Discussion Session 8

Fourier Transform: $X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, e^{jwt}\, dt$$

If $x(t)$ is peiodic signal, you can define the Fourier Transform from Fourier Series. $x(t) \longleftrightarrow \{a_k\} \Rightarrow X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta(w - kw_0)$

Fourier Transfom

$\delta(t) \xleftrightarrow{F} 2\pi$ $\qquad$ $\frac{1}{2\pi} \xleftrightarrow{F} \delta(w)$

$\frac{e^{jw_0t}}{2\pi} \xleftrightarrow{F} \delta(w - w_0)$

$\begin{cases} \cos(w_0t) \xleftrightarrow{F} \pi\,\delta(w-w_0) + \pi\,\delta(w+w_0) \\ \sin(w_0t) \xleftrightarrow{F} \pi j\,\delta(w-w_0) - \pi j\,\delta(w+w_0) \end{cases}$

$e^{-at}\,u(t) \longleftrightarrow \frac{1}{a+jw}$ $\quad$ $\mathrm{Re}(a) > 0$

$e^{-a|t|} \xleftrightarrow{F} \frac{2a}{a^2 + w^2}$ $\quad$ $\mathrm{Re}(a) > 0$

$\begin{cases} \mathrm{rect}(t) \longleftrightarrow \mathrm{sinc}\left(\frac{w}{2\pi}\right) \\ \mathrm{rect}\left(\frac{t}{2T_1}\right) \longleftrightarrow 2T_1\,\mathrm{sinc}\left(\frac{wT_1}{\pi}\right) \end{cases}$ $\qquad$ $\begin{cases} \delta(t) \xleftrightarrow{F} 1 \\ u(t) \xleftrightarrow{F} \frac{1}{jw} + \pi\,\delta(w) \end{cases}$

$\begin{cases} \mathrm{sinc}(t) \longleftrightarrow \mathrm{rect}\left(\frac{w}{2\pi}\right) \\ \frac{B}{2\pi}\,\mathrm{sinc}\left(\frac{Bt}{2\pi}\right) \longleftrightarrow \mathrm{rect}\left(\frac{w}{B}\right) \end{cases}$

Duality property of fourier transform:

If the fourier transform of $x(t)$ is $Y(jw)$, fourier transform of $Y(t)$ is $2\pi\, x(-w)$: $\quad F(x(t)) = Y(jw) \Rightarrow F(Y(t)) = 2\pi\, x(-w)$

Fourier transform and signal tranformation.

$x(t) \xleftrightarrow{F} X(jw)$ $\quad y(t) \xleftrightarrow{F} Y(jw)$

Linearity: $a\,x(t) + b\,y(t) \xleftrightarrow{F} a\,X(jw) + b\,Y(jw)$

Time Shift: $x(t-t_0) \xleftrightarrow{F} e^{-jwt_0}\,X(jw)$

Frequency Shift $x(t)\,e^{jw_0 t} \xleftrightarrow{F} X(j(w-w_0))$

Differentiation. $\frac{dx(t)}{dt} \xleftrightarrow{F} jw\,X(jw)$

Integration: $\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{F} \frac{1}{jw}\,X(jw) + \pi X(0)\,\delta(jw)$

especial case:
$x(-t) \longleftrightarrow X(-jw)$

Time scaling: $x(at) \xleftrightarrow{F} \frac{1}{|a|}\,X\left(\frac{jw}{a}\right)$

Frequency scaling: $\frac{1}{|b|}\,X\left(\frac{t}{b}\right) \xleftrightarrow{F} X(jbw)$

Time expansion lead to frequency compression and time compression lead to frequency expansion.

Convolution in time: $x(t) * y(t) \xleftrightarrow{F} X(jw)\,Y(jw)$

Multiplication in time: $x(t)\,y(t) \xleftrightarrow{F} \frac{1}{2}\int_{-\infty}^{\infty} X(j\theta)\,Y(j(w-\theta))\,d\theta$

$$= \frac{1}{2}\left[X(jw) * Y(jw)\right]$$

Parseval's theorem $\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(jw)|^2\,dw$

Example #1:

a) $\mathcal{F}\{e^{-j4t}\ \text{rect}(2t-1)\}$

b) $\mathcal{F}^{-1}\{4\ \text{rect}(2\omega) - 2\}$

c) $\mathcal{F}\{2\ e^{-j2t}\ \text{sinc}\left(\frac{t-1}{2}\right)\}$

Example: Find $\int_{-\infty}^{\infty} |\text{sinc}(2000t)|^2\, dt$