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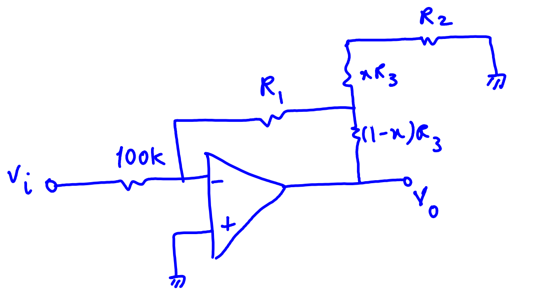
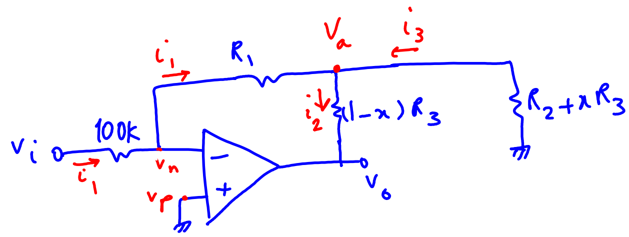
ECE 65 Fall 2018 Final Exam Solutions

Problem 1.

Part a)





ideal op-amp with negative feedback:

$i_n = i_p = 0$

$v_n = v_p$

$i_1 = \frac{v_i}{100k}$, $v_a = -\frac{R_1}{100k} v_i$

$i_3 = \frac{-v_a}{R_2 + xR_3}$; $x = 0\% - 100\%$

KCL : $i_1 + i_3 = i_2$

$i_2 = \frac{v_i}{100k} + \frac{+R_1/(R_2+xR_3)}{100k} v_i$

$v_o = -i_2 \times (1-x) R_3 + v_a$

$$\Rightarrow v_o = \frac{-(1-x)R_3}{100k}\left(1 + \frac{R_1}{R_2 + xR_3}\right) v_i - \frac{R_1}{100k} v_i$$

for $x=1 \longrightarrow \frac{v_o}{v_i} = -\frac{R_1}{100k} = -1\ V/V \longrightarrow \boxed{R_1 = 100k\,\Omega}$

for $x=0 \longrightarrow v_o = \frac{-R_3}{100k}\left(1 + \frac{R_1}{R_2}\right) v_i - \frac{R_1}{100k} v_i = -1\left(1 + \frac{100k}{R_2}\right) v_i - v_i$

$$\frac{v_o}{v_i} = -2 - \frac{100k}{R_2} = -100\ V/V$$

$$\frac{100k}{R_2} = 98 \longrightarrow \boxed{R_2 = 1.02\,k\Omega}$$

Part b) $x = 0.5 \longrightarrow \boxed{\frac{v_o}{v_i} = -2.48\ V/V}$

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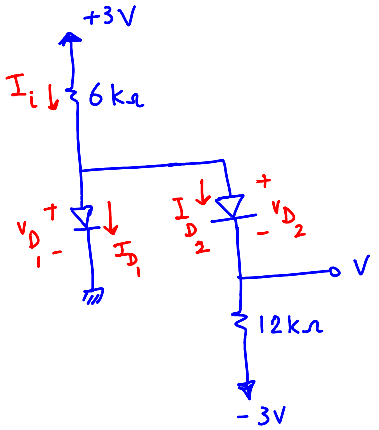
Problem 2.

Part a)



Case 1: Assume D_1 off, D_2 off

$$\boxed{\begin{array}{ll} V_{D_1} < V_{D_0}, & V_{D_2} < V_{D_0} \\ I_{D_1} = 0, & I_{D_2} = 0 \end{array}}$$

$I_i = I_{D_1} + I_{D_2} = 0$

$V_{D_1} = 3V - 6k\Omega \times I_i = 3V > V_{D_0}$

$\Rightarrow$ The assumption was wrong

Case 2: Assume D_1 ON, D_2 off

$$\boxed{\begin{array}{ll} V_{D_1} = V_{D_0}, & V_{D_2} < V_{D_0} \\ I_{D_1} \geq 0, & I_{D_2} = 0 \end{array}}$$

KVL: $V_{D_1} = V_{D_2} + 12k \times I_{D_2} - 3V$

$\rightarrow 0.7V = V_{D_2} + 0 - 3V$

$\rightarrow V_{D_2} = 3.7V > V_{D_0}$

$\Rightarrow$ The assumption was wrong

Case 3: Assume D_1 off, D_2 ON

$$\boxed{\begin{array}{ll} V_{D_1} < V_{D_0}, & V_{D_2} = V_{D_0} \\ I_{D_1} = 0, & I_{D_2} \geq 0 \end{array}}$$

$V_{D_2} = 0.7$, $I_{D_2} = I_i$

KVL: $3V = 6k \times I_{D_2} + V_{D_2} + 12k \times I_{D_2} - 3V$

$$I_{D_2} = \frac{6 - 0.7V}{18k} = 0.29\,mA$$

$V_{D_1} = V_{D_2} + 12k\, I_{D_2} - 3V = 1.23V > V_{D_0}$

our assumption was wrong

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Problem 2.

Case 4: Assume D_1 ON , D_2 ON

$$\boxed{\begin{matrix} V_{D_1} = V_{D_0} , & V_{D_2} = V_{D_0} \\ I_{D_1} \geq 0 , & I_{D_2} \geq 0 \end{matrix}}$$

$I_i = I_{D_1} + I_{D_2}$

$I_i = \dfrac{3V - V_{D_1}}{6k} = \dfrac{3 - 0.7}{6k} \simeq 0.38\,mA$

KVL: $V_{D_1} = V_{D_2} + 12k \times I_{D_2} - 3V \longrightarrow I_{D_2} = \dfrac{3V}{12k} = 0.25\,mA \geq 0$

$I_{D_1} = I_i - I_{D_2} = 0.38\,mA - 0.25\,mA = 0.13\,mA \geq 0$

$I_{D_1} \geq 0$ and $I_{D_2} \geq 0 \longrightarrow$ our assumption was correct.

$\boxed{I = I_{D_1} = 0.13\,mA}$, $V = 12k\Omega \times I_{D_2} - 3V = 0V \longrightarrow \boxed{V = 0V}$

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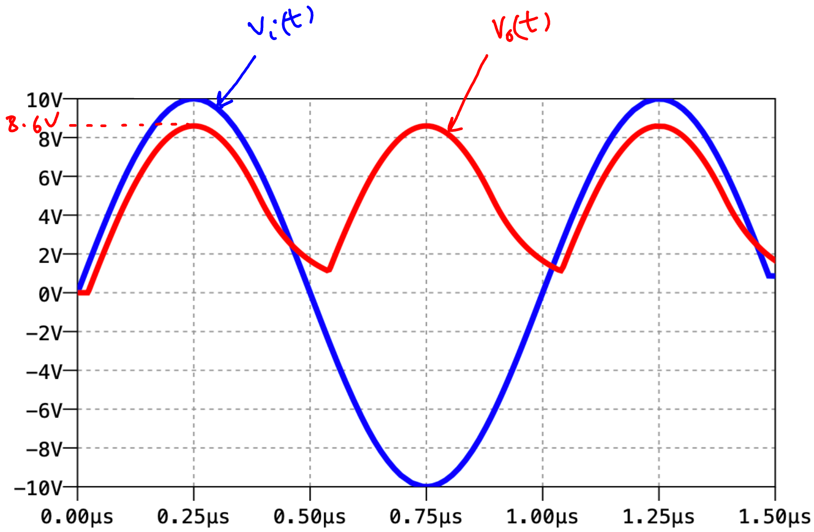
Problem 2. Part b)

ii) $R = 100\Omega$



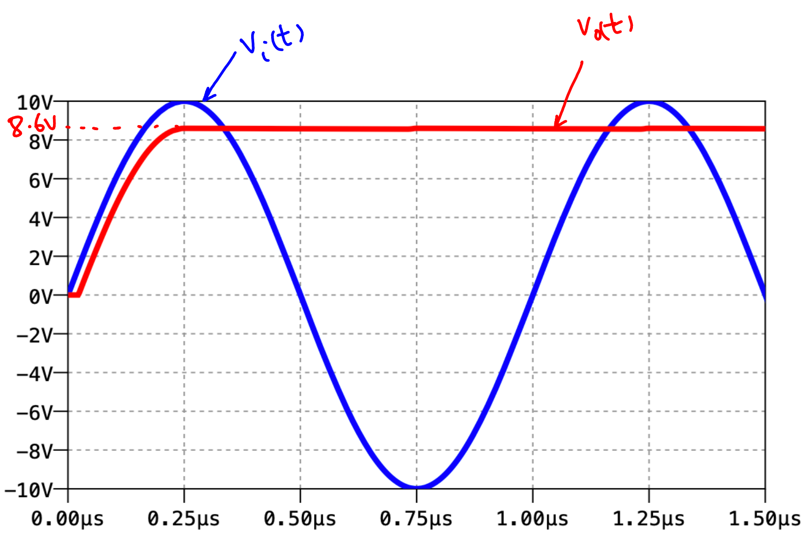
$RC = 100\Omega \times 1nF = 10^{-7}$ s

$T = 10^{-6}$

$RC = 0.1\,T$

$V_{o\,peak} = 10 - 2V_{D_0} = 8.6V$

i) $R = 100k\Omega$



$RC = 100k \times 1nF = 10^{-4}$

$RC = 100\,T$

$V_{o\,peak} = 10 - 2V_{D_0} = 8.6V$

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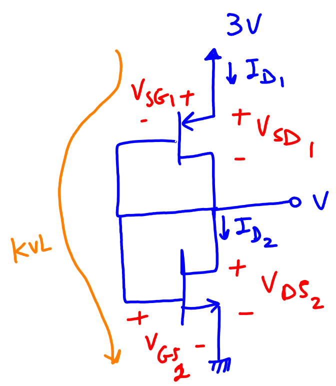
Problem 3.

gate and drain are connected to eachother in both MOSFETs, so they both will operate in saturation.



$$I_{D_1} = I_{D_2} = I$$

$$I_{D_1} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p\left(V_{SG} - |V_{tp}|\right)^2$$

$$I_{D_2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(V_{GS} - V_{tn}\right)^2$$

$$I_{D_1} = I_{D_2} \longrightarrow \mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{ov_p})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{ov_n})^2$$

$$(V_{ov_p})^2 = 3\,(V_{ov_n})^2$$

KVL : $3V = V_{SG_1} + V_{GS_2} = V_{SG_1} - |V_{tp}| + V_{GS_2} - V_{tn} + (|V_{tp}| + V_{tn})$

$$= V_{ov_p} + V_{ov_n} + 2|V_t|$$

$$\begin{cases} V_{ovp} + V_{on} = 2V \\ V_{op} = \sqrt{3}\, V_{ovn} \end{cases} \longrightarrow (\sqrt{3}+1)\, V_{ovn} = 2 \longrightarrow V_{ovn} = 0.732\text{ V}$$

$$V = V_{GS_2} = V_{ov_n} + V_{tn} = 0.732 + 0.5 \longrightarrow \boxed{V = 1.232\text{ V}}$$

$$I = I_{D_2} = \frac{1}{2} \times 270 \times \frac{3}{1} \times (0.732)^2 = 217\ \mu A \longrightarrow \boxed{I = 217\ \mu A}$$

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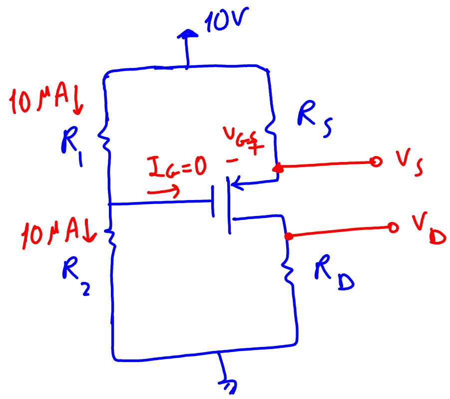
Problem 4.



PMOS is in saturation : $V_{SD} \geq V_{OV}$

at the edge of saturation: $V_{SD} = V_{OV}$

in this problem: $V_{SD} = V_{ov} + 1$

$$R_D = \frac{V_D}{I_D} = \frac{3V}{1mA} \longrightarrow \boxed{R_D = 3k\Omega}$$

$$I_D = \frac{1}{2} k_p V_{ov}^2 \longrightarrow 1mA = \frac{1}{2} \times 0.5\ mA/V^2 \times V_{ov}^2 \longrightarrow V_{ov} = 2V$$

$$V_{SD} = V_{ov} + 1 = 3V$$

$$V_{SD} = V_S - V_D \longrightarrow V_S = 3V + V_D \longrightarrow \boxed{V_S = 6V}$$

$$R_S = \frac{10V - 6V}{1mA} \longrightarrow \boxed{R_S = 4\ k\Omega}$$

$$V_{ov} = V_{SG} - |V_{tp}| = 2V \longrightarrow V_{SG} = 2 + 1 = 3V$$

$$V_{SG} = V_S - V_G = 6V - V_G = 3V \longrightarrow \boxed{V_G = 3V}$$

$$V_G = \frac{R_2}{R_1 + R_2} \times 10 = 3 \longrightarrow R_2 = \frac{3}{10}(R_1 + R_2) \longrightarrow \underline{3R_1 - 7R_2 = 0}$$

$$10V = 10\ \mu A \times (R_1 + R_2) \longrightarrow \underline{R_1 + R_2 = 1\ M\Omega}$$

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Problem 4.

$$\begin{cases} 3R_1 - 7R_2 = 0 \\ R_1 + R_2 = 1\ M\Omega \end{cases} \Longrightarrow \boxed{R_1 = 0.7\ M\Omega} \quad \boxed{R_2 = 0.3\ M\Omega}$$

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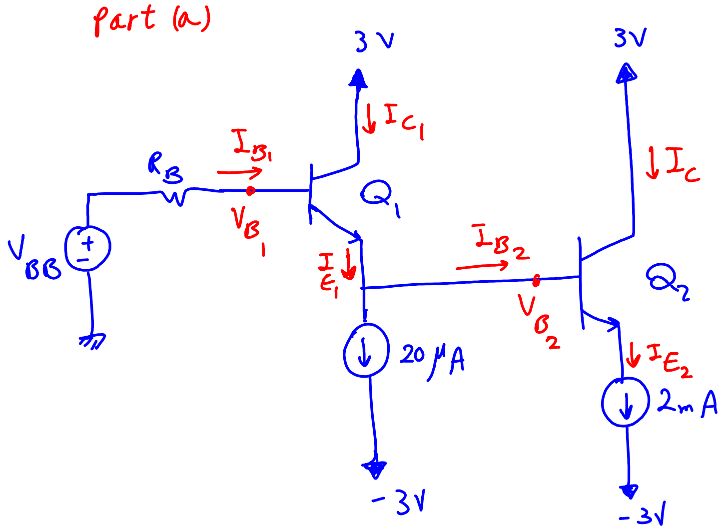
Problem 5.

part (a)



$\boxed{I_{E_2} = 2mA}$

$R_B = 3M\Omega \parallel 1M\Omega = 0.75 M\Omega$

$V_{BB} = \frac{3M\Omega}{3M\Omega + 1M\Omega} \times 3V = 2.25V$

BJTs will be operating in the active mode $\Rightarrow$

$I_{B_2} = \frac{I_{E_2}}{1+\beta_2} = \frac{2mA}{201} \simeq 9.95\mu A$

$I_{E_1} = 20\mu A + I_{B_2} = 20\mu A + 9.95\mu A = 29.95\mu A$

$\boxed{I_{E_1} = 29.95\mu A}$ $I_{B_1} = \frac{I_{E_1}}{1+\beta_1} = \frac{29.95\mu A}{51} \simeq 0.59\mu A$

$V_{B_1} = V_{BB} - R_B I_{B_1} = 2.25 - 0.75M\Omega \times 0.59\mu A \simeq 1.81 V$

$\boxed{V_{B_1} \simeq 1.81 V}$

$V_{B_2} = V_{E_1} = V_{B_1} - 0.7V = 1.11 V \longrightarrow \boxed{V_{B_2} = 1.11 V}$

$V_{CE_1} = V_{C_1} - V_{E_1} = 3V - 1.11V = 1.89 > V_{D_0} \longrightarrow Q_1$ is in active mode

$V_{E_2} = V_{B_2} - 0.7V = 1.11V - 0.7V = 0.41V$

$V_{CE_2} = V_{C_2} - V_{E_2} = 3V - 0.41 = 2.59V > V_{D_0} \longrightarrow Q_2$ is in active mode

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Problem 5.

Part b) $g_{m_1} = \dfrac{I_{C_1}}{V_T}$, $r_{\pi_1} = \dfrac{\beta_1}{g_{m_1}}$

$$I_{C_1} = \frac{\beta_1}{\beta_1 + 1} I_{E_1} = \frac{50}{51} \times 29.95\,\mu A = 29.36\,\mu A$$

$$g_{m_1} = \frac{29.36\,\mu A}{25\,mV} = 1.17\,(mA/V) \longrightarrow \boxed{g_{m_1} = 1.17\,(mA/V)}$$

$$r_{\pi_1} = \frac{50}{1.17} = 42.73\,k\Omega \longrightarrow \boxed{r_{\pi_1} = 42.73\,k\Omega}$$

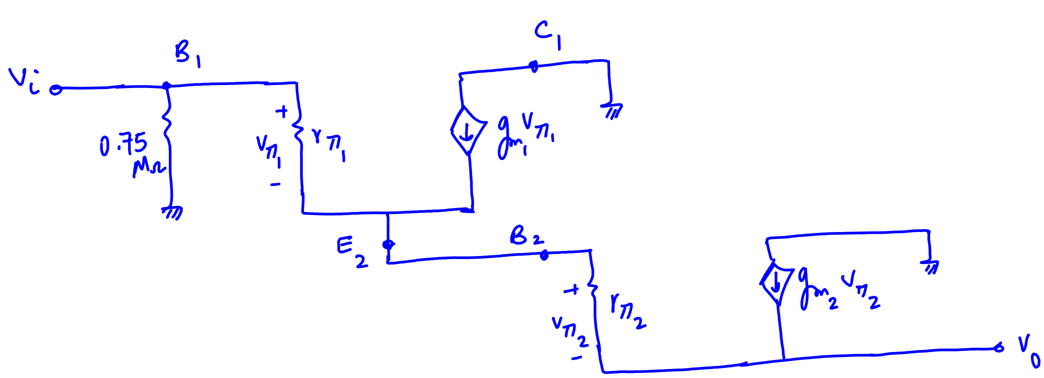
$g_{m_2} = \dfrac{I_{C_2}}{V_T}$, $r_{\pi_2} = \dfrac{\beta_2}{g_{m_2}}$

$$I_{C_2} = \frac{\beta_2}{1+\beta_2} I_{E_2} = \frac{200}{201} \times 2\,mA = 1.99\,mA$$

$$g_{m_2} = \frac{1.99\,mA}{25\,mA} = 79.6\,mA/V \longrightarrow \boxed{g_{m_2} = 79.6\,mA/V}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{200}{79.6\,mA/V} \simeq 2.51\,k\Omega \longrightarrow \boxed{r_{\pi_2} = 2.51\,k\Omega}$$

Part c)



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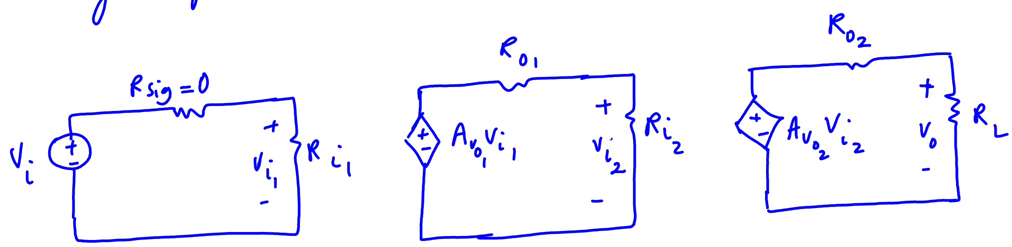
Problem 5.

Part d)

The voltage amplifier model:



We need to find R_{i_1}, R_{i_2}, R_{o_1}, R_{o_2}, $A_{v_{o_1}}$, $A_{v_{o_2}}$. We can use the given equations for these parameters

Both Q_1 and Q_2 are used in the common-collector configuration.

$$A_{v_{o_1}} = \frac{\frac{1}{g_{m_1}} \| r_{\pi_1} \| R_{E_1} \| r_{o_1}}{\frac{1}{g_{m_1}} \| r_{\pi_1}} \longrightarrow A_{v_{o_1}} = \frac{\frac{1}{g_{m_1}} \| r_{\pi_1}}{\frac{1}{g_{m_1}} \| r_{\pi_1}} = 1 \left(\frac{V}{V}\right) \quad \text{also} \quad A_{v_{o_2}} = 1 \left(\frac{V}{V}\right)$$

$$R_{o_1} = \frac{1}{g_{m_1}} \| r_{\pi_1} \| R_{E_1} \| r_{o_1} \longrightarrow R_{o_1} = \frac{1}{1.17\,mA/V} \| 42.73\,k\Omega \simeq 838\,\Omega$$

$$R_{o_2} = \frac{1}{g_{m_2}} \| r_{\pi_2} = \frac{1}{79.6\,mA/V} \| 2.51\,k\Omega \longrightarrow R_{o_2} \simeq 12.5\,\Omega$$

$$R_{i_2} = R_{B_2} \| \left[r_{\pi_2} + (\beta_2 + 1)(R_{E_2} \| r_{o_2} \| R_L) \right], \quad R_{B_2} = \infty,\ R_{E_2} = \infty,\ r_o = \infty,\ R_L = 100\,k\Omega$$

$$R_{i_2} = r_{\pi_2} + (\beta_2 + 1) R_L = 2.51\,k\Omega + 201 \times 100\,k\Omega \simeq 20.1\,M\Omega$$

$$R_{i_1} = R_{B_1} \| \left[r_{\pi_1} + (\beta_1 + 1) R_{i_2} \right] = 0.75\,M\Omega \| \left[42.73\,k\Omega + 51 \times 20.1\,M\Omega \right]$$

$$R_{i_1} \simeq 744\,k\Omega$$

$$\frac{v_o}{v_{sig}} = \left(\frac{R_L}{R_L + R_{o_2}}\right)(A_{v_{o_2}})\left(\frac{R_{i_1}}{R_{i_1} + R_{sig}}\right) A_{v_{o_1}} = \frac{100k}{100k + 838\,\Omega} \times 1 \times 1 \times 1 \simeq 0.99\ \frac{V}{V}$$