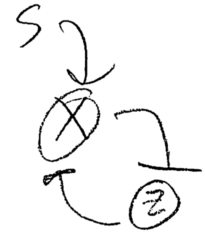
2

a)
$$\frac{d\tilde{X}}{d\tilde{t}} = \frac{\tilde{\alpha}_X + \tilde{\beta}_X S}{1 + S + (\tilde{Z}/\bar{z}_X)^{n_{ZX}}} - \tilde{\delta}_X \tilde{X} \quad \left(\text{from eqn 2}\right)$$

$$\frac{d\tilde{Z}}{d\tilde{t}} = \frac{\tilde{\alpha}_Z}{1 + (\tilde{X}/\bar{X}_Z)^{n_{XZ}} + (\tilde{y}/\bar{y}_Z)^{n_{YZ}}} - \tilde{\delta}_Z Z \quad \left(\text{from eqn 2}\right)$$

mistake? $\tilde{Z}$?

Because the system is just



the ODEs become

$$\boxed{\begin{aligned} \frac{d\tilde{X}}{d\tilde{t}} &= \frac{\tilde{\alpha}_X + \tilde{\beta}_X S}{1 + S + (\tilde{Z}/\bar{z}_X)^{n_{ZX}}} - \tilde{\delta}_X \tilde{X} \quad &\text{eqn (i)} \\ \frac{d\tilde{Z}}{d\tilde{t}} &= \frac{\tilde{\alpha}_Z}{1 + (\tilde{X}/\bar{X}_Z)^{n_{XZ}}} - \tilde{\delta}_Z Z \quad &\text{eqn (ii)} \end{aligned}}$$

should be $\tilde{Z}$?

note from here on I assume the degregation term should have been $\tilde{\delta}_Z \tilde{Z}$

from eqns 3-6 **2B**

$$\frac{dt}{\tilde{\delta}_x} = d\tilde{t}, \quad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} dX = d\tilde{X}, \quad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} dz = d\tilde{z}$$

$\boxed{\text{The mistake } t = \tilde{t}\delta_x \text{ should be } t = \tilde{t}\tilde{\delta}_x}$

$$\tilde{\alpha}_x = \alpha_x \tilde{\alpha}_z, \quad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} X = \tilde{X}, \quad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} z = \tilde{z}$$

$$\frac{\tilde{\alpha}_z}{\tilde{\delta}_x} z_x = \bar{z}_x, \quad \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} X_z = \bar{X}_z, \quad \tilde{\delta}_z = \tilde{\delta}_x \delta_z$$

S and n_{ij} are dimensionless $\qquad \tilde{\beta}_x = \tilde{\alpha}_z \beta_x$

eqn(i) becomes

$$\tilde{\delta}_x \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} \frac{dX}{dt} = \frac{\alpha_x \tilde{\alpha}_z + \tilde{\alpha}_z \beta_x S}{1 + S + \left[\left(\frac{\tilde{\alpha}_z}{\tilde{\delta}_x} z\right)\left(\frac{\tilde{\delta}_x}{\tilde{\alpha}_z z_x}\right)\right]^{n_{zx}}} - \left(\tilde{\delta}_x \left(\frac{\tilde{\alpha}_z}{\tilde{\delta}_x} X\right)\right)$$

$\downarrow$

$$\boxed{\frac{dX}{dt} = \frac{\alpha_x + \beta_x S}{1 + S + \left[\frac{z}{z_x}\right]^{n_{zx}}} - X} \quad \text{eqn(iii)}$$

eqn(ii) becomes

$$\tilde{\delta}_x \frac{\tilde{\alpha}_z}{\tilde{\delta}_x} \frac{dz}{dt} = \frac{\tilde{\alpha}_z}{1 + \left[\left(\frac{\tilde{\alpha}_z}{\tilde{\delta}_x} X\right)\left(\frac{\tilde{\delta}_x}{\tilde{\alpha}_z X_z}\right)\right]^{n_{xz}}} - \tilde{\delta}_x \delta_z \left(\frac{\tilde{\alpha}_z}{\tilde{\delta}_x}\right) z$$

if eqn ii uses $\tilde{z}$ instead of z in the deg term.

$\downarrow$

$$\boxed{\frac{dz}{dt} = \frac{1}{1 + \left(\frac{X}{X_z}\right)^{n_{xz}}} - \delta_z z} \quad \text{eqn(iv)}$$

$\boxed{2C}$ @ S.S. $\frac{d}{dt} = 0$

$\therefore$ from eqn (iii)

$$X = \frac{\alpha_X + \beta_X S}{1 + S + \left[\frac{Z}{Z_X}\right]^{n_{ZX}}}$$

given: $\alpha_X = 1.5$, $\beta_X = 5.0$, $Z_X = 0.4$, $n_{ZX} = 2.7$, $X_Z = 1.5$, $n_{XZ} = 2.7$, $\delta_Z = 1.0$

$$X = \frac{1.5 + 5S}{1 + S + \left[\frac{Z}{0.4}\right]^{2.7}} \quad \text{eqn (v)}$$

from eqn (iv)

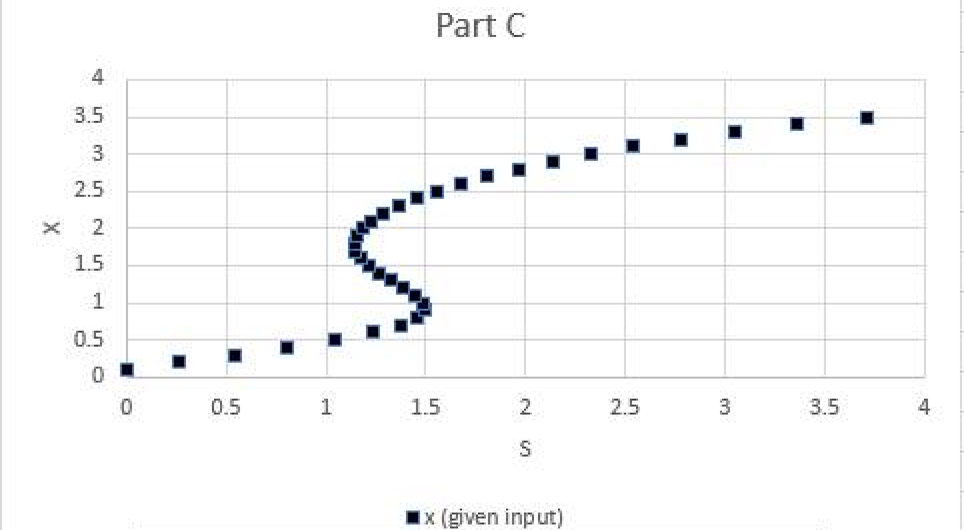
$$Z = \frac{1}{\delta_Z \left(1 + \left(\frac{X}{X_Z}\right)^{n_{XZ}}\right)}$$

$$Z = \frac{1}{1 \left(1 + \left(\frac{X}{1.5}\right)^{2.7}\right)} \quad \text{eqn (vi)}$$

Combining eqn (v) and eqn (vi)

$$X = \frac{1.5 + 5S}{1 + S + \left(2.5\left[\frac{1}{1 + \left(\frac{X}{1.5}\right)^{2.7}}\right]\right)^{2.7}} \quad \text{eqn (vii)}$$

Solved on Excel using a least squares fit and Solver, the Results and concluding text are shown on the following page of this PDF. But, to find the work please look at the Excel workbook 'Problem 2 Part C'.



Qualitatively yes the figure can be reproduced.

$\boxed{2d}$ (from eqn 1)

$$\frac{dX}{dt} = \frac{\alpha_X + \beta_X S}{1 + S + (Z/z_X)^{n_{zx}}} - X$$

$$\frac{dY}{dt} = \frac{\alpha_Y + \beta_Y S}{1 + S + (X/x_y)^{n_{xy}}} - \delta_y Y$$

$$\frac{dZ}{dt} = \frac{1}{1 + (X/x_z)^{n_{xz}} + (Y/y_z)^{n_{yz}}} - \delta_z Z$$

solve for with $S = 0.02,\ 10,\ 10^5$

given: $X_0 = Y_0 = Z_0 = 0$

from table S.1

$\alpha_X = 3.9 \cdot 10^{-2}$

$\alpha_Y = 4.3 \cdot 10^{-3}$

$\beta_X = 6.1$

$\beta_Y = 5.7$

$\delta_Y = 1.05$

$\delta_Z = 1.04$

$z_X = 1.3 \cdot 10^{-5}$

$y_Z = 11 \cdot 10^{-3}$

$X_Z = 12 \cdot 10^{-2}$

$X_Y = 7.9 \cdot 10^{-4}$

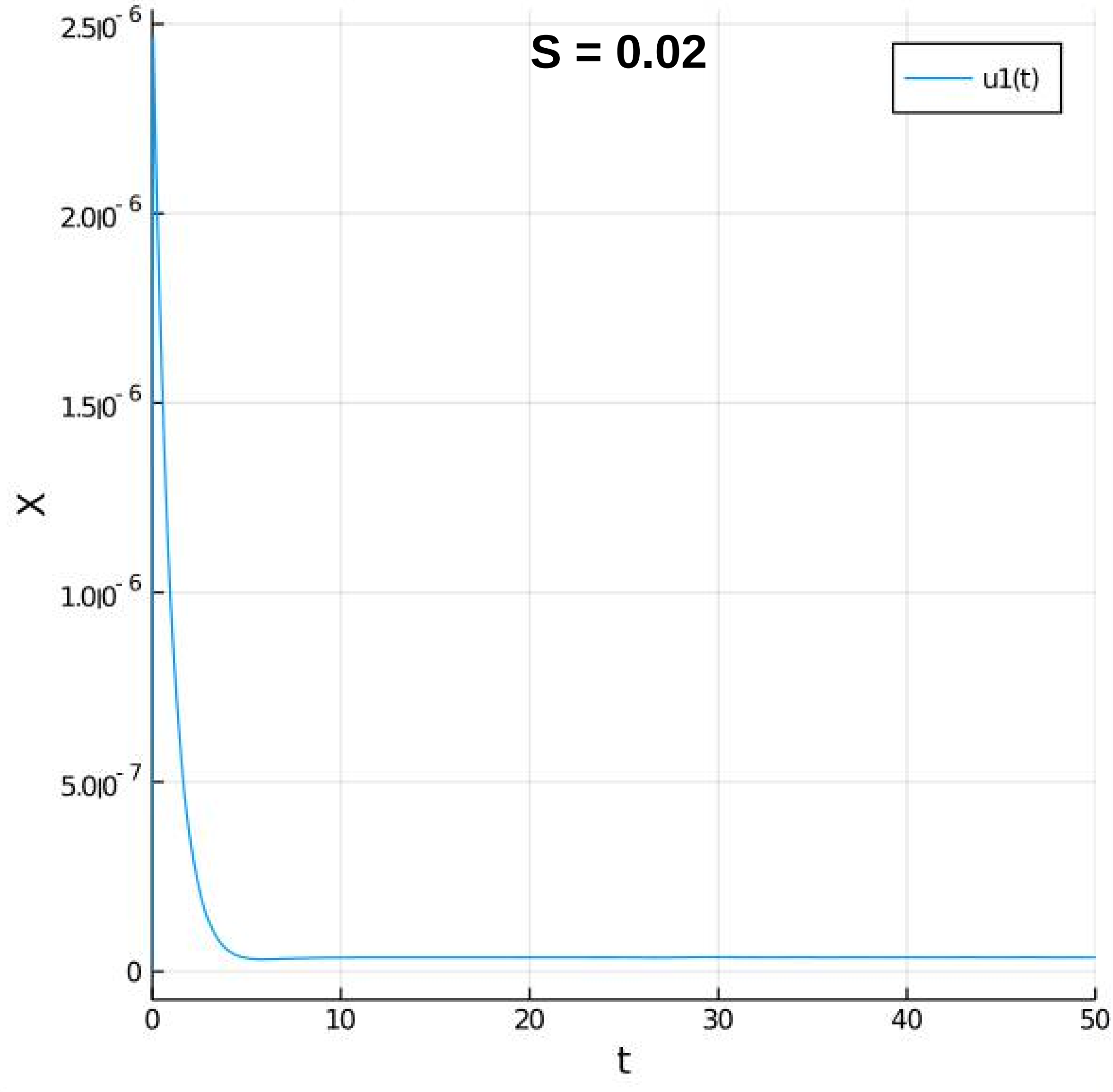
$n_{zx} = 2.32$

$n_{xy} = 2$

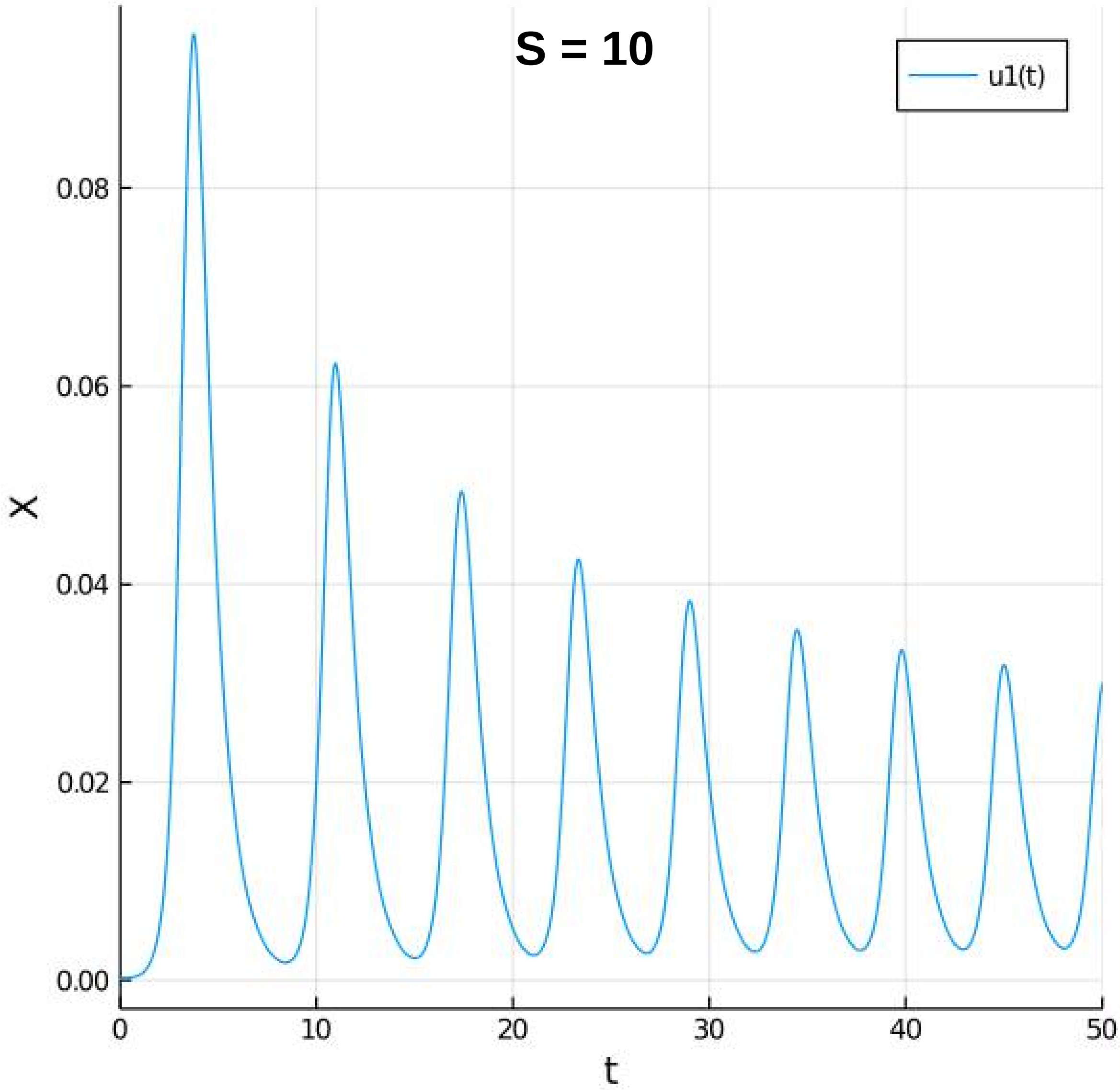
$n_{xz} = 2$

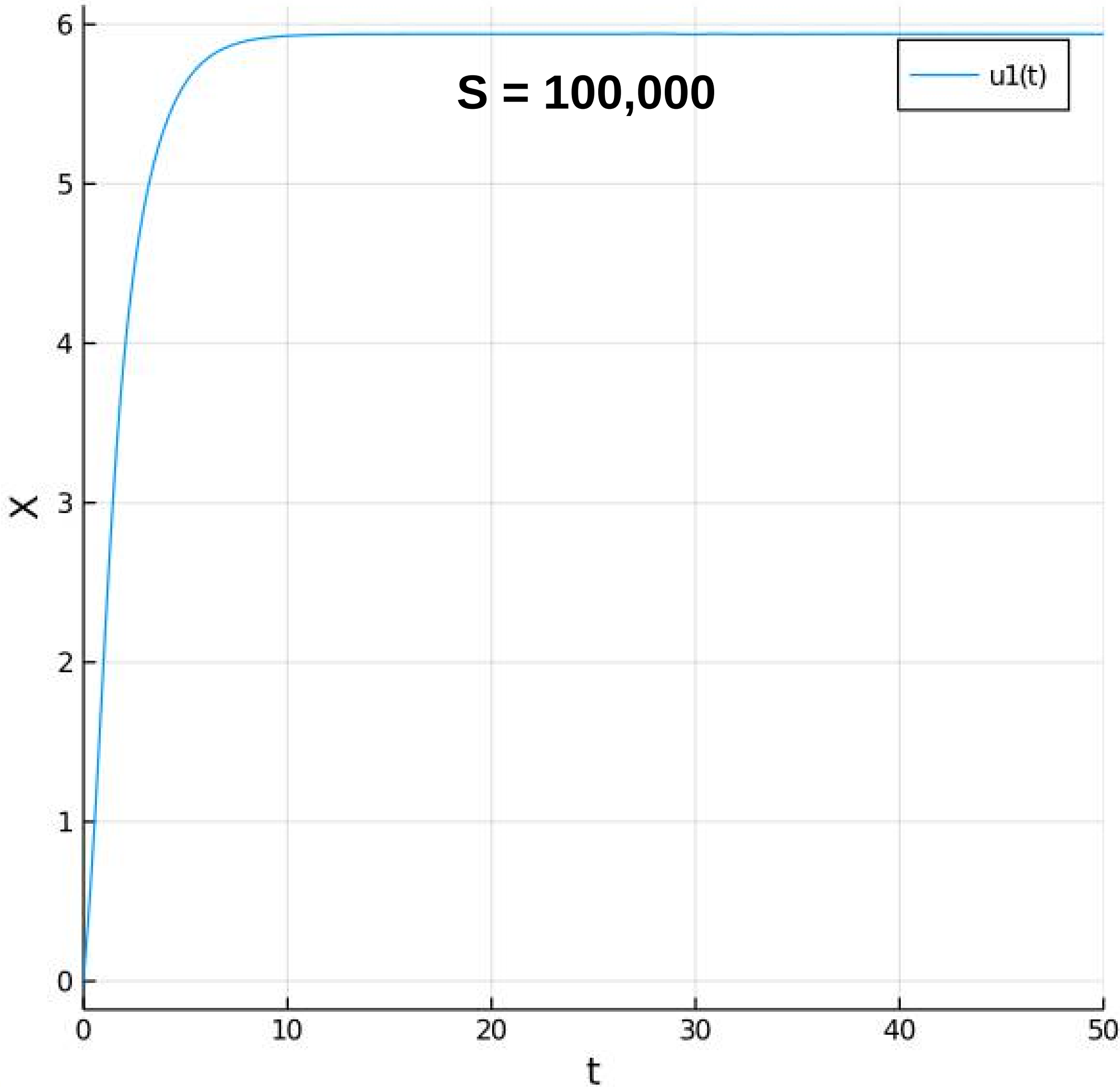
$n_{yz} = 2$

The results can be found on the following pages of this PDF. The code to achieve these results can be found in the Julia file 'Problem2PartD'. The file should be able to be run as(if the S is changed to 0.02, 10, 10E5, respectively) to reproduce a one of the plots at a time.



S = 0.02
u1(t)
2.5|0- 6
2.0|0- 6
1.5|0- 6
1.0|0- 6
5.0|0- 7
0
X
0
10
20
30
40
50
t


S = 10
u1(t)
0.08
0.06
0.04
0.02
0.00
X
0
10
20
30
40
50
t


S = 100,000
u1(t)
X
t
0
1
2
3
4
5
6
0
10
20
30
40
50

2e
Part 1

The hopf bifurcation point occurs @ $\omega S = 5 \cdot 10^{-1} - 6 \cdot 10^{-1}$

$\therefore$ to be safe I choose $\boxed{S = 4.5 \cdot 10^{-1}}$

as shown in pictures my S.S. values are $[X_{ss}, Y_{ss}, Z_{ss}]$

$\hookrightarrow = [0.00132, 0.575, 0.000352]$

The resulting graphs are on the following pages of the PDF. For the code to produce these graphs please see the Julia file that can be run as is in atom called 'Problem2PartE__hopf' to reproduce the graphs.

I set these as the initial condition for each cell multiplied by the cell's respective multiplier, ie $(1, 1.25, 0.75)$ (cell 1, cell 2, cell 3).

I then changed $S = 100$ and combined the two plots for Z vs time.

i.e. flat Z_{ss} plot + Z after S change

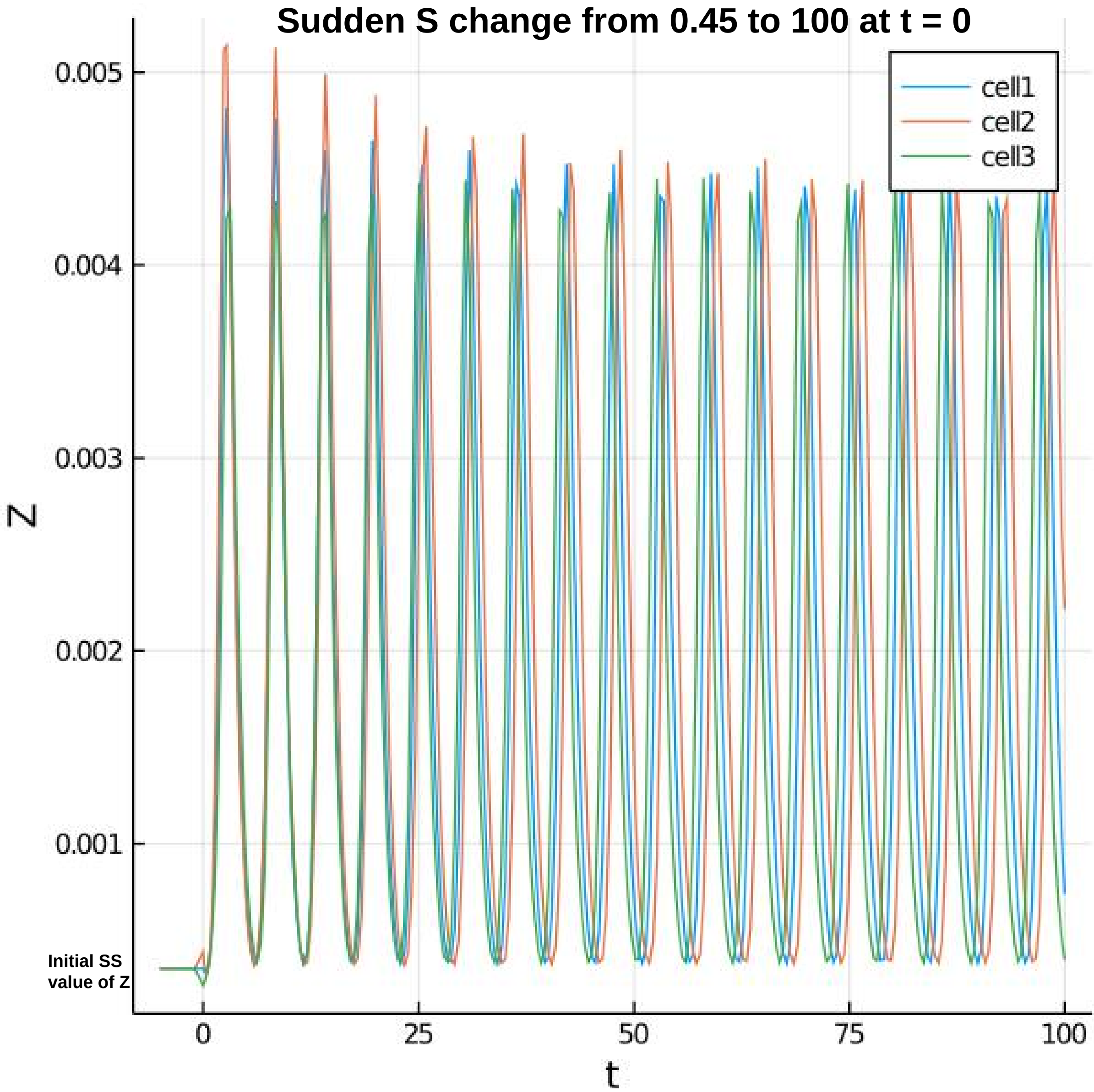
initial states $[0,0,0]$, $S = 0.45$ | initial states $[mX_{ss}, mY_{ss}, mZ_{ss}]$, $S = 100$

Please note for the next page the divergence of the 3 cells just before t = 0. This is caused by setting the initial z of cell 1 = Zss, zinitial cell2 = 1.25Zss , ZinitialCell3 = 0.75Zss. The reason why the initial SS values are a singular line before this is because I suck at Julia plotting and couldn't figure out how to add different data to make it look nicer until Part F. (This does not impact the actual initial starting states of the cells before the change in S, it only impacts how the graph looks before t = 0 and explains the jump to their starting values at t = 0.)

To me they look as if they are starting to get out of phase $\therefore$ incoherent

Please also note the SS values are shown after the Sudden change graph for the hopf bifurcation, but shown before for the saddle bifurcation.

* If the S.S. values are not precise enough or a plot missing include("Problem2PartE__hopf") can be run in atom to regenerate the plots and print the full S.S. values to high percision.



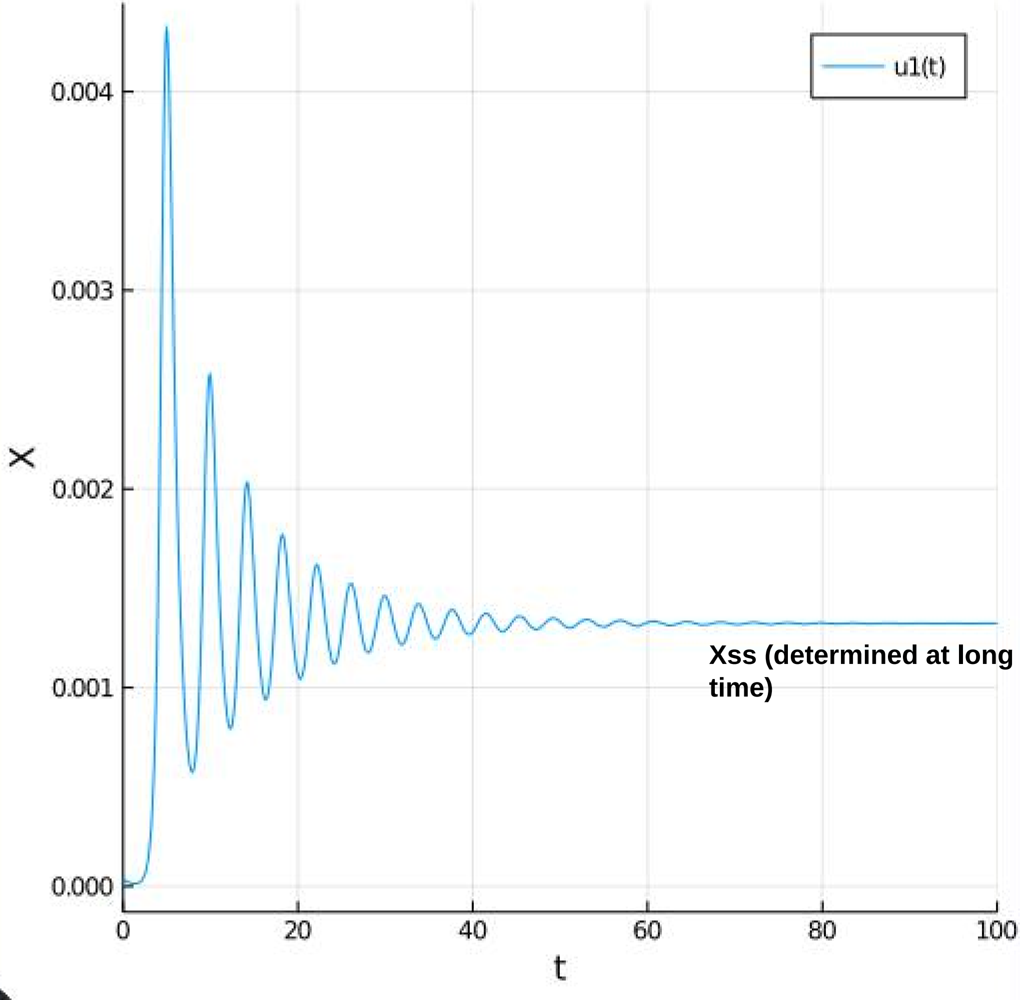
Sudden S change from 0.45 to 100 at t = 0
0.005
0.004
0.003
0.002
0.001
Z
Initial SS
value of Z
0
25
50
75
100
t
cell1
cell2
cell3

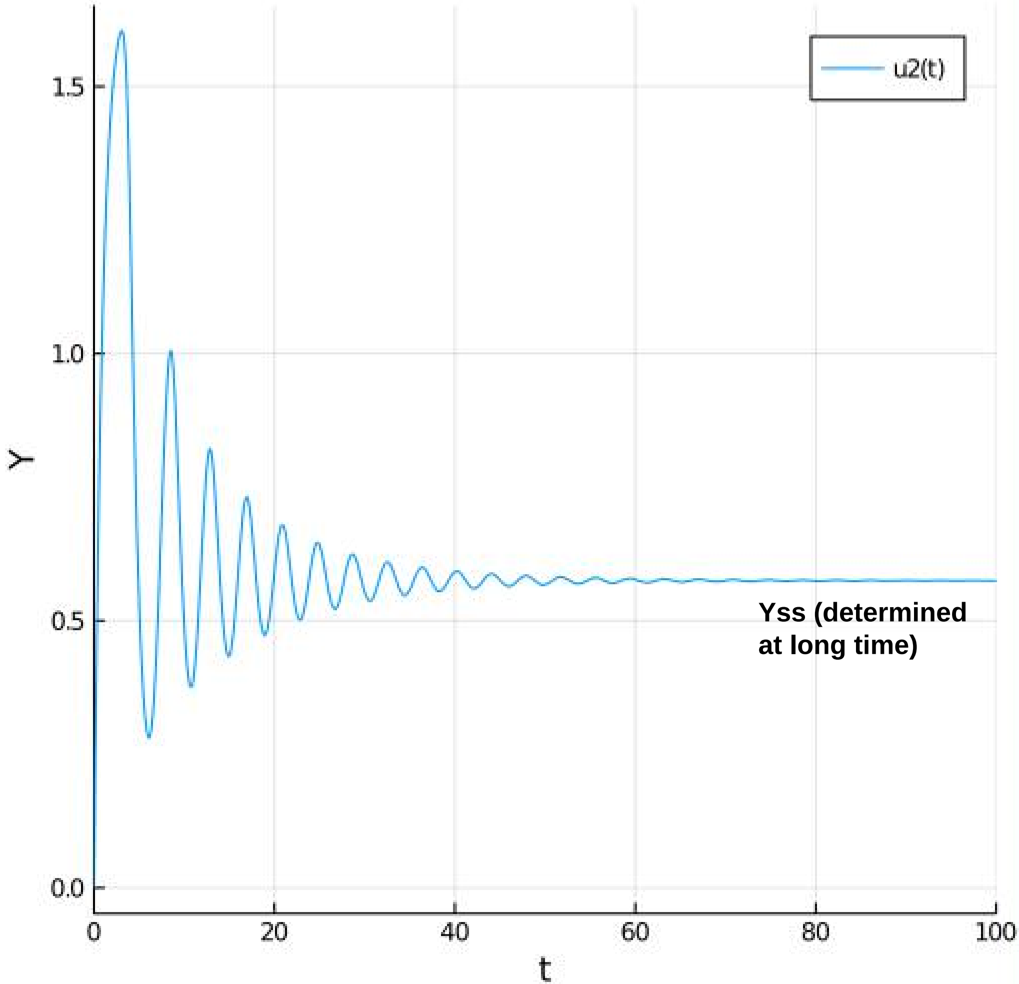
```
using DifferentialEquations
using Plots
gr(size=(500,500), show = true)  #use the gr backend for plotting
#Function for part D
#u[1] = x; u[2] = y; u[3] = z
function partD!(du,u,p,t)
  alphax = 3.9*10^-2
  alphay = 4.3*10^-3
  betax = 6.1
  betay = 5.7
  degy = 1.05
  degz = 1.04
  zx = 1.3*10^-5
  yz = 11*10^-3
  xz = 12*10^-2
  xy = 7.9*10^-4
  nzx = 2.32
  nxy = 2
  nxz = 2
  nyz = 2
  s = 0.45

 du[1] =  (alphax+betax*s)/(1+s+(u[3]/zx)^nzx) - u[1]            #dx/dt
```

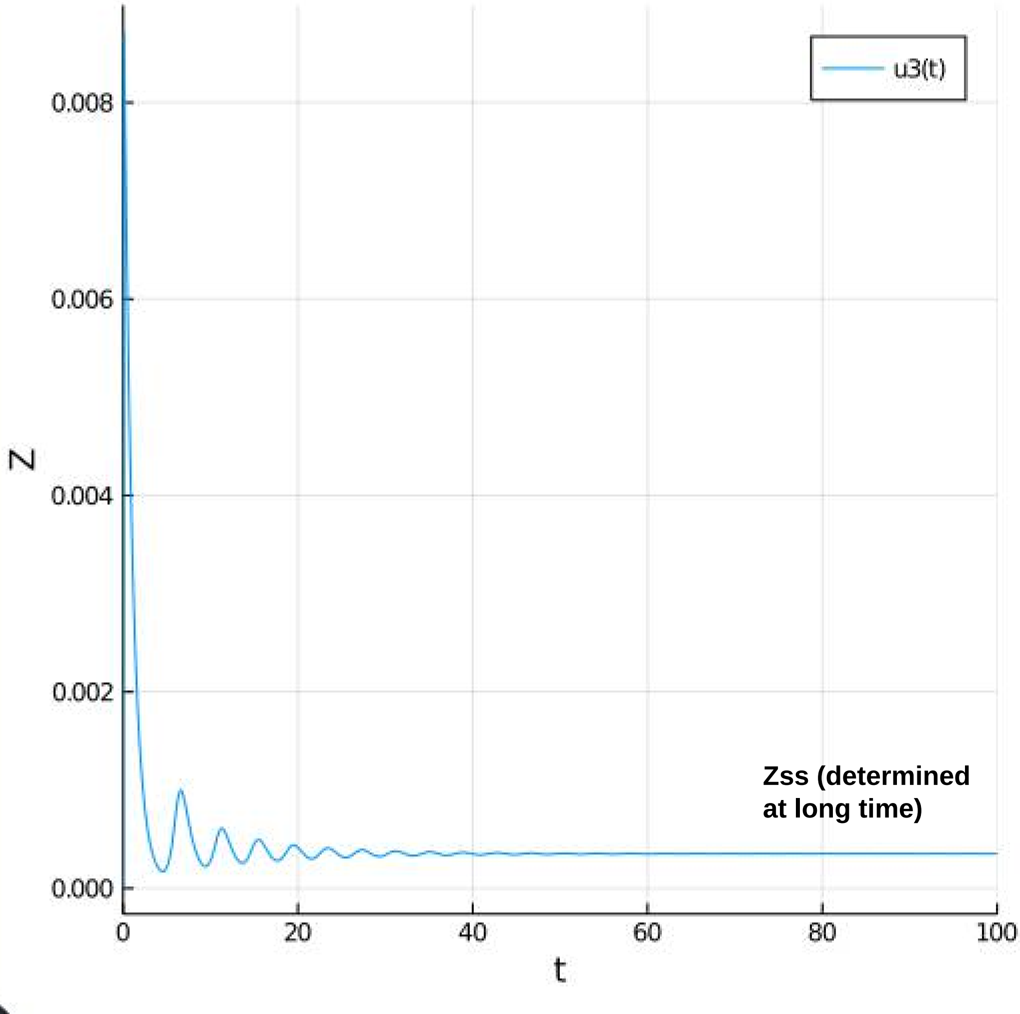
Below hopf bifurcation

```
in expression starting at E:\School Stuff\Junior Save Folder\Spring\5440\Prelim 1\Question 2\Pr

julia> include("Problem2PartE")
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
[0.0013233884934778165, 0.5750087121521615, 0.00035190745055507563]3.52024732661381e-5
julia> include("Problem2PartE")
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
[0.0013233884934778165, 0.5750087121521615, 0.00035190745055507563]
julia> include("Problem2PartE")
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
[0.0013233884934778165, 0.5750087121521615, 0.00035190745055507563]
julia>
```

x_{ss} , y_{ss} , z_{ss}


u1(t)
0.004
0.003
0.002
0.001
0.000
X
0
20
40
60
80
100
t
Xss (determined at long time)


u2(t)
1.5
1.0
0.5
0.0
Y
0
20
40
60
80
100
t
Yss (determined
at long time)



u3(t)
0.008
0.006
0.004
0.002
0.000
Z
0
20
40
60
80
100
t
Zss (determined at long time)

2e

Part 2

The saddle bifurcation point occurs @ $\sim S = 10^4 - 1.7 \cdot 10^4$

so to be safe

I initialy picked $S = 1.85 \cdot 10^4$, but in reality I found oscillations until around $\boxed{S = 35{,}450}$

as shown in my pictures my S.S. values are:

$$[X_{ss}, Y_{ss}, Z_{ss}] = [5.499, 0.0039688, 0.000457637]$$

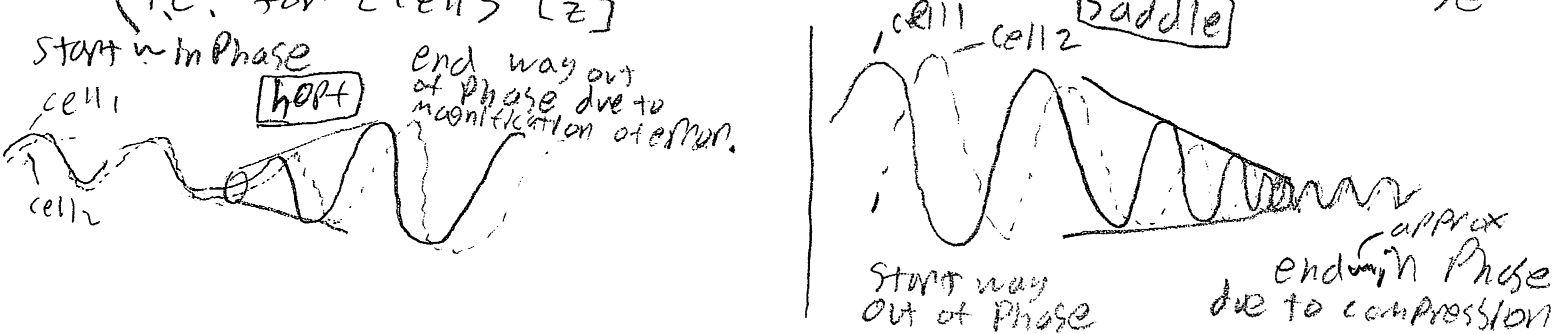
following the same procedure as Part 1 I found that:

As before include("Problem2PartE_ -- Saddle") can be run in atom if plot is missing or ss # not precise enough

The cells seemed to stay in phase ∴ **coherant**. Why are they coherant for transition through the saddle point but not the hopf bifurcation point?

The paper suggests that the hopf point acts to magnify any slight differences already present in the cells causing them to be wildly different and out of phase. On the other hand the paper suggests the saddle point condenses its input reducing initial differences in the cells to yelld an in phase final oscillating state.

approx i.e. for 2 cells [z]



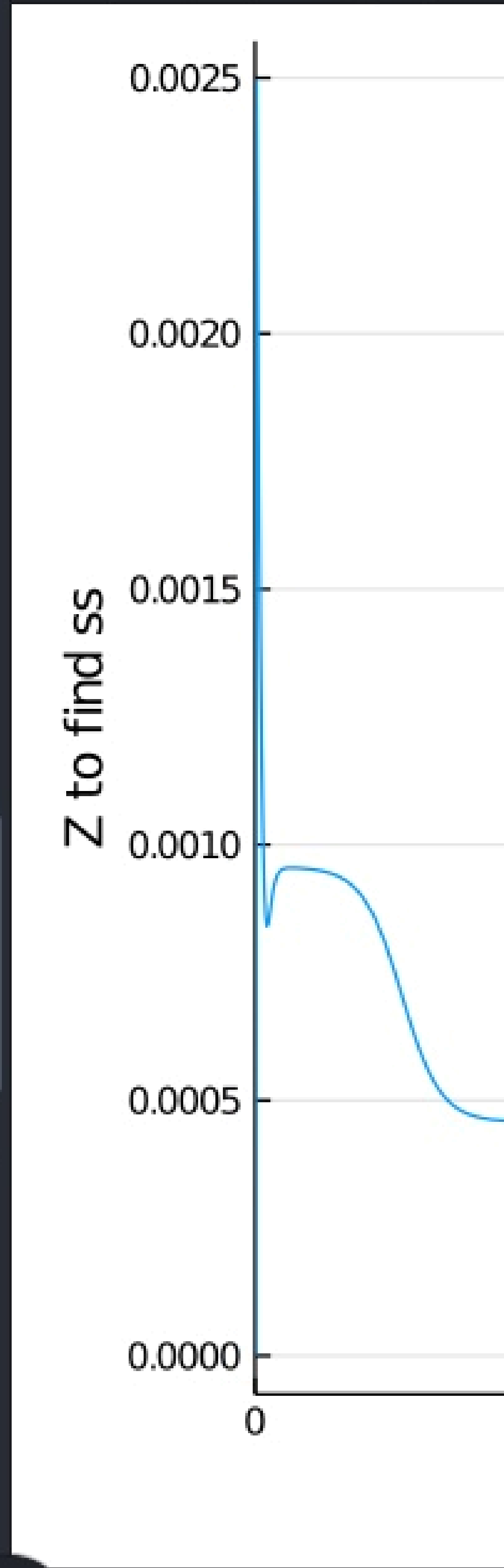
```
betay = 5.7
degy = 1.05
degz = 1.04
zx = 1.3*10^-5
yz = 11*10^-3
xz = 12*10^-2
xy = 7.9*10^-4
nzx = 2.32
nxy = 2
nxz = 2
nyz = 2
s = 35450
```

above saddle bif.

```
 du[1] =  (alphax+betax*s)/(1+s+(u[3]/zx)^nzx) - u[1]            #dx/dt
 du[2] =  (alphay +betay*s)/(1+s+(u[1]/xy)^nxy) - degy*u[2]       #dy/dt
 du[3] =  (1)/(1+(u[1]/xz)^nxz+(u[2]/yz)^nyz) - degz*u[3]        #dz/dt
end
```

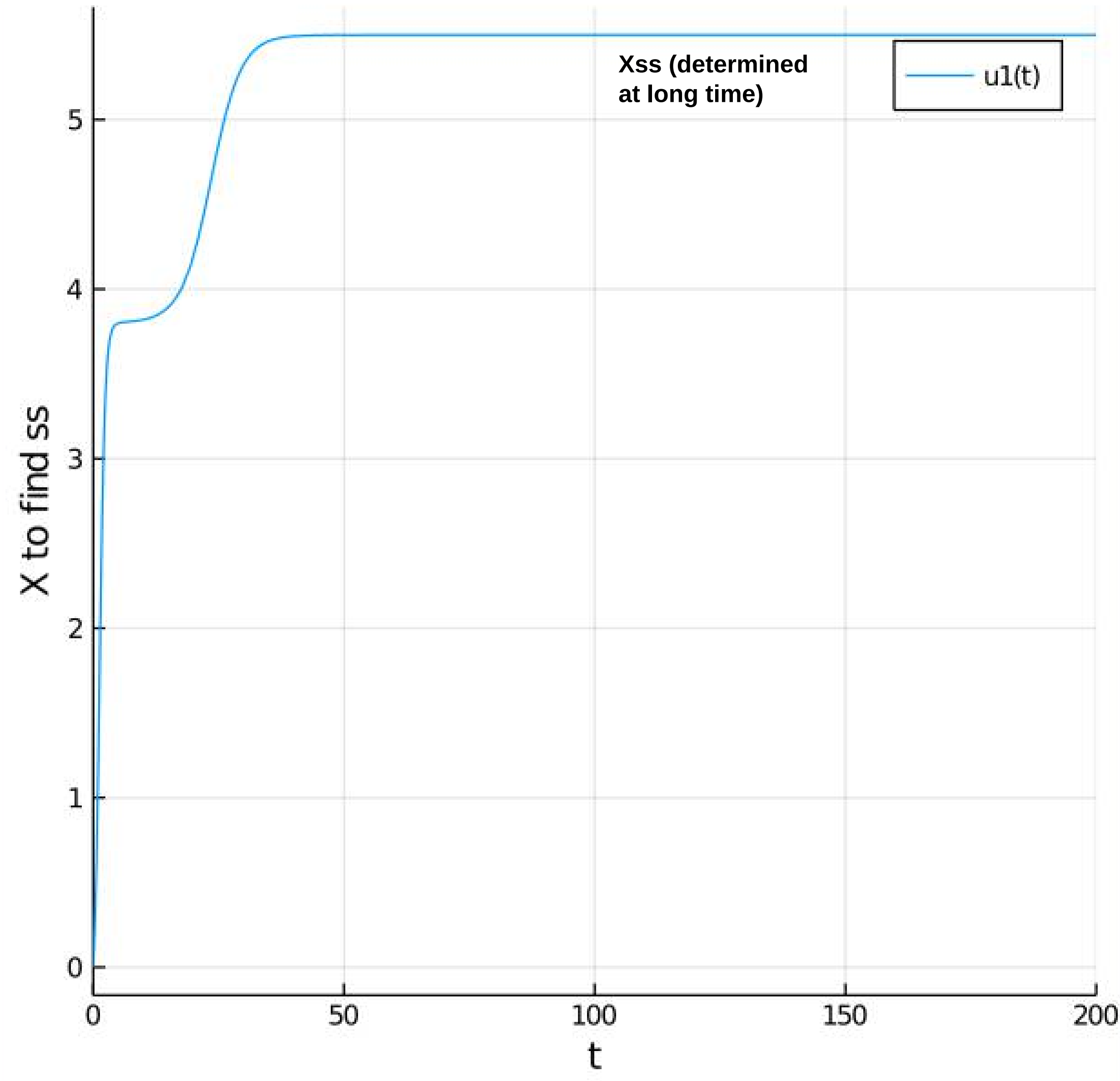
Please know these initial conditions are for the cells before the change of S. After the change of S the initial values were set to the respective multiplier*Zss

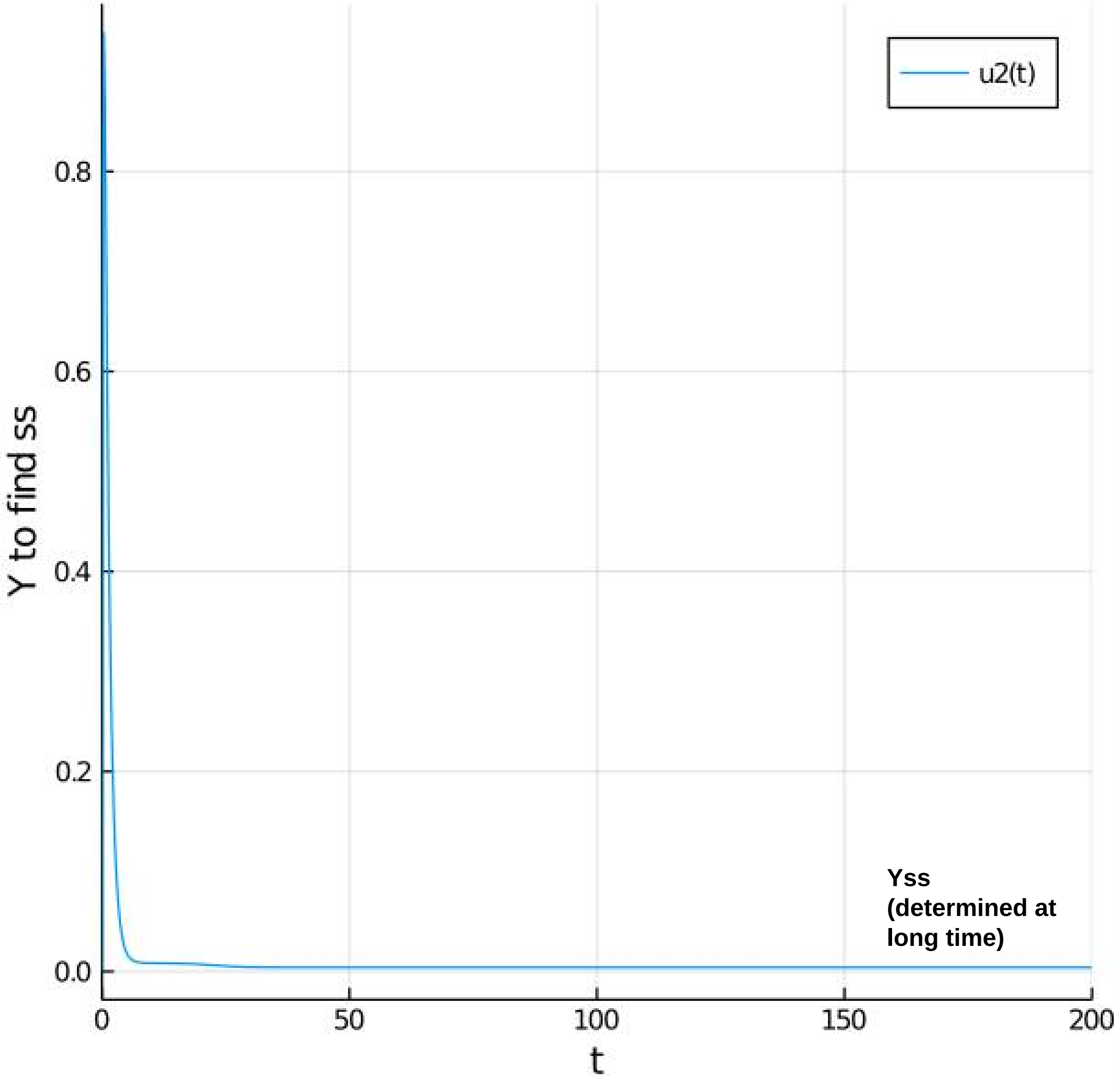
```
u0 = [0.0;0.0;0.0]                            #intial conditions
tspan = (0.0,200)                            #start and end time
prob = ODEProblem(partD!,u0,tspan)      #Create an ODE problem for the PartD funct
sol = solve(prob)                             #Solve the system

#Plot the results; the vars=(0,1) argument specifies to plot X (column 1 of sol)
#vs t (column 0 of sol)
plt5 = plot(sol,vars=(0,1), xaxis="t", yaxis = "X to find ss")
display(plt5)
```



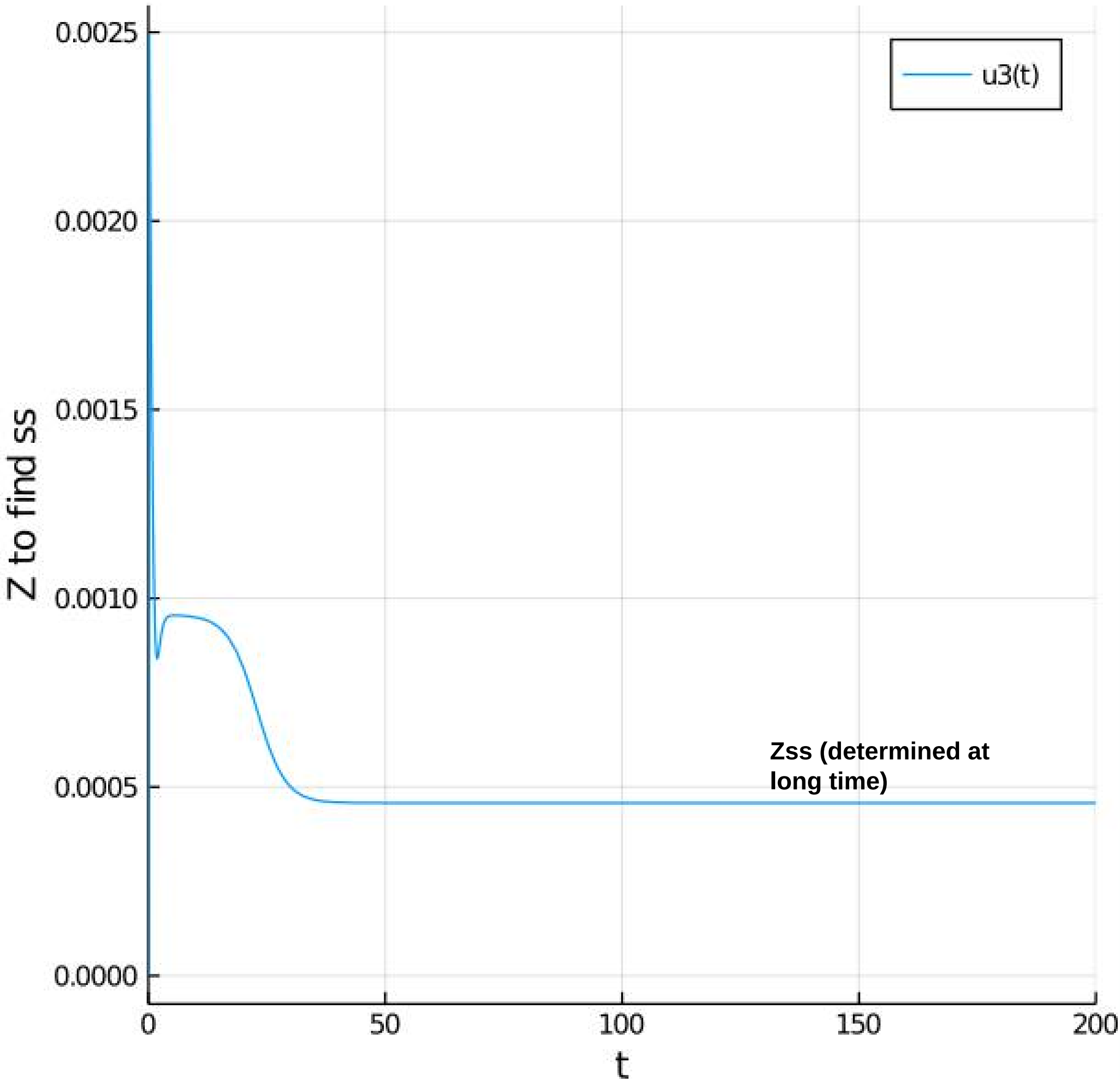
REPL

```
julia> include("Problem2PartE___saddle")
250249250Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
The steady state values of Xss,Yss,Zss are:[5.499044091608997, 0.003968843236107806, 0.0004576369463005712]
julia> include("Problem2PartE___saddle")
Plot{Plots.GRBackend() n=3}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
Plot{Plots.GRBackend() n=1}
The steady state values of Xss,Yss,Zss are:[5.499044091608997, 0.003968843236107806, 0.0004576369463005712]
julia>
```

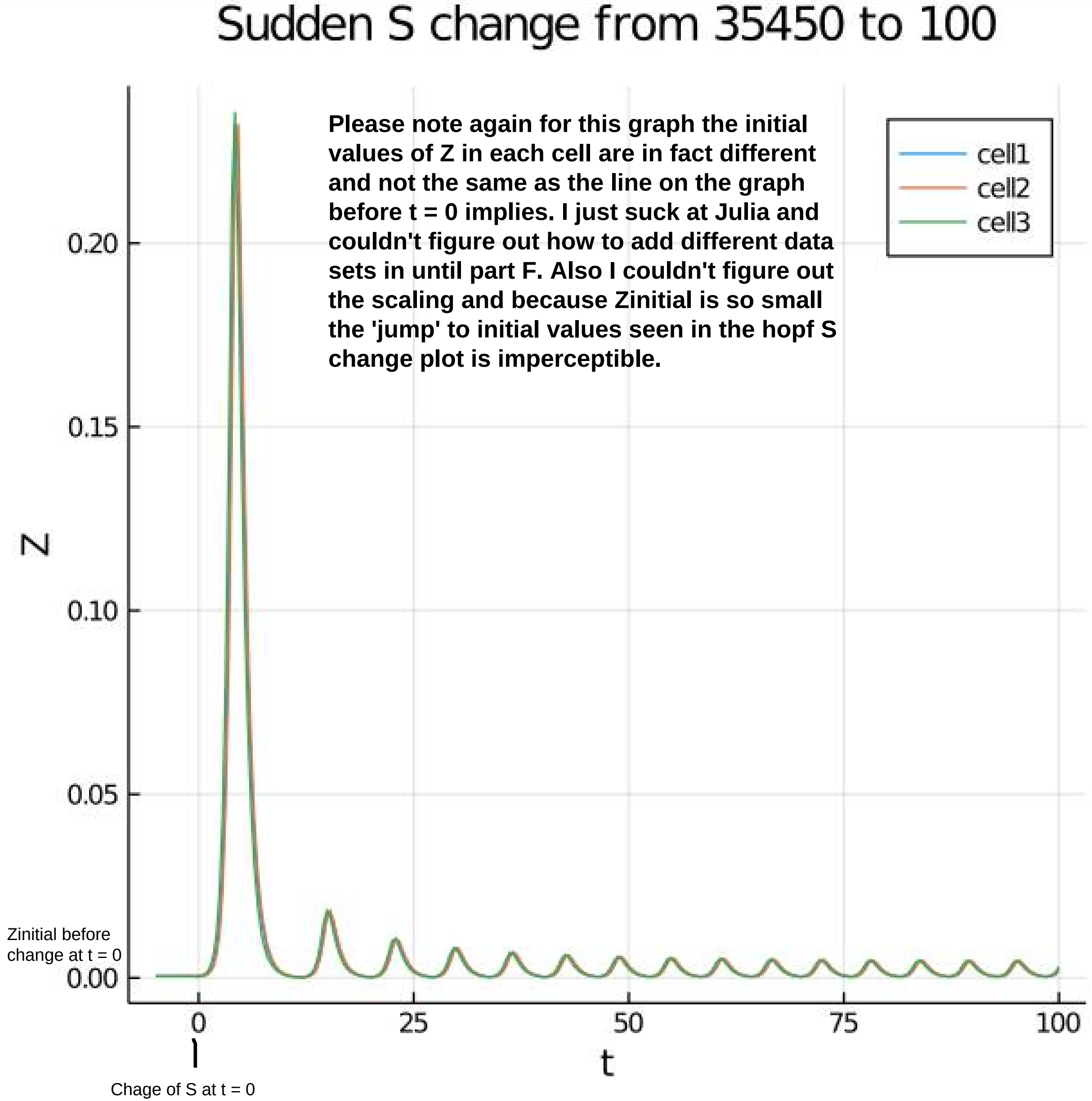

Xss (determined
at long time)
u1(t)
X to find ss
t
0
1
2
3
4
5
0
50
100
150
200



u2(t)
Y to find ss
0.8
0.6
0.4
0.2
0.0
0
50
100
150
200
t
Yss
(determined at
long time)

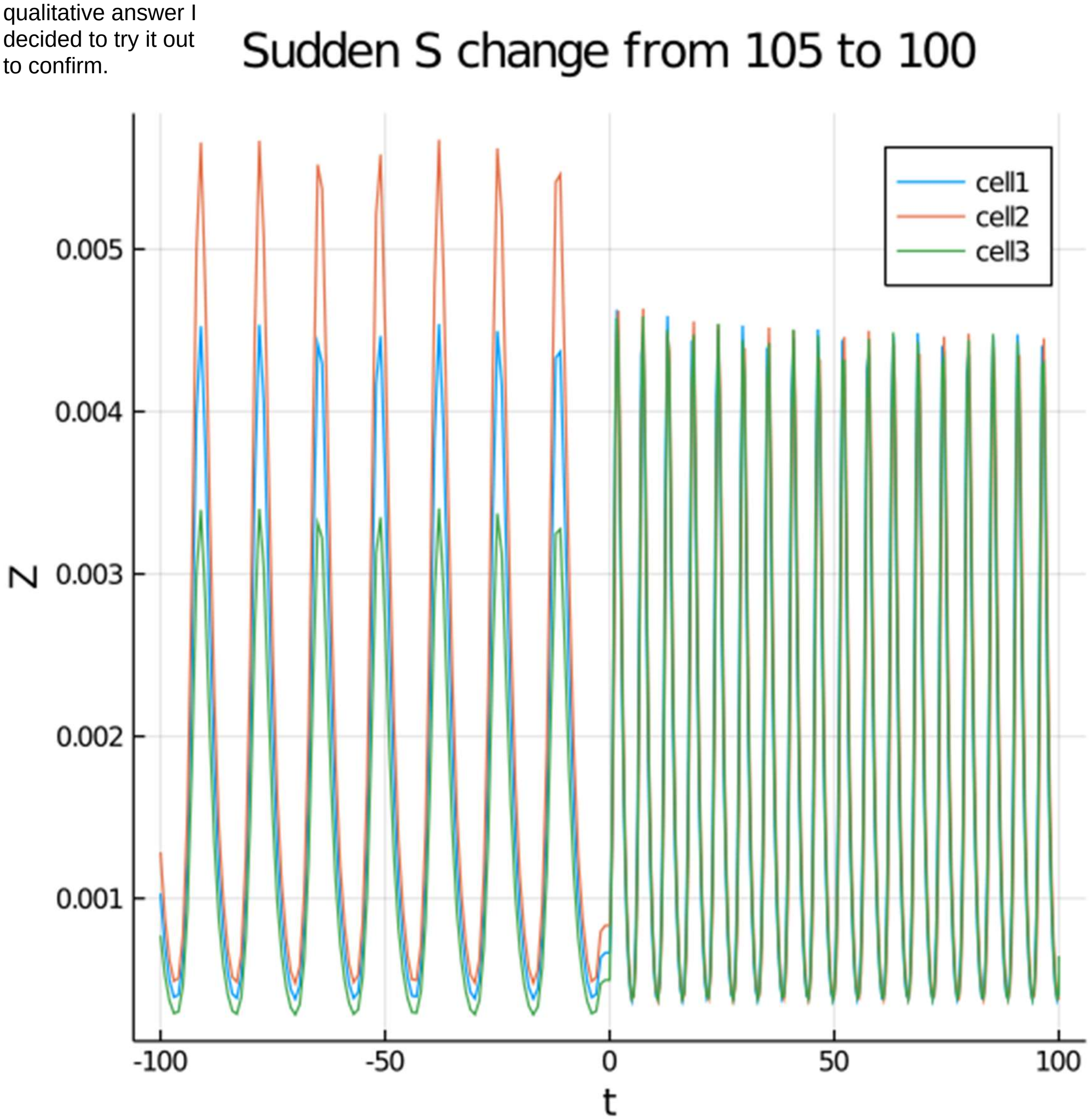


0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
Z to find ss
0
50
100
150
200
t
u3(t)
Zss (determined at long time)


Sudden S change from 35450 to 100
Please note again for this graph the initial values of Z in each cell are in fact different and not the same as the line on the graph before t = 0 implies. I just suck at Julia and couldn't figure out how to add different data sets in until part F. Also I couldn't figure out the scaling and because Zinitial is so small the 'jump' to initial values seen in the hopf S change plot is imperceptible.
cell1
cell2
cell3
Z
0.20
0.15
0.10
0.05
0.00
0
25
50
75
100
t
Zinitial before change at t = 0
Chage of S at t = 0

Part F)

Unsure of my qualitative answer I decided to try it out to confirm.



As shown in the figure a sudden change from 105 to 100 at t = 0 does indeed normalize the concentrations in each cell and produces coherent oscillations. My results agree with figure 3E in the magnitude of the concentration of Z in the period before the change, and in the qualitative analysis of the oscillations after the change. However, my figure does not agree with the concentration of Z after the change suggesting that I messed up, or perhaps they used different input parameters. Thus, as a final response I believe that coherent oscillations from this decrease can be achieved

But, I believe some parameters must have been different than mine as their Z changed drastically after the change while mine did not.