Logic

AI and ML

Representation and Reasoning System (RRS)

A RRS is made up of:

- syntax: specifies the symbols used, and how they can be combined to form legal sentences
- semantics: specifies the meaning of the symbols
- reasoning theory or proof procedure: a (possibly nondeterministic) specification of how an answer can be produced.

Propositional definite clauses

- An atom is a symbol starting with a lower case letter.
- A body is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.
- A definite clause is an atom or is a rule of the form $h \leftarrow b$ where h is an atom and b is a body.
- A knowledge base is a set of definite clauses.

Semantics

- An interpretation I assigns a truth value to each atom.
- A body $b_1 \wedge b_2$ is true in I if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if b is true in I and h is false in I. Otherwise the rule is true.
- A knowledge base KB is true in $I \iff$ every clause in KB is true in I.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are true.
- If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $\mathrm{KB} \models g$, if g is true in every model of KB.
- That is, $\mathrm{KB} \models g$ if there is no interpretation in which KB is true and g is false.

Logical program

Logic programming is based on formulas called Horn rules.

$$\forall_{x_1 \ldots x_k}[A \leftarrow B_1 \wedge B_2 \wedge \ldots B_j], \quad \text{where} \quad k, j \geq 0$$

Non-Horn formulas do not correspond to programs, we need to convert them into Horn form using two methods:

- Logical equivalence
- Skolemization