UNIVERSITY OF CALIFORNIA, SAN DIEGO
Electrical & Computer Engineering Department
ECE 101 - Fall 2023
Linear Systems Fundamentals

MIDTERM EXAM

You are allowed one 2-sided sheet of notes.

No books, no other notes, no calculators.

PRINT YOUR NAME ______________________________

Signature ______________________________

Student ID Number ______________________________

Your signature confirms that you have completed this exam in accordance with the ECE 101 Academic Integrity Agreement.

Problem	Weight	Score
1	32 pts	
2	36 pts	
3	32 pts	
Total	100 pts	

Please do not begin until told.
Show your work.
Use back of previous page and attached scratch sheets as needed.
Tables 3.1 and 3.2 from the textbook are attached to the exam.
Scan, upload, and tag your solutions on Gradescope at the end of the exam.

Good luck!

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Problem 1 [DT Signals] (32 points)

(a) (16 points, 8 points each part)

Indicate whether or not the following statements are True or False. As usual, $\delta[n]$ denotes the unit impulse signal, $u[n]$ denotes the unit step signal, and $*$ denotes the discrete-time (DT) convolution operator.

Justify your answers.

True	False	
☐	☐	$x[n] * y[n-2] = x[n-2] * y[n]$, for all DT signals $x[n]$, $y[n]$.
☐	☐	Let $y[n] = x[n] * u[n-4]$, where $x[n]$ is any discrete-time signal. Then $y[0] = \sum_{n=-\infty}^{4} x[n]$.

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Problem 1 [DT Signals] (cont.)

(b) (16 points, 8 points each part)

Let $x[n] = e^{j\frac{\pi}{5}n} + \sum_{k=-\infty}^{\infty}(-1)^k\delta[n-4k]$.

(i) (8 points)

Determine the fundamental frequency Ω of $x[n]$. Check box below. **Justify your answer.**

☐ $\Omega = \frac{2\pi}{10}$

☐ $\Omega = \frac{2\pi}{20}$

☐ $\Omega = \frac{2\pi}{40}$

☐ $\Omega = \frac{2\pi}{80}$

Problem 1 [DT Signals] (cont.)

(b) (cont.)

Let $x[n] = e^{j\frac{\pi}{5}n} + \sum_{k=-\infty}^{\infty}(-1)^k\delta[n-4k]$.

(ii) (8 points)

Determine the even part of $x[n]$, $f[n] = \mathcal{E}v\{x[n]\}$. Check box below. **Justify your answer.**

- ☐ $f[n] = 2\cos(\frac{\pi}{5}n) + 2\sum_{k=-\infty}^{\infty}(-1)^k\delta[n-4k]$
- ☐ $f[n] = \cos(\frac{\pi}{5}n) + \sum_{k=-\infty}^{\infty}(-1)^k\delta[n-4k]$
- ☐ $f[n] = \cos(\frac{\pi}{5}n) + \sum_{k=-\infty}^{\infty}\delta[n-4k]$
- ☐ $f[n] = \cos(\frac{\pi}{5}n)$

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Problem 2 [DT LTI Systems] (36 points)

Let S be the DT LTI system defined by the difference equation:

$$y[n] = x[n-1] + x[n+1].$$

(a) (6 points)

Determine the impulse response $h[n]$ of system S. Sketch $h[n]$ precisely.

(b) (6 points)

Determine the step response $s[n]$ of system S. Sketch $s[n]$ precisely.

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Problem 2 (cont.)

(c) (6 points)

Determine the system function $H(z)$ of system S.

(d) (6 points)

Does there exist a complex number $z \neq 0$ such that $H(z) = 0$?

If not, why not? If so, give an example of such a number z.

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Problem 2 (cont.)

e) (12 points, 6 points each part)

Check the appropriate box indicating whether or not system S satisfies the specified properties. **Justify your answers.**

True	False	
☐	☐	Invertible

☐	☐	Stable

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Problem 3 [DTFS and DT Filtering] (32 points)

(a) (10 points, 5 points each part)

Let $x[n]$ be the a discrete-time periodic signal with fundamental period $N = 2$, and Fourier Series coefficients a_k. Let $y[n] = x[n-1]$.

The Fourier Series coefficients b_k of $y[n]$ are $b_k = (-1)^k$, for all k.

(i) (5 points)

Determine the Fourier Series coefficients a_k of the signal $x[n]$.

(ii) (5 points)

Identify and sketch the signal $x[n]$ precisely.

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Problem 3 (cont.)

(b) (10 points)

Find the fundamental period N and the DT Fourier Series coefficients a_k, $k = 0, \ldots, N-1$ of the signal $x[n] = \sin(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$. Use the DTFS coefficients to decide if the signal is even or odd.

Justify your answers.

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Problem 3 (cont.)

(c) (12 points)

Let $x[n]$ be a DT signal with fundamental period $N = 4$ and DT Fourier series coefficients $a_0 = 3, a_1 = 1, a_2 = -1, a_3 = 1$. Consider the DT LTI system whose frequency response in the interval $[-\pi, \pi]$ is given by

$$H(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega| \leq \frac{2\pi}{3} \\ 0 & \text{otherwise.} \end{cases}$$

Suppose the signal $x[n]$ is the input to this system. Determine and sketch precisely the output $y[n]$ in the interval $[0, \ldots, 3]$.

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TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\left\{\begin{array}{l} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{array}\right.$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\left\{\begin{array}{ll} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{array}\right.$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{array}{l} x[n] \\ y[n] \end{array}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{array}{l} a_k \\ b_k \end{array}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n - r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n - 1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$