ECE 101: Linear Systems Fundamentals

Summer Session II 2020 - Lecture 10

Saharnaz Baghdadchi

University of California, San Diego

Today's topics

- Continuous-time Fourier series and LTI systems
- Discrete-time Fourier series and LTI systems

  Signals and Systems (2th Edition): section 3.8

CTFS and LTI systems - road map

- Signals in the format of $x(t) = e^{st}$ are eigenfunctions of **LTI systems**

- That means if we apply the input signal

$$x(t) = e^{st}, \quad where \quad s = \alpha + j\omega$$

  to a CT LTI system, the output will be a scaled version of the input signal.

$$y(t) = H(s)e^{st}$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau, \quad H(s) \in \mathbb{C}$$

- We found this using the convolution integral **defined for LTI systems**.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

CTFS and LTI systems - road map

- Think about these eigenfunctions as a set of base functions

$$\boxed{\begin{array}{cccc}
e^{(\alpha_1+j\omega_1)t} & e^{(\alpha_1+j\omega_2)t} & e^{(\alpha_1+j\omega_3)t} & \cdots\cdots \\
e^{(\alpha_2+j\omega_1)t} & e^{(\alpha_2+j\omega_2)t} & e^{(\alpha_2+j\omega_3)t} & \cdots\cdots \\
e^{(\alpha_3+j\omega_1)t} & e^{(\alpha_3+j\omega_2)t} & e^{(\alpha_3+j\omega_3)t} & \cdots\cdots \\
\vdots & \vdots & \vdots &
\end{array}}$$

- If we apply either one of them to an **LTI system**, the output will be the input signal multiplied by a scalar

$$\xrightarrow{e^{(\alpha_1+j\omega_1)t}} \boxed{h(t)} \xrightarrow{H(\alpha_1+j\omega_1)e^{(\alpha_1+j\omega_1)t}}$$

CTFS and LTI systems - road map

- Let's limit ourselves to a subset of the eigenfunctions e^{st} in which $s = jk\omega_0$, where $k \in \mathbb{Z}$

$$e^0 = 1$$

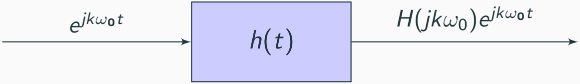
$$e^{-j\omega_0 t} \qquad e^{j\omega_0 t}$$

$$e^{-j2\omega_0 t} \qquad e^{j2\omega_0 t}$$

$$e^{-j3\omega_0 t} \qquad e^{j3\omega_0 t}$$

$$\vdots \qquad \vdots$$

$$e^{jk\omega_0 t} \;, k \in \mathbb{Z}$$



CTFS and LTI systems - road map

- Now, let's look at the Fourier series representation of periodic signals with period T, that have a finite energy in one period.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

CTFS and LTI systems - road map

- Now, let's look at the Fourier series representation of periodic signals with period T, that have a finite energy in one period.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \cdots + a_{-1}e^{-j\omega_0 t} + a_0 e^0 + a_1 e^{j\omega_0 t} + \cdots$$

$$a_{-1}e^{-j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow H(-j\omega_0)e^{-j\omega_0 t}$$

$$a_1 e^{j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow a_1 H(j\omega_0)e^{j\omega_0 t}$$

$$a_k e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow a_k H(jk\omega_0)e^{jk\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)e^{jk\omega_0 t}$$

CTFS and LTI systems - road map

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

- The Fourier series representation of a continuous-time signal will tell us which frequency components (the $k\omega_0$ terms in $e^{jk\omega_0 t}$) are present in the signal
- The Fourier series coefficients of a continuous-time signal will tell us the strength of each frequency component that is present in the signal
- $H(j\omega)$ evaluated at $\omega = k\omega_0$ will tell us the behavior of the LTI system at $k\omega_0$ frequency values

Continuous-time Fourier series and LTI systems

Using CTFS to find the output of CT LTI systems

- In the last lecture, we learned how to find the Fourier series coefficients of the CT periodic signals
- Let's look at how we can use the continuous-time Fourier series to find the output of an LTI system to a periodic input signal

Using CTFS to find the output of CT LTI systems

Example. Consider an LTI system with the impulse response $h(t) = \delta(t-1)$. Use the continuous-time Fourier series to find the output of the system to input signal $x(t) = 1 + 2cos(2\pi t) - cos(3\pi t)$.

1. We will find the Fourier series coefficients of $x(t)$ denoted by a_k
2. We will find the frequency response, $H(j\omega)$, of the system
3. We will find the output using $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$
4. Since $x(t)$ is a periodic signal with the period T and $\omega_0 = \frac{2\pi}{T}$, the output of the system $y(t)$ will also be periodic with the period T.

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4. Since $x(t)$ is a periodic signal with the period T and $\omega_0 = \frac{2\pi}{T}$, the output of the system $y(t)$ will also be periodic with the period T
5. If we show the Fourier series coefficients of $y(t)$ with b_k,

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$b_k = a_k H(jk\omega_0)$$

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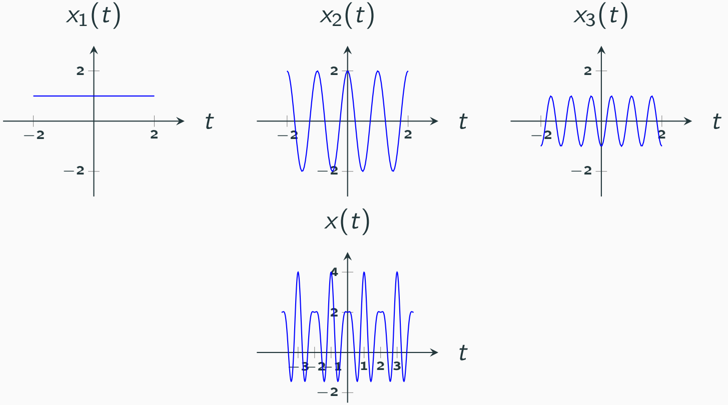
$$x(t) = 1 + 2cos(2\pi t) - cos(3\pi t) = x_1(t) + x_2(t) + x_3(t)$$



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1. We will find the Fourier series coefficients of $x(t)$ denoted by a_k

   $x(t)$ is in the sinusoidal format, so we can write $x(t)$ using complex exponentials and find its Fourier series coefficients directly

$$x(t) = 1 + 2cos(2\pi t) - cos(3\pi t)$$

Using CTFS to find the output of CT LTI systems

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$$\begin{aligned}
x(t) &= 1 + 2cos(2\pi t) - cos(3\pi t) \\
&= 1 \times e^0 + 2 \times (\frac{e^{j2\pi t} + e^{-j2\pi t}}{2}) - 1 \times (\frac{e^{j3\pi t} + e^{-j3\pi t}}{2}) \\
&= 1 \times e^0 + 2 \times \frac{e^{j2\pi t}}{2} + 2 \times \frac{e^{-j2\pi t}}{2} - 1 \times \frac{e^{j3\pi t}}{2} - 1 \times \frac{e^{-j3\pi t}}{2} \\
&= 1 \times e^0 + 1 \times e^{j2\pi t} + 1 \times e^{-j2\pi t} - \frac{1}{2} \times e^{j3\pi t} - \frac{1}{2} \times e^{-j3\pi t}
\end{aligned}$$

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$$\begin{aligned} x(t) &= 1 + 2cos(2\pi t) - cos(3\pi t) \\ &= 1 \times e^0 + 1 \times e^{j2\pi t} + 1 \times e^{-j2\pi t} - \frac{1}{2} \times e^{j3\pi t} - \frac{1}{2} \times e^{-j3\pi t} \end{aligned}$$

Comparing the above expression with the Fourier series representation of $x(t)$, we can find the Fourier series coefficients of $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\pi t}$$

$$a_0 = 1, \quad a_{-2} = 1, \quad a_2 = 1, \quad a_{-3} = -0.5, \quad a_3 = -0.5$$

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2. Let's find the frequency response, $H(j\omega)$, of the system

$$\begin{aligned}
H(j\omega) &= \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau-1) e^{-j\omega\tau} d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau-1) e^{-j\omega} d\tau \\
&= e^{-j\omega} \int_{-\infty}^{\infty} \delta(\tau-1) d\tau \\
&= e^{-j\omega}
\end{aligned}$$

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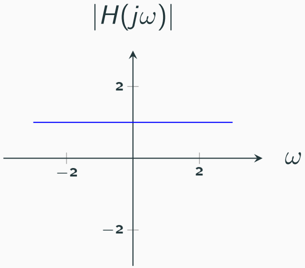
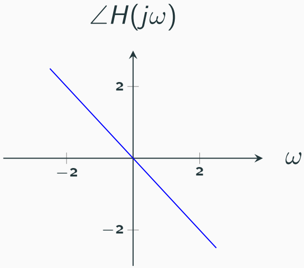
The frequency response, $H(j\omega)$, of the LTI system will tell us the behavior of the system at different frequencies.

$$H(j\omega) = e^{-j\omega}$$





Using CTFS to find the output of CT LTI systems

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3. We will find the output using $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$
4. Since $x(t)$ is a periodic signal with the period T and $\omega_0 = \frac{2\pi}{T}$, the output of the system $y(t)$ will also be periodic with the period T.

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$$a_0 = 1, \quad a_{-2} = 1, \quad a_2 = 1, \quad a_{-3} = -0.5, \quad a_3 = -0.5$$

$$H(j\omega) = e^{-j\omega}, \quad so \quad H(jk\omega) = e^{-jk\omega}$$

- $H(jk\omega) = e^{-jk\omega}$ is a complex number whose magnitude and angle depends on the value of k and ω
- Since $x(t)$ has the fundamental frequency of $\omega_0 = \pi$, in the equation of $H(jk\omega) = e^{-jk\omega}$ we will replace ω with $\omega_0 = \pi$
- Since the Fourier series coefficients of $x(t)$ a_k are non-zero for $k = -3, -2, 0, 2, 3$, we only need to find calculate $H(jk\omega_0) = e^{-jk\omega_0}$ for these k values

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$$H(j\omega) = e^{-j\omega}, \quad so \quad H(jk\omega) = e^{-jk\omega}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k \times e^{-jk\pi} \times e^{jk\pi t}$$

$$y(t) = a_0 + a_{-3}e^{3j\pi}e^{-j3\pi t} + a_3 e^{-3j\pi}e^{j3\pi t} + a_{-2}e^{2j\pi}e^{-j2\pi t} + a_2 e^{-2j\pi}e^{j2\pi t}$$

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$$
\begin{aligned}
y(t) &= a_0 + a_{-3}e^{3j\pi}e^{-j3\pi t} + a_3 e^{-3j\pi}e^{j3\pi t} + a_{-2}e^{2j\pi}e^{-j2\pi t} + a_2 e^{-2j\pi}e^{j2\pi t} \\
&= 1 + \frac{-1}{2}e^{3j\pi}e^{-j3\pi t} + \frac{-1}{2}e^{-3j\pi}e^{j3\pi t} + e^{2j\pi}e^{-j2\pi t} + e^{-2j\pi}e^{j2\pi t} \\
&= 1 + \frac{-1}{2}\left[e^{3j\pi}e^{-j3\pi t} + e^{-3j\pi}e^{j3\pi t}\right] + \left[e^{2j\pi}e^{-j2\pi t} + e^{-2j\pi}e^{j2\pi t}\right] \\
&= 1 + \frac{-1}{2}\left[e^{-j(3\pi t - 3\pi)} + e^{j(3\pi t - 3\pi)}\right] + \left[e^{-j(2\pi t - 2\pi)} + e^{j(2\pi t - 2\pi)}\right] \\
&= 1 - cos(3\pi t - 3\pi) + 2cos(2\pi t - 2\pi) \\
&= 1 - cos(3\pi t - \pi) + 2cos(2\pi t)
\end{aligned}
$$

Note: $cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$

Using CTFS to find the output of CT LTI systems

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$$\begin{aligned}
y(t) &= a_0 + a_{-3}e^{3j\pi}e^{-j3\pi t} + a_3 e^{-3j\pi}e^{j3\pi t} + a_{-2}e^{2j\pi}e^{-j2\pi t} + a_2 e^{-2j\pi}e^{j2\pi t} \\
&= 1 + \frac{-1}{2}e^{3j\pi}e^{-j3\pi t} + \frac{-1}{2}e^{-3j\pi}e^{j3\pi t} + e^{2j\pi}e^{-j2\pi t} + e^{-2j\pi}e^{j2\pi t} \\
&= 1 + \frac{-1}{2}\left[e^{3j\pi}e^{-j3\pi t} + e^{-3j\pi}e^{j3\pi t}\right] + \left[e^{2j\pi}e^{-j2\pi t} + e^{-2j\pi}e^{j2\pi t}\right] \\
&= 1 + \frac{-1}{2}\left[e^{-j(3\pi t - 3\pi)} + e^{j(3\pi t - 3\pi)}\right] + \left[e^{-j(2\pi t - 2\pi)} + e^{j(2\pi t - 2\pi)}\right] \\
&= 1 - cos(3\pi t - 3\pi) + 2cos(2\pi t - 2\pi) \\
&= 1 - cos(3\pi t - \pi) + 2cos(2\pi t)
\end{aligned}$$

The system does not change the amplitude of x(t). It only adds a phase shift to the input signal. The value of phase shift depends on the frequency components of the input signal.

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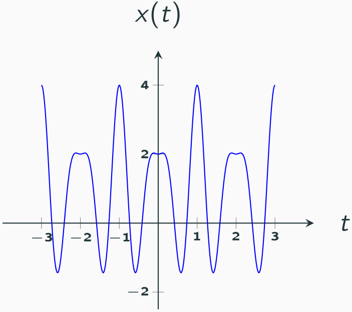
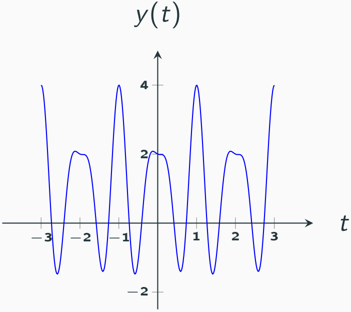
$$y(t) = 1 + 2cos(2\pi t) - cos(3\pi t - \pi)$$





Discrete-time Fourier series and LTI systems

Using DTFS to find the output of DT LTI systems

Example. Consider a DT LTI system with the impulse response h[n]. Assume a periodic input signal, $x[n]$, with period N and fundamental frequency $\omega_0 = \frac{2\pi}{N}$ is applied to the system. Find the output of the system, $y[n]$.

1. We will find the Fourier series coefficients of $x[n]$ denoted by a_k
2. We will find the frequency response, $H(e^{j\omega})$, of the system
3. We will find the output using $y[n] = \sum_{k=<N>} a_k H(e^{jk\omega_0})e^{jk\omega_0 n}$
4. Since $x[n]$ is a periodic signal with the period N and $\omega_0 = \frac{2\pi}{N}$, the output of the system $y[n]$ will also be periodic with the period N.

Using DTFS to find the output of DT LTI systems

Example. Consider a DT LTI system with the impulse response h[n]. Assume a periodic input signal, $x[n]$, with period N and fundamental frequency $\omega_0 = \frac{2\pi}{N}$ is applied to the system. Find the output of the system, $y[n]$.

4. Since $x[n]$ is a periodic signal with the period N and $\omega_0 = \frac{2\pi}{N}$, the output of the system $y[n]$ will also be periodic with the period N
5. If we show the Fourier series coefficients of $y[n]$ with b_k,

$$y[n] = \sum_{k=<N>} b_k e^{jk\omega_0 n}$$

$$y[n] = \sum_{k=<N>} a_k H(e^{jk\omega_0}) e^{jk\omega_0 n}$$

$$b_k = a_k H(e^{jk\omega_0})$$

Example: Using DTFS to find the output of DT LTI systems

- Consider a discrete time LTI system whose frequency response is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}$$

What is the output of this system to a periodic input signal with period of $N = 3$ and Fourier series coefficients $a_0 = 1$, $a_1 = j$, $a_{-1} = -j$.

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$$b_k = a_k H\left(e^{jk\omega_0}\right)$$

$$b_0 = a_0 H\left(e^{j0\times\frac{2\pi}{3}}\right) = a_0 H\left(e^{jk\omega_0}\right)\Big|_{k\omega_0=0} = a_0 H\left(e^{j\omega}\right)\Big|_{\omega=0}$$

$$b_0 = a_0 \times 1 = a_0 = 1$$

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$$b_1 = a_1 H\left(e^{j1\times\frac{2\pi}{3}}\right) = a_1 H\left(e^{jk\omega_0}\right)\Big|_{k\omega_0=\frac{2\pi}{3}} = a_1 H\left(e^{j\omega}\right)\Big|_{\omega=\frac{2\pi}{3}}$$

$$b_1 = a_1 \times 0 = 0$$

note: $\pi/8 < 2\pi/3 < \pi$

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$$b_{-1} = a_{-1} H\left(e^{-j2\pi/3}\right) = a_{-1} H\left(e^{jk\omega_0}\right)\Big|_{k\omega_0 = -2\pi/3} = a_{-1} H\left(e^{j\omega}\right)\Big|_{\omega = -2\pi/3}$$

$$b_{-1} = a_{-1} \times 0 = 0$$

note: $-\pi < -2\pi/3 < -\pi/8$

Example: Using DTFS to find the output of DT LTI systems

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What is the output of this system to a periodic input signal with period of $N = 3$ and Fourier series coefficients $a_0 = 1$, $a_1 = j$, $a_{-1} = -j$.

$b_0 = 1$, $b_1 = 0$, $b_{-1} = 0$

$y[n] = \sum_{k=-1}^{1} b_k e^{jk\omega_0 n} = 1 \longrightarrow y[n] = 1$ for all n

Example: Using DTFS to find the output of DT LTI systems

- Consider a discrete time LTI system whose frequency response is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}$$

  What is the output of this system to a periodic input signal with period of $N = 3$ and Fourier series coefficients $a_0 = 1$, $a_1 = j$, $a_{-1} = -j$.

$$b_k = a_k H\left(e^{jk\omega_0}\right)$$

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Example: Using DTFS to find the output of DT LTI systems

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$$b_1 = a_1 H(e^{j1\times\frac{2\pi}{3}}) = a_1 H(e^{jk\omega_0})\Big|_{k\omega_0=\frac{2\pi}{3}} = a_1 H(e^{j\omega})\Big|_{\omega=\frac{2\pi}{3}}$$

$$b_1 = a_1 \times 0 = 0$$

note: $\pi/8 < 2\pi/3 < \pi$

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- Consider a discrete time LTI system whose frequency response is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}$$

What is the output of this system to a periodic input signal with period of $N = 3$ and Fourier series coefficients $a_0 = 1$, $a_1 = j$, $a_{-1} = -j$.

$b_0 = 1$, $b_1 = 0$, $b_{-1} = 0$

$y[n] = \sum_{k=-1}^{1} b_k e^{jk\omega_0 n} = 1 \longrightarrow y[n] = 1$ for all n