

1.

$$A + B = \begin{bmatrix} 4 & 0 \\ 4 & 4 \\ 2 & 6 \end{bmatrix} \quad \text{and} \quad A - B = \begin{bmatrix} 2 & -4 \\ 0 & -2 \\ 4 & 2 \end{bmatrix}$$

2. (a)

$$A^T = \begin{bmatrix} -3 & 0 & -1 & 2 \\ 1 & -3 & 1 & -2 \end{bmatrix} \quad \text{and} \quad b^T = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}$$

(b)

$$AB = \begin{bmatrix} -4 & 8 & -6 & -5 \\ 3 & -6 & 0 & -12 \\ -2 & 4 & -2 & 1 \\ 4 & -8 & 4 & -2 \end{bmatrix}$$

3. For

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the inverse is

$$\frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Now $ad - bc = 7 \cdot 18 - 10 \cdot 11 = 16$. This gives that

$$A^{-1} = \begin{bmatrix} 18/16 & 10/16 \\ 11/16 & 7/16 \end{bmatrix} = \begin{bmatrix} 9/8 & 5/8 \\ 11/16 & 7/16 \end{bmatrix}$$

4. Consider the product

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \end{bmatrix}$$

Because $\theta = 60^\circ$, $\cos \theta = 1/2$ and $\sin \theta = \sqrt{3}/2 \approx 0.866$. Therefore, this is what happens to one point:

$$(x, y) \mapsto (0.5x - 0.866y, 0.866x + 0.5y).$$

This means that

$$\begin{aligned} (0, 0) &\mapsto (0, 0) \\ (0, 2) &\mapsto (0.5 \cdot 0 - 0.866 \cdot 2, 0.866 \cdot 0 + 0.5 \cdot 2) = (-1.732, 1) \\ (4, 0) &\mapsto (0.5 \cdot 4 - 0.866 \cdot 0, 0.866 \cdot 4 + 0.5 \cdot 0) = (2, 3.464) \\ (4, 2) &\mapsto (0.5 \cdot 4 - 0.866 \cdot 2, 0.866 \cdot 4 + 0.5 \cdot 2) = (0.268, 4.464) \end{aligned}$$

