UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2022

Linear Systems Fundamentals

FINAL EXAM

Preface

- I recognize that final exams are stressful in the best of times, and the past 2 years have been unusually hard times.
- Please remember that despite the stress, I am counting on you to uphold academic integrity while you complete your final exam.
- Posting or seeking exam questions or answers online, or by consulting unauthorized resources, is a gross violation of our principles of integrity and engineering ethics.
- To be fair to all students, any integrity violations discovered during the final exam will be reported to the Dean of Engineering and to the office of Academic Integrity.
- Please make sure you understand and follow the academic integrity guidelines for the exam. If you are not sure, ask me.
- An honest effort, no matter what the outcome, is something to be proud of, especially in these challenging times.
- I am very proud of the commitment and resilience you have displayed. You should feel proud, too.
- **Good luck on the exam!**

PRINT YOUR NAME ______________________________

Student ID Number ______________________________

Signature ______________________________

Your signature confirms that you have completed this exam on your own and in accordance with the Academic Integrity Agreement.

Instructions

- No electronics allowed for problem solving.
- Four 2-sided sheets of your own notes are allowed.
- Time allowed: 3 hours (including Gradescope upload time).
- Write your solutions in the designated spaces in the exam.
- **Justify all of your answers. Credit requires proper justification.**
- If extra pages are needed to show all of your work, use the scratch pages at the end of the exam and submit them.
- **Upload your solutions by 6pm via Gradescope.**

Problem	Weight	Score
1	20 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
6	20 pts	
Total	120 pts	

You've got this!

Name/Student ID: ______________________________

Problem 1 [CTFT Properties] (20 points, 10 points each part)

Let $x(t) = u(t+1) - u(t-2)$. Let $X(j\omega)$ denote its Fourier transform.

(a) Determine $\int_{-\infty}^{\infty} X(j\omega) d\omega$.

(b) Determine $Y(j\frac{\pi}{2})$ where $Y(j\omega)$ is the Fourier transform of $y(t) = \frac{dx(t)}{dt}$.

Write answers to parts (a) and (b) on the following pages.

Justify your answers.

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Problem 1 (cont.)

(a) (10 points)

Determine $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

$$\boxed{\int_{-\infty}^{\infty} X(j\omega)d\omega =}$$

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Problem 1 (cont.)

(b) (10 points)

Determine $Y(j\frac{\pi}{2})$ where $Y(j\omega)$ is the Fourier transform of $y(t) = \frac{dx(t)}{dt}$.

$$Y(j\frac{\pi}{2}) =$$

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Problem 1 (cont.)

Check the box next to the solution you found above, or check **None of the above** if it does not appear on the list.

Problem 1(a):

- ☐ 1
- ☐ 2
- ☐ 2π
- ☐ $\frac{1}{2\pi}$
- ☐ None of the above.

Problem 1(b):

- ☐ 0
- ☐ $j+1$
- ☐ $j\frac{\pi}{2}$
- ☐ $\frac{2}{\pi}$
- ☐ None of the above.

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Problem 2 [DTFT] (20 points; 10 points each part)

Let $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Let $y[n] = \delta[n] + \frac{1}{2}\delta[n-1]$.

(a) Determine the frequency response $H(e^{j\omega})$ of the LTI system that produces output $y[n]$ when $x[n]$ is the input.

(b) Determine the impulse response $h[n]$ of the LTI system that produces output $y[n]$ when $x[n]$ is the input.

Write answers to parts (a) and (b) on the following pages.

Justify your answers.

Name/Student ID: ____________________

Problem 2 (cont.)

Let $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Let $y[n] = \delta[n] + \frac{1}{2}\delta[n-1]$.

(a) (10 points)

Determine the frequency response $H(e^{j\omega})$ of the LTI system that produces output $y[n]$ when $x[n]$ is the input.

$H(e^{j\omega}) =$

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Problem 2 (cont.)

Let $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Let $y[n] = \delta[n] + \frac{1}{2}\delta[n-1]$.

(b) (10 points)

Determine the impulse response $h[n]$ of the LTI system that produces output $y[n]$ when $x[n]$ is the input

$\boxed{h[n] = \qquad\qquad\qquad\qquad}$

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Problem 2 (cont.)

Check the box next to the solution you found above, or check **None of the above** if it does not appear on the list.

Problem 2(a):

- ☐ $\frac{1-\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$
- ☐ $\frac{1+\frac{1}{2}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$
- ☐ $\frac{1}{1-\frac{1}{4}e^{-j2\omega}}$
- ☐ $1-\frac{1}{4}e^{-j2\omega}$
- ☐ None of the above.

Problem 2(b):

- ☐ $\delta[n]-\frac{1}{4}\delta[n-2]$
- ☐ $\delta[n]-\frac{1}{2}\delta[n-2]$
- ☐ $\left(\frac{1}{4}\right)^n u[n]$
- ☐ $\left(\frac{1}{2}\right)^n u[n]+\left(\frac{1}{2}\right)^n u[n-1]$
- ☐ None of the above.

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Problem 3 [Modulation] (20 points, 10 points each part)

Let $x(t) = \frac{\sin(20t)}{\pi t}$.

The signal $x(t)$ is modulated with a carrier signal $\cos(30t)$ to produce the signal $y(t) = x(t)\cos(30t)$. The signal $y(t)$ is passed through a bandpass filter with frequency response

$$H_{BP}(j\omega) = \begin{cases} 1, & 30 < |\omega| < 50 \\ 0, & \text{otherwise.} \end{cases}$$

Let $w(t)$ be the output of the bandpass filter. The signal $w(t)$ is modulated with a carrier signal $\cos(40t)$ to produce $z(t) = w(t)\cos(40t)$. The signal $z(t)$ is then passed through an ideal low-pass filter with gain 2 and cut-off frequency $\omega_c = 20$. Let $r(t)$ be the output of the low-pass filter.

(a) Determine the Fourier transform $R(j\omega)$ of $r(t)$, expressed in terms of $X(j\omega)$.

(b) Determine the output signal $r(t)$.

Write answers to parts (a) and (b) on the following pages.

Justify your answers.

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Problem 3 (cont.)

(a) (10 points)

Determine the Fourier transform $R(j\omega)$ of $r(t)$, expressed in terms of $X(j\omega)$.

$R(j\omega) =$

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Problem 3 (cont.)

(b) (10 points)

Determine the output signal $r(t)$.

$r(t) =$

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Problem 3 (cont.)

Check the box next to the solution you found above, or check **None of the above** if it does not appear on the list.

Problem 3(a):

- ☐ 0
- ☐ $X(j(\omega - 10)) + X(j(\omega + 10))$
- ☐ $\frac{1}{2}X(j\omega)$
- ☐ $X(j2\omega)$
- ☐ None of the above.

Problem 3(b):

- ☐ 0
- ☐ $\frac{\sin(10t)}{\pi t}$
- ☐ $\frac{\sin(10t)}{2\pi t}$
- ☐ $\frac{\sin(20t)}{\pi t}\cos(10t)$
- ☐ None of the above.

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Problem 4 [Sampling Theory] (20 points, 10 points each part)

Let $x(t)$ be a signal with Fourier transform $X(j\omega)$ satisfying

$$X(j\omega) = 0, \text{ for } |\omega| > W.$$

Determine the minimum sampling frequency ω_s required to avoid aliasing and allow reconstruction of $y(t)$ (defined below) from its samples, or write "Reconstruction not possible for any ω_s" if no such ω_s exists. Refer specifically to the formula for $Y(j\omega)$.

(a) $y(t) = (x(3t-1))^2$

(b) $y(t) = x(t) * \frac{\sin(2Wt)}{\pi t}$

Write answers to parts (a) and (b) on the following pages.

Justify your answers.

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Problem 4 (cont.)

Let $x(t)$ be a signal with Fourier transform $X(j\omega)$ satisfying $X(j\omega) = 0$, for $|\omega| > W$. Determine the minimum sampling frequency ω_s required to avoid aliasing and allow reconstruction of $y(t)$ from its samples, or write "Reconstruction not possible for any ω_s" if no such ω_s exists. Refer to $Y(j\omega)$.

(a) (10 points)

$y(t) = (x(3t-1))^2$

$\omega_s >$

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Problem 4 (cont.)

Let $x(t)$ be a signal with Fourier transform $X(j\omega)$ satisfying $X(j\omega) = 0$, for $|\omega| > W$. Determine the minimum sampling frequency ω_s required to avoid aliasing and allow reconstruction of $y(t)$ from its samples, or write "Reconstruction not possible for any ω_s" if no such ω_s exists. Refer to $Y(j\omega)$.

(b) (10 points)

$y(t) = x(t) * \frac{\sin(2Wt)}{\pi t}$

$\omega_s >$

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Problem 4 (cont.)

Check the box next to the solutions you found above, or check **"Reconstruction not possible for any ω_s"** if no such ω_s exists.

Problem 4(a):

- ☐ $3W$
- ☐ $6W$
- ☐ $9W$
- ☐ $12W$
- ☐ Reconstruction not possible for any ω_s.

Problem 4(b):

- ☐ $\frac{W}{2}$
- ☐ W
- ☐ $2W$
- ☐ $4W$
- ☐ Reconstruction not possible for any ω_s.

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Problem 5 [Laplace Transform] (20 points)

Let

$$X(s) = \frac{s+2}{(s^2 - s - 2)} = \frac{s+2}{(s+1)(s-2)}.$$

(a) Determine the number N of distinct signals that have Laplace transform expressed as $X(s)$ in their region of convergence.

(b) Determine the signal $x(t)$ with Laplace transform $X(s)$ that has a Fourier transform.

(c) Determine an explicit formula for the magnitude $|X(j\omega)|$ of the Fourier transform of the signal $x(t)$ in part (b). Express it without using any imaginary numbers.

Write your answers to parts (a), (b), and (c) on the following pages.

Justify your answers.

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Problem 5 (cont.)

Let $X(s) = \frac{s+2}{(s^2-s-2)} = \frac{s+2}{(s+1)(s-2)}$.

(a) (5 points)

Determine the number N of distinct signals that have Laplace transform expressed as $X(s)$ in their region of convergence.

$N =$

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Problem 5 (cont.)

Let $X(s) = \frac{s+2}{(s^2-s-2)} = \frac{s+2}{(s+1)(s-2)}$.

(b) (10 points)

Determine the signal $x(t)$ with Laplace transform $X(s)$ that has a Fourier transform.

$x(t) =$

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Problem 5 (cont.)

Let $X(s) = \frac{s+2}{(s^2-s-2)} = \frac{s+2}{(s+1)(s-2)}$.

(c) (5 points)

Determine an explicit formula for the magnitude $|X(j\omega)|$ of the Fourier transform of the signal $x(t)$ in part (b). Express it without using any imaginary numbers.

$|X(j\omega)| =$

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Problem 5 (cont.)

Check the box next to the solutions you found above, or check **None of the above** if it does not appear on the list.

Problem 5(a):

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ None of the above.

Problem 5(b):

- ☐ $-\frac{1}{3}e^{t}u(t) - \frac{4}{3}e^{-2t}u(-t)$
- ☐ $-\frac{1}{3}e^{-t}u(t) - \frac{4}{3}e^{2t}u(t)$
- ☐ $-\frac{4}{3}e^{t}u(t) - \frac{1}{3}e^{-2t}u(t)$
- ☐ $-\frac{4}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(t)$
- ☐ None of the above.

Problem 5(c):

- ☐ $|X(j\omega)| = \frac{1}{|w+1|}$
- ☐ $|X(j\omega)| = \frac{1}{|w-1|}$
- ☐ $|X(j\omega)| = \sqrt{\frac{1}{\omega^2+1}}$
- ☐ $|X(j\omega)| = \sqrt{\frac{\omega^2+4}{(\omega^2+1)(\omega^2-2)}}$
- ☐ None of the above.

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Problem 6 [LT and LTI Systems] (20 points)

Let $H(s)$ be given by

$$H(s) = \frac{s^2 - 1}{(s+2)(s-(-1+j))(s-(-1-j))}.$$

(a) Determine the region of convergence (ROC) of a causal, stable LTI system S_1 with transfer function $H_1(s) = H(s)$.

(b) Determine a differential equation relating the input $x(t)$ and corresponding output $y(t)$ of the system S_1 in part (a).

(c) The system S_1 is serially concatenated with another causal system S_2 that has impulse response $h_2(t) = \frac{3}{2}e^{t}u(t) - \frac{1}{2}e^{-t}u(t)$. Determine the transfer function $G(s)$ of the system obtained from this concatenation of S_1 and S_2.

Write your answers to parts (a), (b), and (c) on the following pages.

Justify your answers.

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Problem 6 (cont.)

Let $H(s) = \frac{s^2-1}{(s+2)(s-(-1+j))(s-(-1-j))}$.

(a) (5 points)

Determine the region of convergence (ROC) of a causal, stable LTI system S_1 with transfer function $H_1(s) = H(s)$.

$ROC =$

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Problem 6 (cont.)

Let $H(s) = \frac{s^2-1}{(s+2)(s-(-1+j))(s-(-1-j))}$.

(b) (5 points) Determine a differential equation relating the input $x(t)$ and corresponding output $y(t)$ of the system S_1 in part (a).

Differential equation :

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Problem 6 (cont.)

Let $H(s) = \frac{s^2-1}{(s+2)(s-(-1+j))(s-(-1-j))}$.

(c) (10 points)

The system S_1 is serially concatenated with another causal system S_2 that has impulse response $h_2(t) = \frac{3}{2}e^t u(t) - \frac{1}{2}e^{-t}u(t)$. Determine the transfer function $G(s)$ of the concatenation of S_1 and S_2.

$G(s) =$

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Problem 6 (cont.)

Check the box next to the solutions you found above, or check **None of the above** if it does not appear on the list.

Problem 6(a):

- ☐ $-2 < \mathcal{R}e\{s\} < -1$
- ☐ $\mathcal{R}e\{s\} > 1$
- ☐ $\mathcal{R}e\{s\} > -1$
- ☐ $\mathcal{R}e\{s\} > -2$
- ☐ None of the above.

Problem 6(b):

- ☐ $\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 4y(t) = \frac{d^2x(t)}{dt^2} - x(t)$
- ☐ $\frac{d^3y(t)}{dt^3} - 4\frac{d^2y(t)}{dt^2} - 6\frac{dy(t)}{dt} - 4y(t) = \frac{d^2x(t)}{dt^2} - x(t)$
- ☐ $\frac{d^2y(t)}{dt^2} - y(t) = \frac{d^3x(t)}{dt^3} + 4\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} + 4x(t)$
- ☐ $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 6y(t) + 4 = \frac{dx(t)}{dt} - 1$
- ☐ None of the above.

Problem 6(c):

- ☐ $G(s) = s^2 + 2s + 2$
- ☐ $G(s) = \frac{1}{s^2+2s+2}$
- ☐ $G(s) = \frac{s^2-1}{s^2+2s+2}$
- ☐ $G(s) = \frac{(s^2-1)^2}{(s+2)^2(s^2+2s+2)}$
- ☐ None of the above.

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