

1 a) Adjacency

	A	B	C	D	E	F
A	0	1	1	1	0	1
B	1	0	1	0	1	0
C	1	1	0	0	0	1
D	1	0	0	0	1	0
E	0	1	0	1	0	0
F	1	0	1	0	0	0

$$A = \begin{matrix} & C & 1 & 1 & 0 & 0 & 0 & 1 \end{matrix} \quad I = \begin{matrix} & C & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{matrix}$$

$$D = \begin{matrix} & D & 1 & 0 & 0 & 0 & 1 & 0 \end{matrix}$$

$$E = \begin{matrix} & E & 0 & 1 & 0 & 1 & 0 & 0 \end{matrix}$$

$$F = \begin{matrix} & F & 1 & 0 & 1 & 0 & 0 & 0 \end{matrix}$$

Incidence

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ A & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{matrix}$$

$$B = \begin{matrix} & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{matrix}$$

$$C = \begin{matrix} & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{matrix}$$

$$D = \begin{matrix} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

$$E = \begin{matrix} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{matrix}$$

$$F = \begin{matrix} & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{matrix}$$

Degree

	A	B	C	D	E	F
A	4	0	0	0	0	0
B	0	3	0	0	0	0
C	0	0	3	0	0	0
D	0	0	0	2	0	0
E	0	0	0	0	2	0
F	0	0	0	0	0	2

Laplace

$$L = D - A$$

$$= \begin{pmatrix} 4 & -1 & -1 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 2 & 0 \\ -1 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}$$

2a)

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$A \rightarrow B$

$$A^3 = \begin{pmatrix} 4 & 8 & 6 & 6 & 1 & 6 \\ 8 & 2 & 6 & 1 & 5 & 2 \\ 6 & 6 & 4 & 3 & 2 & 5 \\ 6 & 1 & 3 & 0 & 4 & 1 \\ 1 & 5 & 2 & 4 & 0 & 3 \\ 6 & 2 & 5 & 1 & 3 & 2 \end{pmatrix} \rightarrow C$$

$A \rightarrow B$ 3 steps : 8 ways

b)

$$A^2 = \begin{pmatrix} 4 & 1 & 2 & 0 & 2 & 1 \\ 1 & 3 & 1 & 2 & 0 & 2 \\ 2 & 1 & 3 & 1 & 1 & 1 \\ 0 & 2 & 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 & 2 & 0 \\ 1 & 2 & 1 & 1 & 0 & 2 \end{pmatrix}$$

$A \rightarrow 3$ 1 step: 1 way

2 steps: 1 way

3 steps: 8 ways

≤ 3 steps: $1 + 1 + 8 = 10$ ways

c) Start from C: $6 + 6 + 4 + 3 + 2 + 5 = 26$ options

3a) Adjacency:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Incidence

$$I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & -1 & & & & \\ -1 & & 1 & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{pmatrix}$$

Incidence

$$I = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Degree

$$D = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

3b) Laplace

$$L = I I^T = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ -1 & -2 & 4 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix}$$

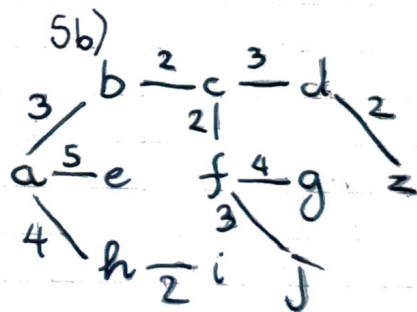
4 a) All nodes have even degrees \Rightarrow Euler circuit

b) 2 nodes have odd degrees \Rightarrow Euler path

c) More than 2 nodes have odd degrees \Rightarrow Neither

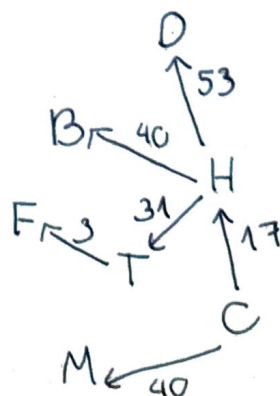
5 a)

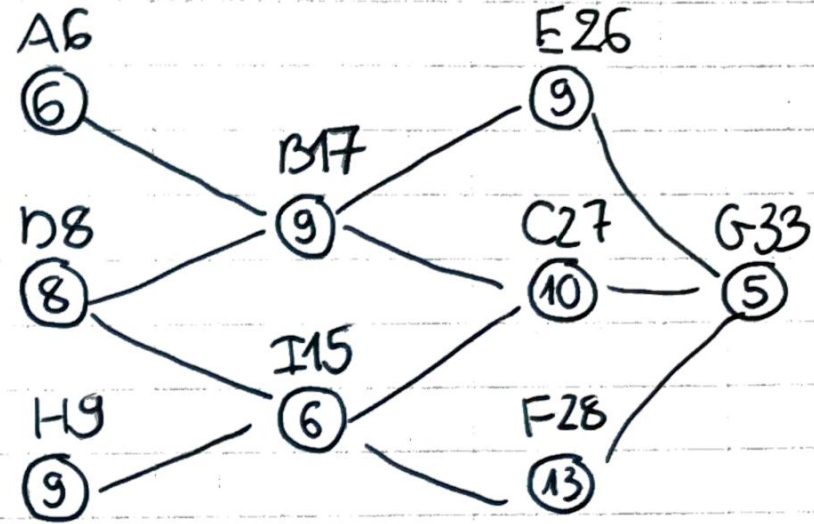
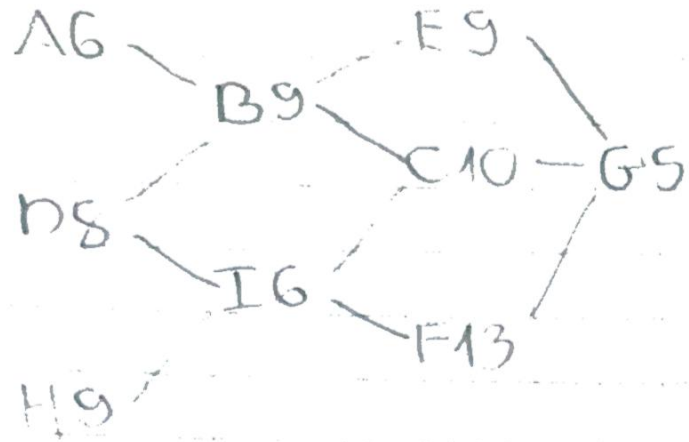
	a	b	c	d	e	f	g	h	i	j	z
a	3				5			4			
b	3	2		5	7						
c		2	3		2	6					
d			3			7					2
e	5	5			4	7					
f		7	2	4	4	5	4	3			
g			6	7	4	4					
h	4				7	5	2				
i						4	2				
j								6	5		
z											5



6 a)

	b	c	d	f	h	m	t
b			15				
c				17	40		
d	17						
f	46					11	
h	40	53				31	
m						8	
t			29	3			





Critical path: H → I → F → G
 Time: 33 days