UC San Diego

Lecture 1

Signals, systems, circuits and phasors

ECE 45

Preview of today's lecture

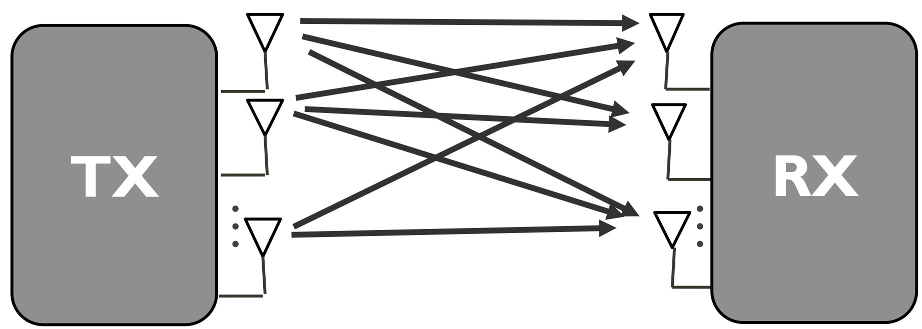
- Introduction
- Context for the course: Tying up loose ends
  - Phasors, complex sinusoids and Fourier
  - Circuits and linear time invariant systems
- Practical details
- Mathematical fundamentals

UC San Diego
NC STATE UNIVERSITY
TEXAS The University of Texas at Austin
Stanford University
University of Virginia

3 textbooks, lab manual, monograph

Robert W. Heath Jr., Ph.D., P.E.
Charles Powell Lee Chair in Wireless Communications
University of California, San Diego

MIMO communication



Quick facts

Graduated 52 Ph.D. students
> 800 papers, 65 US patents
99.5k citations, h-index 149
Fellow of IEEE, AAAS, NAI

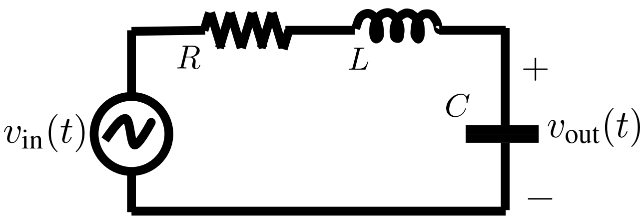
NOKIA
Qualcomm
SAMSUNG
NSF

Context for the course

Learning objectives

- Recall key concepts from ECE 35
- Explain the holes that are plugged by ECE 45

Circuits and linear systems



Analog design
ECE 35

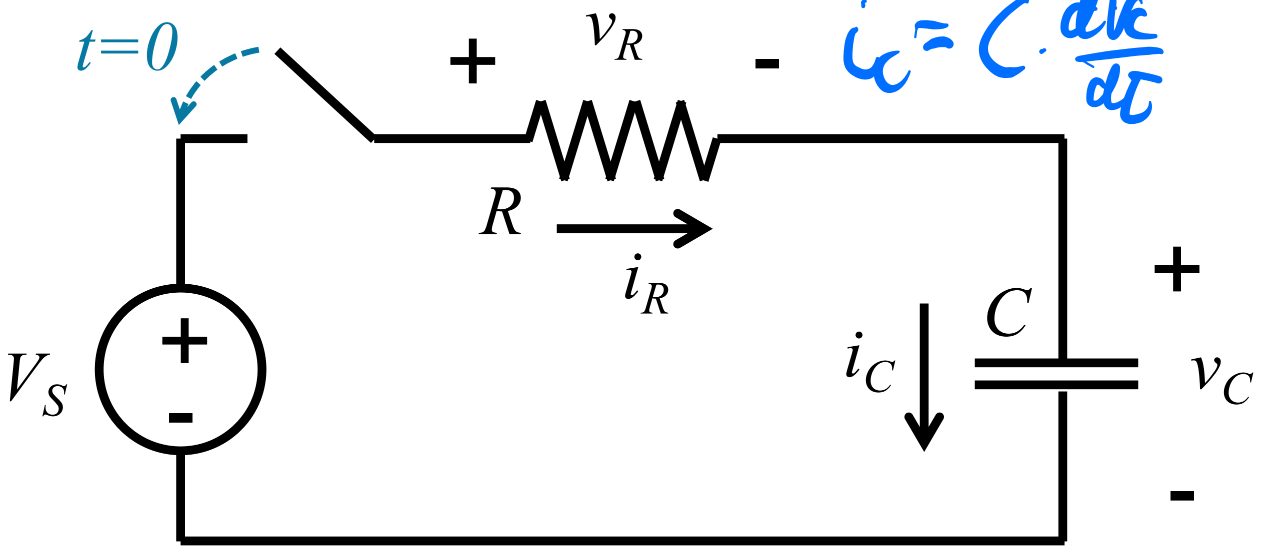
Circuits and systems
ECE 45

Linear systems
ECE 101

ECE 45 serves as an important transition from simple analog circuits to analog and digital signal processing

From C. Schurgers ECE 35 course materials

Solving for voltages and currents in a circuit



$i_C = C \cdot \frac{dv_C}{dt}$

$$i_C = i_R$$

$$V_S = v_R + v_C$$

$$i_R = \frac{v_R}{R}$$

$$i_C = C \cdot \frac{dv_C}{dt}$$

$$C \cdot \frac{dv_C}{dt} = \frac{v_R}{R} = \frac{V_S - v_C}{R}$$

$$RC \cdot \frac{dv_C}{dt} + v_C = V_S$$

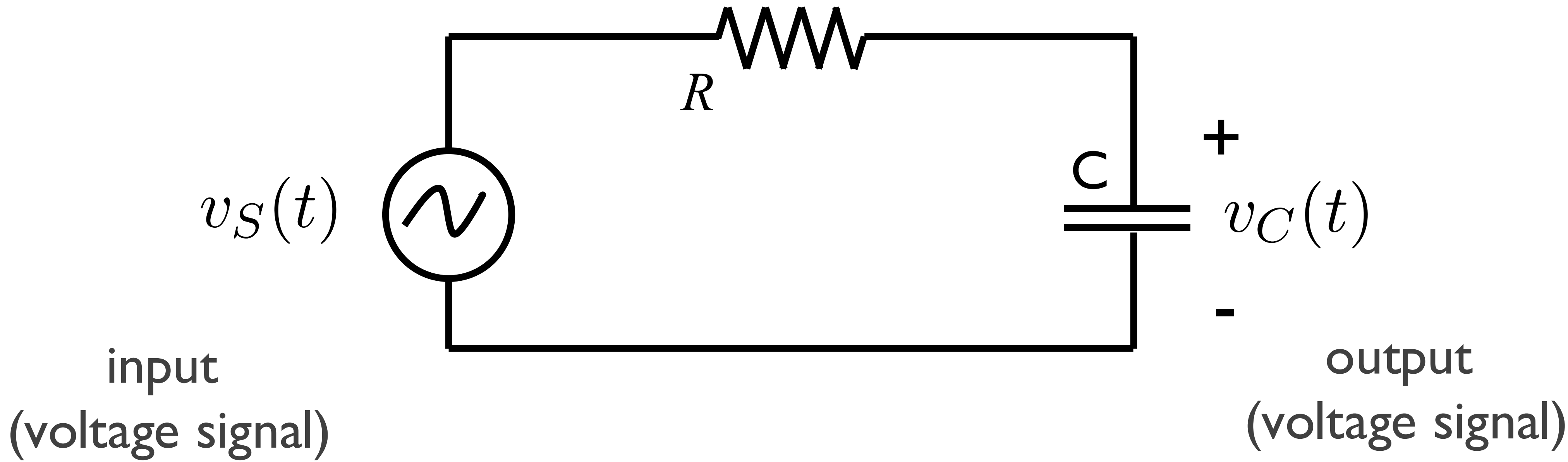
1[st] order differential equation

$v_C(0^-) = V_0$

Initial condition

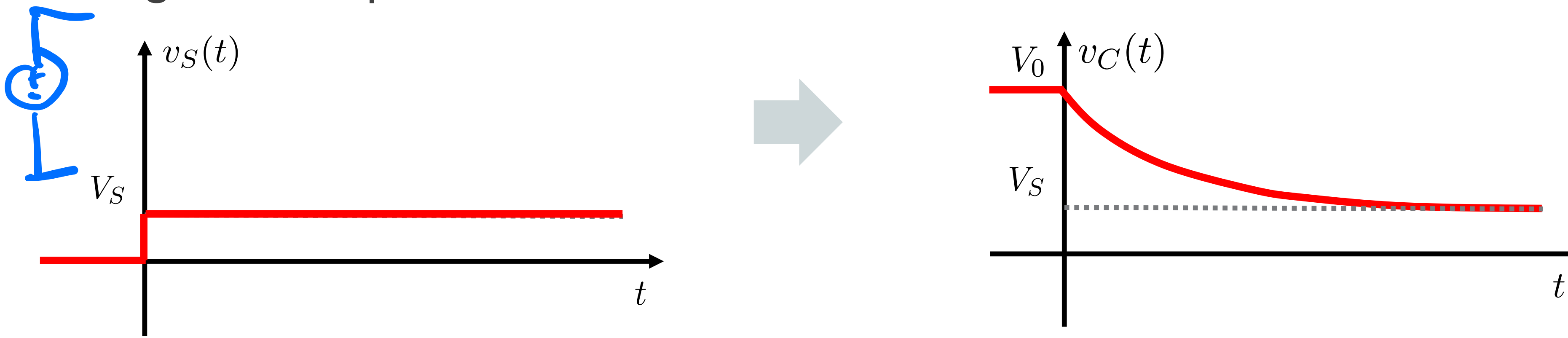
$$v_C(t) = (V_0 - V_S)e^{-\frac{t}{RC}} + V_S$$

The circuit as a system

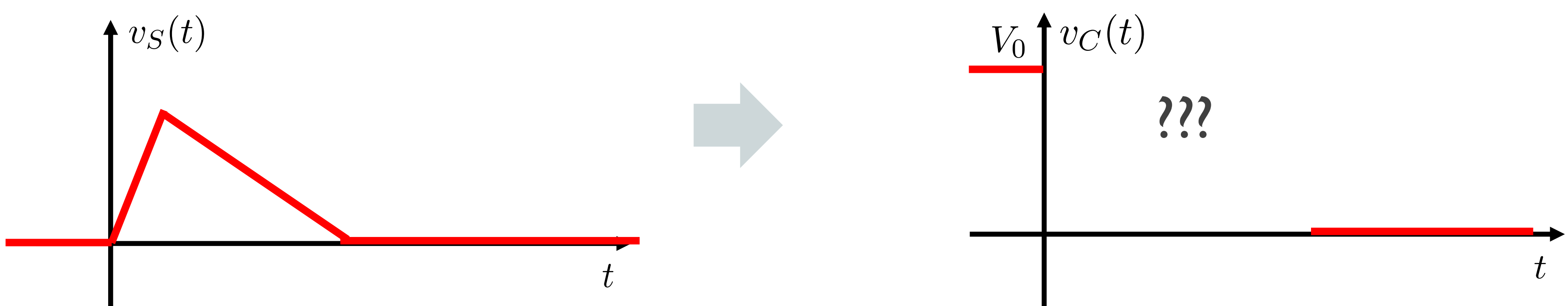


How to compute outputs for other inputs?

In the given example

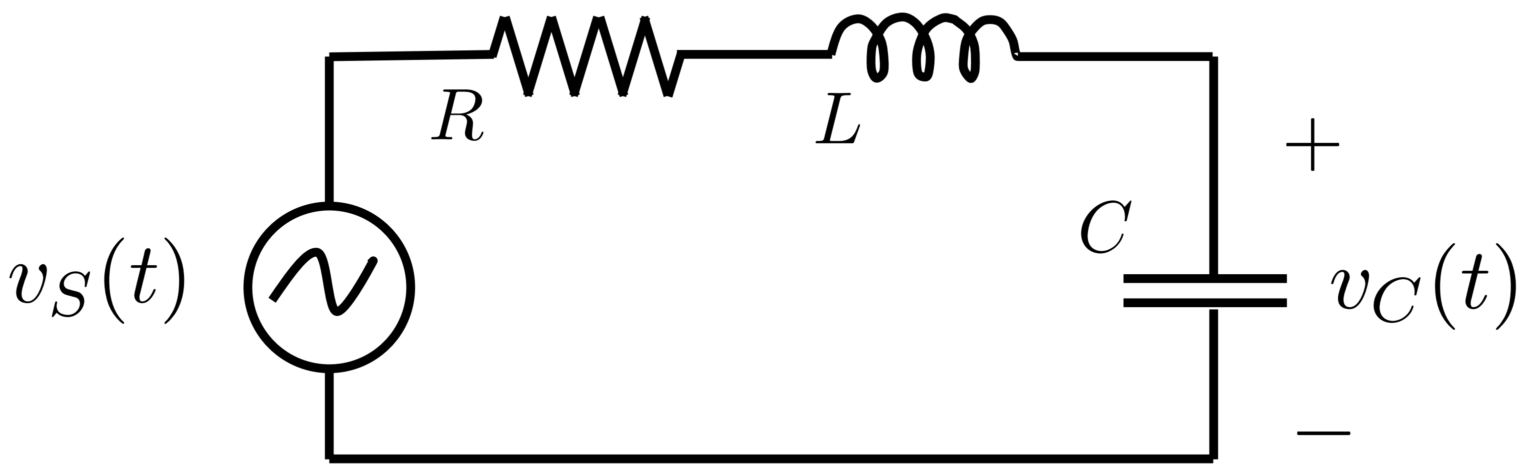


What about for a different input signal?



How to compute outputs for other circuits?

Example RLC circuit



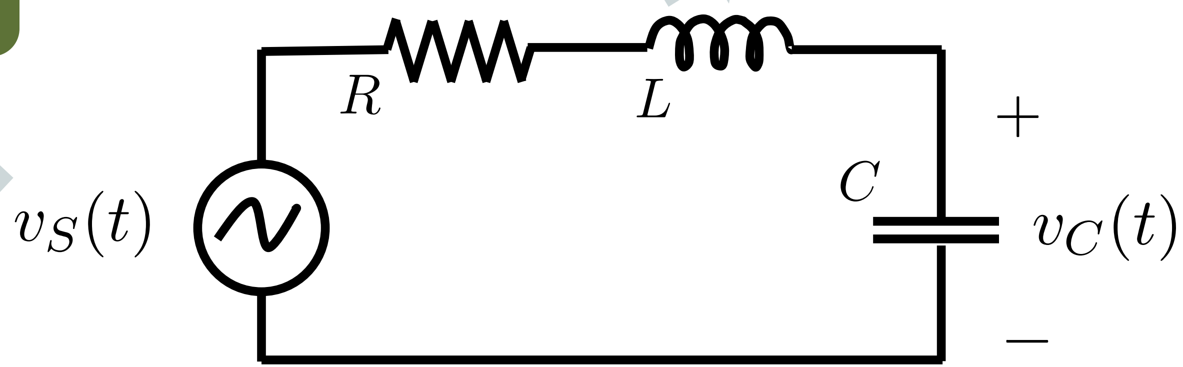
Use KVL to write the differential equation and solve ???

$$LC\frac{d^2 v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = v_S(t)$$

Approach in ECE 45

Recognize circuits as
linear time invariant systems

Build upon phasor
knowledge



Leverage important connections between sinusoids, linear time invariant systems and the Fourier transform

Complete solutions to differential equations, discrete-time signals, and the Laplace and Z transforms are deferred to ECE 101 ☹

Phasors and Fourier

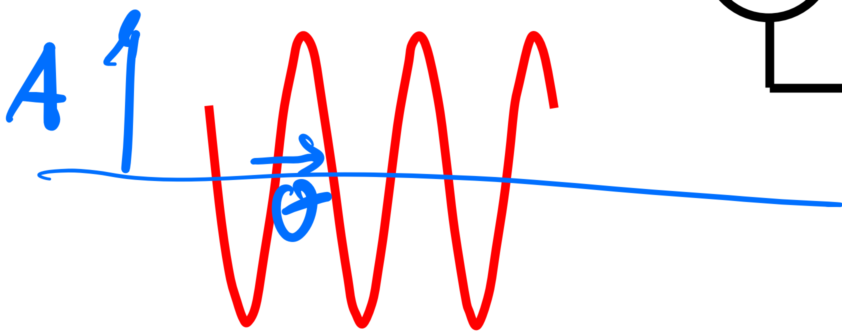
Learning objectives

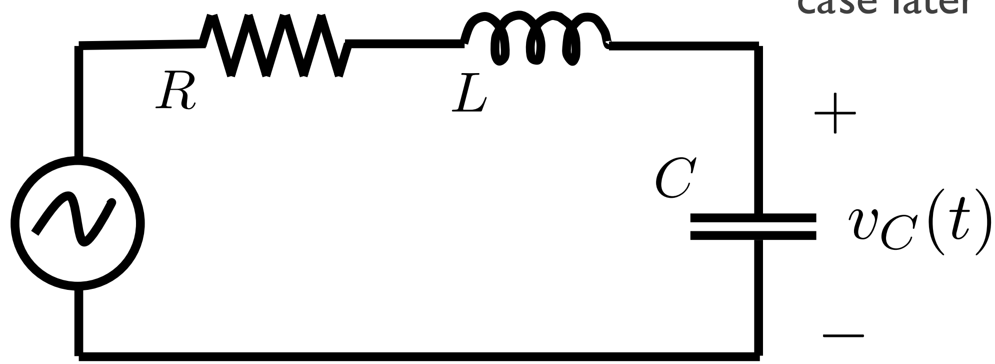
- Recall the use of phasors in AC circuit analysis
- Motivate the use of the Fourier transform in the context of phasors

Alternating current (AC) analysis of a circuit

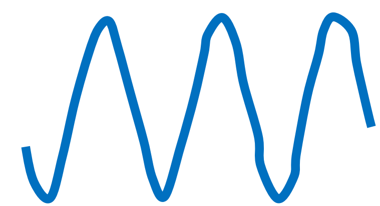
$2\pi f$

$$v(t) = A\cos(\omega t + \theta)$$





* We prove why this is the case later in the course



Solve for desired output voltage or current assuming the input is a sinusoid

Works because all the measured voltages and currents are also sinusoids*

AC analysis of a circuit: Treat the input as a phasor

- Recall that a sinusoid with $A>0$ can be written as

$$A\cos(\omega t+\theta) = \mathrm{Re}\left[Ae^{j\theta}e^{j\omega t}\right]$$

$e^{j(\omega t+\theta)}$

$= e^{j\omega t} e^{j\theta}$

phasor

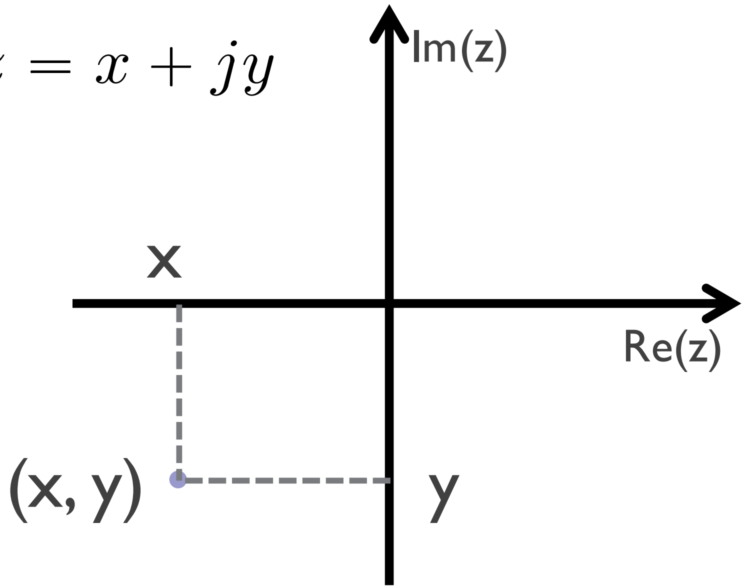
$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$$v(t) = A\cos(\omega t+\theta) \longleftrightarrow \mathbf{V} = Ae^{j\theta}$$

time domain	phasor domain
real function	complex number
depends on time	not a function of time

Complex numbers

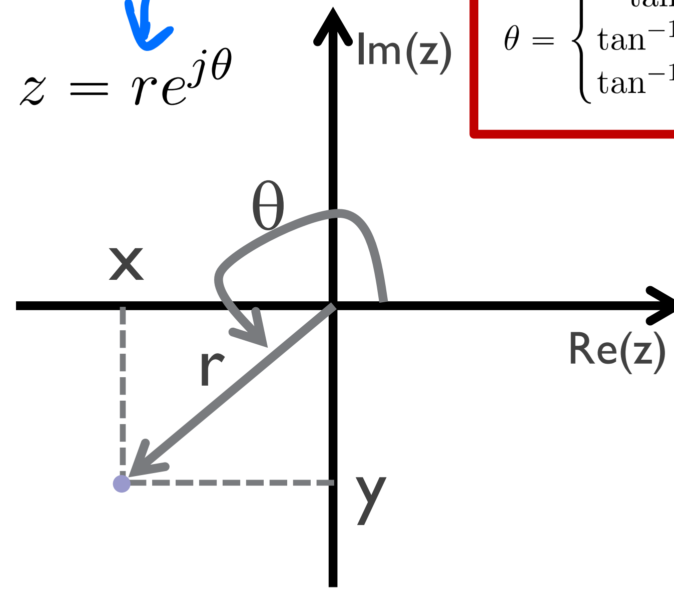
$$z = x + jy$$



Cartesian form

r>0

$$z = re^{j\theta}$$



Polar form

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & x > 0 \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & x < 0, y \geqslant 0 \\ \tan^{-1}\left(\frac{y}{x}\right) - \pi & x < 0, y < 0 \end{cases}$$

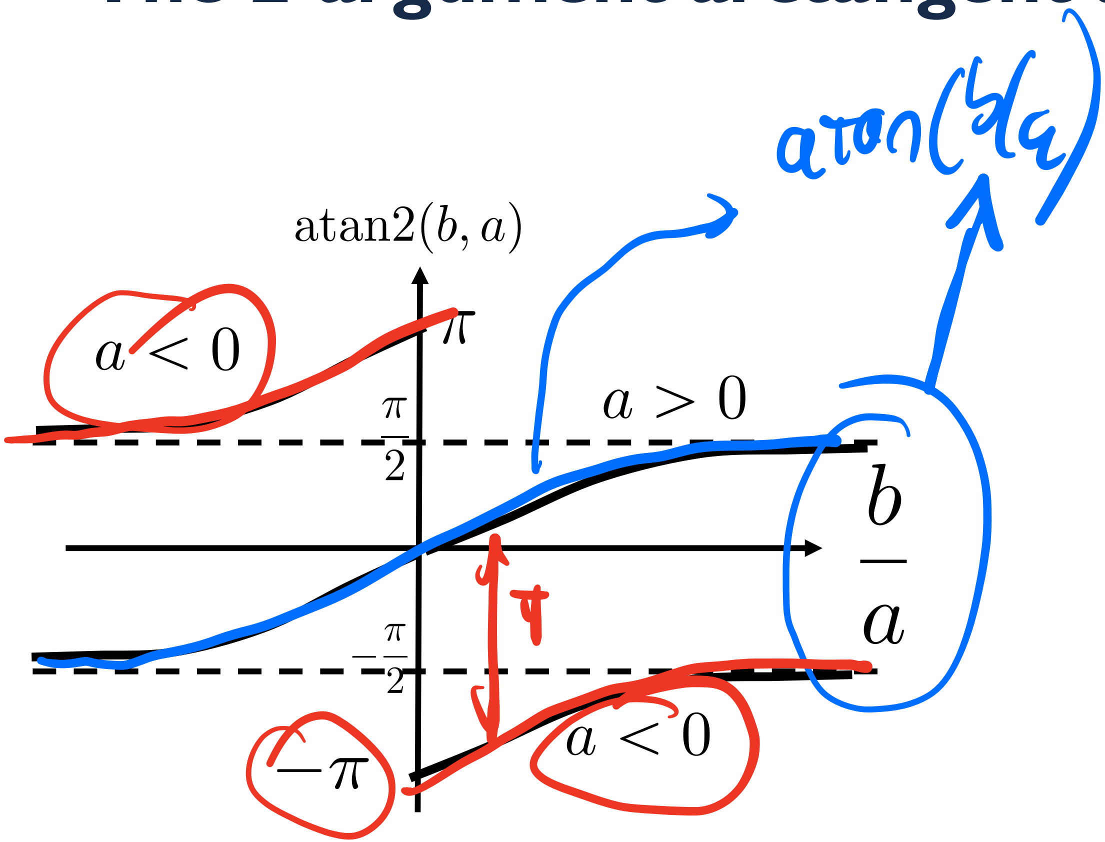
The quadrant matters!

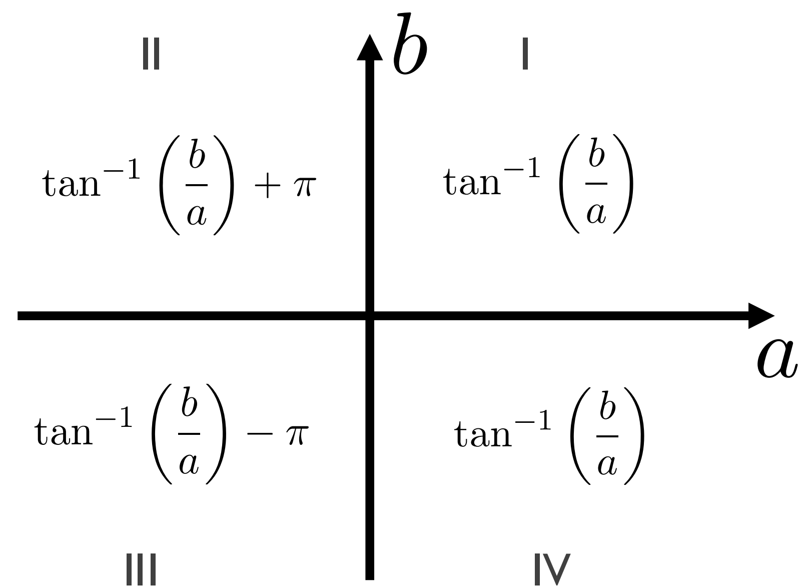
$\mathrm{atan2}(x, y)$

- Complex number is essentially a pair of independent real numbers z = (x, y)
  - Widely used in engineering and science
  - $j = \sqrt{-1}$ is the imaginary number (EE's use j because i is often current)

j

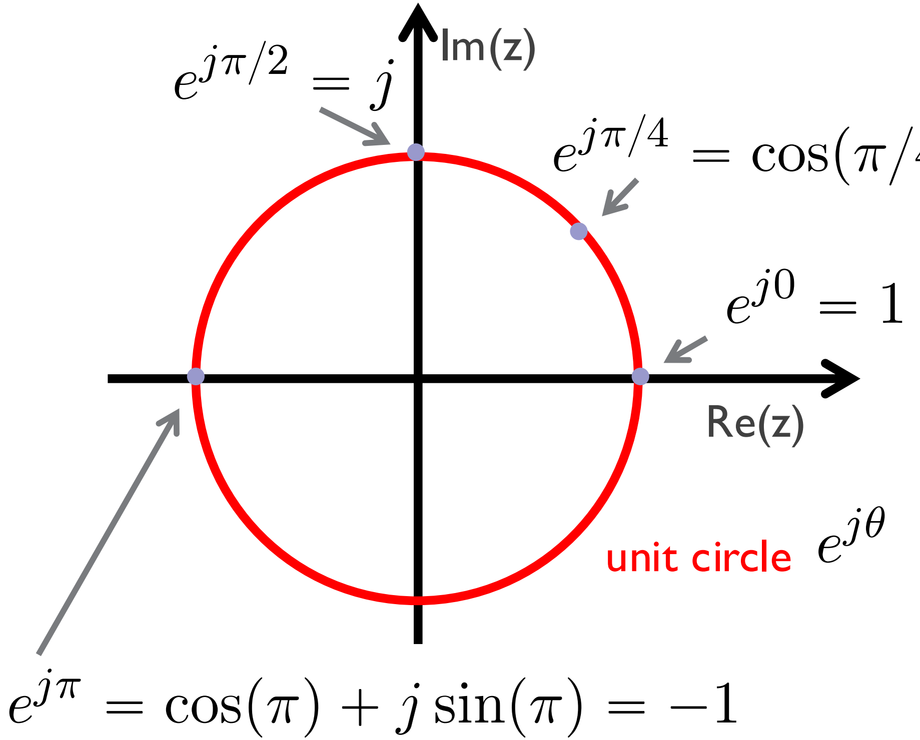
The 2-argument arctangent atan2(b,a)





Euler's formula

$$\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$$



Useful facts

$$e^{jx} = \cos x + j\sin x$$

$$e^{-jx} = \cos x - j\sin x$$

$$e^{jx} + e^{-jx} = 2\cos x$$

$$e^{jx} - e^{-jx} = 2j\sin x$$

$$\boxed{\begin{aligned} \cos(\theta) &= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \\ \sin(\theta) &= \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \end{aligned}}$$

Working with complex numbers

$x(t)$ $x^*(t) = a(t)$
$= a(t) + jb(t)$ $- jb(t)$

consider these two complex numbers

$$z_1 = a + jb = r_1 e^{j\theta_1}$$
$$z_2 = c + jd = r_2 e^{j\theta_2}$$

addition

$$z_1 + z_2 = (a + jb) + (c + jd)$$
$$= (a + c) + j(b + d)$$

real imag

multiplication

$$z_1 z_2 = (a + jb)(c + jd)$$
$$= (ac - bd) + j(bc + ad)$$
$$= r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

conjugate

$$z_1^* = a - jb$$
$$z_1 + z_1^* = a - jb + a + jb$$
$$= 2a$$
$$= 2\mathrm{Re}(z_1)$$
$$z_1 z_1^* = r_1 r_1 e^{j(\theta_1 - \theta_1)}$$
$$= r_1^2$$
$$= x^2 + y^2$$

magnitude squared

division

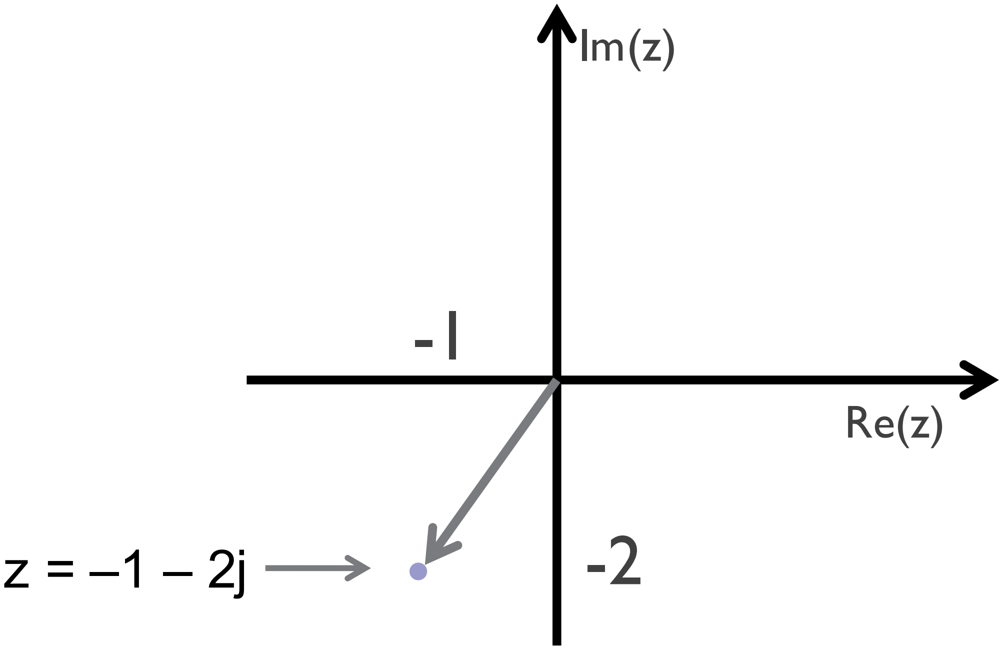
$$z_1/z_2 = (r_1/r_2)e^{j(\theta_1 - \theta_2)}$$

Example

- Consider z = –1 – 2j
  - Plot this complex number
  - Find its polar form

$$r = \sqrt{(-1)^2 + (-2)^2}$$
$$= \sqrt{5}$$

$$\theta = \tan^{-1}(-1/-2) - \pi$$
$$= -2.0344 \quad \text{radians}$$
$$= -0.6476\pi \quad \text{radians}$$



Example

- Let $z_1 = 1 - j2$

  $z_2 = 2 + j3$

- Compute

$$z_1 + z_2$$

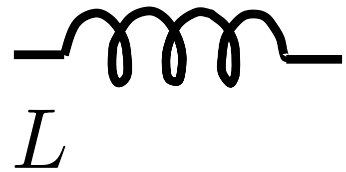
$$z_1 z_2$$

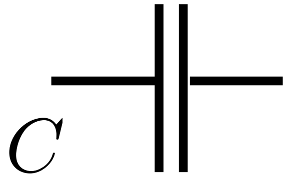
$$z_1 / z_2$$

$$z_1 z_1^*$$

$$z_1 - z_1^*$$

AC analysis of a circuit: Convert to complex impedances





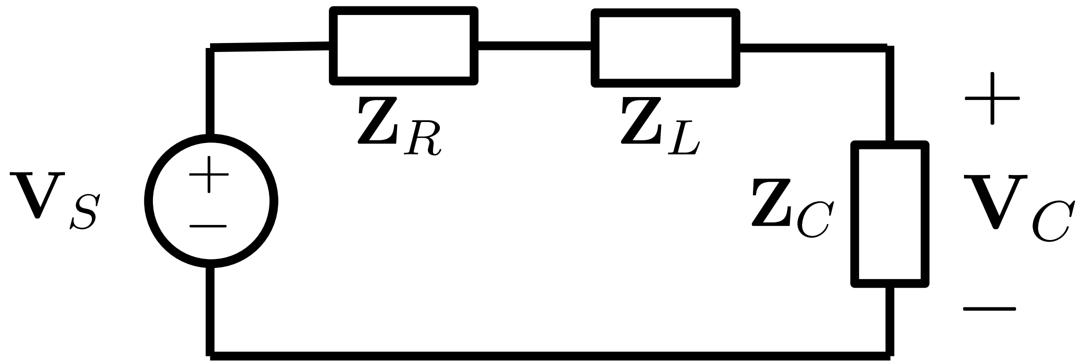
$$v(t) = Ri(t) \qquad v(t) = L\frac{di}{dt} \qquad i(t) = C\frac{dv}{dt}$$

$$\mathbf{V} = R\,\mathbf{I} \qquad \mathbf{V} = j\omega L\,\mathbf{I} \qquad \mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

$$\mathbf{Z}_R = R \qquad \mathbf{Z}_L = j\omega L \qquad \mathbf{Z}_C = \frac{1}{j\omega C}$$

freq

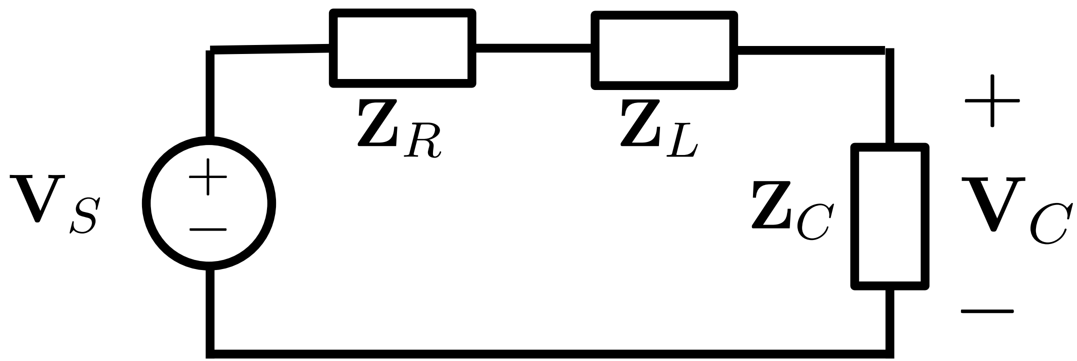
AC analysis of a circuit: Solve with impedances



$V_S = Ae^{j\theta}$

$$V_C = \frac{Z_C}{Z_R + Z_L + Z_C} V_S$$

AC analysis of a circuit: Solve with impedances



voltage divider

$Ae^{j\theta}$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C}\mathbf{V}_S$$

$$= \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}\mathbf{V}_S$$

$$\cdot\frac{j\omega C}{j\omega C}$$

Frequency Response

$$= \boxed{\frac{1}{1 - \omega^2 LC + j\omega RC}}\mathbf{V}_S$$

AC analysis of a circuit: Convert to polar form

$$\mathbf{Z}_E = \frac{1}{1 - \omega^2 LC + jRC\omega}$$

$$\mathbf{V}_C = \mathbf{Z}_E \mathbf{V}_S$$

AC analysis of a circuit: Convert to polar form

$$\mathbf{Z}_E = \frac{1}{1 - \omega^2 LC + jRC\omega} \qquad \mathbf{V}_C = \mathbf{Z}_E \mathbf{V}_S$$

A^2 $\qquad$ 2

$$|\mathbf{Z}_E| = \sqrt{\frac{1}{(1 - \omega^2 LC)^2 + (RC\omega)^2}}$$

$$\frac{Ae^{j\theta_1}}{Be^{j\theta_2}} = \frac{A}{B} e^{j(\theta_1 - \theta_2)}$$

$$\phi_E = -\text{atan2}(1 - \omega^2 LC, \omega RC)$$

why?

AC analysis of a circuit: Convert back to the time domain

$$v_S(t) = A\cos(\omega t + \theta)$$

$$v_C(t) = \mathrm{Re}\left[|\mathbf{Z}_E|e^{j\phi_E}\overbrace{Ae^{j\theta}}^{\text{phasor}}e^{j\omega t}\right]$$

AC analysis of a circuit: Convert back to the time domain

$$v_S(t) = A\cos(\omega t + \theta)$$

$$v_C(t) = \mathrm{Re}\left[|\mathbf{Z}_E|e^{j\phi_E}Ae^{j\theta}e^{j\omega}\right]$$

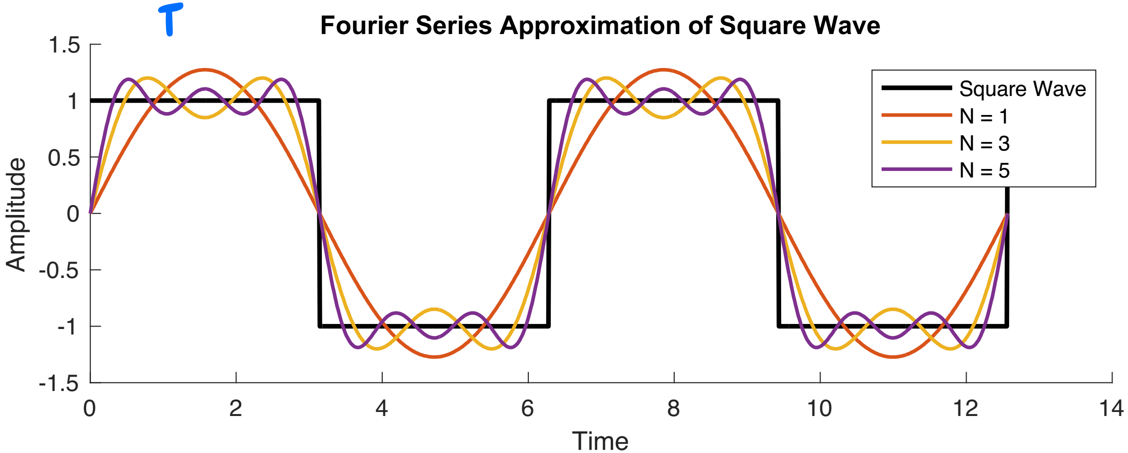
$$= A|\mathbf{Z}_E|\cos(\omega t + \theta + \phi_E)$$

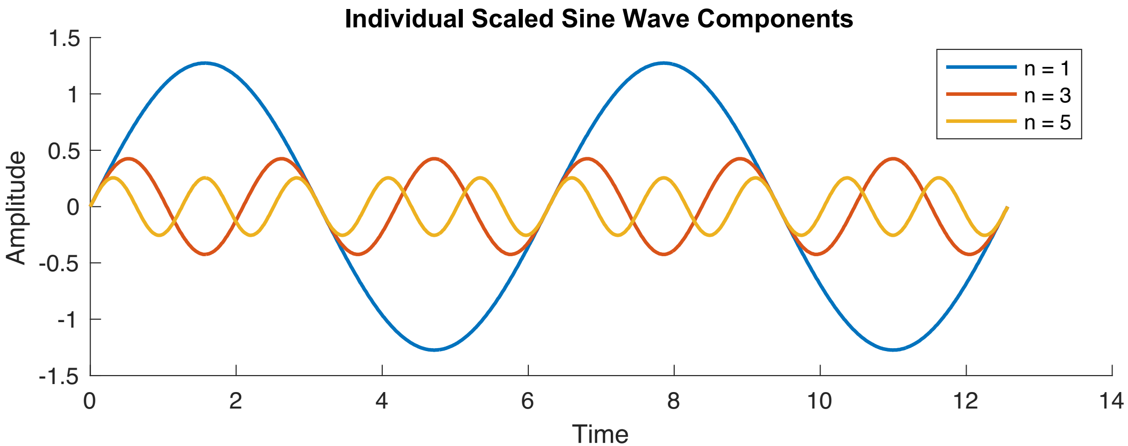
$$= A\sqrt{\frac{1}{(1-\omega^2 LC)^2 + (\omega RC)^2}}\cos(\omega t + \theta + \mathrm{atan2}(1-\omega^2 LC, \omega RC))$$

Amplitude and phase also depend on the frequency (but not time)

Why do we care about sinusoids? Part I

$\frac{2\pi t}{T}$



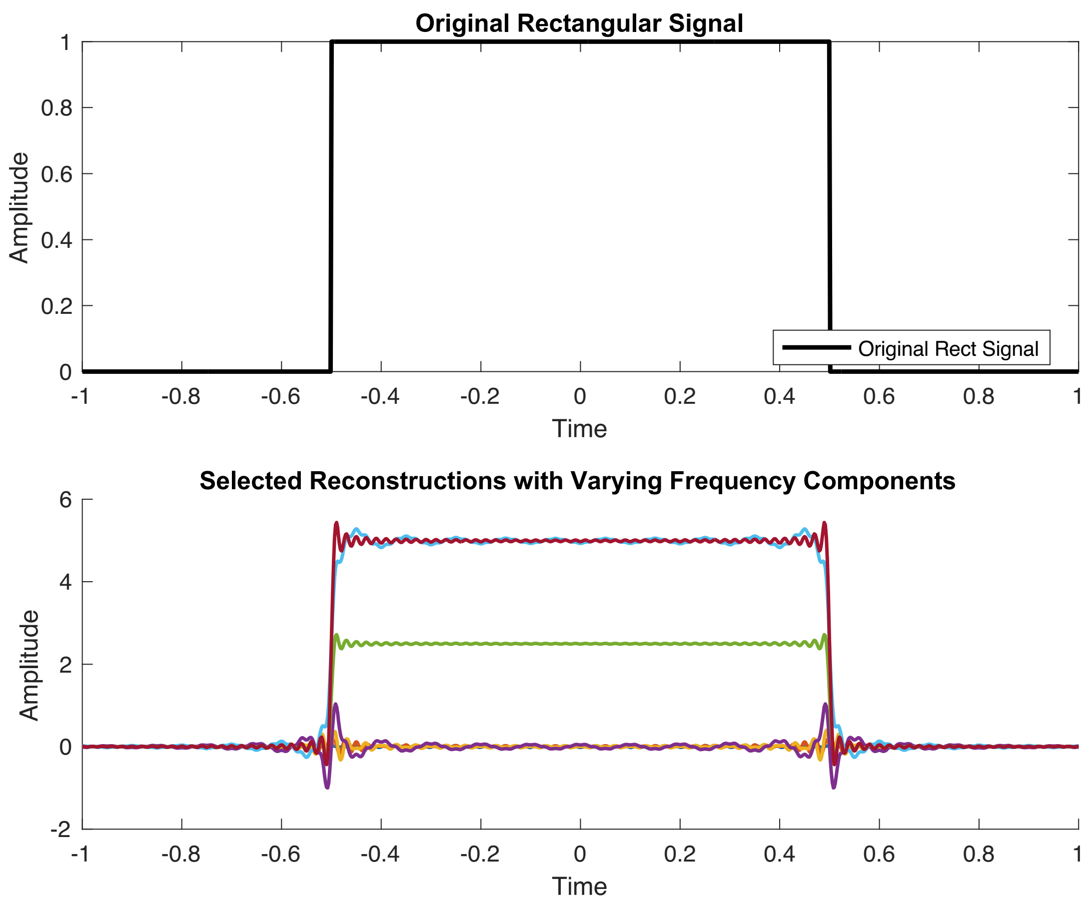


Fourier Series

It is possible to reconstruct periodic signals with sums of sinusoids!

UC San Diego

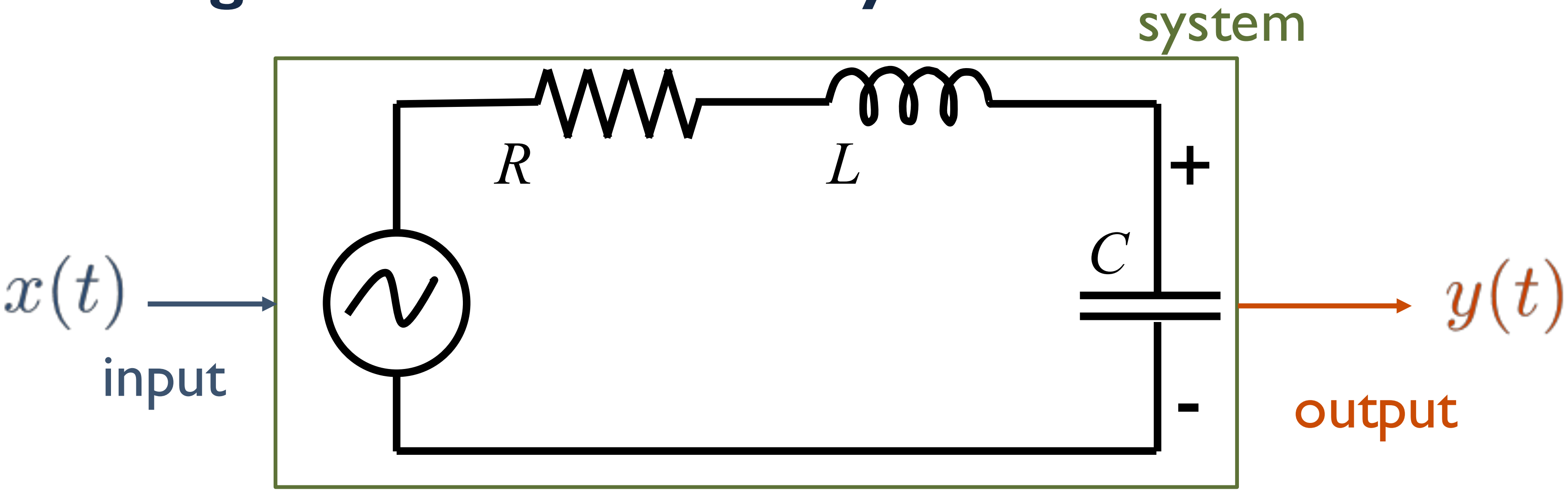
Why do we care about sinusoids? Part II

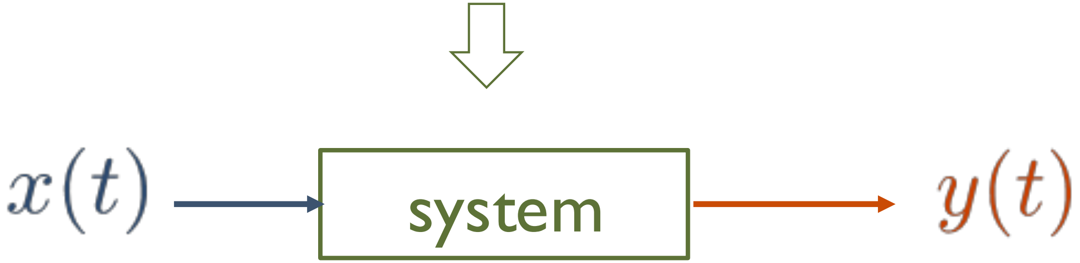


Fourier Transform

It is possible to reconstruct non-periodic signals using integrals of sinusoids!

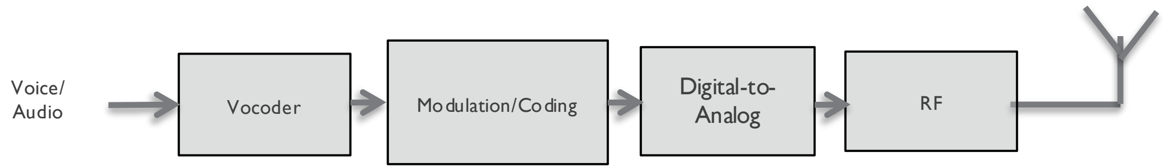
Thinking about circuits as a system

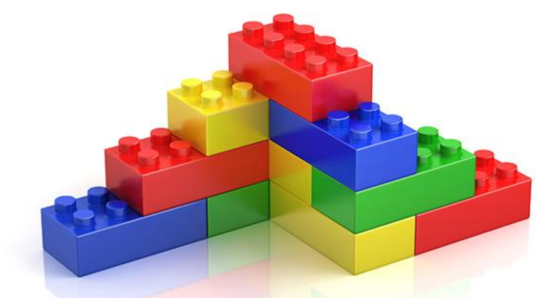


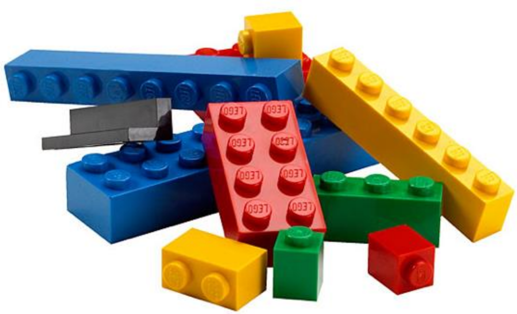


A mathematical description of how the input is transformed into the output

What is "systems thinking"?







A complicated object decomposed into functional blocks that interact

Why do we care about sinusoids? Part III

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

Special systems that satisfy the linear and time invariant (LTI) property like circuits at rest (zero initial conditions) are fully characterized by their frequency response!

$$e^{j\omega t} \longrightarrow \boxed{\text{system}} \longrightarrow \boxed{H(j\omega)}e^{j\omega t}$$

complex sinusoid

frequency response

Why do we care about sinusoids? Part IV

Fourier

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

Component of $x(t)$ represented by $e^{j\omega t}$

$$X(j\omega)e^{j\omega t} \longrightarrow \boxed{\text{system}} \longrightarrow \underbrace{X(j\omega)H(j\omega)}_{Y(j\omega)}e^{j\omega t}$$

Component of $y(t)$ represented by $e^{j\omega t}$

Sinusoidal outputs for LTI systems are easy to compute by using the Fourier Series or the Fourier transform

Connections to back to ECE 45

Lectures 2 - 4 working with signals

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Lectures 5 - 7 LTI systems in the time domain

Lectures 11-12 LTI systems in the frequency domain

$$X(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega)$$

Lectures 8 - 10 Fourier series

Lectures 13 - 17 Fourier transform

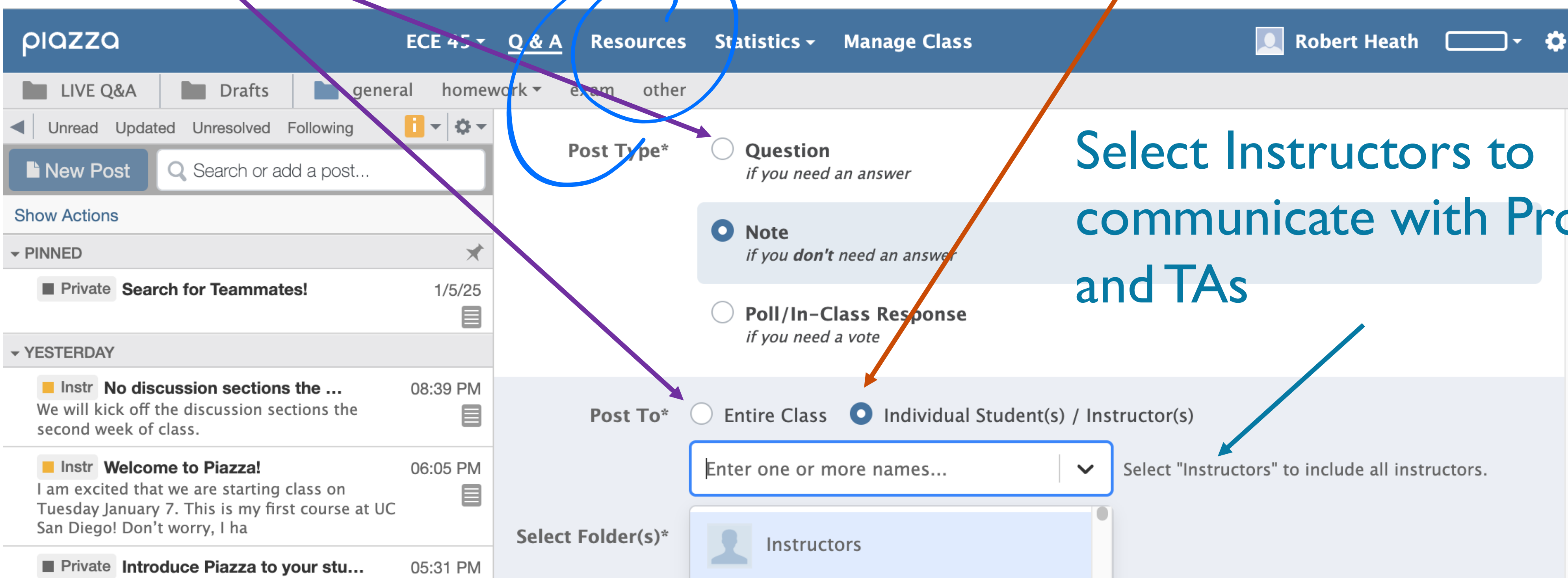
Fourier

Best way to send messages

Preferred for course questions

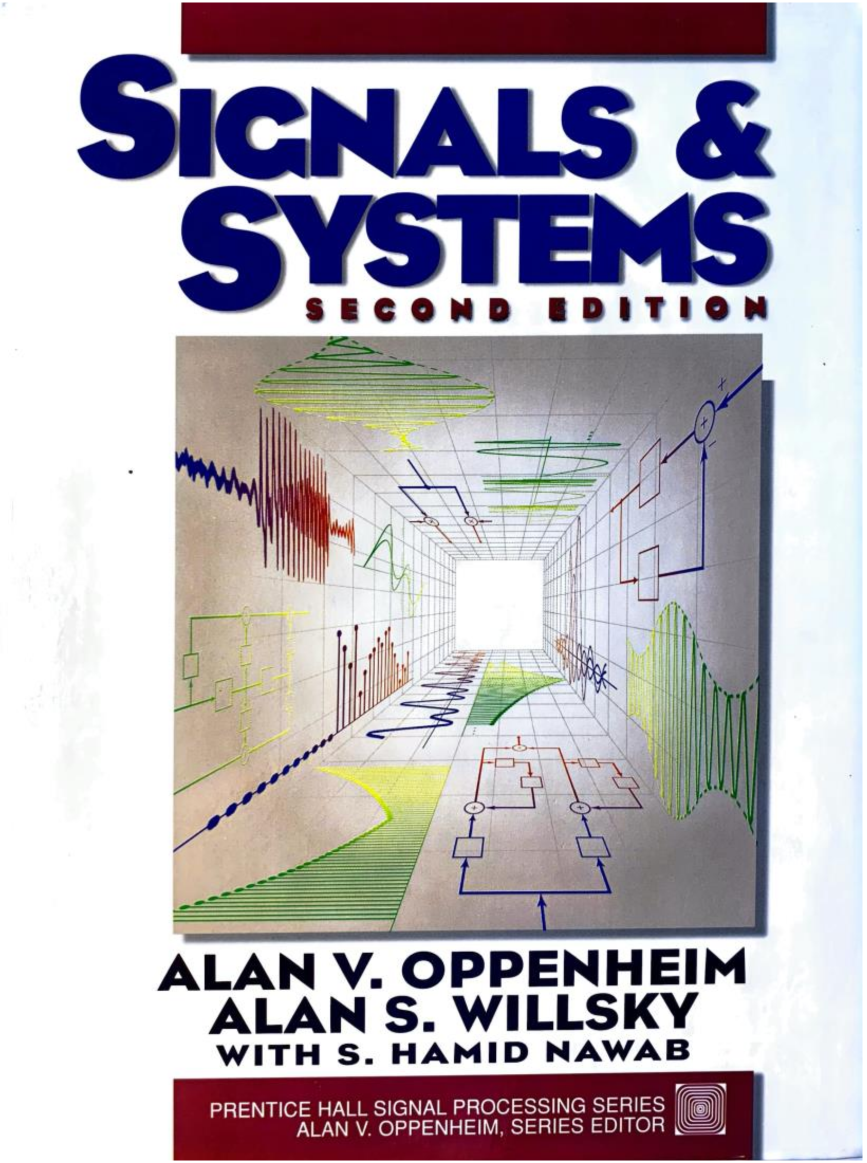
Used for specific communication, preferred over email

Select Instructors to communicate with Prof and TAs



Materials for the course

- Textbook
  - Signals & Systems, 2nd edition
  - Classic book in ECE, worth buying
  - Many examples and problems
- Lecture notes
  - Developed based on the book
  - May have additional examples
- Supplemental materials
  - Material not covered in the book
  - Background material



How this class will work

- Lectures
  - Slides with annotations and handwritten examples
  - Advice: Take notes and be an active thinker
- Discussion sections
  - Practice problem solving
- Piazza will be used for course materials and discussions
  - Files will be uploaded in the resources folder
  - Use relevant tags when posting, e.g. hw1 for homework 1 discussions

	Date	Theme	Topic	Readings	Out	In
1	1/7	Signals	Signals, systems, circuits and phasors	1.1	HW1	
2	1/9	Signals	Signal transformations, periodic, even and odd, rectangle, step	1.2	HW2	HW1
3	1/14	Signals	Exponential, sinusoids, complex exponentials, phasors	1.3		
4	1/16	Signals	Dirac delta, Kronecker delta, Sha function	1.4	HW3	HW2
5	1/21	LTI in time	Linear and time-invariant systems	2.1		
6	1/23	LTI in time	Convolution, convolution with a sinusoid, connection to phasors	2.2	HW4	HW3
7	1/28	LTI in time	Convolution properties	2.3		
8	1/30	Fourier series	Fourier series	3.1 - 3.3	HW5	HW4
	2/4		**Midterm 1**			
9	2/6	Fourier series	Fourier series convergence and properties	3.4	HW6	HW5
10	2/11	Fourier series	Fourier series properties	3.5		
11	2/13	LTI in frequency	Frequency response of LTI systems	3.9	HW7	HW6
12	2/18	LTI in frequency	Filters, bode plots	3.10, 6.2.3		
13	2/20	Fourier transform	Fourier transform	4.1-4.2	HW8	HW7
	2/25		**Midterm 2**			
14	2/27	Fourier transform	Fourier transform properties	4.3	HW9	HW8
15	3/4	Fourier transform	Rectangle and sinc functions	4.3		
16	3/6	Fourier transform	Convolution property	4.4	HW10	
17	3/11	Fourier transform	Multiplication property	4.5		
18	3/13	Sampling	Sampling theorem	7.1		HW10
	3/19		**Final exam Tuesday 3-6pm**			

Note: HW will normally be due at 11:59pm on Thursdays (not this week)

Course Information | Staff | Resources

Edit Resource Sections

Chapter Notes

Manually sort using ≡ Sort on: - -

Chapter Notes	Actions
README_note_about_these_slides.pdf	Edit · Post a note · Update file · Delete
Chapter_1_ECE_45.pdf	Edit · Post a note · Update file · Delete
Chapter_2_ECE_45.pdf	Edit · Post a note · Update file · Delete
Chapter_3_ECE_45.pdf	Edit · Post a note · Update file · Delete
Chapter_4_ECE_45.pdf	Edit · Post a note · Update file · Delete
Chapter_7_ECE_45.pdf	Edit · Post a note · Update file · Delete

Add Links · Add Files

Lecture Notes

Manually sort using ≡ Sort on: - -

How learning will be assessed

Homework

Problems related to material covered the previous week, graded on effort + corrections

Midterm exams

2 midterms, during class, each covering topics up to the midterm

Final exam

A comprehensive exam, held during final exam time

Gradescope

- Used for all homeworks and possibly for in-class assessments
  - Check out general instructions at http://www.gradescope.com
  - Cheat-sheet for scanning your homework is here
- Gradescope is linked to the course through Canvas
- You will upload your assignments at home and scan the areas relevant for the problems (required to receive credit)

Variable-length submissions

In most cases, your instructor will allow you to turn in a variable-length submission for written assignments. This means your answers may span across any number of pages and responses may not be located in the same place on each page. If your instructor will accept a variable-length submission, when you open or start the assignment, you'll have two options for submitting your work:

- **Individual images**, one or more per question
- **One PDF** that contains the whole submission; Gradescope will ask you to mark where each answer is

From https://help.gradescope.com/article/ccbpppziu9-student-submit-work

We use MATLAB, but be aware of Python

- Why Python?
  - It is free to you and your future employer
  - Fast prototyping language, similar to MATLAB, similar syntax
  - Used extensively in machine learning and data sciences
  - Many packages are available for scientific computing including manipulation of matrices (numpy) and plotting data (matlabplotlib), machine learning (scikit-learn)
  - Can be deployed on the cloud such as Amazon AWS
- Tons of free online tutorials: https://www.eecis.udel.edu/~boncelet/ipython.html
- MATLAB will be used but some examples in both languages

Tips for success

Seriously…

- Review your book and notes from ECE 35 and calculus book
  - You will need to perform integrals and derivatives
  - You will need trigonometric identities
  - You will need to master complex numbers and complex exponentials
- Don't fall behind!
  - Keep up with the material every week, read the book each week
  - Attend class and discussion sections
  - Get help: TA, office hours, your peers on Piazza, virtual study group

Attempt the homework problems on your own and get as far as you can using the book and course materials. This is critical to developing the problem-solving and self-teaching skills you need for this class and beyond.

Useful reference material

Trigonometric identities

Euler's theorem: $e^{\pm ju} = \cos u \pm j \sin u$

$\cos u = \frac{1}{2}(e^{ju} + e^{-ju})$

$\sin u = (e^{ju} - e^{-ju})/2j$

$\sin^2 u + \cos^2 u = 1$

$\cos^2 u - \sin^2 u = \cos 2u$

$2 \sin u \cos u = \sin 2u$

$\cos^2 u = \frac{1}{2}(1 + \cos 2u)$

$\sin^2 u = \frac{1}{2}(1 - \cos 2u)$

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \frac{1}{2}[\sin(u - v) + \sin(u + v)]$

From Signals and Systems: Continuous and Discrete by Ziemer, Tranter, and Fannin

Indefinite integrals

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin^2(ax)\,dx = x/2 - \sin(2ax)/4a$$

$$\int \cos^2(ax)\,dx = x/2 + \sin(2ax)/4a$$

$$\int x\sin(ax)\,dx = [\sin(ax) - ax\cos(ax)]/a^2$$

$$\int x\cos(ax)\,dx = [\cos(ax) + ax\sin(ax)]/a^2$$

$$\int x^m \sin(x)\,dx = -x^m\cos(x) + m\int x^{m-1}\cos(x)\,dx$$

$$\int x^m \cos(x)\,dx = x^m\sin(x) - m\int x^{m-1}\sin(x)\,dx$$

$$\int \sin(ax)\sin(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}, \qquad a^2 \neq b^2$$

$$\int \sin(ax)\cos(bx)\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right], \qquad a^2 \neq b^2$$

$$\int \cos(ax)\cos(bx)\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}, \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = e^{ax}/a$$

$$\int x^m e^{ax}\,dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1}e^{ax}\,dx$$

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin(bx) - b\cos(bx)]$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos(bx) + b\sin(bx)]$$

From Signals and Systems: Continuous and Discrete by Ziemer, Tranter, and Fannin

Definite integrals

$$\int_0^\infty \frac{a\,dx}{a^2 + x^2} = \pi/2, \qquad a > 0$$

$$\int_0^{\pi/2} \sin^n(x)\,dx = \int_0^{\pi/2} \cos^n(x)\,dx = \begin{cases} \dfrac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots(n)}\,\dfrac{\pi}{2}, & n \text{ even, } n \text{ an integer} \\ \dfrac{2\cdot 4\cdot 6\cdots(n-1)}{1\cdot 3\cdot 5\cdots(n)}, & n \text{ odd} \end{cases}$$

$$\int_0^\pi \sin^2(nx)\,dx = \int_0^\pi \cos^2(mx)\,dx = \pi/2, \qquad n \text{ an integer}$$

$$\int_0^\pi \sin(mx)\sin(nx)\,dx = \int_0^\pi \cos(mx)\cos(nx)\,dx = 0, \quad m \neq n,\ m \text{ and } n \text{ integer}$$

$$\int_0^\pi \sin(mx)\cos(nx)\,dx = \begin{cases} 2m/(m^2 - n^2), & m + n \text{ odd} \\ 0, & m + n \text{ even} \end{cases}$$

$$\int_0^\infty \frac{\sin(ax)}{x}\,dx = \frac{\pi}{2}, \qquad a > 0$$

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty e^{-a^2x^2}\,dx = \sqrt{\pi}/2a, \qquad a > 0$$

$$\int_0^\infty x^n e^{-ax}\,dx = n!/a^{n+1}, \qquad n \text{ an integer and } a > 0$$

$$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax}\cos(bx)\,dx = \frac{a}{a^2 + b^2}, \qquad a > 0$$

$$\int_0^\infty e^{-ax}\sin(bx)\,dx = \frac{b}{a^2 + b^2}, \qquad a > 0$$

$$\int_0^\infty e^{-a^2x^2}\cos(bx)\,dx = \frac{\sqrt{\pi}}{2a}\,e^{-b^2/4a^2}$$

From Signals and Systems: Continuous and Discrete by Ziemer, Tranter, and Fannin