

Assignment#1 One Note Answers Andrew Plum

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$$1) \sum_{i=0}^{n-1} \sum_{i=0}^{n-1} 1 = \sum_{i=0}^{n-1} (n-1) - 0 + 1 = \sum_{i=0}^{n-1} n =$$

$$= [(n-1) - 0 + 1] n \approx \boxed{\Theta(n^2)}$$

$$2) \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-1} (n-1) - (i+1) + 1 =$$

$$= \sum_{i=0}^{n-1} [n-1-i-1+1] = \sum_{i=0}^{n-1} [n-i-1] =$$

$$= [n-1-0+1][n-1] = n(n-1) \approx \boxed{\Theta(n^2)}$$

$$11) \sum_{i=0}^{n-1} (i^2 + 2)^2 = \sum_{i=0}^{n-1} (i^4 + 4i^2 + 4) = \sum_{i=0}^{n-1} i^4 + \sum_{i=0}^{n-1} 4i^2 + \sum_{i=0}^{n-1} 4 =$$

$$\sum_{i=0}^{n-1} 4 = 4 \sum_{i=0}^{n-1} 1 = 4(n-1-0+1) = 4n \approx \Theta(n)$$

$$\begin{aligned} \sum_{i=0}^{n-1} 4i^2 &= 4 \sum_{i=0}^{n-1} \frac{(n-1)((n-1)+1)(2(n-1)+1)}{6} = \\ &= \frac{(n-1)(n)(2n-1)}{6} = \frac{(2n^2-3n+1)(n)}{6} = \\ &= \frac{2n^3-3n^2+n}{6} \approx \Theta(n^3) \end{aligned}$$

$$\sum_{i=0}^{n-1} i^4 = \frac{1}{4+1} (n-1)^{4+1} = \frac{(n-1)^5}{5} \approx \Theta(n^5)$$

$$\approx \Theta(n^5) + \Theta(n^3) + \Theta(n) \approx \boxed{\Theta(n^5)}$$

$$12) \sum_{i=1}^n [(i+2)2^{i-1}] = \sum_{i=1}^n [(i+2)\frac{2^i}{2}] = \frac{1}{2} \sum_{i=1}^n (i2^i + 2^{i+1}) =$$

$$= \frac{1}{2} \sum_{i=1}^n (i2^i) + \frac{1}{2} \sum_{i=1}^n [2 \times 2^i] = \frac{1}{2} \sum_{i=1}^n (i2^i) + \sum_{i=1}^n (2^i) \approx$$

$$\frac{1}{2} \sum_{i=1}^n (i2^i) = \frac{1}{2} [(n-1)2^{n+1} + 2] = 2^n(n-1) + 1 \approx \Theta(n2^n)$$

$$\sum_{i=1}^n (2^i) = \frac{2^{n+1}-1}{2-1} = 2^{n+1}-1 \approx \Theta(2^n)$$

$$\approx \Theta(n2^n) + \Theta(2^n) \approx \boxed{\Theta(n2^n)}$$