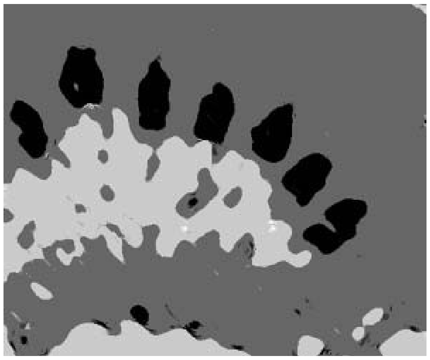

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECT

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CONTENTS

Water Purification

MATHEMATICS CLASSIFICATIONS:
Calculus, Differential Equations

DISCIPLINARY CLASSIFICATIONS:
Water Resources Management

PREREQUISITE SKILLS:
Situation 1. First-term calculus.
Situation 2. SI measurement units, characteristic equation for a second-order linear ODE with constant coefficients via matrix techniques (eigenvalues and eigenvectors).
Situation 3. Solution of a system of simultaneous first- order linear ODEs with constant coefficients.
Situation 4. Calculus of the exponential function.

PHYSICAL CONCEPTS EXAMINED:
Situation 1. Speed of flow, average flow; Requirement 4 uses bending moment and modulus of elasticity.
Situation 2. Forces, Newton's Second Law.
Situation 3. Conversion of measurement units, dilution.
Situation 4. Exponential population growth.

COMPUTING REQUIREMENTS:
1. Software for graphing
2. Software for finding eigenvalues and eigenvectors.

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Contents

1. Setting the Scene

Fruita (pronounced "fruit-ah") is at the western edge of Colorado near Grand Junction. It is a farming community centered on potatoes, sugar beets, and winter wheat. The city operates a wastewater collection and treatment system; the Fruita treatment plant is an aerated lagoon treatment system.

Fruita's population in 2005 was about 9,000 people, and its sewer system was at 74% capacity and cannot be expanded. When a sewer plant reaches 80%, the state health department requires the city begin to plan for either expansion or a new plant. The new sewer system should be able to handle the sewage of up to 20,000 people plus commercial development [Vader 2005].

In July 2005, the Fruita City Council considered options for building a new sewer plant to accommodate growth for the next 20 years, with the cost being from $14 to $20 million.

2. Situation 1: Designing a New Facility

Because of your broad background in environmental engineering and mathematical modeling, the city has asked for your assistance in solving some of the problems involved with the design of the new wastewater treatment facility.

Background: Wastewater does not flow into a municipal wastewater treatment plant at a constant rate; the flow rate varies from hour to hour, reflecting the daily water-use patterns of the area served. On the other hand, a wastewater treatment plant works best when designed to handle a constant inflow rate. A solution to this dilemma is the construction of a flow equalization basin (FEB), which is a tank designed to collect and store wastewater (**Figure 1**).

Such a storage basin allows wastewater to be pumped into the treatment plant at a constant rate, thus maximizing the efficiency of the facility. The level of wastewater in the FEB therefore varies throughout the day, reflecting the constantly changing amounts of wastewater produced from the community.

Figure 1. An aeration tank that might be modified for use as an equalization tank.

The constant influx of wastewater creates a mixing effect that prevents any settling from occurring.

Requirement 1: Flow Equalization

Table 1 shows readings of wastewater flow from Fruita town into the FEB taken at one-hour intervals over a 24-hour period (zero is midnight). The flow readings are cyclical; therefore, the reading at the 24th hour will be the same as the 0th hour. Using these data and a numerical approximation technique for integration, determine the constant flow rate at which the wastewater should be pumped out of the FEB and into the treatment plant. Also, determine the required volume of the FEB.

What concerns would you point out to the Fruita Planning Commission about your confidence in your answers above? What other considerations should be included in the design of your FEB?

Requirement 2: FEB Optimization

The depth of the FEB must be 3 m to accommodate the available equipment designed to clean the FEBs. Additionally, you learn that the cost associated of constructing the equalization basin is a function of its length and width:

$$c(w, \ell) = 1.13\ell^2 + 51w + 34\ell w. \qquad \textbf{(1)}$$

Design an optimal rectangular FEB that would have the lowest cost for the aforementioned sewage flow while providing a 25% excess capacity for unexpected flow variations.

Table 1.
Flow rates.

Hour	Flow(m^3/s) (m^3/s)	Hour (m^3/s)	Flow
0	0.0417	12	0.0725
1	0.0321	13	0.0754
2	0.0236	14	0.0761
3	0.0185	15	0.0775
4	0.0189	16	0.081
5	0.0199	17	0.0839
6	0.0228	18	0.0863
7	0.0369	19	0.0807
8	0.0514	20	0.0781
9	0.0630	21	0.069
10	0.0685	22	0.0584
11	0.0697	23	0.0519

Requirement 3: Pipe Size

You now need to determine the radius of the pipe needed to carry the flow of wastewater between the FEB and the treatment plant (**Figure 2**). The velocity of fluid in a pipe depends on how far the fluid is from the wall of the pipe. For fluid flow in a circular pipe, the function that describes the velocity of the fluid is given by

$$v(r) = a\left(1 - \frac{r^2}{R^2}\right), \tag{2}$$

where

r is the location of the fluid in the pipe measured as a radius of fluid from the center of the pipe,

$v(r)$ is the velocity of the fluid as a function of its location in the pipe measured in m/s,

R is the radius of the pipe in meters, and

a is a constant.

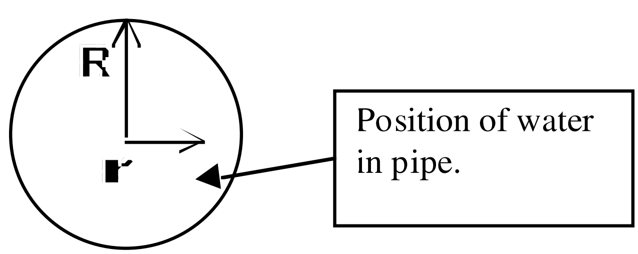

Figure 2. Cross section of the pipe.

First, compare and discuss the velocities of the water at the wall of the pipe and at the center of the pipe. Give a physical interpretation of the meaning of your answers.

The flow rate can be calculated as

$$Q = 2\pi \int_0^x v(r) \cdot r \, dr, \tag{3}$$

where $v(r)$ is the velocity function for a circular pipe as above. Using the flow rate that you calculated in **(2)**, and given **(1)**, determine the radius of the pipe needed to carry the flow of wastewater between the FEB and the treatment plant. Explain your results and discuss why you do not want a smaller/larger pipe.

Requirement 4: Inspection Catwalk

The wastewater treatment personnel require a catwalk to cross over the equalization basin that you designed in **Requirement 2** (**Figures 3–4**). Three-person inspection teams who take readings of the wastewater in the FEB will use the catwalk. It is assumed that each of the personnel will weigh no more than 251 lb.

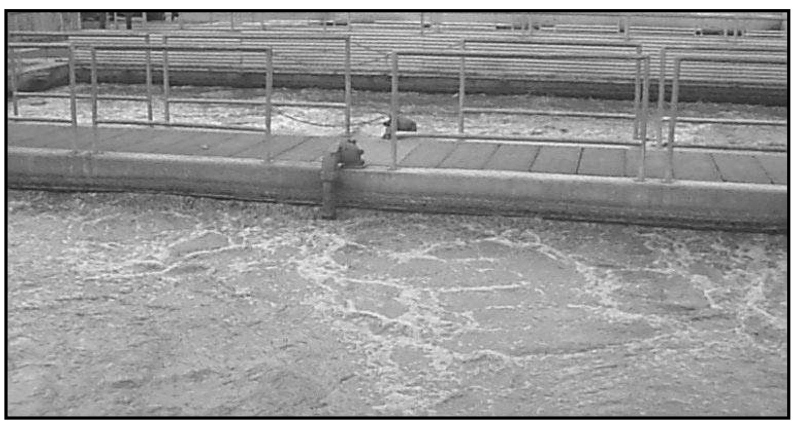

Figure 3. A catwalk.

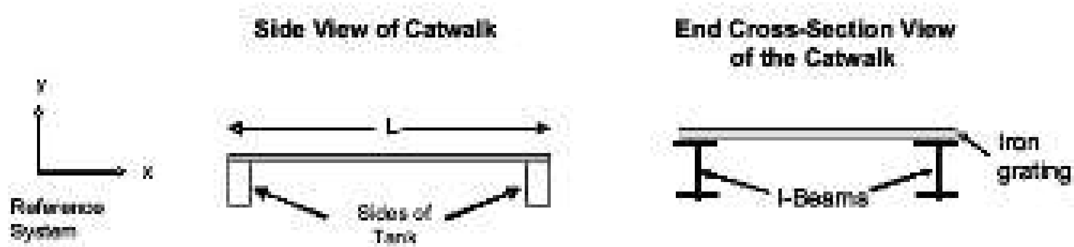

Figure 4. Catwalk construction (not to scale).

For a fixed length and specific beam material, the chief factor in determining the vertical deflection of a beam comes from the moment of inertia (I). The moment of inertia is defined as the sum of the products of each elementary area of the cross-section of the I-beam multiplied by the square of the distance of that area from the assumed axis of rotation. The size of each I-beam has an associated moment of inertia specification. The water treatment personnel have in stock two sections of I-beam with the following specifications:

I-Beam designation:	W18 $\times$ 35
Moment of inertia:	I = 510 in^4
Modulus of elasticity:	E = 29 $\times 10^6$ lb/in^2
Cost:	\$1/linear ft

The differential equation that models the vertical deflection of a beam, from the *Handbook of Engineering Fundamentals* [Tapley 1990], is given by

$$EI\,\frac{d^2y}{dx^2} = M(x), \tag{4}$$

with initial conditions $y(0) = 0$ and $y'(0) = L/2$, where L is the length of the beam. The bending moment $M(x)$ is defined to be

$$M(x) = \frac{1}{2}\int_0^x P\,dx, \qquad 0 \le x \le L/2, \tag{5}$$

where P is is the concentrated load in pounds per I-beam.

First, explain the physical meaning of the two given initial conditions and discuss the reasonableness of these conditions. Then, given the maximum possible load (three-person inspection team standing in the center of the walkway), determine the maximum vertical deflection of the catwalk, noting that the weight of the catwalk itself causes a downward vertical deflection of 1.0527 in. Under this maximum projected load, determine whether the catwalk will deflect more than 1.5 in. Why might the personnel be concerned about deflections greater than 1.5 in?

3. Situation 2: Modeling the Gate System

Since you performed so well for the city of Fruita in the design of their wastewater treatment facility, the mayor has asked you for more assistance in solving two other problems involved with the design of this facility. The town has decided to design the facility with two microbe tanks, operating one microbe tank full time and the secondary tank for half of each day (**Figure 5**).

A solid gate will be placed across the entrance to the secondary tank and opened at specific times of the day to allow excess flow into the secondary tank. The contractors have told you that the gate design calls for a spring latch system that will catch the swinging gate and hold it in the open position (**Figure 6**).

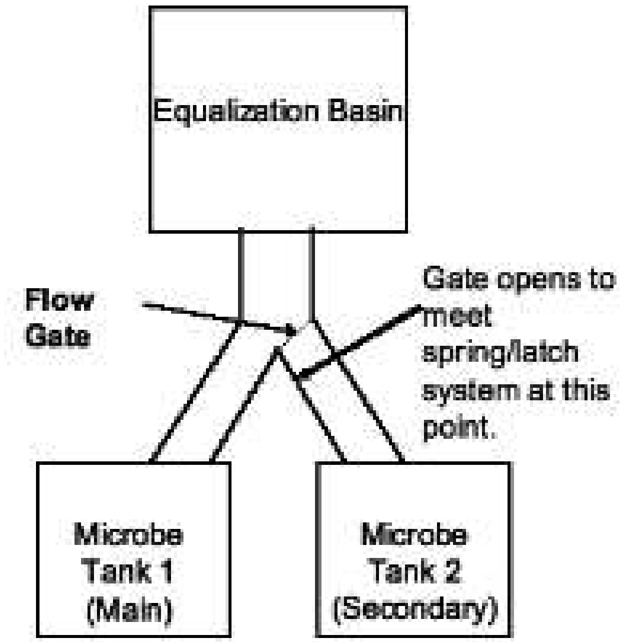

Figure 5. Tank setup.

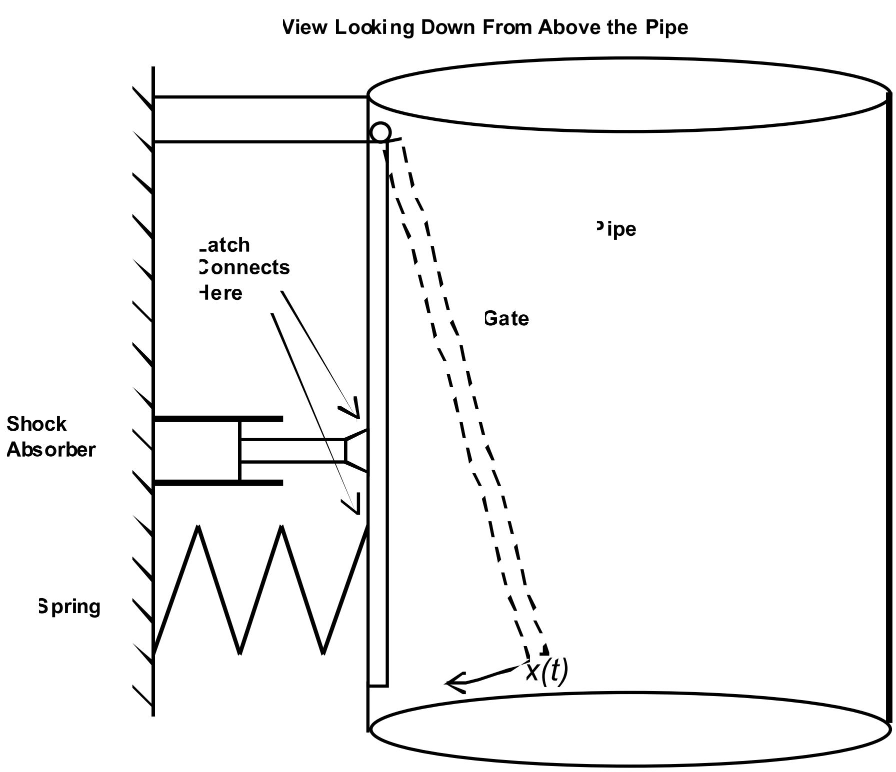

Figure 5. Spring latch system.

The contractors have provided you with the following information about the gate and spring design:

- The gate has a mass of 40 kg.
- The spring in the spring latch system has a spring constant of 1562.5 N/m.
- The system has a shock absorber built into it that, when moved at a velocity of 0.06 m/s, damps the motion of the gate with a resistance of 30 N.

The contractors further explained that the gate will contact the spring latch system with a force that is modeled by the equation

$$f(t) = 3e^{kt},$$

where $f(t)$ is in Newtons and $k = -3\ \mathrm{s}^{-1}$ after the gate makes first contact with the spring latch system. This force, which is due to the moving water, is at a maximum when the gate is opened. The force quickly approaches zero once the gate hits the spring. When the gate impacts the spring latch system, the gate will be moving at 0.8 m/s. This latch system is designed so that as soon as the gate makes contact with the spring and the shock absorber, the gate immediately attaches to them. The designers are concerned about the behavior of this spring system and want you to model it so as to determine the long-term behavior of the system.

Requirement 1: Equation for the Model

Determine the second-order differential equation that models the motion of this spring latch system.

Requirement 2: Behavior

Determine the particular solution to the model. Graph this solution and determine the long-term behavior of this system. Also, classify the damping that occurs in this system.

4. Situation 3: Shutting Down the Old Facility

The Fruita Planning Commission has also asked you to assist in determining how to shut down the current treatment facility. The city council is in favor of flushing out the facility with clean wastewater (i.e., shutting off the flow of waste into the facility). However, the mayor is unsure about how long this will take and does not want to run the old facility for too long after the new one is built, because of cost considerations. The facility engineer of the current treatment facility has provided you with a diagram of a potential model for cleaning out the wastewater treatment facility (**Figure 7**) and the pertinent flow data.

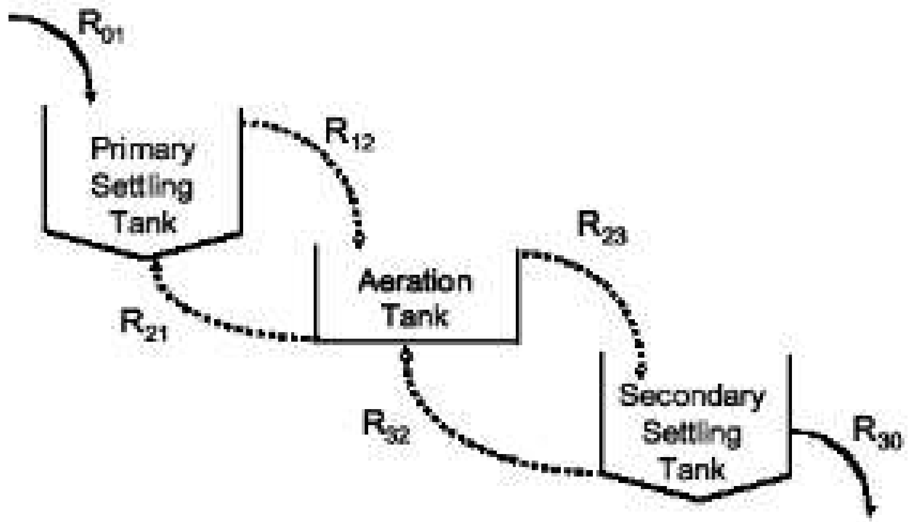

Figure 7. Purification process.

Flow Rates to/from Tanks:

R01: 235 gal/hr
R12: 370 gal/hr
R21: 135 gal/hr
R23: 320 gal/hr
R32: 85 gal/hr
R30: 235 gal/hr

Volumes of Tanks:

Primary	18,500 gal
Aeration	14,250 gal
Secondary	8,500 gal

The estimated initial amount of waste in each tank is

Primary settling tank:	137,600 lb
Aeration tank:	22,800 lb
Secondary settling tank:	44,020 lb

The mayor would like each tank to be at 1% of its initial amount in 21 days. He would like all tanks completely cleaned and ready for shut down in 37 days. Discuss whether or not the current plan will meet his timeline, supporting your answers analytically and graphically.

5. Situation 4: Meeting Demand

One member of the Fruita city council has raised a concern about whether the new wastewater treatment facility will be large enough to support the anticipated needs of the city for the next 20 years. The designers believe so. However, the only support for their claim is their years of experience. Obviously, the town council wants something a bit more analytical to determine if the facility will meet their needs. The town has been growing over the last several years and is expected to continue to grow at approximately the same rate. The town manager has available the data in **Table 2**.

Table 2.
Population of Fruita (source: www.fruita.org/demographics/).

Year	Population
2001	6,570
2002	7,290
2003	8,093

Requirement 1: Population Growth

Based on the given historical population data, develop a model of the Fruita City population growth. Use this model to estimate the population of Fruita in 20 years.

Requirement 2: Time to Capacity

The designers of the wastewater treatment facility have indicated that the average water usage per person in the United States is between 150 and 200 gallons per day. Based on this planning factor and using only 80% of capacity (at which point expansion must be planned), the current wastewater treatment facility can handle a population of between 6,500 and 8,600 people. Using your model above, how long will it take the population of Fruita to exceed the designed capacity (20,000) of the new water treatment facility? Give a best-case scenario and a worst-case scenario.

References

Tapley, Byron D. (ed.). 1990. *Eshbach's Handbook of Engineering Fundamentals*. 4th ed. New York: Wiley-Interscience.

Vader, Marija B. 2005. Fruita looks at options for new sewer. *Daily Sentinel* (Grand Junction, CO) (23 July 2005). http://72.14.207.104/search?q=cache:3JzHVq8xKL8J:www.gjsentinel.com/hp/content/news/stories/2005/07/23/7_23_Fruita_wastewater_WWW.html .