UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2023
Linear Systems Fundamentals

FINAL EXAM

You are allowed two 2-sided sheets of notes.
Tables 3.1, 3.2, 4.1, 4.2, 5.1, 5.2, 9.1, and 9.2 are attached.
No books, no other notes, no calculators.

PRINT YOUR NAME ______________________________

Signature ______________________________

Student ID Number ______________________________

Your signature confirms that you have completed this exam in accordance with the ECE 101 Academic Integrity Agreement.

Problem	Weight	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total	120	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Scan, upload, and tag your solutions on Gradescope at the end of the exam.

Good luck! You've got this!

Name/Student ID: ______________________________

Problem 1 [CTFT Properties] (20 points, 5 points each part)

Let $x(t) = u(t+3) - u(t-1)$. Let $X(j\omega)$ denote its Fourier transform.
In the following problems, refer to the attached tables, as necessary, to explain your solutions.

(a) Determine $X(j\omega)$.

(b) Determine $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

(c) Let $y(t) = x(\frac{t}{2})$, with Fourier transform $Y(j\omega)$. Determine $Y(j0)$.

(d) Determine $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Write answers to parts (a) through (d) on the following pages.

Justify your answers.

Name/Student ID: ____________________

Problem 1 [CTFT Properties] (cont.)

Let $x(t) = u(t+3) - u(t-1)$. Let $X(j\omega)$ denote its Fourier transform.

(a) Determine $X(j\omega)$.

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Problem 1 [CTFT Properties] (cont.)

Let $x(t) = u(t+3) - u(t-1)$. Let $X(j\omega)$ denote its Fourier transform.

(b) Determine $\int_{-\infty}^{\infty} X(j\omega) d\omega$.

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Problem 1 [CTFT Properties] (cont.)

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Name/Student ID: ______________________________

Problem 2 [DTFT] (20 points, 5 points each part)

This problem deals with the DT LTI system S with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{-n} u[-n] + \left(\frac{1}{2}\right)^{n} u[n].$$

(a) Precisely sketch the signal $h[n]$ in the time interval $[-2, 2]$.

(b) Let $x[n] = u[n-1]$ be the input signal to the system S.
Determine the value of the corresponding output signal $y[n]$ at $n = 0$.

(c) Determine the frequency response $H(e^{j\omega})$ of the system S.
Express it in a form that does not use any exponential functions of ω.

(d) Is the system S invertible? If not, why not? If so, determine the frequency response $G(e^{j\omega})$ of its inverse system.

Write answers to parts (a) through (d) on the following pages.

Justify your answers.

Name/Student ID: ______________________________

Problem 2 [DTFT] (cont.)

This problem deals with the DT LTI system S with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{-n} u[-n] + \left(\frac{1}{2}\right)^{n} u[n].$$

(a) Precisely sketch the signal $h[n]$ in the time interval $[-2, 2]$.

Name/Student ID: ____

Problem 2 [DTFT] (cont.)

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$$h[n] = \left(\frac{1}{2}\right)^{-n} u[-n] + \left(\frac{1}{2}\right)^{n} u[n].$$

(b) Let $x[n] = u[n-1]$ be the input signal to the system S.
Determine the value of the corresponding output signal $y[n]$ at $n = 0$.

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(d) Is the system S invertible? If not, why not? If so, determine the frequency response $G(e^{j\omega})$ of its inverse system.

Name/Student ID: ______________________________

Problem 3 [Amplitude Modulation] (20 points, 5 points each part)

Consider the signals $x_1(t) = \frac{\sin(10t)}{\pi t}$ and $x_2(t) = \frac{\sin(5t)}{\pi t}$.

(a) Determine the Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$. Sketch them precisely.

(b) Let signal $y(t) = y_1(t) + y_2(t)$, where $y_1(t) = x_1(t)\cos(20t)$ and $y_2(t) = x_2(t)\cos(40t)$. Express the Fourier transform $Y(j\omega)$ of $y(t)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Sketch $Y(j\omega)$ precisely.

(c) The signal $y(t)$ is demodulated using multiplication by $\cos(20t)$ to form the signal $z(t) = y(t)\cos(20t)$. Express the Fourier transform $Z(j\omega)$ in terms of $Y_1(j\omega)$ and $Y_2(j\omega)$. Then express $Z(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Sketch precisely the portion of $Z(j\omega)$ in the frequency range $[-50, 50]$.

(d) (i) Describe precisely a scheme to recover $x_1(t)$ from $z(t)$.

(ii) Describe a scheme to recover $x_2(t)$ from the signal $y(t)$.

Write answers to parts (a) through (d) on the following pages.

Justify your answers.

Name/Student ID: ____

Problem 3 [Amplitude Modulation] (cont.)

Consider the signals $x_1(t) = \frac{\sin(10t)}{\pi t}$ and $x_2(t) = \frac{\sin(5t)}{\pi t}$.

(a) Determine the Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$. Sketch them precisely.

Name/Student ID: ______________________________

Problem 3 [Amplitude Modulation] (cont.)

(b) Let signal $y(t) = y_1(t) + y_2(t)$, where $y_1(t) = x_1(t)\cos(20t)$ and $y_2(t) = x_2(t)\cos(40t)$. Express the Fourier transform $Y(j\omega)$ of $y(t)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Sketch $Y(j\omega)$ precisely.

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Problem 3 [Amplitude Modulation] (cont.)

(c) The signal $y(t)$ is demodulated using multiplication by $\cos(20t)$ to form the signal $z(t) = y(t)\cos(20t)$. Express the Fourier transform $Z(j\omega)$ in terms of $Y_1(j\omega)$ and $Y_2(j\omega)$. Then express $Z(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Sketch precisely the portion of $Z(j\omega)$ in the frequency range $[-50, 50]$.

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Problem 3 [Amplitude Modulation] (cont.)

(d) (i) Describe precisely a scheme to recover $x_1(t)$ from $z(t)$.

(ii) Describe a scheme to recover $x_2(t)$ from the signal $y(t)$.

Name/Student ID: ____________________________________

Problem 4 [Sampling Theory] (20 points, 5 points each part)

For each of the following signals, determine if a sampling period of $T = \frac{1}{4}$ seconds is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. Justify your answers by explicit reference to the Fourier transforms of the signals.

(a) $x(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$

(b) $x(t) = \cos(\frac{3}{2}\pi t)e^{-j\frac{3}{2}\pi t}$

(c) $x(t) = \frac{\sin(3\pi t)}{\pi t} * \frac{\sin(6\pi t)}{\pi t}$

(d) $x(t) = e^{-3\pi t}u(t)$

Write answers to parts (a) through (d) on the following pages.

Justify your answers.

Name/Student ID: ____

Problem 4 [Sampling Theory] (cont.)

For each of the following signals, determine if a sampling period of $T = \frac{1}{4}$ seconds is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. Justify your answers by explicit reference to the Fourier transforms of the signals.

(a) $x(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$

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Problem 4 [Sampling Theory] (cont.)

For each of the following signals, determine if a sampling period of $T = \frac{1}{4}$ seconds is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. Justify your answers by explicit reference to the Fourier transforms of the signals.

(b) $x(t) = \cos(\frac{3}{2}\pi t)e^{-j\frac{3}{2}\pi t}$

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Problem 4 [Sampling Theory] (cont.)

For each of the following signals, determine if a sampling period of $T = \frac{1}{4}$ seconds is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. Justify your answers by explicit reference to the Fourier transforms of the signals.

(c) $x(t) = \frac{\sin(3\pi t)}{\pi t} * \frac{\sin(6\pi t)}{\pi t}$

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For each of the following signals, determine if a sampling period of $T = \frac{1}{4}$ seconds is sufficiently small to avoid aliasing and allow reconstruction of the signal from its samples. Justify your answers by explicit reference to the Fourier transforms of the signals.

(d) $x(t) = e^{-3\pi t}u(t)$

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Problem 5 [Laplace Transform and LTI Systems] (20 points)

The system function of a stable LTI system S is

$$H(s) = \frac{s}{s^2 + 2s + 2}$$

(a) Determine the poles and zeros and sketch precisely the pole-zero plot.

(b) Let $H(j\omega)$ be the frequency response of the system S. Determine the magnitude $|H(j\omega)|$ at $\omega = 0, 1$, and in the limit as $\omega \to \pm\infty$.

(c) Sketch $|H(j\omega)|$ approximately. Is the system low-pass, high-pass, band-pass, or all-pass in nature?

Write answers to parts (a) through (c) on the following pages. Justify your answers.

Name/Student ID: ____

Problem 5 [Laplace Transform and LTI Systems] (cont.)

The system function of a stable LTI system S is

$$H(s) = \frac{s}{s^2 + 2s + 2}$$

(a) (8 points)
Determine the poles and zeros and sketch precisely the pole-zero plot.

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Problem 5 [Laplace Transform and LTI Systems] (cont.)

The system function of a stable LTI system S is

$$H(s) = \frac{s}{s^2 + 2s + 2}$$

(b) (8 points)
Let $H(j\omega)$ be the frequency response of the system S. Determine the magnitude $|H(j\omega)|$ at $\omega = 0, 1$, and in the limit as $\omega \to \pm\infty$.

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Problem 5 [Laplace Transform and LTI Systems] (cont.)

The system function of a stable LTI system S is

$$H(s) = \frac{s}{s^2 + 2s + 2}$$

(c) (4 points)
Sketch $|H(j\omega)|$ approximately. Is the system low-pass, high-pass, band-pass, or all-pass in nature?

Name/Student ID: ______________________________

Problem 6 [Laplace Transform and Differential Equations] (20 points)

Consider an LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}.$$

(a) Determine the system function $H(s)$ and the possible regions of convergence (ROC). For each region of convergence, determine whether the corresponding system is causal or stable. [Hint: Your answer to Problem 5(a) will be helpful here.]

(b) Determine the impulse response $h(t)$ when the system is causal. Express your answer in the form $Ce^{-\alpha t}\cos(\omega_0 t)u(t) + De^{-\alpha t}\sin(\omega_0 t)u(t)$, where C, D, α, ω_0 are constants. Use partial fraction expansion, and verify your answer using Table 9.2.

(c) Carefully sketch the canonical (direct-form) block diagram representing the causal LTI system in part (b).

Write answers to parts (a) through (c) on the following pages.

Justify your answers.

Name/Student ID: ______________________________

Problem 6 [Laplace Transform and Differential Equations] (cont.)

Consider an LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}.$$

(a) (8 points)
Determine the system function $H(s)$ and the possible regions of convergence (ROC). For each region of convergence, determine whether the corresponding system is causal or stable. [Hint: Your answer to Problem 5(a) will be helpful here.]

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Problem 6 [Laplace Transform and Differential Equations] (cont.)

Let S be the causal LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}.$$

(b) (8 points)
Determine the impulse response $h(t)$ when the system is causal. Express your answer in the form $Ce^{-\alpha t}\cos(\omega_0 t)u(t) + De^{-\alpha t}\sin(\omega_0 t)u(t)$, where C, D, α, ω_0 are constants. Use partial fraction expansion, and verify your answer using Table 9.2.

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Problem 6 [Laplace Transform and Differential Equations] (cont.)

Consider an LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}.$$

(c) (4 points)
Carefully sketch the canonical (direct-form) block diagram representing the causal LTI system in part (b).

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