UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Spring 2025
Linear Systems Fundamentals

MIDTERM EXAM

You are allowed one 2-sided sheet of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME _____

Signature _____

Student ID Number _____

Your signature confirms that you have completed this exam in accordance with the ECE 101 Academic Integrity Agreement.

Problem	Weight	Score
1	32 pts	
2	34 pts	
3	34 pts	
Total	100 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Tables 3.1 and 3.2 from the textbook are attached to the exam.
Scan, upload, and tag your solutions on Gradescope at the end of the exam.

Good luck!

Name/Student ID: _____

Problem 1 [DT Signals and Systems] (32 points)

Consider the discrete-time system whose output $y[n]$ in response to input $x[n]$ is given by:

$$y[n] = x[2n-1] + x[3n-1].$$

(a) (10 points)

Determine and sketch precisely the impulse response $h[n]$ of the system.

(b) (10 points)

Determine and sketch precisely the step response $s[n]$ of the system.

(c) (12 points)

Check the appropriate box indicating whether or not the system satisfies the specified property.

Justify your answers by either proving that the property holds for the system, or giving a counterexample with specific signals if it does not.

True	False	
☐	☐	Invertible
☐	☐	Stable
☐	☐	Linear
☐	☐	Time-invariant

Do not write your answers on this page.

Write your answers to parts (a), (b), and (c) on the following pages.

Name/Student ID: _____

Problem 1 [DT Signals and Systems] (cont.)

Consider the discrete-time system whose output $y[n]$ in response to input $x[n]$ is given by: $y[n] = x[2n-1] + x[3n-1]$.

(a) Determine and sketch precisely the impulse response $h[n]$ of the system.

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Problem 1 [DT Signals and Systems] (cont.)

Consider the discrete-time system whose output $y[n]$ in response to input $x[n]$ is given by: $y[n] = x[2n-1] + x[3n-1]$.

(b) Determine and sketch precisely the step response $s[n]$ of the system.

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(c) Check the appropriate box indicating whether or not the system satisfies the specified property. **Justify your answers by either proving the property holds or giving a counterexample with specific signals.**

True	False	
☐	☐	Invertible
☐	☐	Stable

Name/Student ID: _____

Problem 1 [DT Signals and Systems] (cont.)

Consider the discrete-time system whose output $y[n]$ in response to input $x[n]$ is given by: $y[n] = x[2n-1] + x[3n-1]$.

(c) Check the appropriate box indicating whether or not the system satisfies the specified property. **Justify your answers by either proving the property holds or giving a counterexample with specific signals.**

True	False	
☐	☐	Linear
☐	☐	Time-invariant

Name/Student ID: _____

Problem 2 [DT - LTI Systems] (34 points)

Let S be the discrete-time LTI system with impulse response:

$$h[n] = \left(\frac{1}{4}\right)^n u[n].$$

(a) (10 points)

Determine the step response $s[n]$ of system S.

(b) (12 points)

Determine the transfer function $H(z)$ of S and express it as a ratio of polynomials in z. Does the frequency response of the system vanish at any frequencies ω? If so, determine the frequencies. If not, explain why not.

(c) (12 points) For each of the following input signals $x[n]$, determine the corresponding output signal $y[n]$. Determine $y[0]$ and express it as a complex number in Cartesian form.

(i) $x[n] = 1$

(ii) $x[n] = (\frac{j}{4})^n$

Do not write your answers on this page.

Write your answers to parts (a), (b), and (c) on the following pages.

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Problem 2 [DT - LTI Systems](cont.)

Let S be the DT - LTI system with impulse response: $h[n] = \left(\frac{1}{4}\right)^n u[n]$.

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Problem 2 [DT - LTI Systems] (cont.)

Let S be the DT - LTI system with impulse response: $h[n] = \left(\frac{1}{4}\right)^n u[n]$.

(b) Determine the transfer function $H(z)$ of S and express it as a ratio of polynomials in z. Does the frequency response of the system vanish at any frequencies ω? If so, determine the frequencies. If not, explain why not.

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Problem 2 [DT - LTI Systems] (cont.)

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Name/Student ID: _____

Problem 3 [DTFS and DT Filtering] (34 points)

(a) (12 points)

Let $x[n]$ be a **real-valued** discrete-time periodic signal with fundamental period $N = 4$ and Fourier Series coefficients a_k. Assume

$$a_{-1} = \frac{1}{4}(1-j), a_2 > 0, \sum_{n=0}^{3} x[n] = 0, \sum_{n=0}^{3} |x[n]|^2 = 2.$$

Determine the coefficients a_0, a_1, a_2, a_3.

(b) (12 points)

Consider a discrete-time signal $x[n]$ with fundamental period $N = 4$. Assume $x[0] = 0, x[1] = 1, x[2] = -1, x[3] = 0$. Determine its Fourier series coefficients a_k, for $k = 0, 1, 2, 3$.

(c) (10 points)

The signal $x[n]$ of part (b) is passed through an LTI system with frequency response $H(e^{j\omega}) = e^{-j\omega}$. Express the Fourier series coefficients b_k of the output signal $y[n]$ in terms of the Fourier coefficients a_k of $x[n]$, and evaluate b_k, for $k = 0, 1, 2, 3$. Determine the signal $y[n]$ **without using the synthesis equation**.

Do not write your answers on this page.

Write your answers to parts (a), (b), and (c) on the following pages.

Name/Student ID: _____

Problem 3 [DTFS and DT Filtering] (cont.)

(a) Let $x[n]$ be a **real-valued** discrete-time periodic signal with fundamental period $N = 4$ and Fourier Series coefficients a_k. Assume

$$a_{-1} = \frac{1}{4}(1-j), a_2 > 0, \sum_{n=0}^{3} x[n] = 0, \sum_{n=0}^{3} |x[n]|^2 = 2.$$

Determine the coefficients a_0, a_1, a_2, a_3.

Name/Student ID: _____

Problem 3 [DTFS and DT Filtering] (cont.)

(b) Consider a discrete-time signal $x[n]$ with fundamental period $N = 4$. Assume $x[0] = 0, x[1] = 1, x[2] = -1, x[3] = 0$. Determine its Fourier series coefficients a_k, for $k = 0, 1, 2, 3$.

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Problem 3 [DTFS and DT Filtering] (cont.)

(c) The signal $x[n]$ of part (b) is passed through an LTI system with frequency response $H(e^{j\omega}) = e^{-j\omega}$. Express the Fourier series coefficients b_k of the output signal $y[n]$ in terms of the Fourier coefficients a_k of $x[n]$, and evaluate b_k, for $k = 0, 1, 2, 3$. Determine the signal $y[n]$ **without using the synthesis equation**.

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