BE-130 Problem set 2 - Due Friday March 8

(1) The twitch response for a motor unit (a set of muscle fibers that are co-innervated and thus co-activated) is the force pattern generated by the motor unit following a single neural action potential at the neuromuscular junction. The twitch response for a slow-twitch muscle can be reasonably well approximated by the model:
$h(t) = K(e^{-t/\tau_1} - e^{-t/\tau_2})$ where τ_1 and τ_2 are $50ms$ and $15ms$, respectively, and $K = 0.17N$.
(a) Plot this impulse response for t = 0→300ms. [for example, `use t=[1:300] in MATLAB`]

Since neural action potentials are very narrow (typically 0.5ms in width for the main peak), the twitch response is essentially the same as the impulse response of the motor unit (and its constituent muscle fibers). Thus the response to a series of motor unit action potentials can be accurately approximated as the convolution of a corresponding impulse train and the twitch response. Use MATLAB (or python or whatever) to plot the muscle force of this motor unit in response to an input with four impulses each separated by **(b)** 200ms, **(c)** 50ms, and **(d)** 20ms.

Please assemble all four plots (a-d) as different axes on the same figure with similar x-axis and y-axis scaling (i.e. so that 100ms in plot (a) is at least roughly the same physical length as 100ms in plot (b), etc). Comment on what plots (b-d) look like and the differences between them. [Note that you can perform & plot the convolution for **1b** like this:

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impulse_train = zeros(900,1); impulse_train(200:200:800)=1; force=conv(impulse_train,h); subplot(4,1,2);
plot(force); xlabel('Time (ms)'); ylabel('Force (N)'); xlim([0 1200]); ylim([0 0.23]); title('200ms Interval')
```
]

(2) Recall that minimum jerk motion (i.e. motion that minimizes cumulative squared jerk (CSJ) given boundary conditions on position, velocity, and acceleration) is described by a function in which position, x(t), is a 5th order polynomial in time.
(a) As we did in class, solve for the 6 coefficients of this polynomial for the case of a point-to-point movement with the following boundary conditions: *x(0)=v(0)=a(0)=0* *x(T_f)=x_f, v(T_f)= a(T_f)=0* (where x_f and T_f are pre-specified constants and x_0 and T_0 are taken to be zero). [*You can use mathematica or the symbolic toolbox in MATLAB, or you can solve this by hand as it's not **that** ugly, or use the normalized form we discussed in lecture where* $\hat{t} = t/t_f$ and analogously $\hat{x} = x/x_f$].
(b) For the case of *T_f=0.5* and *x_f=10*, plot *x(t), v(t)* and *a(t)* for *t* in the range *[0, T_f]*
(for this you can use t = [0 : 0.001 : Tf]). Please put these 3 plots on separate panels on the same page.
`[e.g. in MATLAB: subplot(3,1,1); plot(t,x); subplot(3,1,2); plot(t,v); subplot(3,1,3); plot(t,a);]`
(c) Solve for x(t) for an "out & back" movement which passes through x_f at t=T_f/2. To accomplish this, you can split the movement into two segments (before and after the midpoint) and then determine what the optimal boundary conditions would be at the midpoint (x(T_f /2) = x_f, v(T_f /2) =?, a(T_f /2) =?). With a little introspection you can convince yourself at the midpoint velocity should be zero and the two segments should be (anti)symmetric (with x(t) identical but time-reversed for the 2nd segment relative to the 1st one). Thus the two segments would have identical CSJ, and so this problem reduces to finding the value for the midpoint acceleration that minimizes the CSJ for just the 1st segment (and then time-reversing this segment to get the 2nd segment). Thus after solving for the midpoint acceleration you can find the polynomial coefficients and write down x(t) for both segments.
(d) Repeat **2b** for the "out & back" case: For *T_f=0.5sec* and *x_f=10cm*, plot *x(t), v(t)* and *a(t)* for *t* on [*0, T_f*].

(3) (a) Find the general solution to minimum acceleration motion (i.e. motion that minimizes cumulative squared acceleration (CSA) given boundary conditions on position and velocity). [*Hint: like the minimum jerk problem, the solution here is also a polynomial*]
(b) Solve for the coefficients of this polynomial for the case of a point-to-point movement (*x(0)=v(0)=0, x(T_f)=x_f, v(T_f)=0*), where x_f and T_f are pre-specified constants.
(c) For the case of *T_f=0.5sec* and *x_f=10cm*, plot *x(t), v(t)* and *a(t)* for *t* on [*0, T_f*].

(4) (a) Find the general solution to minimum snap motion (i.e. motion that minimizes cumulative squared snap (CSS) given boundary conditions on position, velocity, acceleration, and jerk). Note that snap is the 4th derivative of position wrt time. [*Hint: like the minimum jerk and minimum acceleration problems, the solution here is also a polynomial*]
(b) Solve for the coefficients of this polynomial for the case of a point-to-point movement (*x(0)=v(0)=a(0)=j(0)=0, x(T_f)=x_f, v(T_f)= a(T_f)= j(T_f)=0*), where x_f and T_f are pre-specified constants.
(c) For the case of *T_f=0.5sec* and *x_f=10*,cm plot *x(t), v(t)* and *a(t)* for *t* on [*0, T_f*].
(d) Write a few sentences qualitatively describing what the plots from 2b, 3c, & 4c look like and how they compare to one another, and additionally how 1b compares to the first half of 1d.