UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2021

Linear Systems Fundamentals

FINAL EXAM

Preface

- I recognize that final exams are stressful in the best of times, and this is an unusually hard time.
- Please remember that despite the stress, I am counting on you to uphold academic integrity while you complete your final exam.
- Posting or seeking exam questions or answers online, or by consulting unauthorized resources, is a gross violation of our principles of integrity and engineering ethics.
- To be fair to all students, any integrity violations discovered during the final exam will be reported to the Dean of Engineering and to the office of Academic Integrity.
- Please make sure you understand and follow the academic integrity guidelines for the exam. If you are not sure, ask me.
- An honest effort, no matter what the outcome, is something to be proud of, especially in these challenging times.
- I am very proud of the commitment and resilience you have displayed. You should feel proud, too.
- **Good luck on the exam!**

PRINT YOUR NAME ____

Student ID Number ____

Signature ____

Your signature confirms that you have completed this exam on your own and in accordance with the Academic Integrity Agreement.

Instructions

- No electronics allowed for problem solving.
- Notes and Tables from Chapters 3 ,4, 5, 9 of the textbook are attached.
- Two 2-sided sheets of your own notes are allowed.
- Time allowed: 3 hours (including upload time).
- Write your solutions in the designated spaces in the exam.
- **Justify all of your answers.**
- If extra pages are needed to show all of your work, use the scratch pages at the end of the exam and submit them.
- **Upload your solutions by 6pm via Gradescope.**

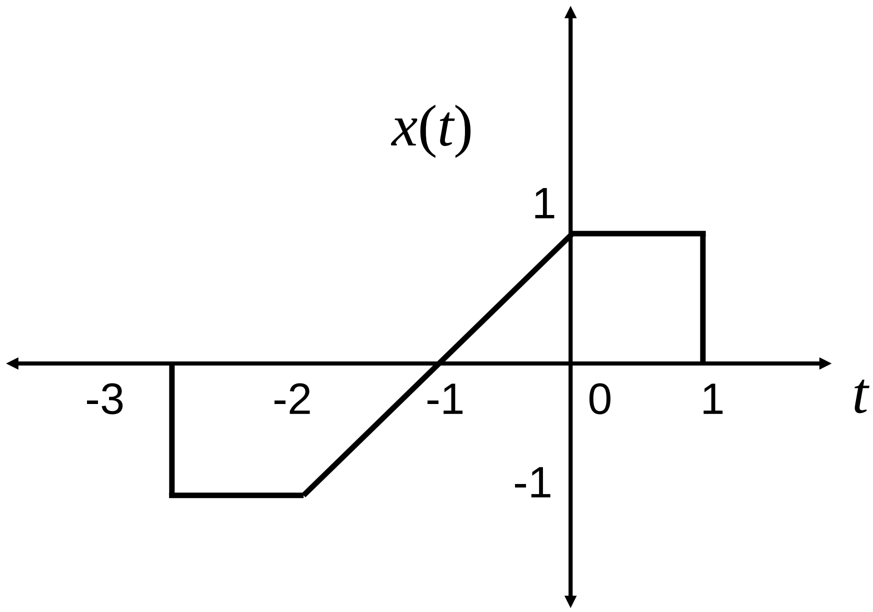
Problem	Weight	Score
1	30 pts	
2	30 pts	
3	30 pts	
4	30 pts	
5	30 pts	
6	30 pts	
Total	180 pts	

Good luck!

Name/Student ID: ____

Problem 1 [CTFT Properties] (30 points, 10 points each part)

Let $x(t)$ be the CT signal below. Let $X(j\omega)$ denote its Fourier transform.



(a) Determine $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

(b) Determine $X(j0)$.

Let $x_1(t) = x(t-1)$, with Fourier transform $X_1(j\omega)$.

(c) Determine $Re\{X_1(j\omega)\}$.

Write answers to parts (a), (b), (c) on the following pages.

Justify your answers.

Name/Student ID: ____________________

Problem 1 (cont.)

(a) (10 points)

Determine $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

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Problem 1 (cont.)

(b) (10 points)

Determine $X(j0)$.

Name/Student ID: ____________________________________

Problem 1 (cont.)

Let $x_1(t) = x(t-1)$, with Fourier transform $X_1(j\omega)$.

(c) (10 points)

Determine $Re\{X_1(j\omega)\}$.

Name/Student ID: ____________________

Problem 2 [DTFT] (30 points; 10 points each part)

Let S be the DT LTI system obtained by concatenating the two DT LTI systems S_1 and S_2 defined by:

$$\begin{aligned} S_1 : \quad y[n] &= x[n] - \frac{1}{2}x[n-1] \\ S_2 : \quad y[n] &= x[n] + \frac{1}{4}x[n-1] \end{aligned}$$

(a) Determine the frequency response $H(e^{j\omega})$ of S and write it in the form

$$\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}.$$

(b) Let S' be the inverse system to S. Determine the frequency response $G(e^{j\omega})$ of S' and write it in the form

$$\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}.$$

(c) Let $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$ be the input signal to the system S'. Determine the output signal $y[n]$.

Write answers to parts (a), (b), (c) on the following pages.

Justify your answers.

Name/Student ID: ____

Problem 2 (cont.)

(a) (10 points)

Determine the frequency response $H(e^{j\omega})$ of S and write it in the form $\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$.

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Problem 2 (cont.)

(b) (10 points)

Let S' be the inverse system to S. Determine the frequency response $G(e^{j\omega})$ of S' and write it in the form $\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$.

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Problem 2 (cont.)

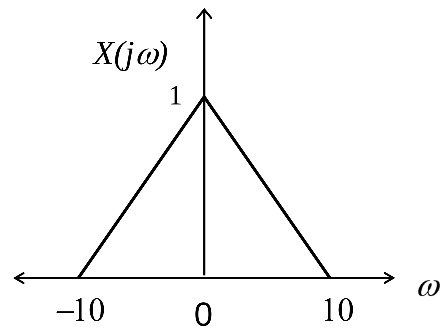
(c) (10 points)

Let $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$ be the input signal to the system S'. Determine the output signal $y[n]$.

Name/Student ID: ______________________________

Problem 3 [Modulation] (30 points, 10 points each part)

Let $x(t)$ be the signal with Fourier transform $X(j\omega)$ shown below.



(a) Let $x_1(t) = x(t)\cos(30t)$. The signal $x_1(t)$ is received and passed through a bandpass filter with frequency response

$$H_{BP}(j\omega) = \begin{cases} 2, & 20 < |\omega| < 30 \\ 0, & \text{otherwise.} \end{cases}$$

Write the CTFT $X_1(j\omega)$ of $x_1(t)$ in terms of $X(j\omega)$.
Let $y_1(t)$ be the output of the bandpass filter. Sketch precisely the Fourier transform $Y_1(j\omega)$ of the signal $y_1(t)$ and write it in terms of $X(j\omega)$.

(b) Using the signal $y_1(t)$ from part (a), define $x_2(t) = y_1(t)\cos(20t)$. The signal $x_2(t)$ is transmitted and then passed through the lowpass filter with frequency response

$$H_{LP}(j\omega) = \begin{cases} 2, & |\omega| < 20 \\ 0, & \text{otherwise.} \end{cases}$$

Let $y_2(t)$ be the output of the lowpass filter. Sketch precisely its Fourier transform $Y_2(j\omega)$ and write it in terms of $X(j\omega)$.

(c) Can the original signal $x(t)$ be recovered from the signal $y_2(t)$ of part (c)? If not, explain why not. If so, describe a demodulation scheme to recover $x(t)$. Write the transform of any intermediate signal you generate from $y_2(t)$ during the recovery process in terms of $X(j\omega)$ and sketch it precisely.

Write your answers to parts (a), (b), (c) on the following pages.

Name/Student ID: ____________________

Problem 3 (cont.)

(a) (10 points)

Let $x_1(t) = x(t)\cos(30t)$. The signal $x_1(t)$ is received and passed through a bandpass filter with frequency response

$$H_{BP}(j\omega) = \begin{cases} 2, & 20 < |\omega| < 30 \\ 0, & \text{otherwise.} \end{cases}$$

Write the CTFT $X_1(j\omega)$ of $x_1(t)$ in terms of $X(j\omega)$.
Let $y_1(t)$ be the output of the bandpass filter. Sketch precisely the Fourier transform $Y_1(j\omega)$ of the signal $y_1(t)$ and write it in terms of $X(j\omega)$.

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Problem 3 (cont.)

(b) (10 points)

Using the signal $y_1(t)$ from part (a), define $x_2(t) = y_1(t)\cos(20t)$. The signal $x_2(t)$ is transmitted and then passed through the lowpass filter of with frequency response

$$H_{LP}(j\omega) = \begin{cases} 2, & |\omega| < 20 \\ 0, & \text{otherwise.} \end{cases}$$

Let $y_2(t)$ be the output of the lowpass filter. Sketch precisely its Fourier transform $Y_2(j\omega)$ and write it in terms of $X(j\omega)$.

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Problem 3 (cont.)

(c) (10 points)

Can the original signal $x(t)$ be recovered from the signal $y_2(t)$ of part (c)? If not, explain why not. If so, describe a demodulation scheme to recover $x(t)$. Write the transform of any intermediate signal you generate from $y_2(t)$ during the recovery process in terms of $X(j\omega)$ and sketch it precisely.

Name/Student ID: ____________________

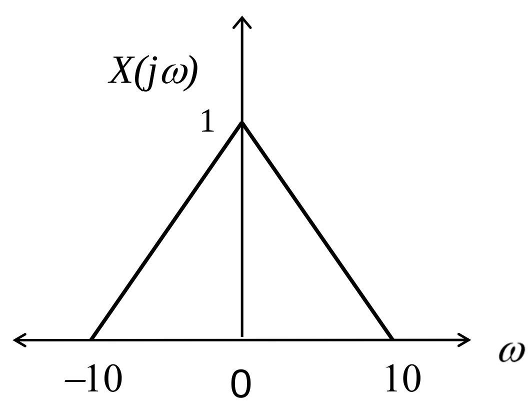
Problem 4 [Sampling Theory] (30 points)

(a) For each of the following signals $x(t)$, determine if a sampling period of $T = \frac{1}{4}$ sec is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. **Justify your answers by determining the Fourier Transforms $X(j\omega)$ of the signals.**

(i) $x(t) = \frac{\sin(3\pi t)}{\pi t}$

(ii) $x(t) = \begin{cases} 1, & |t| < 3\pi \\ 0, & |t| > 3\pi \end{cases}$

(b) Let $x(t)$ be the signal with Fourier transform $X(j\omega)$ shown below.



Let $x_p(t)$ be obtained from $x(t)$ by impulse train sampling with sampling period $T = \frac{\pi}{20}$, with Fourier transform by $X_p(j\omega)$. Let $x_d[n] = x(nT)$ be the DT sampled signal, with Fourier transform $X_d(e^{j\Omega})$.

(i) Write $X_p(j\omega)$ in terms of $X(j\omega)$. Write the formula relating $X_d(e^{j\Omega})$ to $X_p(j\omega)$. Determine ω_0 such that $X_d(e^{j2\pi}) = X_p(j\omega_0)$.

(ii) Sketch precisely $X_p(j\omega)$ in the range $|\omega| \le \omega_0$ and $X_d(e^{j\Omega})$ in the range $|\Omega| \le 2\pi$.

Justify your answers by referring to the formula for $X_p(j\omega)$ and its relation to $X_d(e^{j\Omega})$

Write your answers to parts (a), (b) on the following pages.

Name/Student ID: ____

Problem 4 (cont.)

(a) (16 points)

For each of the following signals $x(t)$, determine if a sampling period of $T = \frac{1}{4}$ sec is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples.

(i) $x(t) = \frac{\sin(3\pi t)}{\pi t}$

(ii) $x(t) = \begin{cases} 1, & |t| < 3\pi \\ 0, & |t| > 3\pi \end{cases}$

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Problem 4 (cont.)

(b) (14 points) Let $x_p(t)$ be obtained from $x(t)$ by impulse train sampling with sampling period $T = \frac{\pi}{20}$, with Fourier transform by $X_p(j\omega)$. Let $x_d[n] = x(nT)$ be the DT sampled signal, with Fourier transform $X_d(e^{j\Omega})$.

(i) (4 points)
Write $X_p(j\omega)$ in terms of $X(j\omega)$. Write the formula relating $X_d(e^{j\Omega})$ to $X_p(j\omega)$. Determine ω_0 such that $X_p(j\omega_0) = X_d(e^{j2\pi})$.

(ii) (10 points)
Sketch precisely $X_p(j\omega)$ in the range $|\omega| \leq \omega_0$ and $X_d(e^{j\Omega})$ in the range $|\Omega| \leq 2\pi$.

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Problem 5 [Laplace Transform and LTI Systems] (30 points)

Consider an LTI system S with transfer function

$$H(s) = \frac{s-1}{(s^2-2s+2)(s+1)}.$$

(a) Sketch precisely the pole-zero plot associated with $H(s)$. Indicate the order of all poles and zeros.

(b) Suppose S is stable.

   (i) Determine $|H(j0)|$ and $\angle H(j0)$.

   (ii) Determine $|H(j\omega)|$ and $\angle H(j\omega)$ in the limit of as $\omega \to \pm\infty$.

   (iii) Is the system lowpass, highpass, bandpass, or allpass?

(c) Suppose S is causal. If the impulse response is $h(t)$, does $h(t)e^{-2t}$ have a Fourier transform?

Write your answers to parts (a), (b), (c) on the following pages.

Name/Student ID: ______________________________

Problem 5 (cont.)

Consider the LTI system function $H(s) = \frac{s-1}{(s^2-2s+2)(s+1)}$.

(a) (10 points)

Sketch precisely the pole-zero plot associated with $H(s)$. Indicate the order of all poles and zeros.

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Problem 5 (cont.)

Consider the LTI system function $H(s) = \frac{s-1}{(s^2-2s+2)(s+1)}$.

(b) (12 points)

Suppose S is stable.

(i) Determine $|H(j0)|$ and $\angle H(j0)$.

(ii) Determine $|H(j\omega)|$ and $\angle H(j\omega)$ in the limit of as $\omega \to \pm\infty$.

(iii) Is the system ~~have~~ lowpass, highpass, bandpass, or allpass?

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Problem 5 (cont.)

Consider the LTI system function $H(s) = \frac{s-1}{(s^2-2s+2)(s+1)}$.

(c) (8 points)

Suppose S is causal. If the impulse response is $h(t)$, does $h(t)e^{-2t}$ have a Fourier transform?

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Problem 6 [LT and Differential Equations] (30 points, 10 points each)

Consider the causal LTI system whose input $x(t)$ and output $y(t)$ satisfy the differential equation

$$\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = \frac{dx(t)}{dt} - x(t).$$

(a) Determine the transfer function $H(s)$ and sketch precisely the pole-zero plot.

(b) Determine the region of convergence (ROC). and determine the impulse response $h(t)$.

(c) Draw a block diagram for the direct form (canonical) realization of the system using a cascade of integrators with feedforward and feedback components.

Write your answers to parts (a), (b), (c), and (d) on the following pages.

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Problem 6 (cont.)

Causal LTI system defined by $\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = \frac{dx(t)}{dt} - x(t)$.

(a) (10 points)

Determine the transfer function $H(s)$ and sketch precisely the pole-zero plot.

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Problem 6 (cont.)

Causal LTI system defined by $\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = \frac{dx(t)}{dt} - x(t)$.

(b) (10 points)

Determine the region of convergence (ROC). and determine the impulse response $h(t)$.

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Problem 6 (cont.)

Causal LTI system defined by $\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = \frac{dx(t)}{dt} - x(t)$.

(c) (10 points)

Draw a block diagram for the direct form (canonical) realization of the system using a cascade of integrators with feedforward and feedback components.

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