Assignment#1 Question#11 Andrew Plum

Wednesday, September 20, 2023 1:39 AM

$$11) \sum_{i=0}^{n-1} (i^2+2)^2 = \sum_{i=0}^{n-1} (i^4+4i^2+4) = \sum_{i=0}^{n-1} i^4 + \sum_{i=0}^{n-1} 4i^2 + \sum_{i=0}^{n-1} 4 =$$

$$\sum_{i=0}^{n-1} 4 = 4\sum_{i=0}^{n-1} 1 = 4(n-1-0+1) = 4n \approx \Theta(n)$$

$$\sum_{i=0}^{n-1} 4i^2 = 4\sum_{i=0}^{n-1} \frac{(n-1)((n-1)+1)(2(n-1)+1)}{6} =$$

$$= \frac{(n-1)(n)(2n-1)}{6} = \frac{(2n^2-3n+1)(n)}{6} =$$

$$= \frac{2n^3-3n^2+n}{6} \approx \Theta(n^3)$$

$$\sum_{i=0}^{n-1} i^4 = \frac{1}{4+1}(n-1)^{4+1} = \frac{(n-1)^5}{5} \approx \Theta(n^5)$$

$$\approx \Theta(n^5) + \Theta(n^3) + \Theta(n) \approx \boxed{\Theta(n^5)}$$