UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2022
Linear Systems Fundamentals

MIDTERM EXAM

You are allowed one 2-sided sheet of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME _____

Signature _____

Your signature confirms that you have completed this exam on your own and in accordance with the ECE 101 Academic Integrity Agreement.

Student ID Number _____

Problem	Weight	Score
1	32 pts	
2	32 pts	
3	36 pts	
Total	100 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Useful facts and Tables 3.1 and 3.2 from the textbook are attached to the exam.

Good luck!

Name/Student ID: _____

Problem 1 [DT Signals] (32 points)

(a) Let $x[n]=u[n+1]-2u[n-1]+u[n-2]$, where $u[n]$ is the unit step signal.

(i) Represent $x[n]$ mathematically as a linear combination of time-shifted unit impulse signals.

Sketch $x[n]$ precisely.

(ii) Let $y[n] = x[n] * u[n]$. Represent $y[n]$ mathematically as a linear combination of time-shifted unit impulse signals.

Sketch $y[n]$ precisely.

(b) Let $x[n] = \delta[n] - \delta[n-1]$.

(i) Let $\mathcal{E}v\{x[n]\}$ be the even part of $x[n]$.

Represent $\mathcal{E}v\{x[n]\}$ mathematically as a linear combination of time-shifted unit impulse signals.

Sketch $\mathcal{E}v\{x[n]\}$ precisely.

(ii) Let $\mathcal{O}dd\{x[n]\}$ be the odd part of $x[n]$.

Represent $\mathcal{O}dd\{x[n]\}$ mathematically as a linear combination of time-shifted unit impulse signals.

Sketch $\mathcal{O}dd\{x[n]\}$ precisely.

Write your answers on the following pages.

Name/Student ID: _____

Problem 1 [DT Signals](cont.)

(a) Let $x[n]=u[n+1]-2u[n-1]+u[n-2]$, where $u[n]$ is the unit step signal.

(i) (8 points)
Represent $x[n]$ mathematically as a linear combination of time-shifted unit impulse signals. Sketch $x[n]$ precisely.

Name/Student ID: _____

Problem 1 [DT Signals](cont.)

(a) Let $x[n]$=$u[n+1]-2u[n-1]+u[n-2]$, where $u[n]$ is the unit step signal.

(ii) (8 points)
Let $y[n] = x[n] * u[n]$.
Represent $y[n]$ mathematically as a linear combination of time-shifted unit impulse signals. Sketch $y[n]$ precisely.

Name/Student ID: _____

Problem 1 [DT Signals] (cont.)

(b) Let $x[n] = \delta[n] - \delta[n-1]$.

(i) (8 points)
Let $\mathcal{E}v\{x[n]\}$ be the even part of $x[n]$. Represent $\mathcal{E}v\{x[n]\}$ mathematically as a linear combination of time-shifted unit impulse signals. Sketch $\mathcal{E}v\{x[n]\}$ precisely.

Name/Student ID: _____

Problem 1 [DT Signals] (cont.)

(b) Let $x[n] = \delta[n] - \delta[n-1]$.

(ii) (8 points)
Let$\mathcal{O}dd\{x[n]\}$ be the odd part of $x[n]$. Represent $\mathcal{O}dd\{x[n]\}$ mathematically as a linear combination of time-shifted unit impulse signals. Sketch $\mathcal{O}dd\{x[n]\}$ precisely.

Name/Student ID: _____

Problem 2 [DT Systems] (32 points)

Let S be the DT system with the property that the output signal $y[n]$ produced by the input signal $x[n]$ is given by:

$$y[n] = x[-n+1].$$

(a) Check the appropriate box indicating whether or not the system S satisfies the specified property.

Justify your answers by either proving that the property holds for the system, or giving a specific counterexample to prove that it does not.

True	**False**	
☐	☐	Causal
☐	☐	Invertible
☐	☐	Stable
☐	☐	Linear
☐	☐	Time-invariant

(b) Determine and sketch precisely the impulse response $h[n]$ of system S. Describe in words the action of the system S on the input signal $x[n]$.

(c) Let S_1 be the **LTI system** that has the same impulse response as S.

Check the appropriate box indicating whether or not the system S_1 satisfies the specified property.

Justify your answers by reference to the impulse response.

True	**False**	
☐	☐	Causal
☐	☐	Invertible
☐	☐	Stable

Describe in words the action of the system S_1 on the input signal $x[n]$.

Write your answers on the following pages.

Name/Student ID: _____

Problem 2 [DT Systems] cont.)

Let S be the DT system with the property that the output signal $y[n]$ produced by the input signal $x[n]$ is given by: $y[n] = x[-n+1]$.

(a) (15 points)

Check the appropriate boxes for the system S. **Justify your answers.**

True	**False**	
☐	☐	Causal
☐	☐	Invertible
☐	☐	Stable
☐	☐	Linear
☐	☐	Time-invariant

Name/Student ID: _____

Problem 2 [DT Systems] (cont.)

Let S be the DT system with the property that the output signal $y[n]$ produced by the input signal $x[n]$ is given by: $y[n] = x[-n+1]$.

(b) (5 points)

Determine and sketch precisely the impulse response $h[n]$ of system S. Describe in words the action of the system S on the input signal $x[n]$.

Name/Student ID: _____

Problem 2 [DT Systems] (cont.)

Let S be the DT system with the property that the output signal $y[n]$ produced by the input signal $x[n]$ is given by: $y[n] = x[-n+1]$.

(c) (12 points)

Let S_1 be the **LTI system** S_1 that has the same impulse response as S.

Check the appropriate boxes for the system S_1.

Justify your answers by reference to the impulse response.

True	**False**	
☐	☐	Invertible
☐	☐	Causal
☐	☐	Stable

Describe in words the action of the system S_1 on the input signal $x[n]$.

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (36 points)

Let $x[n]$ be a periodic DT signal with fundamental period $N = 4$. You are given the following information about the DTFS coefficients of $x[n]$:

$$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 0.$$

(a) **Referring to Table 3.2** (Properties of DTFS), answer the following questions. **Justify your answers.**

(i) Is $x[n]$ an even signal?

(ii) Is $x[n]$ a real signal?

(iii) What is the average power $\frac{1}{4}\sum_{n=0}^{3}|x[n]|^2$ in one period of $x[n]$?

(b) Write one period of the signal $x[n]$: $x[0],\ x[1],\ x[2],\ x[3]$.
Express the values as complex numbers in rectangular form.

(c) A DT LTI system S has impulse response $h[n] = \delta[n] + \delta[n-1]$.

(i) Determine the transfer function $H(z)$ and the frequency response $H(e^{j\omega})$ of S.

(ii) Determine and sketch precisely the phase $\angle H(e^{j\omega})$ and gain $|H(e^{j\omega})|$ over the frequency interval $[0, 2\pi]$. Express both of them in the simplest mathematical form that you can, using only linear and sinusoidal functions of ω.

(iii) When the input to system S is the signal $x[n]$ above, determine the DTFS of the output signal $y[n]$.

(iv) Determine one period of $y[n]$: $y[0],\ y[1],\ y[2],\ y[3]$. Express the values as complex numbers in rectangular form.

Write your answers on the following pages.

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (cont.)

Let $x[n]$ be a periodic DT signal with fundamental period $N = 4$. You are given the following information about the DTFS coefficients of $x[n]$:
$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 0.$

(a) **Referring to Table 3.2** (Properties of DTFS), answer the following questions. **Justify your answers.**

(i) (4 points)
Is $x[n]$ an even signal?

(ii) (4 points)
Is $x[n]$ a real signal?

(iii) (4 points)
What is the average power $\frac{1}{4}\sum_{n=0}^{3}|x[n]|^2$ in one period of $x[n]$?

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (cont.)

Let $x[n]$ be a periodic DT signal with fundamental period $N = 4$. You are given the following information about the DTFS coefficients of $x[n]$:
$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 0.$

(b) (4 points)

Write one period of the signal $x[n]$: $x[0],\ x[1],\ x[2],\ x[3]$.
Express the values as complex numbers in rectangular form.

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (cont.)

(c) A DT LTI system S has impulse response $h[n] = \delta[n] + \delta[n-1]$.

(i) (4 points)
Determine the transfer function $H(z)$ and the frequency response $H(e^{j\omega})$ of S.

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (cont.)

(c) A DT LTI system S has impulse response $h[n] = \delta[n] + \delta[n-1]$.

(ii) (8 points)

Determine and sketch precisely the phase $\angle H(e^{j\omega})$ and gain $|H(e^{j\omega})|$ over the frequency interval $[0, 2\pi]$. Express both of them in the simplest mathematical form that you can, using only linear and sinusoidal functions of ω.

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Problem 3 [DTFS and DT LTI Systems] (cont.)

Let $x[n]$ be a periodic DT signal with fundamental period $N = 4$. You are given the following information about the DTFS coefficients of $x[n]$:
$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 0.$

(c) A DT LTI system S has impulse response $h[n] = \delta[n] + \delta[n-1]$.

(iii) (4 points)

When the input to system S is the signal $x[n]$ above, determine the DTFS of the output signal $y[n]$.

Name/Student ID: _____

Problem 3 [DTFS and DT LTI Systems] (cont.)

Let $x[n]$ be a periodic DT signal with fundamental period $N = 4$. You are given the following information about the DTFS coefficients of $x[n]$:
$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = 0.$

(c) A DT LTI system S has impulse response $h[n] = \delta[n] + \delta[n-1]$.

(iv) (4 points)
Determine one period of $y[n]$: $y[0],\ y[1],\ y[2],\ y[3]$.
Express the values as complex numbers in rectangular form.

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