Discussion week 3:

Convolution: $$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

For discrete-time signals:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

① Flip $h(\tau)$ to get $h(-\tau)$

② Shift $h(-\tau)$ by t to get $h(t-\tau)$

③ Multiply $x(\tau)$ and $h(t-\tau)$

④ Integrate over τ for all time.

Example #1: $x(t) = 2u(t+2)$, $h(t) = \frac{1}{4}\int_{-\infty}^{t+5} \delta(t-1)$, what is $y(t)$?

example # 2 : $x(t) = 4\left[u(t-2) - u(t-7)\right]$, $h(t) = u(t-4)$

Example #3: $x(t) = e^{-2t}\,u(t)$, $h(t) = e^{-3t}\,u(t)$

Example #4: $x(t) = 3\,e^{-t}\,\text{rect}\left(\frac{t}{4}\right)$ $\quad h(t) = \int_{t-1}^{t+1} e^{-(t-\lambda)}\,\delta(\lambda)\,d\lambda$

Example #5: $x(t) = \text{rect}\left(\frac{t}{2}\right)$, $h(t) = \text{rect}(t-1)$

Solve analytically.

Some properties of convolution

① $x(t) * h(t) = h(t) * x(t)$

② $x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$

③ $y(t) = x(t) * h(t) \Rightarrow y'(t) = x(t-t_0) * h(t) = y(t-t_0)$

④ $x(t) * \delta(t) = x(t)$, $x(t) * \delta(t-t_0) = x(t-t_0)$