CSE 167 (WI 2025) Exercise 6

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1 Barycentric Coordinates

We are given the triangle with vertices:

$$p_1 = \begin{bmatrix}0\\0\end{bmatrix}, \quad p_2 = \begin{bmatrix}0\\1\end{bmatrix}, \quad p_3 = \begin{bmatrix}1\\1\end{bmatrix}$$

The barycentric coordinates for the given points are computed as follows:

1.1 Point $a = \begin{bmatrix}2\\1\end{bmatrix}$

Solving for $\lambda_1, \lambda_2, \lambda_3$:

$$\begin{bmatrix}0 & 0 & 1\\0 & 1 & 1\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{bmatrix} = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

Solving the system, we get:

$$\lambda_1 = 0, \quad \lambda_2 = -1, \quad \lambda_3 = 2$$

Since $\lambda_2 < 0$, point a is **outside** the triangle.

1.2 Point $b = \begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\\\frac{2}{3}\end{bmatrix}$

Solving for barycentric coordinates:

$$\begin{bmatrix}0 & 0 & 1\\0 & 1 & 1\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\\1\end{bmatrix}$$

Solving the system, we get:

$$\lambda_1 = \frac{1}{3}, \quad \lambda_2 = \frac{1}{3}, \quad \lambda_3 = \frac{1}{3}$$

Since all values are within $[0, 1]$, point b is **inside** the triangle.

2 Interpolation Under Perspective Distortion

Given two markers at distances:

$$z_1 = 2\text{m}, \quad z_2 = 10\text{m}$$

A bunny appears at the midpoint in the image. Using perspective-correct interpolation:

$$\frac{1}{z_{bunny}} = \frac{1}{2} \cdot \frac{1}{z_1} + \frac{1}{2} \cdot \frac{1}{z_2}$$

Substituting values:

$$\frac{1}{z_{bunny}} = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{10}\right) = \frac{1}{2}\left(\frac{5}{10} + \frac{1}{10}\right) = \frac{1}{2} \cdot \frac{6}{10} = \frac{3}{10}$$

Thus,

$$z_{bunny} = \frac{10}{3} \approx 3.33\text{m}$$

The bunny was **3.33 meters** away from the photographer.