

2. a) i) $134 \bmod 6 = 2$
 $88 \bmod 6 = 4$
 \Rightarrow Not congruence

ii) $1573 \bmod 7 = 5$
 $12850 \bmod 7 = 5$
 $\Rightarrow 1573 \equiv 12850 \pmod{7}$

b) ISBN-10: 026204630X

$$S_d = 143 \equiv 0 \pmod{11}$$

Change 4 to 5. $S_d' = 148 \pmod{11} = 5$
 026205630X

3 a) i)

m	n	r
247	117	13
117	13	0

 $\Rightarrow \gcd(247, 117) = 13$

ii)

m	n	r
1479	272	119
272	119	34
119	34	17
34	17	0

 $\gcd(1479, 272) = 17$

b) i) ~~2173~~ $\gcd(3157, 656) = 41$
 $2173 \bmod 41 = 0$

i	q_i	r_i	x_i	y_i
-1	-1	3157	1	0
0	0	656	0	1
1	4	533	1	-4
2	1	123	-1	5
3	4	41	5	-24
4	3	0	-16	77

$$2173/41 = 53$$

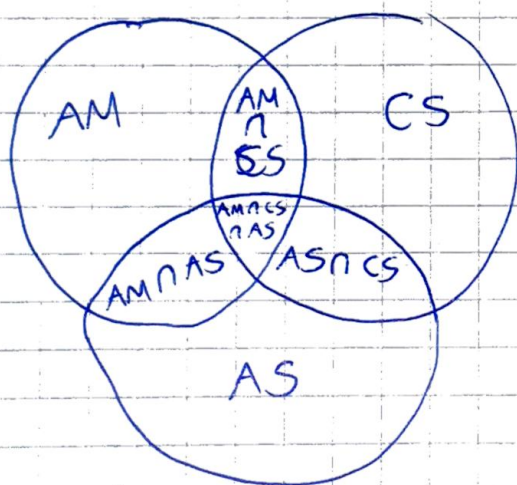
$$x = 265$$

$$y = -1272$$

$$\begin{aligned}
 4. \quad & AM = 120 \\
 & CS = 200 \\
 & AS = 60 \\
 & CS \cap AS = 40 \\
 & AM \cap CS = 80 \\
 & AM \cap AS = 30 \\
 & AM \cap CS \cap AS = 20
 \end{aligned}$$

$$a) AM \cup CS \cup AS = 120 + 200 + 60 - 40 - 80 - 30 + 20 = 250$$

b)



5b)

13 ranks in one deck

Choose 1st rank: 13 ways

Choose 2nd rank: $13 - 1 = 12$ ways

Choose cards for 1st rank:

$$4C3 = 4 \text{ ways}$$

Choose cards for 2nd rank:

$$4C2 = 6 \text{ ways}$$

$$\text{Total} = 13 \times 12 \times 4 \times 6 = 3744 \text{ ways}$$

5a)

1st round:

Each group has $3! = 6$ games.

$$8 \text{ groups} \Rightarrow 8 \cdot 6 = 48 \text{ games}$$

2nd round:

2 teams from 8 groups $\Rightarrow 16$ teams

4 groups w/ 4 teams each

$$4 \text{ groups} \Rightarrow 4 \cdot 6 = 24 \text{ games}$$

5a) 1st round: 8 groups, 4 teams each. Pick 2 from 4: $4C2 = 6$

Each team plays 2 games: $6 \times 2 = 12$ games

Total game from 1st round = $12 \times 8 = 96$ games

Similarly for 2nd round = $12 \times 4 = 48$ games

1st playoffs, 8 teams 1-on-1 = 8 games

2nd playoffs, 4 teams 1-on-1 = 4 games

Final: 2 games

$$\begin{aligned}
 & 96 + 48 + 8 + 4 + 2 \\
 & = 158 \text{ games}
 \end{aligned}$$

3b) iii)

i	q_i	r_i	x_i	y_i
-1		1054	1	0
0		-833	0	1
1	1	221	1	1
2	-3	-170	3	4
3	-1	51	4	5
4	-3	<u>-17</u>	<u>15</u>	<u>19</u>
5	-3	0	49	62

$$2277 \bmod 17 = 0$$

$$\gcd(1054, -833) = 17$$

$$a = 2277/17$$

$$\Rightarrow x = 15 \times \frac{2277}{17}$$

$$y = 19 \times \frac{2277}{17}$$

$$4b) |AM| = 120 - 80 - (30 - 20) = 30$$

$$|CS| = 200 - 80 - (40 - 20) = 100$$

$$|AS| = 60 - 30 - (40 - 20) = 10$$

$$|AM \cap CS| = 80 - 20 = 60$$

$$|AM \cap AS| = 30 - 20 = 10$$

$$|AS \cap CS| = 40 - 20 = 20$$

$$|AM \cap CS \cap AS| = 20$$

$$AU = 30 + 100 + 10 + 60 + 10 + 20 + 20 = 250$$

6 a) 7! letters, 3A's, 2R's, 2L's

$$\frac{7!}{2! \cdot 2! \cdot 3!} \times 6 = 420 \text{ combinations}$$