I. INTRODUCTION

1.1 Circuit Theory Fundamentals

Circuit theory is an approximation to Maxwell's electromagnetic equations in order to simplify analysis of complicated circuits. A circuit is made of several elements (boxes or devices) which are connected to each other with "ideal" wires. The variables in a circuit are currents and voltages in all elements.

The physics of current flow inside each element is captured by its iv characteristics. Since circuit elements communicate with each other only through current and voltages, elements with similar iv characteristics are viewed by the rest of the circuit as being <u>identical</u> regardless of what is inside the element/box.

Two general rules govern what happens when these elements are connected to each other: Kirchhoff current law, KCL, which is conservation of electric charge and Kirchhoff voltage law, KVL, which is a topology-driven constraint (*i.e.,* you get to the same place if you follow a closed loop). These two rules are independent of internal physics of elements and can be applied to non-linear elements.

In a circuit with N elements, we have $2N$ unknowns (i and v of each element) and we need $2N$ equations: N iv characteristics equations for elements and N KVLs and/or KCLs which depend only on how the elements are connected to each other. In the circuit theory, we learned that we can reduce the number of equations to be solved by a large number using "node-voltage" and "mesh-current" methods. As these method really are compact forms of writing KVL and KCLs, they equally apply to circuit with non-linear elements.

A linear circuit is made of "linear" elements (*i.e., iv* characteristics of every element is linear). A linear circuit has many desirable properties such as "proportionality" and "superposition" which are essential for many practical circuits (*e.g.,* an amplifier).

Because of the desirability of linear circuits, circuit theory includes <u>symbols</u> for five fundamental "two-terminal" elements which are linear:

Resistor: $v = Ri$

Capacitor: $i = C\,\dfrac{dv}{dt}$ or $V = \dfrac{1}{j\omega C}\,I$

Inductor: $v = L\,\dfrac{di}{dt}$ or $V = j\omega L\,I$

Independent voltage source: $v = v_s = const.$ for any current

Independent current source: $i = i_s = const.$ for any voltage and four “two-port networks” (boxes with two-wires going in and two-wire coming out): voltage-controlled voltage source, (similar to an independent voltage source but with source strength depending on voltage on another element in the circuit), current-controlled voltage source, voltage-controlled current source, current-controlled current source.

There are two other two-terminal elements that we will use and are special cases of the above elements. They are:

Short Circuit:	$v = 0$	for any current
Open Circuit:	$i = 0$	for any voltage

As can be seen, “short circuit” is a special case of a resistor (with $R = 0$) or a special case of a voltage source (with $v_s = 0$) while “open circuit” is a special case of a resistor (with $R \to \infty$) or a special case of a current source (with $i_s = 0$).

It is essential to remember that the above circuit elements are NOT physical devices. The idealized “resistor” in circuit theory is a symbol for any element whose iv characteristics equation is $v = Ri$. This means that if we have a two-terminal network (box or device) which has a $v = Ri$ characteristics, we can “represent” the network in a circuit with an ideal resistor.

Similarly, if we have a box whose voltage is constant for all currents, it can be represented as an independent voltage source (without any knowledge of what is inside the box). You actually have been doing this in the lab, modeling the power supply (which includes many transistors, diodes, resistors, capacitors) with an independent voltage source.

Physical elements (*i.e.,* resistors in the lab) can only be modeled with one of these idealized circuit elements within a certain accuracy (discussed in Sec. 1.4) and/or within a certain range of parameters: For example take a resistor in the lab. At high enough current, when the resistor is hot enough, the ratio of v/i is not linear anymore. So, a resistor in the lab can be approximated by an “ideal” circuit-theory resistor only for a range of currents or voltages (typically rated by its maximum power).

The bottom line is that the iv characteristics of a two-terminal network is the key. For example, we will make a two-terminal network with transistors which has a linear iv characteristics. From the point view of the rest circuit this two-terminal network is identical to an ideal resistor.

Which Solution Method to Use?

Since we will solve a lot of circuits in this course, it is essential that one should be able to choose the best method to solve a circuit. Basically, one wants to have the smallest number of equations. Assuming that we have "reduced" the circuit (*i.e.,* replaced parallel and series elements), one can readily estimate the number of circuit equations for each method:

KVL & KCL:	$2N_{element}$	equations
Node-voltage method:	$N_{nodes} - 1 - N_{IVS}$	equations
Mesh-Current method:	$N_{loops} - N_{ICS}$	equations

where N_{IVS} denotes the number of independent voltage sources and N_{ICS} the number of independent current sources. Obviously, one should use KVL & KCL only if there are only a few elements. Furthermore, we are mostly interested in voltages in the circuits. As such, usually node-voltage method is preferred as we will have a fair number of voltage sources and the answer is also a voltage. (PSpice uses node-voltage method but with full non-linear iv characteristics equations for elements). Note:

1. You CANNOT mix and match the three methods above!

2. Apply the techniques consistently *e.g.,* always write KCL as sum of currents flowing out of a node = 0. This minimizes the chance for error.

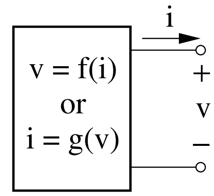
Functional Circuits:

In this course, we will use diodes and transistors to build circuits which do specific functions (and learn to design simple circuits). These circuits are either:

1. **Two-terminal networks,** *i.e.,* a box with two wire coming out of it, such as a power supply, output terminals of an iPod, etc. (Sec. 1.2) or
2. **Two-port networks,** *i.e.,* a box with two wires going in and two wires coming out of it, such as an amplifier, rectifier, etc. (Sec. 1.3)

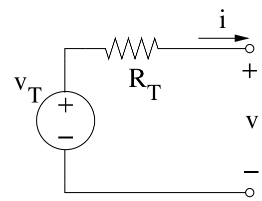
Note that principles of two-port networks (discussed later) can be easily extended to circuits with multiple inputs.

1.2 Two-terminal networks and Thevenin Theorem

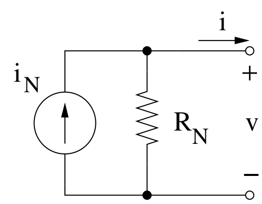


The simplest "functional" circuit is a two-terminal network. Since the rest of the circuit communicates with our network through voltages and currents, we only need to calculate (or measure) its *iv* characteristics. In general, we solve the circuit and find the output voltage v as a function of output current i (*i.e.,* assume output voltage is v and compute i). Note that the convention is to use "active" sign convention for our two-terminal networks.

Our calculation would be much simpler if the *iv* characteristics equation of a two-terminal network is linear. **Thevenin theorem** states that if a two-terminal network contains only linear elements, its *iv* characteristics equation would be linear. Furthermore, if the *iv* characteristics equation of a two-terminal network is linear, we can represent the network with a combination of two of ideal circuit-theory elements.



Thevenin Equivalent



Norton Equivalent

Proof of the Thevenin Theorem is straight forward. A linear *iv* characteristics equation, *i.e.,* $Av + Bi + C = 0$ can be written as $v = v_T - R_T i$ (with $v_T = -C/A$ and $R_T = B/A$). A circuit containing a voltage source v_T and a resistor R_T in series would have the same *iv* characteristics equation and, thus, can used instead of the two-terminal network in circuit analysis. Such a "simple" circuit is called the Thevenin equivalent circuit. Similarly, the linear *iv* characteristics equation can be written as $i = i_N - v/R_N i$ (with $i_N = -C/B$ and $R_N = B/A$) and the Norton equivalent circuit containing a current source i_N and a resistor R_N in parallel can be constructed. Note that $R_T = R_N$ and $i_N R_N = v_T$.

An important corollary to the Thevenin Theorem is that if a two-terminal network does not include an independent source, it can be reduced to a single "resistor" (even if it includes dependent sources).

Not only Thevenin Theorem simplifies calculation of the *iv* characteristics of a two-terminal network (as is discussed below), it allows us to identify a two-terminal network with only two numbers (instead of an *iv* equation).

1.2.1 How to calculate *iv* characteristics and the Thevenin parameters

You have seen a detailed discussion of Thevenin/Norton forms in your circuit theory course(s). In summary, the best method is to calculate two of the following three parameters:

1. Open-circuit voltage, v_{oc} (found by setting $i = 0$)

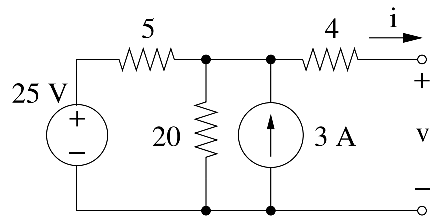
$$v = v_T - R_T i \qquad \rightarrow \qquad v = v_{oc} = v_T - R_T \times 0 = v_T$$

2. Short-circuit current, i_{sc} (found by shorting the terminals of the two-terminal network, *i.e.,* setting $v = 0$),

$$i = i_N - v/R_N \qquad \rightarrow \qquad i = i_{sc} = i_N - 0 = i_N$$

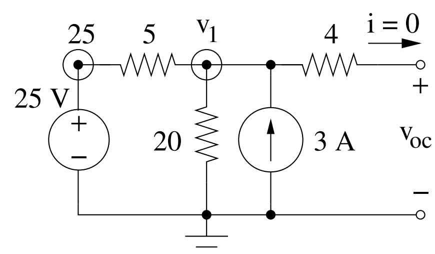
3. Direct calculation of R_T which is the resistance seen at the terminals with the <u>independent</u> sources "zeroed out" (*i.e.,* their strengths set equal to zero). Remember, you should NOT "zero out" dependent sources.

Example 1: Find the Thevenin and Norton Equivalent of this circuit:



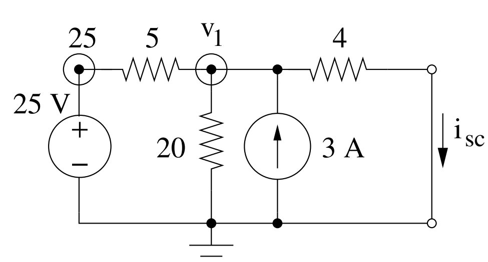
Using Node-voltage-method:

<u>1. v_{oc}</u>: Note that by Ohm's law across 4-Ω resistor $v_{oc} = v_1$.



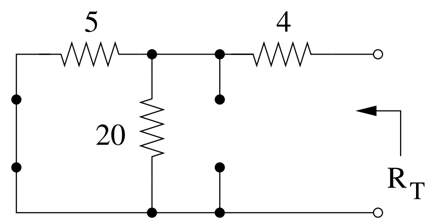
$$\frac{v_1 - 25}{5} - 3 + \frac{v_1}{20} = 0 \qquad \rightarrow \qquad v_1 = 32\text{V}$$
$$v_T = v_{oc} = v_1 = 32\text{V}$$

<u>2. i_{sc}</u>: Note that by Ohm's law across 4-Ω resistor $i_{sc} = v_1/4$.



$$\frac{v_1 - 25}{5} + \frac{v_1}{4} - 3 + \frac{v_1}{20} = 0 \qquad \rightarrow \qquad v_1 = 16\text{V}$$
$$i_N = i_{sc} = 0.25 v_1 = 4\text{A}$$

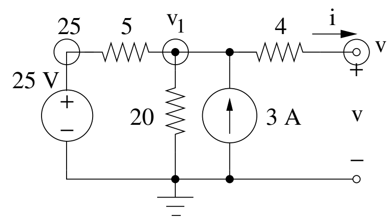
<u>3. R_T (zeroing the independent sources)</u>: From the circuit, we have $R_T = 4 + (5 \parallel 20) = 4 + 4 = 8\ \Omega$.



While Thevenin Theorem leads to a simpler analysis, it cannot be used for circuits with non-linear elements. In general, we need to calculate the iv characteristics of the network directly. The way to do this is to assume the output voltage v is known and compute i in terms of v. If we use the node-voltage method, first we have to compute all node voltages in terms of v. Computing i in terms of node voltages would then give i in terms of v.

In general, computing iv characteristics directly is quite time-consuming. We will solve the circuit of Example 1 to demonstrate how the method works (for this circuit, the solution is rather simple because there is only one unknown node, v_1).

Using Node-voltage method, we first find all node voltages in terms of v:



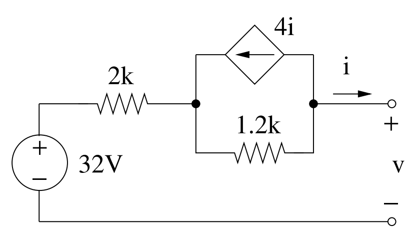
$$\frac{v_1 - 25}{5} - 3 + \frac{v_1}{20} + \frac{v_1 - v}{4} = 0$$
$$v_1 = 0.5v + 16$$

We now calculate i from node voltages. Here, i can be found from Ohm's law across the 4-Ω resistor:

$$4i = v_1 - v = (0.5v + 16) - v = -0.5v + 16 \qquad \rightarrow \qquad v = 32 - 8i$$

Note that since the iv characteristics is linear, this circuit can be reduced to its Thevenin equivalent: $v = 32 - 8i \equiv v_T - R_T i$ which gives $v_T = 32$ V and $R_T = 8\ \Omega$.

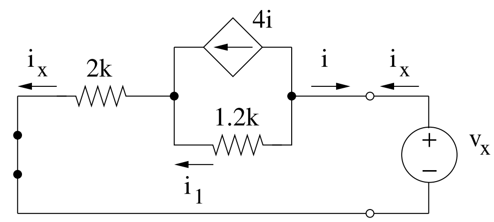
Example2 : Find the Thevenin equivalent of this two-terminal network.



Finding v_{oc} and i_{sc} are left as an exercise (answer: $v_T = v_{oc} = 32$ V and $i_N = i_{sc} = 4$ mA).

Finding R_T: We "zero" out all independent sources in the circuit. However, the resulting circuit cannot be reduced to a simple resistor by series/parallel formulas. To find R_T, we attach a "test" source, v_x to the terminals and calculate current i_x (see circuit). Since the two-terminal network should be reduced to a resistor (R_T), we should get $R_T = v_x/i_x$.

Since the circuit is simple, we proceed to solve it with KVL and KCL. We note that $i = -i_x$ and current i_x flows in the 2 k resistor (see circuit):



KCL: $\quad -i_1 + 4i - i_x = 0 \quad \rightarrow \quad -i_1 + 4(i_x) - i_x = 0 \quad \rightarrow \quad i_1 = 5i_x$

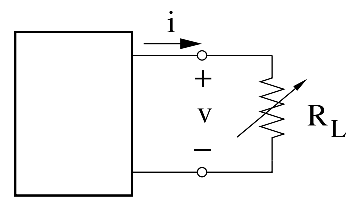
KVL: $\quad 2 \times 10^3 i_x + 1.2 \times 10^3 (5i_x) - v_x = 0$

$$v_x = 8 \times 10^3 i_x \quad \rightarrow \quad R_T = \frac{v_x}{i_x} = 8 \times 10^3 = 8 \text{ k}\Omega$$

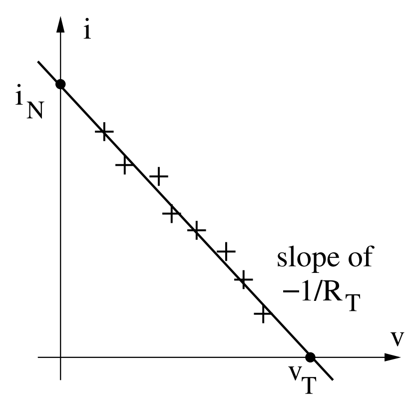
1.2.2 How to measure the iv characteristics and Thevenin parameters

Suppose we have a two-terminal network and want to measure its Thevenin equivalent circuit. In principle, we cannot use the above technique and try to measure v_{oc}, i_{sc}, and R_T. We cannot use an ohm-meter to measure R_T. It is not advisable to short the terminals and measure i_{sc} (there is a good chance that we are going to ruin the circuit if we do that). In principle, we can use a scope (or volt-meter) to measure v_{oc} but care should be taken as it is not known a priori if the internal resistance of the volt-meter (or scope) is large enough to act as an open circuit.

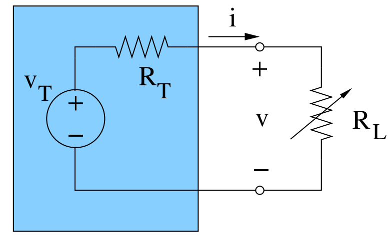
The best way to measure the Thevenin Equivalent parameters is to measure the iv characteristics of the two-terminal network. We can do this by attaching a variable load (a variable resistance) to the box and vary the load (R_L). For each value of R_L, we measure the pair of i and v (highlighted with a + in the figure). We can find the iv characteristics of the device by repeating this measurement with several values of R_L as is shown. Typically this is done with starting from a "large" R_L and gradually reducing the load.



If the circuit is linear, these data point should lie on a line (see the figure). Values of v_T, i_N, and R_T can be read directly from the graph as is shown. This method is specially accurate as one can use a "best-fit" line to the data in order to minimize random measurement errors.



A simpler, but less accurate version of the above method is to measure the output voltage for TWO different values of R_L (*i.e.,* R_{L1} and R_{L2} with measured voltages of v_1 and v_2, respectively). From the circuit:



$$\frac{v_1}{v_T} = \frac{R_{L1}}{R_T + R_{L1}} \qquad \text{and} \qquad \frac{v_2}{v_T} = \frac{R_{L2}}{R_T + R_{L2}}$$

Dividing the two equations gives:

$$\frac{v_1}{v_2} = \frac{R_{L1}}{R_T + R_{L1}} \times \frac{R_T + R_{L2}}{R_{L2}}$$

which can be solved to find R_T. Then, one of the above equations for v_1 or v_2 can be used to find v_T. Typically, we choose R_{L2} to be very large, $R_{L2} \to \infty$ (*e.g.,* internal resistance of scope), then $v_2 = v_{oc}$ (open circuit voltage) and

$$\frac{v_1}{v_{oc}} = \frac{R_{L1}}{R_T + R_{L1}} \qquad \to \qquad \frac{R_T}{R_{L1}} = \frac{v_{oc}}{v_1} - 1$$

In general, we should choose R_{L1} such that v_1 is sufficiently different from v_{oc} for the measurement to be accurate. Typically, experiment is repeated for several values of R_{L1} until v_1/v_{oc} is between 0.3 to 0.7).

Note that the above method has to be modified if the box includes time-dependent sources as R_T may be an impedance. (Exercise: How?)

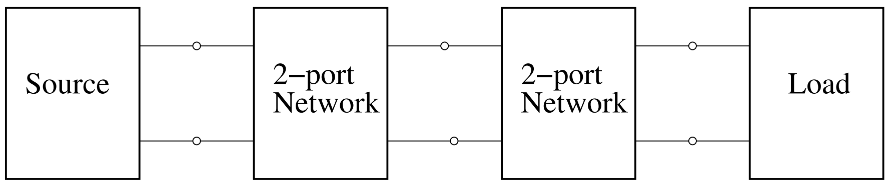
In addition, this method would not give the Thevenin equivalent circuit if $v_T = 0$ (*i.e.,* the circuit reduces to a resistor).

1.2.3 How to find the *iv* characteristics and Thevenin parameters using PSpice

You can use the same technique described above for measuring the Thevenin parameters with PSpice. Attach a "variable" load (using "Parameter" setting in PSpice) to the circuit. Ask PSpice to compute output voltage i for different values of the "parameter" R_L. Plot the output current i versus the output voltage v and you will have the *iv* characteristics of the circuit similar to the figure above (Make sure that you have the current direction correctly!).

1.3 Two-port networks

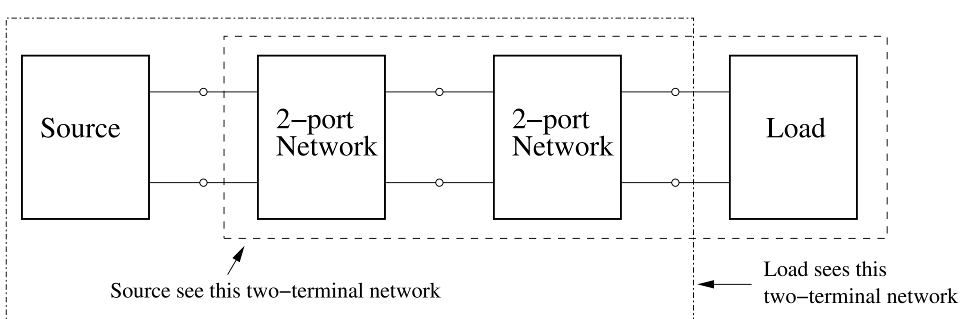
It is not practical to design a complete circuit as a whole from scratch. It is usually much easier to break the circuit into components and design and analyze each component separately. In this manner we can design "building blocks" (amplifiers, filters, *etc.*) that can be used in a variety of devices. A typical analog circuit is composed of a "source," and a "load" both of which are two-terminal networks discussed before. A typical circuit also contains several "two-port networks" as is shown below.



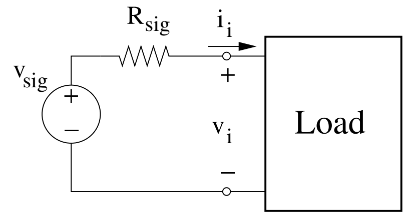
In a two-port network the input signal (either input current or input voltage) is modified by the circuit and an output signal (either output current or output voltage) is generated. For most electronic circuits, we keep the currents low and modify voltages in order to minimize power dissipation in the circuit. As such the relationship between the output voltage and the input voltage dictates the response of the two-port network. This relationship is called the voltage transfer function of the network. For two-port networks with non-linear elements, transfer function is non-linear (*e.g.,* diode waveform shaping circuits of Sec. 2). However, If a two-port network includes only linear elements, its transfer functions is linear (*i.e.,* ratio of v_o/v_i does not depend on v_i). In addition, such a linear two-port network can be modeled by four linear circuit elements (often 3) as we discuss in the transistor amplifier section.

1.3.1 How each sub-circuit sees other elements

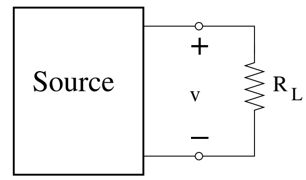
The strategy of dividing a "linear" circuit into individual components works because of the Thevenin Theorem. Recall that any two-terminal network can be replaced by its Thevenin equivalent. In addition, if a two-terminal network does not include an "independent source" it will be reduced to a single "impedance" (even if it includes dependent sources).



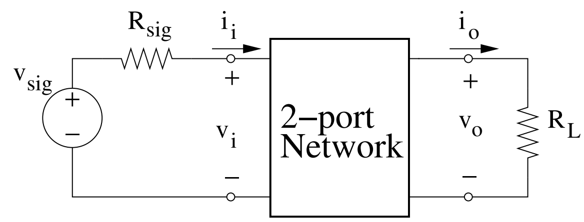
What Load sees: The load sees a two-terminal network. This two-terminal network contains an independent source. So it can be reduced to its Thevenin equivalent.



What Source sees: The source sees a two-terminal network. This two-terminal network does not contain an independent source. So it can be reduced to a single impedance.



What each two-port network sees: Following the logic above, its obvious that each two-port network sees a two-terminal network containing an independent source in the input side (can be reduced to a Thevenin form) and a two-terminal network that does not contain an independent source on the output side (so it can be reduced to a single impedance).



The above observations indicate that we do not need to solve a complete circuit. For a two-terminal network like the source, we only need to find its iv characteristics (or v_T and R_T for a linear network) to be able to predict its response when it is attached to any circuit (here modeled as R_L). For a two-port network, we only need to solve the circuit above with v_{sig}, R_{sig}, and R_L as parameters. We will discuss this in depth in transistor amplifier section.

Note that the concept of two-port networks can be easily extended to circuit with multiple inputs and one output as we will see in our discuss of digital gates.

1.4 Mathematics versus Engineering

You should have learned by now that one cannot achieve "mathematical" accuracy in practical systems. Firstly, our instruments have a finite accuracy. Secondly, we can build components only within a certain accuracy (or tolerance). As a general rule, the higher the accuracy in a component, the more expensive it is. Therefore, we always design and build component based on the accuracy that is needed (to reduce the cost). We use idealized model to describe physical phenomena in practical systems by making "approximations." We also ignore terms in our analysis by arguing that they are small or large. Some of these issues are discussed below.

What does accuracy mean?

When a number (or a measurement), A, has a relative accuracy (or tolerance) of ϵ, it means that its value is between $A(1-\epsilon)$ to $A(1+\epsilon)$. This means that we cannot differentiate between any number in the range $A-\epsilon A$ to $A+\epsilon A$. We would say that all numbers in this range are "approximately equal" to each other:

$$B \approx A \qquad \Leftrightarrow \qquad A - \epsilon A \leq B \leq A + \epsilon A$$

and we can use B and A interchangeably as we cannot distinguish between them.

Concepts of infinity and zero are also meaningless in abstract. They are used in the context of "much bigger" and "much smaller." For example, in the discussion of measuring Thevenin parameters, we arrived at a equation like:

$$\frac{v_o}{v_T} = \frac{R_L}{R_T + R_L}$$

Mathematically, if $R_L \to \infty$, $v_o = v_T$. In engineering, if we assume $R_L \gg R_T$, we have $R_T + R_L \approx R_L$ and $v_o \approx v_T$. This means that we have defined "infinite" R_L as $R_L \gg R_T$.

Similarly, if $R_T = 0$, $v_o = v_T$. If we assume $R_T \ll R_L$, we have $R_T + R_L \approx R_L$ and $v_o \approx v_T$. This means that we have defined "zero" R_T as $R_T \ll R_L$.

So, concepts of large and small (zero and infinite) are defined as "much smaller" or "much larger" than $\cdots$. They also require a frame of reference, *i.e.,* large or small compared to $\cdots$. In the above example, larger and smaller were defined with respect to another resistor value. For example, if $R_T = 1\ \Omega$, a $R_L = 100\ \Omega$ resistor would be large, while if $R_T = 1000\ \Omega$, a $100\ \Omega$ load resistor would actually be small.

Notions of much smaller ($\ll$) and much greater ($\gg$) are defined in term of a given or needed accuracy, ϵ. Consider quantity $B = A + a$. We use the concept of much smaller, $a \ll A$, to write $B \approx A$. From the above definition of approximate, we should have (assuming that a and A are positive):

$$\begin{aligned}
B \approx A \qquad &\rightarrow \qquad A - \epsilon A \leq B \leq A + \epsilon A \\
&\qquad\quad A - \epsilon A \leq A + a \leq A + \epsilon A \qquad \rightarrow \qquad a \leq \epsilon A \\
a \ll A \qquad &\Leftrightarrow \qquad a \leq \epsilon A
\end{aligned}$$

Exercise: Show that with a tolerance of ϵ, $A \gg a$ means $A \geq (1/\epsilon)a$.

Measurement accuracy:

Once calibrated, every instrument has a certain and finite accuracy. As an example, the scopes in ECE65 lab are accurate within 2% ($\epsilon = 0.02$). So, if a scope reads a value of 1.352 V, the "real" value is anywhere between $1.352 \pm 0.02 \times 1.352$ or in the range of 1.325 to 1.379. In this context any number between 1.325 and 1.379 is approximately equal to 1.352 as we CANNOT differentiate among them by our measurements: $1.325 \approx 1.352$ and $1.379 \approx 1.325$.

Note: In the example above, the 4th significant digits in 1.352 is totally meaningless (see the range of numbers we cannot distinguish). It is a poor engineering practice to even report this 4th significant digit! (Still some ECE65 students report their calculations to 8th significant digits, directly writing the number from their calculators!). Similarly, it is poor engineering practice to report numbers in whole fractions (*e.g.,* 4/3). No measuring instrument measure any property in whole numbers!

Component accuracy/tolerance:

Each element/component/system is manufactured to a certain tolerance – the smaller the tolerance, the more expensive is to build that component. For example, resistors we will use in the Lab have a tolerance of 5%. This means that a 1 kΩ has a value of $1,000 \pm 5\% = 1,000 \pm 50$ Ω or somewhere between 950 and 1,050 Ω.

A corollary of this concept is that if you designed a circuit and found that you need a 1,010 Ω resistor, you SHOULD NOT put a 1 kΩ and a 10 Ω resistor (with 5% tolerance) in series. The resultant combination would have a value between 959.5 and 1,060.5 Ω which is no better than a 5% 1 kΩ resistor (*i.e.,* you have wasted the second resistor). If you need to have a 1,010 Ω resistor (*i.e.,* more precision), you should use a 1,010 Ω resistor with a 1% tolerance (which is more expensive).

Accuracy in modeling:

As we discussed before, practical elements/components are "modeled" (*i.e.,* approximated). Obviously, the desired accuracy plays an important role in these approximations. The higher the needed accuracy, the more sophisticated model should be constructed. Since a more complicated model would lead to a more difficult analysis, we always use the simplest model which provide the needed accuracy for the job in hand.

Computer simulation (*e.g.,* PSpice) uses sophisticated models for circuit elements because computer solves the circuit numerically. However, numerical solution of a circuit requires that values of all circuit elements to be prescribed. If we want to understand the impact of the value of an element value on the circuit response, we need to run many many simulations with different values. Even with a circuit with 10 elements, this approach is impractical.

Analytical solution of a circuit (*i.e.,* "paper and pencil" calculation) provides a great insight into the circuit behavior as we can treat values/properties of elements as parameters (instead of numbers). For example, if an element is a resistor, we gain more insight if we have the solution in terms of parameter R as opposed to its numerical value. We can use our "design" criteria to find the value of R which makes the circuit behave as we want. Unfortunately, analytical solutions of circuits can only be found if we use simple and mostly linear models for circuit elements. In many cases, further approximations are needed in order to find a useful "closed-form" answer.

Typically, design of a circuit is done in multiple steps, hand calculations (with simple models and approximations) for the first pass design followed by PSpice simulations with more sophisticated models to confirm and optimize the design.