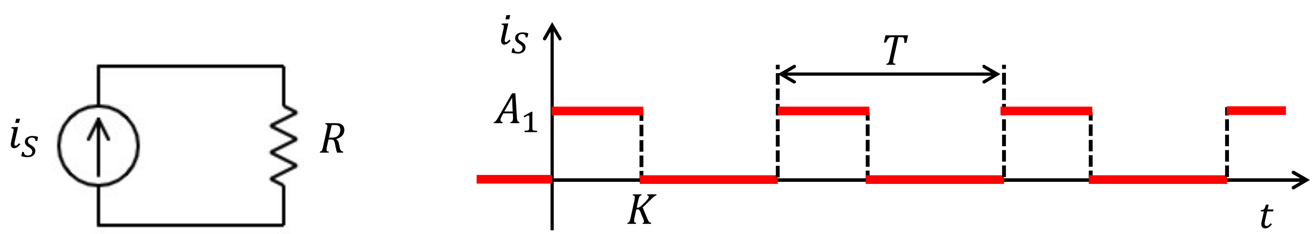
(b) Find the complex power S supplied by the voltage source v_1.

(c) Find the average power P received by that same voltage source v_1.

R:	2 Ω
C:	100 mF
L:	100 mH
k:	3 A/A
A1:	10 V
B1:	-15 degrees
T:	4 s



(d) In the circuit below, the waveform $i_S(t)$ is periodic with period T. The resistor R is the same as in the circuit above. Find the value of K such that the average power received by the resistor in the circuit below is the same as by the resistor in the circuit above.



In the circuit, v_1 and v_2 are AC sources with ω = 100 rad/s . The phasor diagram shows the phasors of v_1 and v_2. It is not drawn to scale. You may assume the system is in steady state.

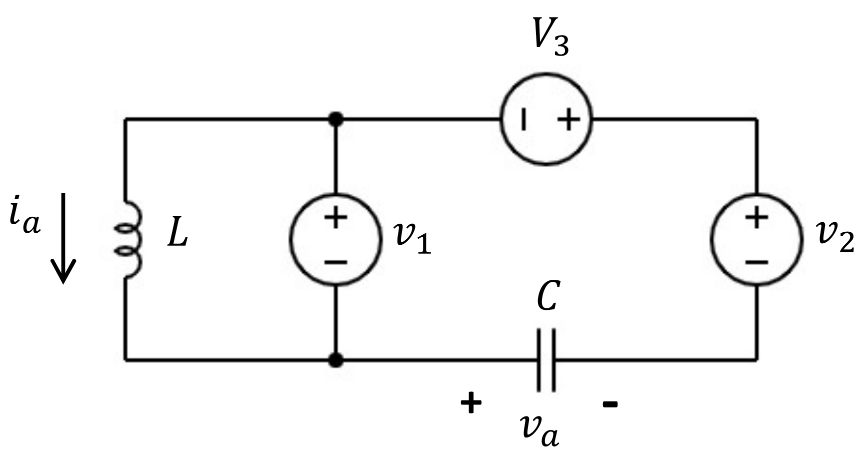
V_3 is a DC source.

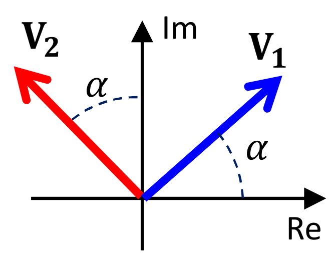
(a) What is $v_2\left(\frac{T}{4}\right)$ where T is the period of the waveform ?

(b) What is the waveform $i_a(t)$?

(c) What is the maximum value of the waveform $v_a(t)$?

|V1|:	2 V
|V2|:	3 V
alpha:	30 degrees
V3:	3 V
C:	10 mF
L:	25 mH





ECE 35, Winter 2022
Final

Your sequence number

Grade

/ 45

Last name

First + middle name(s)

PID

Instructions:

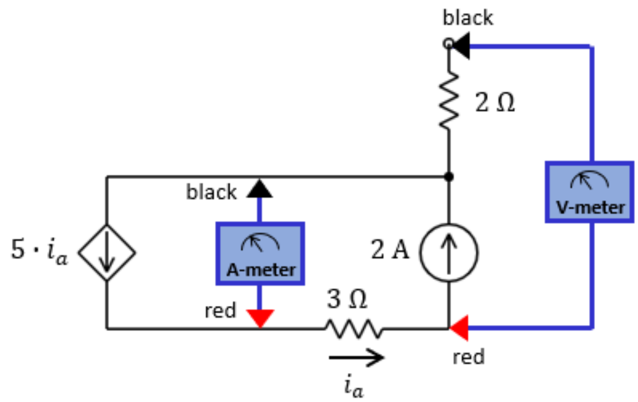
- Do not look at the questions or start writing until it is announced you can do so.
- <u>Make sure you write your PID on EACH page.</u>
- <u>We will only look at the work that is within the dotted box on each page.</u> You can use pages that were part of earlier questions if there is still space. There are also two extra pages at the end that you can use as overflow for any of the questions. Always clearly mark where we can find the work for each question.
- You can use the backside of the pages as scratch paper, but we will not look at this for grading purposes.
- Read each problem completely and thoroughly before beginning.
- Answers without supporting calculations will receive zero credit. If you are using intuition, write a short explanation. Write clearly and make sure your answer is structured properly. We will not hunt for your work or answers.
- Write your final answers in the answer boxes. Make sure you list units.
- The last page of the exam is the equation sheet. You may detach this if you want.
- You must follow the Final Exam Procedures that were posted on Canvas. If you are unsure of anything, ask. As a reminder:
  - Your phone should be turned off and put inside your bag
  - Calculators are not allowed.
  - This is a closed book exam.
  - Follow the Academic Integrity standards.

PID

(1) *(5 points)* In the circuit below, the two meters are digital multimeters (DMMs), with one set to the ammeter setting and the other to the voltmeter setting. You can assume the DMMs act as ideal meters in either setting.

(a) What is the reading X_1 of the DMM in the ammeter setting? X_1

(b) What is the reading Y_1 of the DMM in the voltmeter setting? Y_1



Next, we change the settings of the DMMs, such that the one that was in the voltmeter setting is now in the ammeter setting and vice versa.

(c) What is the reading X_2 of the DMM that is now in the ammeter setting? X_2

(d) What is the reading Y_2 of the DMM that is now in the voltmeter setting? Y_2

PID

PID

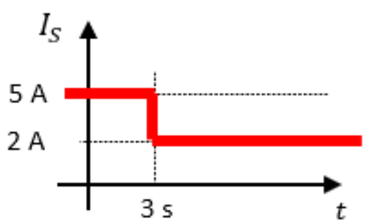
(2) *(5 points)* Consider the circuit below. The current source I_S changes as indicated in the graph. At time $t = 3^{-}$s (just before the current I_S changes), the system is not in steady state, but you are told that $i_a = 9$ A.

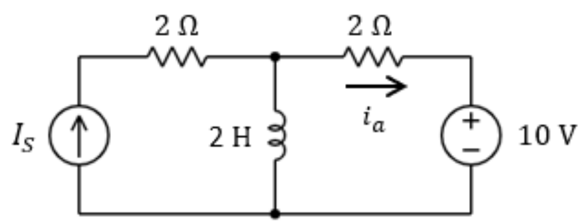
(a) Find the current i_a at time $t = 3^{+}$s (i.e., right after the current I_S changes)

$i_a(3^{+}\mathrm{s})$

(b) Find the current i_a at time $t = \infty$ s.

$i_a(\infty)$





PID

PID

(3) *(8 points)* Consider the circuit below. You are told that $R_1 = 2\ \Omega$ and $k = 2\,\frac{V}{A}$.

(a) What is the power P_1 <u>received</u> by resistor R_1?

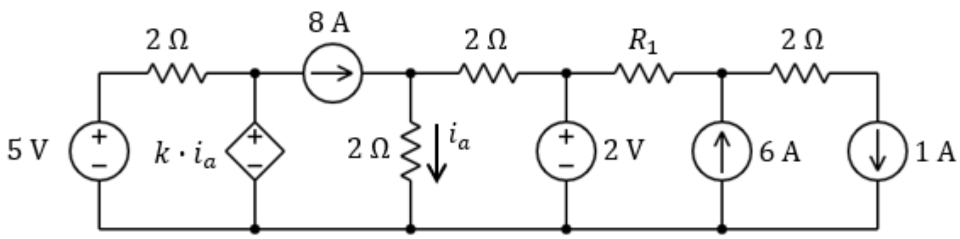
P_1

(b) What is the current i_a? (Hint: consider using superposition)

i_a

(c) What is the power P_2 <u>supplied</u> by the <u>dependent</u> voltage source?

P_2



PID

PID

PID

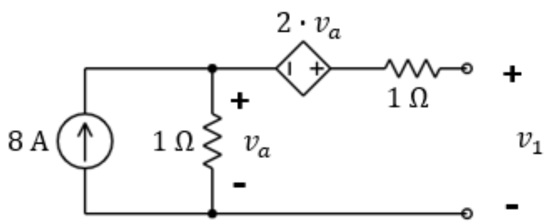
(4) *(7 points)*

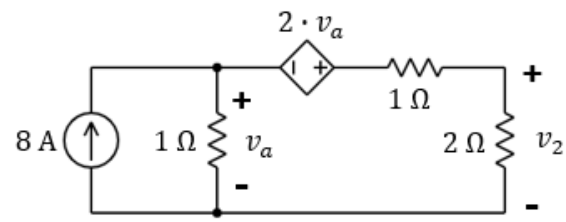
(a) For the circuit below on the left, find the voltage v_1.

v_1

(b) For the circuit below on the right, find the voltage v_2.

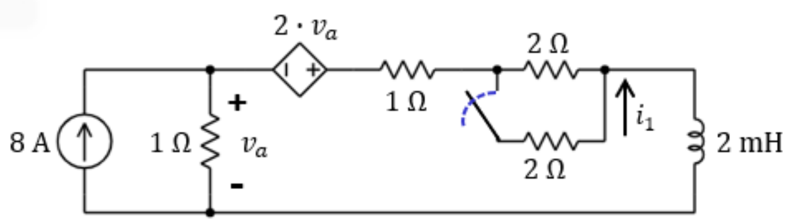
v_2





(c) Consider the circuit below. For $t < 4$ s, the switch is open, and the system has reached steady state. The switch closes at time $t = 4$ s. Find the time constant τ associated with $i_1(t)$ for $t > 4$ s.

τ



PID

PID

(5) *(12 points)* Consider the circuit below. For $t < \frac{\pi}{40}$ s, both switches are in position 1 and the system has reached steady state. At time $t = \frac{\pi}{40}$ s, both switches move from position 1 to position 2 simultaneously.

(a) Find the current i_a at time $t = \frac{\pi}{40}^-$ s (i.e., immediately before the switches move). $i_a\left(\frac{\pi}{40}^- \text{s}\right)$

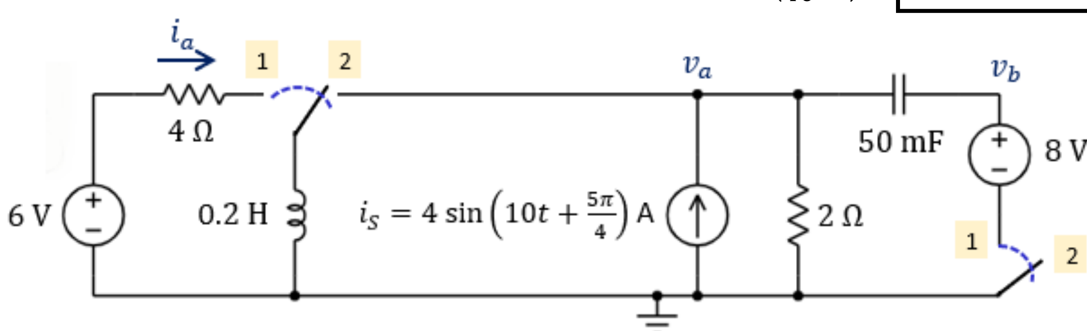
(b) Find the node voltage v_a at time $t = \frac{\pi}{40}^-$ s. $v_a\left(\frac{\pi}{40}^- \text{s}\right)$

(c) Find the node voltage v_b at time $t = \frac{\pi}{40}^-$ s. $v_b\left(\frac{\pi}{40}^- \text{s}\right)$

(d) Find the current i_a at time $t = \frac{\pi}{40}^+$ s (i.e., immediately after the switches move). $i_a\left(\frac{\pi}{40}^+ \text{s}\right)$

(e) Find the node voltage v_a at time $t = \frac{\pi}{40}^+$ s. $v_a\left(\frac{\pi}{40}^+ \text{s}\right)$

(f) Find the node voltage v_b at time $t = \frac{\pi}{40}^+$ s. $v_b\left(\frac{\pi}{40}^+ \text{s}\right)$



PID

PID

PID

(6) *(8 points)* The circuits below are in steady state.

(a) Find the complex power received by the capacitor, in the circuit on the left.

(b) What is the reading X of the voltmeter, in the circuit on the left?

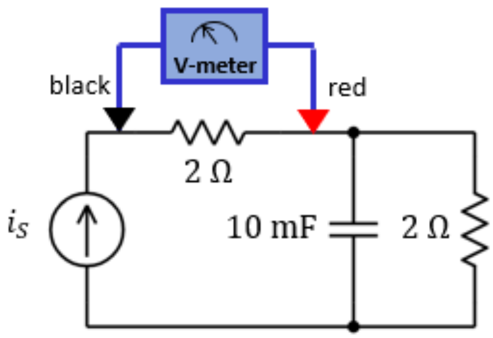
(c) I add a new circuit element in series with the current source as illustrated below on the right. As a result, the reactive power supplied by the current source is reduced to zero. What type of element should this new element be and what is its value (i.e., what is its resistance, capacitance or inductance)?

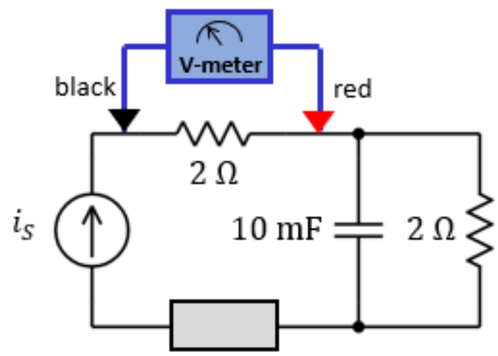
S_C

X

Type of element

value





$$i_S = 5\cos\left(100t + \frac{\pi}{3}\right)\text{ A}$$

PID

PID

ECE35 Equation Sheet

Basics: $i \triangleq \frac{dq}{dt}$ $\quad v_{ab} \triangleq \frac{dw}{dq}$ $\quad R = \rho \frac{l}{A}$

Capacitors: $C = \epsilon \cdot \frac{A}{d}$ $\quad Q = C \cdot v$ $\quad w_C = \frac{1}{2} C v^2$

Inductors: $L = \mu \cdot \frac{N^2 A}{l}$ $\quad B \sim i$ $\quad w_L = \frac{1}{2} L i^2$

AC power: $p(t) = \frac{1}{2} V_m I_m \cdot \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cdot \cos(2\omega t + \theta_v + \theta_i)$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$ $\quad Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$ $\quad X_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt}$

Trigonometry:

$\sin(-\alpha) = -\sin(\alpha)$ $\quad \cos(-\alpha) = \cos(\alpha)$

$\sin(\pi - \alpha) = \sin(\alpha)$ $\quad \cos(\pi - \alpha) = -\cos(\alpha)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$ $\quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$

$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha)$ $\quad \cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha)$

$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha)$ $\quad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$

$\sin(\alpha) \sin(\beta) = 0.5 \cdot (\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos(\alpha) \cos(\beta) = 0.5 \cdot (\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin(\alpha) \cos(\beta) = 0.5 \cdot (\sin(\alpha - \beta) + \sin(\alpha + \beta))$

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin(α):	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan(α):	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞