$\boxed{1A}$ See Excel Sheet "1 Part A" of the Excel file "Problems" for each Calculation.

General method:

$$\beta = \langle M_c \rangle \hat{N}_c V$$

given: V = sample size = 1 mL

$$\hat{N}_c = \frac{\#\ \text{cells}}{\text{mL}}$$

given OD600 = 0.1 $\approx 1\cdot 10^{8} \frac{\text{cells}}{\text{mL}}$

$\therefore$

$$\hat{N}_c = 1\cdot 10^{8} \frac{\text{cells}}{\text{mL}}$$

$\langle M_c \rangle$ = average mass of dry ecoli cell $\left(\frac{\text{gDW}}{\text{cell}}\right)$

↳ from Ecoli Metabolome data base

Cell dry weight = $3\cdot 10^{-13}$ g/cell

↳ Link given in the Excel sheet.

$\therefore \beta = 3\cdot 10^{-6}$ gDW

Specific Volume Basis = $\frac{\langle n \rangle}{\beta}$ ($\langle n \rangle$ converted to mols by dividing by Avogadro's #)

Table given in Excel as well

continued

IPTG(nM)	$\langle n \rangle$ $\frac{\text{mRNA}}{\text{cell}}$	$\frac{\langle n \rangle}{\beta}$ $\left(\frac{\text{nmol}}{\text{gDW}}\right)$
0	19	$1.0517\cdot 10^{-8}$
$5\cdot 10^{-10}$	21	$1.1624\cdot 10^{-8}$
$5\cdot 10^{-9}$	41	$2.269\cdot 10^{-8}$
$1.2\cdot 10^{-8}$	67	$3.709\cdot 10^{-8}$
$5.3\cdot 10^{-8}$	86	$4.76\cdot 10^{-8}$
$2.16\cdot 10^{-7}$	93	$5.1478\cdot 10^{-8}$
$1\cdot 10^{-6}$	93	$5.1478\cdot 10^{-8}$

$\boxed{1B}$

$$\dot{m}_i = r_{x,i}\bar{u}_i - (\mu + \theta_{m,i})m_i$$

$$\frac{dm_i}{dt}$$

@ P.SS $\frac{d}{dt} = 0$

$\therefore$

$$0 = r_x\bar{u} - (\mu + \theta_m)m^*$$

$$m^*(\mu + \theta_m) = r_x\bar{u}$$

$$\boxed{m^* = \underbrace{\left(\frac{r_x}{\mu + \theta_m}\right)}_{\substack{K_x \\ \text{gain function}}}\underbrace{\left(\bar{u}\right)}_{\text{Promoter function}},}$$

$$r_x = K_E R_{XT}\left(\frac{G}{\tau_{x_i}K_{x_i} + (\tau_{x_i}+1)G}\right)$$

$\therefore$ $\underline{K_x = f(G, \text{constants})}$

$$\bar{u} = \frac{w_1 + w_2 f_I}{1 + w_1 + w_2 f_I}, \quad f_I = \frac{I^n}{K_d^n + I^n}$$

$\therefore$ $\underline{\bar{u} = f(I, \text{constants})}$

1 C ⊛ For graph, table, and sources please see the PDF or Exel workbook "Problem 1".

@ $I = \underbrace{\max(I)}_{(\text{large } I)}$ $\quad \bar{u} = 1$

$\therefore$

@ $I = \max(I)$ $\quad M^* = K_x = gaM = \text{constant}$

(large I)

Because we are given $G = \text{constant} = 2$

$\therefore \quad \max(M^*) = K_x$

$\therefore$ from the graph (as shown in sheet "I part (D)")

$$\boxed{gaM = K_x = 5.14779 \cdot 10^{-8} \frac{nmol}{gDW}}$$

@ $I = \min(I)$ (small I) $\quad \bar{u} = \dfrac{w_1}{1+w_1}$ as $f_I(\min) = 0$

$\therefore \min(M^*) = K_x\left(\frac{w_1}{1+w_1}\right)$

from the graph (as shown on sheet "I part (D)")

$\min(M^*) = 1.0517 \cdot 10^{-8}$ $\therefore$

$$\boxed{\frac{w_1}{1+w_1} = 0.2043075^{\text{unitless}} \quad \therefore w_1 = 0.2567_{\text{unitless}}}$$

@ $I = K_d \quad f_I(I = K_d) = \frac{1}{2}$

$\therefore \quad M^*(I = K_d) = K_x\left(\dfrac{w_1 + \frac{1}{2}w_2}{1 + w_1 + \frac{1}{2}w_2}\right)$

Knowing that $\bar{u}(I = \text{Big}) = 1$ and that $f_I(I = \text{Big}) = 1$ this means

$\dfrac{w_1 + w_2}{1 + w_1 + w_2} \simeq 1$ for this to be true

But also have $\frac{w_1}{1+w_1} = 0.204$ $\quad w_1$ must be on the order of 0.1 and w_2 must be $\gg w_1, 1$

1C continued

with $w_2 >> w_1, 1$

$\frac{w_2}{2}$ is also $>> w_1, 1$

$\therefore$

$$m^*(I=K_d) \approx K_x \left(\frac{\frac{1}{2}w_2}{\frac{1}{2}w_2}\right)$$

$$m^*(I=K_d) \approx K_x$$

$\therefore$ $K_d \approx \max(I)$

$\boxed{\text{from the graph } K_d = 0.00001 \text{ nMol}}$

Then plotting the data $\frac{\langle n \rangle}{\beta} \left(\frac{nmol}{gDW}\right)$ vs IPTG (nmol)

on a semilogx plot

and using least squares to fit w_2, n I got — can be seen on sheet "1 part CD"

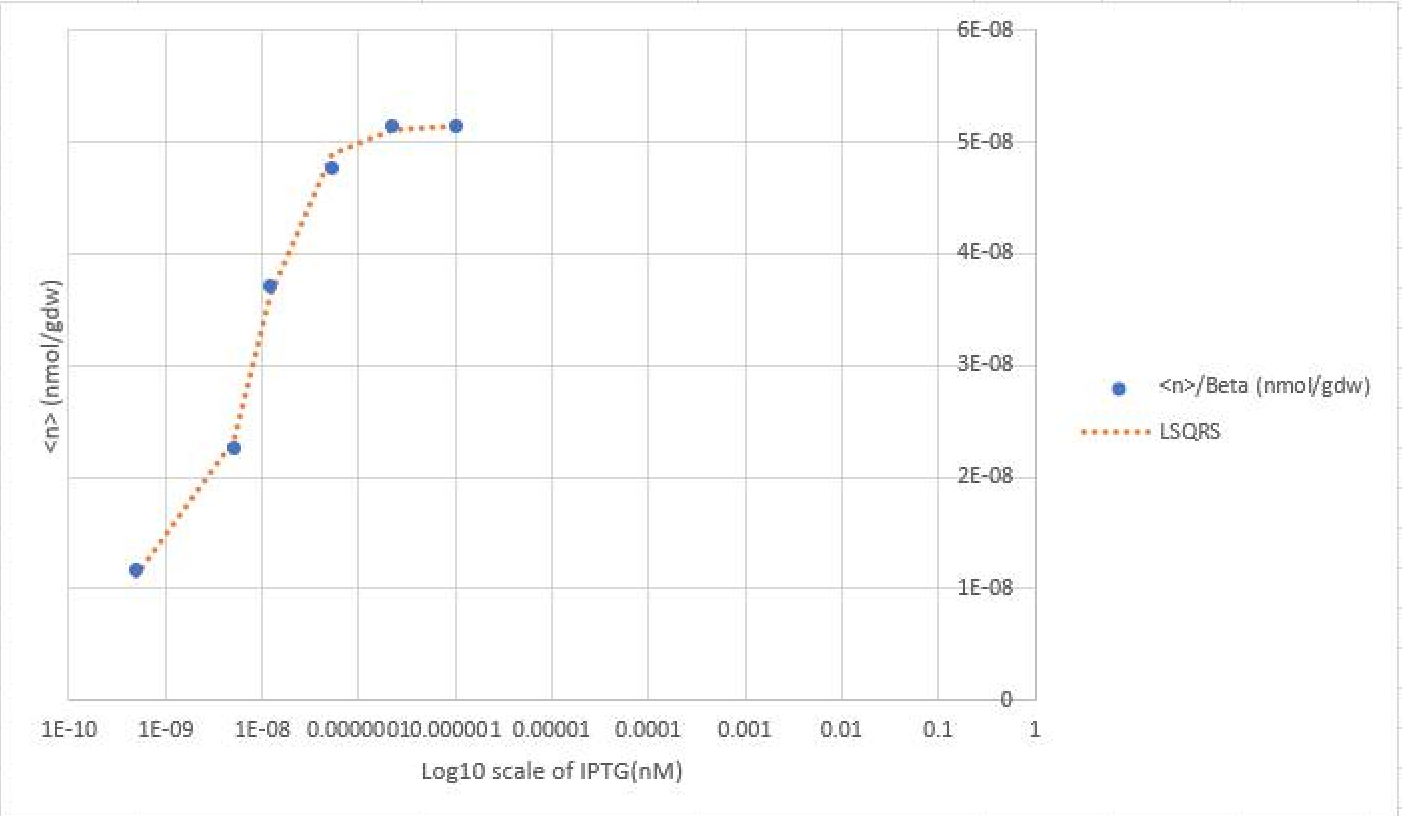
$$\boxed{\begin{aligned} w_2 \text{ (unitless)} &= 1492.7 \\ n \text{ (unitless)} &= 1.484 \end{aligned}}$$

← validates earlier assumption $\frac{w_2}{2} >> 1, w_1$

for the graph, table and sources please see PDF or excel sheet "1 part CD"

1 D) See PDF please.

Property and units	Value	Source (For better explanations please see written work)
Max(m*) (nmol/gdw)	5.14779E-08	From this plot
Min(m*) (nmol/gdw)	1.0517E-08	From this plot
Gain (nmol/gdw)	5.14779E-08	Max(m*) = Gain*u(I=large) = Gain*1 = Gain
W1/(1+W1) (background production) (unitless)	0.204301075	Min(m*) = Gain*u(I=0) = Gain* (W1/(1+W1) Therefore, W1/(1+W1) = Min(m*) / Gain
W1 (unitless)	0.256756757	Solving above entry for W1
kd (nM)	0.000001	When I = K, m* = Gain*[(w1+(0.5)w2)/(1+w1+(0.5)w2)]. Therefore, K = max(I) (Because W2 must be large relative to 1,W1) (Because (W1+W2)/(1+W1+W2) = 1 as u(large I) = 1)
n (unitles)	1.483632025	(From Least Squares Fit)
W2 (unitless)	1492.678306	(From Least Squares Fit)



6E-08
5E-08
4E-08
3E-08
2E-08
1E-08
0
<n> (nmol/gdw)
1E-10
1E-09
1E-08
0.0000001
0.000001
0.00001
0.0001
0.001
0.01
0.1
1
Log10 scale of IPTG(nM)
<n>/Beta (nmol/gdw)
LSQRS

I think the fit is good.

D) n and W2 are the parameters of my fit. In terms of how the curve behaves relative to the change in the various variables: n changes how steep the curve is and W2 changes the concavity of the curve and the steepness. W1 changes the y value of the base of the curve and slightly pulls the other data points with it. Gain is just an overall multiplier, so it just shifts the entire curve up or down.