Lecture. probability.

1. Prob is one way to measure uncertainty and perform prediction.

2. Random varible, prob, and distribution

x → event, data, ...

takes values randomly from a set.

event: dice game. x: results of dice game.

$$x \in \{1, 2, 3, 4, 5, 6\}$$

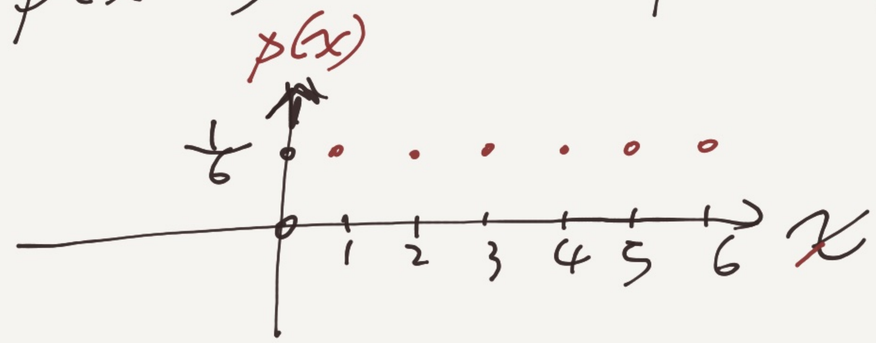
prob:

$p(x) \geq 0$

$\int_{-\infty}^{+\infty} p(x) = 1$

$p(x)$: $p(x=1) = p(x=2) \cdots = p(x=6) = \frac{1}{6}$

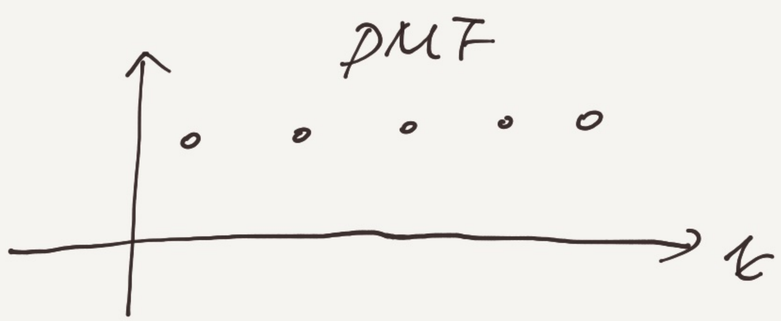
Distribution:



Two types of distributions:

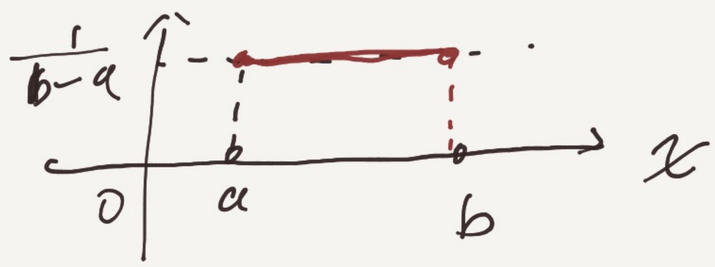
$x \in$ a discrete set, e.g., $\{1, 2, 3, 4, 5, 6\}$, $p(x)$ is called prob. mass function (PMF)

$x \in$ a continuous set, $p(x)$ is called prob. density function (PDF)



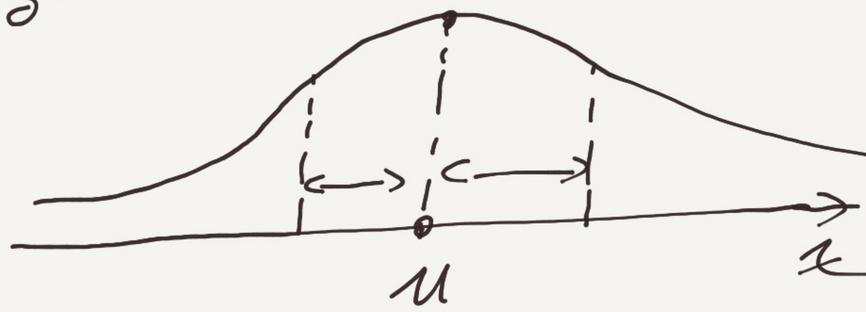
3. Uniform Distribution

$x \in [a, b]$, $b > a$, $p(x) = \frac{1}{b-a} \geq 0$ $\int_a^b p(x) = 1$



4. Gaussian Distribution / normal distribution

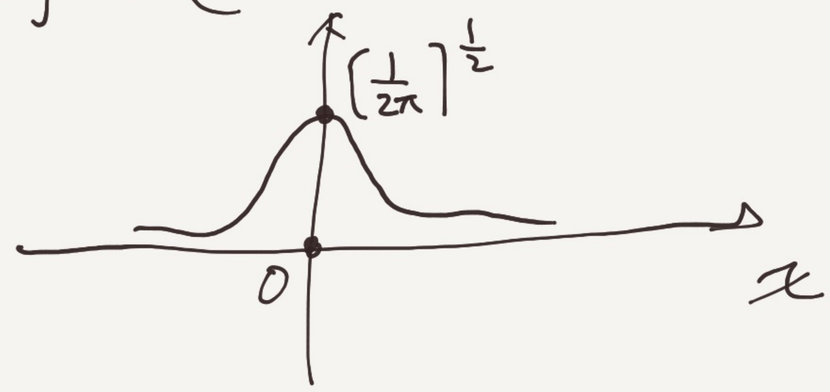
$$\mathcal{N}(x;\mu,\delta^2)=\left[\frac{1}{2\pi\delta^2}\right]^{\frac{1}{2}}\cdot e^{-\frac{(x-\mu)^2}{2\delta^2}}$$



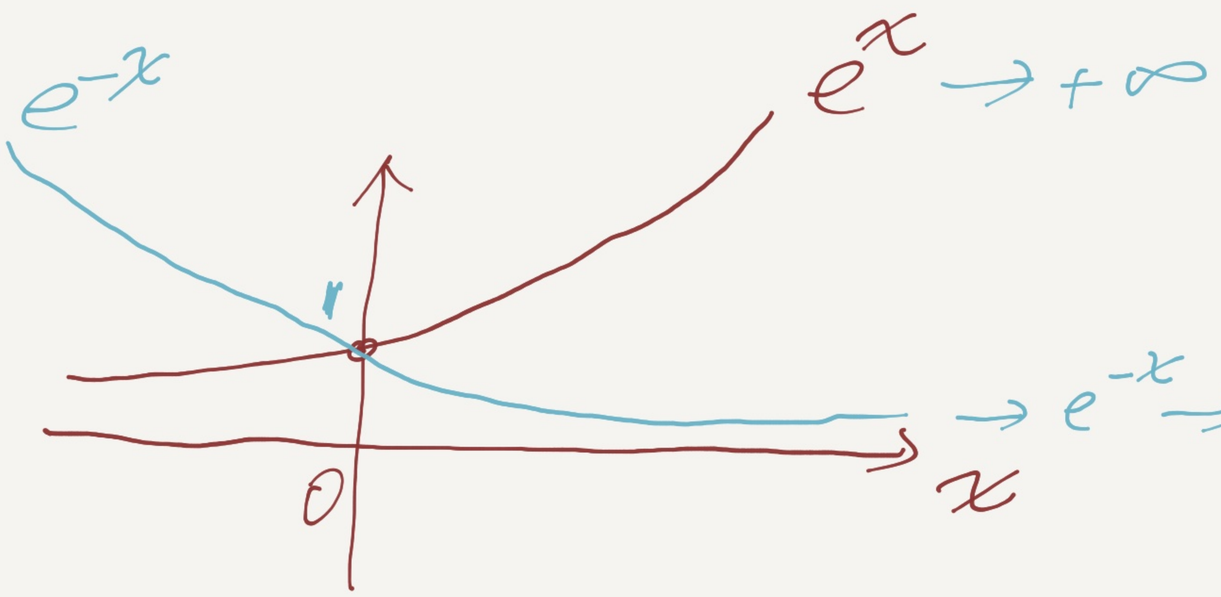
μ : mean.

δ : std

if, $\mu=0$, $\delta^2=1$. $\quad \mathcal{N}(x;0,1)=\left[\frac{1}{2\pi}\right]^{1/2}\cdot e^{-\frac{x^2}{2}}$



2D Gaussian distribution ??



$f(x)=e^{-x}$

$Df(x)=-e^{-x}$

5. Bayes' Formula.

x, $p(x)$,

Joint prob. $\underline{p(x, y)}$, $p(x=1, y=2) = ?$ (dice game)

$$p(x, y) = p(x) \cdot \underline{p(y|x)}$$
$$= p(y) \cdot \underline{p(x|y)}$$

(conditional prob. $\underline{p(y|x)}$, $p(x|y)$

↳ prob of $x=?$ given the condition of $y=?$.

if. x and y are independent:

$p(y|x) = p(y)$ ∴ $p(x|y) = p(x)$.

Bayes' formula:

$$p(y|x) = \frac{p(x|y) \cdot p(y)}{p(x)}$$

$p(x|y)$ → likelihood

$p(y)$ → prior prob.

$p(x)$ → evidence.

Bayes' Decision Rule ??

'Dog vs. cat' task.