Problem Set 1 – BE-130 / ES-249 Due: Thursday Feb 14

Basic Robotics:

(1) Forward kinematics:
(a) Derive the forward kinematics $[x_1 \ x_2]^T$ as a function of $[\Theta_1 \ \Theta_2]^T$ and the Jacobian matrix $[dx/d\Theta]$ for a two joint planar arm where the joint angles are defined "absolutely" with respect to the x-axis.
(b) Repeat for relative joint angles Θ where the angle of each joint (after the first one) is defined with respect to the angle of the previous limb segment.
(c-d) Repeat for a three joint planar arm for both absolute and relative joint angles.

(2) Inverse kinematics, jacobians, & dynamics:
For the two joint absolute joint angle case above (1a):
(a) Derive the inverse kinematics $[\Theta_1 \ \Theta_2]^T$ as a function of $[x_1 \ x_2]^T$ (note that this is surprisingly tricky…)
[Hint: note that mathematically inverting the forward kinematic equations is quite challenging, and an easier approach is to sketch out the geometry and apply the law of cosines].
(b) Derive the inverse Jacobian matrix $[d\Theta/dx]$.
(c) Comment briefly on how these compare with the forward kinematic equations derived in 1a.
(d) Create an interactive graphical simulation showing the 2-joint inverse kinematics computed from x-y mouse input, i.e. continually adjust the joint angles of a 2-link arm so that its endpoint position matches the mouse cursor position. Please see the example MATLAB m-file **draw_arm_starter1.m** for some starter code that implements the sensing and graphics, so that this is less tedious to do.
(e) Analytically compute (i) the minimum and (ii) the maximum Force levels that can be produced at any point in the workspace across all force directions for the 2-link arm simulated above, assuming that the max Torque level = T_{MAX} for both joints.
(f) Set T_{MAX}=1, and draw a pair of 2-D plots (images) showing (i) the minimum and (ii) the maximum Force levels that can be produced at any point in the workspace across all force directions for the 2-link arm simulated above.

Bonus: Create an interactive graphical simulation, like that in **2d**, that displays the maximum force levels that can be produced at any point in the workspace for the 2-link arm simulated above. Display this as an ellipse centered at the endpoint showing the max force that can be produced in each direction.