Assignment 1

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SOEN 331

1 Problem 1: Propositional Logic (7 pts)

1.1 (3 pts)

P: Humans are nice to Sofia
Q: Sofia is nice to humans $P \rightarrow Q$

- $P \rightarrow Q$ If humans are nice to Sofia, then Sofia is nice to humans.
- $\neg P$ Humans are not nice to Sofia.
- Therefore, $\neg Q$ Sofia is not nice to humans.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

As seen from this truth table, it is not implied that when humans are not nice, then Sofia will not be nice to humans. Indeed, there are two possibilities for it. Therefore, there is an inverse error and you can not determine Sofia is a threat to humans.

1.2 (4 pts)

P: Each man has a definite set of rules of conduct
Q: Man is no better than a machine

- $P \rightarrow Q$ If men have strict rules, then they are like machines.
- $\neg P$ There are no such rules.
- Therefore, $\neg Q$ Men cannot be machines.

This argument follows once again falls in the inverse error category, which is an invalid form of reasoning. In general, from $P \rightarrow Q$ and $\neg P$, we cannot logically conclude $\neg Q$.

2 Problem 2: Predicate logic (8 pts)

2.1 (4 pts)

(a) $\forall x \in \mathbb{R} : \text{number}(x) \to \neg\text{rational}(x)$
For all x, if x is a real number, then x is not rational.
Type: E

(b) $\exists x \in \mathbb{R} : \text{number}(x) \wedge \text{rational}(x)$
There exists a number that is real and rational.
Type: I

2.1 (4 pts)

(a) $\forall x \in \mathbb{R}, \text{number}(x) \to \text{rational}(x)$
Type: E

(b) $\exists x \in \mathbb{R}, \text{number}(x) \wedge \neg\text{rational}(x)$
Type: I

3 Problem 3: Linear Temporal Logic 1 (15 pts)

3.1 (3 pts)

$$\bigcirc\Box(\phi \oplus \psi) \rightarrow \bigcirc^2\Diamond(\chi W \tau) \tag{1}$$

3.2 (3 pts)

$$(\phi \oplus \psi) \rightarrow \bigcirc\Box(\chi \rightarrow (\kappa R \omega)) \tag{2}$$

3.3 (3 pts)

$$\neg(\neg\varphi \vee \neg\psi) \rightarrow \Diamond\Box\tau \wedge \bigcirc^2\Diamond\pi \tag{3}$$

If φ and ψ are true, then τ will eventually become invariant and starting from time $i+2$, π will eventually happen.

3.4 (3 pts)

If starting at time $i+2$, τ eventually becomes invariant and at time i, x becomes true and $/psi$ becomes true at time $i+1$, then starting at time $i+2$, τ will be true at all states until and including when β becomes true, infinitely often.

3.5 (3 pts)

If at time $i+1$, either one but not both of α and β are true, then starting at time $i+1$, τ will eventually be true and remain until κ becomes true. (κ will become true)

4 Problem 4: Linear Temporal Logic 2 (15 pts)

4.1 Visualizing all models of behavior (9 pts)

4.2 (4 pts)

$$\langle(\phi \wedge \omega), \kappa, \kappa, \lambda\rangle$$
$$\langle(\phi \wedge \chi \wedge \omega \wedge \alpha), (\alpha \wedge \kappa), (\alpha \wedge \kappa), (\alpha \wedge \beta \wedge \lambda)\rangle$$
$$\langle(\phi \wedge \psi \wedge \omega), (\tau \wedge \kappa \wedge \epsilon), (\tau \wedge \kappa \wedge \epsilon), (\pi \wedge \lambda \wedge \epsilon), \epsilon\rangle$$

4.3 (2 pts)

Model 1: The first model fully terminates. Additionally, it is consistent.
Model 2: The second model fully terminates. Additionally, it is consistent.
Model 3: The third model fully terminates. Additionally, it is consistent.

5 Problem 5: Unordered Structures (10 pts)

1. $Crew = \mathbb{P}\ Names$

2. *Spock* would not be a legitimate value, as all values of *Crew* must be a set of names. *Spock* is not (it does not have curly braces).

3. {Kirk} is a valid subset of *Names*. In $\mathbb{P}$ Names, it is the set of all subsets of S. {Kirk} is one of those subsets which makes this valid.

4. No, it is not. Writing *Crew:* $\mathbb{P}$ *Names* is interpreted as the variable *Crew* can assume any value supported by the Powerset of *Names*. However, $Crew = \mathbb{P}\ Names$ means that the variable Crew is the Powerset of *Names*.

5. No, the elements in *Names* aren't sets. Therefore, they can't be subsets of $\mathbb{P}$ *Names* if they are individual names.

6. $\{Names\} = \{\{Kirk,\ Spock,\ ...\}\}$ is not an element of Powerset of *Names*.

7. No, the elements of the product are actually ordered pairs.

8. Yes, because ever Powerset has an empty set.

9. The variable *Commander* can assume any value supported by the Powerset of *Crew*. For example, can be the empty set {}, one element of the said or the entire set.

10. NEED TO CHANGE FOR CREW X CREWThey must be ordered pairs. Therefore, in this case it is false.

6 Problem 6: Ordered Structures (10 pts)

6.1 (4 pts)

To implement a Queue Abstract Data Type with element $\langle 1, 2, 3 \rangle$, I would utilize the Stack data type. However, these two different data types operate differently. Queues work with the priniciple of first-in and first-out. Stacks work by adding removing elements from the same end. Therefore, using two lists of stacks will help us modify this difference by utilizing the push and pop operations. In addition, this proposed model will allow us to have elements of generic type T.

6.2 (6 pts)

7 Problem 7: Binary Relations, Functions, and Orderings (15 pts)

7.1 Poset proofs and Hasse diagrams

- (a) $\big(\{2,3,4,6,24,36,72\}, /\big)$ is a poset (2 pts)
  Hasse diagram, maximal/minimal elements
- (b) $\big(\mathcal{P}\{1,2,3\}, \subseteq\big)$ is a poset (1 pt)
  Hasse diagram, maximal/minimal elements

7.2 Analyzing variable `map` (12 pts)

8 Problem 8: Binary Relations, Functions, and Orderings 2 (10 pts)

8.1 (2 pts)

8.2 (8 pts)

9 Problem 9: Construction Techniques (10 pts)

9.1 Function $\texttt{map}(f, \Lambda)$ (5 pts)

- (a) Transform into computable function
- (b) Define function recursively
- (c) Unfold your definition for $\texttt{map}(f, \langle a, b, c \rangle)$

9.2 Function $\texttt{insert}(x, \Lambda)$ (5 pts)

- (a) Transform into computable function
- (b) Define function recursively
- (c) Unfold example: $\texttt{insert}(5, \langle 1, 3, 5, 7 \rangle)$